Tutorial Robotics IN2067

Exercise Sheet 02

Formulae

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

Formulae

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} = \cdots$$

For $\forall a \in \mathbb{R}$:

Adding a times a column to another column does not change the determinant.

Adding a times a row to another row does not change the determinant.

P01

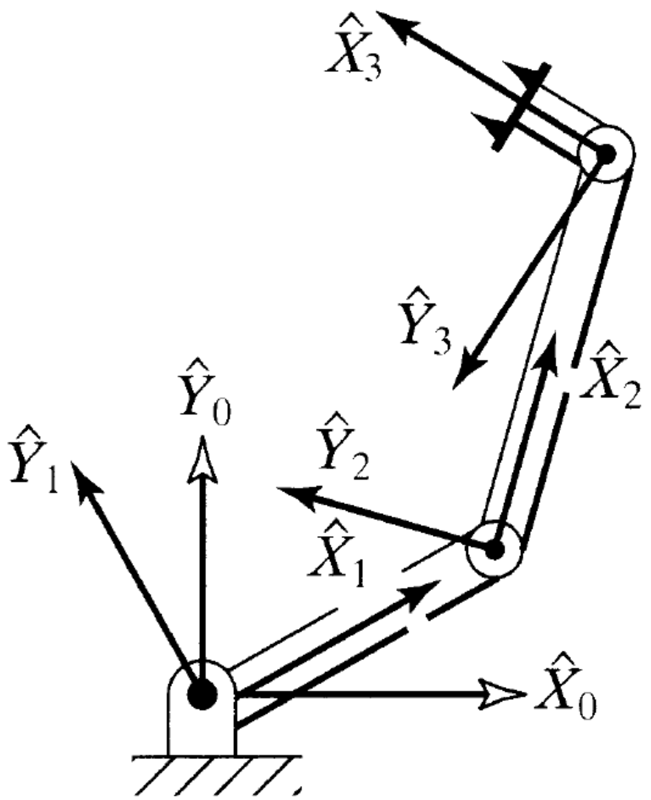


Figure 1: 3*R* *Robot (Problem 1)*

Problem 1

For the 3R manipulator shown in Figure 1, solve the following problems:

a) Compute the forward kinematics, i.e., the position and orientation, of the end effector, for this manipulator. Note that the manipulator has an especially simple configuration, because all rotation axes are parallel. The robot endeffector position can be described by specifying a planar position x, y and the rotation angle Θ_{tip}. The three roboter parameters are denoted by $\Theta_1, \Theta_2, \Theta_3$, the lengths of the robot links are given by l_1, l_2, l_3.

b) Determine the Jacobian of the manipulator.

c) Express $\dot{p}$ as a function of

$$\Theta_1, \Theta_2, \Theta_3, \dot{\Theta}_1, \dot{\Theta}_2, \dot{\Theta}_3$$

d) Determine the singularities of the manipulator.

e) For each singularity, determine which degrees of freedom are lost, and try to give an intuitive explanation for that.

P01

position

- $\text{fkm}(\theta) = \begin{pmatrix} p_x \\ p_y \\ p_z \\ o_x \\ o_y \\ o_z \end{pmatrix}$ in three-dimensional space

orientation.

- $\text{fkm}(\theta) = \begin{pmatrix} p_x \\ p_y \\ o_z \end{pmatrix}$ in two-dimensional space

In the text: The robot end-effector position can be described by specifying a planar position x, y and the rotation angle θ_{tip}

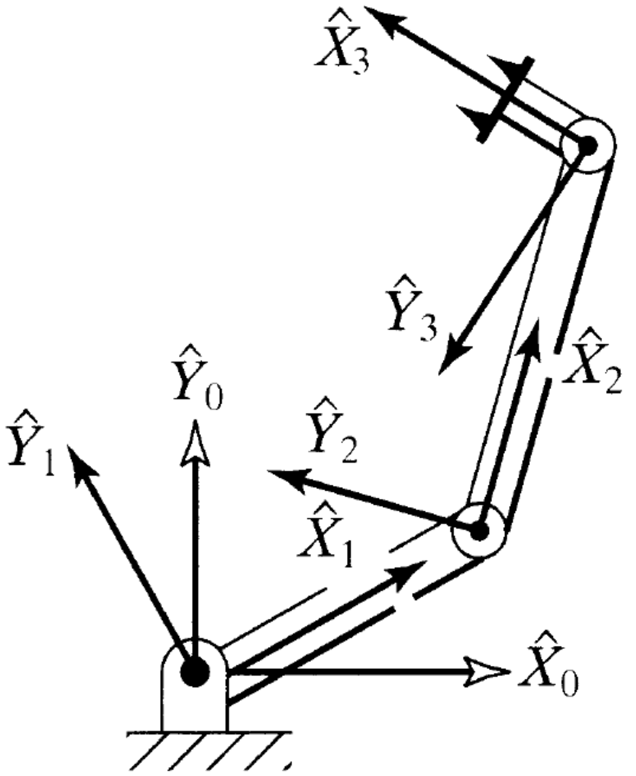


Figure 1: 3*R Robot* (*Problem* 1)

P01

$$fkm(\theta) = \begin{pmatrix} p_x \\ p_y \\ o_z \end{pmatrix}$$

P1 3 Joints $\theta_1, \theta_2, \theta_3 \Rightarrow fkm(\theta) = \begin{pmatrix} p_x \\ p_y \\ p_\theta \end{pmatrix} \in \mathbb{R}^3$

DH	α	a	d	θ
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	0	l_2	0	θ_3
EE/4	0	l_3	0	0

$$\Rightarrow {}^0_4T = \begin{pmatrix} C_{123} & -S_{123} & 0 & l_1C_1 + l_2C_{12} + l_3C_{123} \\ S_{123} & C_{123} & 0 & l_1S_1 + l_2S_{12} + l_3S_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow fkm(\theta) = \begin{pmatrix} l_1c_1 + l_2C_{12} + l_3C_{123} \\ l_1S_1 + l_2S_{12} + l_3S_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{pmatrix}$$

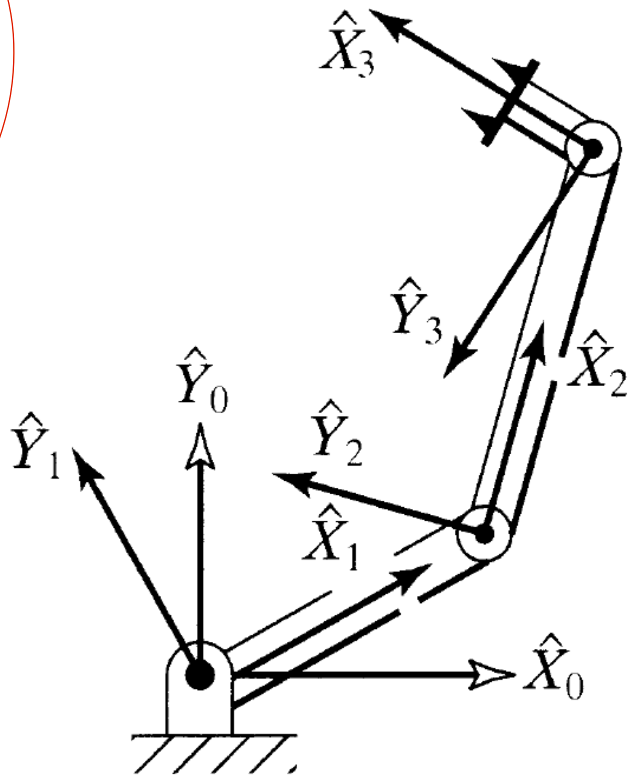


Figure 1: *3R Robot (Problem 1)*

P01

$$f(\theta) = \begin{pmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{pmatrix}$$

$$J_{i,j}(\theta) = \frac{\partial f(\theta)_i}{\partial \theta_j}$$

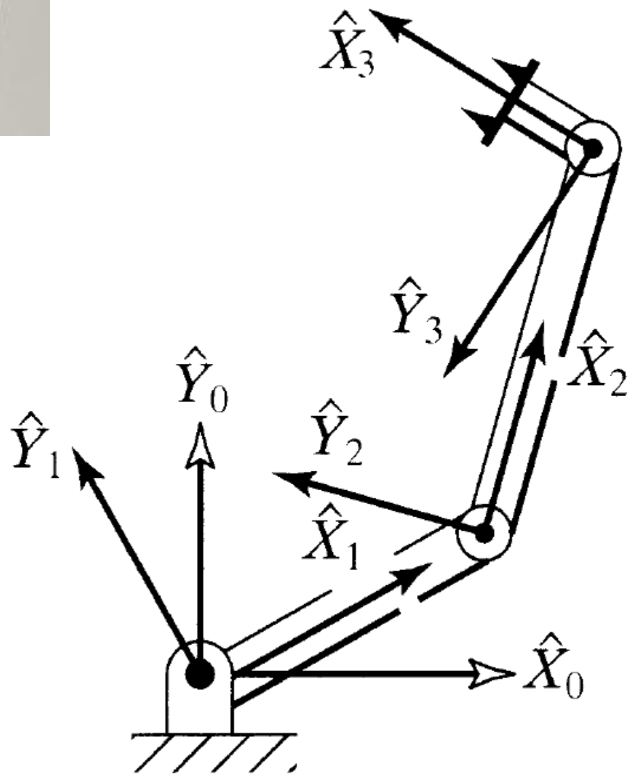


Figure 1: 3*R Robot (Problem 1)*

P01

$$f(\theta) = \begin{pmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{pmatrix}$$

$$J_{i,j}(\theta) = \frac{\partial f(\theta)_i}{\partial \theta_j} \Rightarrow J(\theta) = \begin{pmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{pmatrix}$$

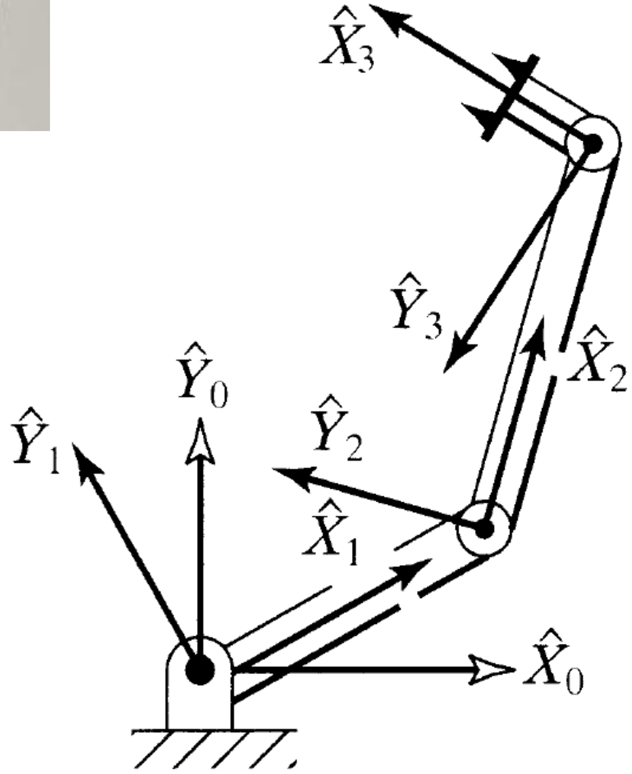


Figure 1: 3*R Robot (Problem 1)*

P01

$$f(\theta) = \begin{pmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{pmatrix}$$

$$J_{i,j}(\theta) = \frac{\partial f(\theta)_i}{\partial \theta_j} \Rightarrow J(\theta) = \begin{pmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{pmatrix}$$

$$\boxed{\dot{p} = J(\theta) \cdot \dot{\theta}} = \begin{pmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix}$$

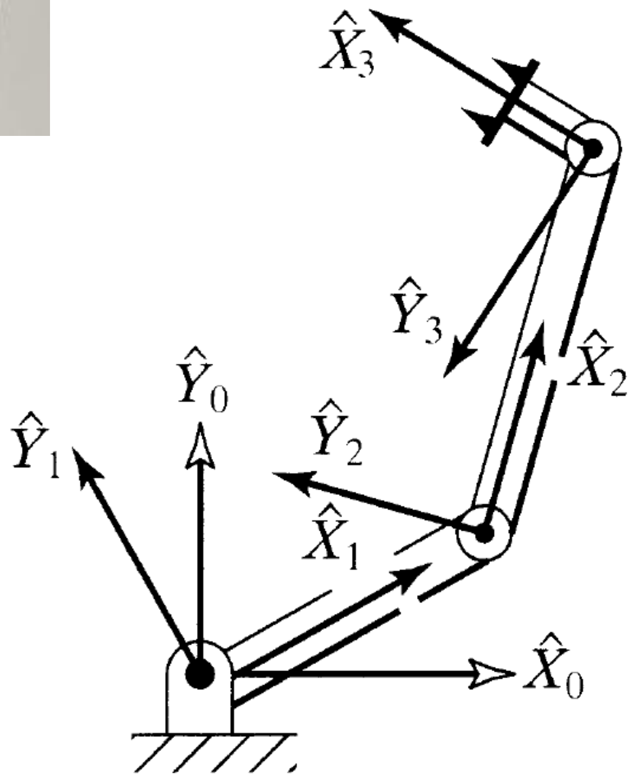


Figure 1: *3R Robot (Problem 1)*

P1

$$\det(J(\theta)) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

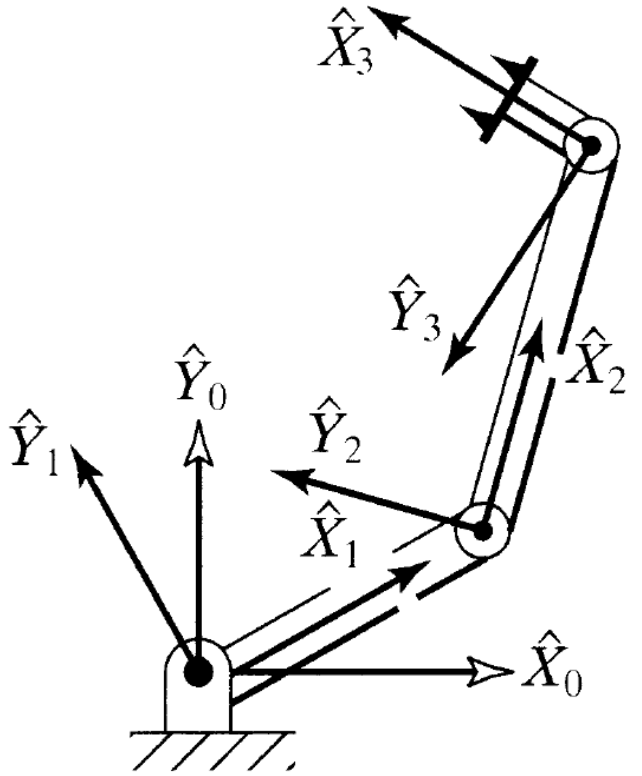


Figure 1: *3R Robot (Problem 1)*

P1

$$\det(J(\theta)) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 1 & 1 \end{vmatrix} =$$

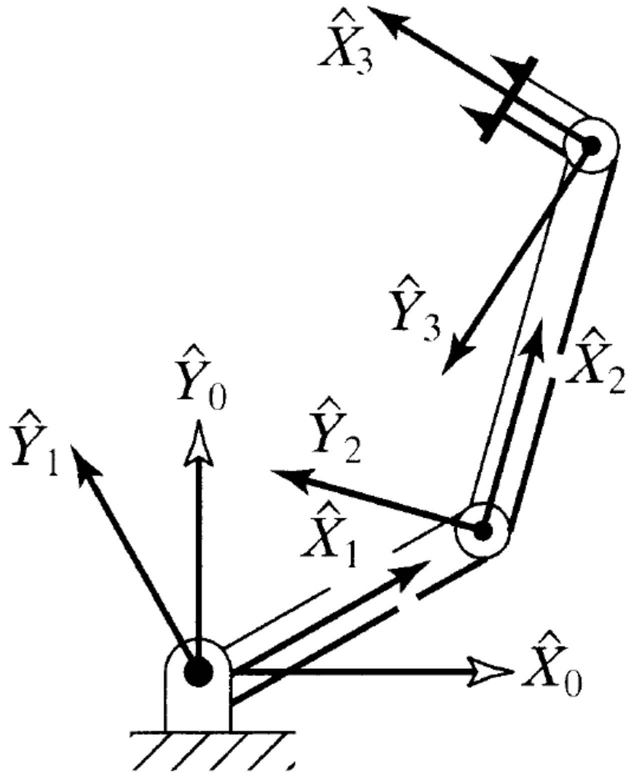


Figure 1: 3*R Robot (Problem 1)*

P1

$$\det(J(\theta)) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} & -l_3 s_{123} \\ l_1 c_1 & l_2 c_{12} & l_3 c_{123} \\ 0 & 0 & 1 \end{vmatrix} = -l_1 l_2 s_1 c_{12} + l_1 l_2 c_1 s_{12} =$$

$$= l_1 l_2 s_2$$

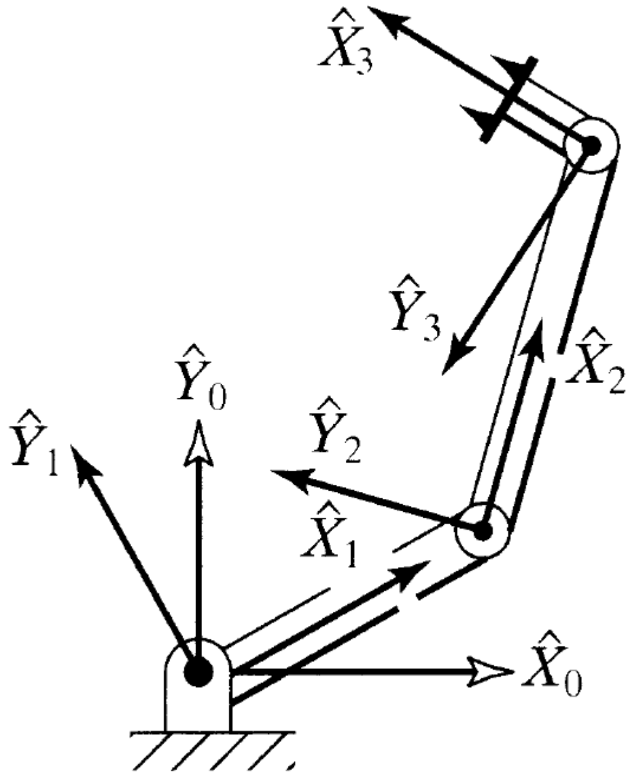


Figure 1: *3R Robot (Problem 1)*

P1

$$\det(J(\theta)) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} & -l_3 s_{123} \\ l_1 c_1 & l_2 c_{12} & l_3 c_{123} \\ 0 & 0 & 1 \end{vmatrix} = -l_1 l_2 s_1 c_{12} + l_1 l_2 c_1 s_{12} =$$

$$= l_1 l_2 s_2 = 0$$

$$\boxed{l_1 l_2 s_2 = 0 \iff \begin{cases} l_1 = 0 & \text{or} \\ l_2 = 0 & \text{or} \\ s_2 = 0 \iff \theta_2 \in \{0^\circ, 180^\circ\} \end{cases}}$$

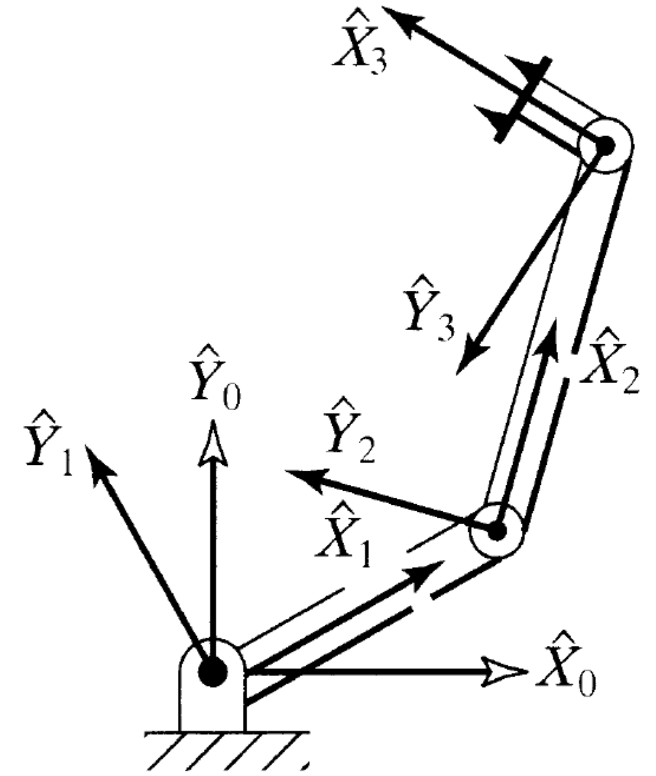


Figure 1: 3*R Robot (Problem 1)*

P01

- It is easier to determine singularities (and usually also to interpret them) in the last coordinate frame

- For this robot: determine ^3J

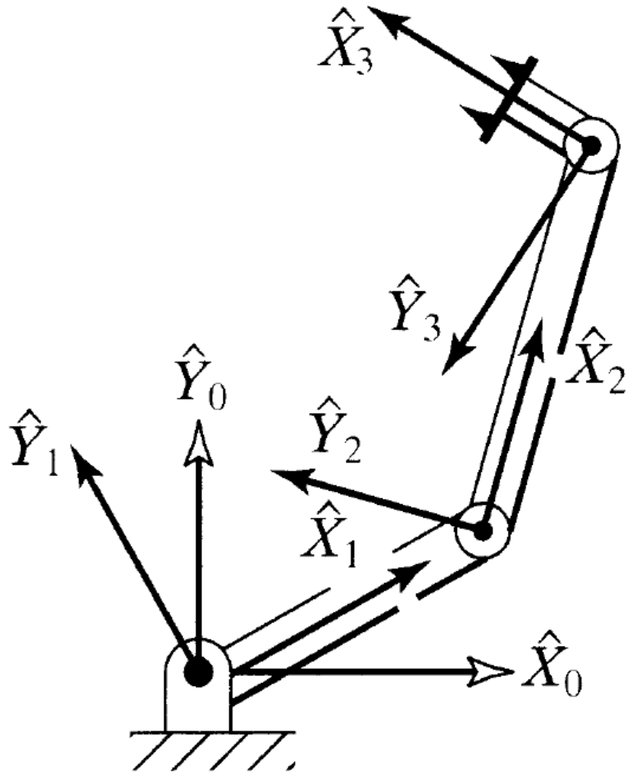


Figure 1: 3*R Robot (Problem 1)*

P01

(P1) $${}^0J = \begin{pmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{pmatrix}$$

$$\boxed{{}^3J = {}^3_0R\,{}^0J} \; ; \; {}^0_3R = \begin{pmatrix} c_{123} & -s_{123} & 0 \\ s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow {}^3_0R = \begin{pmatrix} c_{123} & s_{123} & 0 \\ -s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{{}^3J} = \begin{pmatrix} l_1 s_{23} + l_2 s_3 & l_2 s_3 & 0 \\ l_1 c_{23} + l_2 c_3 + l_3 & l_2 c_3 + l_3 & l_3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\det({}^3J) = l_1 l_2 s_2 = 0 \Leftrightarrow l_1 = 0 \text{ or } l_2 = 0 \text{ or } \theta_2 \in \{0°, 180°\}$$

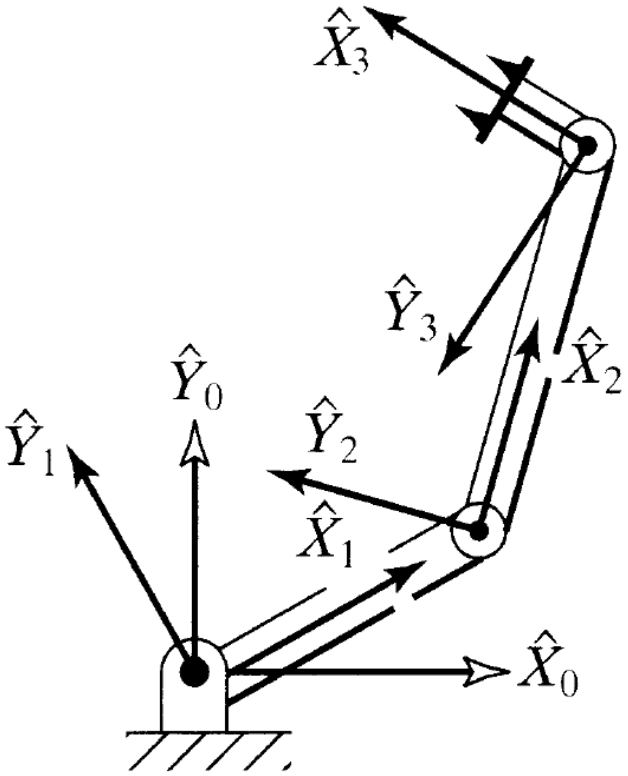


Figure 1: *3R Robot (Problem 1)*

P01

for $l_1 = 0$: $${}^3J = \begin{pmatrix} l_2 s_3 & l_2 s_3 & 0 \\ l_2 c_3 + l_3 & l_2 c_3 + l_3 & l_3 \\ 1 & 1 & 1 \end{pmatrix}$$

for $l_2 = 0$: $${}^3J = \begin{pmatrix} l_1 s_{23} & 0 & 0 \\ l_1 c_{23} + l_3 & l_3 & l_3 \\ 1 & 1 & 1 \end{pmatrix}$$

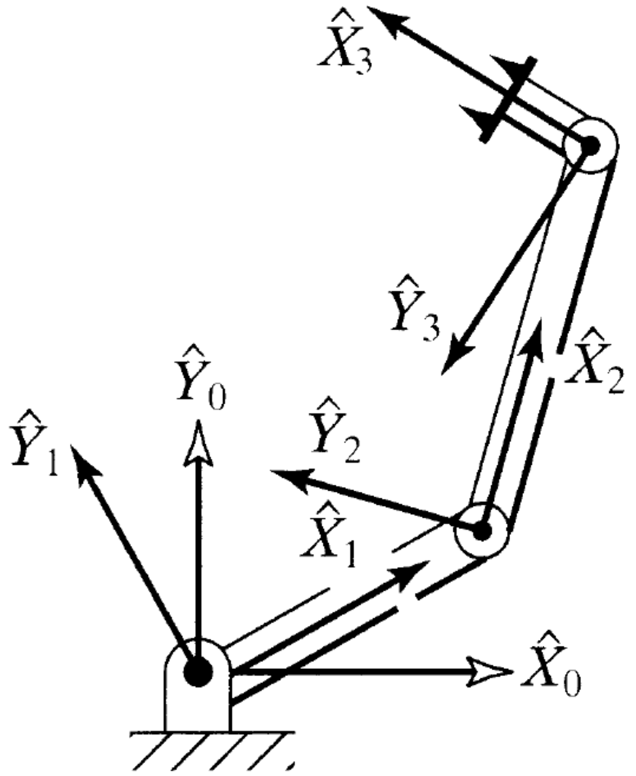


Figure 1: 3*R* *Robot (Problem* 1*)*

Joints are same

P01

for $l_1 = 0$: ${}^{3}J = \begin{pmatrix} \boxed{\begin{matrix} l_2 s_3 \\ l_2 c_3 + l_3 \\ 1 \end{matrix}} & \boxed{\begin{matrix} l_2 s_3 \\ l_2 c_3 + l_3 \\ 1 \end{matrix}} & \begin{matrix} 0 \\ l_3 \\ 1 \end{matrix} \end{pmatrix}$

$=$

for $l_2 = 0$: ${}^{3}J = \begin{pmatrix} \begin{matrix} l_1 s_{23} \\ l_1 c_{23} + l_3 \\ 1 \end{matrix} & \boxed{\begin{matrix} 0 \\ l_3 \\ 1 \end{matrix}} & \boxed{\begin{matrix} 0 \\ l_3 \\ 1 \end{matrix}} \end{pmatrix}$

$=$

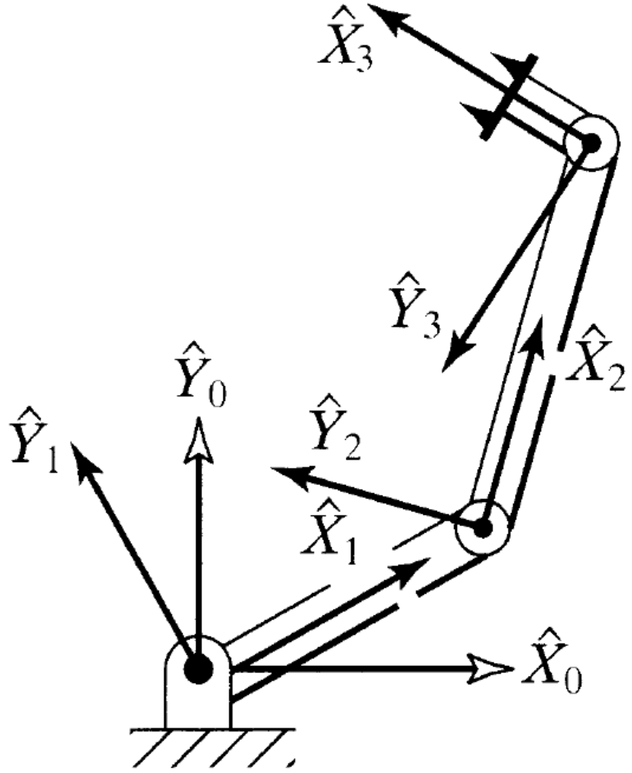


Figure 1: 3*R* *Robot (Problem 1)*

P01

for $\theta_2 \in \{0°, 180°\}$: ${}^3J = \begin{pmatrix} (l_1+l_2)s_3 & l_2 s_3 & 0 \\ (l_1+l_2)c_3 + l_3 & l_2 c_3 + l_3 & l_3 \\ 1 & 1 & 1 \end{pmatrix}$

- Robot loses degree of freedom if rows are linearly dependent
- Extra: Workspace boundary singularity
  If a row of the robot is 0.

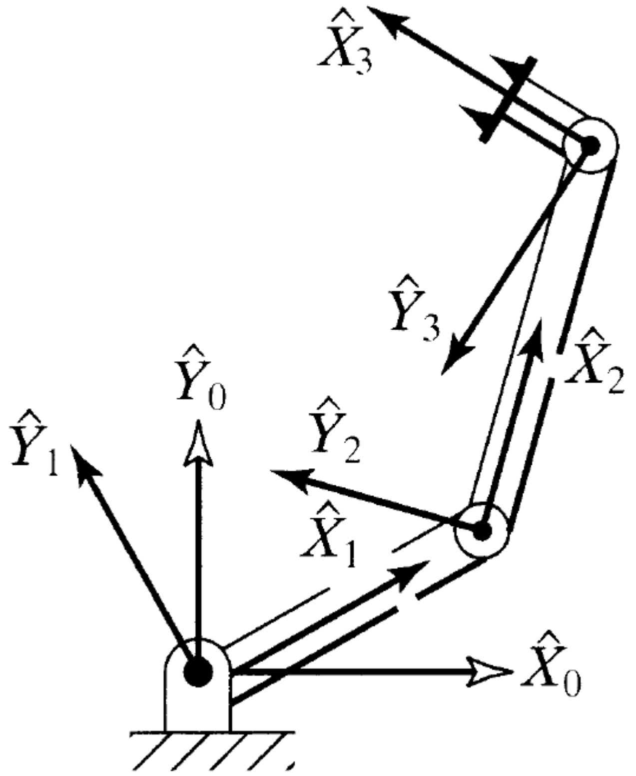


Figure 1: *3R Robot (Problem 1)*

P01

for $\theta_2 \in \{0°, 180°\}$: $${}^{3}J = \begin{pmatrix} (l_1+l_2)s_3 & l_2 s_3 & 0 \\ (l_1+l_2)c_3 + l_3 & l_2 c_3 + l_3 & l_3 \\ 1 & 1 & 1 \end{pmatrix}$$

Workspace boundary singularity:

- Row 1 $= \; 0 \Leftrightarrow \sin(\theta_3) = 0 \Leftrightarrow \theta_3 \in \{0°, 180°\}$
  No motion in x_3-axis direction
- Row 2 $= \; 0 \Leftrightarrow l_3 = 0 \wedge \cos(\theta_3) = 0 \Leftrightarrow$
  $\Leftrightarrow l_3 = 0 \wedge \theta_3 \in \{90°, 270°\}$
  No motion in y_3-axis direction

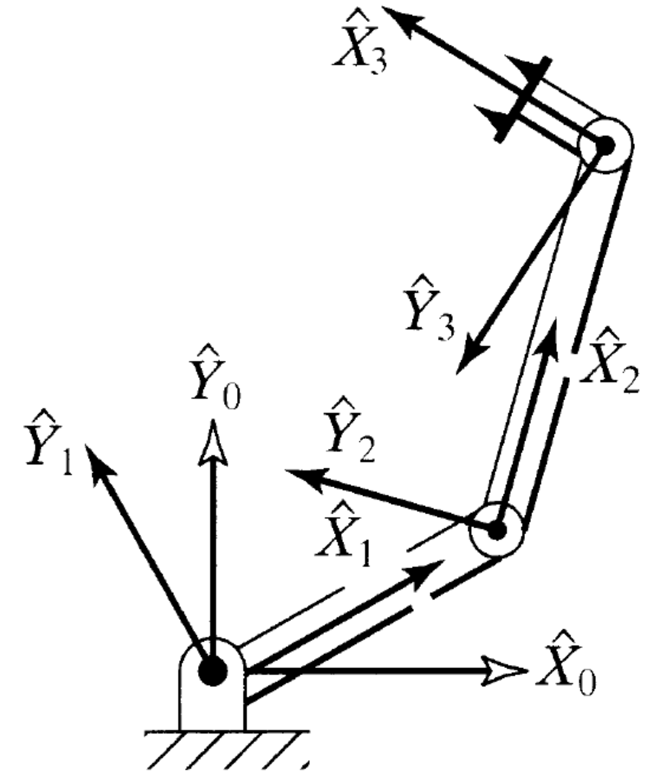


Figure 1: 3*R Robot* (*Problem* 1)

P01

for $\theta_2 \in \{0^\circ, 180^\circ\}$: ${}^3J = \begin{pmatrix} (l_1+l_2)s_3 & l_2 s_3 & 0 \\ (l_1+l_2)c_3 + l_3 & l_2 c_3 + l_3 & l_3 \\ 1 & 1 & 1 \end{pmatrix}$

Linearly Dependent Rows:

- Rows 1 and 2 $\Leftrightarrow l_3 = 0 \wedge \cos(\theta_3) = \sin(\theta_3)$
  $\Leftrightarrow l_3 = 0 \wedge \theta_3 \in \{45°, 225°\}$
  Coupled degrees of freedom: x_3- and y_3-axis velocity
- Rows 2 and 3 $\Leftrightarrow \cos(\theta_3) = 0 \Leftrightarrow \theta_3 \in \{90°, 270°\}$
  Coupled degrees of freedom:
  y_3-axis velocity and angular velocity are coupled

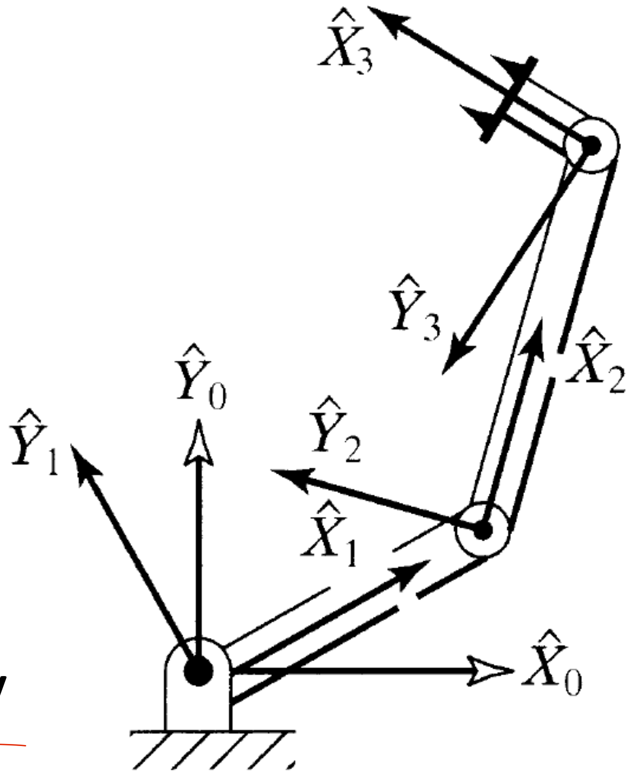


Figure 1: 3*R Robot (Problem 1)*

P02

Problem 2

A manipulator may have special configurations, called "isotropic points," that are characterized by the Jacobi matrix having orthogonal columns of equal length, thus $J^T J = \delta I$ for some $\delta \in \mathbb{R}$. Now consider the 2R manipulator shown in Figure 2. It's Jacobian (with respect to gripper position only, ignoring gripper orientation) looks like this:

$$^3J(\Theta) = \begin{pmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{pmatrix}$$

Determine the manipulator's isotropic points. Draw the manipulator in the corresponding configuration(s). Can you give an interpretation of the special role that the isotropic configuration plays?

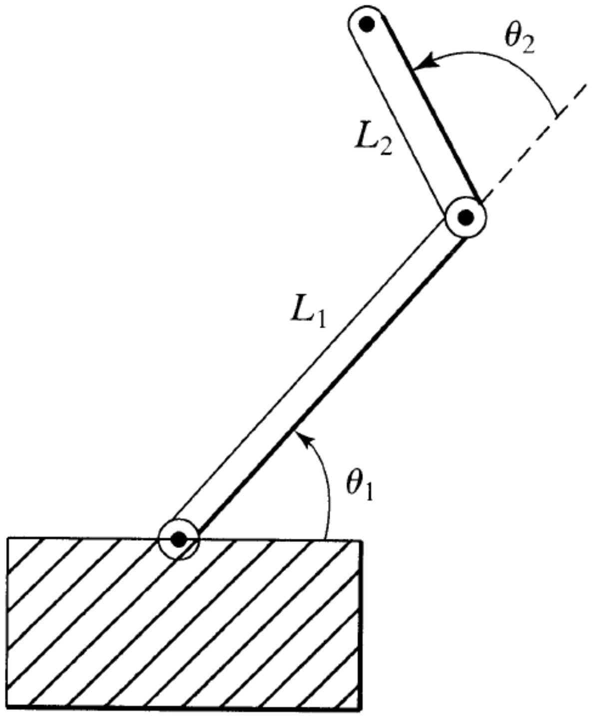


Figure 2: *2R Robot (Problem 2)*

P02

$$^{3}J = \begin{pmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{pmatrix}$$

$$^{3}J^{T}\,J = \begin{pmatrix} (L_1 s_2)^2 + (L_1 c_2 + L_2)^2 & L_1 L_2 c_2 + L_2^2 \\ L_1 L_2 c_2 + L_2^2 & L_2^2 \end{pmatrix}$$

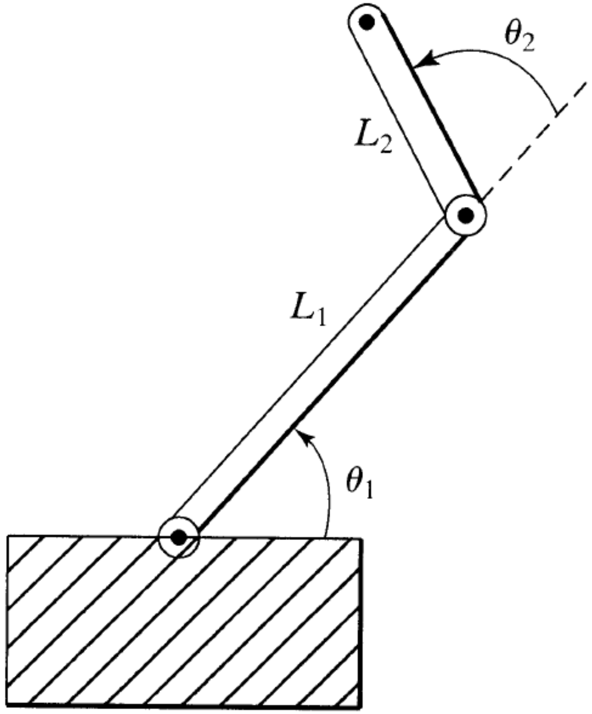


Figure 2: *2R Robot (Problem 2)*

P02

$$^3J = \begin{pmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{pmatrix}$$

$$^3J^T\, {}^3J = \begin{pmatrix} (L_1 s_2)^2 + (L_1 c_2 + L_2)^2 & L_1 L_2 c_2 + L_2^2 \\ L_1 L_2 c_2 + L_2^2 & L_2^2 \end{pmatrix} = \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix}, \delta \in \mathbb{R}$$

$$\Rightarrow L_2^2 = -L_1 L_2 c_2 \Leftrightarrow \begin{cases} L_2 = 0 \\ L_2 = -L_1 c_2 \end{cases}$$

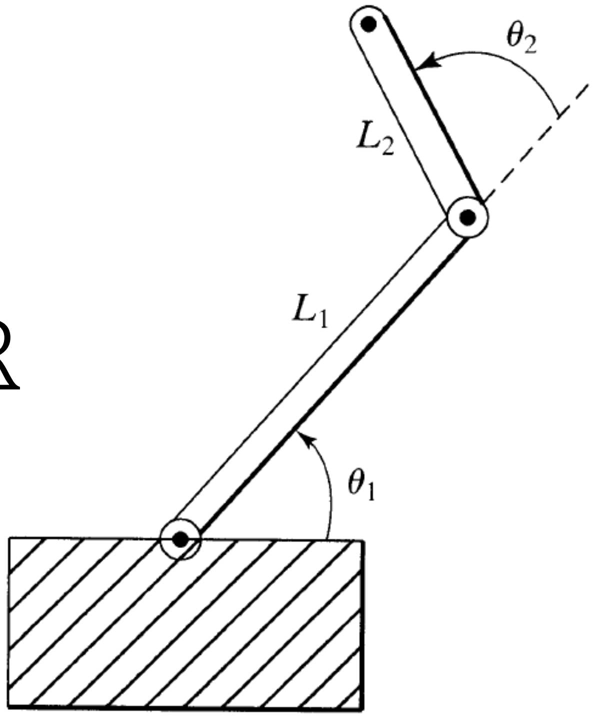


Figure 2: *2R Robot (Problem 2)*

P02

$$^3J = \begin{pmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{pmatrix}$$

$$^3J^T J = \begin{pmatrix} (L_1 s_2)^2 + (L_1 c_2 + L_2)^2 & L_1 L_2 c_2 + L_2^2 \\ L_1 L_2 c_2 + L_2^2 & L_2^2 \end{pmatrix} = \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix}, \delta \in \mathbb{R}$$

$$\Rightarrow L_2^2 = -L_1 L_2 c_2 \Leftrightarrow \begin{cases} L_2 = 0 \\ L_2 = -L_1 c_2 \end{cases} \Rightarrow c_2 = \frac{-L_2}{L_1}$$

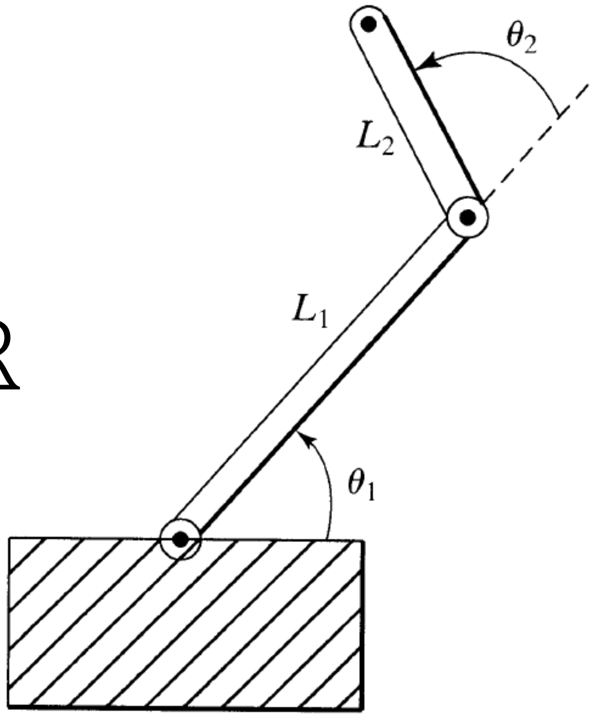


Figure 2: *2R Robot (Problem 2)*

P02

$$^3J = \begin{pmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{pmatrix}$$

$$^3J^T J = \begin{pmatrix} (L_1 s_2)^2 + (L_1 c_2 + L_2)^2 & L_1 L_2 c_2 + L_2^2 \\ L_1 L_2 c_2 + L_2^2 & L_2^2 \end{pmatrix} = \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix}, \delta \in \mathbb{R}$$

$$\Rightarrow L_2^2 = -L_1 L_2 c_2 \Leftrightarrow \begin{cases} L_2 = 0 \\ L_2 = -L_1 c_2 \end{cases} \Rightarrow c_2 = \frac{-L_2}{L_1}$$

$$(L_1 s_2)^2 + (L_1 c_2 + L_2)^2 = (L_1 s_2)^2 = L_2^2$$

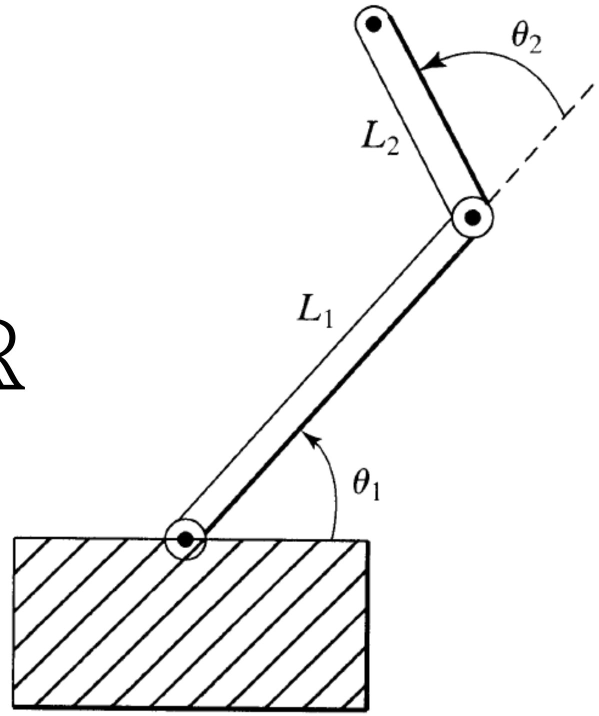


Figure 2: *2R Robot (Problem 2)*

P02

$$^3J = \begin{pmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{pmatrix}$$

$$^3J^T J = \begin{pmatrix} (L_1 s_2)^2 + (L_1 c_2 + L_2)^2 & L_1 L_2 c_2 + L_2^2 \\ L_1 L_2 c_2 + L_2^2 & L_2^2 \end{pmatrix} = \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix}, \delta \in \mathbb{R}$$

$$\Rightarrow L_2^2 = -L_1 L_2 c_2 \Leftrightarrow \begin{cases} L_2 = 0 \\ L_2 = -L_1 c_2 \end{cases} \Rightarrow c_2 = \frac{-L_2}{L_1}$$

$$(L_1 s_2)^2 + (L_1 c_2 + L_2)^2 = (L_1 s_2)^2 = L_2^2 \Leftrightarrow s_2 = \pm \frac{L_2}{L_1}$$

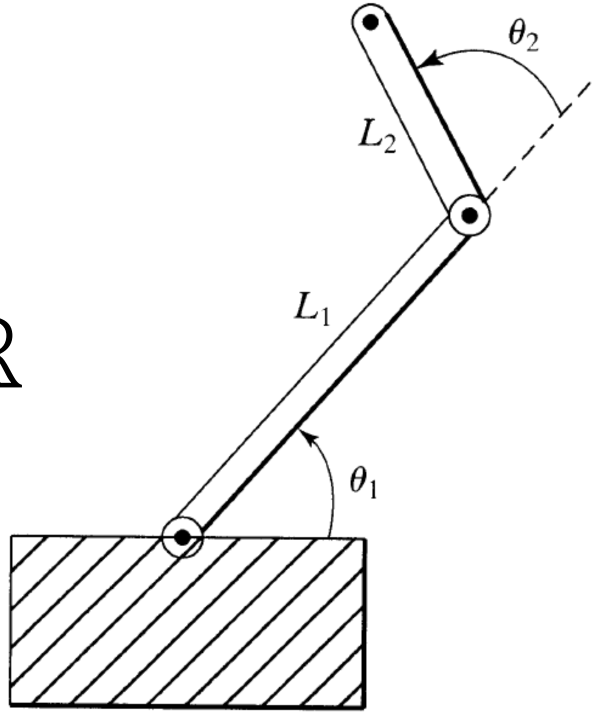


Figure 2: *2R Robot (Problem 2)*

P02

$$C_2 = \frac{-L_2}{L_1}$$

$$S_2 = \pm \frac{L_2}{L_1}$$

$$\hookrightarrow C_2 = \sqrt{1 - S_2^2} = \frac{\sqrt{L_1^2 - L_2^2}}{L_1}$$

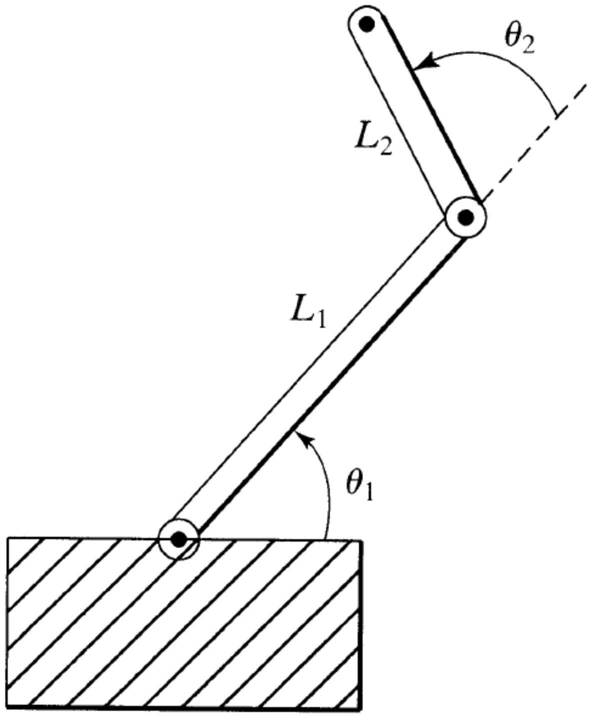


Figure 2: *2R Robot (Problem 2)*

P02

$$C_2 = \frac{-L_2}{L_1}$$

$$S_2 = \pm \frac{L_2}{L_1}$$

$$C_2 = \sqrt{1 - S_2^2} = \frac{\sqrt{L_1^2 - L_2^2}}{L_1}$$

$$\Rightarrow$$

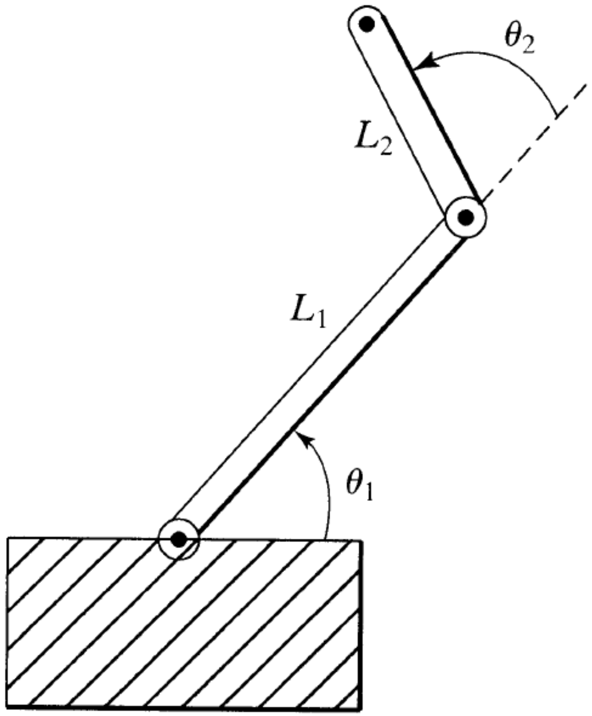


Figure 2: *2R Robot (Problem 2)*

P02

$$C_2 = \frac{-L_2}{L_1}$$

$$S_2 = \pm \frac{L_2}{L_1}$$

$$\hookrightarrow C_2 = \sqrt{1 - S_2^2} = \frac{\sqrt{L_1^2 - L_2^2}}{L_1}$$

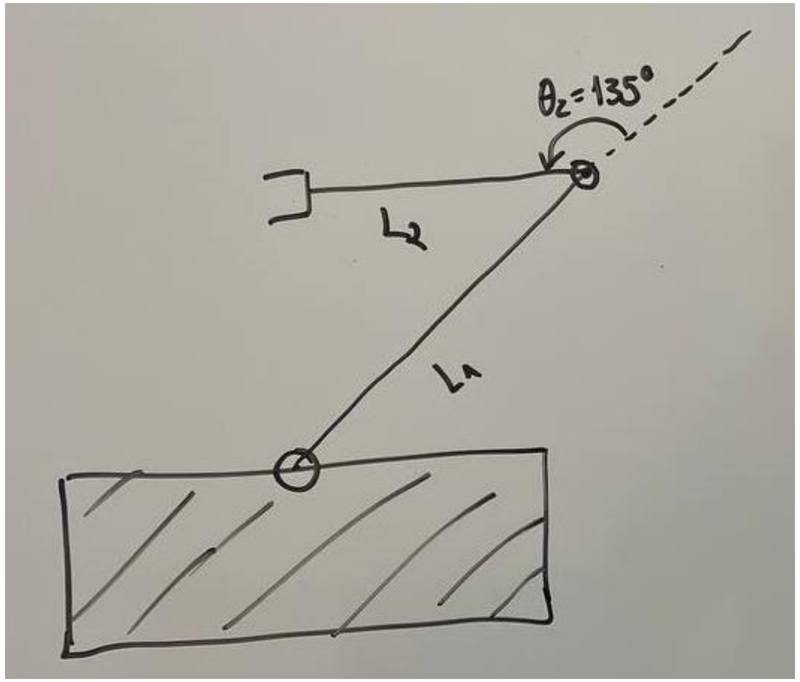
$$\Rightarrow L_1^2 - L_2^2 = L_2^2 \Rightarrow L_1 = L_2\sqrt{2} \Rightarrow \left.\begin{array}{l} C_2 = \frac{-\sqrt{2}}{2} \\ S_2 = \pm\frac{\sqrt{2}}{2} \end{array}\right\} \Rightarrow \theta_2 = \pm 135^\circ$$

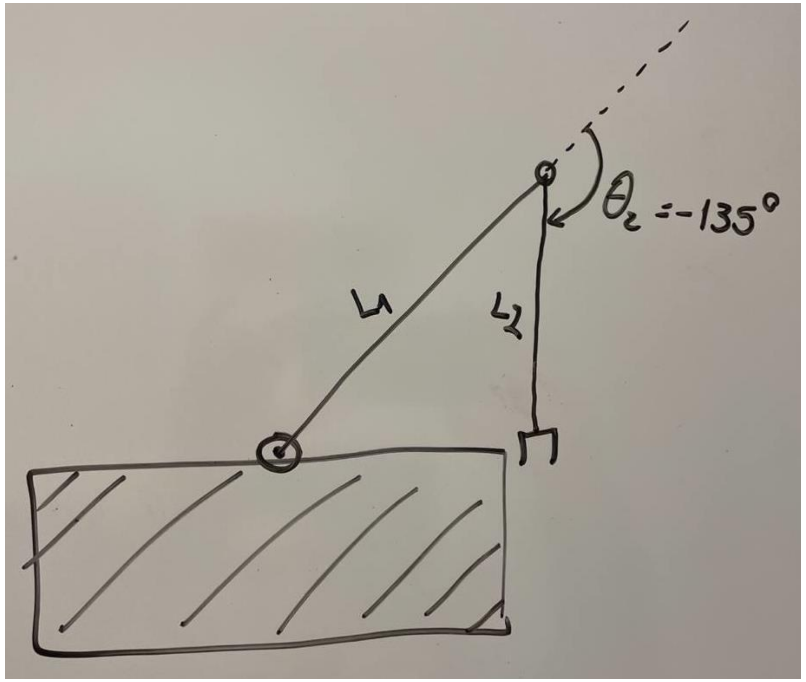
$$^3J(\theta) = \begin{pmatrix} \pm L_2 & 0 \\ 0 & L_2 \end{pmatrix}$$

P02

- Each joint is responsible for moving the end-effector along a specific axis

$$ {}^{3}J(\theta) = \begin{pmatrix} \pm L_2 & 0 \\ 0 & L_2 \end{pmatrix} $$





P03

Problem 3

Show that determining singularities of the 2R robot from problem 2 is significantly easier based on the Jacobian relative to frame $\{3\}$ than based on the Jacobian relative to frame $\{0\}$. The Jacobian ${}^{0}J$ can be computed using two different methods:

- Direct computation (through differentiation of gripper position).
- Transforming ${}^{3}J$ into ${}^{0}J$ by exploiting the Jacobian transformation relation.

Perform the computation of ${}^{0}J$ using both methods. Determine singularities based on ${}^{0}J$. How is it easier to compute singularities based on ${}^{3}J$?

P03

$$^0_3T = \begin{pmatrix} c_{12} & -s_{12} & 0 & L_1c_1 + L_2c_{12} \\ s_{12} & c_{12} & 0 & L_1s_1 + L_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^0p = fkm(\theta) = \begin{pmatrix} L_1c_1 + L_2c_{12} \\ L_1s_1 + L_2s_{12} \\ 0 \\ 0 \\ 0 \\ \theta_1 + \theta_2 \end{pmatrix} \Rightarrow \dot{fkm}(\theta) = \begin{pmatrix} -L_1s_1\dot\theta_1 - L_2s_{12}(\dot\theta_1 + \dot\theta_2) \\ L_1c_1\dot\theta_1 + L_2c_{12}(\dot\theta_1 + \dot\theta_2) \\ 0 \\ 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 \end{pmatrix}$$

$$\Rightarrow {}^0J = \begin{pmatrix} -L_1s_1 - L_2s_{12} & -L_2s_{12} \\ L_1c_1 + L_2c_{12} & L_2c_{12} \end{pmatrix}$$

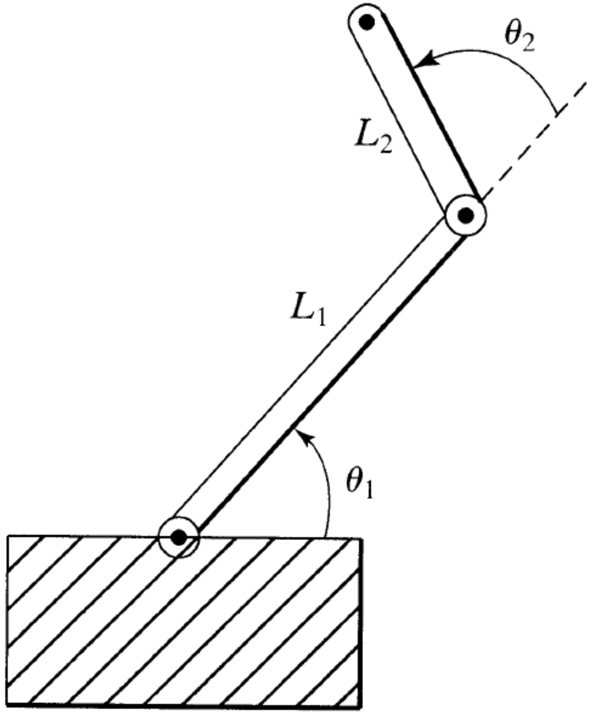


Figure 2: *2R Robot (Problem 2)*

P03

$${}^0J \text{ through } {}^3J$$

$${}^0J = \underbrace{{}^0_3R}_{\text{2D rotation}}\,{}^3J = \begin{pmatrix} c_{12} & -s_{12} \\ s_{12} & c_{12} \end{pmatrix} \begin{pmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{pmatrix} = \begin{pmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{pmatrix}$$

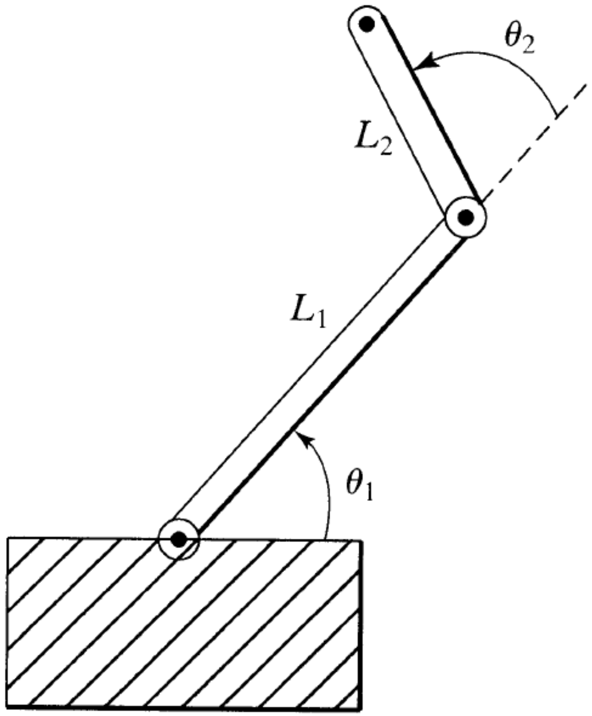


Figure 2: *2R Robot (Problem 2)*

P03

$$\det({}^{0}J(\theta)) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{vmatrix} = \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} \\ l_1 c_1 & l_2 c_{12} \end{vmatrix} =$$

$$= l_1 l_2 s_2$$

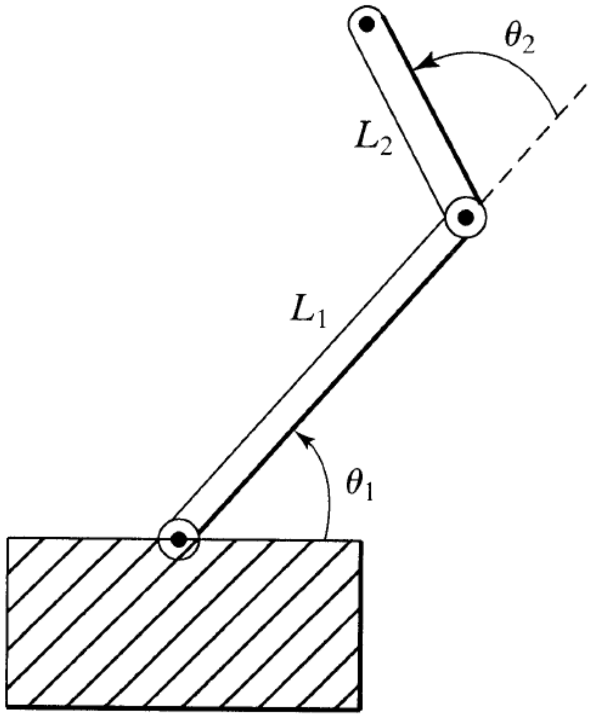


Figure 2: *2R Robot (Problem 2)*

P03

$$\det({}^0J(\theta)) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{vmatrix} = \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} \\ l_1 c_1 & l_2 c_{12} \end{vmatrix} =$$

$$= l_1 l_2 s_2$$

$$\det({}^3J(\theta)) = \begin{vmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{vmatrix} = l_1 l_2 s_2$$

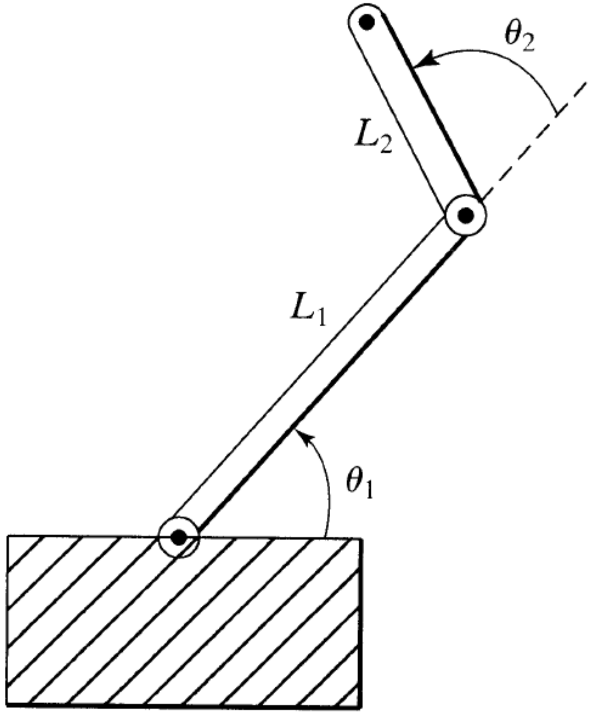


Figure 2: *2R Robot (Problem 2)*

P03

$$\det\left({}^{0}J(\theta)\right) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{vmatrix} = \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} \\ l_1 c_1 & l_2 c_{12} \end{vmatrix} =$$

$$= l_1 l_2 s_2$$

$$\det\left({}^{3}J(\theta)\right) = \begin{vmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{vmatrix} = l_1 l_2 s_2$$

singularities: $\det\left(J(\theta)\right) = 0 = l_1 l_2 s_2 \Leftrightarrow \begin{cases} l_1 = 0 \text{ or} \\ l_2 = 0 \text{ or} \\ s_2 = 0 \Leftrightarrow \theta_2 = \{0^\circ, 180^\circ\} \end{cases}$

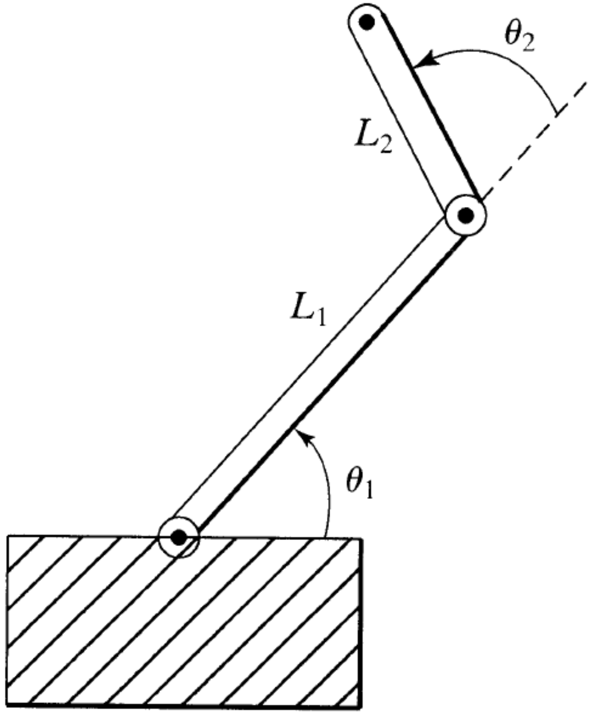


Figure 2: $2R$ *Robot (Problem 2)*

P03

- Easier to compute singularities on ${}^{3}J$
  because of the proximity to the tool coordinate frame (there are no transformation from the previous frames to consider)

$$\det\left({}^{0}J(\theta)\right) = \begin{vmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{vmatrix} = \begin{vmatrix} -l_1 s_1 & -l_2 s_{12} \\ l_1 c_1 & l_2 c_{12} \end{vmatrix} =$$

$$= l_1 l_2 s_2$$

$$\det\left({}^{3}J(\theta)\right) = \begin{vmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{vmatrix} = l_1 l_2 s_2$$

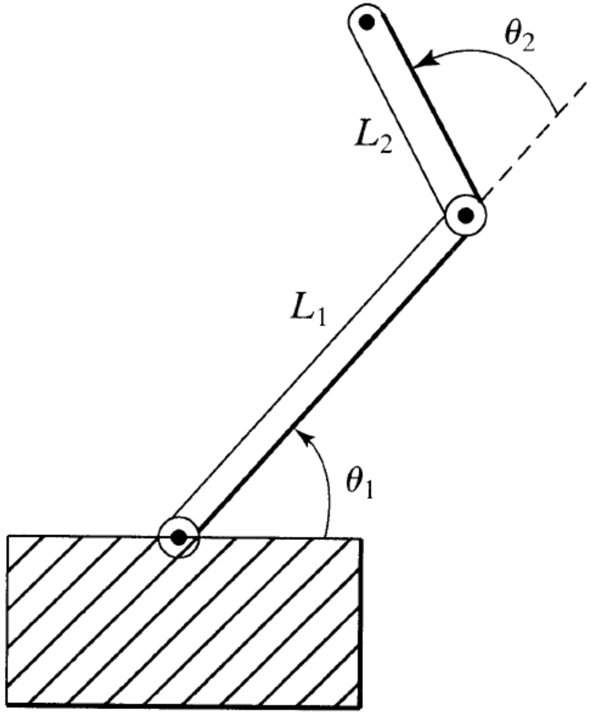


Figure 2: *2R Robot (Problem 2)*