Jacobians

Partial derivatives

$$
\begin{aligned}
\delta y_1 &= \frac{\partial f_1}{\partial x_1}\delta x_1 + \frac{\partial f_1}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f_1}{\partial x_6}\delta x_6, \\
\delta y_2 &= \frac{\partial f_2}{\partial x_1}\delta x_1 + \frac{\partial f_2}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f_2}{\partial x_6}\delta x_6, \\
&\vdots \\
\delta y_6 &= \frac{\partial f_6}{\partial x_1}\delta x_1 + \frac{\partial f_6}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f_6}{\partial x_6}\delta x_6,
\end{aligned}
$$

$$\delta Y = J(X)\delta X.$$

$$
{}^0J = \begin{pmatrix}
\frac{\partial p_1}{\partial x_1} & \frac{\partial p_1}{\partial x_2} & \cdots & \frac{\partial p_1}{\partial x_n} \\
\frac{\partial p_2}{\partial x_1} & \frac{\partial p_2}{\partial x_2} & \cdots & \frac{\partial p_2}{\partial x_n} \\
\frac{\partial p_3}{\partial x_1} & \frac{\partial p_3}{\partial x_2} & \cdots & \frac{\partial p_3}{\partial x_n}
\end{pmatrix}
$$

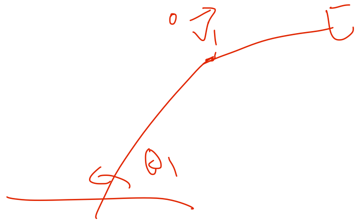
$$
\begin{aligned}
{}^{i}v_{i+1} &= {}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1} \\
{}^{i+1}v_{i+1} &= {}^{i+1}R_i \left({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1}\right) \\
{}^{i+1}\omega_{i+1} &= {}^{i+1}_{i}R \cdot {}^{i}\omega_i + \dot{\Theta}_{i+1} \cdot {}^{i+1}\hat{Z}_{i+1}.
\end{aligned}
$$

$$
{}^{0}J = \begin{pmatrix}
\frac{\partial p_1}{\partial x_1} & \frac{\partial p_1}{\partial x_2} & \dots & \frac{\partial p_1}{\partial x_n} \\
\frac{\partial p_2}{\partial x_1} & \frac{\partial p_2}{\partial x_2} & \dots & \frac{\partial p_2}{\partial x_n} \\
\frac{\partial p_3}{\partial x_1} & \frac{\partial p_3}{\partial x_2} & \dots & \frac{\partial p_3}{\partial x_n} \\
{}^{0}\hat{Z}_1 & {}^{0}\hat{Z}_2 & \dots & {}^{0}\hat{Z}_n
\end{pmatrix}
$$

$$
\begin{aligned}
{}^{n}\omega_n &= {}^{n}_{n-1}R \cdot {}^{n-1}\omega_{n-1} + \dot{\Theta}_n \cdot {}^{n}\hat{Z}_n \\
&= {}^{n}_{n-1}R \cdot ({}^{n-1}_{n-2}R \cdot {}^{n-2}\omega_{n-2} + \dot{\Theta}_{n-1} \cdot {}^{n-1}\hat{Z}_{n-1}) + \dot{\Theta}_n \cdot {}^{n}\hat{Z}_n \\
&= {}^{n}_{n-1}R \cdot ({}^{n-1}_{n-2}R \cdot ({}^{n-2}_{n-3}R \cdot {}^{n-3}\omega_{n-3} + \dot{\Theta}_{n-2} \cdot {}^{n-2}\hat{Z}_{n-2}) + \dot{\Theta}_{n-1} \cdot {}^{n-1}\hat{Z}_{n-1}) + \dot{\Theta}_n \cdot \\
&= {}^{n}_{1}R \cdot \dot{\Theta}_1 \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} + {}^{n}_{2}R \cdot \dot{\Theta}_2 \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} + {}^{n}_{3}R \cdot \dot{\Theta}_3 \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} + \ldots + {}^{n}_{n}R \cdot \dot{\Theta}_n \cdot \begin{pmatrix}0\\0\\1\end{pmatrix}
\end{aligned}
$$

$$
\begin{bmatrix} {}^{A}v \\ {}^{A}\omega \end{bmatrix} = \left[\begin{array}{c|c} {}^{A}_{B}R & 0 \\ \hline 0 & {}^{A}_{B}R \end{array}\right] {}^{B}J(\Theta)\dot{\Theta} \qquad {}^{A}J(\Theta) = \left[\begin{array}{c|c} {}^{A}_{B}R & 0 \\ \hline 0 & {}^{A}_{B}R \end{array}\right] {}^{B}J(\Theta).
$$

$$\Rightarrow {}^3\vec{V} = \underbrace{{}^3\begin{pmatrix} & \end{pmatrix}}_{{}^3\tilde{J}} \begin{pmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{pmatrix} \qquad {}^x w = \tilde{J}_w \dot{\Theta}$$

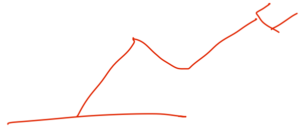


$${}^0\vec{V} = \vec{0} \qquad {}^0\vec{w} = \vec{0}$$

$${}^1\vec{v}_1 = {}^1_0\hat{R}\, {}^0\vec{V}_0 + {}^1\vec{w}_1 \times l_1 \qquad {}^A\hat{\vec{Z}}_A = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

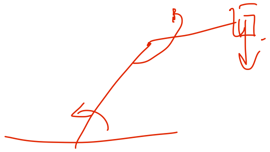
$${}^1\vec{w}_1 = {}^1_0\hat{R}\, {}^0\vec{w}_0 + {}^1\hat{\vec{z}}_1 \dot{\theta}_1$$

$${}^0 w_n = \boxed{\underbrace{\begin{pmatrix} {}^0\hat{z}_1, & {}^0\hat{z}_2, & \dots & {}^0\hat{z}_n \end{pmatrix}}_{{}^0 J_w}} \dot{\vec{\Theta}}$$



$$J_w = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tau = \begin{pmatrix} \vec{\tau}_1 \\ \vec{\tau}_2 \end{pmatrix}$$



$F = mg$

$$W = Fs = F\,\partial x \qquad W_c = \tau \cdot \partial\theta$$

$$\vec{F}^T \cdot \partial\vec{X} = \vec{\tau}^T \cdot \partial\vec{\theta}$$

$$\tilde{J}^T \vec{F} = \boxed{\vec{\tau}}$$

third method for calculting J

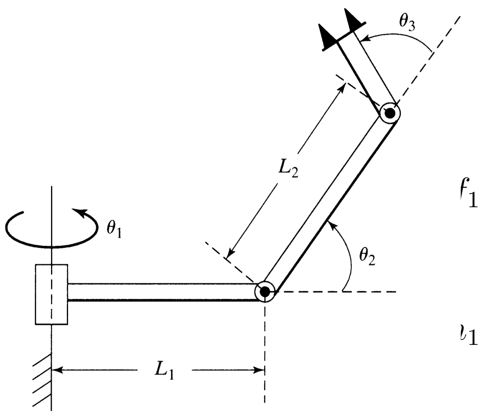
Set 3 Problem 2

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \vdots\\ \tau_n\end{pmatrix} = \tau = {}^A J^T\, {}^A\mathcal{F} = {}^A J^T \begin{pmatrix}{}^A f\\ {}^A n\end{pmatrix}$$

$${}^4 f_4 = \begin{pmatrix}F_1\\ F_2\\ F_3\end{pmatrix} \qquad {}^4 n_4 = \begin{pmatrix}N_1\\ N_2\\ N_3\end{pmatrix}$$

$$J^T = \begin{pmatrix} l_3 \sin\Theta_3 & l_3 & l_2 \cdot \cos\Theta_3 & 0 & 0 & 1\end{pmatrix} \begin{pmatrix}F_1\\ F_2\\ F_3\\ N_1\\ N_2\\ N_3\end{pmatrix}$$

$${}^3 f_3 = \begin{pmatrix}F_1\\ F_2\\ F_3\end{pmatrix} \qquad {}^3 n_3 = \begin{pmatrix}N_1\\ N_2 - F_3\, l_3\\ N_3 + F_2\, l_3\end{pmatrix}$$



$${}^2 f_2 = \begin{pmatrix}F_1\, \cos\Theta_3 - F_2\, \sin\Theta_3\\ F_1\, \sin\Theta_3 + F_2\, \cos\Theta_3\\ F_3\end{pmatrix} \qquad {}^2 n_2 = \begin{pmatrix}N_1\, \cos\Theta_3 - (N_2 - F_3\, l_3)\, \sin\Theta_3\\ N_1\, \sin\Theta_3 + (N_2 - F_3\, l_3)\, \cos\Theta_3 - l_2\, F_3\\ \underline{l_2\, (F_1\, \sin\Theta_3 + F_2\, \cos\Theta_3) + N_3 + F_2\, l_3}\end{pmatrix}$$

τ_2

$$f_1 = \begin{pmatrix}F_1\, \cos(\Theta_3 + \Theta_2) - F_2\, \sin(\Theta_3 + \Theta_2)\\ -F_3\\ F_1\, \sin(\Theta_3 + \Theta_2) + F_2\, \cos(\Theta_3 + \Theta_2)\end{pmatrix}$$

$$n_1 = \begin{pmatrix}(F_3\, l_3 - N_2)\, \sin(\Theta_3 + \Theta_2) + N_1\, \cos(\Theta_3 + \Theta_2) + l_2\, F_3\, \sin\Theta_2\\ -F_1\, l_1\, \sin(\Theta_3 + \Theta_2) - l_1\, F_2\, \cos(\Theta_3 + \Theta_2) - F_1\, l_2\, \sin\Theta_3 - F_2\, l_2\, \cos\Theta_3 - N_3 - F_2\, l_3\\ N_1\, \sin(\Theta_3 + \Theta_2) + (N_2 - F_3\, l_3)\, \cos(\Theta_3 + \Theta_2) - l_2\, F_3\, \cos\Theta_2 - l_1\, F_3\end{pmatrix}$$

$${}^4 J^{\mathrm{T}} \begin{pmatrix}F_1\\ F_2\\ F_3\\ N_1\\ N_2\\ N_3\end{pmatrix} = \begin{pmatrix}N_1\, \sin(\Theta_3 + \Theta_2) + F_3\, (-l_3\, \cos(\Theta_3 + \Theta_2) - l_2\, \cos\Theta_2 - l_1) + N_2\, \cos(\Theta_3 + \Theta_2)\\ F_1\, l_2\, \sin\Theta_3 + F_2\, (l_2\, \cos\Theta_3 + l_3) + N_3\\ N_3 + F_2\, l_3\end{pmatrix}$$

Denavit-Hartenberg Convention
Modified DH Paramers (proximal)

Modified Denavit-Hartenberg Convention

- Number the joints from 1 to n starting with the base and ending with the end-effector.
- *Establish the base coordinate system.* Establish a right-handed orthonormal coordinate system (X_0, Y_0, Z_0) at the supporting base with Z_0 axis lying along the axis of motion of joint 1.
- *Establish joint axis.* Align the Z_i with the axis of motion (rotary or sliding)
- *Establish the origin of the (i-1)th coordinate system.* Locate the origin of the (i-1)th coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- *Establish X_{i-1} axis.* Establish $= \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$ or along the common normal between the Z_{i-1} & Z_i axes when they are parallel.
- Find the link and joint parameters

Link and Joint Parameters

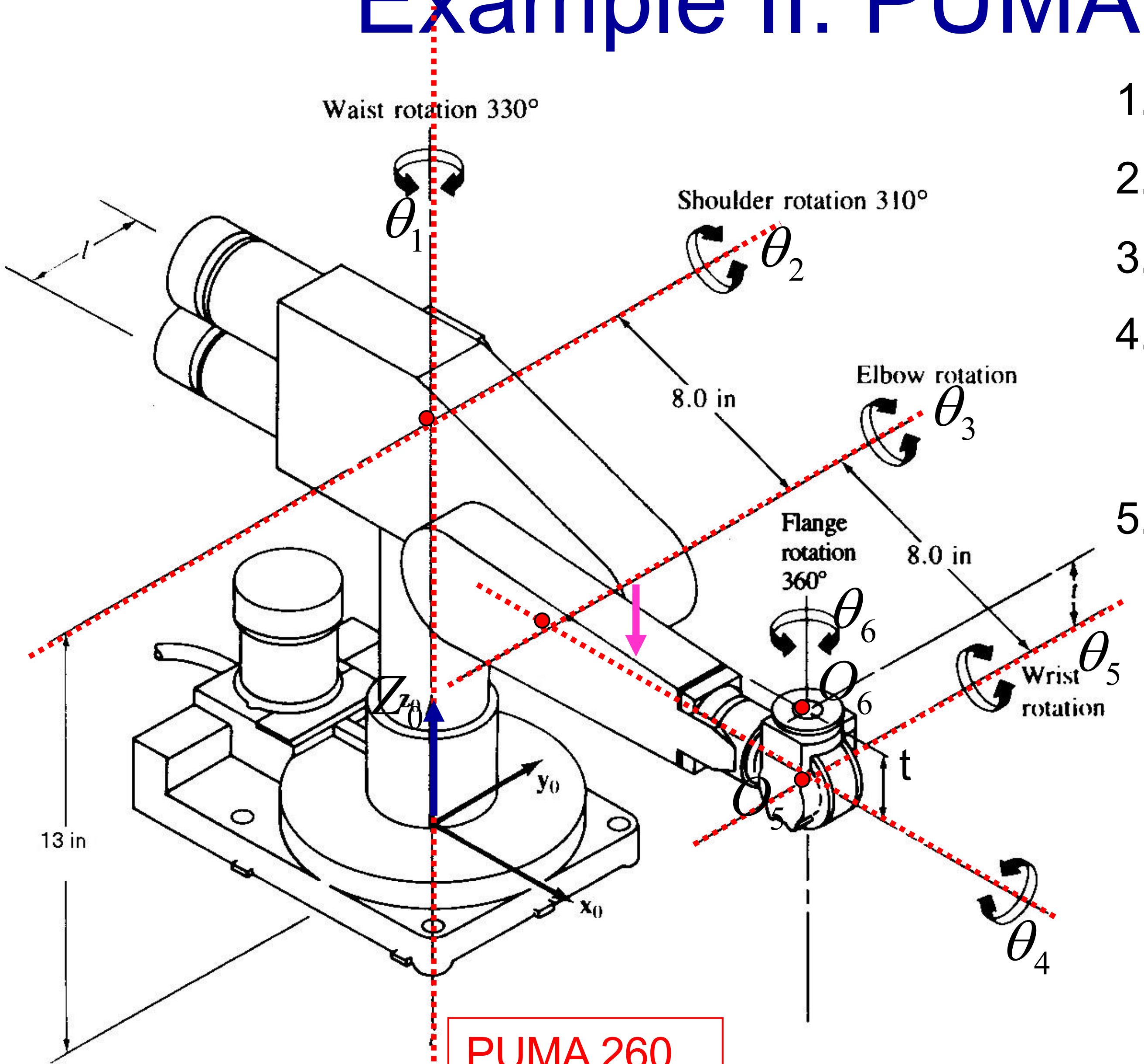
- *Joint angle* θ_i: the angle of rotation from the X_{i-1} axis to the X_i axis about the Z_{i-1} axis. It is the joint variable if joint i is rotary.

- *Joint distance* d_i : the distance from the origin of the (i-1) coordinate system to the intersection of the Z_{i-1} axis and the X_i axis along the Z_{i-1} axis. It is the joint variable if joint i is prismatic.

- *Link length* a_{i-1} : the distance from the intersection of the Z_{i-1} axis and the X_i axis to the origin of the ith coordinate system along the X_i axis.

- *Link twist angle* α_{i-1} : the angle of rotation from the Z_{i-1} axis to the Z_i axis about the X_i axis.

Example II: PUMA 260



PUMA 260

1. Number the joints
2. Establish base frame
3. Establish joint axis Z_i
4. Locate origin, (intersect. of Z_i & Z_{i-1}) OR (intersect of common normal & Z_i)
5. Establish X_i,Y_i