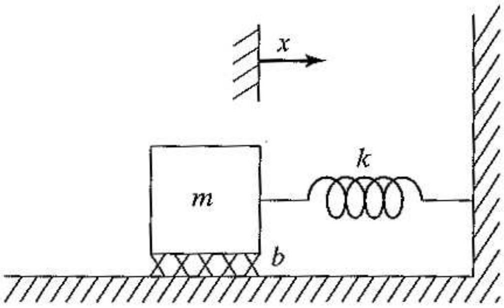

Figure 1: *Simple mass-spring-system.*

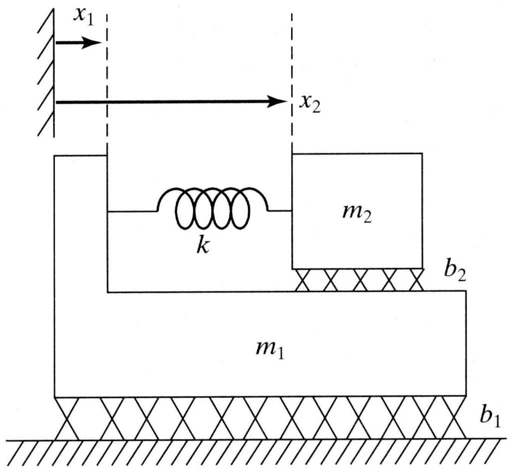

Figure 2: *Complex mass-spring-system (Problem 2)*

Problem 1

Consider a simple mass-spring system (Figure 1) with one object of mass $m = 1$, attached to a spring with stiffness $k = 5$ and affected by friction with a friction constant $b = 4$. The system has a resonant frequency of $\omega_{\text{res}} = 6.0$. Determine k_v and k_p such that the system is critically damped.

Problem 2

Derive a PD controlling scheme for the system shown in Figure 2 that allows following of trajectories for both objects and critically damps the error. The steps you should perform are the following:

- Determine forces that apply to both objects, derive equations of motion.
- Apply the control law partitioning principle. Explicitely show model-based portion and servo portion of the control law.
- Formulate the error equation.

①
$$f_2 = m_2 \ddot{x}_2 + b_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)$$

$$f_1 = m_1 \ddot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) + b_1 \dot{x}_1 - k_2(x_2 - x_1)$$

②
$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} b_1 + b_2 & -b_2 \\ -b_2 & b_2 \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

③
$$f' = \ddot{x}_d + k_v \dot{e} + k_p e = \ddot{x}$$

$$\ddot{e} + k_v \dot{e} + k_p e = 0$$

1 $m\ddot{x} + b\dot{x} + kx = f = -k_p x - k_v \dot{x}$

$$m\ddot{x} + (b+k_v)\dot{x} + (k+k_p) = 0$$

$$\ddot{x} + \frac{b+k_v}{m}\dot{x} + \frac{k+k_p}{m} = 0$$

critically damped: $b' = 2\sqrt{mk'}$

$$(b+k_v)^2 = 4m(k+k_p)$$

$$b = -k_v \pm \sqrt{4m(k+k_p)}$$

$$k_v = -b + 2\sqrt{k+k_p}$$

$$k_v = -4 + 2\sqrt{5+k_p}$$

$$k_p = 4 \quad k_v = 2$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \sqrt{\frac{k+k_p}{m}} \le \frac{1}{2}\omega_{res} = 3$$

$$0 \le k+k_p \le 9$$

$$-5 \le k_p \le 4$$