

Distance between lines

$$d(r, r_1) = \frac{\begin{vmatrix} a - a_1 & b - b_1 & c - c_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix}}{\sqrt{\begin{vmatrix} m & n \\ m_1 & n_1 \end{vmatrix}^2 + \begin{vmatrix} l & n \\ l_1 & n_1 \end{vmatrix}^2 + \begin{vmatrix} l & m \\ l_1 & m_1 \end{vmatrix}^2}} \quad Q \equiv (a, b, c) \quad Q_1 \equiv (a_1, b_1, c_1)$$

Where $r = \vec{a}(l, m, n)$ and $r_1 = \vec{a}_1(l_1, m_1, n_1)$ passing through Q and Q_1

Common perpendicular between two lines

$$\begin{vmatrix} X - a & Y - b & Z - c \\ \beta_1 & \beta_2 & \beta_3 \\ l & m & n \end{vmatrix} = 0$$

$$\begin{vmatrix} X - a_1 & Y - b_1 & Z - c_1 \\ \beta_1 & \beta_2 & \beta_3 \\ l_1 & m_1 & n_1 \end{vmatrix} = 0$$

Where $\langle \beta_1, \beta_2, \beta_3 \rangle = \langle \vec{a} \wedge \vec{a}_1 \rangle$

Distance between a plane and a point

$$d(P_0, \pi) = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} \quad P_0 \equiv (x_0, y_0, z_0)$$

Distance between a point and a line

$$d(P_0, r) = \frac{\begin{vmatrix} y_0 - b & z_0 - c \\ m & n \end{vmatrix}^2 + \begin{vmatrix} x_0 - a & z_0 - c \\ l & n \end{vmatrix}^2 + \begin{vmatrix} x_0 - a & y_0 - b \\ l & m \end{vmatrix}^2}{\sqrt{l^2 + m^2 + n^2}} \quad P_0 \equiv (x_0, y_0, z_0)$$

Angle between two planes

$$\cos \varphi = \frac{\langle n, n_1 \rangle}{\|n\| \|n_1\|} = \frac{AA_1 + BB_1 + CC_1}{\sqrt{A^2 + B^2 + C^2} \sqrt{A_1^2 + B_1^2 + C_1^2}}$$

Angle between a line and a plane

$$\sin \varphi = \frac{Al + Bm + Cn}{\sqrt{A^2 + B^2 + C^2} \sqrt{l^2 + m^2 + n^2}}$$