Fundamentals of Artificial Intelligence
Exercise 6: Inference in First Order Logic

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Friday 15$^{\text{th}}$ December, 2023

Problem 6.1: The man in the painting

A man stands in front of a painting and says the following:

> *Brothers and sisters have I none, but that man's father is my father's son.*

What is the relationship between the man in the painting and the speaker?

Problem 6.1: The man in the painting

To solve the riddle with first-order logic, use the predicates

$$\begin{aligned} Male(x) &: x \text{ is male.} \\ Father(x, y) &: x \text{ is the father of } y. \\ Son(x, y) &: x \text{ is a son of } y. \\ Parent(x, y) &: x \text{ is a parent of } y. \\ Child(x, y) &: x \text{ is a child of } y. \\ Sibling(x, y) &: x \text{ is a sibling of } y \end{aligned}$$

and the knowledge

- A sibling is another child of one's parents.

$$\forall x, y \quad Sibling(x, y) \;\Leftrightarrow\; x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)$$

- Parent and child are inverse relations.

$$\forall p, c \quad Parent(p, c) \;\Leftrightarrow\; Child(c, p)$$

Problem 6.1: The man in the painting

Problem 6.1.1: Define the rule which relates *son*, *child*, and *male*. Define also the rule which relates *father*, *parent*, and *male*.

- Every son is a male child, and every male child is a son:

$$\forall s, p \quad Son(s,p) \Leftrightarrow (Male(s) \land Child(s,p))$$

- Every father is a male parent, and every male parent is a father:

$$\forall c, f \quad (Father(f,c) \Leftrightarrow (Parent(f,c) \land Male(f)))$$

Problem 6.1: The man in the painting

Problem 6.1.2: Using the constants *Me* for the speaker and *That* for the person depicted in the painting, formalize the sentences regarding the sexes of the people in the puzzle.

Male(That)

Male(Me)

Problem 6.1: The man in the painting

Problem 6.1.3: Formalize the sentences "Brothers and sisters have I none" and "That man's father is my father's son" in first-order logic.

- Brothers and sisters have I none: I don't have siblings

$$\forall x\ \neg Sibling(x, Me) \underbrace{\wedge \neg Sibling(Me, x)}$$

↳ we need this part, because the computer does not know that "Sibling of" is a symmetric relation

- $\underbrace{\underbrace{\text{That man's}}_{That}\text{ father}}_{f_1}$ is $\underbrace{\underbrace{\text{my}}_{Me}\text{ father's}}_{f_2}$ son:

$$\exists f_1, f_2\quad Father(f_1, That) \wedge Father(f_2, Me) \wedge Son(f_1, f_2)$$

Problem 6.1: The man in the painting

Problem 6.1.4: Solve this puzzle informally and decide what is the relation between the man in the painting and the speaker.

"... but that man's father is $\underbrace{\text{my father's son}}_{\text{Me or one of my siblings}}$."

But: "Brothers and sisters have I none"

↳ No siblings ⤳ My father's son = Me

Man in painting: Son of Me
That

Me is father of man in painting

Son(That, Me)

Alternative:
(Father(Me, That))

Problem 6.1: The man in the painting

Problem 6.1.5: Using the resolution technique for first-order logic, prove your answer.

We want to show

$$KB \models \alpha$$

where $\alpha = Son(That, Me)$

Reminder: Conversion to CNF

The following steps need to be performed to convert a first-order logic formula into Conjunctive Normal Form:

- **Eliminate implications** → like propositional logic $A \Rightarrow B \equiv \neg A \lor B$
- **Move $\neg$ inwards** → $\neg \forall x \; \varphi \equiv \exists x \; \neg\varphi$
  → $\neg \exists x \; \varphi \equiv \forall x \; \neg\varphi$ + De Morgan for $\land$ and $\lor$
- **Standardize variables**
- **Skolemization** → Replace $\exists$ with function / constant
- **Drop universal quantifiers** → make universal quantification implicit
- **Distribute $\lor$ over $\land$** → like propositional logic

$(\underline{\forall x} \; P(x)) \land (\underline{\exists x} \; Q(x))$

↪ $(\forall x \; P(x)) \land (\exists y \; Q(y))$

Problem 6.1: The man in the painting

"A sibling is another child of one's parents."

$$\forall x, y \quad Sibling(x, y) \Leftrightarrow x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)$$

Problem 6.1: The man in the painting

"A sibling is another child of one's parents."

$$\begin{aligned}
&\forall x, y \quad Sibling(x, y) \Leftrightarrow x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y) \\
&\equiv \forall x, y \quad [Sibling(x, y) \Rightarrow x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)] \\
&\quad \land [Sibling(x, y) \Leftarrow x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)]
\end{aligned}$$

Problem 6.1: The man in the painting

"A sibling is another child of one's parents.", direction $\Rightarrow$

$$\forall x, y \quad Sibling(x, y) \Rightarrow x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)$$

Problem 6.1: The man in the painting

"A sibling is another child of one's parents.", direction $\Rightarrow$

$$\forall x, y \quad Sibling(x, y) \ \Rightarrow \ x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)$$
$$\equiv \forall x, y \quad \neg Sibling(x, y) \lor [x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)]$$

Problem 6.1: The man in the painting

“A sibling is another child of one’s parents.”, direction $\Rightarrow$

$$\forall x, y \quad Sibling(x, y) \;\Rightarrow\; x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)$$
$$\equiv \forall x, y \quad \neg Sibling(x, y) \vee [x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)]$$
$$\equiv \forall x, y \quad \neg Sibling(x, y) \vee [x \neq y \wedge Parent(F(x, y), x) \wedge Parent(F(x, y), y)]$$

Sholemization

Let's play a game! You tell me any natural number x.

I win, if I can always tell you a number that is larger than x.

You win, if you find a number where I cannot do that.

Formally: I win, if $\underline{\forall x}\ \underline{\exists y}\ y > x$ is a valid sentence. $\longleftrightarrow\ \forall x\ F(x) > x$

I can always win, but with which strategy?

If you tell me x, I say $x+1$

↳ $\text{Strategy}(x) = x+1$

$\Rightarrow$ My strategy defines a mapping (function)

from the values you choose (universally quantified variables)

to the value I have to choose (existentially quantified variable)

If the sentence is valid, this Sholem function always exists

Problem 6.1: The man in the painting

"A sibling is another child of one's parents.", direction $\Rightarrow$

$$
\begin{aligned}
&\forall x, y \quad Sibling(x, y) \;\Rightarrow\; x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y) \\
&\equiv \forall x, y \quad \neg Sibling(x, y) \lor [x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)] \\
&\equiv \forall x, y \quad \neg Sibling(x, y) \lor [x \neq y \land Parent(F(x, y), x) \land Parent(F(x, y), y)] \\
&\equiv \forall x, y \quad (\neg Sibling(x, y) \lor (x \neq y)) \land (\neg Sibling(x, y) \lor Parent(F(x, y), x)) \\
&\qquad\quad \land (\neg Sibling(x, y) \lor Parent(F(x, y), y))
\end{aligned}
$$

Problem 6.1: The man in the painting

"A sibling is another child of one's parents.", direction $\Leftarrow$

$$\forall x, y \quad Sibling(x, y) \Leftarrow x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)$$
$$\equiv \forall x, y \quad x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y) \Rightarrow Sibling(x, y)$$

Problem 6.1: The man in the painting

"A sibling is another child of one's parents.", direction $\Leftarrow$

$$\forall x, y \quad Sibling(x, y) \ \Leftarrow \ x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)$$
$$\equiv \forall x, y \quad x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y) \ \Rightarrow \ Sibling(x, y)$$
$$\equiv \forall x, y \quad \neg(x \neq y \land \exists p \quad Parent(p, x) \land Parent(p, y)) \ \lor \ Sibling(x, y)$$

Problem 6.1: The man in the painting

"A sibling is another child of one's parents.", direction $\Leftarrow$

$$\forall x, y \quad Sibling(x, y) \;\Leftarrow\; x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)$$
$$\equiv \forall x, y \quad x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y) \;\Rightarrow\; Sibling(x, y)$$
$$\equiv \forall x, y \quad \neg(x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)) \;\vee\; Sibling(x, y)$$
$$\equiv \forall x, y \quad (x = y \vee \forall p \quad \neg Parent(p, x) \vee \neg Parent(p, y)) \;\vee\; Sibling(x, y)$$

Problem 6.1: The man in the painting

“A sibling is another child of one’s parents.”, direction $\Leftarrow$

$$
\begin{aligned}
&\forall x, y \quad Sibling(x, y) \;\Leftarrow\; x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y) \\
&\equiv \forall x, y \quad x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y) \;\Rightarrow\; Sibling(x, y) \\
&\equiv \forall x, y \quad \neg(x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)) \;\vee\; Sibling(x, y) \\
&\equiv \forall x, y \quad (x = y \vee \forall p \quad \neg Parent(p, x) \vee \neg Parent(p, y)) \;\vee\; Sibling(x, y) \\
&\equiv \forall x, y, p \quad \underbrace{x = y \vee \neg Parent(p, x) \vee \neg Parent(p, y) \;\vee\; Sibling(x, y)}
\end{aligned}
$$

Problem 6.1: The man in the painting

“That man’s father is my father’s son.”

$$\underline{\exists f_1, f_2} \quad Father(f_1, That) \wedge Father(f_2, Me) \wedge Son(f_1, f_2)$$

Skolemize: Constants F_1, F_2

Problem 6.1: The man in the painting

“That man’s father is my father’s son.”

$$\begin{aligned} \exists f_1, f_2 \quad & Father(f_1, That) \wedge Father(f_2, Me) \wedge Son(f_1, f_2) \\ \equiv \quad & Father(F_1, That) \wedge Father(F_2, Me) \wedge Son(F_1, F_2) \end{aligned}$$

Problem 6.1: The man in the painting

Goal: $\alpha = Son(That, Me)$

↳ Add negated goal to KB

$\neg Son(That, Me)$

Recall: Resolution is proof by contradiction ("reductio ad absurdum")

Steps: 1. Assume $\neg\alpha$

2. Derive a contradiction (empty clause)

3. Conclude that, since we could derive a contradiction, our initial assumption must have been wrong, so α holds

Problem 6.1: The man in the painting

$\forall x, y$

$\boxed{Sibling(x, y) \lor (x = y) \lor \neg Parent(p, x) \lor \neg Parent(p, y)}$

$\forall x, y$ $\boxed{\neg Sibling(x, y) \lor (x \neq y)}$

$\boxed{\neg Sibling(x, y) \lor Parent(F(x, y), x)}$

$\boxed{\neg Sibling(x, y) \lor Parent(F(x, y), y)}$

$\boxed{\neg Parent(p, c) \lor Child(c, p)}$

$\boxed{\neg Child(c, p) \lor Parent(p, c)}$

$\boxed{\neg Son(s, p) \lor Child(s, p)}$ $\boxed{\neg Son(s, p) \lor Male(s)}$

$\boxed{Son(s, p) \lor \neg Child(s, p) \lor \neg Male(s)}$

$\boxed{\neg Father(p, c) \lor Parent(p, c)}$

$\boxed{\neg Father(p, c) \lor Male(p)}$

$\boxed{Father(p, c) \lor \neg Parent(p, c) \lor \neg Male(p)}$

$\boxed{\neg Sibling(Me, x)}$ $\boxed{\neg Sibling(x, Me)}$

$\boxed{Father(F_1, That)}$ $\boxed{Father(F_2, Me)}$ $\boxed{Son(F_1, F_2)}$

$\boxed{Male(That)}$ $\boxed{Male(Me)}$ $\boxed{\neg Son(That, Me)}$

$A(c) \lor B(x)$ $\neg A(x)$

$B(x)$

$\cancel{B(c)}$

$KB \models \alpha$ iff $KB \Rightarrow \alpha$ is valid

α is valid iff $\neg\alpha$ is unsatisfiable

Reminder: Resolution Inference Rule

The resolution rule of propositional logic can be lifted to first-order logic:

Resolution rule for first-order logic

$$\frac{l_1 \vee \ldots \vee l_k, \quad m_1 \vee \ldots \vee m_n}{\texttt{Subst}(\theta, l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)},$$

where $\texttt{Unify}(l_i, \neg m_j) = \theta$.

Example: We can resolve the two clauses

$$[Animal(F(x)) \vee Loves(G(x), x)] \text{ and } [\neg Loves(u, v) \vee \neg Kills(u, v)]$$

by eliminating the complementary literals $Loves(G(x), x)$ and $\neg Loves(u, v)$, with unifier $\theta = \{u/G(x), v/x\}$, to produce the **resolvent** clause

$$[Animal(F(x)) \vee \neg Kills(G(x), x)].$$

Problem 6.1: The man in the painting

$Sibling(x, y) \lor (x = y) \lor \neg Parent(p, x) \lor \neg Parent(p, y)$

$\neg Sibling(x, y) \lor (x \neq y)$

$\neg Sibling(x, y) \lor Parent(F(x, y), x)$

$\neg Sibling(x, y) \lor Parent(F(x, y), y)$

$\neg Parent(p, c) \lor Child(c, p)$

$\neg Child(c, p) \lor Parent(p, c)$

$\neg Son(s, p) \lor Child(s, p)$ | $\neg Son(s, p) \lor Male(s)$

$Son(s, p) \lor \neg Child(s, p) \lor \neg Male(s)$

$\neg Father(p, c) \lor Parent(p, c)$

$\neg Father(p, c) \lor Male(p)$

$Father(p, c) \lor \neg Parent(p, c) \lor \neg Male(p)$

$\neg Sibling(Me, x)$ | $\neg Sibling(x, Me)$

$Father(F_1, That)$ | $Father(F_2, Me)$ | $Son(F_1, F_2)$

$Male(That)$ | $Male(Me)$ | $\neg Son(That, Me)$

$\boxed{\neg Son(That, Me)}$

$\{S/That,\ P/Me\}$

$\boxed{\neg Child(That, Me) \lor \neg Male(That)}$

$\emptyset$

$\boxed{Male(That)}$

$\boxed{\neg Child(That, Me)}$

$\{c/That,\ P/Me\}$

$\boxed{\neg Parent(Me, That)}$

...

next slide

Problem 6.1: The man in the painting

$\neg \mathbf{Son}(\mathbf{s},\mathbf{p}) \vee Child(s,p)$	$Son(F_1,F_2)$
$\neg \mathbf{Child}(\mathbf{c},\mathbf{p}) \vee Parent(p,c)$	$Child(F_1,F_2)$
$Sibling(x,y) \vee (x=y) \vee \neg \mathbf{Parent}(\mathbf{p},\mathbf{x}) \vee \neg Parent(p,y)$	$Parent(F_2,F_1)$
$\neg Father(p,c) \vee \mathbf{Parent}(\mathbf{p},\mathbf{c})$	$Sibling(F_1,y) \vee (F_1=y) \vee \neg Parent(F_2,y)$
$\mathbf{Father}(\mathbf{F_2},\mathbf{Me})$	$Sibling(F_1,y) \vee (F_1=y) \vee \neg Father(F_2,y)$
$\neg \mathbf{Sibling}(\mathbf{x},\mathbf{Me})$	$Sibling(F_1,Me) \vee (F_1=Me)$
$\mathbf{Father}(\mathbf{F_1},\mathbf{That})$	$F_1 = Me$
$\neg \mathbf{Father}(\mathbf{p},\mathbf{c}) \vee Parent(p,c)$	$Father(Me,That)$
	$Parent(Me,That)$

Previous slide:

... ...

$\boxed{\neg Parent(Me, That)}$

$\emptyset$

$KB \models Son(That, Me)$

Problem 6.2: Backward chaining

work backward from goal

Horn Clauses

$A_1 \wedge A_2 \wedge \dots \wedge A_n \Rightarrow B$

$\forall x_1, \dots x_n .\ A_1(x_1) \wedge A_2(x_2) \dots \wedge A_n(x_n) \Rightarrow B$

$A_1 \vee A_2 \Rightarrow B$

↳ we have to choose whether to prove A_1 or A_2

Similar for existential quantifiers

Suppose you are given the following axioms:

1. $0 \leq 3$
2. $7 \leq 9$
3. $\forall x \quad x \leq x$
4. $\forall x \quad x \leq x + 0$
5. $\forall x \quad x + 0 \leq x$
6. $\forall x, y \quad x + y \leq y + x$
7. $\forall w, x, y, z \quad w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$
8. $\forall x, y, z \quad x \leq y \wedge y \leq z \Rightarrow x \leq z.$

Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that lead to success, not the irrelevant steps.

Reminder: Backward-Chaining Algorithm

function FOL-BC-Ask (KB,$goals$,θ) **returns** a set of substitutions

inputs: KB, a knowledge base
$goals$, a list of conjuncts forming a query (θ already applied)
θ, the current substitution, initially the empty substitution $\emptyset$

local variables: $answers$, a set of substitutions, initially empty

if $goals$ is empty **then return** $\{\theta\}$
$q' \leftarrow \texttt{Subst}(\theta, \texttt{First}(goals))$
for each sentence r **in** KB where $\texttt{Standardize-Apart}(r) = (p_1 \wedge \ldots \wedge p_n \Rightarrow q)$
and $\theta' \leftarrow \texttt{Unify}(q, q')$ succeeds
    $new\_goals \leftarrow [p_1, \ldots, p_n | \texttt{Rest}(goals)]$
    $answers \leftarrow \texttt{FOL-BC-Ask}(KB, new\_goals, \texttt{Compose}(\theta', \theta)) \cup answers$
return $answers$

Step 1: $(\underline{7 \leq 3+9}) \Leftarrow (\underline{7 \leq y} \wedge \underline{y \leq 3+9})$

goals : $\{\underline{7 \leq 3+9}\}$

$q' \leftarrow \textsc{Subst}(\underline{\emptyset},$ $7 \leq 3+9$ $)$

$\hookrightarrow$ $\underline{7 \leq 3+9}$

Using rule 8: $\boxed{\forall \underline{x_8}, \underline{y_8}, \underline{z_8} \quad \underline{x_8 \leq y_8} \wedge \underline{y_8 \leq z_8} \Rightarrow \underline{x_8 \leq z_8}}$

$\theta' \leftarrow \{x_8/7,\ z_8/3+9\}$

new goals $\leftarrow \{x_8 \leq y_8,\ y_8 \leq z_8\}$

Step 2: $(7 \leq y \wedge y \leq 3+9) \Leftarrow (\mathit{True} \wedge 7+0 \leq 3+9)$ $\Leftarrow (7+0 \leq 3+9)$

goals : $\{x_8 \leq y_8, y_8 \leq z_8\}$

$q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9\},\ x_8 \leq y_8\ )$

$7 \leq y_8$

Using rule 4: $\boxed{\forall x_4\ x_4 \leq x_4 + 0}$

$7 \leq 7+0$

$\theta' \leftarrow \{x_4/7,\ y_8/7+0\}$

new goals $\leftarrow \{y_8 \leq z_8\}$

Step 3: $(7 + 0 \leq 3 + 9) \Leftarrow (7 + 0 \leq y \land y \leq 3 + 9)$

goals : $\{y_8 \leq z_8\}$

$q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3{+}9,\ x_4/7,\ y_8/7{+}0\},\quad)$

$7+0 \leq 3+9$

Using rule 8: $\boxed{\forall x_8', y_8', z_8' \quad x_8' \leq y_8' \land y_8' \leq z_8' \;\Rightarrow\; x_8' \leq z_8'}$

$\theta' \leftarrow \{x_8'/7+0,\ z_8'/3+9\}$

new goals $\leftarrow \{x_8' \leq y_8',\ y_8' \leq z_8'\}$

Step 4: $(7+0 \leq y \wedge y \leq 3+9) \Leftarrow (\mathit{True} \wedge 0+7 \leq 3+9)$
$\Leftarrow (0+7 \leq 3+9)$

goals : $\{x_8' \leq y_8', y_8' \leq z_8'\}$

$q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x_8'/7+0,$

$z_8'/3+9\},\qquad)$

$\hookrightarrow\ 7+0 \leq y_8'$

Using rule 6: $\boxed{\forall x_6, y_6\ \ x_6 + y_6 \leq y_6 + x_6}$

$\theta' \leftarrow \{x_6/7,\ y_6/0,\ y_8'/0+7\}$

new goals $\leftarrow \{y_8' \leq z_8'\}$

Step 5: $(0 + 7 \leq 3 + 9) \Leftarrow (0 \leq 3 \wedge 7 \leq 9)$

goals : $\{y'_8 \leq z'_8\}$

$$q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x'_8/7+0,$$
$$z'_8/3+9,\ y'_8/0+7,\ x_6/7,\ y_6/0\},\ \ y_8' \leq z_8' \quad)$$

$\hookrightarrow 0+7 \leq 3+9$

Using rule 7:

$$\boxed{\forall w_7, x_7, y_7, z_7 \quad w_7 \leq y_7 \wedge x_7 \leq z_7 \Rightarrow \quad w_7 + x_7 \leq y_7 + z_7}$$

$\theta' \leftarrow \{w_7/0,\ x_7/7,\ y_7/3,\ z_7/9\}$

new goals $\leftarrow \{w_7 \leq y_7,\ x_7 \leq z_7\}$

Step 6: $(0 \leq 3 \wedge 7 \leq 9) \Leftarrow (\mathit{True} \wedge 7 \leq 9) \Leftarrow (7 \leq 9)$

goals : $\{w_7 \leq y_7, x_7 \leq z_7\}$

$$q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x'_8/7+0,\ z'_8/3+9,\ y'_8/0+7,\ x_6/7,\ y_6/0,\ w_7/0,\ y_7/3,\ x_7/7,\ z_7/9\},\ w_7 \leq y_7)$$

$0 \leq 3$

Using rule 1: $\boxed{0 \leq 3}$

$\theta' \leftarrow \emptyset$

new goals $\leftarrow \{x_7 \leq z_7\}$

Step 7: $(7 \leq 9) \Leftarrow \mathit{True}$

goals : $\{x_7 \leq z_7\}$

$$q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x_8'/7+0,\ z_8'/3+9,\ y_8'/0+7,\ x_6/7,\ y_6/0,\ w_7/0,\ y_7/3,\ x_7/7,\ z_7/9\},\quad x_7 \leq z_7 \quad)$$

$\hookrightarrow 7 \leq 9$

Using rule 2: $\boxed{7 \leq 9}$

$\theta' \leftarrow \emptyset$

new goals $\leftarrow \emptyset$

$KB \models 7 \leq 3+9$