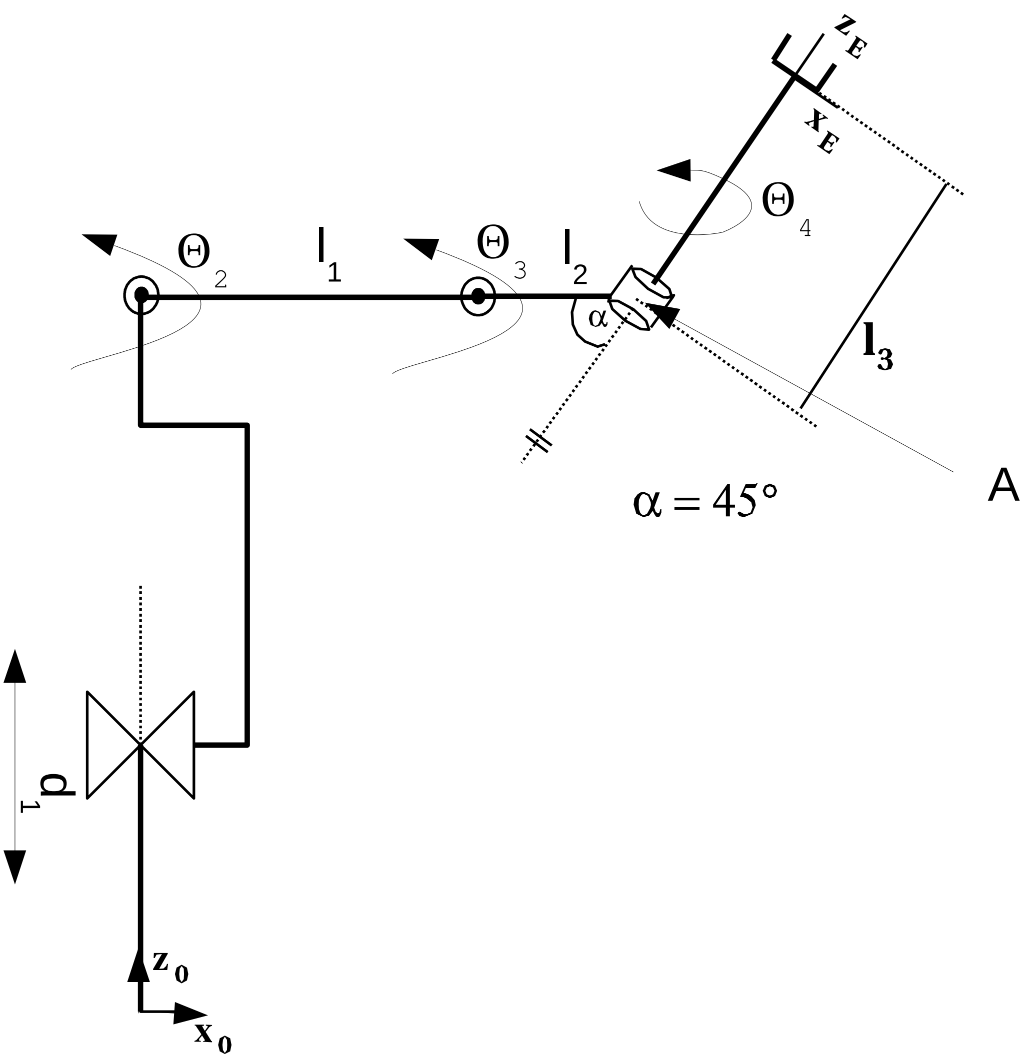
Problem 1: (19 points)

For a given manipulator with a prismatic joint d_1 and 3 rotatory joins $\Theta_{2\text{-}4}$

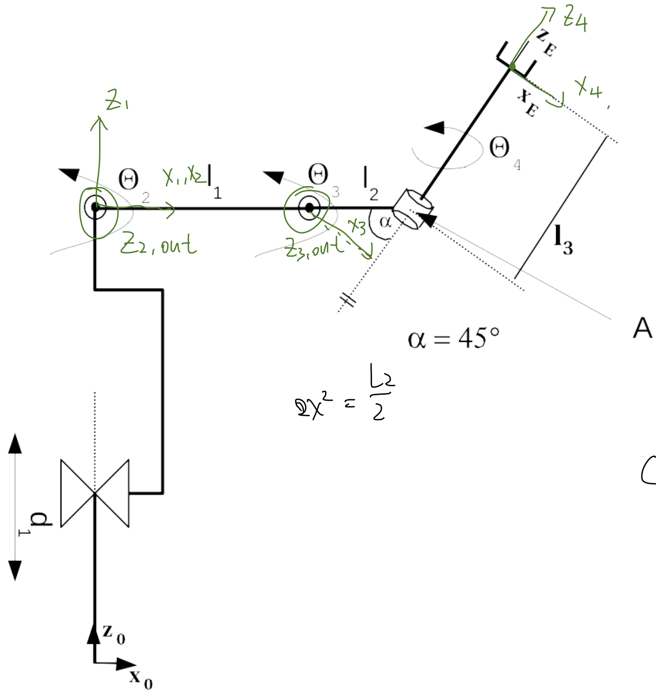


a) Enter in the above picture the directions of the z- and x- axes according to Denavitt-Hartenberg convention (4 points).

b) Explain with a drawing the meaning of the DH-Parameter. (3 points)

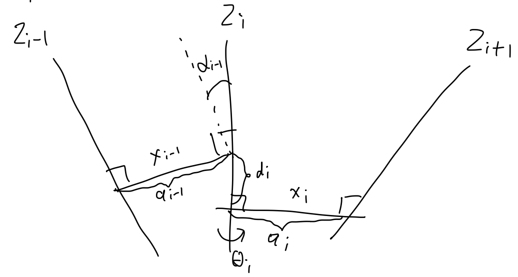
c) Enter the above DH parameter in a table (4 points)

d) How many degrees of freedom are in the configuration space and how many DoF has the robot? (2 points)



$\partial x^2 = \frac{L_2}{2}$

PRRR



c)

J	a_{i-1}	α_{i-1}	d_i	θ_i	Value
1	0	0	d_1	0	$\geq 0m$
2	0	$90°$	0	θ_2	$0°$
3	L_1	0	0	θ_3	$-45°$
4	$\frac{\sqrt{2}}{2}L_2$	$-90°$	$\frac{\sqrt{2}}{2}L_2+L_3$	θ_4	$0°$
EE	0	0	0	0	

d) 3 DoF 3 DOF

e) $${}^0p_A = \begin{pmatrix} L_2 C_{32} + L_1 C_2 \\ L_2 S_{32} + L_1 S_2 + d_1 \end{pmatrix}$$

PRRR

$${}^0J_v = {}^0\dot{p}_{EE} = {}^0\dot{J}_{km} = {}^0J_v \cdot \dot{\Theta}$$

$$\begin{pmatrix} 0 & -L_2 S_{23} - L_1 S_2 & -L_2 S_{23} & 0 \\ 1 & L_1 C_2 + L_2 C_{23} & L_2 C_{23} & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \xRightarrow{\text{subsystem}} \begin{pmatrix} 0 & -L_1 S_2 - L_2 S_{23} & -L_2 S_{23} \\ 1 & L_1 C_2 + L_2 C_{23} & L_2 C_{23} \\ 0 & 1 & 1 \end{pmatrix}$$

f) Robot loss at least one degree of freedom, i.e. it can't still move along there direction

$\det |J(\Theta)| = 0$

$$\begin{vmatrix} 0 & -L_1 S_2 - L_2 S_{23} & -L_2 S_{23} \\ 1 & L_1 C_2 + L_2 C_{23} & L_2 C_{23} \\ 0 & 1 & 1 \end{vmatrix} = \cancel{-L_2 S_{23}} + L_1 S_2 + \cancel{L_2 S_{23}} = L_1 S_2 = 0$$

$L_1 = 0$ column 2 and 3 are linear dependent

$\Rightarrow$ Joint 2 and 3's velocity and torque are coupled.

$\Theta_2 = 0, 180$ $${}^0J_A = \begin{pmatrix} 0 & -L_2 S_3 & -L_2 S_3 \\ 1 & L_1 + L_2 C_3 & L_2 C_3 \\ 0 & 1 & 1 \end{pmatrix}$$

Row 1 $= 0 \Leftrightarrow \Theta_3 = \{0, 180°\}$

No motion in X-Direction

e) The position of point A in the {0} coordinate frame is given as:

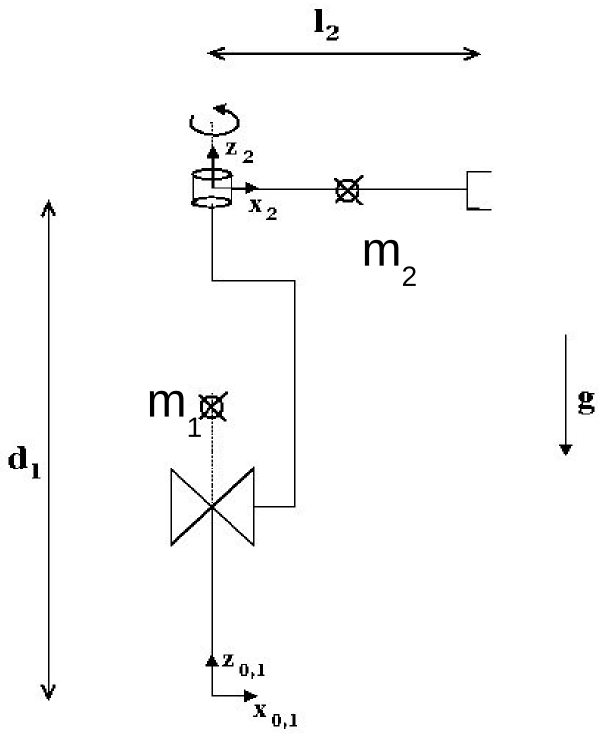
$$P_E(\theta)=\begin{pmatrix} l_2 c_{32} + l_1 c_2 \\ l_2 s_{32} + l_1 s_2 + d_1 \end{pmatrix}$$

with s_x and c_x being the sine and cosine expressions of the angles. Calculate the Jacobian ${}^{o}J$ for this subsystem. What is the generic form how to calculate the elements of the Jacobian? (3 points)

f) Explain the geometrical meaning of a singularity. How can we estimate the singularities of a system? Does the above system have singular configurations? (3 points)

Problem 2: (18 points)

In a following manipulator with a prismatic and rotatory joint:



The links have the mass m_1 and m_2 . The center of mass are at

$$^{1}P_{C_1} = \begin{pmatrix} 0 \\ 0 \\ d_1/2 \end{pmatrix} \quad \text{und} \quad ^{2}P_{C_2} = \begin{pmatrix} l_2/2 \\ 0 \\ 0 \end{pmatrix} .$$

The inertial Tensoren have a following form:

$$^{C_1}I_{C_1} = \begin{pmatrix} I_{XX_1} & 0 & 0 \\ 0 & I_{YY_1} & 0 \\ 0 & 0 & I_{ZZ_1} \end{pmatrix} , \quad ^{C_2}I_{C_2} = \begin{pmatrix} I_{XX_2} & 0 & 0 \\ 0 & I_{YY_2} & 0 \\ 0 & 0 & I_{ZZ_2} \end{pmatrix}$$

The system is supposed be analysed with Newton-Euler Method,

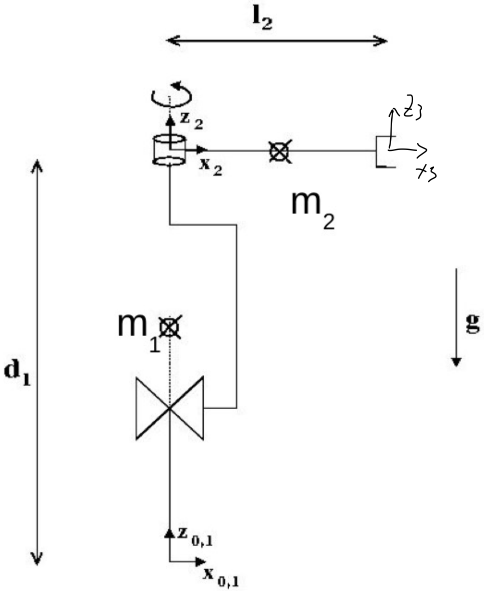
a) Estimate the velocity- and force- components for the first iteration of the Newton-Euler method (7 points)

b) Estimate the forces and torques (f_i, n_i) in the joints of each subsystem. (the direction of gravity g is depicted). (5 points)

c) Estimate the torques for the actuators τ. (2 points)

d) How can we group the elemenst from problem 2c)? What is the physical meaning of the resulting martrices (3 points)

MVG



J	a_{i-1}	α_{i-1}	d_i	θ_i	Value
1	0	0	0	0	
2	0	0	d_1	θ_2	
EE	l_2	0	0	0	

$$^0_1T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad ^1_2T = \begin{pmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad ^2_{EE}T = \begin{pmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$^0\dot v_0 = -G = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \quad ^0v_0 = {^0w_0} = {^0\dot w_0} = 0$$

PR

$$^1w_1 = \cancel{^1_0R\,^0w_0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$^1\dot w_1 = \cancel{^1_0R\,^0\dot w_0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$^1v_1 = {^1_0R}(\cancel{^0v_0} + \cancel{^0w_0 \times {^0P_1}}) + \begin{pmatrix} 0 \\ 0 \\ \dot d_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot d_1 \end{pmatrix}$$

$$^1\dot v_1 = \cancel{^1_0R}(\cancel{^0\dot w_0 \times {^0P_1}} + \cancel{^0w_0 \times (^0w_0 \times {^0P_1})} + {^0\dot v_0}) + 2\,\cancel{^1w_1 \times \begin{pmatrix} 0 \\ 0 \\ \dot d_1 \end{pmatrix}} + \begin{pmatrix} 0 \\ 0 \\ \ddot d_1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \ddot d_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ g+\ddot d_1 \end{pmatrix}$$

$$^1\dot v_{C_1} = \cancel{^1\dot w_1 \times {^1P_{C1}}} + \cancel{^1w_1 \times (^1w_1 \times {^1P_{C_1}})} + {^1\dot v_1} = \begin{pmatrix} 0 \\ 0 \\ g+\ddot d_1 \end{pmatrix}$$

$$^1F_1 = m_1 \cdot {^1\dot v_{C_1}} = \begin{pmatrix} 0 \\ 0 \\ m_1(g+\ddot d_1) \end{pmatrix}$$

$$^1N_1 = \cancel{^{C_1}I_1 \cdot {^1\dot w_1}} + \cancel{^1w_1 \times {^{C_1}I_1}\,{^1w_1}} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$^2_1R = \begin{pmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$^2w_2 = \cancel{^2_1R\,^1w_1} + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix}$$

$$^2\dot w_2 = \cancel{^2_1R\,^1\dot w_1} + \cancel{^2_1R\,^1w_1 \times \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix}} + \begin{pmatrix} 0 \\ 0 \\ \ddot\theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \ddot\theta_2 \end{pmatrix}$$

$$^2v_2 = {^2_1R}(^1v_1 + \cancel{^1w_1 \times {^1P_2}})$$

$$= \begin{pmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot d_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot d_1 \end{pmatrix}$$

$$^2\dot v_2 = {^2_1R}(\cancel{^1\dot w_1 \times {^1P_2}} + \cancel{^1w_1 \times (^1w_1 \times {^1P_2})} + {^1\dot v_1})$$

$$= \begin{pmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ g+\ddot d_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ g+\ddot d_1 \end{pmatrix}$$

$$^2\dot v_{C_2} = {^2\dot w_2} \times {^2P_{C_2}} + {^2w_2} \times (^2w_2 \times {^2P_{C_2}}) + {^2\dot v_2}$$

$$= \begin{pmatrix} 0 \\ 0 \\ \ddot\theta_2 \end{pmatrix} \times \begin{pmatrix} \frac{L_2}{2} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} \times \begin{pmatrix} \frac{L_2}{2} \\ 0 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ g+\ddot d_1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \frac{1}{2}L_2\ddot\theta_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ \frac{1}{2}L_2\dot\theta_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ g+\ddot d_1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{2}L_2\dot\theta_2^2 \\ \frac{1}{2}L_2\ddot\theta_2 \\ g+\ddot d_1 \end{pmatrix}$$

$$ {}^2F_2 = m_2 \cdot {}^2\dot{v}_{C_2} = \begin{pmatrix} -\frac{1}{2} m_2 L_2 \dot{\theta}_2^2 \\ \frac{1}{2} m_2 L_2 \ddot{\theta}_2 \\ m_2(g+\ddot{d}_1) \end{pmatrix} $$

$$ {}^2N_2 = {}^{C_2}I_2 \cdot {}^2\dot{\omega}_2 + {}^2\omega_2 \times {}^{C_2}I_2 \cdot {}^2\omega_2 $$

$$ = \begin{pmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy2} & 0 \\ 0 & 0 & I_{zz2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} \times \left[\begin{pmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy2} & 0 \\ 0 & 0 & I_{zz2} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} \right] $$

$$ = \begin{pmatrix} 0 \\ 0 \\ I_{zz2} \cdot \ddot{\theta}_2 \end{pmatrix} + \cancel{\begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ I_{zz2} \dot{\theta}_2 \end{pmatrix}} $$

$$ = \begin{pmatrix} 0 \\ 0 \\ I_{zz2} \cdot \ddot{\theta}_2 \end{pmatrix} $$

$$ {}^2f_2 = \cancel{{}^2_3R\,{}^3f_3} + {}^2F_2 = \begin{pmatrix} -\frac{1}{2} m_2 L_2 \dot{\theta}_2^2 \\ \frac{1}{2} m_2 L_2 \ddot{\theta}_2 \\ m_2(g+\ddot{d}_1) \end{pmatrix} $$

$$ {}^2n_2 = {}^2N_2 + \cancel{{}^2_3R\,{}^3n_3} + {}^2P_{C_2} \times {}^2F_2 + \cancel{{}^2P_3 \times {}^2_3R\,{}^3f_3} $$

$$ = \begin{pmatrix} 0 \\ 0 \\ I_{zz2} \cdot \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} \frac{L_2}{2} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -\frac{1}{2} m_2 L_2 \dot{\theta}_2^2 \\ \frac{1}{2} m_2 L_2 \ddot{\theta}_2 \\ m_2(g+\ddot{d}_1) \end{pmatrix} $$

$$ {}^1_2R = \begin{pmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ = \begin{pmatrix} 0 \\ -\frac{1}{2} m_2 L_2 (g+\ddot{d}_1) \\ \frac{1}{4} m_2 L_2^2 \ddot{\theta}_2 + I_{zz2} \cdot \ddot{\theta}_2 \end{pmatrix} $$

$$ {}^1f_1 = {}^1_2R\,{}^2f_2 + {}^1F_1 $$

$$ = \begin{pmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{2} m_2 L_2 \dot{\theta}_2^2 \\ \frac{1}{2} m_2 L_2 \ddot{\theta}_2 \\ m_2(g+\ddot{d}_1) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ m_1(g+\ddot{d}_1) \end{pmatrix} $$

$$ = \begin{pmatrix} -\frac{1}{2} m_2 L_2 c_2 \dot{\theta}_2^2 - \frac{1}{2} m_2 L_2 s_2 \ddot{\theta}_2 \\ -\frac{1}{2} m_2 L_2 s_2 \dot{\theta}_2^2 + \frac{1}{2} m_2 L_2 c_2 \ddot{\theta}_2 \\ (m_1+m_2)(g+\ddot{d}_1) \end{pmatrix} $$

$$ {}^1n_1 = \cancel{{}^1N_1} + {}^1_2R\,{}^2n_2 + {}^1P_{C_1} \times {}^1F_1 + {}^1P_2 \times {}^1_2R\,{}^2f_2 $$

$$ = \begin{pmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{1}{2} m_2 L_2 (g+\ddot{d}_1) \\ \frac{1}{4} m_2 L_2^2 \ddot{\theta}_2 + I_{zz2} \cdot \ddot{\theta}_2 \end{pmatrix} + \cancel{\begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} d_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ m_1(g+\ddot{d}_1) \end{pmatrix}} + \begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix} \times \begin{pmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{2} m_2 L_2 \dot{\theta}_2^2 \\ \frac{1}{2} m_2 L_2 \ddot{\theta}_2 \\ m_2(g+\ddot{d}_1) \end{pmatrix} $$

$$ \begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix} \times \begin{pmatrix} -\frac{1}{2} m_2 L_2 c_2 \dot{\theta}_2^2 - \frac{1}{2} m_2 L_2 s_2 \ddot{\theta}_2 \\ -\frac{1}{2} m_2 L_2 s_2 \dot{\theta}_2^2 + \frac{1}{2} m_2 L_2 c_2 \ddot{\theta}_2 \\ m_2(g+\ddot{d}_1) \end{pmatrix} $$

$$ \begin{pmatrix} \frac{1}{2} m_2 L_2 s_2 d_1 \dot{\theta}_2^2 - \frac{1}{2} m_2 L_2 c_2 d_1 \ddot{\theta}_2 \\ -\frac{1}{2} m_2 L_2 c_2 d_1 \dot{\theta}_2^2 - \frac{1}{2} m_2 L_2 s_2 d_1 \ddot{\theta}_2 \\ 0 \end{pmatrix} $$

$$ = \begin{pmatrix} \frac{1}{2} m_2 L_2 s_2 (g+\ddot{d}_1) \\ -\frac{1}{2} m_2 L_2 c_2 (g+\ddot{d}_1) \\ \frac{1}{4} m_2 L_2^2 \ddot{\theta}_2 + I_{zz2} \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} m_2 L_2 s_2 d_1 \dot{\theta}_2^2 - \frac{1}{2} m_2 L_2 c_2 d_1 \ddot{\theta}_2 \\ -\frac{1}{2} m_2 L_2 c_2 d_1 \dot{\theta}_2^2 - \frac{1}{2} m_2 L_2 s_2 d_1 \ddot{\theta}_2 \\ 0 \end{pmatrix} $$

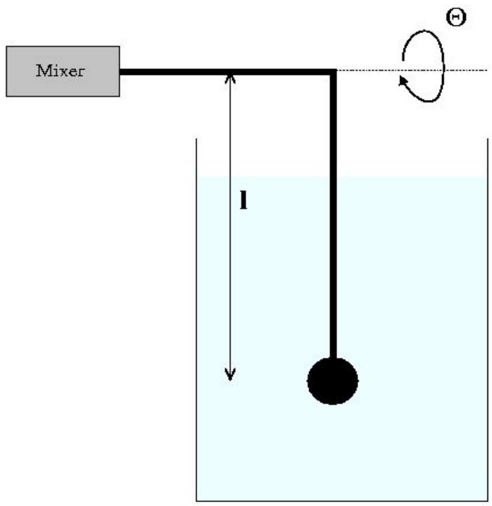
$$ = \begin{pmatrix} \frac{1}{2} m_2 L_2 \left[s_2 (g+\ddot{d}_1) + s_2 d_1 \dot{\theta}_2^2 - c_2 d_1 \ddot{\theta}_2 \right] \\ -\frac{1}{2} m_2 L_2 \left[c_2 (g+\ddot{d}_1) + c_2 d_1 \dot{\theta}_2^2 + s_2 d_1 \ddot{\theta}_2 \right] \\ \frac{1}{4} m_2 L_2^2 \ddot{\theta}_2 + I_{zz2} \ddot{\theta}_2 \end{pmatrix} $$

PR $\quad \tau_1 = {}^1f_1^T\,{}^1\hat{z}_1 = (m_1+m_2)(g+\ddot{d}_1)$

$\quad \tau_2 = {}^2n_2^T\,{}^2\hat{z}_2 = \frac{1}{4} m_2 L_2^2 \ddot{\theta}_2 + I_{zz2} \cdot \ddot{\theta}_2$

Problem 3: (16 points)

Consider a system in which a motor can rotate a sphere with mass m_{kugel} in a viscous fluid. The length of the lever is l and the sphere has a



radius r_{kugel}. The individual sub-systems have the following inertia tensors for a rotation about the center of gravity:

$$I_{Stick} = \frac{m_{Stick}}{12} l^2 \quad \text{and} \quad I_{Kugel} = \frac{2}{5} m_{Kugel} r_{Kugel}^2$$

a) What is the inertia tensor for the given rotation of, in which the rotational axes of the individual sub-objects are displaced in parallel in each case (2 points)

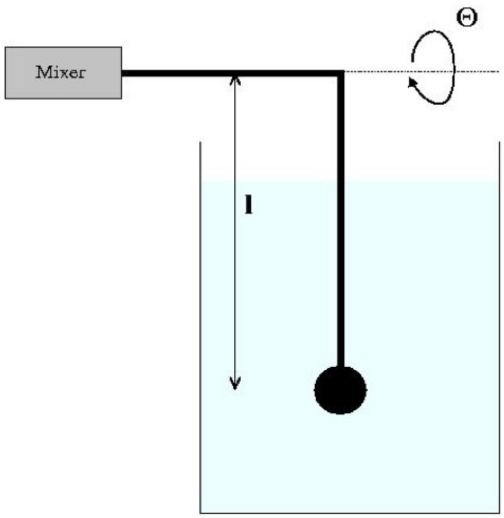
The formula for viscous friction depends on the translational velocity v and is at low speed estimated to

$$F = -6 \cdot \pi \cdot \eta \cdot r \cdot v$$

b) Calculate τ, the torque that must be applied by the motor to stabilize the ball to a specific angle Θ? (The ball is constantly in the liquid) (5 points)
   - How can τ be estimated for this system?
   - How is the friction force F related to τ ? (for the viscous friction only the influence of the ball should be taken into account).
     How does the gravitational force on τ? Again, the weight of the stick is to be mstick neglected.

c) the paddle is to be kept at a specific angular position. What is an appropriate controller for this application? Draw the controller structure (5 points)

d) Explain briefly the principle of control law partitioning at this example. (3 points)



$$I_{Stick} = \frac{m_{Stick}}{12} l^2 \quad \text{and} \quad I_{Kugel} = \frac{2}{5} m_{Kugel} r_{Kugel}^2$$

a)

$$I' = I + md^2$$

$$I' = \frac{m_{stick}}{12} l^2 + m_{stick}\left(\frac{1}{2}l\right)^2 + \frac{2}{5} m_{kugel} r_{kugel}^2 + m_{kugel} \cdot l^2$$

$$= \frac{1}{3} m_{stick} l^2 + m_{kugel}\left(\frac{2}{5} r_{kurgel}^2 + l^2\right)$$

b)

$$F = -6 \cdot \pi \cdot \eta \cdot r \cdot v = -6\pi\eta r_{kurger} \cdot l \cdot \dot{\Theta}$$

$$\tau = m_{kurgel}\, g\, l \cdot \sin\theta + I\ddot{\theta} + 6\pi\eta r_{kurgel} \cdot l^2 \dot{\theta}$$

$$N_{\hat{z}} = I \cdot \ddot{\theta} = \tau + F_{riction} \cdot l + F_G \cdot \sin\theta \cdot l$$

$$I \cdot \ddot{\theta} = \tau - 6\pi \cdot \eta r_{kugel} \cdot l^2 \cdot \dot{\theta} - m_{kugel} \cdot g \cdot l^2 \sin\theta$$

$$\tau = I\ddot{\theta} + 6\pi\eta \cdot r_{kurgel} \cdot l^2 \cdot \dot{\theta} + m_{kurgel} \cdot g \cdot l^2 \sin\theta$$

c) Trajectr Folln

$$I\ddot{\theta} + 6\pi\eta r_{kurgel} \cdot L^2 \cdot \dot{\theta} + m_{kurgel} \cdot g \cdot l \cdot \sin\theta = \tau = \alpha\tau' + \beta$$

$$\begin{cases} \alpha = I \\ \beta = 6\pi\eta r_{kurgel} \cdot L^2 \dot{\theta} + m_{kurgel} \cdot g \cdot l \cdot \sin\theta . \\ \tau' = \ddot{\theta} = -k_v \dot{\theta} + k_p e \end{cases}$$