Conditional Probability

Statistical Inference

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## Conditional probability, motivation

* The probability of getting a one when rolling a (standard) die is usually assumed to be one sixth
* Suppose you were given the extra information that the die roll was an odd number (hence 1, 3 or 5)
* *conditional on this new information*, the probability of a one is now one third

## Conditional probability, definition

* Let be an event so that
* Then the conditional probability of an event given that has occurred is
* Notice that if and are independent, then

## Example

* Consider our die roll example

## Bayes' rule

## Diagnostic tests

* Let and be the events that the result of a diagnostic test is positive or negative respectively
* Let and be the event that the subject of the test has or does not have the disease respectively
* The **sensitivity** is the probability that the test is positive given that the subject actually has the disease,
* The **specificity** is the probability that the test is negative given that the subject does not have the disease,

## More definitions

* The **positive predictive value** is the probability that the subject has the disease given that the test is positive,
* The **negative predictive value** is the probability that the subject does not have the disease given that the test is negative,
* The **prevalence of the disease** is the marginal probability of disease,

## More definitions

* The **diagnostic likelihood ratio of a positive test**, labeled , is , which is the
* The **diagnostic likelihood ratio of a negative test**, labeled , is , which is the

## Example

* A study comparing the efficacy of HIV tests, reports on an experiment which concluded that HIV antibody tests have a sensitivity of 99.7% and a specificity of 98.5%
* Suppose that a subject, from a population with a .1% prevalence of HIV, receives a positive test result. What is the probability that this subject has HIV?
* Mathematically, we want given the sensitivity, , the specificity, , and the prevalence

## Using Bayes' formula

* In this population a positive test result only suggests a 6% probability that the subject has the disease
* (The positive predictive value is 6% for this test)

## More on this example

* The low positive predictive value is due to low prevalence of disease and the somewhat modest specificity
* Suppose it was known that the subject was an intravenous drug user and routinely had intercourse with an HIV infected partner
* Notice that the evidence implied by a positive test result does not change because of the prevalence of disease in the subject's population, only our interpretation of that evidence changes

## Likelihood ratios

* Using Bayes rule, we have
* and

## Likelihood ratios

* Therefore
* ie
* Similarly, relates the decrease in the odds of the disease after a negative test result to the odds of disease prior to the test.

## HIV example revisited

* Suppose a subject has a positive HIV test
* The result of the positive test is that the odds of disease is now 66 times the pretest odds
* Or, equivalently, the hypothesis of disease is 66 times more supported by the data than the hypothesis of no disease

## HIV example revisited

* Suppose that a subject has a negative test result
* Therefore, the post-test odds of disease is now of the pretest odds given the negative test.
* Or, the hypothesis of disease is supported times that of the hypothesis of absence of disease given the negative test result