FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE
Exercise 6: Inference in First Order Logic – Problems   Winter semester 2023/2024
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Problem 6.1: The man in the painting

(The following puzzle appears in [1] Exercise 9.10.) A man stands in front of a painting and says the following:

> Brothers and sisters have I none, but that man's father is my father's son.

What is the relationship between the man in the painting and the speaker? Use the predicates

$$\begin{aligned} Male(x) &: x \text{ is male.} \\ Father(x,y) &: x \text{ is the father of } y. \\ Son(x,y) &: x \text{ is a son of } y. \\ Parent(x,y) &: x \text{ is a parent of } y. \\ Child(x,y) &: x \text{ is a child of } y. \\ Sibling(x,y) &: x \text{ is a sibling of } y \end{aligned}$$

and the knowledge

- A sibling is another child of one's parents.

$$\forall x, y \quad Sibling(x,y) \;\Leftrightarrow\; x \neq y \land \exists p \quad Parent(p,x) \land Parent(p,y)$$

- Parent and child are inverse relations.

$$\forall p, c \quad Parent(p,c) \;\Leftrightarrow\; Child(c,p)$$

to solve the riddle with first-order logic.

Problem 6.1.1: Define the rule which relates *son*, *child*, and *male*. Define also the rule which relates *father*, *parent*, and *male*.

Problem 6.1.2: Using the constants Me for the speaker and $That$ for the person depicted in the painting, formalize the sentences regarding the sexes of the people in the puzzle.

Problem 6.1.3: Formalize the sentences "Brothers and sisters have I none" and "That man's father is my father's son" in first-order logic.

Problem 6.1.4: Solve this puzzle informally and decide what is the relation between the man in the painting and the speaker.

Problem 6.1.5: Using the resolution technique for first-order logic, prove your answer. You can use the two diagrams on the next page to structure your proof.

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6.1.1

① Every Son is a male child, vice versa

$\forall s,p \ Son(s,p) \Leftrightarrow Male(s) \wedge Child(s,p)$

② Every father is a male parents, vice versa

$\forall s,f \ Father(f,s) \Leftrightarrow Male(f) \wedge parent(f,s)$

6.1.2

$Male(Me) \ / \ Male(That)$

Problem 6.1.3: Formalize the sentences "Brothers and sisters have I none" and "That man's father is my father's son" in first-order logic.

$\forall x \ \neg Sibling(x, Me) \wedge \neg sibling(Me, x)$

$\exists f_1, f_2 \quad Father(f_1, That) \wedge Father(f_2, Me) \wedge Son(f_1, f_2)$

Problem 6.1.4: Solve this puzzle informally and decide what is the relation between the man in the painting and the speaker.

$Son(That, Me)$

$\neg\alpha = \neg Son(That, Me)$

$\underline{\exists f_1, f_2} \quad Father(f_1, That) \wedge Father(f_2, Me) \wedge Son(f_1, f_2)$

$\equiv Father(F_1, That) \wedge Father(F_2, Me) \wedge Son(F_1, F_2)$

Problem 6.1.5: Using the resolution technique for first-order logic, prove your answer. You can use the two diagrams on the next page to structure your proof.

$\forall x, y \quad Sibling(x, y) \Leftrightarrow x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)$

$\equiv \forall x,y \ [Sibling(x,y) \Rightarrow x \neq y \wedge \exists p \ Parent(p,x) \wedge parent(p,y)] \wedge$

$[Sibling(x,y) \Leftarrow x \neq y \wedge \exists p \ Parent(p,x) \wedge parent(p,y)]$

$\forall x,y \ \neg Sibling(x,y) \vee [x \neq y \wedge \exists p \ Parent(p,x) \wedge parent(p,y)]$

$\forall x,y \ \neg Sibling(x,y) \vee [x \neq y \wedge Parent(F(x,y), x) \wedge Parent(F(x,y), y)]$

① $[\neg sibling(x,y) \vee x \neq y] \wedge [\neg sibling(x,y) \vee Parent(F(x,y), y)] \wedge [\neg sibling(x,y) \vee Parent(F(x,y), y)]$

$\forall (x,y) \ [x \neq y \wedge \exists p \ Parent(p,x) \wedge parent(p,y)] \Rightarrow Sibling(x,y)$

$[\neg x \neq y \vee \forall p \ \neg Parent(p,x) \vee \neg parent(p,y)] \vee Sibling(x,y)$

$\forall \cancel{(x,y,p)} \ x = y \vee \neg Parent(p,x) \vee \neg parent(p,y) \vee Sibling(x,y)$

②

$\cancel{Son}(s,p) \vee \neg Child(s,p) \vee \neg Male(s)$	$\neg \cancel{Son}(That, Me)$
$\cancel{Male}(That)$	$\neg Child(That, Me) \vee \neg \cancel{Male}(That)$
$\neg Parent(p,c) \vee \cancel{Child}(c,p)$	$\cancel{\neg Child(That, Me)}$
	$\neg Parent(Me, That)$

	$Son(F_1, F_2)$
	$Parent(Me, That)$

Problem 6.2: Backward chaining

(The following exercise is taken from [1] Exercise 9.9.) Suppose you are given the following axioms:

1. $0 \leq 3$
2. $7 \leq 9$
3. $\forall x \quad x \leq x$
4. $\forall x \quad x \leq x + 0$
5. $\forall x \quad x + 0 \leq x$
6. $\forall x, y \quad x + y \leq y + x$
7. $\forall w, x, y, z \quad w \leq y \land x \leq z \;\Rightarrow\; w + x \leq y + z$
8. $\forall x, y, z \quad x \leq y \land y \leq z \;\Rightarrow\; x \leq z.$

Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that lead to success, not the irrelevant steps.

References

[1] S. Russell and P. Norvig, *Artificial Intelligence: A Modern Approach*. Prentice Hall, 2010.

① $\{7 \le 3+a\}$

$q' \leftarrow subset(\phi,\ 7 \le 3+a)$

$7 \qquad 3+9$

$\Downarrow \qquad \Downarrow$

$\forall x_8, y_8, z_8 \quad x_8 \le y_8 \wedge y_8 \le z_8 \Rightarrow x_8 \le z_8$

$\theta' \leftarrow \{x_8/7,\ z_8/3+9\}$

new goal $\leftarrow \{x_8 \le y_8,\ y_8 \le z_8\}$

② $\{x_8 \le y_8,\ y_8 \le z_8\}$

$\Leftarrow \quad 7 \le y_8$

$q' \leftarrow subset(\{x_8/7,\ z_8/3+9\},\ x_8 \le y_8)$

$\forall x_4 \quad x_4 \le x_4 + 0$

$\theta' \leftarrow \{x_4/7,\ y_8/7+0\}$

new goal $\leftarrow \{y_8 \le z_8\}$

$7+0 \le 3+9$

$\Downarrow$

③ $\{y_8 \le z_8\}$

$q' \leftarrow subset(\{x_4/7,\ y_8/7+0,\ x_8/7,\ z_8/3+9\},\ y_8 \le z_8)$

$\forall x_8', y_8', z_8' \quad x_8' \le y_8' \wedge y_8' \le z_8' \Rightarrow x_8' \le z_8'$

$\theta' \leftarrow \{x_8'/7+0,\ z_8'/3+a\}$

new goal $\leftarrow \{x_8' \le y_8',\ y_8' \le z_8'\}$

④ $\{x_8' \le y_8'\}$

$7+0$

$\Downarrow$

$q' \leftarrow subset(\{x_4/7,\ y_8/7+0,\ x_8/7,\ z_8/3+9,\ x_8'/7+0,\ z_8'/3+9\},\ x_8' \le y_8')$

$\forall x_6, y_6 \quad x_6 + y_6 \le y_6 + x_6$

$\theta' \leftarrow \{x_6/7,\ y_6/0,\ y_8'/y_6+x_6\}$

new goal $\leftarrow \{y_8' \le z_8'\}$

⑤ $\{y_8' \le z_8'\}$