FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE
Exercise 10: Making Simple Decisions – Problems Winter Semester 2023/24
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Problem 10.1: Expected Utility

(Taken from [1] Exercise 16.15) Consider a student who has the choice to buy or not buy a textbook for a course. We'll model this as a decision problem with one Boolean decision node, B, indicating whether the agent chooses to buy the book, and two Boolean chance nodes, M, indicating whether the student has mastered the material in the book and P, indicating whether the student passes the course. Of course, there is also a utility node, U. A certain student, Sam, has an additive utility function: 0 for not buying the book and €-100 for buying it; and €2000 for passing the course and 0 for not passing. Sam's conditional probability estimates are as follows:

$$\begin{aligned} P(p|b,m) &= 0.9 \\ P(p|b,\neg m) &= 0.5 \\ P(p|\neg b,m) &= 0.8 \\ P(p|\neg b,\neg m) &= 0.3 \\ P(m|b) &= 0.9 \\ P(m|\neg b) &= 0.7 \end{aligned}$$

You might think that P would be independent of B given M, but this course has an open-book final—so having the book helps.

Problem 10.1.1: Draw the decision network for this problem.

Problem 10.1.2: Compute the expected utility of buying the book and of not buying it.

Problem 10.1.3: What should Sam do?

Problem 10.2: Optimal Decision

An investor is considering whether he should buy some stocks of company A. He can also ask a professional stock consultant for investment advice. Asking the stock consultant will cost €50. The investor can decide to ask the consultant and then, depending on the feedback, decide whether to buy the stock. $B \in \{b, \neg b\}$ indicates whether the investor is going to buy the stock. $C \in \{c, \neg c\}$ indicates whether he is going to ask the consultant.

The stock can be a high quality stock (will earn some money) or a low quality stock(will lose some money) ($Q \in \{q, \neg q\}$), and the consultant might help to indicate what kind of stock it is. The stock costs €1500, and its market value is €2000 if it is a high quality stock. If not, its market value is €1300. We assume that the stock quality wouldn't be affected by this investor's decision. The investor estimates that the stock has a 70% chance of being a high quality stock, which means, $P(q) = 0.7$.

Problem 10.2.1: Draw the decision network that represents this problem.

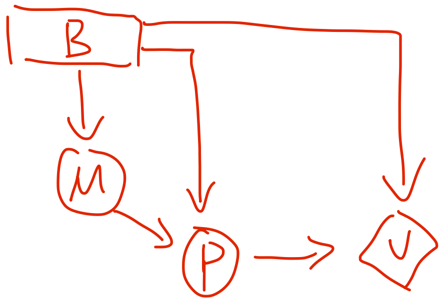
Problem 10.2.2: Draw the decision tree that represents this problem.

Problem 10.2.3: Calculate the expected utility of buying the stock, without asking the consultant.

Problem 10.1: Expected Utility

b

(Taken from [1] Exercise 16.15) Consider a student who has the choice to buy or not buy a textbook for a course. We'll model this as a decision problem with one Boolean decision node, B, indicating whether the agent chooses to buy the book, and two Boolean chance nodes, M, indicating whether the student has mastered the material in the book and P, indicating whether the student passes the course. Of course, there is also a utility node, U. A certain student, Sam, has an additive utility function: 0 for not buying the book and €-100 for buying it; and €2000 for passing the course and 0 for not passing. Sam's conditional probability estimates are as follows:



$B < M < P < U$

$$U(b,p) = 1900$$
$$U(b,\neg p) = -100$$
$$U(\neg b, p) = 2000$$
$$U(\neg b, \neg p) = 0$$

$$\begin{aligned}
P(p|b,m) &= 0.9 \\
P(p|b,\neg m) &= 0.5 \\
P(p|\neg b,m) &= 0.8 \\
P(p|\neg b,\neg m) &= 0.3 \\
P(m|b) &= 0.9 \\
P(m|\neg b) &= 0.7
\end{aligned}$$

$$U(b) = -100$$
$$U(\neg b) = 0$$
$$U(p) = 2000$$
$$U(\neg p) = 0$$

You might think that P would be independent of B given M, but this course has an open-book final—so having the book helps.

$$EU(a) = \sum_{s'} P(Result(a) = s')\, U(s')$$
$$EU(a|e) = \sum_{s'} P(s'|a,e)\, U(s')$$

Problem 10.1.1: Draw the decision network for this problem.

Problem 10.1.2: Compute the expected utility of buying the book and of not buying it.

Problem 10.1.3: What should Sam do?

$$\begin{aligned}
P(p|b) &= \sum_m P(p,m|b) = \sum_m P(p|b,m)\cdot P(m|b) \\
&= P(p|b,m)\cdot P(m|b) + P(p|b,\neg m)\cdot P(\neg m|b) \\
&= 0.9\cdot 0.9 + 0.5\cdot 0.1 \\
&= 0.86
\end{aligned}$$

$$\begin{aligned}
E(b) &= \sum_p P(p|b)\cdot U(p,b) \\
&= P(p|b)\cdot U(p,b) + P(\neg p|b)\cdot U(\neg p,b) \\
&= 0.86\cdot 1900 + (1-0.86)\cdot(-100) \\
&= 1620
\end{aligned}$$

$$\begin{aligned}
P(p|\neg b) &= \sum_m P(p,m|\neg b) = \sum_m P(p|\neg b,m)\cdot P(m|\neg b) \\
&= P(p|\neg b,m)\cdot P(m|\neg b) + P(p|\neg b,\neg m)\cdot P(\neg m|\neg b) \\
&= 0.8\cdot 0.7 + 0.3\cdot(1-0.7) \\
&= 0.65
\end{aligned}$$

$$\begin{aligned}
E(\neg b) &= \sum_p P(p|\neg b)\cdot U(p,\neg b) \\
&= P(p|\neg b)\cdot U(p,\neg b) + P(\neg p|\neg b)\cdot U(\neg p,\neg b) \\
&= 0.65\cdot 2000 + 0 \\
&= 1300
\end{aligned}$$

$$1620 > 1300$$

$$\pi^*(B) = b$$

Problem 10.2: Optimal Decision

An investor is considering whether he should buy some stocks of company A. He can also ask a professional stock consultant for investment advice. Asking the stock consultant will cost €50. The investor can decide to ask the consultant and then, depending on the feedback, decide whether to buy the stock. $B \in \{b, \neg b\}$ indicates whether the investor is going to buy the stock. $C \in \{c, \neg c\}$ indicates whether he is going to ask the consultant.
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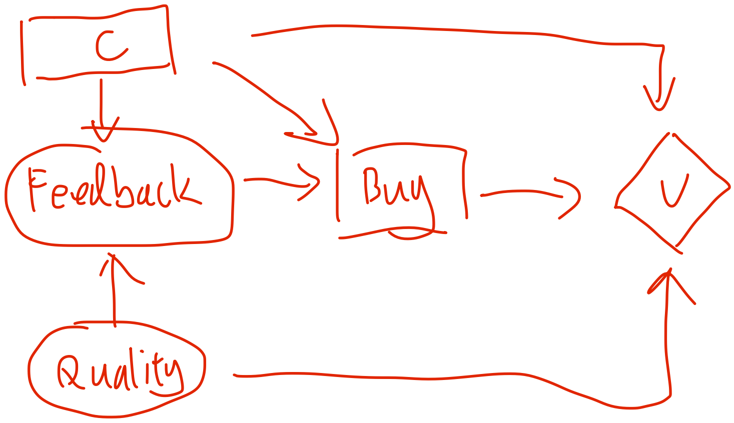
$U(c) = -50$
$U(\neg c) = 0$
$U(b) = -1500$
$U(\neg b) = 0$
$U(q) = 2000$
$U(\neg q) = 1300$

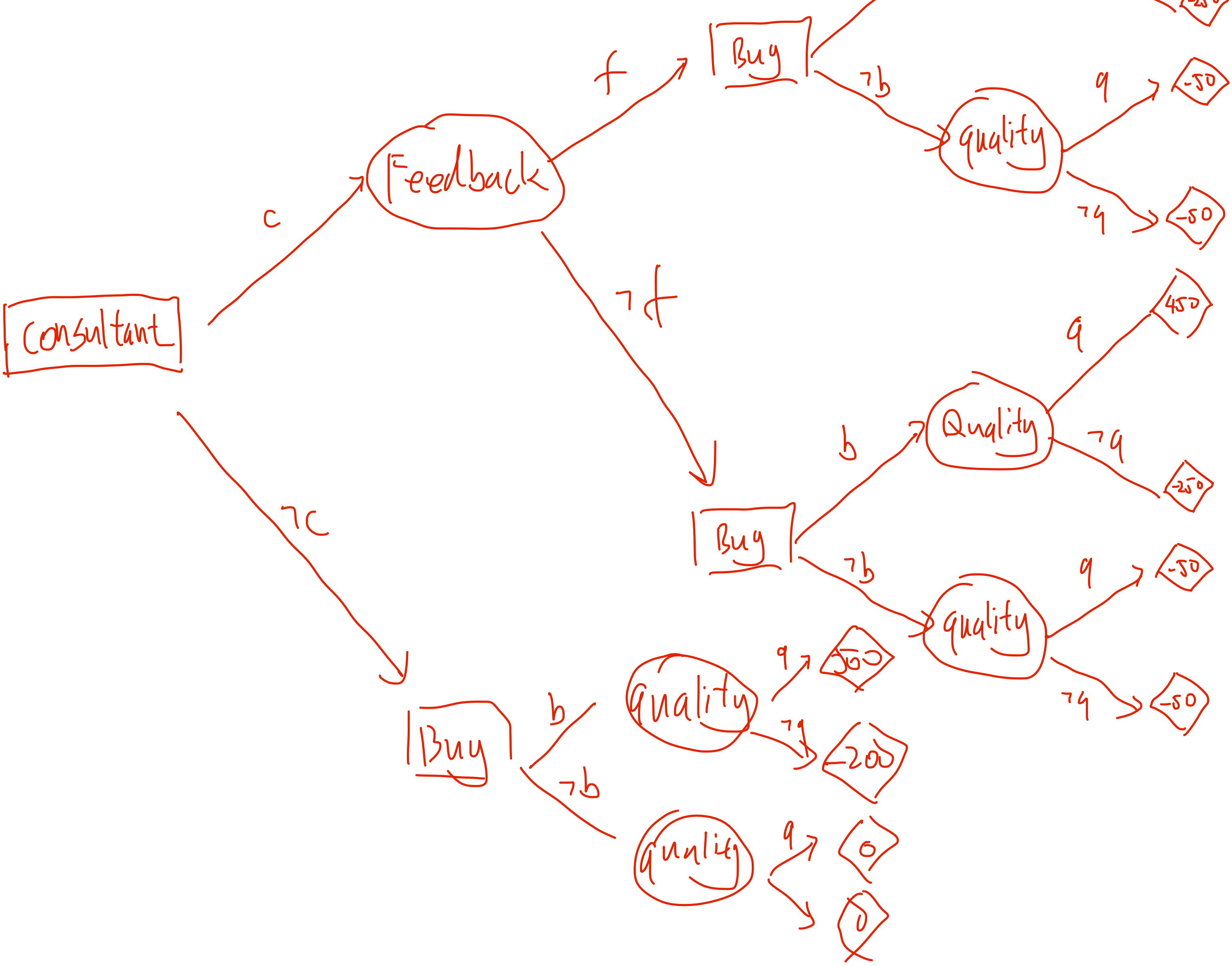
Problem 10.2.1: Draw the decision network that represents this problem.

Problem 10.2.2: Draw the decision tree that represents this problem.

Problem 10.2.3: Calculate the expected utility of buying the stock, without asking the consultant.



$C < F < B < Q$



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$U(c) = -50$
$U(\neg c) = 0$
$U(b) = 1500$
$U(\neg b) = 0$
$U(q) = 2000$
$U(\neg q) = 1300$

Problem 10.2.1: Draw the decision network that represents this problem.

Problem 10.2.2: Draw the decision tree that represents this problem.

Problem 10.2.3: Calculate the expected utility of buying the stock, without asking the consultant.

$$EU(b, \neg c) = \sum_Q P(Q \mid b, \neg c) \cdot U(Q, b, \neg c)$$
$$= P(q) \cdot U(q, b, \neg c) + P(\neg q) \cdot U(\neg q, b, \neg c)$$
$$= 0.7 \cdot 500 + 0.3 \cdot (-200)$$
$$= 290$$

The consultant's advice can be described by the probability that the feedback from consultant is positive or not ($F \in \{f, \neg f\}$) given that the stock will be high quality or low quality. Which means, we have the following information:

$$P(f|q) = 0.85 \qquad P(f|\neg q) = 0.05$$
$$P(\neg f|q) = 0.15 \qquad P(\neg f|\neg q) = 0.95$$

Problem 10.2.4: Derive an optimal conditional plan for the investor. Start with determining the optimal decisions whether to buy the stock given no consultation, a positive feedback or a negative feedback. Calculate the value of information of the consultation.

$$EU(\neg b, \neg c) = \sum_Q P(Q \mid \neg b, \neg c) \cdot U(Q, \neg b, \neg c)$$
$$= P(q) \cdot U(q, \neg b, \neg c) + P(\neg q) \cdot U(\neg q, \neg b, \neg c)$$
$$= 0$$
$$\pi^*(B \mid \neg c) = b$$

$$EU(b, c, f) = \sum_Q P(Q \mid b, c, f) \cdot U(Q, b)$$
$$= \underline{P(q|f)} \cdot U(q, b) + \underline{P(\neg q|f)} \cdot U(\neg q, b) - 50$$
$$= \alpha\, P(f|q) \cdot P(q)$$
$$\langle 0.85 \cdot 0.7,\ 0.05 \cdot 0.3 \rangle = \langle 0.595,\ 0.015 \rangle = \langle 0.9754,\ 0.0246 \rangle$$

$= 0.9754 \cdot 500 + 0.0246 \cdot (-200) - 50$

$= 432.78$

$$EU(\neg b, c, f) = \sum_Q P(Q|f) \cdot U(Q, f) - 50 = -50$$

$$\pi^*(B | c, f) = b$$

$$EU(b, c, \neg f) = \sum_Q P(Q|\neg f)\, U(Q, \neg f)$$

$= \underline{P(q|\neg f)} \cdot U(q, \neg f) + P(\neg q|\neg f)\, U(\neg q, \neg f) - 50$

$=$

$= \alpha\, P(\neg f | q) \cdot P(q)$

$= \langle 0.15 \cdot 0.7,\ 0.95 \cdot 0.3 \rangle = \langle 0.105,\ 0.285 \rangle$

$= \underline{\langle 0.2692,\ 0.7308 \rangle}$

$= 0.2692 \cdot 500 + 0.7308 \cdot (-200) - 50$

$= -61.56$

$$EU(\neg b, c, \neg f) = -50.$$

$$\pi^*(B | c, \neg f) = \neg b$$

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2 should consultant ?

$EU(c) = \sum_Q P(Q$

The consultant's advice can be described by the probability that the feedback from consultant is positive or not ($F \in \{f, \neg f\}$) given that the stock will be high quality or low quality. Which means, we have the following information:

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Problem 10.2.4: Derive an optimal conditional plan for the investor. Start with determining the optimal decisions whether to buy the stock given no consultation, a positive feedback or a negative feedback. Calculate the value of information of the consultation.

References

[1] S. Russell and P. Norvig, *Artificial Intelligence: A Modern Approach*. Prentice Hall, 2010.