Fundamentals of Artificial Intelligence
Exercise 10: Making Simple Decisions

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Learning Outcomes of this Exercise

- You are able to draw the decision network of a decision problem.
- You are able to compute expected utilities (EUs).
- You understand rational decision making based on EUs.
- You are able to draw the decision tree of a decision problem.
- You are able to compute the value of information (VOI).
- You understand how the VOI influences the agent's decisions.

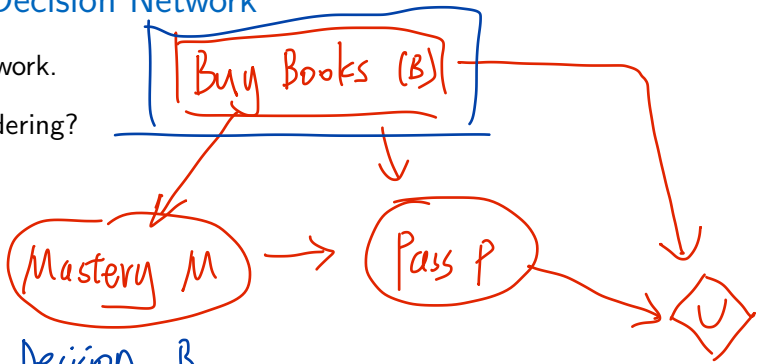
Task

Sam has the choice to buy or not buy a textbook for his course.

Problem 10.1.1: Decision Network

Draw the decision network.

What is the partial ordering?



1. Identify 1st Decision B
2. - - - next Decision. all random variables
3. Add all random variables that are not included

Problem 10.1.2: Expected Utility

$$EU(a) = \sum_{s'} P(\texttt{Result}(a) = s' | a) U(s').$$

Approach: Condition on decision variable, marginalize out random variables later in the partial ordering.

$$U(b) = -100€, \quad U(\neg b) = 0€ \quad U(p) = 2000€, \quad U(\neg p) = 0€$$

$$U(b, p) = ?, \quad U(\neg b, p) = ? \quad U(b, \neg p) = ?, \quad U(\neg b, \neg p) = ?$$

1900 2000 -100 0

Problem 10.1.2: Expected Utility

$EU(a|e) = \sum_{s'} P(s'|a,e) \cdot U(s')$

1) Expected utility of buying the book:

$P(p|b,m) = 0.9$
$P(p|b,\neg m) = 0.5$
$P(p|\neg b,m) = 0.8$
$P(p|\neg b,\neg m) = 0.3$
$P(m|b) = 0.9$
$P(m|\neg b) = 0.7$

$P(p|b) = \sum_m P(p|b,m) \cdot P(m|b)$

$= P(p|b,m) \cdot P(m|b) + P(p|b,\neg m) \cdot P(\neg m|b)$

$= 0.9 \cdot 0.9 + 0.5 \cdot 0.1 = 0.86$

$EU(b) = \sum_P P(p|b)\, U(p,b) = P(p|b) \cdot U(p,b) + P(\neg p|b) \cdot U(\neg p,b)$

$= 0.86 \cdot 1900 + 0.14 \cdot (-100)$

$= 1620$

Problem 10.1.2: Expected Utility

2) Expected utility of not buying the book:

$$P(p|b,m) = 0.9$$
$$P(p|b,\neg m) = 0.5$$
$$P(p|\neg b,m) = 0.8$$
$$P(p|\neg b,\neg m) = 0.3$$
$$P(m|b) = 0.9$$
$$P(m|\neg b) = 0.7$$

$$EU(\neg b) = \sum_p P(p|\neg b) \cdot U(p,\neg b)$$
$$= P(p|\neg b) \cdot U(p,\neg b) + P(\neg p|\neg b) \cdot U(\neg p,\neg b)$$
$$P(p|\neg b) = \sum_m P(p|\neg b,m) \cdot P(m|\neg b)$$
$$= P(p|\neg b,m) \cdot P(m|\neg b) + P(p|\neg b,\neg m) \cdot P(\neg m|\neg b)$$
$$= 0.8 \cdot 0.7 + 0.3 \cdot 0.3 = 0.65$$
$$= 0.65 \cdot 2000 + 0.35 \cdot 0 = 1300$$

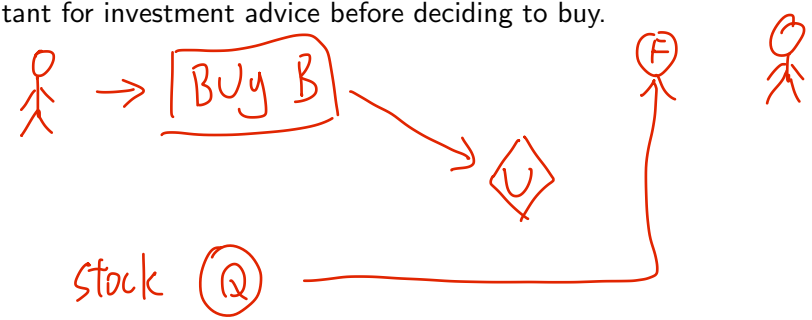
Problem 10.1.3: Making Simple Decision

What should Sam do?

$EU(b) = 1620 > EU(\neg b) = 1300$

Task

An investor has to decide whether he should buy a stock and whether he should ask a consultant for investment advice before deciding to buy.

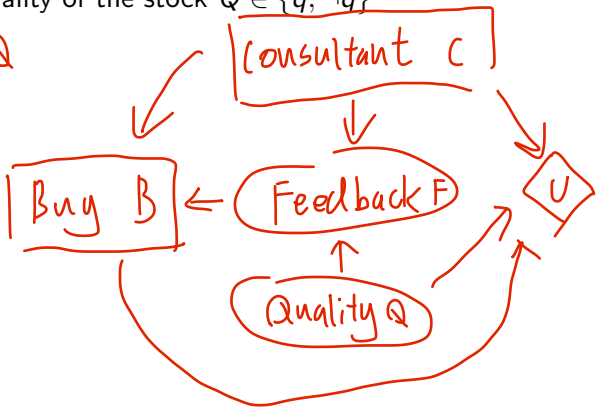


Problem 10.2.1: Decision Network

Draw the decision network:

Buy the stocks $B \in \{b, \neg b\}$, ask a consultant $C \in \{c, \neg c\}$, feedback from the consultant $F \in \{f, \neg f\}$, quality of the stock $Q \in \{q, \neg q\}$

$C \prec F \prec B \prec Q$



Problem 10.2.2: Decision Tree

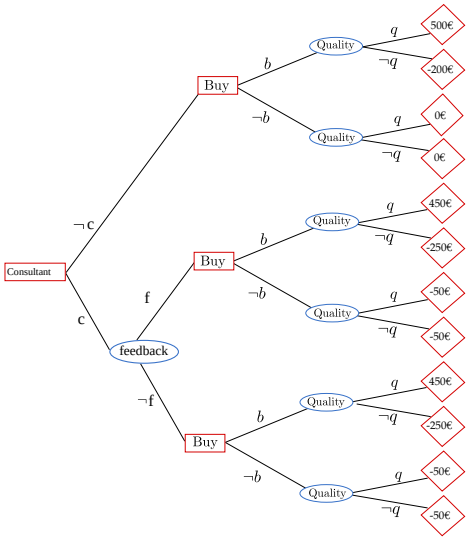
Draw the decision tree:

Utilities:

$$U(b) = -1500€, \quad U(\neg b) = 0€, \quad U(q) = 2000€ \quad U(\neg q) = 1300€$$
$$U(c) = -50€, \quad U(\neg c) = 0€$$

Compact form?

Problem 10.2.2: Decision Tree



Problem 10.2.3: Expected Utility

Calculate the expected utility of buying the stock, given no consultation:

$$P(q) = 0.7$$
$$P(f|q) = 0.85$$
$$P(f|\neg q) = 0.05$$
$$P(\neg q) = 0.3$$
$$P(\neg f|q) = 0.15$$
$$P(\neg f|\neg q) = 0.95$$

$$EU(b|\neg c) = \sum_Q P(Q|b,\neg c) \cdot V(Q,b,\neg c)$$
$$= P(q) \cdot V(q,b,\neg c) + P(\neg q) \cdot V(\neg q,b,\neg c)$$
$$= 0.7 \cdot 300 + 0.3 \cdot -200$$
$$\approx 210$$

Problem 10.2.4: Optimal Plan

Derive an optimal conditional plan for the investor.

Start with determining the optimal decisions whether to buy the stock given no consultation, a positive feedback or a negative feedback.

Calculate the **value of information** of the consultation.

$$VOI_e(E_j) = \left(\sum_u P(E_j = e_{ju} \mid e)\, MEU(a_{e_{ju}} \mid e, E_j = e_{ju}) \right) - MEU(a \mid e)$$

prior evidence; new evidence has value e_{ju} (information); new EU; current EU

Problem 10.2.4: Optimal Plan

Find a decision maximizing the expected utility for each decision variable:

$$MEU(d_{1:n}) = \max_{d_1} \sum_{x_1} \ldots \max_{d_n} \sum_{x_n} \prod_{i=1}^{n} P(x_i | x_{1:i-1}, d_{1:i}) U(x_{1:n}, d_{1:n})$$

What is the partial ordering?

C < F < B < A

Problem 10.2.4: Optimal Plan

The **last decision** is: should the investor **buy the stock?**

There are three cases:

- Case 1: Not asking the consultant.

$$\pi^*(B|\neg c) = \underset{B}{\operatorname{argmax}}\, EU(B|\neg c, none)$$

- Case 2: Ask the consultant, the feedback is positive.

$$\pi^*(B|c, f) = \underset{B}{\operatorname{argmax}}\, EU(B|c, f)$$

- Case 3: Ask the consultant, the feedback is negative.

$$\pi^*(B|c, \neg f) = \underset{B}{\operatorname{argmax}}\, EU(B|c, \neg f)$$

Problem 10.2.4: Optimal Plan

$$EU(a) = \sum_{s'} P(s'|a) \cdot U(s', e)$$

Case 1: Not asking the consultant. $(\neg c, none)$

We already calculated $EU(b|\neg c) = 290€$.

What is $EU(\neg b|\neg c)$?

$$EU(\neg b|\neg c) = 0$$

$$EU(b|\neg c) = 290$$

Problem 10.2.4: Optimal Plan

Case 2: Ask the consultant, the feedback is positive. (c, f).

$P(q) = 0.7$
$P(f|q) = 0.85$
$P(f|\neg q) = 0.05$
$P(\neg q) = 0.3$
$P(\neg f|q) = 0.15$
$P(\neg f|\neg q) = 0.95$

$$P(q|f) = \frac{P(f|q) \cdot P(q)}{P(f)} = \alpha P(f|q) \cdot P(q) = \alpha \cdot 0.595$$

$$P(\neg q|f) = \alpha P(f|\neg q) \cdot P(\neg q) = \alpha \cdot 0.015$$

We need to calculate $EU(b|c, f)$ and $EU(\neg b|c, f)$!

$$= \begin{bmatrix} 0.9754 \\ 0.6246 \end{bmatrix}$$

Problem 10.2.4: Optimal Plan

$$EU(b|c,f) = P(q|c,f)\,U(q,b) + P(\neg q|c,f)\cdot U(\neg q,b)$$

$$= 0.975 \cdot 500 + 0.025 \cdot (-200) - 50$$

$$= 432.5$$

$$EU(\neg b|c,f) = P(q|c,f)\,\underbrace{U(q,\neg b)}_{0} + P(\neg q|c,f)\cdot \underbrace{U(\neg q,\neg b)}_{0} - 50$$

$$= -50$$

Problem 10.2.4: Optimal Plan

Case 3: Ask the consultant, the feedback is negative $(c, \neg f)$.

$P(q) = 0.7$
$P(f|q) = 0.85$
$P(f|\neg q) = 0.05$
$P(\neg q) = 0.3$
$P(\neg f|q) = 0.15$
$P(\neg f|\neg q) = 0.95$

$p(q|\neg f) = \alpha p(\neg f|q) \cdot p(q) = \alpha \cdot 0.105$

$p(\neg q|\neg f) = \alpha p(\neg f|\neg q) \cdot p(\neg q) = \alpha \cdot 0.285$

$\begin{bmatrix} 0.2692 \\ 0.7308 \end{bmatrix}$

We need to calculate $EU(b|c, \neg f)$ and $EU(\neg b|c, \neg f)$!

$EU(b|c,\neg f) = p(q|c,\neg f) \cdot U(b,q) + p(\neg q|c,\neg f) \cdot U(b,\neg q)$

$= 0.269 \cdot 500 + 0.73 \cdot (-200) - 50$

$= -61.7$

Problem 10.2.4: Optimal Plan

$EV(\text{table}, \text{left}) = -50$

Problem 10.2.4: Optimal Plan

Second last decision: Should the investor **ask the consultant?**

How much is the Info val

There are two cases:

- Case 1: Asking the consultant (c).
- Case 2: Not asking the consultant ($\neg c$).

Problem 10.2.4: Optimal Plan

Case 1: Asking the consultant.

$$EU(c) = \sum_F P(F \mid c) \max_{\mathcal{B}} \sum_Q P(Q \mid F, c)\, U(Q, c, \mathcal{B})$$

$P(F \mid c)$: known

$\max_{\mathcal{B}}$: $\pi(\mathcal{B} \mid c, F)$

$P(Q \mid F, c)$: known

Problem 10.2.4: Optimal Plan

Case 2 : Not asking the consultant.

Problem 10.2.4: Value of Information

Calculate the value of perfect information for the consultation.

$VOI(F) =$