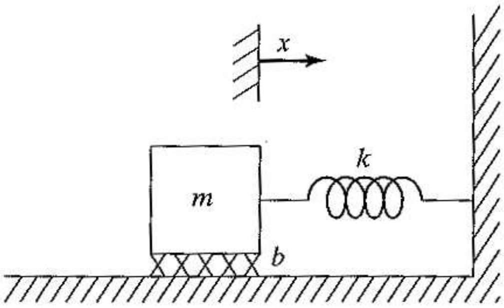


Figure 1: *Simple mass-spring-system.*

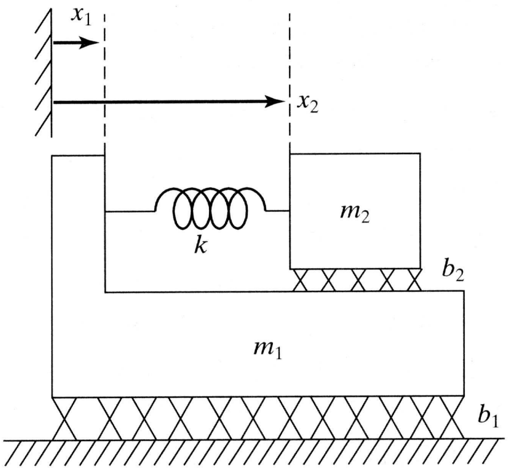


Figure 2: *Complex mass-spring-system (Problem 2)*

Problem 1

Consider a simple mass-spring system (Figure 1) with one object of mass $m = 1$, attached to a spring with stiffness $k = 5$ and affected by friction with a friction constant $b = 4$. The system has a resonant frequency of $\omega_{\text{res}} = 6.0$. Determine k_v and k_p such that the system is critically damped.

Problem 2

Derive a PD controlling scheme for the system shown in Figure 2 that allows following of trajectories for both objects and critically damps the error. The steps you should perform are the following:

- Determine forces that apply to both objects, derive equations of motion.
- Apply the control law partitioning principle. Explicitely show model-based portion and servo portion of the control law.
- Formulate the error equation.

P_1 $\quad m=1 \quad k=5 \quad b=4 \quad \omega_R = 6.0$

$$m\ddot{x} + b\dot{x} + kx = f = -k_v\dot{x} - k_p x$$

$$\ddot{x} + (4+k_v)\dot{x} + (5+k_p)x = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = 5 + k_p$$

$$\omega_n = \sqrt{5+k_p} \le \frac{1}{2}\omega_r.$$

$$5+k_p \le \frac{1}{4}\omega_r^2$$

$$5+k_p \le 9$$

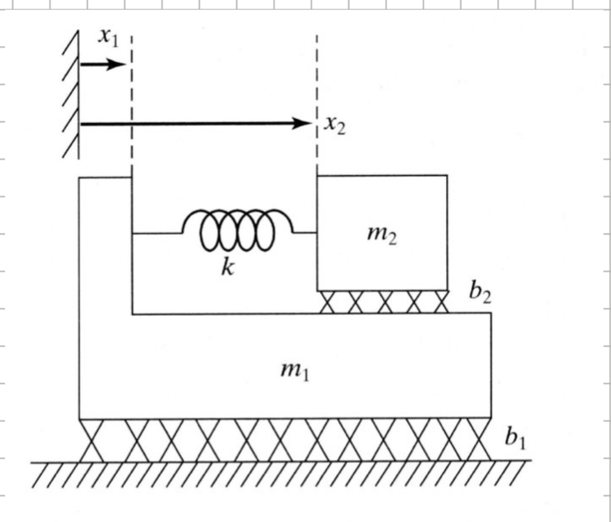
$$k_p \le 4.$$

$$(4+k_v)^2 = 4(5+k_p)$$

$$4+k_v = 2\sqrt{5+k_p}.$$

$$k_v = 2\sqrt{5+4} - 4$$
$$= 6-4$$

$$k_v = 2. \quad k_p = 4.$$



$$m_2\ddot{x}_2 + b_2\cdot(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \quad = f$$

$$m_1\ddot{x}_1 + b_1\dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) = 0 = f.$$

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} \cdot & \cdot \end{pmatrix}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} - \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$M\ddot{x} + B\dot{x} + Kx = \alpha f' + \beta$$

$$\begin{cases} \alpha = M \\ \beta = B\dot{x} + Kx \\ f' = \ddot{x} = \ddot{x}_d + k_v\dot{e} + k_p e \end{cases}$$

$$\Rightarrow \ddot{e} + k_v\dot{e} + k_p e = 0 \qquad e = x_d - x$$

$$k_{v_i} = 2\sqrt{k_{p_i}}$$