DCIT 203
DIGITAL AND LOGIC SYSTEM DESIGN

Session – Number System

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COURSE DESCRIPTION

- This course provides a modern introduction to logic design and the basic building blocks used in digital systems, in particular digital computers.
- The course will provide an overview of principles and Techniques of modern digital systems.
- It exposes students to a wide array of classic as well as state of the art digital electronics technology.
- The course will further explore the theories and operations of the fundamental building blocks of digital electronics.

COURSE DESCRIPTION

- The course will also expose students to the design process and construction of combinational and sequential logic.
- Emphasis will be placed on Technologies and Application of wide array of digital components used within state-of-the-art IT Systems.
- An understanding of the applications of such digital devices embedded within telecommunications systems, storage systems, computing systems, multimedia systems, and computer networks.

LEARNING OUTCOMES

At the end of the course, students are should be able to;

- Demonstrate a basic understanding of digital terminology, digital components, and systems
- Apply digital circuit theory in a laboratory setting as it is applied to a work situation;
- Formulate and employ a Karnaugh Map to reduce Boolean expressions and logic circuits to their simplest forms
- Evaluate logic circuit outputs, describe the operation of logic gates, write truth tables for logic gates, logic gate simplification;
- Appreciate different circuit types (combinational and Sequential circuit) and their design principles
- Understand the differences in synchronous and asynchronous logic circuits

LEARNING OUTCOMES

- Illustrate the operation of encoders, decoders, multiplexers, shift registers, and wave generating circuits
- Explain the operation of flip flops, D-Flip-Flop, J-K Flip-Flop, Flip-Flop used as a shift register;
- Design and evaluate a solution to a digital design problem.
- Understand the design and operations of Finite State Machine/ Automaton
- Synthesize a circuit using a logic compiler software on a personal computer;

COURSE EVALUATION

The assessment of students on this course will be constituted by the following components:

COMPONENTS OF THE GRADING SYSTEM	
Grading component	Percentage (%)
Assignments	10%
Quizzes	10%
Mid Semester Examination	20%
End-of-Semester Examination	60%
	100%

COURSE SESSIONS OVERVIEW

- The course is presented session by session
- Each session provides learning outcomes and what you will be expected to know by the end of it.
- Each session also provides an overview of the topics in that session.

Number Systems

OVERVIEW

Many number systems are in use in digital technology. The most common are the binary and hexadecimal number systems. In digital systems, the physical quantities can assume only discrete values.

In this session you will learn how bits can be used to represent numbers and characters. You will learn about different numbers systems and the conversion between them. In this session you will learn about the decimal number system, binary numbers, decimal to binary conversion, binary arithmetic and 1’s and 2’s complement of binary numbers.

LEARNING OUTCOME

At the end of the session, the student should be able to:

- Understand the binary number system and its similarity to the decimal system
- Convert from binary to decimal and from decimal to binary
- Apply arithmetic operations to binary numbers
- Determine the 1's and 2's complement of a binary number

SESSION OUTLINE

The key topics to be covered in the session include:

- Decimal Number Systems
- Binary Number Systems
- Octal Number Systems
- Hexadecimal Number System
- Number System conversion
- Complement arithmetic

READING LIST

S. Salivahanan & S. Arivazhagan (2018), Digital Circuits and Design.

NUMBER SYSTEM

- Number system is a set of values used to represent different quantities
- The total number of digits used in a number system is called its base or radix.
- The base is written after the number as subscript
- Example: $number_{radix}$

NUMBER SYSTEM

Some important number systems are as follows.

- Decimal number system
- Binary number system
- Octal number system
- Hexadecimal number system

The decimal number system is used in general. However, the computers use binary number system.

The octal and hexadecimal number systems are also used in the computer.

NUMBER SYSTEM

Numbering System		
System	Base	Digits
Binary	2	0, 1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

DECIMAL NUMBER SYSTEM

- An unsigned number A can be represented using n digits in base b:

  $A=(a_{n-1}a_{n-2}\ldots a_2\, a_1\, a_0.a_{-1}a_{-2}\ldots a_{-n})_b$
- This representation is called positional representation
- Each number in this system consists of digits which are located at different positions.
- The position of first digit towards left side of the decimal point is 0.

- The position of second digit towards left side of the decimal point is 1
- Similarly, the position of first digit towards right side of decimal point is -1
- The position of second digit towards right side of decimal point is -2 and so on.
- Decimal number system has 10 distinct symbols, starting from 0-9

- Given n digits number A, in base *b*, the number of possible values(N) that can be addressed is given by:

$$N = b^n$$

- largest value that can be addressed A_{max}, is given by

$$A_{\max} = b^n - 1$$

- For example, the largest number that can be obtained using 3 digits in decimal is given by

$$A_{\max} = 10^3 - 1$$
$$= 1000 - 1$$
$$= 999$$

Thus we have the values in the range [000:999]

- 4 bits in base 2 is given by

$$A_{\max} = 2^4 - 1$$
$$= 16 - 1$$
$$= 15$$

- 16 different numbers can be represented
- In this case, decimal numbers ranging from 0 to 15 (corresponding to binary 0000 to 1111) can be represented.

NUMBER SYSTEM

- The decimal value of the integer number A is given by

$$A = \sum_{i=0}^{n-1} a_i * b^i$$

E.g., A=451 can be represented in decimal as

$4*10^2+5*10^1+1*10^0$

$4*100 + 5*10 + 1*1$

400+50+1=451

NUMBER SYSTEM

The above formula can be generalized for real numbers as:

$$X = \sum_{i=-m}^{k-1} x_i * b^i = x_{k-1}b^{k-1} + x_{k-2}b^{k-2} + \ldots.. + x_1 b^1 + x_0 b^0 + x_{-1}b^{-1} + \ldots.. + x_{-m}b^{-m}$$

Example: given the decimal number 65.375 can be written as

$$X = 6*10^1 + 5*10^0 + 3*10^{-1} + 7*10^{-2} + 5*10^{-3}$$

Example: The weights and positions of each digit of the number 542 are as follows:

Face value	5	4	2
Position	2	1	0
Weight	10^2	10^1	10^0

The above table indicates that:

The value of digit 5 = $5x10^2$ = 500

The value of digit 4 = $4x10^1$ = 40

The value of digit 2 = $2x10^0$ = 2

The actual number can be found by adding the values obtained by the digits as follows:

$500 + 40 + 2 = 542_{10}$

To do:

Find the weight presentation of each of the following numbers and compute the actual number

1. 6877
2. 357.74
3. 23.375

BINARY NUMBER SYSTEMS

- Digital computer represents all kinds of data and DFS information in the binary system.
- Binary Number System consists of two distinct symbols: 0 and 1.
- It is also referred to as base 2.
- Each bit in binary number system can be 0 or 1.

BINARY NUMBER SYSTEMS

An n-bit binary number b= $b_{n-1}\, b_{n-2} \ldots b_1 b_0$ can represent 2^n different values.

Call b_{n-1} the most significant bit (msb), b_0 the least significant bit (lsb).

Example:

2-bit register can represent 4 different values (2^2)

3-bit register can represent 8 different values(2^3)

2-bit presentation:*10, 01, 11, 00*

3-bit presentation:*000, 001, 010, 100,101, 110, 011, 111*

OCTAL NUMBER SYSTEM

- Octal number system has a base or radix 8.
- Eight digits are used : 0, 1, 2, 3, 4, 5, 6, 7

Addition of Octal Number

$(162)_8 + (537)_8$

Solution:

```
  1 1        <---- carry
  1 6 2
  5 3 7
 -------
  7 2 1
 -------
```

Therefore, sum = 721_8

OCTAL NUMBER SYSTEM

$(136)_8 + (636)_8$

Solution:

$$\begin{array}{r} 1 \\ 1\,3\,6 \\ \underline{6\,3\,6} \\ 7\,7\,4 \end{array} \quad \texttt{<----} \text{ carry}$$

Therefore, sum = 774_8

OCTAL NUMBER SYSTEM

$(25.27)_8 + (13.2)_8$

Solution:

$$
\begin{array}{r l}
1 & \text{<---- carry} \\
25.27 & \\
\underline{13.2} & \\
\underline{40.47} &
\end{array}
$$

Therefore, sum = $(40.47)_8$

OCTAL NUMBER SYSTEM

Compute the following:

1. $(67.5)_8 + (45.6)_8$
2. $(7563)_8 + (6176)_8$
3. $(36.75)_8 + (47.64)_8$

HEXADECIMAL NUMBER SYSTEM

- The Hexadecimal Number System consists of 16 digits from 0 to 9 and A to F
- Thus: Uses 10 digits and 6 letters 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10. A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
- Also called base 16 number system.

- The first position from the right in a hexadecimal number represents a 0 power of the base (16).
- Example 16^0
- Last position in a hexadecimal number represents an x power of the base (16).
- Example 16^x where x represents the last position - 1.

Example

$13A_{16}=1*16^2+3*16^1+A*16^0$

$C6F8_{16}=C*16^3+6*16^2+F*16^1+8*16^0$

HEXADECIMAL OPERATIONS

Addition in hexadecimal

1-->$B_{16}+4_{16}=F_{16}$

2-->$C_{16}+D_{16=}19_{16}$

3-->$A3_{16}+B5_{16}=158_{16}$

4-->$4A6_{16}+1B3_{16}=659_{16}$

5-->$(8\ A\ 5\ C)_{16}$ and $(F\ 3\ 9\ A)_{16}$

1	0	0	1	←	Carry
	8	A	5	C	
	F	3	9	A	
1	7	D	F	6	

HEXADECIMAL OPERATIONS

Subtraction in hexadecimal

1-->F-A=5

2-->AB-2C=7F

3-->1586_{16}-243_{16}=1343_{16}

4-->5CD2-2A0=5A32

5→4A6-1B3=?

6→ABC_{16}-$A3B_{16}$=?

RADIX CONVERSION

A radix conversion algorithm is used to convert a number representation in each radix, r1, into another representation in a different radix, r2.

The following conversion schemes are available

- Decimal to other bases
- Other bases to Decimal
- Decimal to Hexadecimal
- Binary to Octal
- Binary to Hexadecimal
- Octal to Hexadecimal

DECIMAL CONVERSION

In order to convert from decimal to other bases

- *Step 1* – Divide the decimal number to be converted by the value of the new base.
- *Step 2* – Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.
- *Step 3* – Divide the quotient of the previous divide by the new base.
- *Step 4* – Record the remainder from Step 3 as the next digit (to the left) of the new base number.
- Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.
- The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.

DECIMAL BINARY

- Conversion from decimal to binary can be performed by successively dividing the decimal number by 2 and using each remainder as a bit of the desired binary number.

Example:

77_{10}=1001101_2

65_{10}=111110_2

127_{10}=?

251_{10} = ?

Convert the following to their binary equivalent

$(25.375)_{10}$

$(56.75)_{10}$

DECIMAL BINARY

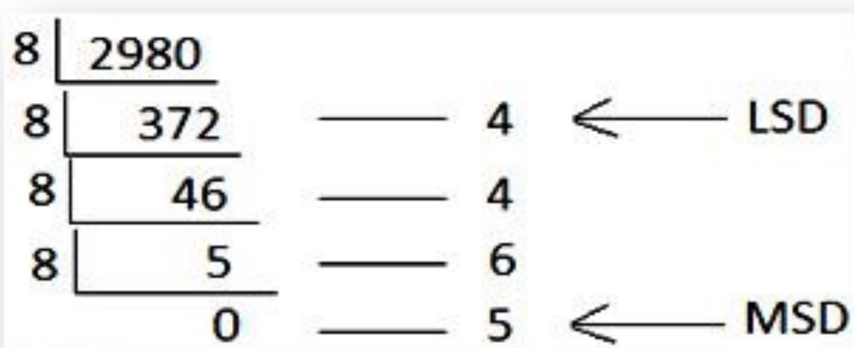
2	4215		
2	2107	1	← LSB
2	1053	1	
2	526	1	
2	263	0	
2	131	1	
2	65	1	
2	32	1	
2	16	0	
2	8	0	
2	4	0	
2	2	0	
2	1	0	
	0	1	← MSB

DECIMAL TO OCTAL

- Conversion from decimal to Octal can be performed by successively dividing the decimal number by 8 and using each remainder as a digit of the desired octal number.

Example:

Convert 2980_{10} to base 8



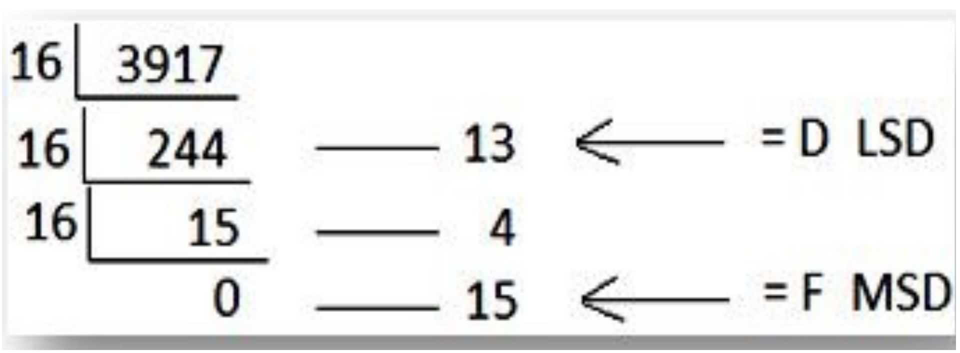
Therefore $2980_{10} = 5644_8$

DECIMAL TO HEXADECIMAL

Conversion from decimal to Hexadecimal can be performed by successively dividing the decimal number by 16 and using each remainder as a digit of the desired hexadecimal number.

Example:

Convert 3917_{10} to its hexadecimal equivalent



Therefore, $3917_{10} = F4D_{16}$

DECIMAL TO HEXADECIMAL

Convert the following decimal numerals to hexadecimal

$759_{10} = 2F7_{16}$

$1789_{10} = ??$

$356.50_{10} = ??$

$64.680_{10} = ??$

BINARY TO DECIMAL

Thus, the decimal equivalent of a binary number has the general form;

$a_{n-1} \times 2^{n-1} + a_{n-2} \times 2^{n-2} + \ldots.. + a_i \times 2^i + \ldots. + a_1 \times 2^1 + a_0 \times 2^0 + a_{-1} \times 2^{-1} + a_{-2} \times 2^{-2} + \ldots. + a_{-p} \times 2^{-p}$

BINARY TO DECIMAL

$110_2=(1*2^2)+ (1*2^1)+(0*2^0)=4+2+0=6_{10}$

OCTAL TO DECIMAL

$12570_8=(1*8^4)+(2*8^3) +(5*8^2)+(7*8^1)+ (0*8^0)=43976_{10}$

HEXADECIMAL TO DECIMAL

Convert the following from hex to decimal

$41CF_{16}=4*16^3+1*16^2+C*16^1+ F*16^{0=}16847_{10}$

BINARY TO HEXADECIMAL

- Since a string of 4 bits has 16 different permutations each 4-bit string represents a hexadecimal digit uniquely.
- Thus, to convert a binary number to its hexadecimal equivalent we arrange the bits into groups of 4 starting at the binary point and move towards the MSB.
- We then replace each group by the corresponding hexadecimal digit.

BINARY TO HEXADECIMAL

Example

$11\ 1110\ 1101_2$ = add zeros to the left of MSB

$0011\quad 1110\quad 1101 = 3ED_{16}$

Convert the following into hexadecimal

a. 10110101110_2 =?

b. 1001011111110_2 =?

HEXADECIMAL TO BINARY

Convert the following to binary equivalents:

$A748_{16}$

Solution:

$A748_{16} = 1010\ 0111\ 0100\ 1000$

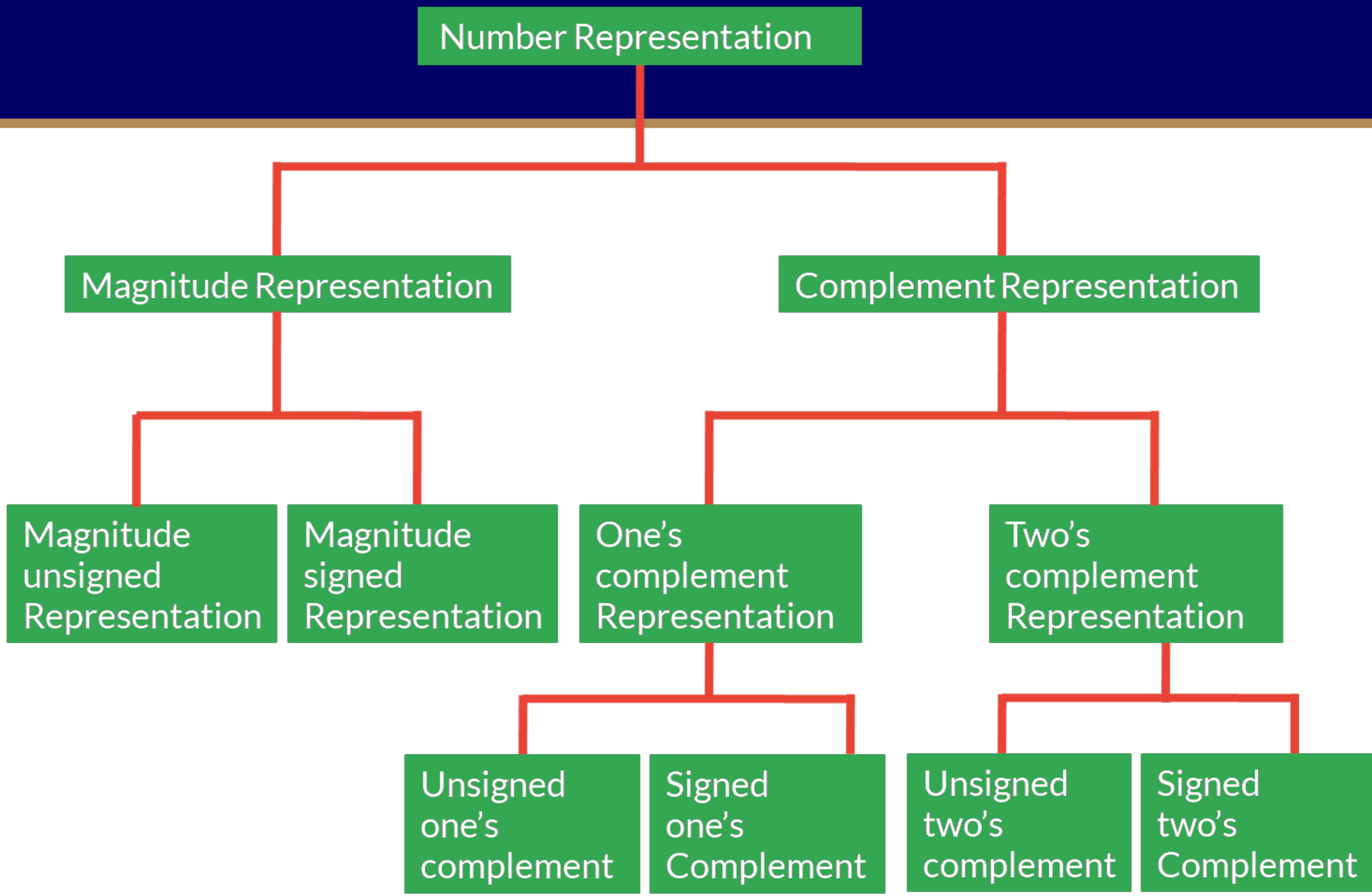
$= 1010011101001000_2$

Convert the following hexadecimal to binary

a. $FFDE_{16}$=?

b. $5ACF_{16}$=?

NUMBER REPRESENTATION SCHEME USING MAGNITUDE AND COMPLEMENT


Number Representation
Magnitude Representation
Complement Representation
Magnitude unsigned Representation
Magnitude signed Representation
One’s complement Representation
Two’s complement Representation
Unsigned one’s complement
Signed one’s Complement
Unsigned two’s complement
Signed two’s Complement

MAGNITUDE REPRESENTATION

Number systems such Binary, Octal, Decimal and Hexadecimal can be represented using both signed and unsigned magnitude.

Unsigned Numbers:

- Unsigned numbers don't have any sign, these can contain only magnitude of the number.
- Representation of unsigned binary numbers are all positive numbers only.
- For example, representation of positive decimal numbers are positive by default.
- We always assume that there is a positive sign symbol in front of every number.

Example: +6=6=110, +10=10=1010,

-6=not possible

MAGNITUDE REPRESENTATION

Unsigned magnitude representation:

- Since there is no sign bit in this unsigned binary number, so N bit binary number represent its magnitude only. Every number in unsigned number representation has only one unique binary equivalent form
- The range of unsigned binary number is from 0 to (2^n-1)
- Find range of 5-bit unsigned binary numbers. Also, find minimum and maximum value in this range.

Solution: Since, range of unsigned binary number is from 0 to (2^n-1). Therefore, range of 5-bit unsigned binary number is from 0 to (2^5-1) which is equal from minimum value 0 (i.e., 00000) to maximum value 31 (i.e., 11111).

MAGNITUDE REPRESENTATION

Signed Numbers

- Signed numbers contain sign flag, this representation distinguish positive and negative numbers.
- This technique contains both sign bit and magnitude of a number.
- The MSB represent the sign bit
- Positive numbers have MSB as 0 and Negative numbers have MSB as 1
- In representing negative decimal numbers, the negative symbol is put in front of given number.

MAGNITUDE REPRESENTATION

Signed Numbers

- Generally, sign bit is a most significant bit (MSB) of representation.
- The range of Sign-Magnitude form is from $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$.
- Example: find the range of 6-bit signed magnitude number

  Solution $-(2^5-1)$ to $(2^5-1) = -31$ to 31

COMPLEMENT ARITHMETIC

- The complements are used to make the arithmetic operations such as subtraction in digital system easier
- The computer system use complements in most of the number system discussed in order to make subtraction operation on numbers easier.
- For each radix-r system (radix r represents base of number system) there are two types of complements.

Radix Complement

The radix complement is referred to as the r's complement

Diminished Radix Complement

The diminished radix complement is referred to as the (r-1)'s complement

BINARY SYSTEM COMPLEMENT

- As the binary system has base r = 2.
- So, the two types of complements for the binary system are 2's complement and 1's complement.
- One’s complement and two’s complement are two important binary concepts.

COMPLEMENT ARITHMETIC

Example of complement

- 1's and 2's complement for binary numbers
- 7's and 8's complement for octal numbers
- 9's and 10's complement for decimal numbers
- 15's and 16's complement for hexadecimal numbers

1’s complement and 2’s complement

One’s Complement

If all bits in a binary number are inverted by changing each 1 to 0 and each 0 to 1, we have formed the one’s complement of the number

Example:

10011001 --> 01100110

10000001 --> 01111110

11110000 --> 00001111

2’s Complement

- The two’s complement is a method for representing positive and negative integer values in binary.

Rule:

- To form the two’s complement, add 1 to the one’s complement.

Two's Complement

We obtain 2's complement by adding 1 to the 1's complement

Example:

01100110+1=01100111(2's complement)

01111110+1=01111111(2's complement)

00001111+1=00010000 (2's complement)

1’s complement and 2’s complement

Convert the following into 2’s complement

- -12_{10}
- -65_{10}

First, we convert to binary (forget about sign for now)

$12_{10}=1100_2$

find 1’s complement==>0011

Find 2’s complement==> [0011+1] =0100

Let consider the following problems

Using 1's complement compute the value of

a-->110101_2-100101_2=010000_2

b-->101011_2-111001_2=-001110_2

Using 2's complement compute the value of the problem above

Find 1's and 2's complement of 15_{10}-10_{10}=………$_2$

DECIMAL COMPLEMENT ARITHMETIC

9's and 10's complement

- This complement system provides an easy way of performing subtraction operation on decimal numbers
- To work in complement arithmetic , we need to convert the decimal number to its 9's or 10's complement equivalence.
- 9′s complement of decimal number can be obtained by ((10^n - 1)-number) where n represents the number of digits in given number.
- 10′s complement can be obtained by (10^n – number) where n represents the number of digits in given number.

DECIMAL COMPLEMENT ARITHMETIC

Example 9's complement of 115 is given by

$((10^3-1)-115)=884$

10's complement of 115 is given by

$884+1=885$

9's complement of 1984 is given by

$((10^4-1)-1984)=8015$

10's complement of 1984 is given by

$8015+1=8016$

Find 9's and 10's complement of the following

a. 2000
b. 1234
c. 48

Using 9's and 10's complement arithmetic to compute 251-185

251-185=>251+(-185)

Convert 185 to 9's complement=>(999-185)=814

10's complement of 185=>814+1=815

251+815

$$
\begin{array}{r}
251 \\
\underline{815} \\
\underline{1}066
\end{array}
$$

Drop the carry (1) so we have 66 which is equivalent to 251-185

Note: If the final answer of the summation does not have carry, we find 10's complement of the answer and add negative sign

TAKE HOME EXERCISE

Perform the following operations with the given complement arithmetic

a) $447_8 - 415_8$ using 8's complement

b) $AFD_{16} - BC5_{16}$ using 16's complement

c) $981_{10} - 519_{10}$ using 2's complement

d) $FAB_{16} - DAD_{16}$ using 9's complement


THE END