Vehicle Routing Problem-206069T

1. **Decision variables, Parameters and indices.**

   Decision Variable

   - x_ij = 1 if a truck goes from node i to node j (binary) else 0
   - f_ij = number of units in a truck going from node i to node j

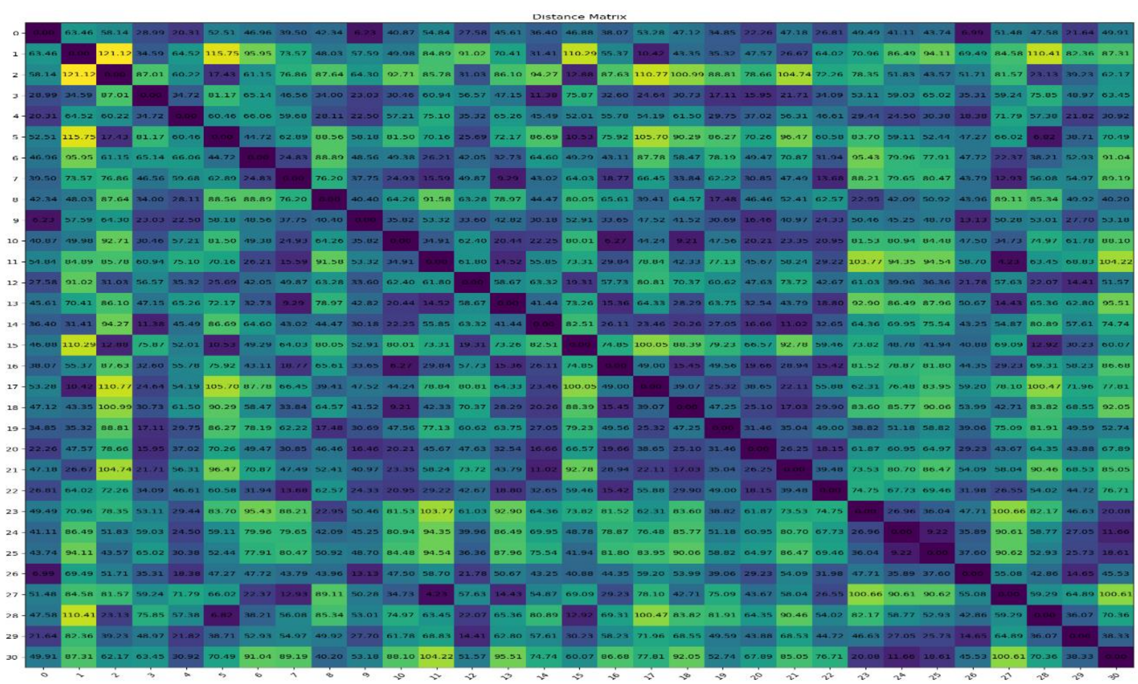
   Parameters

   - n is the number of clients/Nodes
   - N is set of clients/nodes, with $N=\{1,2,...,n\}$
   - V is set of vertices (or nodes), with $V=\{0\}\cup N$
   - A is set of arcs, with $A=\{(i,j)\in V2:i\neq j\}$
   - cij is cost of travel over arc $(i,j)\in A$
   - Q is the truck capacity.
   - qi is the amount that has to be delivered to nodes $i\in N$

   Indices

   - i,j = Indices representing the source and destination nodes.

2. **Distance matrix.**



Euclidean distance formula was used to calculate the distances among nodes.

3. **Mixed-Integer Linear Programming model.**

Objective function:

$$Minimize \quad Z = \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subjected to

$$\sum_{j \in V, j \neq i} x_{ij} = 1 \qquad i \in N$$

$$\sum_{i \in V, i \neq j} x_{ij} = 1 \qquad j \in N$$

$$\text{if } x_{ij} = 1 \Rightarrow u_i + q_j = u_j \qquad i, j \in A : j \neq 0, i \neq 0$$

$$q_i \leq u_i \leq Q \qquad i \in N$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in A$$

- the objective is to minimize the total distance traveled. The sum of $c_{ij}x_{ij}$ represents the total distance traveled, where x_{ij} is a binary decision variable indicating whether there is a route from node **i** to node **j**, and c_{ij} is the distance between nodes **i** and **j**.
- The first constraint represents the number of routes entering node **i**, and it is constrained to be equal to 1.
- The second constraint represents the number of routes leaving node j, and it is constrained to be equal to 1.
- The third constraints ensure that if there is a route from node i to node j, the cumulative demand at node i plus the demand at node j must be equal to the cumulative demand at node j
- The fourth constraint is to restrict cumulative demand at each customer node does not exceed its demand and also does not exceed the vehicle capacity 'Q'
- additionally, a time limit constraint is given for model for optimization process.

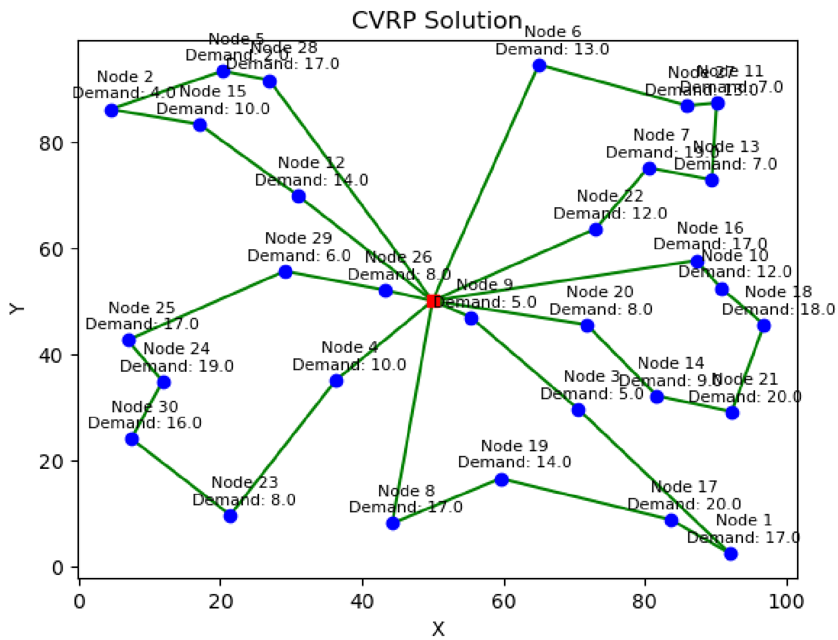
4. **Assumptions and Implications**

Assumptions

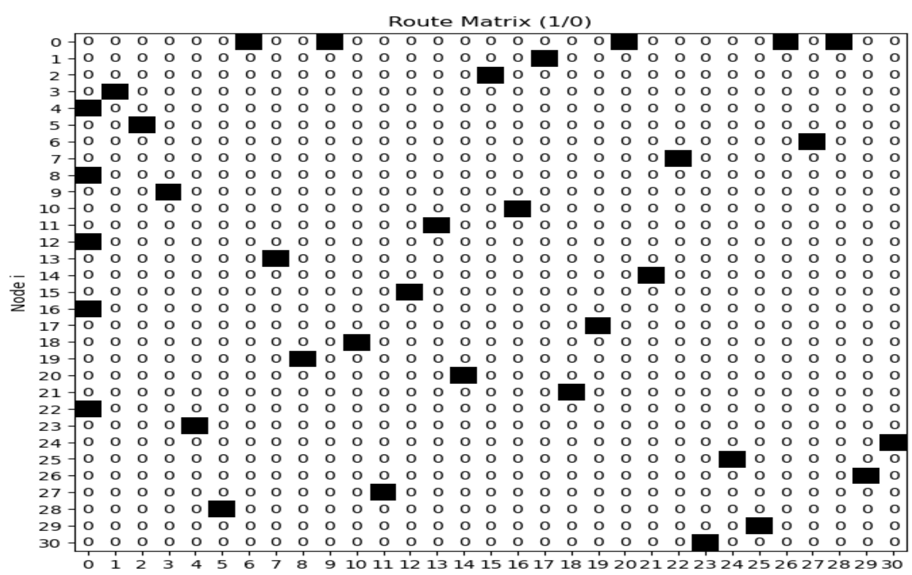
- Euclidean distance formula is used to calculate distance
- All trucks have the same capacity
- Single depot (node 0) is where all vehicle start and return

Implications

- Total Distance Traveled: 687.4340550788327
- Active Arcs (Optimal Routes): [(0, 6), (0, 9), (0, 20), (0, 26), (0, 28), (1, 17), (2, 15), (3, 1), (4, 0), (5, 2), (6, 27), (7, 22), (8, 0), (9, 3), (10, 16), (11, 13), (12, 0), (13, 7), (14, 21), (15, 12), (16, 0), (17, 19), (18, 10), (19, 8), (20, 14), (21, 18), (22, 0), (23, 4), (24, 30), (25, 24), (26, 29), (27, 11), (28, 5), (29, 25), (30, 23)]



- Xij-Route Matrix



- Fij – Flow of unit matrix

Outflows:

[51. 20. 10. 17. 0. 4. 13. 12. 0. 5. 17. 7. 0. 19. 20. 14. 0. 14.
 12. 17. 9. 18. 0. 10. 16. 19. 6. 7. 2. 17. 8.]

Inflows:

[0. 17. 4. 5. 10. 2. 13. 19. 17. 5. 12. 7. 14. 7. 9. 10. 17. 20.
 18. 14. 8. 20. 12. 8. 19. 17. 8. 13. 17. 6. 16.]

Balance:

[-51. -3. -6. -12. 10. -2.   0.   7. 17.   0. -5.   0. 14. -12.
 -11. -4. 17.   6.   6. -3. -1.   2. 12. -2.   3. -2.   2.   6.
  15. -11.   8.]

Demand:

[17. 4. 5. 10. 2. 13. 19. 17. 5. 12. 7. 14. 7. 9. 10. 17. 20. 18.
 14. 8. 20. 12. 8. 19. 17. 8. 13. 17. 6. 16.]

- Based on the results the truck should travel 5 time to satisfy the demand of all nodes which will minimize the distance and the trips could be completed by 687.4340550788327