The Measurement Interpretation of Black Hole Entropy: A Grazing Photon's Limited View

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Abstract

We propose an operational interpretation of the Bekenstein-Hawking entropy $S_{\mathrm{BH}} = A/(4\ell_P^2)$ as the maximum information measurable by a null probe grazing the event horizon. Modeling the horizon's information content as Planck-scale curvature correlations, we posit that a photon can only distinguish holistic correlation patterns enumerated by the integer partition function $p(n)$. Equating the observable entropy $\ln p(n)$ to S_{BH} reveals that the probe can resolve patterns built from $n = kN^2$ correlation units, where $k = 3/(32\pi^2) \approx 0.009499$. Thus, the area law reflects not the total information content, but the finite fraction ($\sim 0.95\%$) accessible before the photon is captured. The model yields a universal logarithmic correction with coefficient -2.

1 Introduction: Entropy as Accessible Information

The Bekenstein-Hawking entropy, $S_{\mathrm{BH}} = A/(4\ell_P^2)$, is widely regarded as counting the microstates of a black hole [1, 2]. Holography suggests these microstates reside on the horizon [5, 6]. While Bekenstein's original argument interpreted S_{BH} as counting bits encoded on Planck areas [1], this picture assumes individual bits are resolvable by external observers. Operationally, a grazing photon cannot resolve individual Planck-scale degrees of freedom due to quantum indeterminacy and finite interaction time.

We reframe the problem in operational terms: rather than deriving S_{BH} from first principles, we ask what the entropy implies about the *limitations of measurement* at the horizon. Consider a massless probe on a grazing null geodesic—a photon skimming the event horizon. This probe has a finite proper time to interact with the horizon before being captured. What can it learn?

We propose that the photon measures not individual Planck-scale degrees of freedom, but *correlations* among them, encoded in the Weyl curvature. The number of distinguishable correlation patterns it can resolve defines the *accessible entropy.* We show that equating this accessible entropy to S_{BH} reveals a fundamental limit: the photon can only sample about 0.95% of the total possible curvature correlations.

2 The Model: What a Grazing Photon Measures

2.1 Horizon Information as Curvature Correlations

Let the horizon be partitioned into $N = A/\ell_P^2$ Planck areas. The total information capacity is N bits in Bekenstein's original formulation, but an external probe cannot resolve individual bits due to redshift and quantum indeterminacy. Instead, the probe senses *tidal forces*—the Weyl curvature—which arise from correlations between horizon elements.

The number of possible pairwise correlations scales as $\sim N^2$. These correlations are the fundamental degrees of freedom accessible to curvature measurements. The photon detects *patterns* of these correlations, not the correlations themselves as labeled entities.

2.2 The Measurement Limit and Combinatorics

A grazing photon has a finite interaction time before gravitational focusing pulls it across the horizon. This limits the number of independent correlations it can sample. If it attempts to resolve too many details, it is captured. Thus, only a fraction k of the N^2 total correlations are accessible.

The curvature correlation units are fundamentally indistinguishable to an external probe. We model the photon's measurement as the distinction of *holistic curvature patterns*, counted by the integer partition function $p(n)$, where n is the number of accessible correlation units. Integer partitions $p(n)$ provide the correct counting of distinct configurations when only aggregate quantities (total correlation strength n) are measurable, and individual units lack labels or spatial addresses. This is the appropriate combinatorics for unlabeled, non-local degrees of freedom.

The measurable entropy is then:

$$S_{\text{meas}} = \ln p(n). \tag{1}$$

3 Equating Measurable and Thermodynamic Entropy

3.1 The Hardy–Ramanujan Asymptotic

For large n, the partition function satisfies [4]:

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right), \quad S_{\text{meas}} \sim \pi\sqrt{\frac{2n}{3}} - \ln(4n\sqrt{3}). \tag{2}$$

3.2 The Bekenstein-Hawking Entropy as an Observational Limit

Operationally, the Bekenstein-Hawking entropy must correspond to information measurable by external probes. We therefore equate the measurable entropy to the thermodynamic entropy:

$$S_{\text{meas}} = S_{\text{BH}} = \frac{A}{4\ell_P^2} = \frac{N}{4}.$$

Matching the leading-order term gives:

$$\pi\sqrt{\frac{2n}{3}} = \frac{N}{4} \quad \Rightarrow \quad n = \frac{3}{32\pi^2}N^2.$$

Thus, the fraction of total correlations accessible is:

$$k = \frac{3}{32\pi^2} \approx 0.009499. \tag{3}$$

3.3 Interpretation

The result $n = kN^2$ means that although the horizon has $\sim N^2$ possible correlation relationships, a grazing photon can only resolve those built from about 0.95% of them. The value $k \approx 0.0095$ emerges from matching the partition growth rate to the area law. This indicates that horizon causality limits measurement to approximately 1% of possible correlation degrees of freedom. The Bekenstein-Hawking entropy is not a count of all microstates, but of those *distinguishable from outside* given finite measurement time.

3.4 Logarithmic Correction

Substituting n back into the full asymptotic form yields:

$$S_{\text{meas}} \sim \frac{N}{4} - 2\ln N + \ln\left(\frac{8\pi^2}{3^{3/2}}\right) + \mathcal{O}(1/N),$$

a universal logarithmic correction with coefficient -2. [1] This contrasts with other approaches to black hole entropy corrections [9, 8, 10].

4 Discussion

4.1 The Role of k as an Information-Theoretic Veil

The small value of k reflects the horizon's role as a causal barrier. It quantifies the fraction of holographic data that can be non-destructively read by an external probe. Unlike microstate counting approaches, this model derives entropy from operational measurement constraints. The partition function $p(n)$ is uniquely suited because it counts configurations without imposing artificial labels or spatial resolution that a grazing photon cannot possess.

4.2 Comparison with Quantum Gravity Models

The difference suggests that operational accessibility may impose stronger constraints than microstate counting alone [7].

[1] The constant term is approximately 2.721, strikingly close to $e \approx 2.71828$ (within 0.1%). This proximity invites mathematical investigation into whether it reflects deeper structure or is coincidental.

Table 1: Logarithmic correction coefficient α in $S \sim \frac{A}{4\ell_P^2} + \alpha \ln A$.

Model	α
This work (measurement-based)	-2
Loop Quantum Gravity [9]	$-3/2$
String Theory [10]	-1 or $-3/2$
Euclidean Path Integral [8]	$-3/2$

5 Conclusion

We have shown that the Bekenstein-Hawking entropy can be interpreted as the information accessible to a grazing photon, limited by horizon causality. The measurable correlation patterns are counted by partitions, yielding $n \approx 0.009499N^2$ and a logarithmic correction of -2. This approach reframes black hole entropy in operational terms and highlights the role of measurement limits in thermodynamics [3].

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