

Problem 1

In classic geometry, a rotation is characterized as a length-preserving and orientation-preserving linear transformation. Starting from this characterization, it can also be shown that rotations preserve angles between vectors, which is a fact that you should use to solve the following problem.

The angle φ between two vectors \vec{x}, \vec{y} in a standard euclidean space (with standard scalar product) can be defined through the relation:

$$\cos \varphi = \frac{\vec{x}^T \vec{y}}{|\vec{x}| \cdot |\vec{y}|}$$

Using this definition, show that any rotation matrix R is orthonormal, or $R \cdot R^T = I_3$.

Problem 2

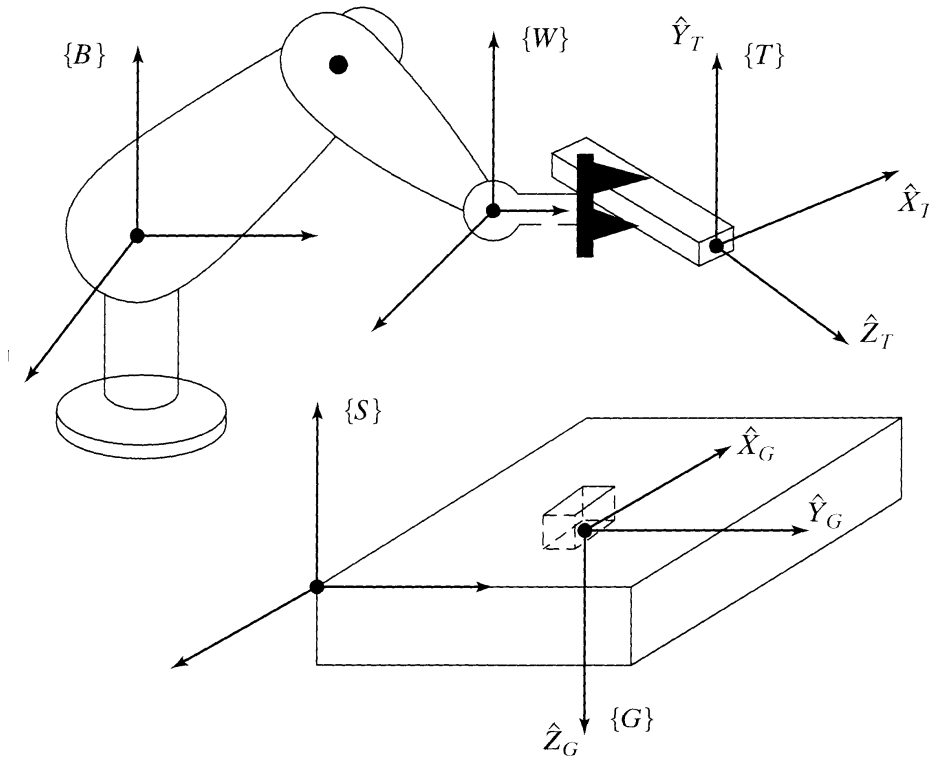


Figure 1: *Coordinate systems (Problem 2)*

Consider the situation shown in Figure 1. There are 5 coordinate frames

$$\{B\}, \{W\}, \{T\}, \{S\}, \{G\}.$$

We are interested in determining the transformation ${}^W_T T$. This can be motivated as follows: Imagine that the robot has previously picked up the tool, and that the position

P1 $\cos(\angle(\vec{p}, \vec{q})) = \frac{\vec{p}^T \vec{q}}{|\vec{p}| \cdot |\vec{q}|} = \frac{(R\vec{p})^T (R\vec{q})}{|R\vec{p}| \cdot |R\vec{q}|} = \cos(\angle(R\vec{p}, R\vec{q}))$

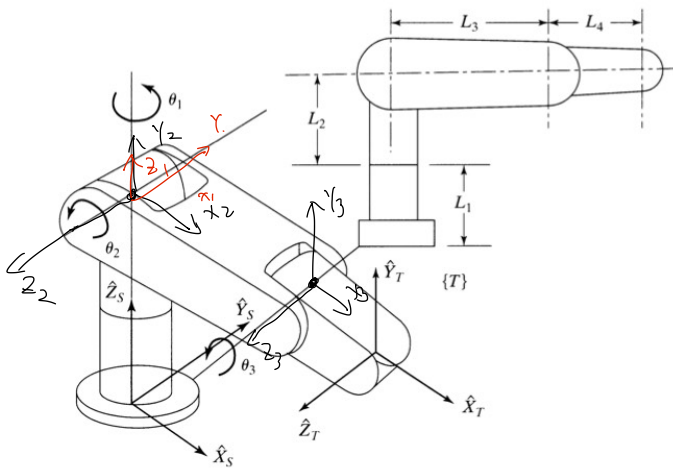
$$\vec{p}^T \vec{q} = \vec{p}^T R^T R \vec{q} \Rightarrow R^T R = I$$

P2 known: ${}^B_S T, {}^S_G T, {}^B_W T$

$$\{G\} = \{T\}$$

$${}^W_T T = {}^W_G T = {}^B_W T^{-1} \cdot {}^B_S T \cdot {}^S_G T$$

P3



j	a_{j-1}	α_{j-1}	d_j	θ_j	Value
1	0	0	$L_1 + L_2$	θ_1	0
2	0	90°	L_2	θ_2	0
3	L_3	0	0	θ_3	0
4	L_4	0	0	0	

$${}^0_1 T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2 T = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3 T = \begin{bmatrix} C_3 & -S_3 & 0 & L_3 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_4 T = \begin{bmatrix} 1 & 0 & 0 & L_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

of the tool in the gripper is not known very well, but we would like to know it exactly. However, the robot might be equipped with a force sensor and it could try to “feel” around to fit the tool to the goal. Assuming that the transformations ${}^B_S T$, ${}^S_G T$, ${}^B_W T$ are known, and that the coordinate frames $\{G\}$ (goal) and $\{T\}$ (tool) are “calibrated”, i.e., coincident, how can we determine the unknown transformation ${}^W_T T$?

Problem 3

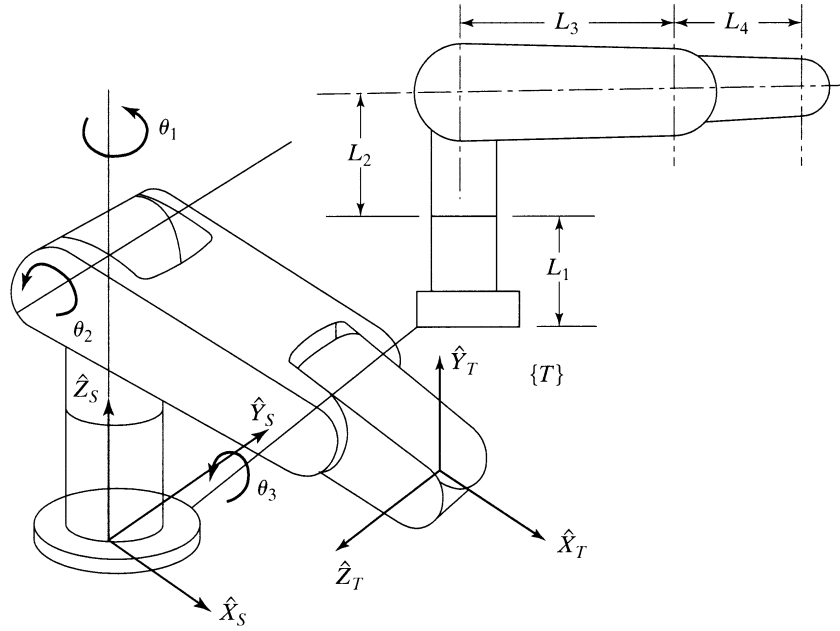


Figure 2: Nonplanar 3R Robot (Problem 3)

Figure 2 shows a robot arm with three rotational axes, thus three degrees of freedom. As shown in the diagram, all robot joints are in zero position. Determine the positions of the robot arm coordinate systems according to the Denavit-Hartenberg convention and derive the corresponding coordinate transformations.

Problem 4

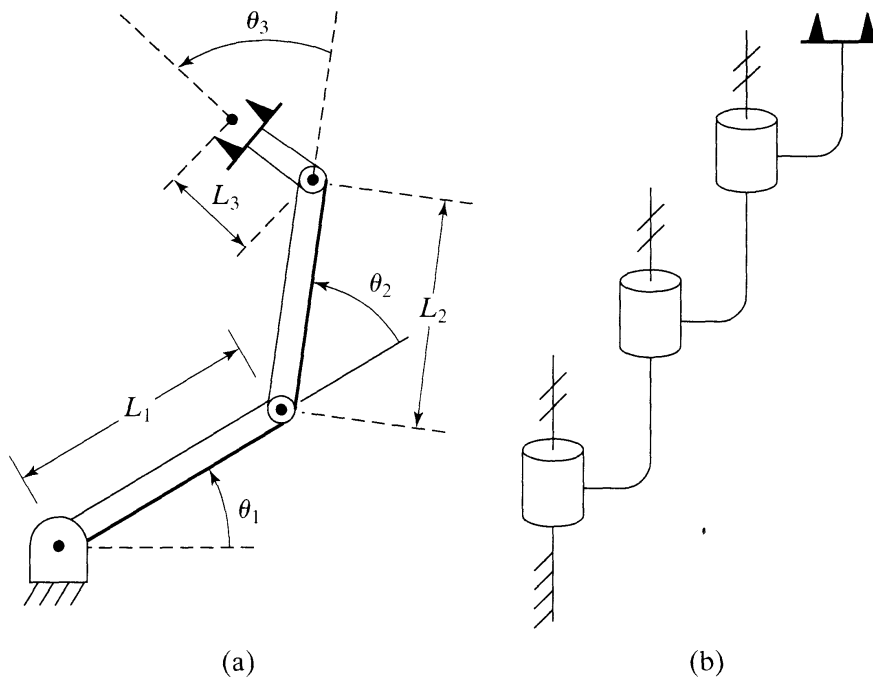
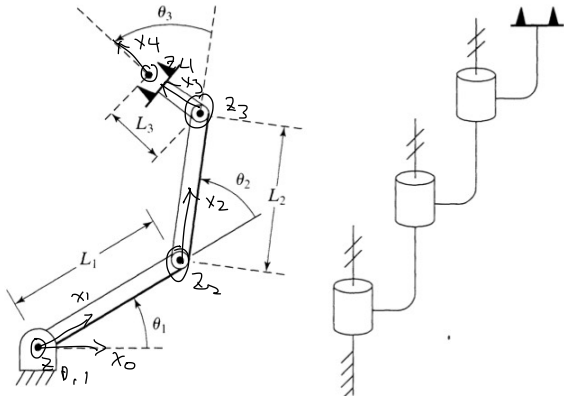


Figure 3: *Planar 3R Robot (Problem 4)*

Consider the planar 3R robot shown in Figure 3. All axes of rotation are parallel, as indicated by the hash marks.

- Identify the DH parameters of this system.
- Determine its forward kinematics.
- Given a position (cartesian coordinates of gripper center and gripper angle), can you think of a simple way to find out if such a position is reachable for the robot?

P4



	a_{i-1}	α_{i-1}	d_i	θ_i	Value
1	0	0	0	θ_1	
2	L_1	0	0	θ_2	
3	L_2	0	0	θ_3	
4	L_3	0	0	0	

$${}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} C_2 & -S_2 & 0 & L_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & L_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_4T = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_2T = \begin{bmatrix} C_1C_2 - S_1S_2 & -C_1S_2 - S_1C_2 & S_1C_2 & L_1C_1 \\ S_1C_2 + C_1S_2 & -S_1S_2 + C_1C_2 & C_1C_2 & L_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_3T = \begin{bmatrix} C_1C_2C_3 - S_1S_2C_3 & -C_1S_2C_3 - S_1C_2C_3 & S_1C_2C_3 & L_1C_1 + L_2C_1 \\ S_1C_2C_3 + C_1S_2C_3 & -S_1S_2C_3 + C_1C_2C_3 & C_1C_2C_3 & L_1S_1 + L_2S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_4T = \begin{bmatrix} C_1C_2C_3 & -S_1S_2C_3 & 0 & L_1C_1 + L_2C_1C_2 + L_3C_1C_2C_3 \\ S_1C_2C_3 & C_1C_2C_3 & 0 & L_1S_1 + L_2S_1C_2 + L_3S_1C_2C_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0P_4 \quad \theta_4 \quad {}^0P_3 = {}^0P_4 - L_3 \begin{pmatrix} C\theta_4 \\ S\theta_4 \\ 0 \end{pmatrix}$$

3轴太难算

$$|L_1 - L_2| \leq \|{}^0P_3\| \leq L_1 + L_2$$

$$\Rightarrow |L_1 - L_2| \leq \|{}^0P_4 - L_3 \begin{pmatrix} C\theta_4 \\ S\theta_4 \\ 0 \end{pmatrix}\| \leq L_1 + L_2$$

Problem 5

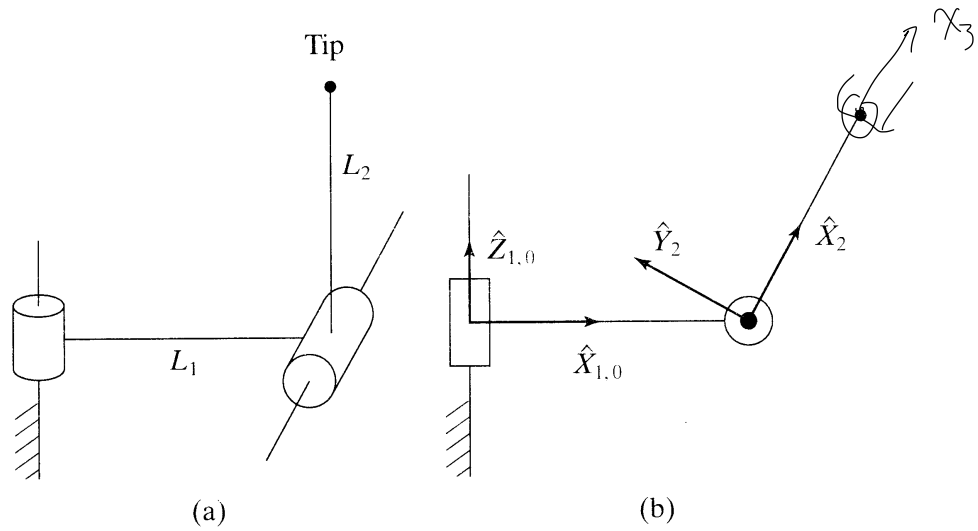


Figure 4: 2R Robot, problem 5

Consider the 2R robot shown in Figure 4. The link transformation matrices have been determined, and the coordinate transformation for the second link is:

$${}^0_2T = \begin{pmatrix} c_1 c_2 & -c_1 s_2 & s_1 & l_1 c_1 \\ s_1 c_2 & -s_1 s_2 & -c_1 & l_1 s_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The link-frame assignments are shown in Figure 4. Note that the frames $\{0\}$ and $\{1\}$ are coincident when $\Theta_1 = 0$. The length of the second link is l_2 . Find an expression for the vector ${}^0P_{\text{tip}}$, which locates the tip of the arm relative to the $\{0\}$ frame.

$$P_T$$

j	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	L_1	$q\theta^0$	0	θ_2
3	L_2	0	0	0

$${}^2_3T = \left[\begin{array}{c|c} I & \begin{matrix} L_2 \\ 0 \\ 0 \end{matrix} \\ \hline 0 & 0 \end{array} \right]$$

$${}^0_3T = \left[\begin{array}{c|c} {}^0_3R & \begin{matrix} L_1C_1 + L_2C_1C_2 \\ L_1S_1 + L_2S_1C_2 \\ \textcolor{red}{L_2S_2} \end{matrix} \\ \hline 0 & 1 \end{array} \right]$$

$${}^0P_{tip} = \begin{pmatrix} L_1C_1 + L_2C_1C_2 \\ L_1S_1 + L_2S_1C_2 \\ \textcolor{red}{L_2S_2} \end{pmatrix}$$

Problem 6

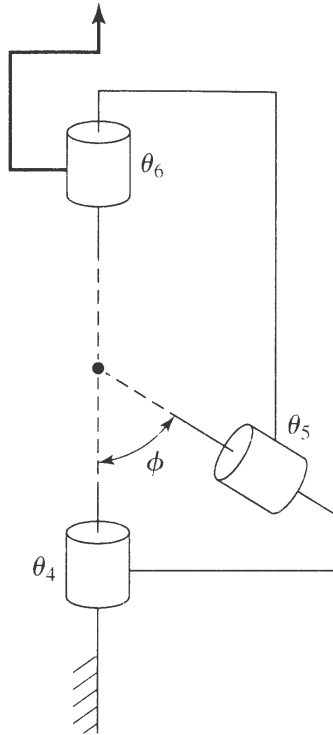


Figure 5: *3R nonorthogonal-axis robot (problem 6)*

Figure 5 shows a schematic of a wrist that has three intersecting axes that are not orthogonal. Assign link frames to the wrist (as if it were a regular 3R manipulator) and give the link parameters.

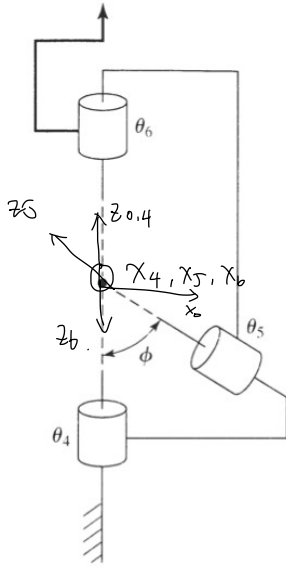
Problem 7

The formula for determining the transformation between consecutive frames within the Denavit-Hartenberg-Convention is given as:

$${}^{i-1}_i\mathbf{T} = \begin{pmatrix} \cos \theta_i & -\sin \theta_i & 0 & a_{i-1} \\ \sin \theta_i \cos \alpha_{i-1} & \cos \theta_i \cos \alpha_{i-1} & -\sin \alpha_{i-1} & -\sin \alpha_{i-1} d_i \\ \sin \theta_i \sin \alpha_{i-1} & \cos \theta_i \sin \alpha_{i-1} & \cos \alpha_{i-1} & \cos \alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

Is it possible to express arbitrary rigid body transformations this way?

P6



J	a_{i-1}	α_{i-1}	d_i	θ_i	Value
4	0	0	0	θ_4	-90°
5	0	ϕ	0	θ_5	0
6	0	$180-\phi$	0	θ_6	0

P7

$${}^{i-1}_i T = \begin{pmatrix} \cos \theta_i & -\sin \theta_i & 0 & a_{i-1} \\ \sin \theta_i \cos \alpha_{i-1} & \cos \theta_i \cos \alpha_{i-1} & -\sin \alpha_{i-1} & -\sin \alpha_{i-1} d_i \\ \sin \theta_i \sin \alpha_{i-1} & \cos \theta_i \sin \alpha_{i-1} & \cos \alpha_{i-1} & \cos \alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

No $p = (x, y, z, \alpha, \beta, \gamma)$

DH only have 4 parameters, lose 2 parameter

e.g. can not show the Y translation and rotation

Problem 8

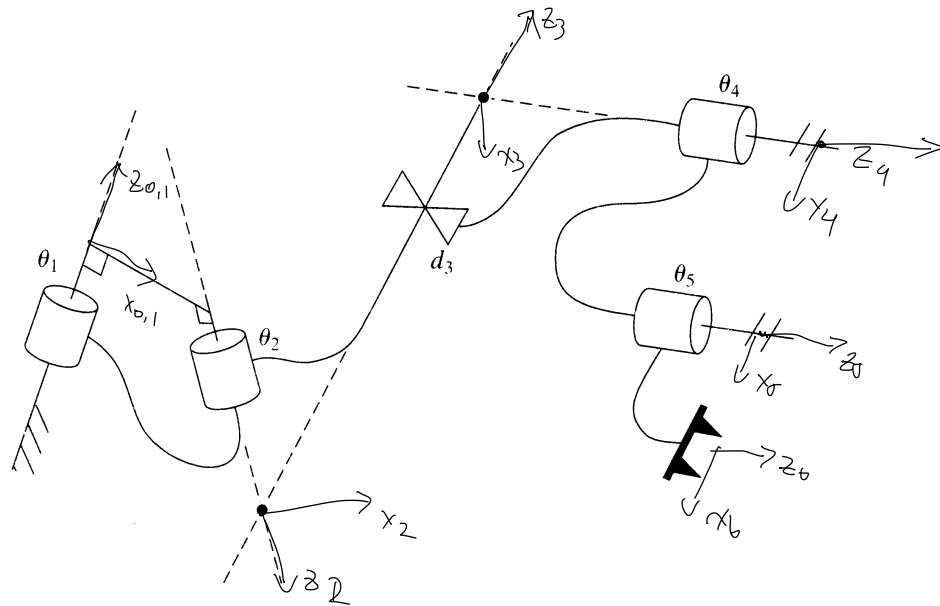


Figure 6: Schematic of a 2RP2R manipulator (problem 8).

Show the attachment of link frames for the manipulator shown in Figure 6.

Problem 9

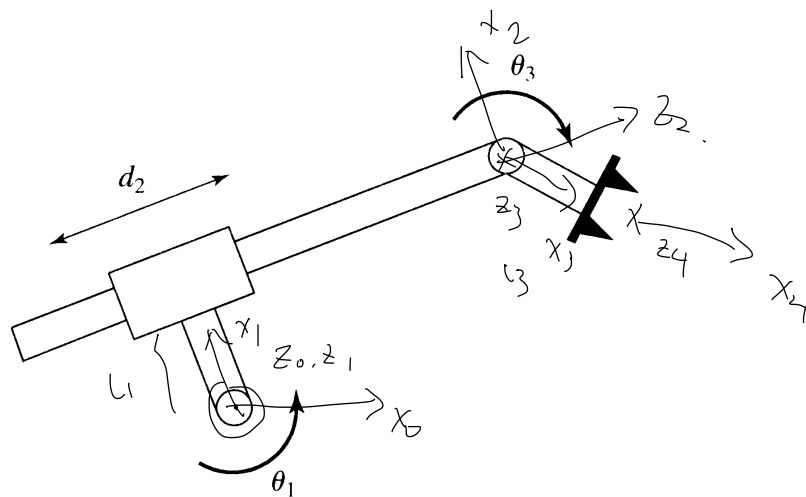


Figure 7: RPR planar robot (problem 9)

Assign link frames to the robot shown in Figure 7, and give approximate linkage parameters.

j	a_{j-1}	d_{j-1}	d_j	θ_j	Value
1	0	0	0	θ_1	135°
2	L_1	90°	d_2	0	
3	0	90°	0	θ_3	135°
4	L_2	0	0	0	

Problem 10

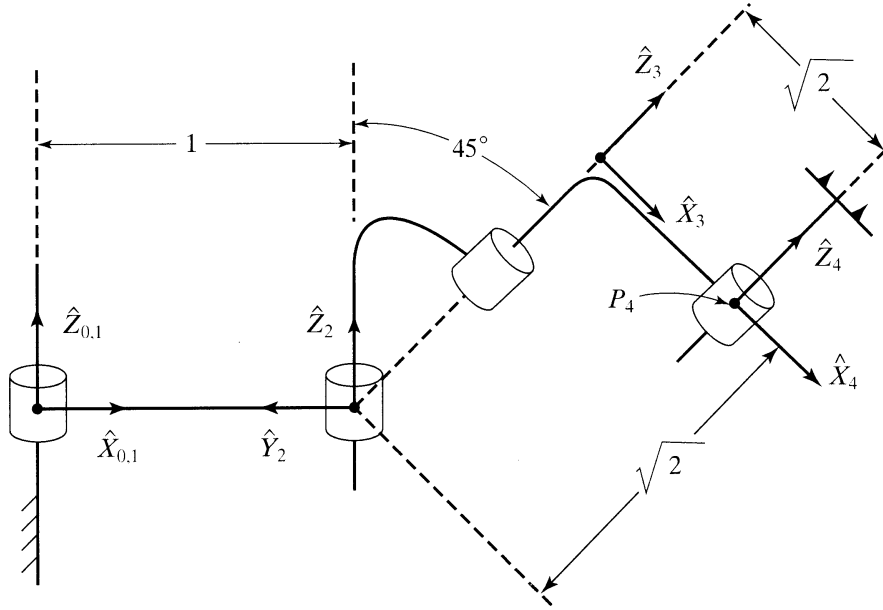


Figure 8: 4R Robot (Problem 10)

For the 4R manipulator shown in Figure 8, we have the following set of DH parameters:

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0°	0	θ_1
2	1	0°	0	θ_2
3	0	45°	$\sqrt{2}$	θ_3
4	$\sqrt{2}$	0°	0	0

The manipulator is shown for the joint configuration

$$\Theta = [0^\circ, 90^\circ, -90^\circ, 0^\circ].$$

We ignore collisions between arms, so we assume that all joints can move freely within the range $[-180^\circ, 180^\circ]$. Determine values of θ_3 such that positions

$${}^0P_{4\text{ORG}} = [\bullet, \bullet, 1.707]^T$$

are reachable for the fourth frame.

$${}^0_1T = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1_2T = \begin{pmatrix} c_2 & -s_2 & 0 & 1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2_3T = \begin{pmatrix} c_3 & -s_3 & 0 & 0 \\ \frac{\sqrt{2}}{2}s_3 & \frac{\sqrt{2}}{2}c_3 & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2}s_3 & \frac{\sqrt{2}}{2}c_3 & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^3_4T = \begin{pmatrix} 1 & 0 & 0 & \sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0°	0	θ_1
2	1	0°	0	θ_2
3	0	45°	$\sqrt{2}$	θ_3
4	$\sqrt{2}$	0°	0	0

$${}^0_2T = \begin{pmatrix} c_1 c_2 - s_1 s_2 & -c_1 s_2 - s_1 c_2 & 0 & c_1 \\ s_1 c_2 + c_1 s_2 & -s_1 s_2 + c_1 c_2 & 0 & s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_3T = \begin{pmatrix} c_1 c_3 - \frac{\sqrt{2}}{2} s_1 s_3 & -c_1 s_3 - \frac{\sqrt{2}}{2} s_1 c_3 & \frac{\sqrt{2}}{2} s_1 & s_1 + c_1 \\ s_1 c_3 + \frac{\sqrt{2}}{2} c_1 s_3 & -s_1 s_3 + \frac{\sqrt{2}}{2} c_1 c_3 & -\frac{\sqrt{2}}{2} c_1 & -c_1 + s_1 \\ \frac{\sqrt{2}}{2} s_3 & \frac{\sqrt{2}}{2} c_3 & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_4T = \begin{pmatrix} s_1 + c_1 + \sqrt{2} c_1 c_3 - s_1 s_3 & -c_1 + s_1 + \sqrt{2} s_1 c_3 + c_1 s_3 & s_3 + 1 & 1 \end{pmatrix}$$

$$s_3 + 1 = 1.707$$

$$s_3 = 0.707 \Rightarrow \begin{cases} \theta_3 = 45^\circ \\ \theta_3 = 135^\circ \end{cases}$$