# Multiple View Geometry: Exercise 3

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**Image Formation**

We are looking at the formation of an image in camera coordinates $\mathbf{X}_C = (X\ Y\ Z\ 1)^\top$. The following relation of homogeneous pixel coordinates $\mathbf{x}'$ and $\mathbf{X}$ holds:

$$\lambda \mathbf{x}' = K \Pi_0 \mathbf{X} \tag{1}$$

with the intrinsic camera matrix $K$.

*Extra Infos on intrinsic camera matrix:*

If the camera is not centered at the optical center, we have an additional translation $o_x$ , $o_y$ and if pixel coordinates do not have unit scale, we need to introduce an additional scaling in x- and y -direction by $s_x$ and $s_y$ . If the pixels are not rectangular, we have a skew factor $s_\theta$ . You can assume that focal lengths along the u and v axes are identical. Accordingly, they are both denoted by $f$. To clearly differentiate between camera coordinates and pixel coordinates, call the pixel coordinates $u$ and $v$: $\mathbf{x}' = (u\ v\ 1)^\top$. The pixel coordinates $(u, v, 1)$ as a function of homogeneous camera coordinates $\mathbf{X}$ are then given by

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} s_x & s_\theta & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_s} \underbrace{\begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_f} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\equiv \Pi_0} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \tag{2}$$

After the perspective projection $\Pi_0$ (with focal length 1), we have an additional transformation which depends on the (intrinsic) camera parameters. This can be expressed by the intrinsic parameter matrix $K = K_s K_f$.

Furthermore, let the non-homogeneous camera coordinates be $\tilde{\mathbf{X}} := \Pi_0 \mathbf{X} = (X\ Y\ Z)^\top$. (1) is then equivalent to

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \tilde{\mathbf{X}} \, . \tag{3}$$

Let $s_x = s_y = 1$ and $s_\theta = 0$ in the intrinsic camera matrix.

1. Compute $\lambda$ and show that (3) is equivalent to

$$u = \frac{fX}{Z} + o_x \, , \quad v = \frac{fY}{Z} + o_y \, . \tag{4}$$

2. A classic ambiguity of the perspective projection is that one cannot tell an object from another object that is exactly *twice as big but twice as far*. Explain why this is true.

3. For a camera with $f = 540$, $o_x = 320$ and $o_y = 240$, compute the pixel coordinates $u$ and $v$ of a point $\tilde{\mathbf{X}} = (60\ 100\ 180)^\top$. Explain with the help of (b) why the units of $\tilde{\mathbf{X}}$ are not needed for this task. Will the projected point be in the image if it has dimensions $640 \times 480$?

We define the generic projection $\pi$ of $\tilde{\mathbf{X}}$ to 2D coordinates as follows:

$$\pi(\tilde{\mathbf{X}}) := \begin{pmatrix} X/Z \\ Y/Z \end{pmatrix} \tag{5}$$

4. Using the generic projection $\pi$, show that (4) — and therefore also (1) and (3) — is equivalent to

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} . \tag{6}$$

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} s_x & s_\theta & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_s} \underbrace{\begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_f} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\equiv \Pi_0} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & o_x \\ 0 & 1 & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix} (I \mid 0) \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} fX + o_x Z \\ fY + o_y Z \\ Z \end{pmatrix} = \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

$$u = \frac{fX}{Z} + o_x \qquad v = \frac{fY}{Z} + o_y$$

b) $X \to 2X$, $Y \to 2Y$, $Z \to 2Z$

$u = \frac{fX}{Z} + o_x$, $v' = v$ 不变

c) 3. For a camera with $f = 540$, $o_x = 320$ and $o_y = 240$, compute the pixel coordinates $u$ and $v$ of a point $\tilde{\mathbf{X}} = (60\ 100\ 180)^\top$. Explain with the help of (b) why the units of $\tilde{\mathbf{X}}$ are not needed for this task. Will the projected point be in the image if it has dimensions $640 \times 480$?

We define the generic projection $\pi$ of $\tilde{\mathbf{X}}$ to 2D coordinates as follows:

$$\pi(\tilde{\mathbf{X}}) := \begin{pmatrix} X/Z \\ Y/Z \end{pmatrix} \tag{5}$$

$$u = \frac{540 \cdot 60}{180} + 320 = 500$$

Scaling isnt change

$$v = \frac{540 \cdot 100}{180} + 240 = 540$$

4. Using the generic projection $\pi$, show that (4) — and therefore also (1) and (3) — is equivalent to

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} . \tag{6}$$

$$\begin{pmatrix} X/Z \\ Y/Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & O_x \\ 0 & 1 & O_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix} \begin{pmatrix} \pi(\tilde{x}) \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{fx}{z} + 0x \\ \frac{fy}{z} + 0y \\ 1 \end{pmatrix} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$