Distance between lines

$$d(r,r_1)=\frac{\begin{vmatrix} a-a_1 & b-b_1 & c-c_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix}}{\sqrt{\begin{vmatrix} m & n \\ m_1 & n_1 \end{vmatrix}^2+\begin{vmatrix} l & n \\ l_1 & n_1 \end{vmatrix}^2+\begin{vmatrix} l & m \\ l_1 & m_1 \end{vmatrix}^2}}\ Q\equiv(a,b,c)\ Q_1\equiv(a_1,b_1,c_1)$$

Where $r=<\vec{a}(l,m,n)>$ and $r_1=<\vec{a_1}(l_1,m_1,n_1)>$ passing through Q and Q_1

Common perpendicular between two lines

$$\begin{vmatrix} X-a & Y-b & Z-c \\ \beta_1 & \beta_2 & \beta_3 \\ l & m & n \end{vmatrix}=0$$

$$\begin{vmatrix} X-a_1 & Y-b_1 & Z-c_1 \\ \beta_1 & \beta_2 & \beta_3 \\ l_1 & m_1 & n_1 \end{vmatrix}=0$$

Where $<\beta_1,\beta_2,\beta_3>=<\vec{a}\times\vec{a_1}>$

Distance between a plane and a point

$$d(P_0,\pi)=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}_{P_0\equiv(x_0,y_0,z_0)}$$

Distance between a point and a line

$$d(P_0,r)=\frac{\begin{vmatrix} y_0-b & z_0-c \\ m & n \end{vmatrix}^2+\begin{vmatrix} x_0-a & z_0-c \\ l & n \end{vmatrix}^2+\begin{vmatrix} x_0-a & y_0-b \\ l & m \end{vmatrix}^2}{\sqrt{l^2+m^2+n^2}}_{P_0\equiv(x_0,y_0,z_0)}$$

Angle between two planes

$$\cos\varphi=\frac{<n,n_1>}{\|n\|\,\|n_1\|}=\frac{AA_1+BB_1+CC_1}{\sqrt{A^2+B^2+C^2}\sqrt{A_1^2+B_1^2+C_1^2}}$$

Angle between a line and a plane

$$\sin\varphi=\frac{Al+Bm+Cn}{\sqrt{A^2+B^2+C^2}\sqrt{l^2+m^2+n^2}}$$

Plane passing for three points

$$\pi = \begin{vmatrix} X-x_0 & Y-y_0 & Z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} = 0 \; {}_{P_0 \equiv (x_0,y_0,z_0), P_1 \equiv (x_1,y_1,z_1), P_2 \equiv (x_2,y_2,z_2)}$$

Parametric equations of a plane

$$\begin{aligned} x &= q_1 + a_1 u + b_1 v \\ y &= q_2 + a_2 u + b_2 v \\ z &= q_3 + a_3 u + b_3 v \end{aligned}$$

Where $\left\{ \vec{a}(a_1, a_2, a_3), \vec{b}(b_1, b_2, b_3) \right\}$ is the basis and $Q \equiv (q_1, q_2, q_3)$ is a point

Cartesian equation of a plane

$$\begin{vmatrix} X-q_1 & Y-q_2 & Z-q_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$$

With $A = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}$, $B = -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}$, $C = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$ one has the following:

$$A\left(X - q_1\right) + B\left(Y - q_2\right) + C\left(Z - q_3\right) = 0$$

Directional vector of a line

$$r = \begin{cases} AX + BY + CZ + D = 0 \\ A'X + B'Y + C'Z + D' = 0 \end{cases}$$

$$l = \begin{vmatrix} B & C \\ B' & C' \end{vmatrix}, \; m = -\begin{vmatrix} A & C \\ A' & C' \end{vmatrix}, \; n = \begin{vmatrix} A & B \\ A' & B' \end{vmatrix}$$

Cartesian equations of a line

$$\begin{pmatrix} X-a & Y-b & Z-c \\ l & m & n \end{pmatrix}_{Q \equiv (a,b,c), \vec{v}(l,m,n)}$$

The equatons are the minors of the above matrix equal to zero:

$$\begin{aligned} m\left(X - a\right) - l\left(Y - b\right) &= 0 \\ n\left(X - a\right) - l\left(Z - c\right) &= 0 \end{aligned}$$

Vectorial product

let $\left\{\vec{i}, \vec{j}, \vec{k}\right\}$ be an orthonormal basis of $\mathbb{R}^3$ then we have the following:

$$
\begin{gathered}
\vec{i} \times \vec{j} = -\vec{j} \times \vec{i} = \vec{k} \\
\vec{j} \times \vec{k} = -\vec{k} \times \vec{j} = \vec{i} \\
\vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j} \\
\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = \vec{0}
\end{gathered}
$$

The **v.p.** between two arbitrary vectors $\vec{v}(v_1, v_2, v_3)$ and $\vec{w}(w_1, w_2, w_3)$ of $\mathbb{R}^3$ is:

$$\vec{v} \times \vec{w} = (v_2 w_3 - v_3 w_2)\,\vec{i} + (v_3 w_1 - v_1 w_3)\,\vec{j} + (v_1 w_2 - v_2 w_1)\,\vec{k} \quad \triangle$$

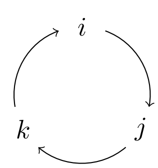
That can be seen as:

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

By theory the $\triangle$ is defined by the minors of second order taken with the criteria $+, -, +$ of the matrix:

$$\begin{pmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}$$

One can remind easily the propreties of $\vec{i}, \vec{j}, \vec{k}$ with the scheme:



More propreties about the v.p.

Let be $\mathfrak{v}, \mathfrak{w}, \mathfrak{u} \in \mathbb{R}^3$ and $c \in \mathbb{R}$ then one has

$$
\begin{gathered}
\mathfrak{v} \times \mathfrak{w} = -\mathfrak{w} \times \mathfrak{v} \\
\mathfrak{v} \times (\mathfrak{w} + \mathfrak{u}) = \mathfrak{v} \times \mathfrak{w} + \mathfrak{v} \times \mathfrak{u} \\
c\,(\mathfrak{v} \times \mathfrak{w}) = (c\mathfrak{v}) \times \mathfrak{w} = \mathfrak{v} \times (c\mathfrak{w})
\end{gathered}
$$