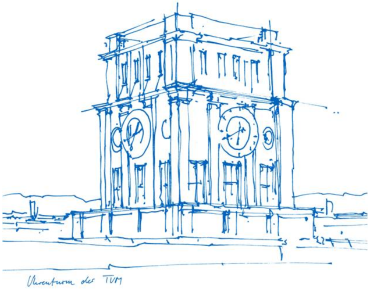
Computer Vision II: Multiple View Geometry (IN2228)

Chapter 05 Correspondence Estimation
(Part 1 Small Motion)

Dr. Haoang Li

11 May 2023 11:00 to 11:45



[?] 课程内容和练习环节
- 今年，我们有新的讲师和新的教学助理。讲座的幻灯片是全新的，但练习题部分是基于往年的材料。
- 我将尝试在今后的练习环节中引入更多的详细知识要求。

Announcement

[?] 课程内容和考试
- 考试问题将与课程内容最为一致，所以与往年的问题会有部分不同。(注意：仍然会有一些重叠。)
- 考试中涉及的所有知识都会在我们的课堂上明确介绍。因此，只要你理解了我们课堂上介绍的知识，你应该获得一个好成绩。
- 我将准备一堂课来复习考试中的重要知识（暂定在7月13日）。P.S.: 考试是在8月4日。

➢ Course Content, Exercise Session, and Exam

✓ Course content and exercise session

- This year, we have new lecturers and new teaching assistants. Slides for lectures are totally new, but exercise questions are partly based on the materials from the previous years.
- **I will try to introduce more detailed knowledge required by the exercise session in the future**.

✓ Couse content and exam

- **Exam questions will be most aligned to the course content**, so they will be partly different from questions from previous years. (Note: there still will be some overlaps.)
- All the involved knowledge in the exam will be clearly introduced in our class. Therefore, **as long as you understand the knowledge introduced in our class, you should obtain a good grade**.
- I will prepare **a class to review knowledge important for the exam (tentatively on 13 July).** P.S.: Exam is on 04 August.

Announcement

☐ 编程任务
- 我们收到了一些反馈，我们讨论了潜在的解决方案。
- 例如，我们的助教将在样本测试中增加额外的反馈，以帮助学生更容易通过测试。
- 请注意，还有一些隐藏的测试案例，你的解决方案正在被评估。

☐ 奖金
- 从我的角度来看，这个奖金不是很容易获得。
- 如果我们认为你要花的时间比你预期的多得多，也就是说，这不是很 "经济"，请主要关注我们的讲座内容。
- 即使没有奖金，你也可以获得一个满意的成绩。

➢ Programming Assignments and Bonus

✓ Programming assignment

- We received some feedback and we have discussed potential solutions.
- For example, our teaching assistants will add additional feedback on the sample tests to help students pass the tests more easily.
- Please note that there are also hidden test cases that your solution is being evaluated against.

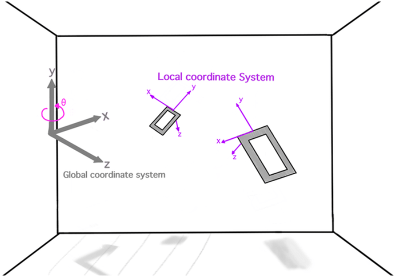
✓ Bonus

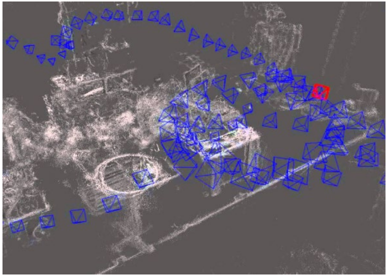
- From my perspective, this bonus is not very easy to obtain.
- If we think you have to spend much more time than you expected, i.e., it is not very “economical”, please mainly focus on the content of our lecture.
- You can still obtain a satisfactory grade even without bonus.

Clarification

➢ Single Camera and Multiple Camera Frames

- ✓ In VO/SLAM/SFM, we use a **single camera** to obtain **multiple images** from different view points. **Multiple view points** correspond to **multiple camera frames**.
- ✓ However, in practice, we may do not differentiate between "cameras" and "camera frames".





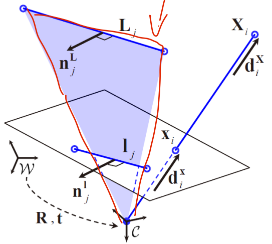
Clarification

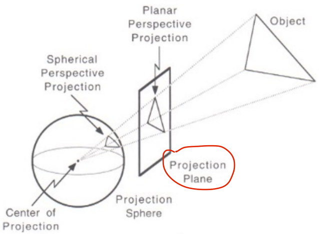
? 严格来说，投影平面是指由坐标系的原点和三维线/二维线定义的平面。
? 然而，在实践中，投影平面也可能对应于图像平面。
? 投影射线是指由坐标系的原点和一条3D线/2D线定义的3D方向。3D点/2D点。

➢ Projection Plane and Image Plane

- ✓ Strictly, projection plane refers to the plane defined by the origin of a coordinate system and a 3D line/2D line.
- ✓ However, in practice, projection plane may also correspond to the image plane.
- ✓ Projection ray refers to the 3D direction defined by the origin of a coordinate system and a 3D point/2D point.

projection plan





Explanation

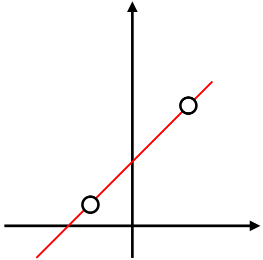
➢ Homogeneous Coordinates of 2D line

Two representative methods introduced in the middle school

✓ $y=kx+b$

✓ $ax+by+c=0$

- (a, b, c) is the homogenous coordinates of 2D line
- $(1, 2, 3)$ is equivalent to $(2, 4, 6)$



- Two points (x_0, y_0) and (x_1, y_1) determine a 2D line

$$\begin{cases} ax_0+by_0+c=0 \\ ax_1+by_1+c=0 \end{cases}$$

To solve this linear system, we can choose an arbitrary value of c

- We can directly obtain (a, b, c) by the cross product between $(x_0, y_0, 1)$ and $(x_1, y_1, 1)$

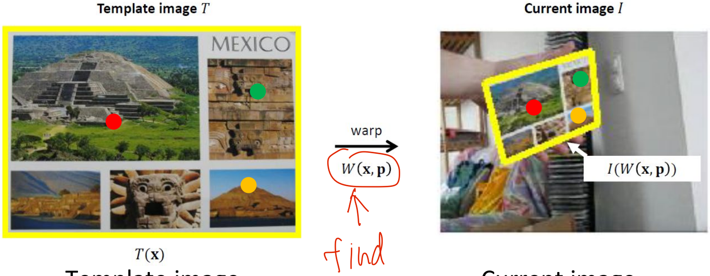
Today's Outline

- Overview of Matching/Tracking Problem
- KLT Tracker for Small Motion
- Simplified Case: Pure Translation
- General Case

Overview of Matching/Tracking Problem

➢ Problem Formulation

✓ A practical task: estimate the transformation W (warping) between a template image T and the current image I.



Template image   Current image

✓ Clue: All the (inlier) 2D-2D point correspondences should satisfy the same warping model.

Overview of Matching/Tracking Problem

➢ Problem Formulation

✓ The warping estimation problem can be reformulated as the correspondence finding problem.

✓ Example of Euclidian transformation

$$x' = x\cos(a_3) - y\sin(a_3) + a_1$$
$$y' = x\sin(a_3) + y\cos(a_3) + a_2$$

2 translation 1 rotation

(x,y) and (x',y') constitute a pair of unknown-but-sought correspondence

chicken-egg problem

$\mathbf{p} = (a_1, a_2, \ldots, a_n)$ are warping parameters to estimate

✓ Two types of solutions to find correspondences exit: indirect and direct methods.

Overview of Matching/Tracking Problem

➢ Problem Formulation

2 method

✓ Indirect methods (next week)

- They work by detecting and matching features (points or lines)
- Pros: They can cope with large frame-to-frame motions (large basin of convergence) and strong illumination changes
- Cons: They are slow due to costly feature extraction, matching, and outlier removal (e.g., RANSAC)

Matched points

Overview of Matching/Tracking Problem

➢ Problem Formulation

✓ Indirect methods (next week)

1. Detect and match features that are invariant to scale, rotation, view point changes (e.g., SIFT)

2. Geometric verification (RANSAC) (e.g., 4 point RANSAC for planar objects, or 5 or 8 point RANSAC for 3D objects)

3. Refine estimate by minimizing the sum of squared reprojection errors between the observed feature $\boldsymbol{f}^i$ in the current image and the warped corresponding feature $W(\mathbf{x}^i,\mathbf{p})$ from the template

$$\mathbf{p} = argmin_{\mathbf{p}} \sum_{i=1}^{N} \left\| W(\mathbf{x}^i,\mathbf{p}) - \boldsymbol{f}^i \right\|^2$$

Feature distance

Overview of Matching/Tracking Problem

- 优点：可以利用图像中的所有信息（精确度更高，对运动模糊和弱质地（即弱梯度）的鲁棒性也更高）。

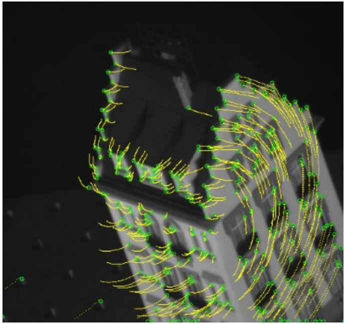
- 优点：提高相机帧率可以减少每帧的计算成本（不需要RANSAC）。

- 缺点：对初始值非常敏感，帧与帧之间的运动有限（收敛的小盆地）。

➢ Problem Formulation

✓ Direct methods (today)

- Pros: All information in the image can be exploited (higher accuracy, higher robustness to motion blur and weak texture (i.e., weak gradients))
- Pros: Increasing the camera frame rate reduces computational cost per frame (no RANSAC needed)
- Cons: Very sensitive to initial value limited frame to frame motion (small basin of convergence)



Tracked points

Overview of Matching/Tracking Problem

- Problem Formulation

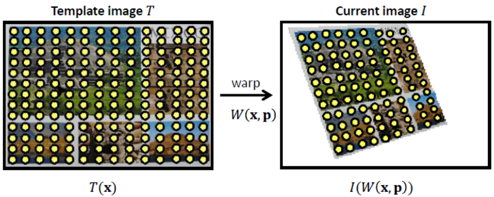
✓ Direct methods (today)

use all pixels

- They work by directly processing pixel intensities.
- Technically, they estimate the parameters p of the transformation $W(\mathbf{x},\mathbf{p})$ that minimize the Sum of Squared Differences:

$$\mathbf{p} = argmin_{\mathbf{p}} \sum_{\mathbf{x} \in \mathbf{T}} \left[I\left(W(\mathbf{x}, \mathbf{p})\right) - T(\mathbf{x}) \right]^2$$

Intensity distance



Every yellow dot in this image denotes a pixel

Overview of Matching/Tracking Problem

➢ Assumptions of Direct Methods

? 亮度恒定
- 追踪的像素的强度在连续的帧中没有太大变化
- 它不能应对强烈的光照变化

✓ Brightness constancy

- The intensity of the pixels to track does not change much over consecutive frames
- It does not cope with strong illumination changes

✔

✘

Overview of Matching/Tracking Problem

➢ Assumptions of Direct Methods

? 时间上的一致性
- 帧与帧之间的小运动（1-2像素）。
- 它不能应对大的帧到帧的运动。然而，这可以用从粗到细的多尺度实现来解决（见后文）。

✓ Temporal consistency

- Small frame-to-frame motion (1-2 pixels).
- It does not cope with large frame to frame motion. However, this can be addressed using coarse to fine multi scale implementations (see later)

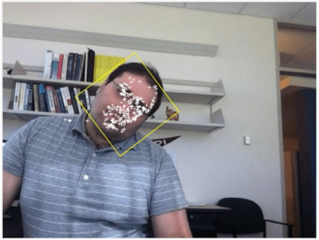
Overview of Matching/Tracking Problem

➢ Assumptions of Direct Methods

空间上的一致性
- 模板中的所有像素都经历了相同的变换（即它们大致位于同一个三维表面上）。

✓ Spatial coherency

- All pixels in the template undergo the same transformation (i.e., they roughly lie on the same 3D surface)



Overview of Matching/Tracking Problem

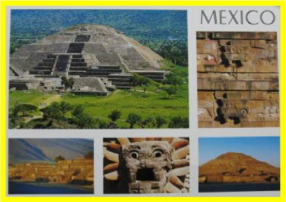
空间上的一致性

➢ Assumptions of Direct Methods

- 无遮挡：整个模板在输入图像中是可见的。
- 模板图像边界没有错误：只有要追踪的物体出现在模板图像中

✓ Spatial coherency

- No errors in the template image boundaries: only the object to track appears in the template image
- No occlusion: the entire template is visible in the input image



✔

Foreground and background have different motions

✘

Occlusion

✘

KLT Tracker for Small Motion

➢ Overview

The Kanade-Lucas-Tomasi (KLT) tracker tackles two problems:

✓ How should we select features? select reliable pixels

- Tomasi-Kanade: Method for choosing the best feature (image patch) for tracking

✓ How should we track them from frame to frame?

- Lucas-Kanade: Method for aligning (tracking) an image patch

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✓ Structure of our introduction

- Simplified case: pure translation
- General case

KLT Tracker for Small Motion

? 考虑在图像II0中以(x , yy)为中心的参考斑块和在图像E1中以($x + u$, $yy + vv$)为中心的移位斑块。该补丁的大小为Ω。

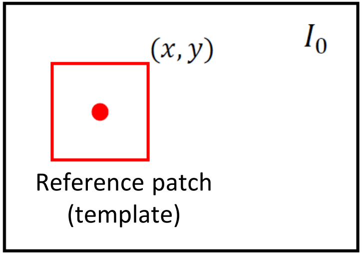
? 我们希望找到运动矢量（uu , vv ），使平方差之和最小。

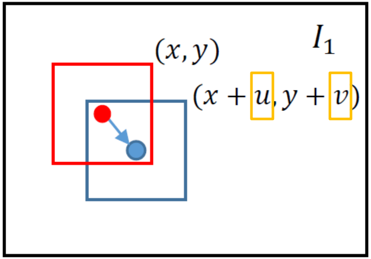
(SSD)对强度的影响最小（基于强度不变性假设）：

➢ Simplified Case: Pure Translation

✓ Consider the reference patch centered at (x,y) in image I_0 and the shifted patch centered at $(x+u,y+v)$ in image I_1. The patch has size Ω.

✓ We want to find the motion vector (u,v) that minimizes the Sum of Squared Differences (SSD) w.r.t. the intensity (based on the **intensity invariance assumption**):





KLT Tracker for Small Motion

➢ Simplified Case: Pure Translation

✓ Recap on mathematical knowledge

- Derivative and gradient

Function: $f(x)$ Single **univariate** function

Derivative: $f'(x) = \frac{df}{dx}$, where x is a scalar

Function: $f(x_1, x_2, \dots, x_n)$ Single **multivariate** function

Gradient: $\nabla f(x_1, x_2, \dots, x_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right)$

Multiple **multivariate** function

$F(x_1, x_2, \dots, x_n) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) \\ \vdots \\ f_m(x_1, x_2, \dots, x_n) \end{bmatrix}$ is a $\boxed{\text{vector-valued}}$ function

The derivative in this case is called Jacobian $\frac{\partial F}{\partial \mathbf{x}}$:

$$\frac{\partial F}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}, \dots, \frac{\partial f_1}{\partial x_n} \\ \vdots \\ \frac{\partial f_m}{\partial x_1}, \dots, \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

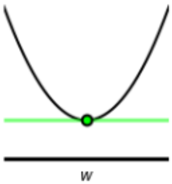
KLT Tracker for Small Motion

- Simplified Case: Pure Translation

✓ Recap on mathematical knowledge

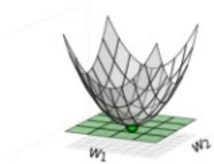
- First-order optimality condition

Zero-valued derivative(s) with respect to the unknown parameter(s) correspond to the minimum.



$$\frac{\mathrm{d}}{\mathrm{d}w} g(w) = 0$$

1D function



2D function

$$\frac{\partial}{\partial w_1} g(\mathbf{v}) = 0$$
$$\frac{\partial}{\partial w_2} g(\mathbf{v}) = 0$$
$$\vdots$$
$$\frac{\partial}{\partial w_N} g(\mathbf{v}) = 0$$

$$\Rightarrow \quad \nabla g(\mathbf{v}) = \mathbf{0}_{N \times 1}$$

Gradient notation

KLT Tracker for Small Motion

➤ Simplified Case: Pure Translation

✓ Cost function (quadratic function) w.r.t. two variables (u,v)

Two images

$$SSD(u,v) = \sum_{x,y\in\Omega} (I_0(x,y) - I_1(x+u,y+v))^2$$

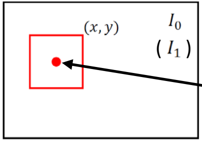
$$I_1(x+u,y+v) \cong I_1(\boxed{x,y}) + I_x u + I_y v$$

Directional derivative

$$\Rightarrow SSD(u,v) \cong \sum_{x,y\in\Omega} (\boxed{I_0(x,y) - I_1(x,y)} - I_x u - I_y v)^2$$

$$\Rightarrow SSD(u,v) \cong \sum_{x,y\in\Omega} (\boxed{\Delta I} - I_x u - I_y v)^2$$

unkown para.



Intensity difference at (x, y)

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-\boxed{a})^n$$

$$f(a) + \frac{f'(a)}{1!}(x-\boxed{a})$$

The first-order Taylor polynomial

KLT Tracker for Small Motion

➢ Simplified Case: Pure Translation

$$\Rightarrow SSD(u,v) \cong \sum_{x,y\in\Omega} (\Delta I - I_x u - I_y v)^2$$

✓ To minimize it, we differentiate it with respect to (u,v) and equate it to zero:

$$\frac{\partial SSD}{\partial u} = 0 \,, \; \frac{\partial SSD}{\partial v} = 0$$

$$\frac{\partial SSD}{\partial u} = 0 \;\Rightarrow\; \underline{-2} \sum \underline{I_x} \underline{(\Delta I - I_x u - I_y v)} = 0$$

$$\frac{\partial SSD}{\partial v} = 0 \;\Rightarrow\; \underline{-2} \sum \underline{I_y} \underline{(\Delta I - I_x u - I_y v)} = 0$$

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➢ Simplified Case: Pure Translation

$$\sum I_x(\Delta I - I_x u - I_y v) = 0$$

$$\sum I_y(\Delta I - I_x u - I_y v) = 0$$

✓ Linear system of two equations w.r.t. two unknown parameters (u,v)
✓ We can write them in matrix form:

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{\text{M}} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x \Delta I \\ \sum I_y \Delta I \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} u \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}^{-1}}_{\text{Inverse of M}} \begin{bmatrix} \sum I_x \Delta I \\ \sum I_y \Delta I \end{bmatrix}$$

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- ➢ Simplified Case: Pure Translation

不然没有逆了

Eigenvector

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$$

Eigenvalue

$$\det(A) = \lambda_1\lambda_2\cdots\lambda_n$$

- ✓ For M to be invertible, $\det(M)$ should be non zero, which means that its eigenvalues should be large (i.e., not a flat region, not an edge)
- ✓ In practice, it should be a corner or more generally contain texture

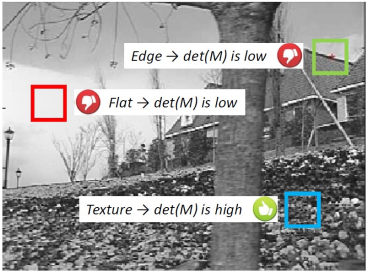
[?] 对于M来说，det(MM)应该是非零的，这意味着它的特征值应该很大（即不是一个平坦的区域，不是一个边缘）
[?] 在实践中，它应该是一个角落，或者更广泛地包含纹理

Eigenvalues

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

Eigenvectors

I_x and I_y: Directional derivatives



KLT Tracker for Small Motion

➢ Simplified Case: Pure Translation

✓ Answer to our two main tasks

- How should we select features?

Patch whose associated M matrix has large eigen values.

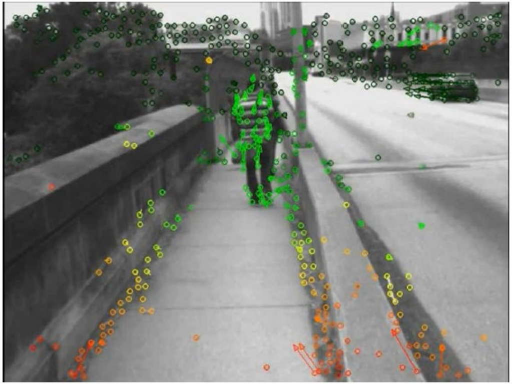
$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

- How should we track them from frame to frame?

(u, v) is the displacement vector.

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}^{-1} \begin{bmatrix} \sum I_x \Delta I \\ \sum I_y \Delta I \end{bmatrix}$$

Intensity difference at (x, y)



Color encodes motion direction

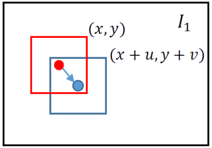
KLT Tracker for Small Motion

➢ General Case

✓ Relationship between pure translation and general motion

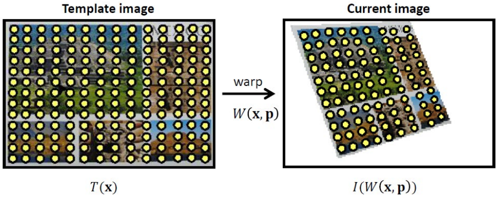
- Definition of cost function

$$SSD(u,v) = \sum_{x,y\in\Omega} (I_0(x,y) - \underbrace{I_1(x+u, y+v)}_{\text{Pure translation}})^2$$



$$SSD = \sum_{\mathbf{x}\in\mathbf{T}} [I(W(\mathbf{x},\mathbf{p})) - T(\mathbf{x})]^2$$

General transformation (warping)
a vector-valued function
p represents the warping parameters



KLT Tracker for Small Motion

➢ General Case

✓ Relationship between pure translation and general motion

- Minimization of cost function

$$\underbrace{I_1(x+u, y+v))^2}$$

$$I_1(x+u, y+v) \cong I_1(x, y) + I_x u + I_y v$$

Recap on first-order approximation in pure translation case

Similarity:

In both case, we apply a first order approximation of the warping.

$$\frac{\partial SSD}{\partial u} = 0\ ,\ \frac{\partial SSD}{\partial v} = 0$$

Recap on zero-valued partial derivatives in pure translation case

Difference:

In pure translation case, we equate partial derivatives to zero and directly obtain solutions (u, v).

In the general case, we leverage Gauss-Newton method to minimize the SSD iteratively.

在纯平移的情况下，我们将偏导数等效为零，直接得到解（u, v）。

在一般情况下，我们利用高斯-牛顿方法来迭代最小化SSD

KLT Tracker for Small Motion

$$SSD = \sum_{\mathbf{x} \in \mathrm{T}} [I(W(\mathbf{x}, \mathbf{p})) - T(\mathbf{x})]^2$$

- General Case

✓ Overview

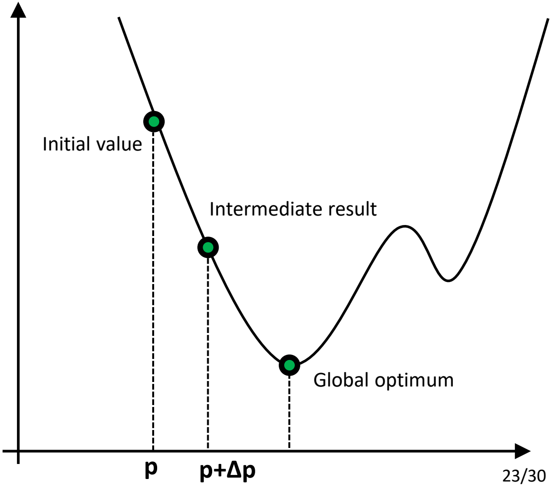
We incrementally update the warping parameters p to continuously reduce the value of cost function.

✓ One iteration

Assume that an initial estimate of **p** is known. Then, we want to find the increment $\Delta\mathbf{p}$ that minimizes

unknown

$$SSD = \sum_{\mathbf{x} \in \mathrm{T}} [I(W(\mathbf{x}, \mathbf{p} + \boxed{\Delta\mathbf{p}})) - T(\mathbf{x})]^2$$



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$$SSD = \sum_{\mathbf{x}\in\mathbf{T}} \left[\boxed{I(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p}))} - T(\mathbf{x}) \right]^2$$

➢ General Case

✓ One iteration

First-order Taylor approximation of $I(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p}))$ yields to:

$$I(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})) \cong I(W(\mathbf{x}, \mathbf{p})) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta\mathbf{p}$$

$\nabla I = [I_x, I_y]$ = Image gradient evaluated at $W(\mathbf{x}, \mathbf{p})$

Jacobian of the warp $W(\mathbf{x}, \mathbf{p})$

$\nabla f(x_1, x_2, \ldots, x_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$ is the image gradient

$F(x_1, x_2, \ldots, x_n) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_m(x_1, x_2, \ldots, x_n) \end{bmatrix}$ is a vector-valued function

Jacobian $\frac{\partial F}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}, \ldots, \frac{\partial f_1}{\partial x_n} \\ \vdots \\ \frac{\partial f_m}{\partial x_1}, \ldots, \frac{\partial f_m}{\partial x_n} \end{bmatrix}$

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$$SSD = \sum_{\mathbf{x}\in\mathbf{T}} \left[\boxed{I\left(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})\right)} - T(\mathbf{x}) \right]^2$$

➢ General Case

✓ One iteration

Substitute Taylor approximation of $I\left(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})\right)$ into SSD cost, we have new cost function:

$$SSD = \sum_{\mathbf{x}\in\mathbf{T}} \left[I\left(W(\mathbf{x}, \mathbf{p})\right) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right]^2$$

✓ How do we minimize new cost?

Briefly, we differentiate SSD with respect to $\Delta\mathbf{p}$ and we equate it to zero, i.e., $\frac{\partial SSD}{\partial \Delta\mathbf{p}} = 0$

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➢ General Case

$$SSD = \sum_{\mathbf{x}\in\mathbf{T}}\left[I(W(\mathbf{x},\mathbf{p}))+\nabla I\frac{\partial W}{\partial \mathbf{p}}\Delta\mathbf{p} - T(\mathbf{x})\right]^2$$

✓ How do we minimize new cost?

By differentiating SSD with respect to $\Delta\mathbf{p}$ and setting the result as 0, we have

Derivative:

distributive laws

$$2\sum_{\mathbf{x}\in\mathbf{T}}\left[\nabla I\frac{\partial W}{\partial \mathbf{p}}\right]^{\mathrm{T}}\left[\underline{I(W(\mathbf{x},\mathbf{p}))}+\nabla I\frac{\partial W}{\partial \mathbf{p}}\Delta\mathbf{p}\ \underline{- T(\mathbf{x})}\right] = 0 \Rightarrow \Delta\mathbf{p} = H^{-1}\sum_{\mathbf{x}\in\mathbf{T}}\underline{\left[\nabla I\frac{\partial W}{\partial \mathbf{p}}\right]^{\mathrm{T}}\left[T(\mathbf{x}) - I(W(\mathbf{x},\mathbf{p}))\right]} =$$

$$H = \sum_{\mathbf{x}\in\mathbf{T}}\underline{\left[\nabla I\frac{\partial W}{\partial \mathbf{p}}\right]^{\mathrm{T}}\left[\nabla I\frac{\partial W}{\partial \mathbf{p}}\right]}$$

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➢ Another Derivation in General Case (for Exercise Session)

✓ Brightness Constancy

- Video (sequential images) is w.r.t. the time t and a tracked point's position is also w.r.t. the time t.

$$I(x(t), t) = \text{const.} \quad \forall t,$$

- Based on the brininess consistency assumption, we set derivative as 0.

$$\frac{d}{dt} I(x(t), t) = \nabla I^\top \left(\frac{dx}{dt} \right) + \frac{\partial I}{\partial t} = 0.$$ Called "optical flow constraint"

$$V = \frac{dx}{dt}$$ Velocity

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➢ Another Derivation in General Case (for Exercise Session)

✓ Constant motion in a neighborhood:

- We assume that assumes that the velocity v is constant over a neighborhood W(x) of the point x

pixels in same patch
have same transformers model

$$\nabla I^\top \left(\frac{dx}{dt}\right) + \frac{\partial I}{\partial t} = 0.$$

$$\Downarrow$$

$$\nabla I(x', t)^\top v + \frac{\partial I}{\partial t}(x', t) = 0 \quad \forall x' \in W(x).$$

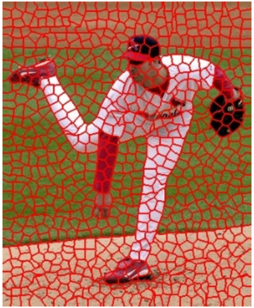

Image patches

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- Another Derivation in General Case (for Exercise Session)

✓ Compute the best velocity vector v for the point x by minimizing the least squares error

$$\nabla I(x',t)^\top v + \frac{\partial I}{\partial t}(x',t) = 0 \quad \forall x' \in W(x). \quad \Rightarrow \quad E(v) = \int_{W(x)} \left|\nabla I(x',t)^\top v + I_t(x',t)\right|^2 dx'.$$

This will be used in exercise session.

✓ Expanding the terms and setting the derivative to zero:

$$\frac{dE}{dv} = 2\underline{M}v + 2\underline{q} = 0 \quad \text{where} \quad \underline{M} = \int_{W(x)} \nabla I \, \nabla I^\top dx', \quad \text{and} \quad \underline{q} = \int_{W(x)} I_t \, \nabla I \, dx'.$$

$$\Rightarrow \quad v = -M^{-1} q.$$

Summary

- ➢ Overview of Tracking Problem
- ➢ KLT Tracker for Small Motion
- • Simplified case: pure translation
- • General case

Thank you for your listening!
If you have any questions, please come to me :-)