

### Distance between lines

$$d(r, r_1) = \frac{\begin{vmatrix} a - a_1 & b - b_1 & c - c_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix}}{\sqrt{\begin{vmatrix} m & n \\ m_1 & n_1 \end{vmatrix}^2 + \begin{vmatrix} l & n \\ l_1 & n_1 \end{vmatrix}^2 + \begin{vmatrix} l & m \\ l_1 & m_1 \end{vmatrix}^2}} \quad Q \equiv (a, b, c) \quad Q_1 \equiv (a_1, b_1, c_1)$$

Where  $r = \langle \vec{a}(l, m, n) \rangle$  and  $r_1 = \langle \vec{a}_1(l_1, m_1, n_1) \rangle$  passing through  $Q$  and  $Q_1$

### Common perpendicular between two lines

$$\begin{vmatrix} X - a & Y - b & Z - c \\ \beta_1 & \beta_2 & \beta_3 \\ l & m & n \end{vmatrix} = 0$$

$$\begin{vmatrix} X - a_1 & Y - b_1 & Z - c_1 \\ \beta_1 & \beta_2 & \beta_3 \\ l_1 & m_1 & n_1 \end{vmatrix} = 0$$

Where  $\langle \beta_1, \beta_2, \beta_3 \rangle = \langle \vec{a} \wedge \vec{a}_1 \rangle$

### Distance between a plane and a point

$$d(P_0, \pi) = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} \quad P_0 \equiv (x_0, y_0, z_0)$$

### Distance between a point and a line

$$d(P_0, r) = \frac{\begin{vmatrix} y_0 - b & z_0 - c \\ m & n \end{vmatrix}^2 + \begin{vmatrix} x_0 - a & z_0 - c \\ l & n \end{vmatrix}^2 + \begin{vmatrix} x_0 - a & y_0 - b \\ l & m \end{vmatrix}^2}{\sqrt{l^2 + m^2 + n^2}} \quad P_0 \equiv (x_0, y_0, z_0)$$

### Angle between two planes

$$\cos \varphi = \frac{\langle n, n_1 \rangle}{\|n\| \|n_1\|} = \frac{AA_1 + BB_1 + CC_1}{\sqrt{A^2 + B^2 + C^2} \sqrt{A_1^2 + B_1^2 + C_1^2}}$$

### Angle between a line and a plane

$$\sin \varphi = \frac{Al + Bm + Cn}{\sqrt{A^2 + B^2 + C^2} \sqrt{l^2 + m^2 + n^2}}$$

### Plane passing for three points

$$\pi = \begin{vmatrix} X - x_0 & Y - y_0 & Z - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} = 0 \quad P_0 \equiv (x_0, y_0, z_0), P_1 \equiv (x_1, y_1, z_1), P_2 \equiv (x_2, y_2, z_2)$$

### Parametric equations of a plane

$$\begin{aligned} x &= q_1 + a_1 u + b_1 v \\ y &= q_2 + a_2 u + b_2 v \\ z &= q_3 + a_3 u + b_3 v \end{aligned}$$

Where  $\{\vec{a}(a_1, a_2, a_3), \vec{b}(b_1, b_2, b_3)\}$  is the basis and  $Q \equiv (q_1, q_2, q_3)$  is a point

### Cartesian equation of a plane

$$\begin{vmatrix} X - q_1 & Y - q_2 & Z - q_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$$

With  $A = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}$ ,  $B = -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}$ ,  $C = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$  one has the following:

$$A(X - q_1) + B(Y - q_2) + C(Z - q_3) = 0$$

### Directional vector of a line

$$\begin{aligned} r &= \begin{cases} AX + BY + CZ + D = 0 \\ A'X + B'Y + C'Z + D' = 0 \end{cases} \\ l &= \begin{vmatrix} B & C \\ B' & C' \end{vmatrix}, \quad m = -\begin{vmatrix} A & C \\ A' & C' \end{vmatrix}, \quad n = \begin{vmatrix} A & B \\ A' & B' \end{vmatrix} \end{aligned}$$

### Cartesian equations of a line

$$\begin{pmatrix} X - a & Y - b & Z - c \\ l & m & n \end{pmatrix}_{Q \equiv (a, b, c), \vec{v}(l, m, n)}$$

The equatons are the minors of the above matrix equal to zero:

$$\begin{aligned} m(X - a) - l(Y - b) &= 0 \\ n(X - a) - l(Z - c) &= 0 \end{aligned}$$