Dedução do TMM para o modo TM

A dedução será feita de maneira análoga à dedução do TMM para o modo TE.

Tomando como base o Orfanidis p. 401

TM modes

The TM modes are obtained by solving Eqs. (9.3.10) in each region and applying the boundary conditions. Assuming x-dependence only, we must solve in each region:

$$(\partial_x^2 + k_f^2)E_z = 0\,, \quad E_x = -\frac{j\beta}{k_f^2}\partial_x E_z\,, \quad H_y = \frac{1}{\eta_{TM}}E_x\,, \quad \eta_{TM} = \frac{\beta}{\omega\epsilon}$$

Considere $K_f^2 = \beta^2 - K_0^2 n_j^2$

O fato de podermos assumir dependência apenas de x, permite:

$$\frac{dE_z}{dx} + \beta^2 - K_0^2 n_j^2 = 0 \quad \ldots (1)$$

cujo pol. característico é:

$$r^2 + (\beta^2 - K_0^2 n_j^2) = 0$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\pm\sqrt{-\cancel{4}(\beta^2 - K_0^2 n_j^2)}}{\cancel{2}}$$

se $\beta^2 > K_0^2 n_j^2$

$$\alpha = 0$$

$$\omega = \pm\sqrt{\beta^2 - K_0^2 n_j^2}$$

e a solução de (1) é:

$$E_z^j(x) = C_j e^{i w_j (x - t_j)} + D_j e^{-i w_j (x - t_j)}$$

Condições de fronteira

The boundary conditions require the continuity of the normal component of displacement field $D_x = \epsilon E_x$ across the interfaces at $x = \pm a$, which is equivalent to the continuity of the tangential field H_y because $H_y = E_x/\eta_{TM} = \epsilon E_x \omega/\beta = D_x\omega/\beta$. Thus, the boundary conditions at $x = \pm a$ require:

Pela citação anterior do Orfanidis, sabemos q Hy está diretamente relacionado com a derivada de E_z, assim, as condições de fronteira, são:

$$(I)\ E_z^{j}(t_{j+1}) = E_z^{j+1}(t_{j+1})$$

$$(II)\ \frac{dE_z^{j}}{dx}(t_{j+1}) = \frac{dE^{j+1}}{dx}(t_{j+1})$$

(I):

$$C_j e^{i w_j (t_{j+1} - t_j)} + D_j e^{-i w_j (t_{j+1} - t_j)} = C_{j+1} e^{i w_j (t_{j+1} - t_{j+1})} + B_{j+1} e^{i w_j (t_{j+1} - t_{j+1})}$$

seja: $w_j (t_{j+1} - t_j) = \sigma_j$, temos:

$$C_j e^{i\sigma_j} + D_j e^{-i\sigma_j} = C_{j+1} + D_{j+1}$$

$$\frac{dE_z^{j}}{dx} = i w_j C_j e^{i w (x - t_j)} - i w_j B_j e^{-i w (x - t_j)}$$

$$\frac{dE_z^{j+1}}{dx} = i w_{j+1} C_{j+1} e^{i w (x - t_{j+1})} - i w_{j+1} D_{j+1} e^{-i w (x - t_{j+1})}$$

Igualando as derivadas acima em $x = t_{j+1}$, e $w_j (t_{j+1} - t_j) = \sigma_j$

$$i w_j C_j e^{i\sigma_j} - i w_j e^{-i\sigma_j} = i w_{j+1} C_{j+1} e^{0} - i w_{j+1} D_{j+1} e^{0}$$

$$\frac{\cancel{i} w_j C_j e^{i\sigma_j}}{\cancel{i} w_{j+1}} - \frac{\cancel{i} w_j e^{-i\sigma_j}}{\cancel{i} w_{j+1}} = C_{j+1} - D_{j+1}$$

temos o conjunto:

$$\begin{cases} C_j e^{i\sigma_j} + D_j e^{-i\sigma_j} = C_{j+1} + D_{j+1} & \dots Ⓐ \\ \dfrac{w_j C_j e^{i\sigma_j}}{w_{j+1}} - \dfrac{w_j e^{-i\sigma_j}}{w_{j+1}} = C_{j+1} - D_{j+1} & \dots Ⓑ \end{cases}$$

(A) + (B) :

$$C_{j+1} = \frac{C_j e^{i\sigma_j}}{2}\left(1+\frac{w_j}{w_{j+1}}\right) + \frac{D_j e^{-i\sigma_j}}{2}\left(1-\frac{w_j}{w_{j+1}}\right) \quad \dots (C)$$

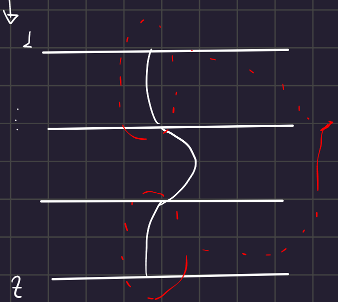
(A) − (B) :

$$D_{j+1} = \frac{C_j e^{i\sigma_j}}{2}\left(1-\frac{w_j}{w_{j+1}}\right) + D_j e^{-i\sigma_j}\left(1+\frac{w_j}{w_{j+1}}\right) \quad \dots (D)$$

Escrevendo (C) e (D) na forma matricial:

$$\begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} e^{i\sigma_j}\left(1+\frac{w_j}{w_{j+1}}\right) + & e^{-i\sigma_j}\left(1-\frac{w_j}{w_{j+1}}\right) \\ e^{i\sigma_j}\left(1-\frac{w_j}{w_{j+1}}\right) + & e^{-i\sigma_j}\left(1+\frac{w_j}{w_{j+1}}\right) \end{bmatrix} \begin{vmatrix} C_j \\ D_j \end{vmatrix}$$

em que $w = \sqrt{\beta^2 - k_0^2 n_j^2}$ e $\sigma = w_j \cdot (t_{j+1} - t_j)$



Os campos das camadas 1 e 2 são evanescentes, i.e., não não transmitidos.

Ou seja, quando $x \to \infty$, $C = 0$. Quando $x \to -\infty$, $D = 0$.

Assim,

$$\begin{bmatrix} C \\ D \end{bmatrix}_1 = T_{wg}\begin{bmatrix} 1 \\ 0 \end{bmatrix} a, \quad (E)$$

$$\begin{bmatrix} C \\ D \end{bmatrix}_2 = T_{wg}\begin{bmatrix} 0 \\ 1 \end{bmatrix} b. \quad (F)$$

(E) e (F) podem ser utilizadas da seguinte forma: os coeficientes da camada 2 são determinados pelo produtório das camadas intermediárias (T_{wg}), com

Os coeficientes da camada 1.

$$\begin{bmatrix} c \\ b \end{bmatrix}_2 = T_{ws} \begin{bmatrix} c \\ b \end{bmatrix}_1$$

Q.E.D.