1. $A = \{(x,y,z) \in \mathbb{R}^3 : 4+2y^2 \leq x \leq 9-z^2\}$

$$\mu_3(A) = \iint_B dz\,dy\,(9-z^2-4-2y^2)$$

$$B = \{(y,z) \in \mathbb{R}^3 : 4+2y^2 \leq 9-z^2\}$$
$$2y^2+z^2 \leq 5$$

$$r: \begin{cases} y = \frac{\rho}{\sqrt{2}}\cos\theta = r_1(\rho,\theta) \\ z = \rho\sin\theta = r_2(\rho,\theta) \end{cases}$$

$$|\det J_r| = \begin{vmatrix} \frac{\cos\theta}{\sqrt{2}} & -\frac{\rho\sin\theta}{\sqrt{2}} \\ \sin\theta & \rho\cos\theta \end{vmatrix} = \frac{\rho}{\sqrt{2}}\cos^2\theta + \rho\frac{\sin^2\theta}{\sqrt{2}} = \frac{\rho}{\sqrt{2}}$$

$$B = \{(\rho,\theta) \in \mathbb{R}^3 : \rho^2 \leq 5\}$$

$$0 \leq \rho \leq \sqrt{5}$$

$$\mu_3(A) = \int_0^{2\pi} d\theta \int_0^{\sqrt{5}} d\rho\, \frac{\rho}{\sqrt{2}}(9-z^2-4-2y^2) =$$

$$= \int_0^{2\pi} d\theta \int_0^{\sqrt{5}} d\rho\, \frac{\rho}{\sqrt{2}}(9-\rho^2\sin^2\theta-4-\rho^2\cos^2\theta) =$$

$$= \frac{2\pi}{\sqrt{2}}\int_0^{\sqrt{5}} d\rho\, \cancel{\rho}(5-\rho^2) =$$

$$= \frac{2\pi}{\sqrt{2}}\left[\frac{5}{2}\rho^2 - \frac{\rho^4}{4}\right]_0^{\sqrt{5}} =$$

$$= \frac{2\pi}{\sqrt{2}}\left[\frac{25}{2} - \frac{25}{4}\right] = \frac{2\pi}{\sqrt{2}}\left(\frac{25}{4}\right)\cdot\frac{\sqrt{2}}{\sqrt{2}} =$$

$$= \frac{25}{4}\pi\sqrt{2}$$

2. $f(x,y,z) = xyz$

$A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + 4y^2 + z^2 - 12 = 0\}$

$\mathcal{L} = xyz - \lambda(x^2 + 4y^2 + z^2 - 12)$

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial x} = yz - 2\lambda x = 0 \\ \dfrac{\partial \mathcal{L}}{\partial y} = xz - 8\lambda y = 0 \\ \dfrac{\partial \mathcal{L}}{\partial z} = xy - 2\lambda z = 0 \\ \dfrac{\partial \mathcal{L}}{\partial \lambda} = -x^2 - 4y^2 - z^2 + 12 = 0 \end{cases}$$

$x \neq 0!$

$$\begin{cases} \lambda = \dfrac{yz}{2x} \\ xz = 8\left(\dfrac{yz}{2x}\right)y \\ xy = \cancel{2}\left(\dfrac{yz}{\cancel{2}x}\right)z \\ x^2 + 4y^2 + z^2 = 12 \end{cases}$$

$xz = \cancel{8}4\dfrac{y^2}{x\cancel{{}^2}}z$

- Caso $z = 0$
- Caso $z \neq 0$ $\rightsquigarrow x^2 = 4y^2$ $(y \neq 0)$

- Caso $z = 0$

$$\begin{cases} \lambda = 0 \\ 0 = 0 \\ xy = 0 & \rightsquigarrow y = 0 \\ x^2 + 4y^2 + 0 = 12 & \rightsquigarrow x^2 = 12 \rightsquigarrow x = \pm 2\sqrt{3} \end{cases}$$

$(\pm 2\sqrt{3}, 0, 0)$ $\quad f(\pm 2\sqrt{3}, 0, 0) = 0$ poco interessante

- Caso $x^2 = 4y^2$

$$\begin{cases} \lambda = \dfrac{yz}{2x} \\ x^2 = 4y^2 \\ \cancel{4}x^2\cancel{y} = \cancel{y}z^2 \\ 2x^2 + z^2 = 12 & \rightsquigarrow 3z^2 = 12 \rightsquigarrow z = \pm 2 \end{cases}$$

$x^2 = z^2 = 4 \rightsquigarrow x = \pm 2$

$x^2 = 4y^2 \rightsquigarrow 4 = 4y^2 \rightsquigarrow y = \pm 1$

$P_\pm = (\pm 2, \pm 1, \pm 2)$

$f(P_\pm) = \pm 4$ (dipende dalle combinazioni che si prendono)

3. $f(x,y,z) = 7x^2y^2 + 3xz^2 + 2z^2 + 5x^2$

$\nabla f = (14xy^2 + 3z^2 + 10x,\ 14x^2y,\ 6xz + 4z) = \underline{0} \iff$

$$\begin{cases} 14xy^2 + 3z^2 + 10x = 0 \\ 14x^2y = 0 \longrightarrow x = 0 \text{ oppure } y = 0 \\ 6xz + 4z = 0 \end{cases}$$

$\nabla f(-\frac{2}{3}, 0, \pm\frac{2\sqrt{5}}{3}) =$

- Caso $x=0$

$$\begin{cases} 0 + 3z^2 + 0 = 0 \\ x = 0 \\ 0 + 4z = 0 \end{cases} \rightsquigarrow \begin{matrix} z = 0 \\ y \text{ libero} \end{matrix} \quad \boxed{(0, y, 0)}$$

- Caso $y=0$

$$\begin{cases} 0 + 3z^2 + 10x = 0 \\ y = 0 \\ 6xz + 4z = 0 \end{cases} \rightsquigarrow \begin{cases} 3z^2 + 10x = 0 \\ y = 0 \\ z(6x + 4) = 0 \end{cases}$$

$z = 0$, oppure $6x + 4 = 0 \rightsquigarrow x = -\frac{2}{3}$

– Caso $z=0$

$$\begin{cases} 10x = 0 \\ y = 0 \\ z = 0 \end{cases} \rightsquigarrow (0,0,0) \text{ ricade in } (0,y,0)$$

- Caso $x = -\frac{2}{3}$

$3z^2 + 10\left(-\frac{2}{3}\right) = 0 \rightsquigarrow z^2 = \frac{20}{9} \rightsquigarrow z = \pm\frac{2\sqrt{5}}{3}$

$\boxed{\left(-\frac{2}{3}, 0, \pm\frac{2\sqrt{5}}{3}\right)}$

$$H_f = \begin{bmatrix} 14y^2+10 & 28xy & 6z \\ 28xy & 14x^2 & 0 \\ 6z & 0 & 6x+4 \end{bmatrix}$$

• Studio $(0,y,0)$

$$H_f(0,y,0) = \begin{bmatrix} 14y^2+10 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Semidefinita positiva → sella o minimo locale

$f(0,y,0) = 0$

$f(x,y,z) = 7x^2y^2 + 3xz^2 + 2z^2 + 5x^2 =$

$= \underbrace{7x^2y^2}_{\geq 0} + z^2(3x+2) + \underbrace{5x^2}_{\geq 0}$

$3x+2 \geq 0 \rightsquigarrow x \geq -\frac{2}{3} \rightsquigarrow \exists\, \delta$ t.c. minimo

$(0,y,0)$ minimo locale

- Studio $\left(-\frac{2}{3}, 0, \pm\frac{2\sqrt{5}}{3}\right)$

$$H_f = \begin{array}{ccc} \text{det}>0 & \text{det}>0 & \\ \end{array}$$
$$H_f = \begin{bmatrix} 10 & 0 & 6\left(\pm\frac{2\sqrt{5}}{3}\right) \\ 0 & 14\cdot\frac{4}{9}=\frac{56}{9} & 0 \\ 6\left(\pm\frac{2\sqrt{5}}{3}\right) & 0 & 0 \end{bmatrix}$$

$$|\det H_f| = 6\left(\pm\frac{2\sqrt{5}}{3}\right)\left[0 - 6\left(\pm\frac{2\sqrt{5}}{3}\right)\frac{56}{9}\right] =$$

$$= 6\left(\pm\frac{2\sqrt{5}}{3}\right)\left(\mp\frac{2\sqrt{5}}{3}\cdot 6\,\frac{56}{9}\right) < 0 \rightsquigarrow \text{indefinita}$$

$\rightsquigarrow \left(-\frac{2}{3}, 0, \pm\frac{2\sqrt{5}}{3}\right)$ sella

4. $f(x,y,z) = \frac{x^{2y}}{2} - \frac{z^{x+2y}}{8}$ $\hat{\nu}$ normale a

$\Gamma = \{(x,y,z)\in\mathbb{R}^3 : \underbrace{xy^2 - 2xyz + 3xz^2}_{=g} = 16\}$

$Q = (2,2,2)$

$\langle\hat{\nu}(Q), \hat{k}\rangle > 0$

$\nabla g = (y^2 - 2yz + 3z^2, 2xy - 2xz, -2xy + 6xz\}$

$\nabla g(2,2,2) = (8, 0, 16)$ ✓ OKAY

$$\hat{\nu} = \frac{\nabla g}{\|\nabla g\|} = \left(\frac{8}{8\sqrt{5}}, 0, \frac{16}{8\sqrt{5}}\right) = \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$$

$$\nabla f = \left(\frac{\cancel{2}y}{\cancel{2}}x^{2y-1} - \frac{\log(z)}{8}z^{x+2y}, \frac{\cancel{2}\log(x)}{\cancel{2}}x^{2y} + \right.$$
$$\left. - \frac{2\log(z)}{8}z^{x+2y}, -\frac{x+2y}{8}z^{x+2y-1}\right)$$

$$= \left(16 - \frac{\log(2)}{8}\,64\cdot 8, \cancel{\log(2)}\ldots, -\cancel{48}\ 24\right)$$

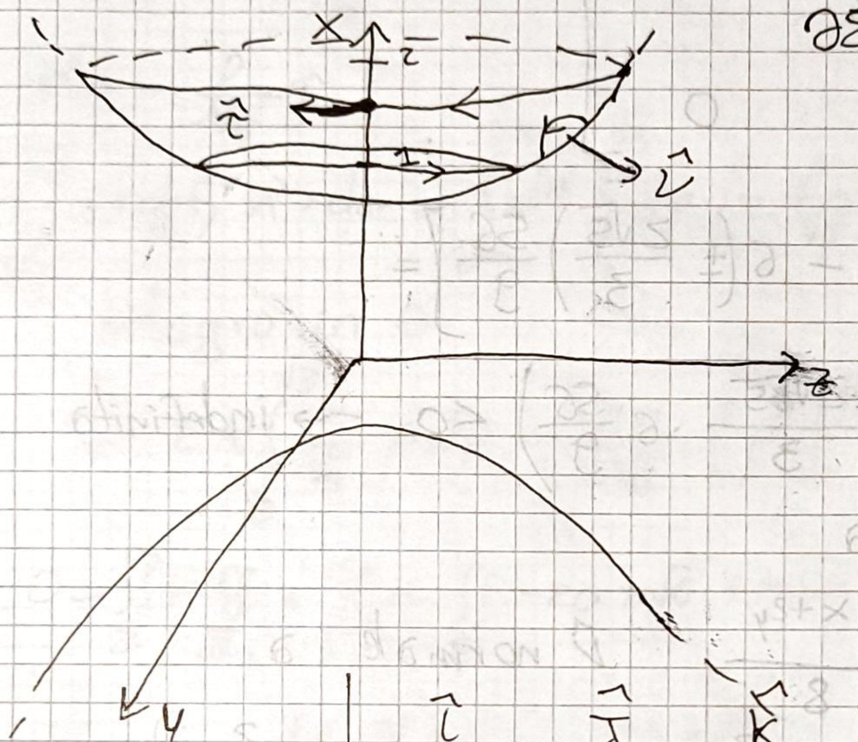
$$\frac{\partial f}{\partial \hat{v}} = \langle \nabla f, \hat{v} \rangle = \left\langle \left(16 - 8\log(2), \dots, -\cancel{48}\,24\right), \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right) \right\rangle =$$

$$= -\frac{\cancel{80}\,32}{\sqrt{5}} - \frac{8\log(2)}{\sqrt{5}} = \left[-\frac{\cancel{16}\,32}{5} - \frac{8}{5}\log(2)\right]\sqrt{5}$$

5. $\vec{F}(x,y,z) = 4xy\,\hat{\imath} + 2x^2z\,\hat{\jmath} - 3y^2\,\hat{k}$

$\Sigma = \left\{(x,y,z) \in \mathbb{R}^3 : 3y^2 + 2z^2 = 4x^2 - 1,\ 1 \le x \le 2\right\}$

$\partial\Sigma = \{(1,y,z)$



$$\operatorname{rot} F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial x & \partial y & \partial z \\ 4xy & 2x^2z & -3y^2 \end{vmatrix} =$$

$$= \hat{\imath}\left[-6y - 2x^2\right] - \hat{\jmath}\left[0 - 0\right] + \hat{k}\left[4xz - 4x\right]$$

$$\frac{\partial r}{\partial t} = \left(1, \frac{4t}{\sqrt{4t^2-1}}\frac{\cos\theta}{\sqrt{3}}, \frac{4t}{\sqrt{4t^2-1}}\frac{\sin\theta}{\sqrt{2}}\right)$$

$$\frac{\partial r}{\partial \theta} = \left(0, -\frac{\sqrt{4t^2-1}}{\sqrt{3}}\sin\theta, \frac{\sqrt{4t^2-1}}{\sqrt{2}}\cos\theta\right)$$

$$\frac{\partial r}{\partial t} \wedge \frac{\partial r}{\partial \theta} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & \frac{4t}{\sqrt{*}} \frac{\cos\theta}{\sqrt{3}} & \frac{4t}{\sqrt{*}} \frac{\sin\theta}{\sqrt{2}} \\ 0 & -\frac{\sqrt{*}}{\sqrt{3}} \sin\theta & \frac{\sqrt{*}}{\sqrt{2}} \cos\theta \end{bmatrix} =$$

$$= \hat{\imath}\left[\frac{4t}{\sqrt{6}}\cos^2\theta + \sin^2\theta \frac{4t}{\sqrt{6}}\right] + $$
$$-\hat{\jmath}\left[\frac{\sqrt{4t^2-1}}{\sqrt{2}} \cos\theta\right] + \hat{k}\left[-\frac{\sqrt{4t^2-1}}{\sqrt{3}} \sin\theta\right] =$$

$$= \hat{\imath}\,\frac{4t}{\sqrt{6}} - \hat{\jmath}\,\frac{\sqrt{4t^2-1}}{\sqrt{2}}\cos\theta - \hat{k}\,\frac{\sqrt{4t^2-1}}{\sqrt{3}}\sin\theta$$

× non compatibile

$$\iiint \operatorname{div} \operatorname{rot} f \, dx\,dy\,dz = 0 = \iint_{\Sigma} + \iint_{\Sigma_1} + \iint_{\Sigma_2}$$

$$\Sigma_1 = \left\{ (2, y, z) \in \mathbb{R}^3 : 3y^2 + 2z^2 \leq 15 \right\}$$
$$\rho^2 \leq 15 \rightsquigarrow 0 \leq \rho \leq \sqrt{15}$$

$$r: \begin{cases} x = 2 \\ y = \frac{\rho}{\sqrt{3}} \cos\theta \\ z = \frac{\rho}{\sqrt{2}} \sin\theta \end{cases}$$

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & \frac{\cos\theta}{\sqrt{3}} & \frac{\sin\theta}{\sqrt{2}} \\ 0 & -\frac{\rho \sin\theta}{\sqrt{3}} & \frac{\rho\cos\theta}{\sqrt{2}} \end{vmatrix} = \hat{\imath}\, \frac{\rho}{\sqrt{6}}$$ ✓ OKAY

~~rotf = ...~~

$$\iint_{\Sigma_1} \langle \operatorname{rot} f, \vec{n} \rangle \, d\sigma = \int_0^{2\pi} d\theta \int_0^{\sqrt{15}} d\rho \, \frac{\rho}{\sqrt{6}} \left(\underbrace{-6 \frac{\rho}{\sqrt{3}} \cos\theta}_{\text{va via}} - 2 \cdot 4 \right)$$ Integro in θ

$$= 2\pi \int_0^{\sqrt{15}} d\rho \, \rho \left(-\frac{8}{\sqrt{6}} \right) = -\frac{16\pi}{\sqrt{6}} \left[\frac{\rho^2}{2} \right]_0^{\sqrt{15}} =$$

$$= -\frac{\cancel{16}^{8}}{\sqrt{6}}\pi \cdot \frac{15}{\cancel{2}} = -\frac{120}{6}\pi\sqrt{6} = -20\pi\sqrt{6}$$

$$\Sigma_2 = \{(1, y, z) \in \mathbb{R}^3 : 3y^2 + 2z^2 \leq 3\}$$

$$\rho^2 \leq 3 \rightarrow 0 \leq \rho \leq \sqrt{3}$$

$$r: \begin{cases} x = 1 \\ y = \frac{\rho}{\sqrt{3}}\cos\theta \\ z = \frac{\rho}{\sqrt{2}}\sin\theta \end{cases}$$

$$\vec{n} = -\frac{\rho}{\sqrt{6}} i$$

$$\iint_{\Sigma_2} \langle \mathrm{rot} f, \vec{n} \rangle d\sigma = \int_0^{2\pi} d\theta \int_0^{\sqrt{3}} d\rho \left(-\frac{\rho}{\sqrt{6}}\right)\left(-6\frac{\rho}{\sqrt{3}}\underbrace{\cos\theta}_{\text{va via}} - 2\right)$$

Integro in θ

$$= 2\pi \int_0^{\sqrt{3}} \frac{2}{\sqrt{6}}\rho\, d\rho = \frac{2\pi}{\sqrt{6}}\left[\rho^2\right]_0^{\sqrt{3}} = \frac{2\pi}{\sqrt{6}} \cdot 3 = \pi\sqrt{6}$$

$$0 = \iint_{\Sigma} + \iint_{\Sigma_1} + \iint_{\Sigma_2} = \iint_{\Sigma} - 20\pi\sqrt{6} + \pi\sqrt{6}$$

$$\rightarrow \iint_{\Sigma} \langle \mathrm{rot} f, \vec{n} \rangle d\sigma = 19\pi\sqrt{6}$$

Altro metodo

$$\iint_{\Sigma} \left\langle \left(-6\frac{\sqrt{4t^2-1}}{\sqrt{3}}\cos\theta, -2t^2, 0, 4t\frac{\sqrt{4t^2-1}}{\sqrt{2}}\sin\theta - 4t\right), \left(\frac{4t}{\sqrt{6}}, -\frac{\sqrt{4t^2-1}}{\sqrt{2}}\cos\theta, -\frac{\sqrt{4t^2-1}}{\sqrt{3}}\sin\theta\right)\right\rangle d\sigma =$$

$$= \iint_{\Sigma} -2t^2\frac{4t}{\sqrt{6}} - \frac{4t(4t^2-1)\sin^2\theta}{\sqrt{6}}\, d\sigma =$$

$$= \int_1^2 -16\pi\frac{t^3}{\sqrt{6}} - \frac{4\pi}{\sqrt{6}}t(4t^2-1) =$$

$$= \left[-\frac{4\pi}{\sqrt{6}}t^4 - \frac{\pi}{2\sqrt{6}}(4t^2-1)^2\right]_1^2 =$$

$$\frac{\pi}{\sqrt{6}}\left[-\cancel{4}t^4 - \frac{(4t^2-1)^2}{2}\right]_1^2 =$$

$$= \frac{\pi}{\sqrt{6}}\left[-\cancel{64} - \frac{225}{2} + \cancel{4} + \frac{9}{2}\right] = -\frac{\pi}{\sqrt{6}} 168 = -28\pi\sqrt{6}$$

Cambiato di segno: $28\pi\sqrt{6}$

$$\partial\Sigma_1 = \{(2,y,z) \in \mathbb{R}^3 : 3y^2 + 2z^2 = 15\}$$

$$r: \begin{cases} x = 2 \\ y = \frac{\sqrt{15}}{\sqrt{3}}\cos\theta \\ z = \frac{\sqrt{15}}{\sqrt{2}}\sin\theta \end{cases} \qquad \frac{\partial r}{\partial\theta} = \left(0, -\sqrt{5}\sin\theta, \frac{\sqrt{15}}{\sqrt{2}}\cos\theta\right)$$

$$\int_0^{2\pi} d\theta\,(2x^2z)(-\sqrt{5}\sin\theta) + (-3y^2)\left(\frac{\sqrt{15}}{\sqrt{2}}\cos\theta\right) =$$

$$= \int_0^{2\pi} d\theta - 8\frac{\sqrt{15}}{\sqrt{2}}\sqrt{5}\sin^2\theta - \underbrace{3(15\cos^2\theta)\left(\frac{\sqrt{15}}{\sqrt{2}}\cos\theta\right)}_{\to 0} =$$

$$= -8\cdot 5\cdot\frac{\sqrt{3}}{\sqrt{2}}\pi = -\frac{40}{2}\sqrt{6}\pi = -20\pi\sqrt{6}$$

$$\partial\Sigma_2 = \{(1,y,z) \in \mathbb{R}^3 : 3y^2 + 2z^2 = 3\}$$

$$r: \begin{cases} x = 1 \\ y = \cos\theta \\ z = \frac{\sqrt{3}}{\sqrt{2}}\sin\theta \end{cases} \qquad \frac{\partial r}{\partial\theta} = \left(0, -\sin\theta, \frac{\sqrt{3}}{\sqrt{2}}\cos\theta\right)$$

$$\int_0^{2\pi} d\theta\,(2x^2z)(-\sin\theta) + \underbrace{(-3y^2)\left(\frac{\sqrt{3}}{\sqrt{2}}\cos\theta\right)}_{\to 0} =$$

$$= \int_0^{2\pi} d\theta\left(2\frac{\sqrt{3}}{\sqrt{2}}\sin\theta\right)(-\sin\theta) = -\pi\sqrt{6}$$ ✓ OKAY