Fundamentals of Artificial Intelligence
Exercise 10: Making Simple Decisions

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Learning Outcomes of this Exercise

- You are able to draw the decision network of a decision problem.
- You are able to compute expected utilities (EUs).
- You understand rational decision making based on EUs.
- You are able to draw the decision tree of a decision problem.
- You are able to compute the value of information (VOI).
- You understand how the VOI influences the agent's decisions.

Task 1

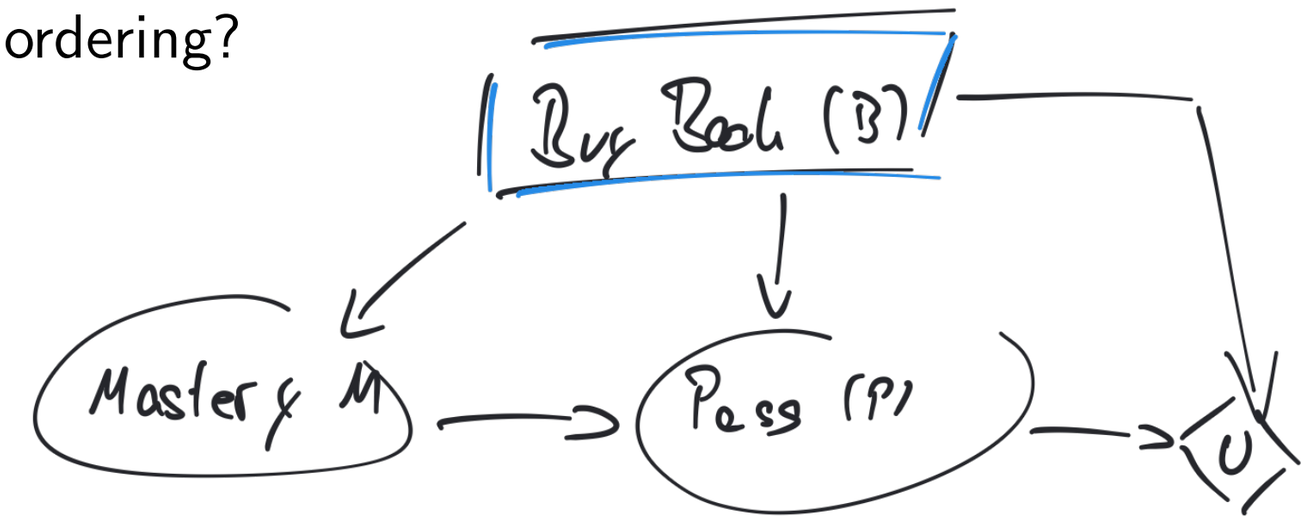
Sam has the choice to buy or not buy a textbook for his course.

Problem 10.1.1: Decision Network

$B < \{M, P\}$

Draw the decision network.

What is the partial ordering?



1. Identify 1st decision
2. " next decision + all random variables that are revealed
3. Add all random variables that are not yet included

Problem 10.1.2: Expected Utility

$$EU(a) = \sum_{s'} P(\texttt{Result}(a) = s'|a)U(s').$$

Approach: Condition on decision variable, marginalize out random variables later in the partial ordering.

$$U(b) = -100€, \quad U(\neg b) = 0€ \quad U(p) = 2000€, \quad U(\neg p) = 0€$$

$$U(b,p) = ?, \quad U(\neg b,p) = ? \quad U(b,\neg p) = ?, \quad U(\neg b,\neg p) = ?$$

1900 € 2000 € -100 € 0 €

Problem 10.1.2: Expected Utility $EU(a|e) = \sum_{s'} P(s'|a,e)\,U(s')$

1) Expected utility of buying the book:

$$P(p|b,m) = 0.9$$
$$P(p|b,\neg m) = 0.5$$
$$P(p|\neg b,m) = 0.8$$
$$P(p|\neg b,\neg m) = 0.3$$

$$P(m|b) = 0.9$$
$$P(m|\neg b) = 0.7$$

$$P(p|b) = \sum_{M \in \{m,\neg m\}} P(p|b,M)\,P(M|b)$$

$$= P(p|b,m)\,P(m|b) + P(p|b,\neg m)\,P(\neg m|b)$$

$$= 0.9 \cdot 0.9 + 0.5 \cdot 0.1$$

$$= 0.86$$

$$EU(b) = \sum_{P \in \{p,\neg p\}} P(P|b)\,U(P,b) = P(p|b)\,U(p,b) + P(\neg p|b)\,U(\neg p,b)$$

$$= 0.86 \cdot (2000 - 100) + 0.14 \cdot (-100)$$

$$= 1620$$

Problem 10.1.2: Expected Utility

2) Expected utility of not buying the book:

$$P(p|b,m) = 0.9$$
$$P(p|b,\neg m) = 0.5$$
$$P(p|\neg b,m) = 0.8$$
$$P(p|\neg b,\neg m) = 0.3$$
$$P(m|b) = 0.9$$
$$P(m|\neg b) = 0.7$$

$$EU(\neg b) = \sum_p P(p|\neg b)\, U(p,\neg b)$$

$$P(p|\neg b) = \sum_M P(p|\neg b, M)\, P(M|\neg b)$$

$$= P(p|\neg b, m)\, P(m|\neg b) + P(p|\neg b, \neg m)\, P(\neg m|\neg b)$$

$$= 0.65$$

$$EU(\neg b) = P(p|\neg b)\, U(p,\neg b) + P(\neg p|\neg b)\, U(\neg p,\neg b)$$

$$= 0.65 \cdot 2000 + 0.35 \cdot 0$$

$$= 1200$$

Problem 10.1.3: Making Simple Decision

What should Sam do?

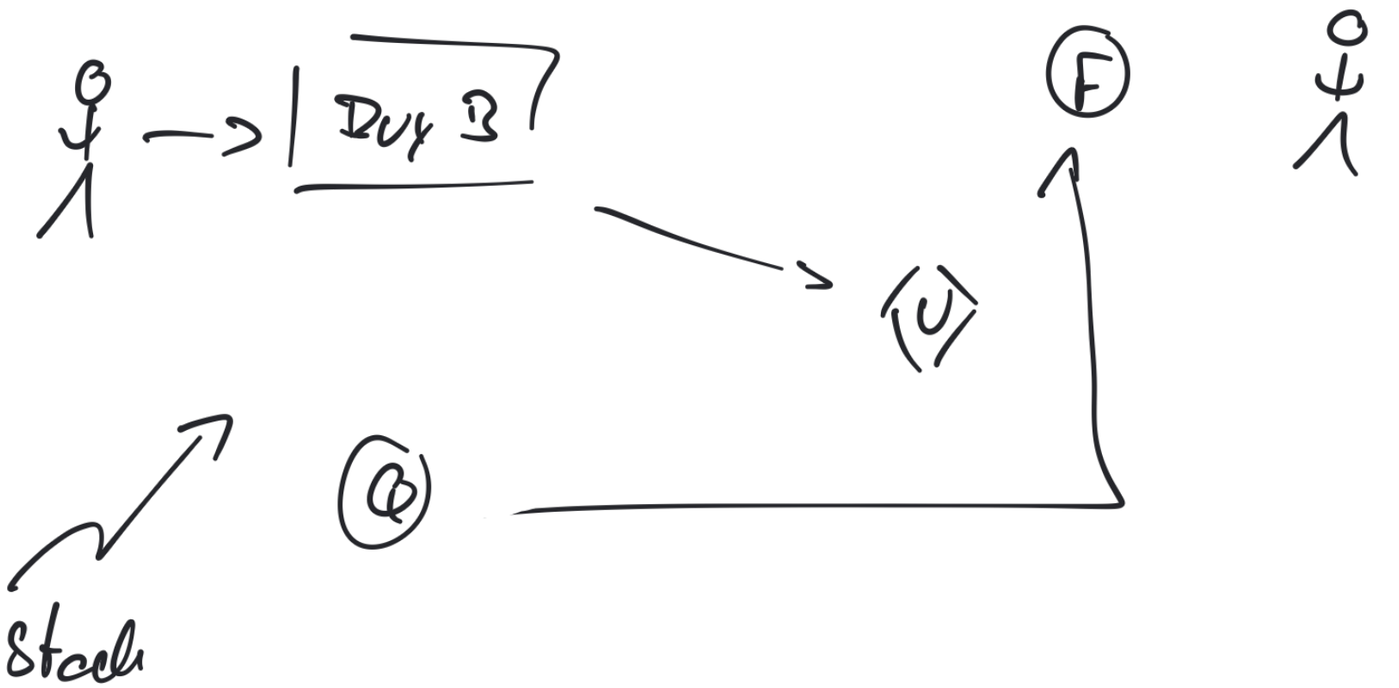
$EU(b) = 1620€ > 1200€ = EU(\neg b)$

$\rightsquigarrow$ Rational Decision: Buy the book

$\pi^*(B) = b$

Task

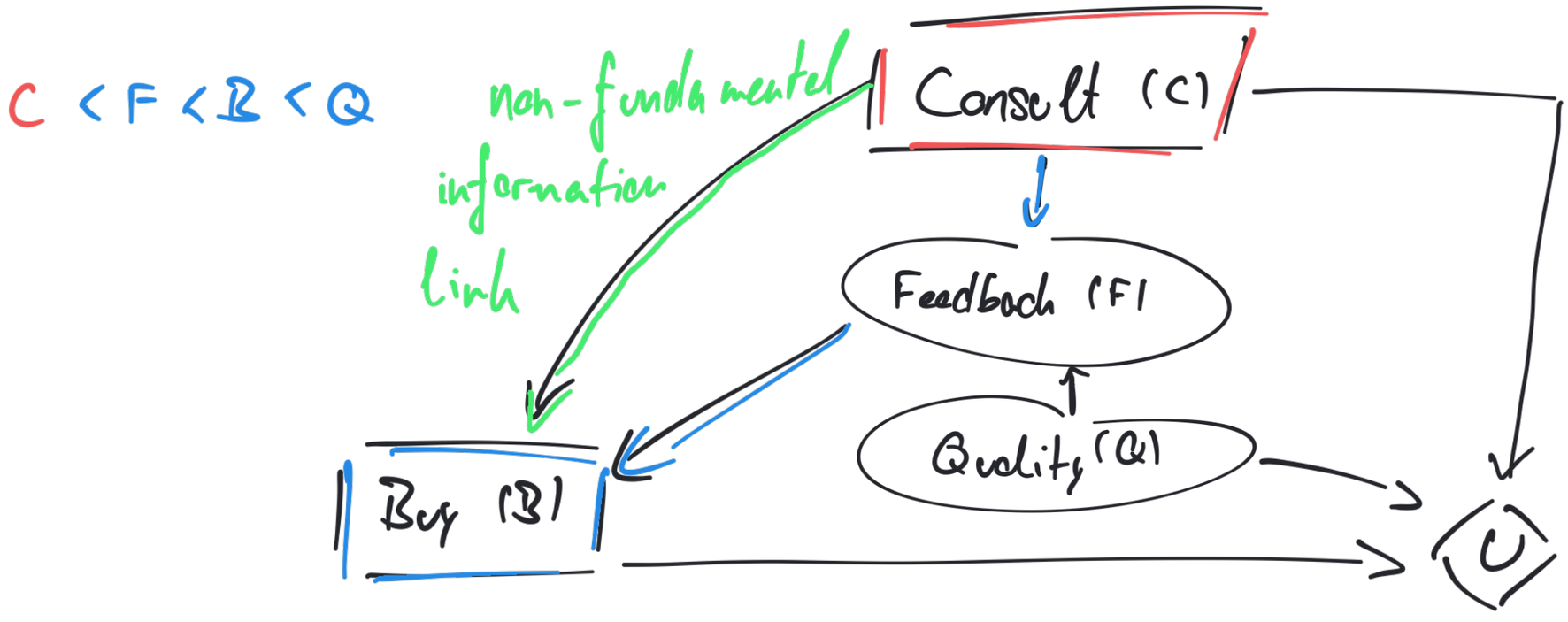
An investor has to decide whether he should buy a stock and whether he should ask a consultant for investment advice before deciding to buy.



Problem 10.2.1: Decision Network

Draw the decision network:

Buy the stocks $B \in \{b, \neg b\}$, ask a consultant $C \in \{c, \neg c\}$, feedback from the consultant $F \in \{f, \neg f\}$, quality of the stock $Q \in \{q, \neg q\}$



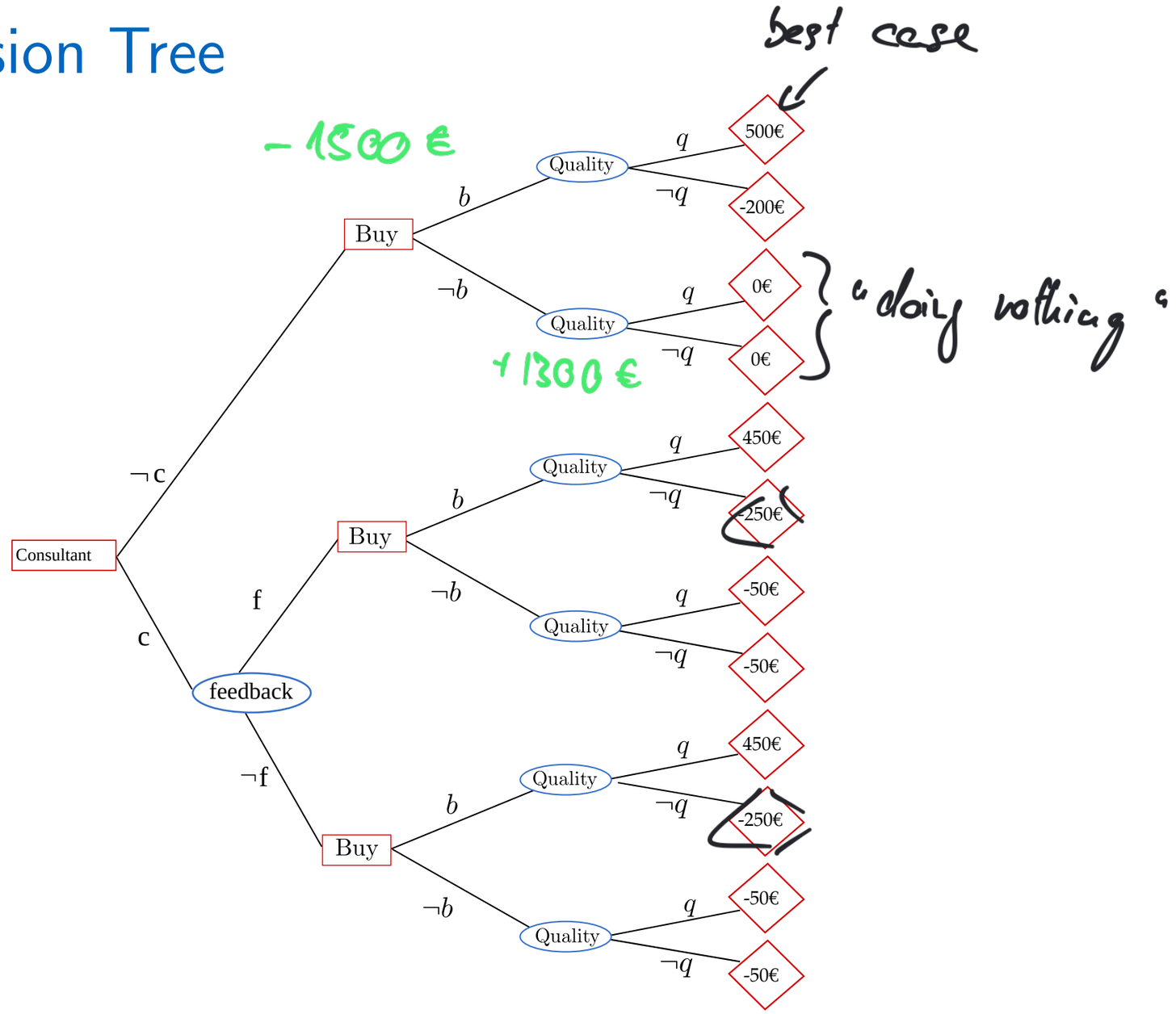
Problem 10.2.2: Decision Tree

Draw the decision tree:

Utilities:

$$U(b) = -1500€, \quad U(\neg b) = 0€, \quad U(q) = 2000€ \quad U(\neg q) = 1300€$$
$$U(c) = -50€, \quad U(\neg c) = 0€$$

Problem 10.2.2: Decision Tree



Problem 10.2.3: Expected Utility

Calculate the expected utility of buying the stock, given no consultation:

$$P(q) = 0.7$$
$$P(f|q) = 0.85$$
$$P(f|\neg q) = 0.05$$
$$P(\neg q) = 0.3$$
$$P(\neg f|q) = 0.15$$
$$P(\neg f|\neg q) = 0.95$$

$$EU(b|\neg c) = \sum_Q P(Q|b,\neg c)\,U(Q,b,\neg c)$$
$$= \sum_Q P(Q)\,U(Q,b,\neg c)$$
$$= 0.7 \cdot 500€ + 0.3 \cdot (-200€)$$
$$= 290€$$

Problem 10.2.4: Optimal Plan

τ^*

Derive an optimal conditional plan for the investor.

Start with determining the optimal decisions whether to buy the stock given no consultation, a positive feedback or a negative feedback. $\tau^*(B|\neg c)$; $\tau^*(B|c,f)$; $\tau^*(B|c,\neg f)$

Calculate the **value of information** of the consultation.

$$VOI_e(E_j) = \left(\sum_k P(E_j = e_{jk} \mid e)\, MEU(a_{e_{jk}} \mid e, E_j = e_{jk}) \right) - MEU(a \mid e)$$

E_j: new evidence (information)

e: prior evidence

e_{jk}: new evidence has value e_{jk}

$MEU(a_{e_{jk}} \mid e, E_j = e_{jk})$: new EU

$MEU(a \mid e)$: current EU

Problem 10.2.4: Optimal Plan

Find a decision maximizing the expected utility for each decision variable:

$$MEU(d_{1:n}) = \max_{d_1} \sum_{x_1} \dots \max_{d_n} \sum_{x_n} \prod_{i=1}^{n} P(x_i | x_{1:i-1}, d_{1:i}) U(x_{1:n}, d_{1:n})$$

What is the partial ordering?

$$C \leq F \leq B \leq Q$$

Problem 10.2.4: Optimal Plan

The **last decision** is: should the investor **buy the stock?**

There are three cases:

- Case 1: Not asking the consultant.

$$\pi^*(B|\neg c) = \underset{B}{\text{argmax}}\, EU(B|\neg c, none)$$

- Case 2: Ask the consultant, the feedback is positive.

$$\pi^*(B|c, f) = \underset{B}{\text{argmax}}\, EU(B|c, f)$$

- Case 3: Ask the consultant, the feedback is negative.

$$\pi^*(B|c, \neg f) = \underset{B}{\text{argmax}}\, EU(B|c, \neg f)$$

Problem 10.2.4: Optimal Plan

Case 1: Not asking the consultant. $(\neg c, \mathit{none})$

We already calculated $EU(b|\neg c) = 290€$.

What is $EU(\neg b|\neg c)$? 0 €

$$EU(a) = \sum_{s'} P(s'|a)\,U(s', e)$$

$EU(\neg b|\neg c) = 0€$

$EU(b|\neg c) = 290€ > EU(\neg b|\neg c)$

$\pi^*(B|\neg c) = b$

Problem 10.2.4: Optimal Plan

Case 2: Ask the consultant, the feedback is positive. (c, f).

$P(q) = 0.7$
$P(f|q) = 0.85$
$P(f|\neg q) = 0.05$
$P(\neg q) = 0.3$
$P(\neg f|q) = 0.15$
$P(\neg f|\neg q) = 0.95$

tedious, can be replace w. $\frac{1}{\alpha}$ (Bayes rule of norm.)

$$P(q|f) = \frac{P(f|q)}{P(f)} P(q) = \alpha P(f|q) \cdot P(q) = \alpha \cdot 0.595$$

$$P(\neg q|f) = \alpha \cdot P(f|\neg q) P(\neg q) = \alpha \cdot 0.015$$

We need to calculate $EU(b|c, f)$ and $EU(\neg b|c, f)$!

$P(q|f) + P(\neg q|f) = 1 = 0.61\alpha \rightarrow \alpha = \frac{1}{P(f)} \cong 1.639$

$= 0.595 + 0.015$

$P(\neg f) = 1 - \frac{1}{1.639}$

$P(q|f) = 0.975$

$P(\neg q|f) = 0.025$

$$\Rightarrow P(q|\neg f) = \frac{P(\neg f|q)}{P(\neg f)} P(q) = 0.269$$

Problem 10.2.4: Optimal Plan

consultation

$$EU(b \mid c, f) = P(q \mid c, f)\, U(q, b) + P(\neg q \mid c, f)\, U(\neg q, b) - 50\,€$$

$$= 0.975 \cdot 500\,€ + 0.025 \cdot (-200) - 50\,€$$

$$= 432.5\,€$$

$$EU(\neg b \mid c, f) = P(q \mid c, f)\, U(q, \neg b) + P(\neg q \mid c, f)\, U(\neg q, \neg b) - 50\,€$$

$$= -50\,€$$

$$\pi^*(B \mid c, f) = b$$

Problem 10.2.4: Optimal Plan

Case 3: Ask the consultant, the feedback is negative $(c, \neg f)$.

$$P(q) = 0.7$$
$$P(f|q) = 0.85$$
$$P(f|\neg q) = 0.05$$
$$P(\neg q) = 0.3$$
$$P(\neg f|q) = 0.15$$
$$P(\neg f|\neg q) = 0.95$$

We need to calculate $EU(b|c, \neg f)$ and $EU(\neg b|c, \neg f)$!

Problem 10.2.4: Optimal Plan

$$EU(b \mid c, \neg f) = P(q \mid c, \neg f)\, U(q, b) + P(\neg q \mid c, \neg f)\, U(\neg q, b) - 50\,€$$

$$= 0{,}269 \cdot 500\,€ + (1 - 0{,}269)\,(-200\,€) - 50\,€$$

$$= -61{,}7\,€$$

$$EU(\neg b \mid c, \neg f) = -50\,€$$

$$\pi^*(B \mid c, \neg f) = \neg b$$

Problem 10.2.4: Optimal Plan

Second last decision: Should the investor **ask the consultant?**

There are two cases:

- Case 1: Asking the consultant (c).
- Case 2: Not asking the consultant ($\neg c$).

1) How much is to be gained from asking? VOI
2) Is this more than the cost for the information?

Problem 10.2.4: Optimal Plan

Case 1: Asking the consultant.

$$EU(c) = \sum_F P(F|c) \max_B \sum_Q P(Q|F,c)\, U(Q,c,B)$$

$P(F|c)$: known

$\max_B$: $\pi(B|c,F)$

$P(Q|F,c)$, $U(Q,c,B)$: known

$$= 244.5\,€$$

$$EU(\neg c) = \sum_Q P(Q|\neg c)\, U(\pi^*(B|\neg c), Q)$$

$$= 290\,€$$

$$EU(\neg c) > EU(c)$$

$$\pi^*(B,c) = (b, \neg c)$$

Problem 10.2.4: Optimal Plan

Case 2 : Not asking the consultant.

Problem 10.2.4: Value of Information

Calculate the value of perfect information for the consultation.

$$VOI(F) = EU(c) - EU(\neg c) + 50€$$

$$= 244.5€ + 50€ - 290€$$

$$= 4.5€$$