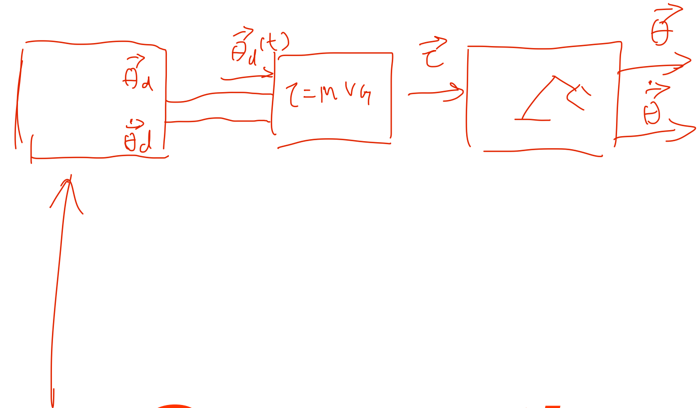

Trajectory Generation

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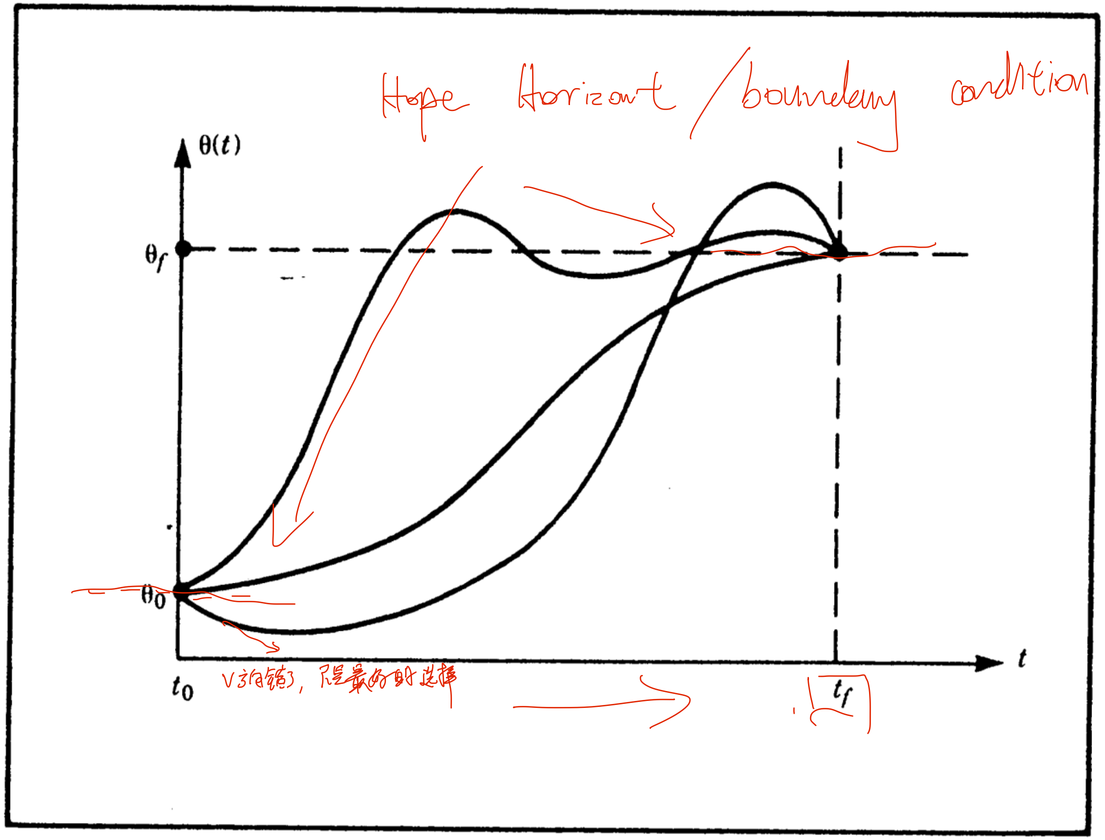


$$\theta(0) = \theta_0,$$

$$\theta(t_f) = \theta_f.$$

$$\dot{\theta}(0) = 0,$$

$$\dot{\theta}(t_f) = 0.$$

$$\theta_0 = a_0,$$

$$\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3,$$

$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2$

$$0 = a_1,$$

$t = 0 \Rightarrow \dot{\theta}_f = a_1 = 0$

$\theta_0 = a_0$

$$0 = a_1 + 2a_2 t_f + 3a_3 t_f^2.$$

$$a_0 = \theta_0,$$

$$a_1 = 0,$$

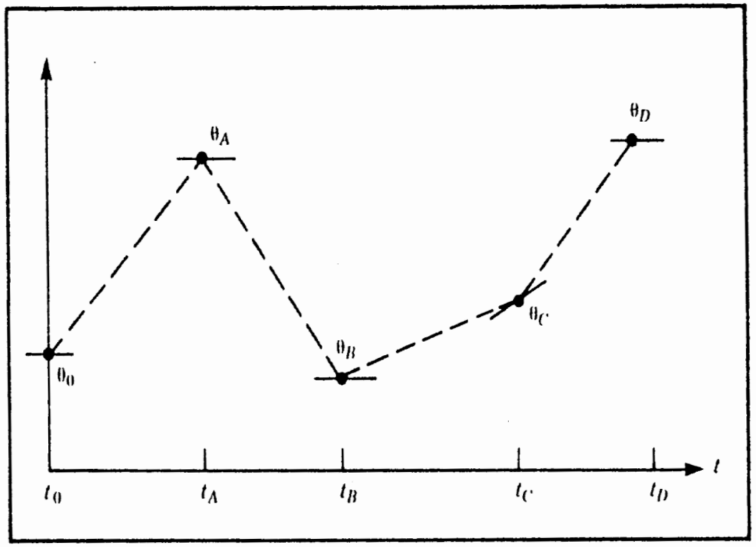
$0 = 0 + 2a_2 t_f + 3a_3 t_f^2$

free choose

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0),$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0).$$

Via Points



$$\theta_0 = a_0,$$

$$\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3,$$

$$\dot{\theta}_0 = a_1,$$

$$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2.$$

Solving these equations for the a_i we obtain

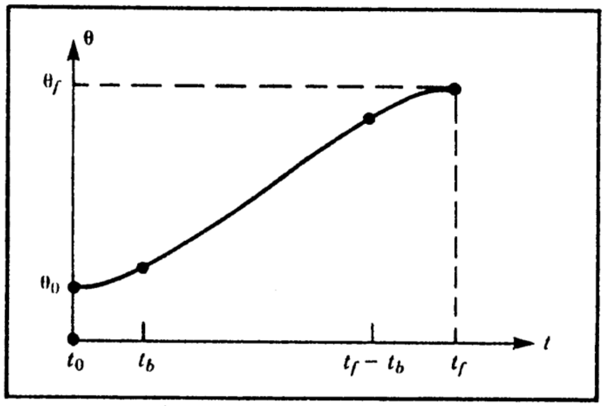
$$a_0 = \theta_0,$$

$$a_1 = \dot{\theta}_0,$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f,$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0).$$

Parabolic blends



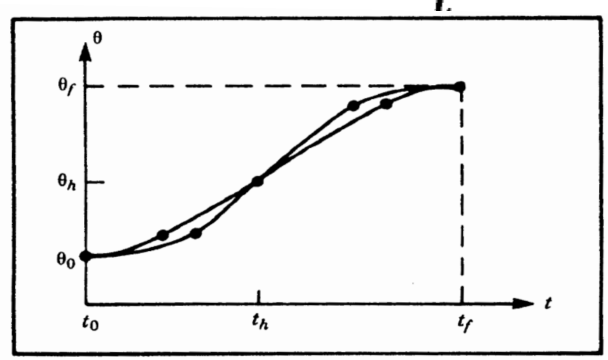
$$\boxed{\ddot{\theta} t_b} = \frac{\theta_h - \theta_b}{t_h - t_b},$$

$$\theta_b = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2.$$

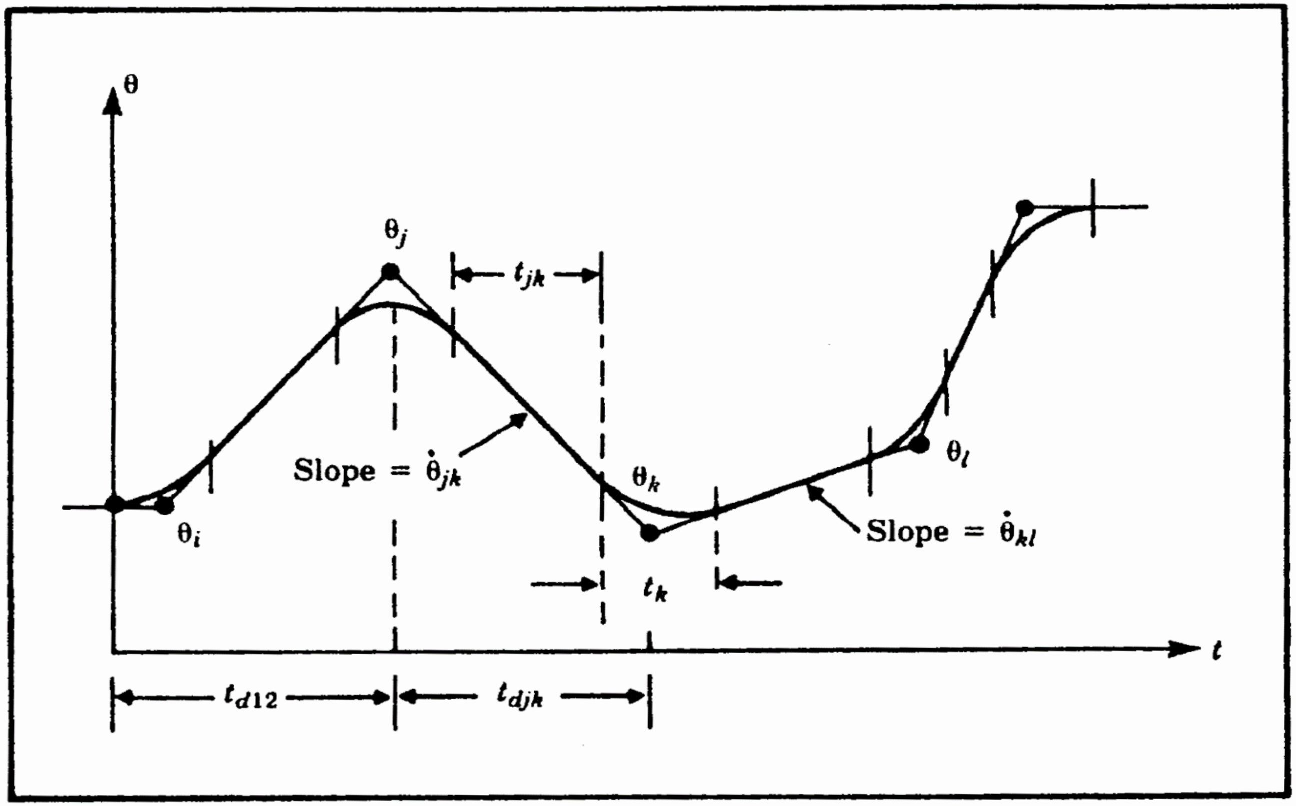
$$\ddot{\theta} t_b^2 - \ddot{\theta} t t_b + (\theta_f - \theta_0) = 0,$$

$$\ddot{\theta} \geq \frac{4(\theta_f - \theta_0)}{t^2}.$$

$$t_b = \frac{t}{2} - \frac{\sqrt{\ddot{\theta}^2 t^2 - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}}.$$



Multisegment Linear Path with multiple blends



$$\dot{\theta}_{jk} = \frac{\theta_k - \theta_j}{t_{djk}},$$

$$\ddot{\theta}_k = SGN(\dot{\theta}_{kl} - \dot{\theta}_{jk}) \left| \ddot{\theta}_k \right|,$$

$$t_k = \frac{\dot{\theta}_{kl} - \dot{\theta}_{jk}}{\ddot{\theta}_k},$$

$$t_{jk} = t_{djk} - \frac{1}{2} t_j - \frac{1}{2} t_k.$$

For the first segment, we solve for t_1 by equating two expressions for the velocity during the linear phase of the segment:

$$\frac{\theta_2 - \theta_1}{t_{d12} - \frac{1}{2} t_1} = \ddot{\theta}_1 t_1. \qquad (7.25)$$

This can be solved for t_1, the blend time at the initial point, and then $\dot{\theta}_{12}$ and t_{12} are easily computed:

$$
\begin{aligned}
\ddot{\theta}_1 &= SGN(\theta_2 - \theta_1)\left|\ddot{\theta}_1\right|, \\
t_1 &= t_{d12} - \sqrt{t_{d12}^2 - \frac{2(\theta_2 - \theta_1)}{\ddot{\theta}_1}}, \\
\dot{\theta}_{12} &= \frac{\theta_2 - \theta_1}{t_{d12} - \frac{1}{2}t_1}, \\
t_{12} &= t_{d12} - t_1 - \frac{1}{2}t_2.
\end{aligned}
\tag{7.26}
$$

Likewise, for the last segment (the one connecting points $n-1$ and n) we have

$$
\frac{\theta_{n-1} - \theta_n}{t_{d(n-1)n} - \frac{1}{2}t_n} = \ddot{\theta}_n t_n, \tag{7.27}
$$

which leads to the solution

$$
\begin{aligned}
\ddot{\theta}_n &= SGN(\theta_{n-1} - \theta_n)\left|\ddot{\theta}_n\right|, \\
t_n &= t_{d(n-1)n} - \sqrt{t_{d(n-1)n}^2 + \frac{2(\theta_n - \theta_{n-1})}{\ddot{\theta}_n}}, \\
\dot{\theta}_{(n-1)n} &= \frac{\theta_n - \theta_{n-1}}{t_{d(n-1)n} - \frac{1}{2}t_n}, \\
t_{(n-1)n} &= t_{d(n-1)n} - t_n - \frac{1}{2}t_{n-1}.
\end{aligned}
\tag{7.28}
$$