# Robotik (Robotics)

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| **Exam:** | IN2067 / Retake 1 | **Date:** | Saturday 27th June, 2020 |
| **Examiner:** | Prof. Darius Burschka | **Time:** | 10:45 – 11:55 |

## Working instructions

- This exam consists of **14 pages** with a total of **4 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 83 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
- **Finish the voluntary identification process on the next page before solving the exam**
- **Subproblems marked by * can be solved without results of previous subproblems.**
- **Do not write with red or green colors nor use pencils.**
- **Due to dificulty in online entry, we ask often just for the final results. If you have time, you can add intermediate steps for longer derivations as a photo of your handwriting on the last 2 pages of the exam.**
- **Return all pages of the PDF!**

## Problem 1 Authentication (0 credits)

The process is meant to help in disputes about possible fraud accusations later. You are supposed to sit in the room alone.

0
1

a) (This step is voluntary proctoring) Take two pictures of your empty room from diagonal back corners showing your desk with the TUMexam page visible open and insert them into the box below. We may require further BBB processing if not provided.

0
1

b) Write in your own handwriting (using mouse or pen in your PDF editor) the following text hand-written by you: *Solved without any further help*

# Problem 2 Kinematics (33 credits)

A 6DoF manipulator depicted in Fig. 2.1 is used in the following analysis of kinematics. Base frame {0} is shown in the figure as xyz at the bottom of the base.

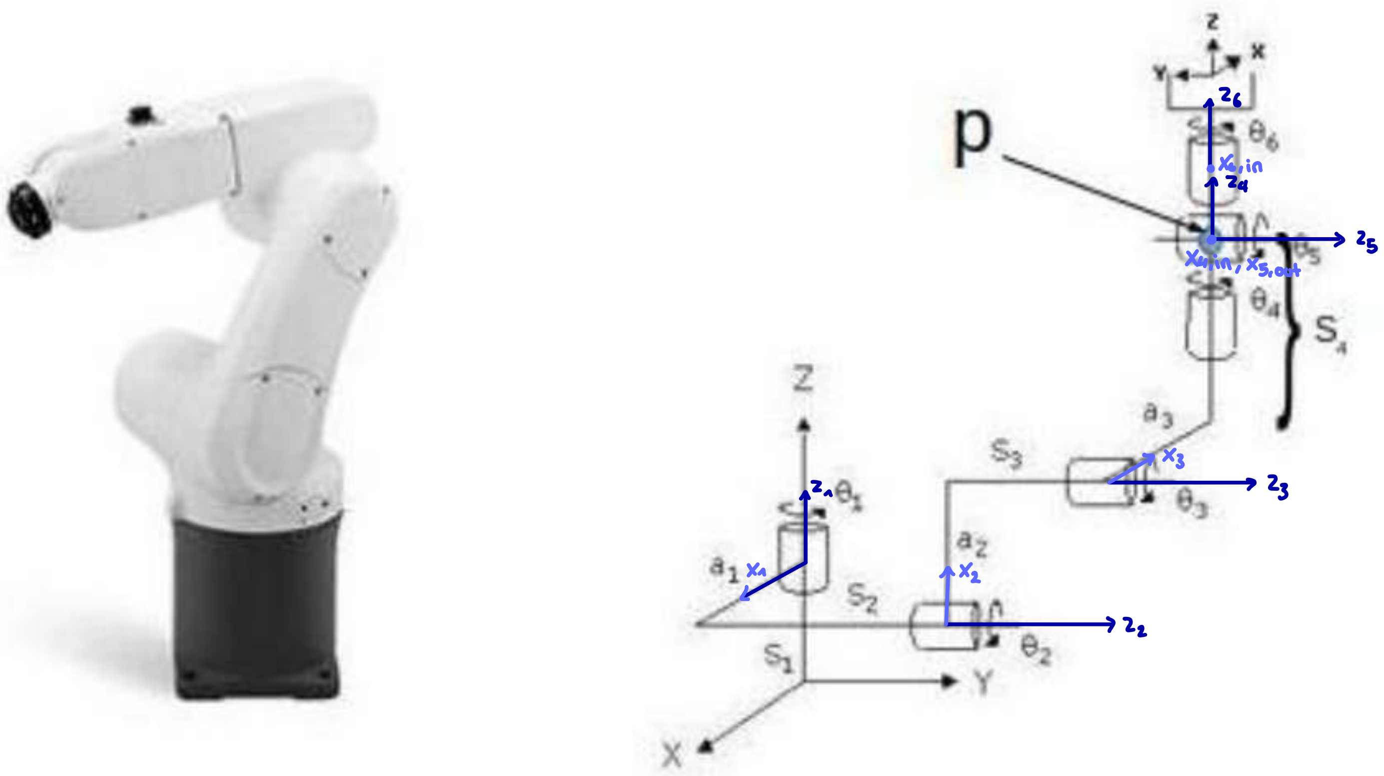



Figure 2.1: 6Dof Robot

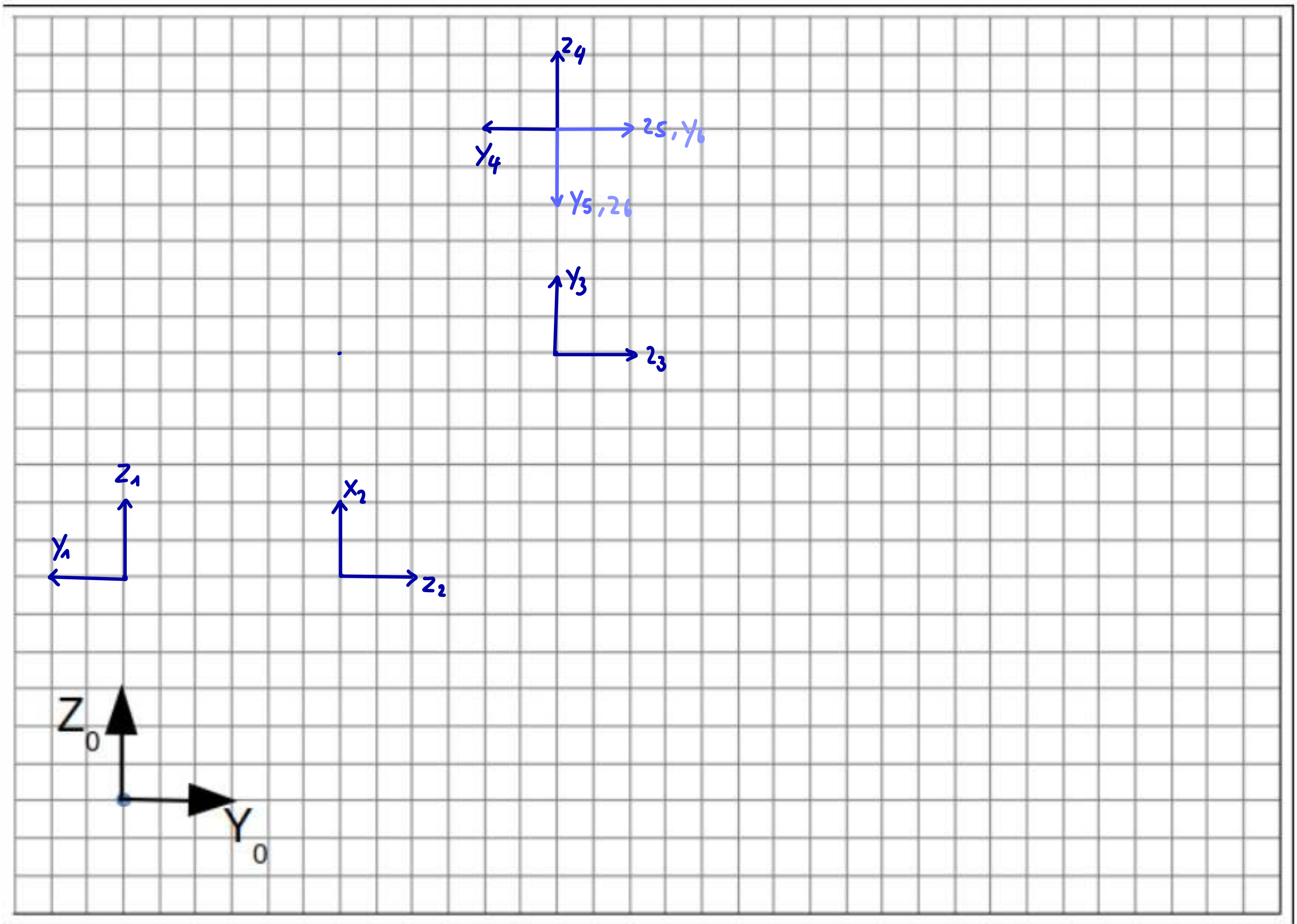
a)* Write the DH parameters (Crag's notation) for the robot in Fig. 2.1 in a DH-table. Give the value of $\Theta_i$ for a configuration depicted in the Fig. 2.1 for each line in the DH-table.

0 1 2 3 4 5 6

| | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | $S_1$ | $\theta_1 = 0$ |
| 2 | -90 | $a_1$ | $S_2$ | $\theta_2 = -90°$ |
| 3 | 0 | $a_2$ | $S_3$ | $\theta_3 = -90°$ |
| 4 | -90 | $a_3$ | $S_4$ | $\theta_4 = 0$ |
| 5 | 90° | 0 | 0 | $\theta_5 = 0$ |
| 6 | -90° | 0 | 0 | $\theta_6 = 0$ |

0 1 2 3 4 5 6

b)* Assign the position and orientation of the link coordinate frames for the manipulator above. Assuming following length parameters ($S_{\{1-4\}}$ = 3[*units*]) and ($a_{\{1-3\}}$ = 3[*units*]) draw the coordinate frames on the solution sheet with 1[unit]=2squares. Project frame origins, which are not within the yz-plane of the base system, into the plane of the drawing under the assumption that the depicted $\Theta_1 = 0°$. Draw only the arrows for coordinates in the image plane as shown for the frame {0}. (You can replace the grid with a picture of your drawing but the size needs to fit into the box and the grid on your photographed sheet needs to be visible).




0 1 2 3

c)* Do all joint angles $\Theta_i$ contribute to the estimation of the position of point P in the coordinates of the base frame in Fig. 2.1? How do we call such structure on the manipulator and what is it used for? What is the function of the joints contributing to this special structure?

Only $\theta_1$, $\theta_2$ and $\theta_3$ contribute to the position and rotation of P.

$\theta_4$, $\theta_5$ and $\theta_6$ intersect in 1 point and represent a wrist.

$\theta_4$, $\theta_5$ and $\theta_6$ adjust the orientation of the wrist.

d)* How do the parameter from the DH-table contribute to the transformation matrix $T_i$ according to the distal notation in Craig's book (equation for the matrix)?

0
1

$$ {}^{i-1}_{\ \ 1}T = \begin{pmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, S_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, S_i \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

e) Write the equations, how you calculate consecutive positions of the joint origins with a symbolic (just symbols, no numbers) equation. Estimate the position of the point P in coordinate frame of the base coordinate system. Write just resulting position vectors of all the coordinate frame origins long the way (here a symbolic and numerical values using structural length from above).

0
1
2
3
4
5
6
7

The consecutive position of the joint origins can be calculated by multiplying with the transformation matrix $\quad {}^0P_i = {}^0_iT\ {}^iP_i = {}^0_1T\ {}^1_2T \dots {}^{i-1}_{\ \ i}T\ {}^iP_i$

$$ {}^0P_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} $$

**Notation**

$C_i = \cos(\theta_i)$
$S_i = \sin(\theta_i)$
$d_i = S_i$

$$ {}^0_1T = \begin{pmatrix} \cos 1 & \sin 1 & 0 & 0 \\ -\sin 1 & \cos 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow {}^0_1P = \begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix} $$

$$ {}^0_2T = \begin{pmatrix} C_1C_2 & -C_1S_2 & -S_1 & a_1C_1 - d_2S_1 \\ C_2S_1 & -S_1S_2 & C_1 & d_2C_1 + a_1S_1 \\ -S_2 & -C_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow {}^0_2P = \begin{pmatrix} a_1C_1 - d_2S_1 \\ d_2C_1 + a_1S_1 \\ d_1 \end{pmatrix} = 3 \cdot \begin{pmatrix} C_1 - S_1 \\ C_1 + S_1 \\ 1 \end{pmatrix} $$

$$ {}^0_3T = \begin{pmatrix} C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1C_3S_2 & -S_1 & a_1C_1 - d_2S_1 - d_3S_1 + a_2C_1C_2 \\ C_1C_3S_1 - S_1S_2S_3 & -C_2S_1S_3 - C_3S_1S_2 & C_1 & d_2C_1 + d_3C_1 + a_1S_1 + a_2C_2S_1 \\ -C_2S_3 - C_3S_2 & S_2S_3 - C_2C_3 & 0 & d_1 - a_2S_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ \Rightarrow {}^0_3P = \begin{pmatrix} a_1C_1 - d_2S_1 - d_3S_1 + a_2C_1C_2 \\ d_2C_1 + d_3C_1 + a_1S_1 + a_2C_2S_1 \\ d_1 - a_2S_2 \end{pmatrix} $$

$$ \Rightarrow {}^0_4P = \begin{pmatrix} a_1C_1 - d_2S_1 - d_3S_1 + a_2C_1C_2 \\ d_2C_1 + d_3C_1 + a_1S_1 + a_2C_2S_1 \\ d_1 - a_2S_2 \end{pmatrix} $$

$$ {}^0_4T = \begin{pmatrix} S_1S_4 - C_4(C_1S_2S_3 - C_1C_2C_3) & -C_1C_2S_3 - C_1C_3S_2 & -S_1 & a_1C_1 - d_2S_1 - d_3S_1 + a_2C_1C_2 \\ -C_1S_4 - C_4(S_1S_2S_3 - C_2C_3S_1) & -C_2S_1S_3 - C_3S_1S_2 & C_1 & d_2C_1 + d_3C_1 + a_1S_1 + a_2C_2S_1 \\ -C_2S_3 - C_3S_2 & S_2S_3 - C_2C_3 & 0 & d_1 - a_2S_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

0 1 2 3 4 5 6

f)* Estimate the translational and angular velocities (${}^i\vec{v}_i, {}^i\vec{\omega}_i$) along the kinematic chain for i=1,2,3. Write the generic equations, how to iteratively do it and give just the vector entries of the results. Assume ${}^0\vec{v}_0 = \vec{0}, {}^0\vec{\omega}_0 = \vec{0}$.

$$ {}^1\omega_1 = {}^1_0R \cdot {}^0\omega_0 + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot\theta_1 \end{pmatrix} $$

$$ {}^2\omega_2 = {}^2_1R \cdot {}^1\omega_1 + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} = \begin{pmatrix} C_2 & 0 & -S_2 \\ S_2 & 0 & -C_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot\theta_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} = \begin{pmatrix} -S_2\dot\theta_1 \\ -C_2\dot\theta_1 \\ \dot\theta_2 \end{pmatrix} $$

$$ {}^3\omega_3 = {}^3_2R \cdot {}^2\omega_2 + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_3 \end{pmatrix} = \begin{pmatrix} c_3 & S_3 & 0 \\ -S_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -S_2\dot\theta_1 \\ -C_2\dot\theta_1 \\ \dot\theta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_3 \end{pmatrix} = \begin{pmatrix} -C_3S_2\dot\theta_1 - S_3C_2\dot\theta_1 \\ +S_3S_2\dot\theta_1 - C_3C_2\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{pmatrix} $$

$$ {}^1v_1 = {}^1_0R\,(0+0) + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} $$

$$ {}^2v_2 = {}^2_1R\,(0 + {}^1\omega_1 \cdot {}^1_2P) = {}^2_1R\left( \begin{pmatrix} 0 \\ 0 \\ \dot\theta_1 \end{pmatrix} \times \begin{pmatrix} a_1 \\ d_2 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} C_2 & 0 & -S_2 \\ -S_2 & 0 & -C_2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -\dot\theta_1 d_2 \\ \dot\theta_1 a_1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\dot\theta_1 d_2 C_2 \\ +\dot\theta_1 d_2 S_2 \\ \dot\theta_1 a_1 \end{pmatrix} $$

$$ {}^3v_3 = {}^3_2R\,({}^2v_2 + {}^2\omega_2 \cdot {}^2_3P) = {}^3_2R\left( \begin{pmatrix} -\dot\theta_1 d_2 c_2 \\ \dot\theta_1 d_2 S_2 \\ \dot\theta_1 a_1 \end{pmatrix} + \begin{pmatrix} -S_2\dot\theta_1 \\ -C_2\dot\theta_1 \\ \dot\theta_2 \end{pmatrix} \times \begin{pmatrix} a_2 \\ 0 \\ d_3 \end{pmatrix} \right) = $$

$$ = \begin{pmatrix} c_3 & S_3 & 0 \\ -S_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\dot\theta_1 d_2 C_2 - C_2\dot\theta_1 d_3 \\ \dot\theta_1 d_2 S_2 - S_2\dot\theta_1 d_3 + \dot\theta_2 a_2 \\ \dot\theta_1 a_1 - C_2\dot\theta_1 a_2 \end{pmatrix} = \begin{pmatrix} -\dot\theta_1 d_2 C_2C_3 - \dot\theta_1 d_3 C_2C_3 + \dot\theta_1 d_2 S_2S_3 - \dot\theta_1 d_3 S_2S_3 + S_3\dot\theta_2 a_2 \\ \dot\theta_1 d_2 S_3C_2 + \dot\theta_1 d_3 S_3C_2 + \dot\theta_1 d_2 S_2C_3 - \dot\theta_1 d_3 S_2C_3 + \dot\theta_2 a_2 C_3 \\ \dot\theta_1 a_1 - C_2\dot\theta_1 a_2 \end{pmatrix} = $$

$$ = \begin{pmatrix} -\dot\theta_1 d_2 C_{23} - \dot\theta_1 d_3 C_{23} + S_3\dot\theta_2 a_2 \\ \dot\theta_1 d_2 S_{32} + \dot\theta_1 d_3 S_{23} + \dot\theta_2 a_2 C_3 \\ \dot\theta_1 a_1 - C_2\dot\theta_1 a_2 \end{pmatrix} = \begin{pmatrix} -\dot\theta_1 d_{23} C_{23} + S_3\dot\theta_2 a_2 \\ \dot\theta_1 d_{23} S_{23} + \dot\theta_2 a_2 C_3 \\ \dot\theta_1 a_1 - \dot\theta_1 C_2 a_2 \end{pmatrix} $$

0 1 2 3 4

g) We are interested just in the structure built from the first 3 joints i=1,2,3 until point P. Write the matrix entries for ${}^3J_3$ using results from 2f) . How can this matrix be transformed into ${}^0J_3$ representation?

$$ {}^3J_3 = \begin{pmatrix} -d_{23}C_{23} & S_3 a_2 & 0 \\ d_{23}S_{23} & a_2 C_3 & 0 \\ a_1 - C_2 a_2 & 0 & 0 \\ -S_{23} & 0 & 0 \\ -C_{23} & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad {}^0J_3 = \begin{pmatrix} {}^0_3R & 0 \\ 0 & {}^0_3R \end{pmatrix} {}^3J_3 $$

## Problem 3 Dynamics (33 credits)

An RRP robot is in Fig. 3.1. The gravitational force applies in negative $z_0$ direction. The entire base structure rotates around a vertical axis $\Theta_1$ and the positive rotation direction of the second joint is shown as $\Theta_2$.

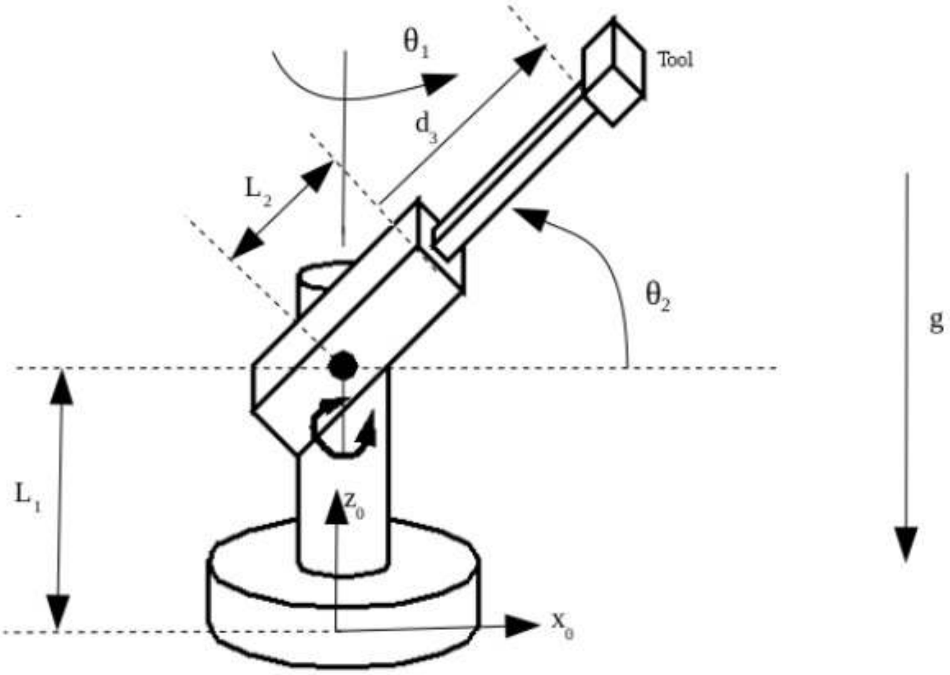



Figure 3.1: RRP Robot

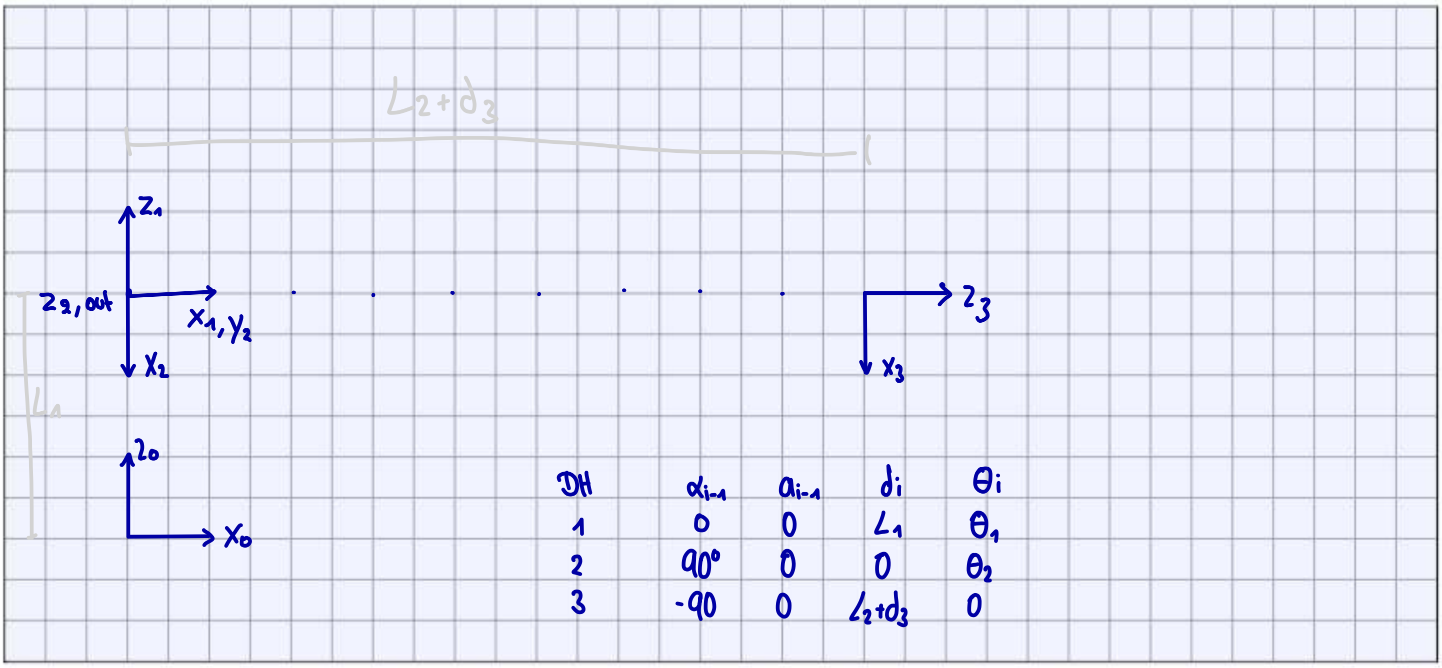
The inertia tensors for each link $i \in \{1, 2, 3\}$ have identical form:

$$^{C_i}I_i = \begin{pmatrix} I_{xxi} & 0 & 0 \\ 0 & I_{yyi} & 0 \\ 0 & 0 & I_{zzi} \end{pmatrix}$$

The masses of the robot's links are $(m_1, m_2, m_3)$ and the centers of mass of the links are located in the middle of the rigid link structure $(L_1/2, L_2/2, (L_3 + d_3)/2)$.

a)* Draw the coordinate frames for this robot for assuming that the depicted configuration has $\Theta_1 = 0$ and for $\Theta_2 = 0°$, $L_1 = L_2 = 2[units]$, $d_3 = 4[unit]$. Use 3 grid elements for 1 [unit].

0 1 2 3 4




| DH | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | $L_1$ | $\theta_1$ |
| 2 | 90° | 0 | 0 | $\theta_2$ |
| 3 | -90 | 0 | $L_2+d_3$ | 0 |

0 1 2 3 4 5 6 7 8 9

b)* Calculate the velocities (${}^0v_i, {}^0v_{C_i}, {}^i\omega_i$). Give just the resulting vectors.

$${}^0_1T = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad {}^1_2T = \begin{pmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^2_3T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_2+d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^0\dot{v}_0 = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$

Positions of the centers of mass

$${}^0p_{C_1} = \begin{pmatrix} 0 \\ 0 \\ L_1/2 \end{pmatrix} \qquad {}^0p_{C_2} = \begin{pmatrix} L_2/2 \cdot c_1 c_2 \\ L_2/2 \cdot s_1 c_2 \\ L_1 + L_2/2\, s_2 \end{pmatrix} \qquad {}^0p_{C_3} = \begin{pmatrix} \left(L_2 + \frac{L_3+d_3}{2}\right) \cdot c_1 c_2 \\ \left(L_2 + \frac{L_3+d_3}{2}\right) s_1 c_2 \\ L_1 + \left(L_2 + \frac{L_3+d_3}{2}\right) s_2 \end{pmatrix}$$

$$L_2 + L_3 = L$$

Positions of the end positions of the links

$$L_2 + \frac{L_3}{2} := \frac{L}{2}$$

$${}^0p_1 = \begin{pmatrix} 0 \\ 0 \\ L_1 \end{pmatrix} \qquad {}^0p_2 = \begin{pmatrix} L_2 c_1 c_2 \\ L_2 s_1 c_2 \\ L_1 + L_2 s_2 \end{pmatrix} \qquad {}^0p_3 = \begin{pmatrix} (L_2+L_3+d_3)\, c_1 c_2 \\ (L_2+L_3+d_3)\, s_1 c_2 \\ L_1 + (L_2+L_3+d_3)\, s_2 \end{pmatrix}$$

$\underline{i=1}$

$${}^0v_1 = \frac{d}{dt}\, {}^0p_1 = \vec{0} \qquad {}^0v_{C_1} = \frac{d}{dt}\, {}^0p_{C_1} = \vec{0} \qquad {}^1\omega_1 = {}^1_0R\, {}^0\omega_0 + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot\theta_1 \end{pmatrix}$$

$\underline{i=2}$

$${}^0v_2 = \begin{pmatrix} -L_2 s_1 \dot\theta_1 c_2 - L_2 c_1 s_2 \dot\theta_2 \\ L_2 c_1 \dot\theta_1 c_2 - L_2 s_1 s_2 \dot\theta_2 \\ L_2 c_2 \dot\theta_2 \end{pmatrix} \qquad {}^0v_{C_2} = \frac{1}{2}\, {}^0v_2$$

$${}^2\omega_2 = {}^2_1R\, {}^1\omega_1 + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} = \begin{pmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot\theta_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\theta_2 \end{pmatrix} = \begin{pmatrix} s_2 \dot\theta_1 \\ c_2 \dot\theta_1 \\ \dot\theta_2 \end{pmatrix}$$

$\underline{i=3}$

$${}^0v_3 = \begin{pmatrix} -(L+d_3)\, s_1 \dot\theta_1 c_2 - (L+d_3)\, c_1 s_2 \dot\theta_2 + \dot d_3\, c_1 c_2 \\ (L+d_3)\, c_1 \dot\theta_1 c_2 - (L+d_3)\, s_1 s_2 \dot\theta_2 + \dot d_3\, s_1 c_2 \\ (L+d_3)\, c_2 \dot\theta_2 + \dot d_3\, s_2 \end{pmatrix} \qquad {}^0v_{C_3} = \frac{1}{2}\, {}^0v_C$$

$${}^3\omega_3 = {}^3_2R\, {}^2\omega_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s_2 \dot\theta_1 \\ c_2 \dot\theta_1 \\ \dot\theta_2 \end{pmatrix} = \begin{pmatrix} s_2 \dot\theta_1 \\ -\dot\theta_2 \\ c_2 \dot\theta_1 \end{pmatrix}$$

c) Calculate the torques and forces in the joints that are necessary to move the robot using the Lagrangian method. (Give the kinetic energies and potential energy expression for the 3 joints, generic equation for generalized torque $\tau_i$ and the resulting expressions for torques or forces for the joints.

$\underline{i=1}$ $\quad k_1 = \frac{1}{2} m_1 \cdot 0 + \frac{1}{2} {}^1\omega_1 \, {}^{C_1}I_1 \, {}^1\omega_1 = \frac{1}{2} I_{zz_1} \dot{\theta}_1^2$

$U_1 = + m_1 g \frac{L_1}{2}$

$\underline{i=2}$ $\quad k_2 = \frac{1}{8} m_2 L_2^2 \left( (s_1 c_1 \dot{\theta}_1 - c_1 s_2 \dot{\theta}_2)^2 + (c_1 \dot{\theta}_1 c_2 - s_1 s_2 \dot{\theta}_2)^2 + (c_2 \dot{\theta}_2)^2 \right) + \frac{1}{2} \left( I_{xx_2} (s_2 \dot{\theta}_1)^2 + I_{yy_2} (c_2 \dot{\theta}_1)^2 + I_{zz_2} \dot{\theta}_2^2 \right)$

$= \frac{m_2}{8} L_2^2 \left( c_1^2 \dot{\theta}_1^2 + s_2^2 \dot{\theta}_2^2 + 2 s_1 c_1 \dot{\theta}_1 c_1 s_2 \dot{\theta}_2 - 2 c_1 c_2 s_1 s_2 \dot{\theta}_1 \dot{\theta}_2 + (c_2 \dot{\theta}_2)^2 + \frac{1}{2} I_{xx_2} s_2^2 \dot{\theta}_1^2 + I_{yy_2} c_2^2 \dot{\theta}_1^2 + I_{zz_2} \dot{\theta}_2^2 \right.$

$= \frac{m_2}{8} L_2^2 \left( c_1^2 \dot{\theta}_1^2 + \dot{\theta}_2^2 - 2 s_1 c_1 \dot{\theta}_1 \dot{\theta}_2 s_2 (c_1 + s_1) \right) + \ldots$

$U_2 = m_2 \left( L_1 + \frac{L_2}{2} s_2 \right) g$

...

## Problem 4 Control (17 credits)

An analogy often used to simulate Spring-Mass-Damper (SMD) systems is depicted in Fig. 4.1. The current **i** flowing through the coil L, resistor R and capacity C creates voltages shown next to the corresponding arrows. The sum of the voltages sums up to $U_e$.

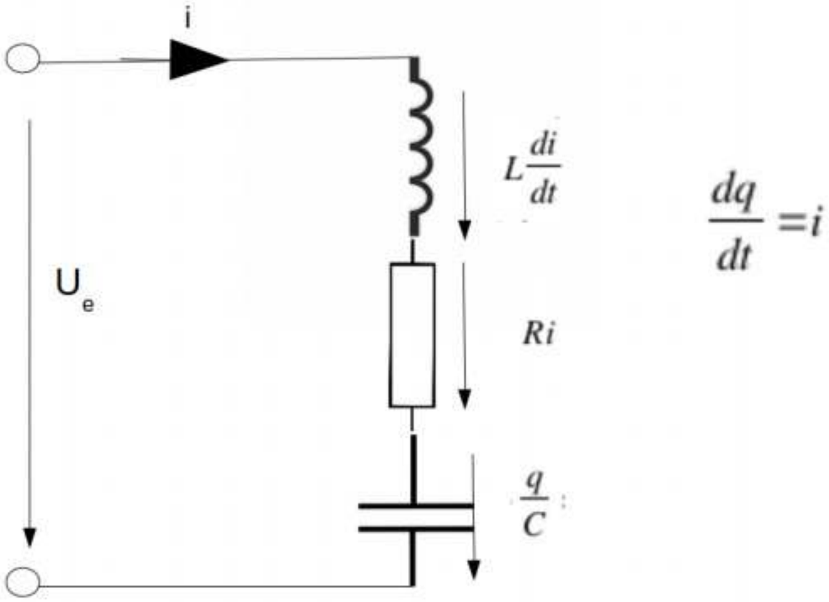



Figure 4.1: Electrical SMD simulation.

a)* Write an equation for $U_e$ as the sum of the other voltages along the arrows on the right side. Use the relation between q and i in the image to convert q in the voltage on the resistor R into an expression using just i.

$$U_e = L\frac{di}{dt} + Ri + \frac{q}{c}$$

$$= L\frac{di}{dt} + R\cdot i + \frac{\int i\, dt}{c}$$

b) You should have now above a second order differential equation. Which values from the circuit in Fig. 4.1 above correspond to mass m, stiffness k, and damping b in SMD?

0 1 2 3 4 5 6

$$\dot{U}_e = L\ddot{i} + R\dot{i} + \frac{1}{q} i$$

$$L\ddot{i} + R\dot{i} + \frac{1}{q} i - \dot{U}_e = 0$$

$$m\ddot{x} + b\dot{x} + kx = 0$$

$m = L$

$b = R$

$k = \frac{1}{q}$

c)* Explain the idea behind control-law partitioning (2 sentences + 1 equation)

0 1 2 3

$$m\ddot{x} + b\dot{x} + kx = \alpha f' + \beta$$

Decouple mass-dependent part from the equation with $f = \alpha f' + \beta$. $f'$ is new input to system.

To make the system appear as unit mass, choose: $\alpha = m$, $\beta = \dot{b}x + kx$, $\ddot{x} = f'$

d) What are the $\alpha$ and $\beta$ expression for this problem? Which control law f controlling the current applies to keep the current i on a specified time evolution $(\ddot{i}_d(t), \dot{i}_d(t), i_d(t))$.

0 1 2 3 4 5

$f' = \ddot{i}$ $\alpha = L$ $\beta = R\dot{i} + \frac{1}{q} i$

$f = \ddot{i}_d + k_v \dot{e} + k_p e$ $e = i_d - i$ $\dot{e} = \dot{i}_d - \dot{i}$