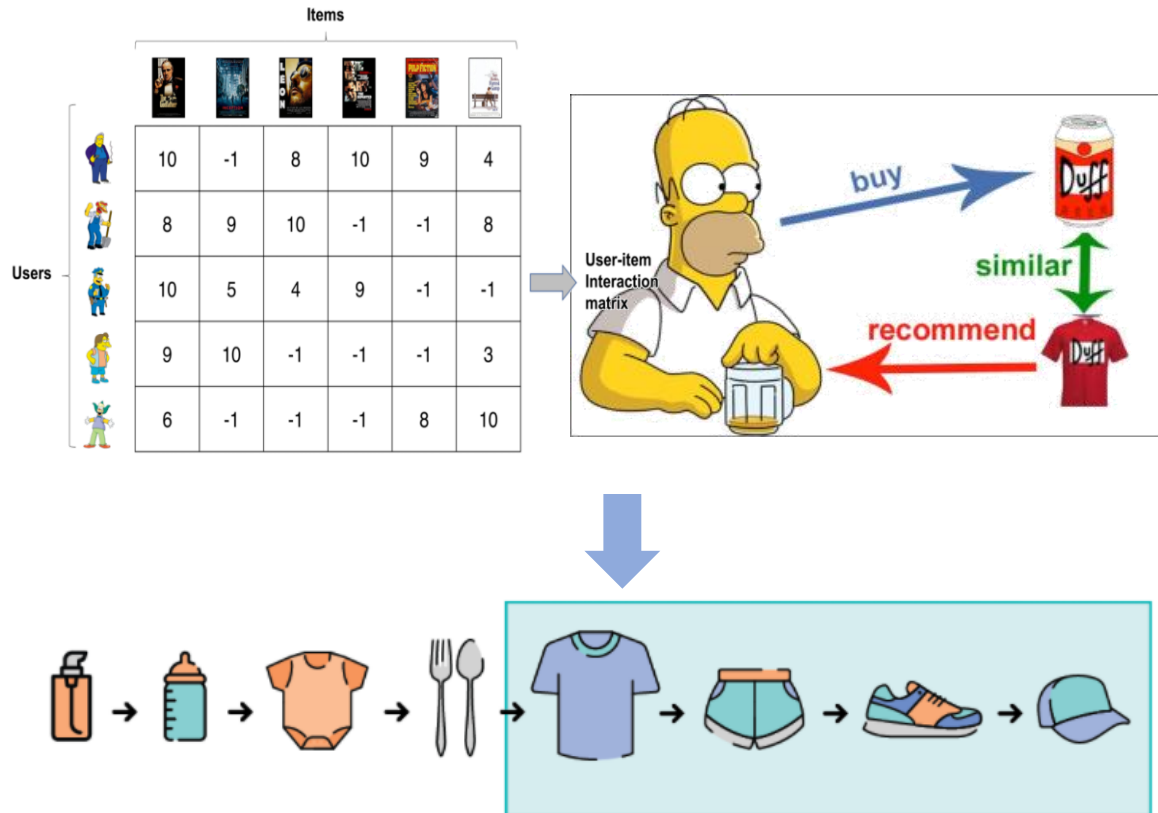


# **Session-based Recommender Systems**

# Introduction

## Session-based recommendations



[fig 01] Difference between CFRS and SBRS

- Content-based and CF Underly assumption that all of the historical interactions of a user are equally important to her current preference.
  - Limitation
    - [1] time-sensitive context (e.g., the recently viewed or purchased items)
    - [2] Dynamic user's preference for items
- **Session-based Recommender Systems (SBRSS)** have emerged with increasing attention in recent years.

# Introduction

## SRS vs SBRS

### Session Example



### Sequence Example

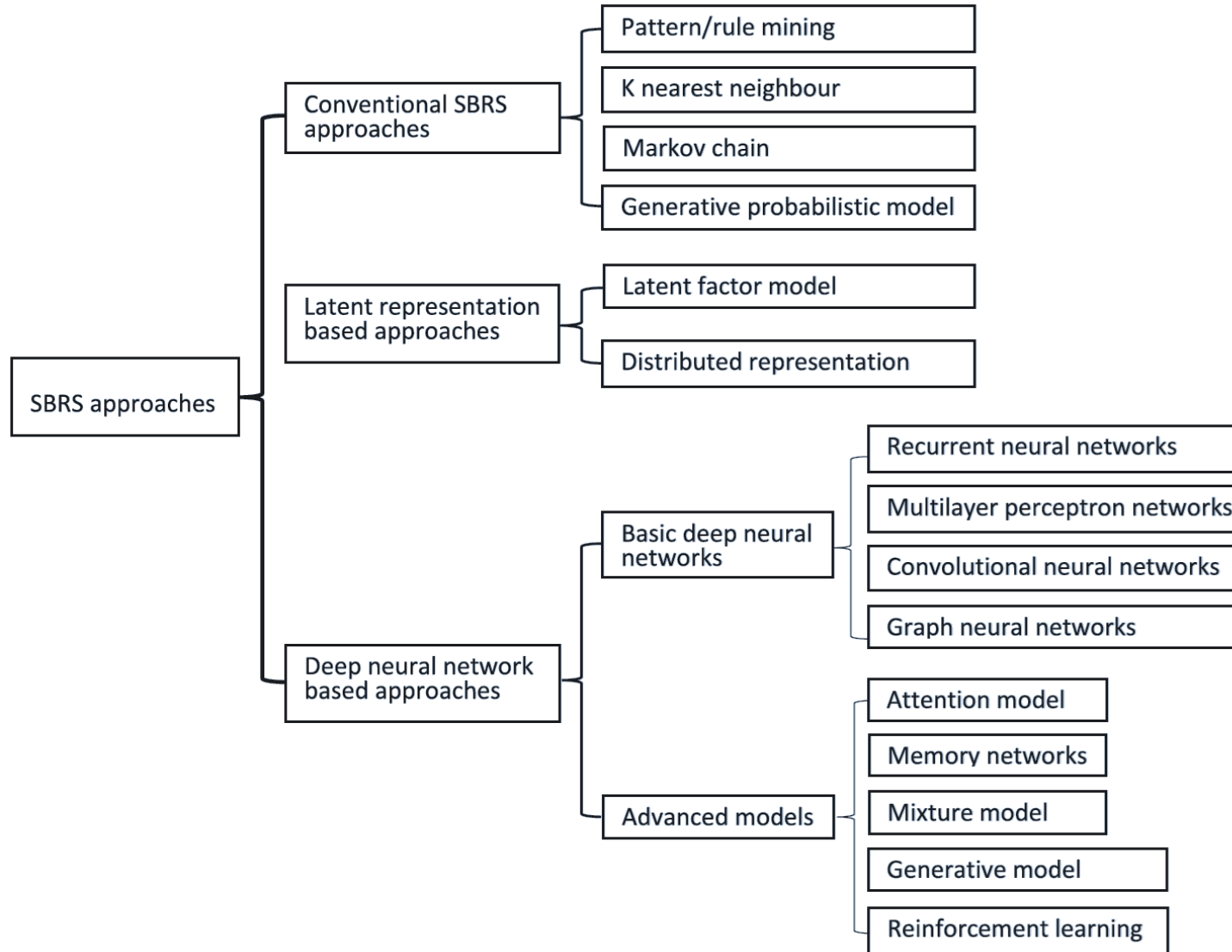


Data Type		Boundary	Order	Time Interval	Main relations embedded	Prediction
Session Data	Unordered session	Multiple	No	Non-identical	Co-occurrence-based dependencies	Unknown part(e.g.,an item or batch of items ), Future session(e.g.,the next-basket)
	Ordered session	Multiple	Yes	Non-identical	Co-occurrence-based dependencies and sequential dependencies	
Sequence data		Single	Yes	Not included	Sequeuntial dependencies	Future item

[fig 02] Session data vs Sequence data

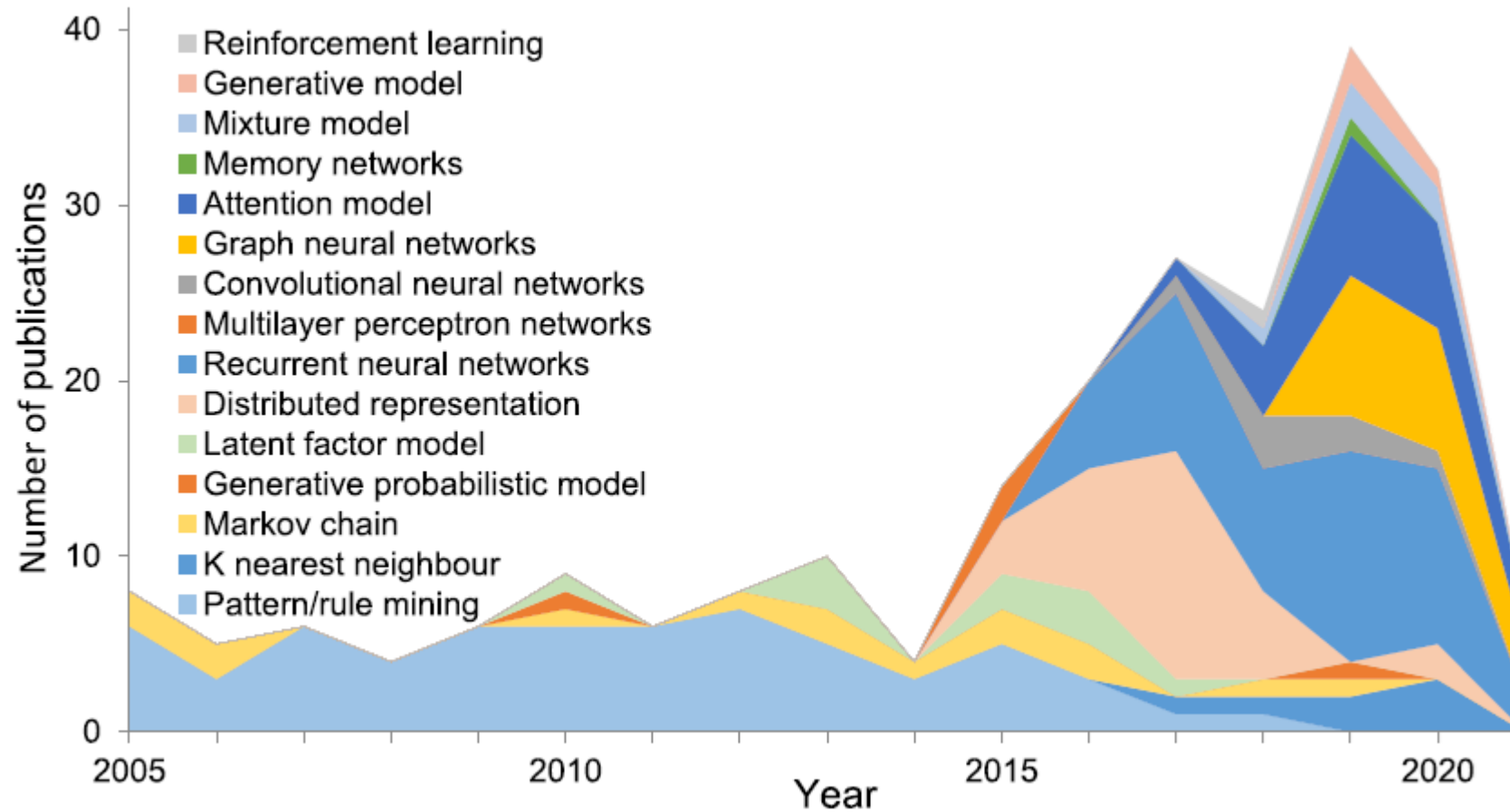
# Introduction

## Taxonomy



# Introduction

## Number of the publications on each class of SBRS



Date : 20 March, 2021 (google scholar)

# Conventional SBRS

## K nearest Neighbor-based SBRSs

### Item-KNN Example (IKNN)

[1] Item-KNN (IKNN)

[2] Session-KNN (SKNN)

Session #	Session content (ordered list of item IDs)
Session 1	[Item 12, Item 23, Item 7, Item 562, Item 346, Item 85]
Session 2	[Item 23, Item 65, Item 12, Item 3, Item 9, Item 248]
Session 3	[Item 248, Item 12, Item 7, Item 9, Item 346]
Session 4	[Item 85, Item 65, Item 248, Item 12, Item 346, Item 9]
Session 5	[Item 346, Item 7, Item 9, Item 3, Item 12]

Current  
Session



Session #	3	7	9	12	23	65	85	248	346	562
Session 1	0	1	0	1	1	0	1	0	1	1
Session 2	1	0	1	1	1	1	0	1	0	0
Session 3	0	1	1	1	0	0	0	1	1	0
Session 4	0	0	1	1	0	1	1	1	1	0
Session 5	1	1	1	1	0	0	0	0	1	0

- Each item is encoded into a binary vector where each element indicates whether the item occurs (set to "1") in a specific session or not (set to "0").

# Conventional SBRS

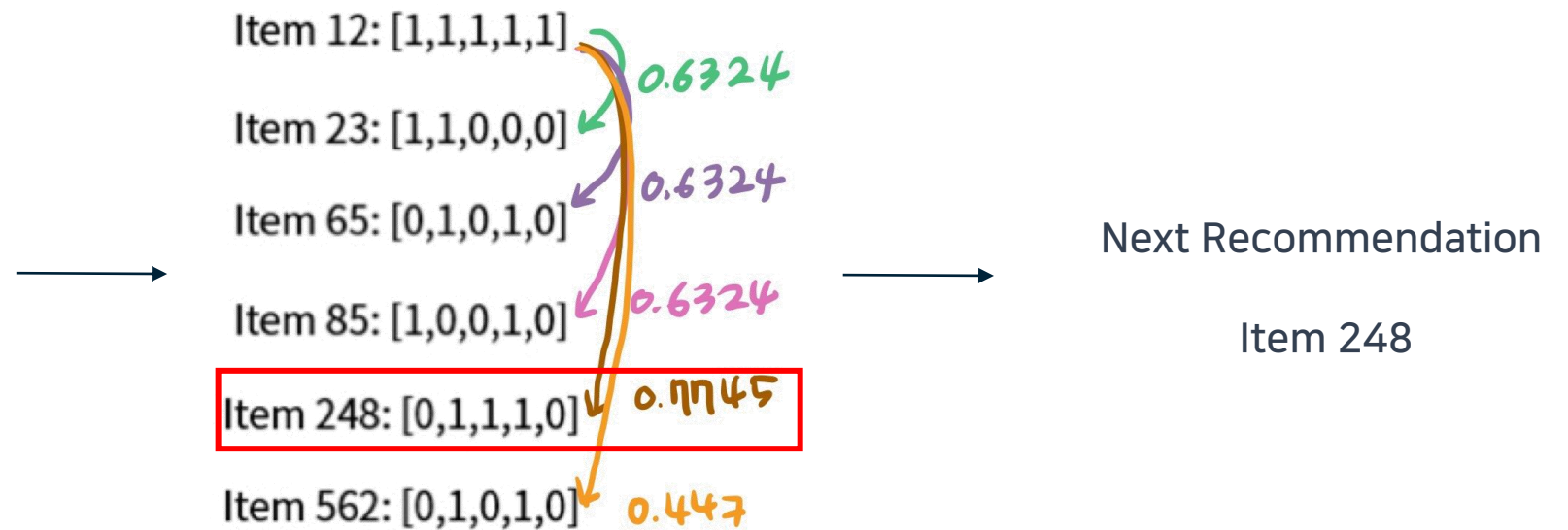
## K nearest Neighbor-based SBRSs

### Item-KNN Example (IKNN)

Session 5	[Item 346, Item 7, Item 9, Item 3, <b>Item 12</b> ]								
-----------	---	--	--	--	--	--	--	--	--

Session #	3	7	9	12	23	65	85	248	346	562
Session 1	0	1	0	1	1	0	1	0	1	1
Session 2	1	0	1	1	1	1	0	1	0	0
Session 3	0	1	1	1	0	0	0	1	1	0
Session 4	0	0	1	1	0	1	1	1	1	0
Session 5	1	1	1	1	0	0	0	0	1	0

$$\text{cosine similarity} = S_C(A, B) := \cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{\sum_{i=1}^n A_i B_i}{\sqrt{\sum_{i=1}^n A_i^2} \sqrt{\sum_{i=1}^n B_i^2}},$$



- During the recommendation, only the last item(Item 12) is considered for the current session(Session 5).
- The similarity between items can be calculated on their vectors with a certain similarity measure, like cosine similarity.

# Conventional SBRS

## K nearest Neighbor-based SBRSs

### Session-KNN Example (SKNN)

Session #	Session content (ordered list of item IDs)
Session 1	[Item 12, Item 23, Item 7, Item 562, Item 346, Item 85]
Session 2	[Item 23, Item 65, Item 12, Item 3, Item 9, Item 248]
Session 3	[Item 248, Item 12, Item 7, Item 9, Item 346]
Session 4	[Item 85, Item 65, Item 248, Item 12, Item 346, Item 9]
Session 5	[Item 346, Item 7, Item 9, Item 3, Item 12]

Session #	Session content (ordered list of item IDs)
Session 1	[Item 12, Item 23, Item 7, Item 562, Item 346, Item 85]
Session 2	[Item 23, Item 65, Item 12, Item 3, Item 9, Item 248]
Session 3	[Item 248, Item 12, Item 7, Item 9, Item 346]
Session 4	[Item 85, Item 65, Item 248, Item 12, Item 346, Item 9]
Session 5	[Item 346, Item 7, Item 9, Item 3, Item 12]

[fig 03] Left: IKNN vs Right: SKNN

- IKNN considers only the last event(e.g., ordered item 12) in current Session, the SKNN method compares the entire current session with the past sessions in the training data to determine the items to be recommended.



# Conventional SBRS

## K nearest Neighbor-based SBRSs

### Session-KNN Example (SKNN)

Session #	3	7	9	12	23	65	85	248	346	562
Session 1	0	1	0	1	1	0	1	0	1	1
Session 2	1	0	1	1	1	1	0	1	0	0
Session 3	0	1	1	1	0	0	0	1	1	0
Session 4	0	0	1	1	0	1	1	1	1	0
Session 5	1	1	1	1	0	0	0	0	1	0

1. Suppose that the number of sessions is five and  
K=4



Session #	3	7	9	12	23	65	85	248	346	562
Session 1	0	1	0	1	1	0	1	0	1	1
Session 2	1	0	1	1	1	1	0	1	0	0
Session 3	0	1	1	1	0	0	0	1	1	0
Session 4	0	0	1	1	0	1	1	1	1	0
Session 5	1	1	1	1	0	0	0	0	1	0

Similarity scores for Session 1 (K=4):

- Session 2: 0.8
- Session 3: 0.5477
- Session 4: 0.5477
- Session 5: 0.5477

2. Calculate the similarity between each session binary vector by Cosine similarity  
(If k=1, Only session 3 is considered when calculating item scores)

# Conventional SBRS

## K nearest Neighbour-based SBRSs

### Session-KNN Example (SKNN)

3. Calculate the recommendation score for each item i

Recommendation Score func:  $score_{SKNN}(i, s) = \sum_{n \in N_s} sim(s, n) \cdot 1_n(i)$

$N_s$  : Neighbor session, s: current session,  $1_n(i)$ : returns 1 if session n contains item i and 0 otherwise

Session #	3	7	9	12	23	65	85	248	346	562
Session 1	0	1	0	1	1	0	1	0	1	1
Session 2	1	0	1	1	1	1	0	1	0	0
Session 3	0	1	1	1	0	0	0	1	1	0
Session 4	0	0	1	1	0	1	1	1	1	0
Session 5	1	1	1	1	0	0	0	0	1	0

Session 5	[Item 346, Item 7, Item 9, Item 3, Item 12]
-----------	---

$$score(\text{item 23, session 5}) = 0.547 \cdot 0 + 0.547 \cdot 1 + 0.8 \cdot 1 + 0.547 \cdot 1 = 1.894$$

$$score(\text{item 65, session 5}) = 0.547 \cdot 1 + 0.547 \cdot 1 + 0.8 \cdot 0 + 0.547 \cdot 0 = 1.094$$

$$score(\text{item 85, session 5}) = 0.547 \cdot 0 + 0.547 \cdot 1 + 0.8 \cdot 0 + 0.547 \cdot 1 = 1.094$$

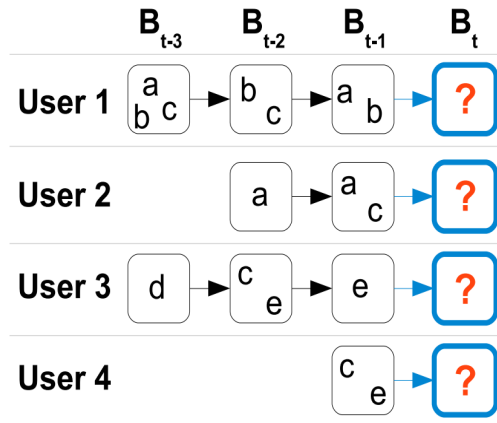
$$score(\text{item 248, session 5}) = 0.547 \cdot 0 + 0.547 \cdot 1 + 0.8 \cdot 1 + 0.547 \cdot 1 = 1.894$$

$$score(\text{item 562, session 5}) = 0.547 \cdot 1 + 0.547 \cdot 0 + 0.8 \cdot 0 + 0.547 \cdot 0 = 0.547$$

Next Recommendation: Item 23, Item 248

# Conventional SBRS

## Markov Chain



- $U = \{u_1, \dots, u_{|U|}\}$  : users
- $I = \{i_1, \dots, i_{|I|}\}$  : items
- $\mathcal{B}^u = (B_1^u, \dots, B_{t_u-1}^u)$  : historical baskets of user  $u$  with  $B_t^u \subseteq I$
- $\mathcal{B} = \{\mathcal{B}^{u_1}, \dots, \mathcal{B}^{u_{|U|}}\}$  : purchase history of all users

$$p(i \in B_t | B_{t-1}) := \frac{1}{|B_{t-1}|} \sum_{l \in B_{t-1}} p(i \in B_t | l \in B_{t-1})$$

### Transition Matrix

to	a	b	c	d	e	#
a	0.5	0.5	1	0	0	2
b	0.5	1	0.5	0	0	2
c	0.3	0.7	0.3	0	0.3	3
d	0	0	1	0	1	1
e	0	0	0	0	1	1

### Example

$$P(o_1 \rightarrow o_2 \rightarrow o_3) = P(o_1) * P(o_2 | o_1) * P(o_3 | o_2). \longrightarrow P(A \rightarrow B \rightarrow C) = P(A) * P(B | A) * P(C | B)$$

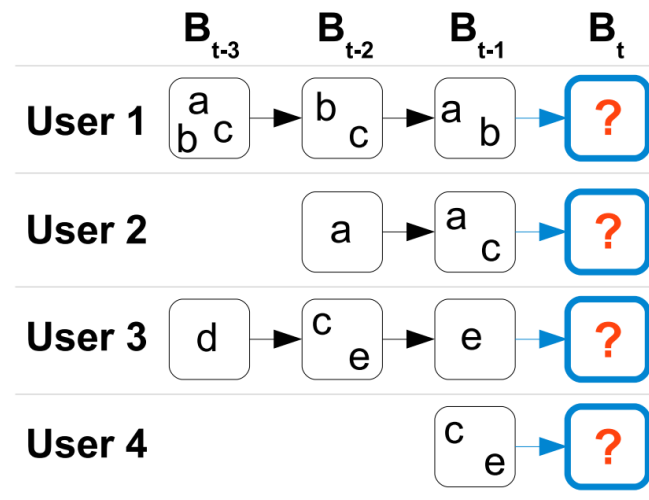
$$= \frac{4}{16} * 0.5 * 0.5 = 0.0625$$

# Latent Representation Approaches for SBRs

## Latent Representation Approach for SBRs

### Factorized Personalized Markov Chain (FPMC)

#### Problem of Markov Chain

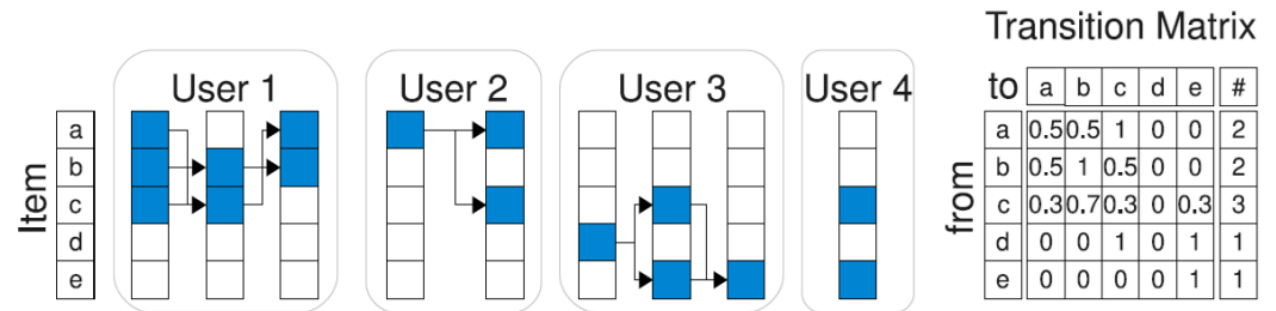


- $U = \{u_1, \dots, u_{|U|}\} : \text{users}$
- $I = \{i_1, \dots, i_{|I|}\} : \text{items}$
- $\mathcal{B}^u = (B_1^u, \dots, B_{t_u}^u) : \text{historical baskets of user } u \text{ with } B_t^u \subseteq I$
- $\mathcal{B} = \{\mathcal{B}^{u_1}, \dots, \mathcal{B}^{u_{|U|}}\} : \text{purchase history of all users}$

$$p(i \in B_t | B_{t-1}) := \frac{1}{|B_{t-1}|} \sum_{l \in B_{t-1}} p(i \in B_t | l \in B_{t-1}) = \frac{\hat{p}(i \in B_t \wedge l \in B_{t-1})}{\hat{p}(l \in B_t)}$$

$$= \frac{|(B_t, B_{t-1}) : i \in B_t \wedge l \in B_{t-1}|}{|(B_t, B_{t-1}) : l \in B_{t-1}|}$$

#### Markov Chain Predict Formula



[fig 04] Non-Personalized Problem in Markov Chain

# Latent Representation Approaches for SBRs

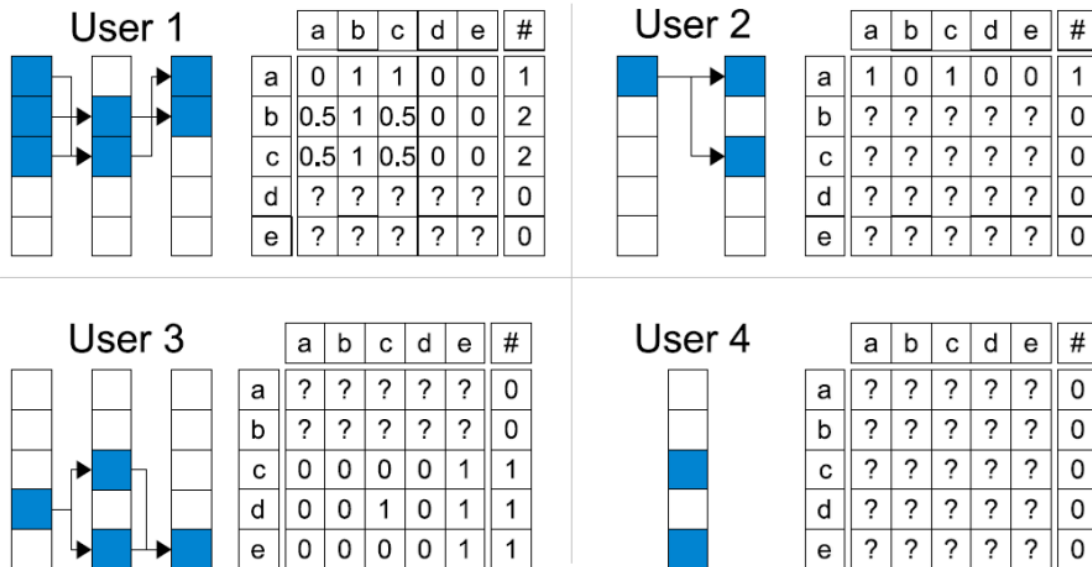
## Latent Representation Approach for SBRs

### Factorized Personalized Markov Chain (FPMC)

#### Problem of Markov Chain

$$p(i \in B_t | B_{t-1}) := \frac{1}{|B_{t-1}|} \sum_{l \in B_{t-1}} p(i \in B_t | l \in B_{t-1}) \longrightarrow p(i \in B_t^u | B_{t-1}^u) := \frac{1}{|B_{t-1}^u|} \sum_{l \in B_{t-1}^u} p(i \in B_t^u | l \in B_{t-1}^u)$$

change the non-personalized formula to **user-specific formula**, so prediction depends only on the users transitions.



[fig 05] **Personalized Markov Chains**

#### - Problems of personalized ML-Estimation:

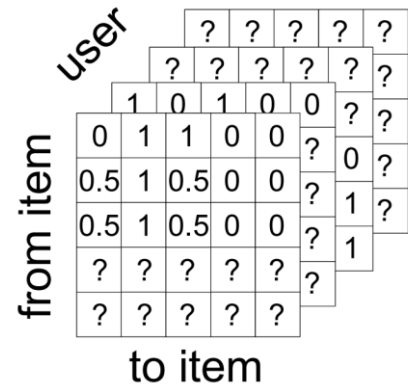
[1] Many of the parameters cannot be estimated.

[2] Theoretical properties are easily fail since the data is extremely sparse.

# Latent Representation Approaches for SBRs

## Latent Representation Approach for SBRs

### Factorized Personalized Markov Chain (FPMC)



"Personalized transition cube"

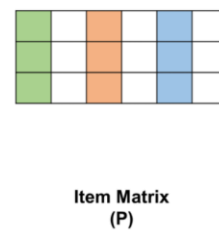


Diagram illustrating the User-item Interaction Matrix (R), showing a matrix where rows represent users and columns represent items. The matrix contains numerical values representing interactions.

	10	-1	8	10	9	4
	8	9	10	-1	-1	8
	10	5	4	9	-1	-1
	9	10	-1	-1	-1	3
	6	-1	-1	-1	8	10

User-item Interaction Matrix (R)

Idea (MF)



$$p(i \in B_t^u | B_{t-1}^u) := \frac{1}{|B_{t-1}^u|} \sum_{l \in B_{t-1}^u} p(i \in B_t^u | l \in B_{t-1}^u)$$

$$\langle v_u^{U,I}, v_i^{I,U} \rangle + \langle v_i^{I,L}, v_l^{L,I} \rangle + \langle v_u^{U,L}, v_l^{L,U} \rangle$$

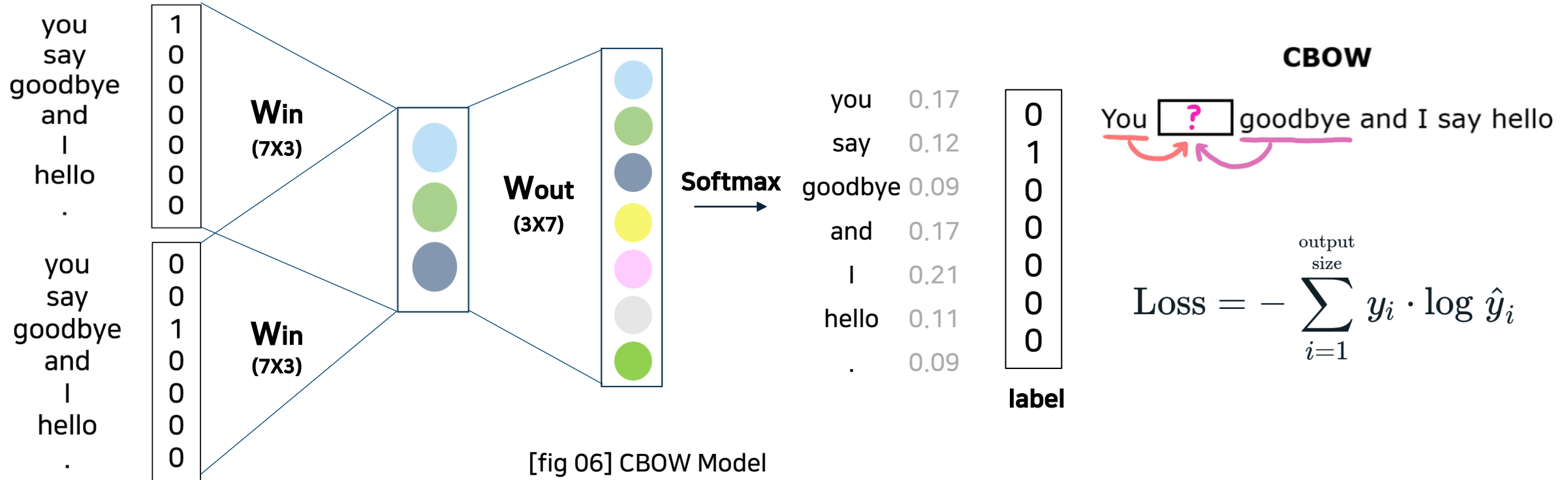
$$\hat{p}(i \in B_t^u | B_{t-1}^u) = \frac{1}{|B_{t-1}^u|} \sum_{l \in B_{t-1}^u} (\langle v_u^{U,I}, v_i^{I,U} \rangle + \langle v_i^{I,L}, v_l^{L,I} \rangle + \langle v_u^{U,L}, v_l^{L,U} \rangle)$$

FPMC = MF + MC

# Latent Representation Approaches for SBRs

## Distributed Representation-based SBRs

### Session-based Wide-In-Wide-Out (SWIWO)



- The SWIWO model is inspired by Skip-gram and CBOW model.
- Skip-gram and CBOW is the representative methods of Word2Vec in Natural Language Processing.

# Latent Representation Approaches for SBRs

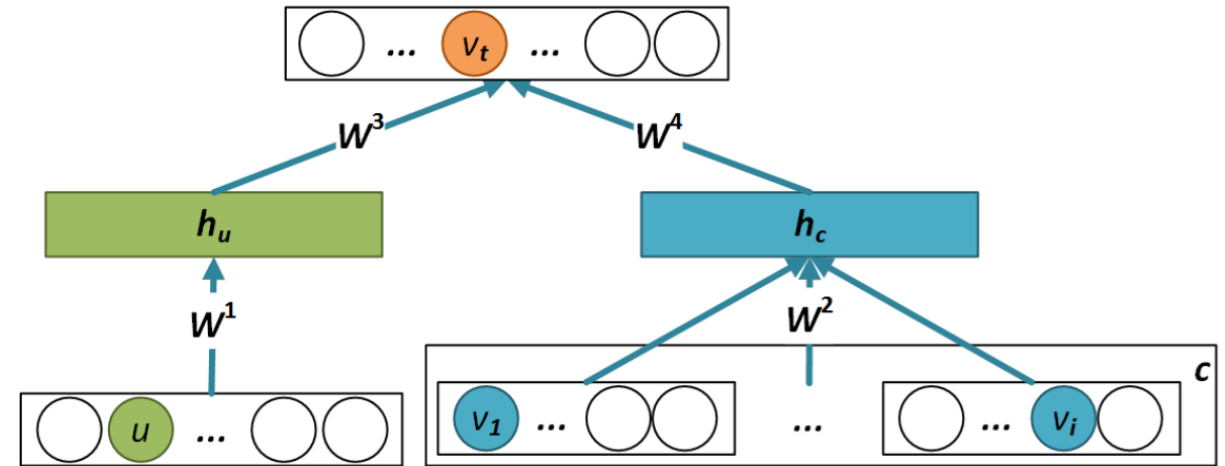
## Distributed Representation-based SBRs

### Session-based WIWO (SWIWO)

- MF and FPMC easily suffers from the data sparsity issue.
- SWIWO propose to diversify **personalized recommendation** results according to user-session contexts.
- SWIWO networks are constructed and trained as probabilistic classifiers that learn to predict a conditional probability distribution.

$$P(v_t | \mathbf{c}) \text{ where } \mathbf{c} \subseteq s$$

c: context s: session vt: item



[fig 07] SWIWO Architecture



# Latent Representation Approaches for SBRs

## Distributed Representation-based SBRs

### Session-based WIWO (SWIWO)

#### 1. Define Formula

$\mathbf{U} = \{u_1, u_2, \dots, u_{|\mathbf{U}|}\}$  user set

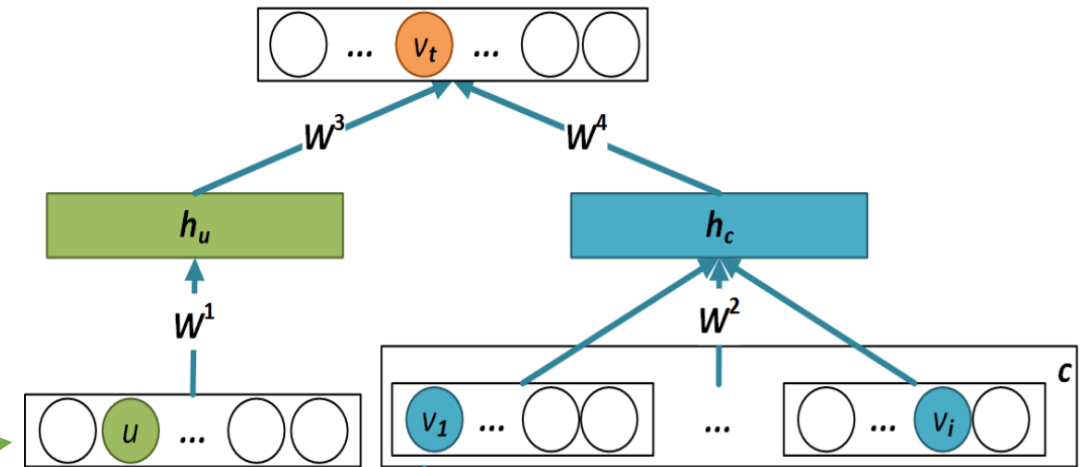
$\mathbf{V} = \{v_1, v_2, \dots, v_{|\mathbf{V}|}\}$  item set

$\mathbf{S} = \{s_1, s_2, \dots, s_{|\mathbf{S}|}\}$  session set

#### 2. Create an embedding layer to map those sparse one-hot user and item vectors

e.g.,  $u_i$ :  $|\mathbf{U}| \times 1$  dimension one-hot vector

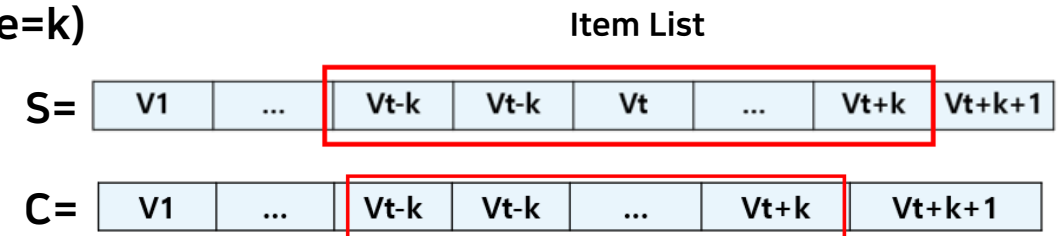
$v_i$ :  $|\mathbf{V}| \times 1$  dimension one-hot vector



#### 3. Define context window size (e.g., predict $V_t$ score & window size= $k$ )

$s = \{v_{t-k}, \dots, v_{t+k}\}$  session

$\mathbf{C} = \{v_{t-k}, \dots, v_{t-1}, v_{t+1}, \dots, v_{t+k}\}$  context



# Latent Representation Approaches for SBRs

## Distributed Representation-based SBRs

### Session-based WIWO (SWIWO)

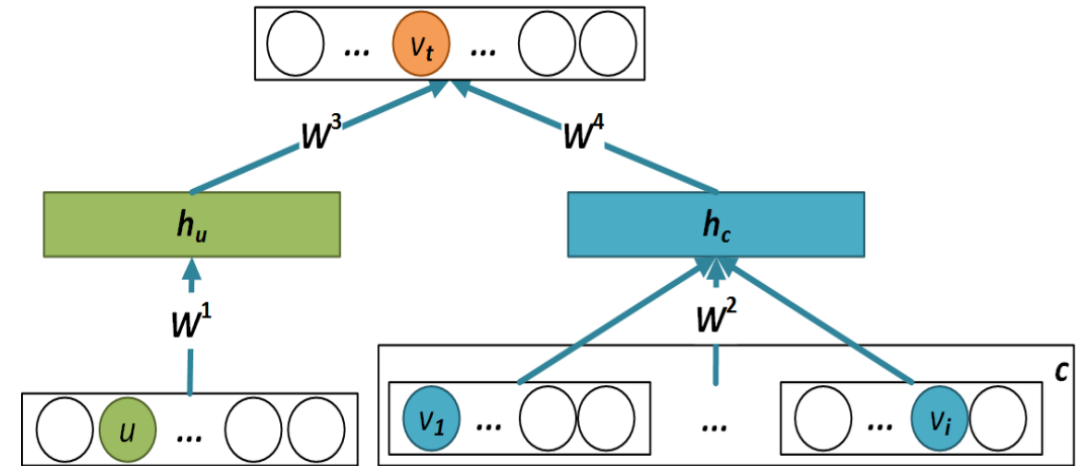
#### 4. Hidden Layer

$$\mathbf{h}_u = \sigma(\mathbf{W}_{:,u}^1)$$

$$\mathbf{h}_v = \sigma(\mathbf{W}_{:,v}^2)$$

$$\mathbf{h}_c = \sum_{v \in \mathbf{c}} w_v \mathbf{h}_v \equiv \sum_{v \in \mathbf{c}} w_v \sigma(\mathbf{W}_{:,v}^2)$$

$$w_v \propto \exp[-\lambda(|v - t| - 1)]$$



#### 5. Compute vt score

$$S_{v_t}(u, \mathbf{c}) = \mathbf{W}_{t,:}^3 \mathbf{h}_u + \mathbf{W}_{t,:}^4 \mathbf{h}_c$$

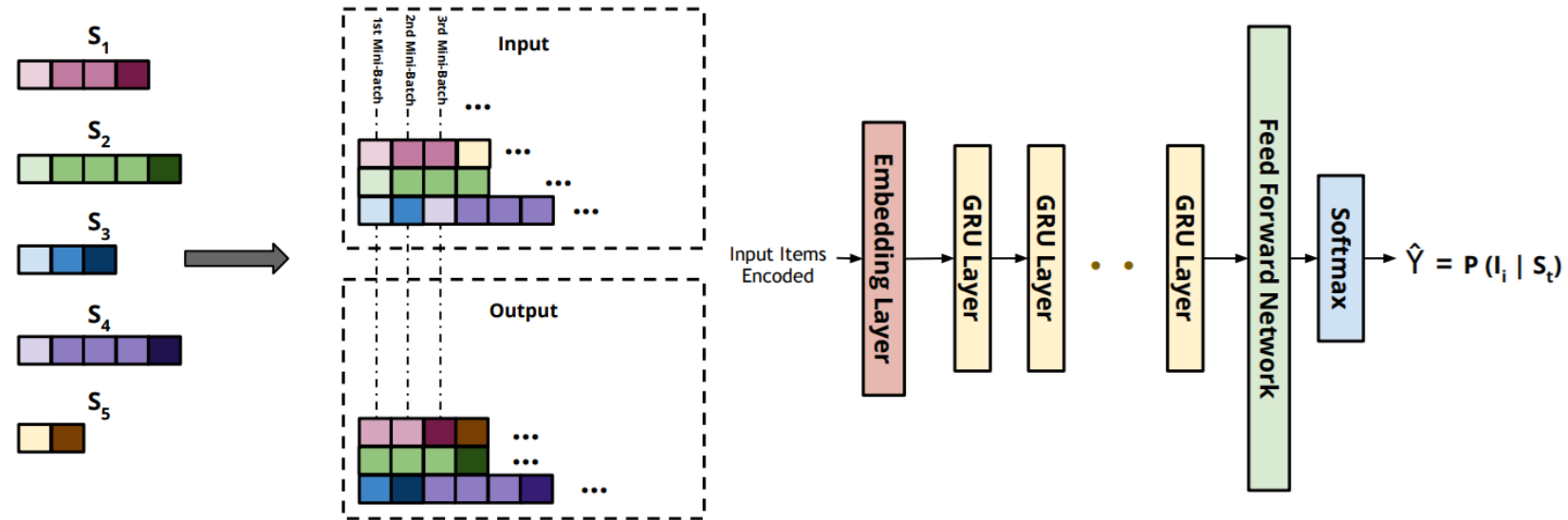
#### 6. Compute the conditional distribution with softmax function

$$P_{\Theta}(v_t | u, \mathbf{c}) = \frac{\exp(S_{v_t}(u, \mathbf{c}))}{Z(u, \mathbf{c})} \quad Z(u, \mathbf{c}) = \sum_{v \in \mathbf{V}} \exp(S_{v_t}(u, \mathbf{c}))$$

# Deep Neural Network Approaches for SBRs

## Recurrent Neural Networks

### GRU4Rec - Architecture



[fig 08] GRU4Rec Architecture

- Input: actual state of the session (item) → Embedding: one-hot encoding (length: the number of items)
- Output: score on items for being the next in the event stream
- GRU based RNN (RNN is worse, LSTM is slower)

# Deep Neural Network Approaches for SBRs

## Recurrent Neural Networks

### GRU4Rec - Minibatch

- **Motivation**

High variance in the length of sessions (from 2  
100s of events)

The goal is to capture how sessions evolve

- **Minibatch**

Input: current events

Output: next events

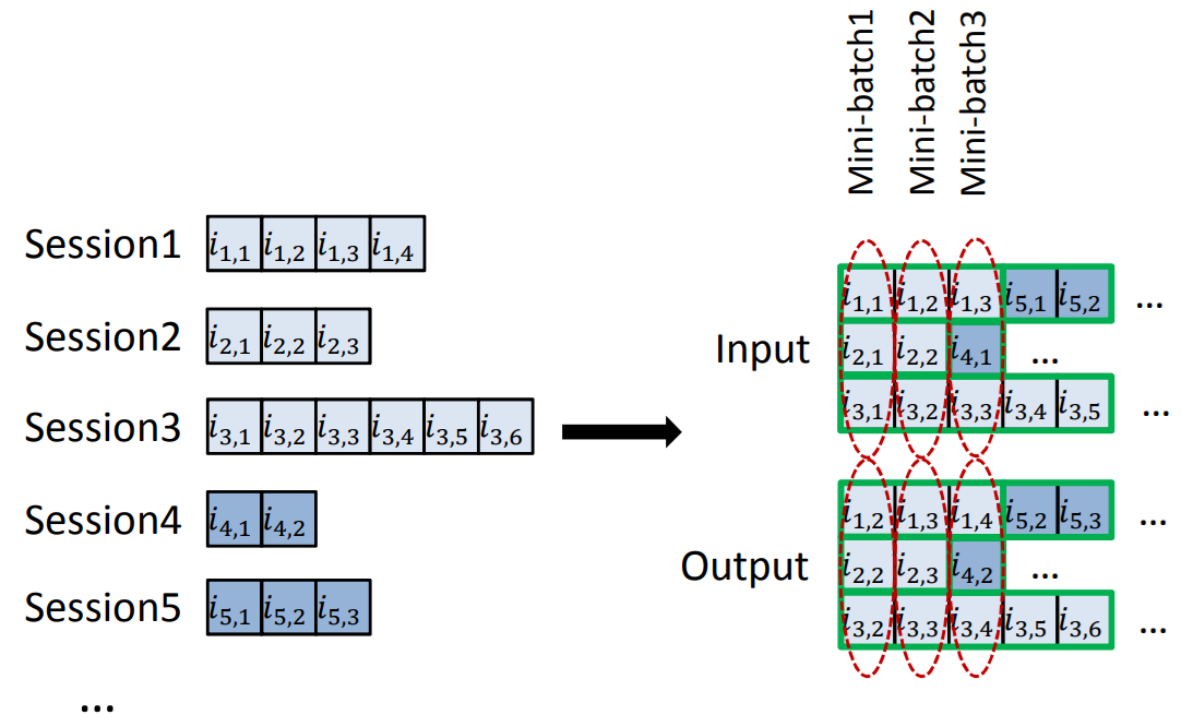
- **Active sessions**

First X

Finished sessions replaced by the next available

Sessions are assumed to be independent, thus reset

the appropriate hidden state when session switch occurs



[fig 09] GRU4Rec Minibatch

# Deep Neural Network Approaches for SBRs

## Recurrent Neural Networks

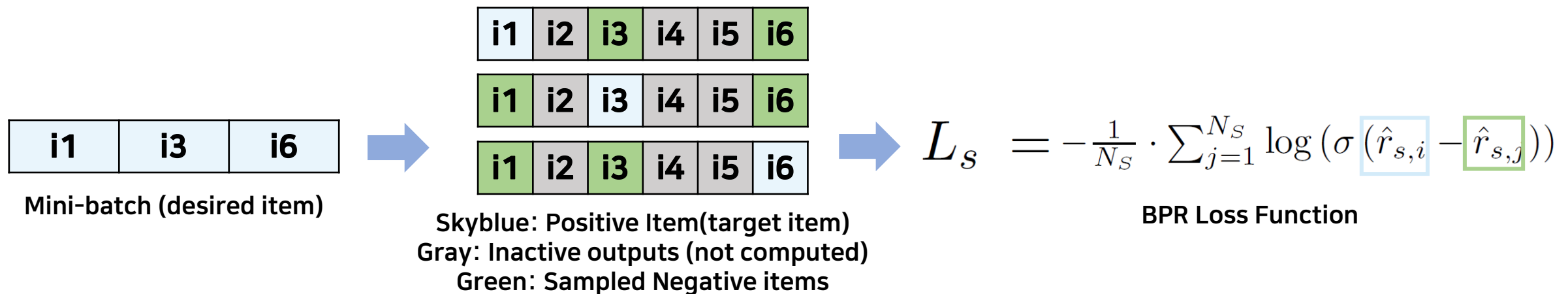
### GRU4Rec – Sampling the output Example

- **Motivation**

Calculate a score for each item is very costly → effective negative sampling

- **Sampling negative items**

Assume that the user didn't interact some items because the user didn't like it → Consider this popularity when sampling → It can further reduce computational times by skipping the sampling.

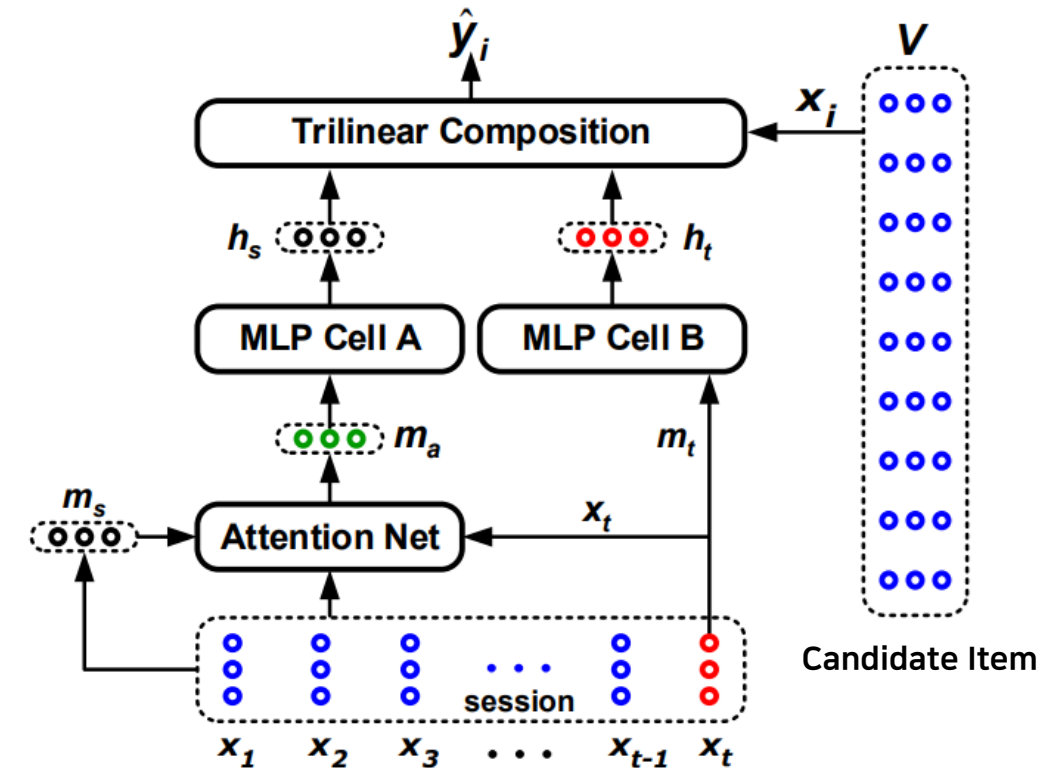


# Deep Neural Network Approaches for SBRs

## Attention + MLP

### STAMP (Short-Term Attention/Memory Priority Model)

- STAMP model captures not only user's **general interests** from the **long-term memory** of a session context, but also taking into account user's **current interests** from the **short-term memory** of the last-clicks.
- Model architecture is simple, but achieves state-of-the-art performance for three benchmark datasets.



[fig 10] STAMP Architecture

# Deep Neural Network Approaches for SBRs

## Attention + MLP

### STAMP (Short-Term Attention/Memory Priority Model)

1. External memory of the session ( $m_s$ ): the user's interests in general with respect to current session

$$\mathbf{m}_s = \frac{1}{t} \sum_{i=1}^t \mathbf{x}_i$$

2. Attention composite function (ai)

$$\alpha_i = \mathbf{W}_0 \sigma(\mathbf{W}_1 \mathbf{x}_i + \mathbf{W}_2 \mathbf{x}_t + \mathbf{W}_3 \mathbf{m}_s + \mathbf{b}_a)$$

Obtain the attention coefficients vector

$$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_t)$$

3.  $m_a$ : Attention based user's interests

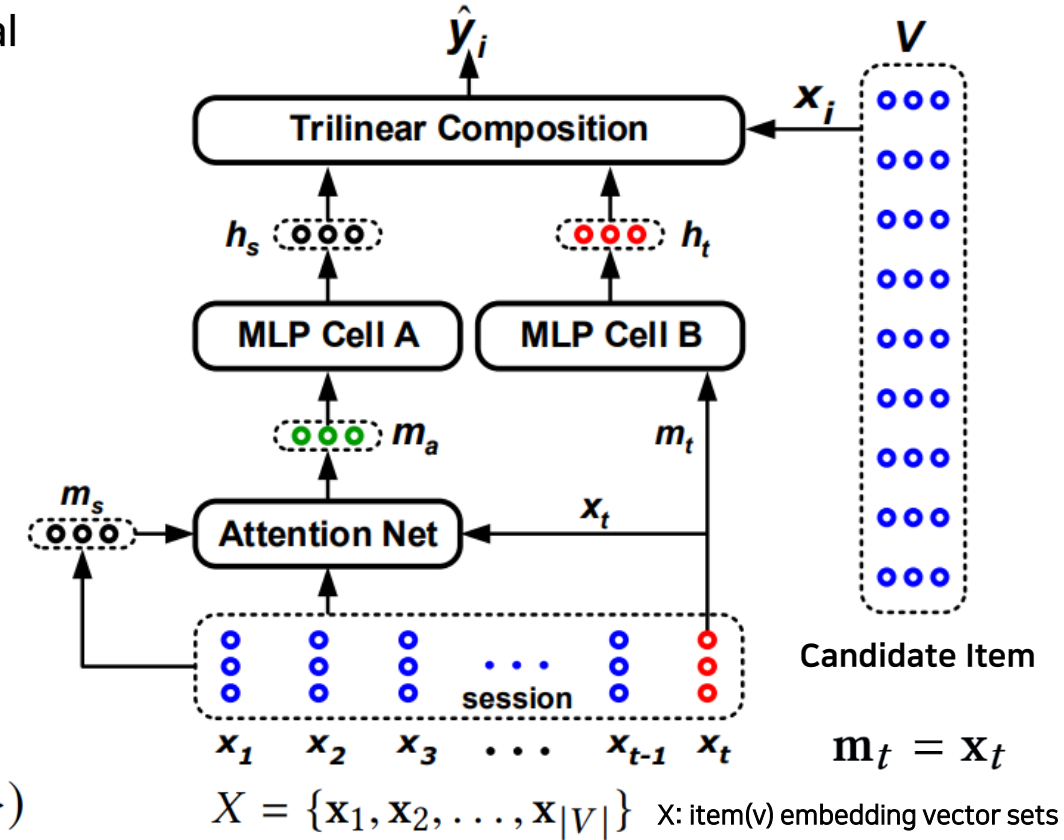
$$\mathbf{m}_a = \sum_{i=1}^t \alpha_i \mathbf{x}_i$$

4. Hidden state & unnormalized cosine similarity

$$\mathbf{h}_s = f(\mathbf{W}_s \mathbf{m}_a + \mathbf{b}_s) \quad \mathbf{h}_t = f(\mathbf{W}_t \mathbf{x}_t + \mathbf{b}_t) \quad \hat{\mathbf{z}}_i = \sigma(\langle \mathbf{h}_s, \mathbf{h}_t, \mathbf{x}_i \rangle)$$

5. output (next-click probability for item  $v_i$ )

$$\hat{\mathbf{y}} = \text{softmax}(\hat{\mathbf{z}})$$

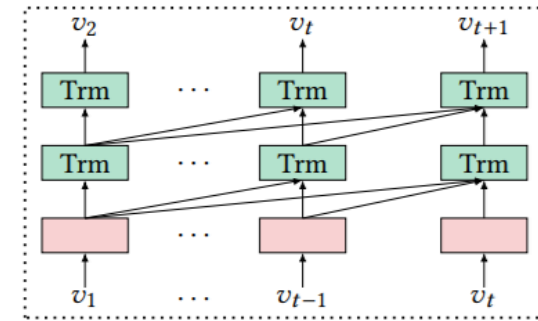
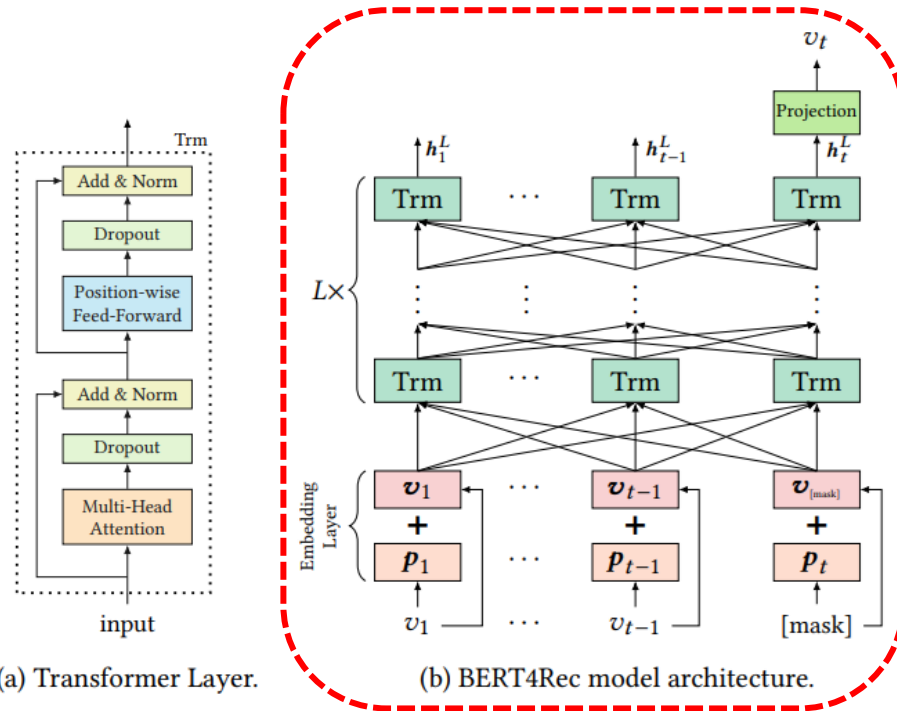


[fig ] STAMP Architecture

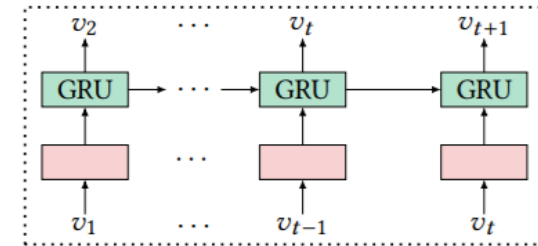
# Deep Neural Network Approaches for SBRs

BERT (self-attention)

BERT4REC



(c) SASRec model architecture.



(d) RNN based sequential recommendation methods.



# Deep Neural Network Approaches for SBRs

## BERT (self-attention)

### BERT4REC Example

#### 1. Embedding layer $h_i^0 = v_i + p_i$

- Example : user 01 sequential items = {i3,i9,i12,i15,i18}

embedding window = W (hyperparameter)

Item list length (N) > W: recently purchase N

Item list length (N) < W : zero-padding

ex) if W=4, i3,i9,i12,i15,i18

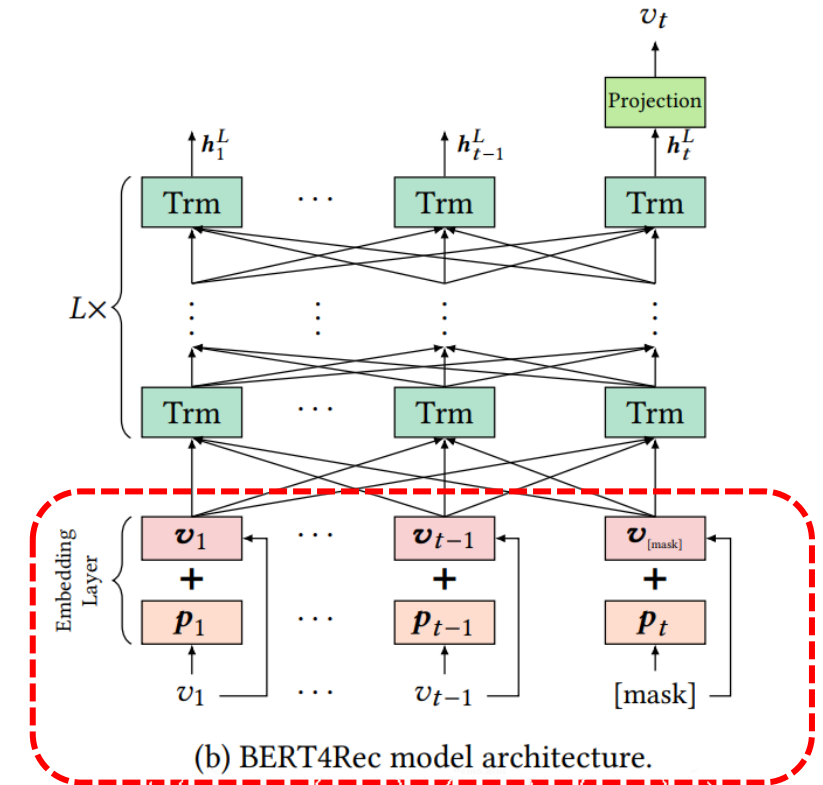
if W=6 0,i3,i9,i12,i15,i18 zero-padding

- embedding = item embedding + positional embedding

Item embedding

i3	i9	i12	i15	i18
0	1	2	3	4

Positional embedding



$$p_{i,j} = \begin{cases} \sin\left(\frac{i}{10000^{\frac{j}{d_{emb-dim}}}}\right) & \text{if } j \text{ is even} \\ \cos\left(\frac{i}{10000^{\frac{j-1}{d_{emb-dim}}}}\right) & \text{if } j \text{ is odd} \end{cases}$$

Cf) Transformer positional encoding

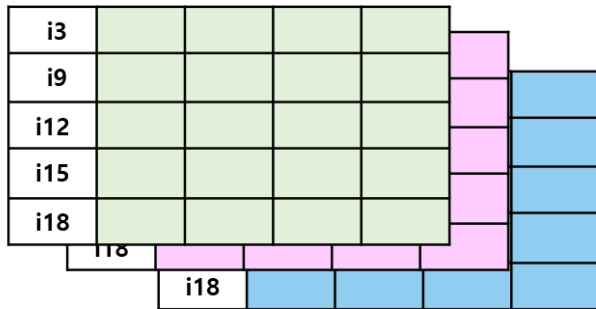
# Deep Neural Network Approaches for SBRs

## BERT (self-attention)

### BERT4REC Example

#### 2. Trm Layer := Transformer encoder

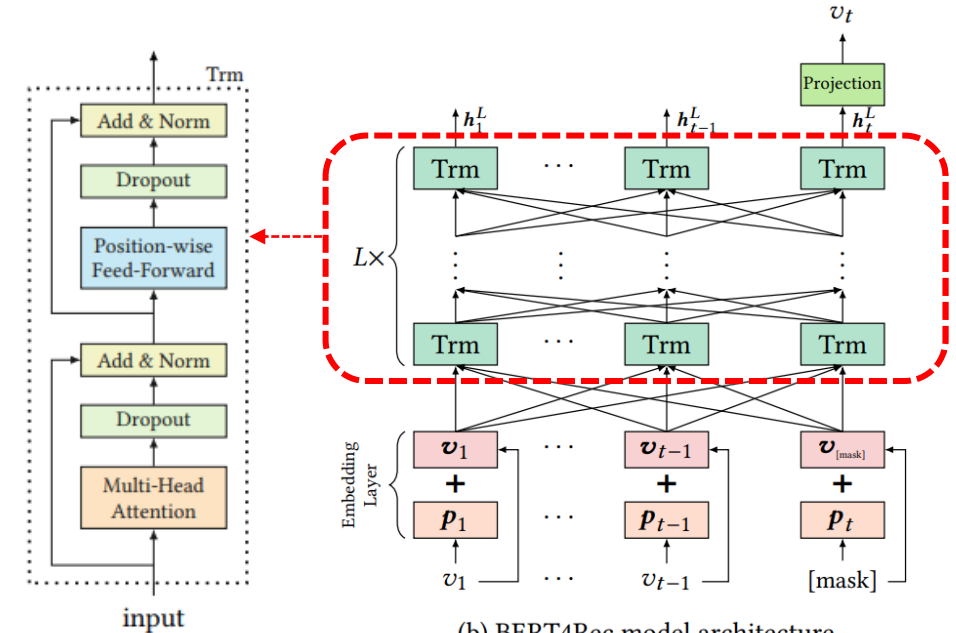
Trm layer is the same step as the transformer encoder's Multi-head attention process.



Q K V matrix (embedding\*W<sub>q</sub>, embedding\*W<sub>k</sub>, embedding\*W<sub>v</sub>)

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d/h}}\right)V$$
$$\text{PFFN}(H^l) = [\text{FFN}(h_1^l)^T; \dots; \text{FFN}(h_t^l)^T]^T$$
$$\text{FFN}(x) = \text{GELU}(xW^{(1)} + b^{(1)})W^{(2)} + b^{(2)}$$
$$\text{GELU}(x) = x\Phi(x)$$

Difference from Transformer: Activation Function → GELU



(b) BERT4Rec model architecture.

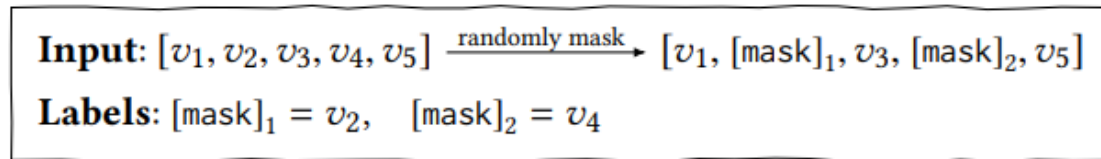
# Deep Neural Network Approaches for SBRs

## BERT (self-attention)

### BERT4REC Example

#### 3. Model Train

Masks are randomly placed on the item by a percentage (Hyperparameter).



{i3,i9,i12,i15,i18}

The process of learning the [mask] item is sequentially performed.(mask 01 → mask 02)

i3				
<ma sk1>				
i2				
<ma sk2>				
i8				

<ma sk1>				
-------------	--	--	--	--

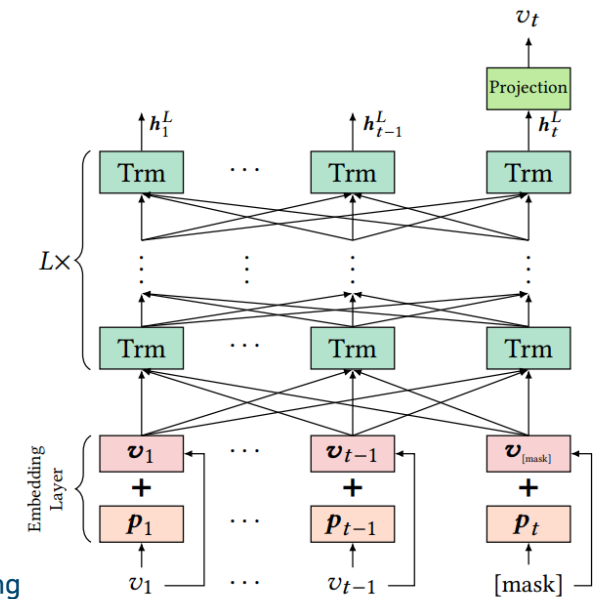


i9				
i8				
i13				
i17				
i20				
i22				

Growth truth  
Score high

Negative Sampling  
Score low

Random sampling among the top 100 popularity.



(b) BERT4Rec model architecture.

Loss function - masked target negative log likelihood

$$\mathcal{L} = \frac{1}{|S_u^m|} \sum_{v_m \in S_u^m} -\log P(v_m = v_m^* | \hat{S}_u)$$

$\hat{S}_u$  User behavior history's masked version  
 $v_m^*$  Masked item  $v_m$  true item

# Deep Neural Network Approaches for SBRs

## BERT (self-attention)

### BERT4REC Example

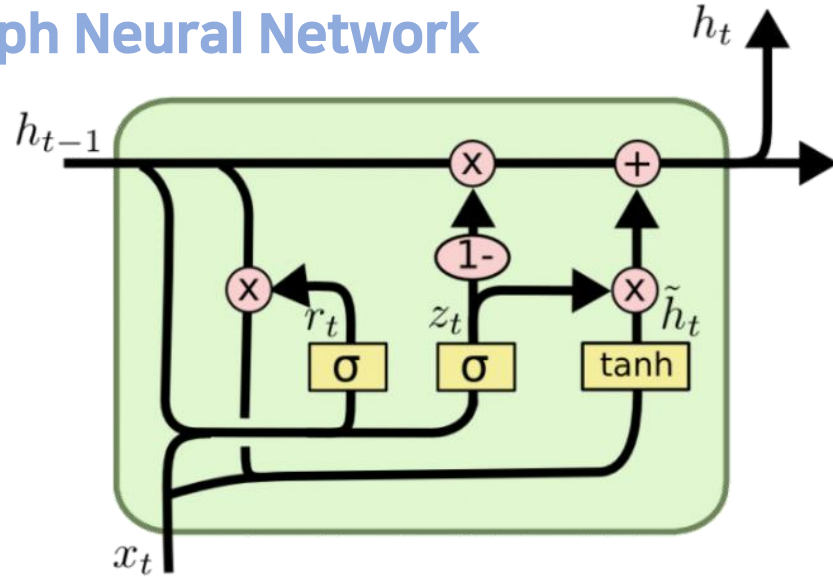
#### Experiments

Datasets	Metric	POP	BPR-MF	NCF	FPMC	GRU4Rec	GRU4Rec <sup>+</sup>	Caser	SASRec	BERT4Rec	Improv.
Beauty	HR@1	0.0077	0.0415	0.0407	0.0435	0.0402	0.0551	0.0475	<u>0.0906</u>	<b>0.0953</b>	5.19%
	HR@5	0.0392	0.1209	0.1305	0.1387	0.1315	0.1781	0.1625	<u>0.1934</u>	<b>0.2207</b>	14.12%
	HR@10	0.0762	0.1992	0.2142	0.2401	0.2343	0.2654	0.2590	<u>0.2653</u>	<b>0.3025</b>	14.02%
	NDCG@5	0.0230	0.0814	0.0855	0.0902	0.0812	0.1172	0.1050	<u>0.1436</u>	<b>0.1599</b>	11.35%
	NDCG@10	0.0349	0.1064	0.1124	0.1211	0.1074	0.1453	0.1360	<u>0.1633</u>	<b>0.1862</b>	14.02%
	MRR	0.0437	0.1006	0.1043	0.1056	0.1023	0.1299	0.1205	<u>0.1536</u>	<b>0.1701</b>	10.74%
Steam	HR@1	0.0159	0.0314	0.0246	0.0358	0.0574	0.0812	0.0495	<u>0.0885</u>	<b>0.0957</b>	8.14%
	HR@5	0.0805	0.1177	0.1203	0.1517	0.2171	0.2391	0.1766	<u>0.2559</u>	<b>0.2710</b>	5.90%
	HR@10	0.1389	0.1993	0.2169	0.2551	0.3313	0.3594	0.2870	<u>0.3783</u>	<b>0.4013</b>	6.08%
	NDCG@5	0.0477	0.0744	0.0717	0.0945	0.1370	0.1613	0.1131	<u>0.1727</u>	<b>0.1842</b>	6.66%
	NDCG@10	0.0665	0.1005	0.1026	0.1283	0.1802	0.2053	0.1484	<u>0.2147</u>	<b>0.2261</b>	5.31%
	MRR	0.0669	0.0942	0.0932	0.1139	0.1420	0.1757	0.1305	<u>0.1874</u>	<b>0.1949</b>	4.00%
ML-1m	HR@1	0.0141	0.0914	0.0397	0.1386	0.1583	0.2092	0.2194	<u>0.2351</u>	<b>0.2863</b>	21.78%
	HR@5	0.0715	0.2866	0.1932	0.4297	0.4673	0.5103	0.5353	<u>0.5434</u>	<b>0.5876</b>	8.13%
	HR@10	0.1358	0.4301	0.3477	0.5946	0.6207	0.6351	<u>0.6692</u>	0.6629	<b>0.6970</b>	4.15%
	NDCG@5	0.0416	0.1903	0.1146	0.2885	0.3196	0.3705	0.3832	<u>0.3980</u>	<b>0.4454</b>	11.91%
	NDCG@10	0.0621	0.2365	0.1640	0.3439	0.3627	0.4064	0.4268	<u>0.4368</u>	<b>0.4818</b>	10.32%
	MRR	0.0627	0.2009	0.1358	0.2891	0.3041	0.3462	0.3648	<u>0.3790</u>	<b>0.4254</b>	12.24%
ML-20m	HR@1	0.0221	0.0553	0.0231	0.1079	0.1459	0.2021	0.1232	<u>0.2544</u>	<b>0.3440</b>	35.22%
	HR@5	0.0805	0.2128	0.1358	0.3601	0.4657	0.5118	0.3804	<u>0.5727</u>	<b>0.6323</b>	10.41%
	HR@10	0.1378	0.3538	0.2922	0.5201	0.5844	0.6524	0.5427	<u>0.7136</u>	<b>0.7473</b>	4.72%
	NDCG@5	0.0511	0.1332	0.0771	0.2239	0.3090	0.3630	0.2538	<u>0.4208</u>	<b>0.4967</b>	18.04%
	NDCG@10	0.0695	0.1786	0.1271	0.2895	0.3637	0.4087	0.3062	<u>0.4665</u>	<b>0.5340</b>	14.47%
	MRR	0.0709	0.1503	0.1072	0.2273	0.2967	0.3476	0.2529	<u>0.4026</u>	<b>0.4785</b>	18.85%

# Deep Neural Network Approaches for SBRs

## Graph Neural Networks

### Gated Graph Neural Network

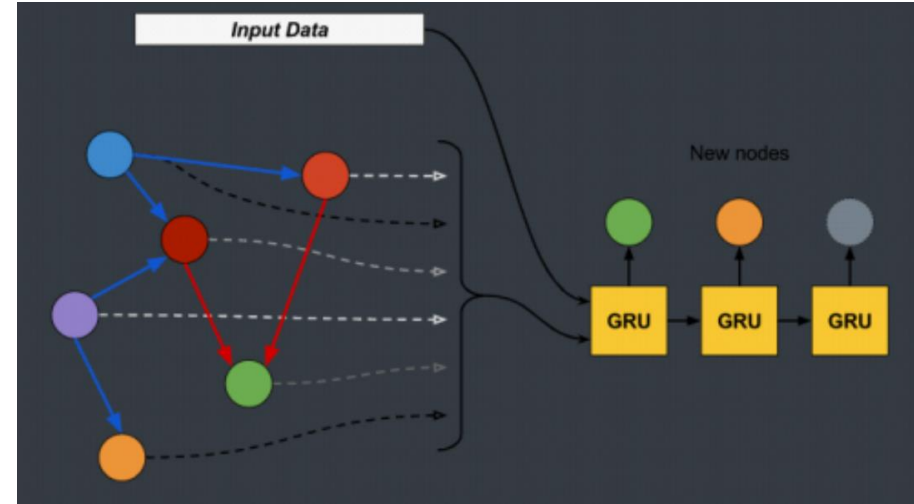


Reset gate  $r^{(t)} = \sigma \left( W_r h^{(t-1)} + U_r x^{(t)} \right)$

Update gate  $u^{(t)} = \sigma \left( W_u h^{(t-1)} + U_u x^{(t)} \right)$

Candidate gate  $\tilde{h}^{(t)} = \tau \left( W h^{(t-1)} * r^{(t)} + U x^{(t)} \right)$

$$h^{(t)} = (1 - u^{(t)}) * h^{(t-1)} + u^{(t)} * \tilde{h}^{(t)}$$

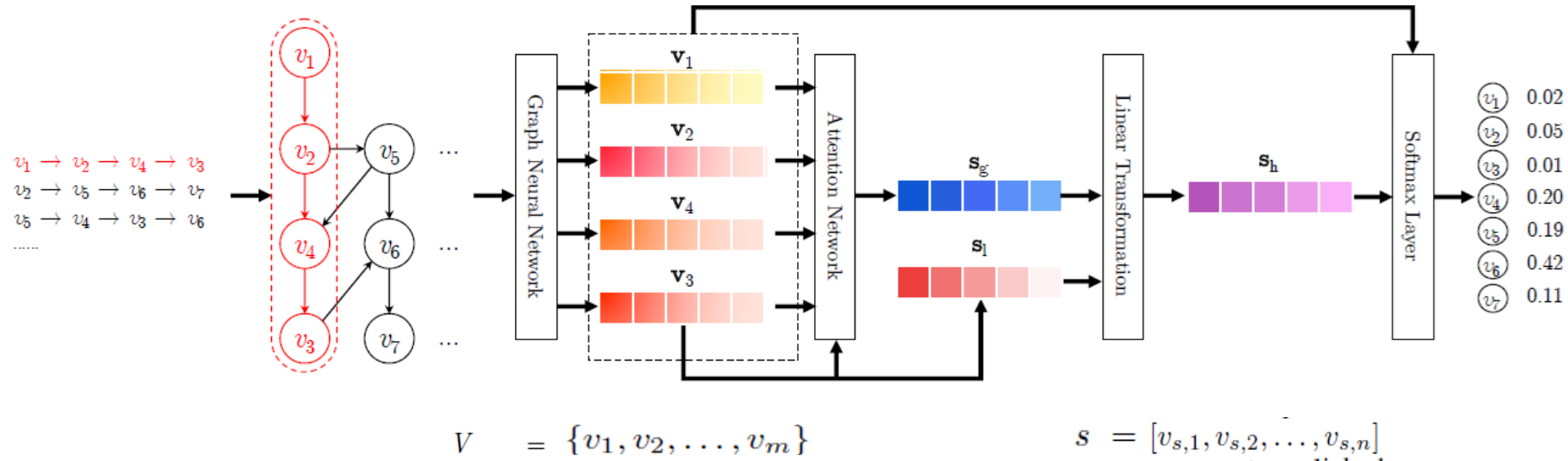


$$\begin{aligned} \mathbf{a}^{(t)} &= \mathbf{A} \mathbf{h}^{(t-1)} + \mathbf{b} \\ \mathbf{r}_v^t &= \sigma \left( \mathbf{W}^r \mathbf{a}_v^{(t)} + \mathbf{U}^r \mathbf{h}_v^{(t-1)} \right) \\ \mathbf{z}_v^t &= \sigma \left( \mathbf{W}^z \mathbf{a}_v^{(t)} + \mathbf{U}^z \mathbf{h}_v^{(t-1)} \right) \\ \widetilde{\mathbf{h}}_v^{(t)} &= \tanh \left( \mathbf{W} \mathbf{a}_v^{(t)} + \mathbf{U} \left( \mathbf{r}_v^t \odot \mathbf{h}_v^{(t-1)} \right) \right) \\ \mathbf{h}_v^{(t)} &= (1 - \mathbf{z}_v^t) \odot \mathbf{h}_v^{(t-1)} + \mathbf{z}_v^t \odot \widetilde{\mathbf{h}}_v^{(t)} \end{aligned}$$

# Deep Neural Network Approaches for SBRs

## Graph Neural Networks

### SR-GNN



[fig 11] SR-GNN Architecture

- SR-GNN aims to clarify the representation of the items by grasping many possible relationships between items through GNN.
- Directed graphs are constructed from past sequences (each sequence can be referred to as a subgraph)

# Deep Neural Network Approaches for SBRs

## Graph Neural Networks

### SR-GNN

#### 1. Graph Neural Network Layer → Gated Graph Neural Network

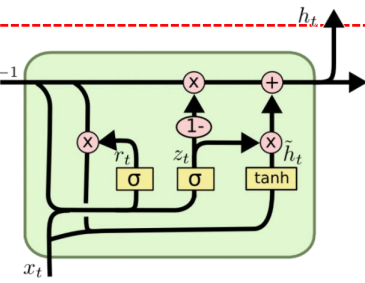
list of the node vectors in session  $s$

$$\mathbf{a}_{s,i}^t = \mathbf{A}_{s,i} [\mathbf{v}_1^{t-1}, \dots, \mathbf{v}_n^{t-1}]^\top \mathbf{H} + \mathbf{b},$$

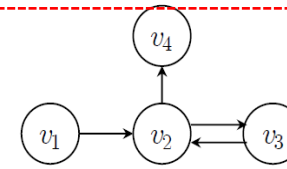
$$\mathbf{z}_{s,i}^t = \sigma(\mathbf{W}_z \mathbf{a}_{s,i}^t + \mathbf{U}_z \mathbf{v}_i^{t-1}), \text{ Update gate}$$

$$\mathbf{r}_{s,i}^t = \sigma(\mathbf{W}_r \mathbf{a}_{s,i}^t + \mathbf{U}_r \mathbf{v}_i^{t-1}), \text{ reset gate}$$

$$\tilde{\mathbf{v}}_i^t = \tanh(\mathbf{W}_o \mathbf{a}_{s,i}^t + \mathbf{U}_o (\mathbf{r}_{s,i}^t \odot \mathbf{v}_i^{t-1})),$$

$$\mathbf{v}_i^t = (1 - \mathbf{z}_{s,i}^t) \odot \mathbf{v}_i^{t-1} + \mathbf{z}_{s,i}^t \odot \tilde{\mathbf{v}}_i^t,$$


$$S = [v_1, v_2, v_3, v_2, v_4]$$

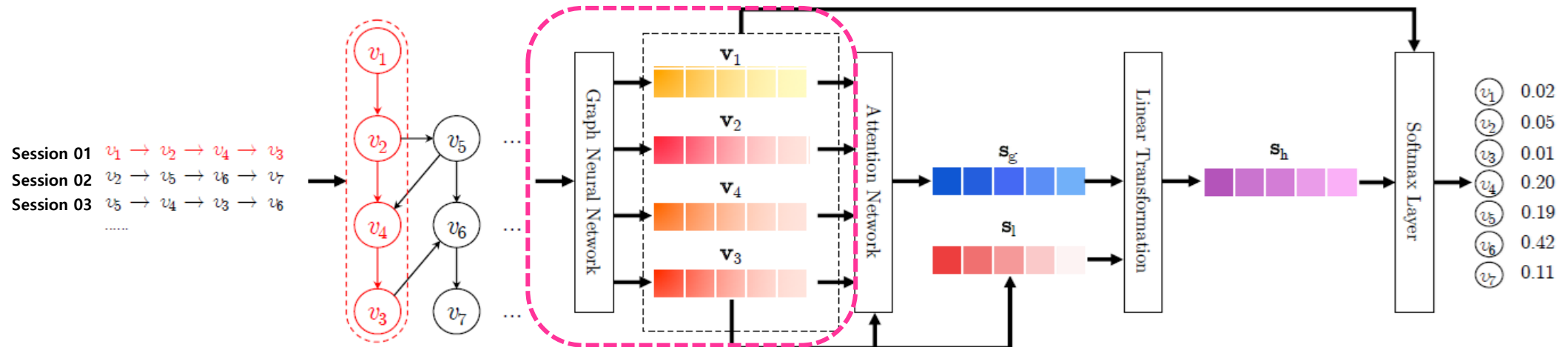


Idea

	Outgoing edges				Incoming edges			
	1	2	3	4	1	2	3	4
1	0	1	0	0	0	0	0	0
2	0	0	1/2	1/2	1/2	0	1/2	0
3	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	0	0

matrix  $\mathbf{A}_s$

[Connected Matrix]





# Deep Neural Network Approaches for SBRs

## Graph Neural Networks

### SR-GNN

#### 2. Generating Session Embeddings: global session embedding( $S_g$ ) & local session embedding ( $S_l$ )

##### - Global session embedding ( $S_g$ )

Aggregate nodes  $\rightarrow$  Soft-Attention Mechanism

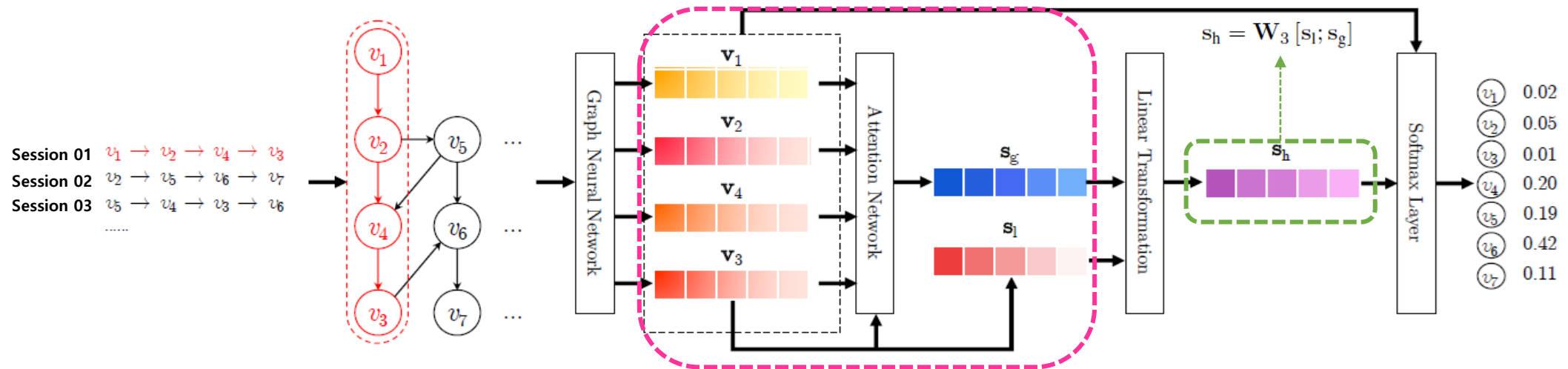
\* Soft-Attention Mechanism

$$\alpha_i = \mathbf{q}^\top \sigma(\mathbf{W}_1 \mathbf{v}_n + \mathbf{W}_2 \mathbf{v}_i + \mathbf{c}),$$
$$\mathbf{s}_g = \sum_{i=1}^n \alpha_i \mathbf{v}_i,$$

##### - Local session embedding( $S_l$ )

The last item( $v_s$ ) user clicked embedding vector  $\mathbf{s}_l = \mathbf{v}_n$

#### 3. $s_g, s_l$ concatenate = $S_h$





# Deep Neural Network Approaches for SBRs

## Graph Neural Networks

### SR-GNN

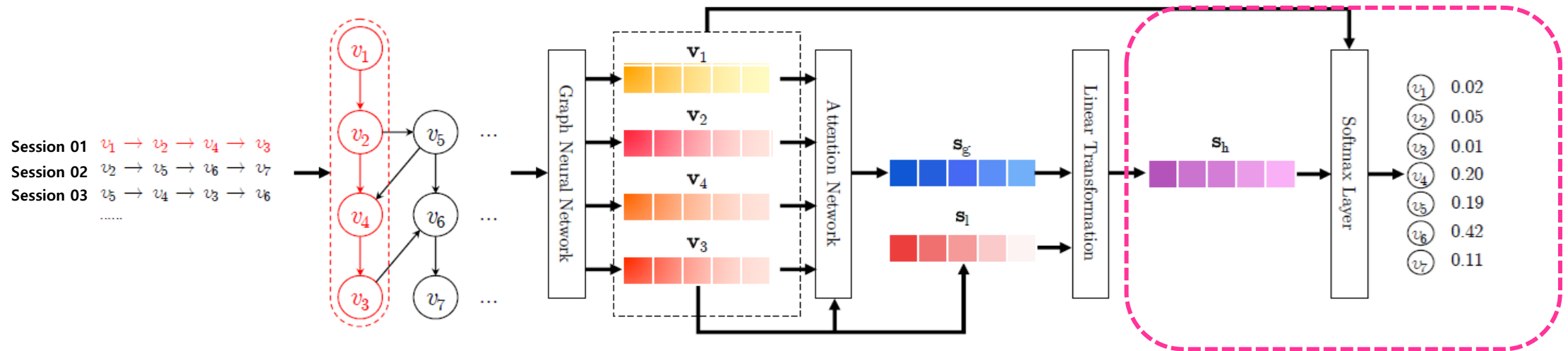
#### 4. Prediction Next Item

$$\hat{\mathbf{z}}_i = \mathbf{s}_h^\top \mathbf{v}_i \dashrightarrow \hat{y} = \text{softmax}(\hat{\mathbf{z}})$$

#### \* Loss Function: Cross-Entropy Loss

$$\mathcal{L}(\hat{y}) = - \sum_{i=1}^m y_i \log(\hat{y}_i) + (1 - y_i) \log(1 - \hat{y}_i)$$

$y$ : denotes the one-hot encoding vector of the ground truth item



# References

- [1] Wang, Shoujin, et al. "A survey on session-based recommender systems." *ACM Computing Surveys (CSUR)* 54.7 (2021): 1-38.
- [2] Ludewig, Malte, and Dietmar Jannach. "Evaluation of session-based recommendation algorithms." *User Modeling and User-Adapted Interaction* 28.4 (2018): 331-390.
- [3] Rendle, Steffen, Christoph Freudenthaler, and Lars Schmidt-Thieme. "Factorizing personalized markov chains for next-basket recommendation." *Proceedings of the 19th international conference on World wide web*. 2010.
- [4] Hu, Liang, et al. "Diversifying Personalized Recommendation with User-session Context." *IJCAI*. 2017.
- [5] Hidasi, Balázs, et al. "Session-based recommendations with recurrent neural networks." *arXiv preprint arXiv:1511.06939* (2015).
- [6] Liu, Qiao, et al. "STAMP: short-term attention/memory priority model for session-based recommendation." *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*. 2018.
- [7] Li, Yujia, et al. "Gated graph sequence neural networks." *arXiv preprint arXiv:1511.05493* (2015).
- [8] Sun, Fei, et al. "BERT4Rec: Sequential recommendation with bidirectional encoder representations from transformer." *Proceedings of the 28th ACM international conference on information and knowledge management*. 2019.
- [9] Wu, Shu, et al. "Session-based recommendation with graph neural networks." *Proceedings of the AAAI conference on artificial intelligence*. Vol. 33. No. 01. 2019.