Chair of Robotics, Artificial Intelligence and Real-time Systems
Department of Informatics
Technical University of Munich

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Robotik (Robotics)

Exam:	IN2067 / Endterm	**Date:**	Monday 17th February, 2020
Examiner:	Prof. Darius Burschka	**Time:**	17:00 – 18:30

Working instructions

- This exam consists of **12 pages** with a total of **3 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 111 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ______ to ______ / Early submission at ______

Problem 1 **Kinematics (34 credits)**

Shown below is a real-type robotic arm PUMA 560 with the base frame and a link lengths given (not to scale).

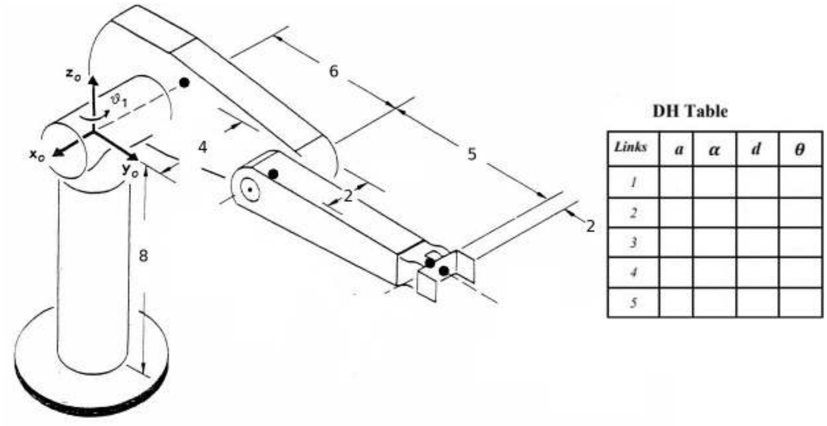
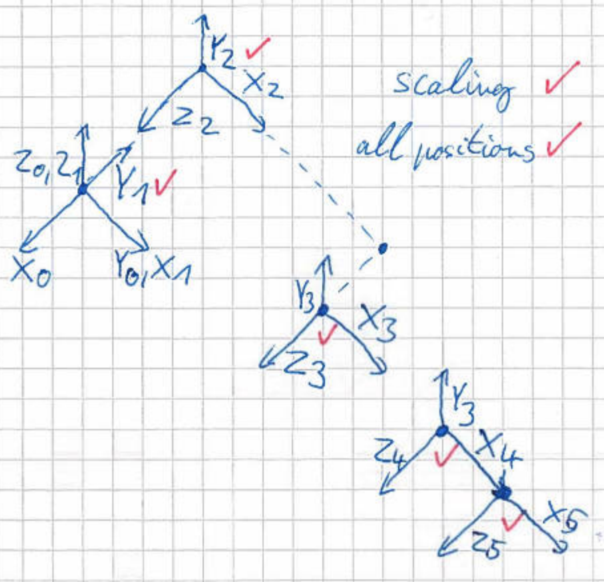


Figure 1.1: Joint representation of the PUMA arm. Axis centers highlighted. Empty DH-table

0 1 2 3 4 5 6 7 8 9 10 11 12

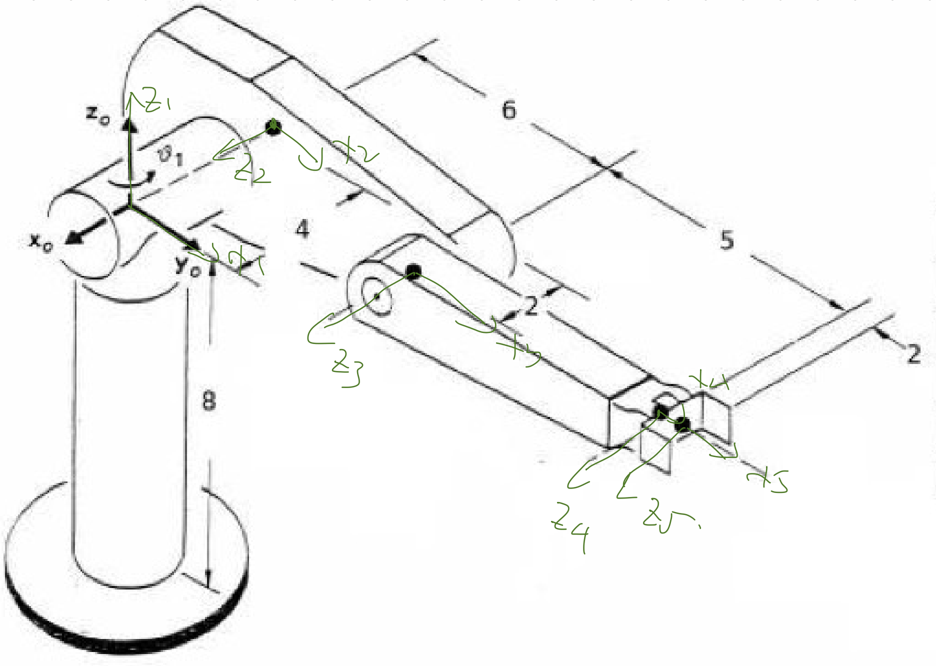
a)*
Draw the robot's coordinate frames in the current state. Use scaling 1 length = 2 squares (1 diagonal). Use lengths from the figure above and pick useful entries where appropriate. Complete the DH-table in above figure.



scaling ✓
all positions ✓

Joint	α_i	a_{i-1}	d_i	θ_i	
1	0°	0	0	90°	✓
2	90°	0	−4*	θ_2	✓
3	0°	6	2*	θ_3	✓
4	0°	5	0	θ_4	✓
5	0°	2	0	0	✓

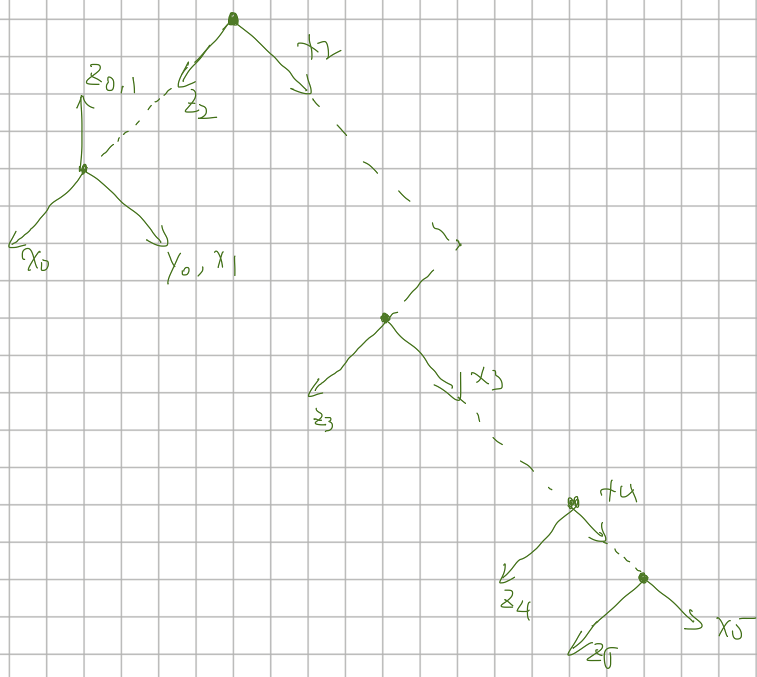
* will be set to zero in the next question



DH Table

Links	a	α	d	θ
1	0	0	0	θ_1
2	0	90°	-4	θ_2
3	6	0	2	θ_3
4	5	0	0	θ_4
5	2	0	0	0

90°



(b) $\theta_1 = 0$

① except z_1 (z_0), all z-axis are parallelle

② all x are ~~paralle~~ parallel

③ EE move in 2-D space

④ can chose origin to make $d=0$

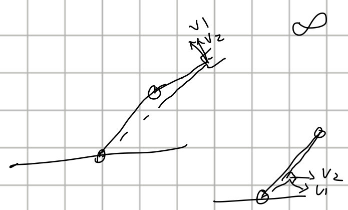
⑤ All rotation occurs on same axis

$$\theta_{EE} = \theta_2 + \theta_3 + \theta_4$$

$$^0P_{EE} = \begin{pmatrix} 6S_2 + 5S_{23} + 2S_{234} \\ \theta_2+\theta_3+\theta_4 \\ 6C_2 + 5C_{23} + 2C_{234} \end{pmatrix}$$

(c) singularity $\det(J(q)) = 0$.

∞ Velocity



① When singularity occurs, means robot has lose at least one dimension on EE

②. If row 1 in J(q) is, then lose neither [illegible] in x-dim
Same for row 2 — y dim
row 3 — z dim

③ If row 1 and row 2 is linear depend, then the v and w in EE are coupled
same for others

④ If colum 1 and colum 2 is lin dep, then Joint 1 and [illegible] can [illegible]

b) Given the robot above and setting $\Theta_1 = 0$. What is special about the other joints? How does that influence kinematics? Use geometric reasoning as in the tutorials to determine forward kinematics for the robot (Hint: you don't need DH-tables here).

0 1 2 3 4 5 6 7 8

All Z-Axes are parallel. ✓
The end effector moves in a 2D-plane. ✓ .
The Axis origin can thus be moved on the Z-Axis so that d_i is zero. ✓
All rotations occur around the same axis ✓ with the resulting angle $\Theta = \Theta_1 + \Theta_2 + \Theta_3$ ✓

$$ {}^{0}p(\Theta) = \begin{pmatrix} 2c_{123} + 5c_{12} + 6c_1 \checkmark \\ 2s_{123} + 5s_{12} + 6s_1 \checkmark \\ \Theta_1 + \Theta_2 + \Theta_3 \checkmark \end{pmatrix} $$

c) For a different, planar PRR robot, the kinematics (X,Y, Θ) are as follows:

$$ {}^{0}p(d_1, \Theta_2, \Theta_3) = \begin{pmatrix} d_1 + l_2 c_2 + l_3 c_{23} \\ l_2 s_2 + l_3 s_{23} \\ \Theta_2 + \Theta_3 \end{pmatrix} $$

Calculate the jacobian for the robot. How is a singular configuration determined? Does the robot have any? What kinds of freedoms could be lost in singularities in general and which here especially?

0 1 2 3 4 5 6 7 8 9 10

$$ {}^{0}J(\Theta) = \begin{pmatrix} 1 & -l_2 s_2 - l_3 s_{23} & -l_3 s_{23} \checkmark \\ 0 & l_2 c_2 + l_3 c_{23} & l_3 c_{23} \checkmark \\ 0 & 1 & 1 \end{pmatrix} $$

Set determinant of the Jacobian to zero and solve. ✓
$det(J(\Theta)) = l_2 c_2$ ✓
Yes, for $c_2 = 0$ OR $\Theta_2 = \{-90, 90\}$. ✓ One of the two answers is enough.
The robot is either at a workspace boundary ✓ where it can not move further in/out ✓
or at workspace interior singularity ✓ , where the angle of the endeffector cannot be chosen freely. ✓
Accept other definitions/types, too.
Here, joint movement is directly coupled (d_1 and Θ_1), losing a degree of freedom. ✓

0 1 2 3 4

d) Which additional constraint limits movement near a singularity? Add a figure to your explanation.

Joint speeds and accelerations are limited ✓ ,
thus limiting the speeds on the next link. ✓
Figure: show added velocity/acceleration of the next joint depending on joint speed (in the form of a paralellogram). ✓ ✓

c $J(\theta) = \begin{pmatrix} 1 & -L_2 S_2 - L_3 S_{23} & -L_3 S_{23} \\ 0 & L_2 C_2 + L_3 C_{23} & L_3 C_{23} \\ 0 & 1 & 1 \end{pmatrix}$

$\det(J(\theta)) = 0$

$\det(J(\theta)) = L_2 C_2 + L_3 \cancel{C_{23}} - L_3 \cancel{C_{23}} = L_2 C_2$

If $L_2 = 0$ or $\theta_2 = \pm 90^\circ$

robot has singularity

Joint 2 and join 3 are coupled.

Problem 2 **Lagrange (50 credits)**

A 2-DoF robot with one prismatic joint is shown below. Note that the prismatic joint is in the direction of angle $\phi = 45°$ relative to the center-line of the first link. The position of the center of masses C_1 and C_2 is given in the figure at half of the link length $\frac{l_i}{2}$. The first actuator fixed to the base produces a torque τ about the first joint, while the second actuator located at the tip of the first link generates linear force f acting on the second link. The world frame (its origin aligns with Joint 1) x-axis is horizontal pointing to the right and the y-axis is vertical pointing up. All inertia matrices are equal to: ${}^{C}I_i = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_1 & 0 \\ 0 & 0 & I_1 \end{pmatrix}$

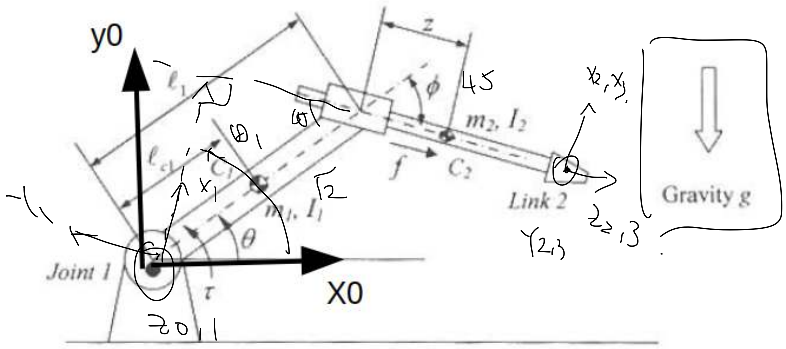
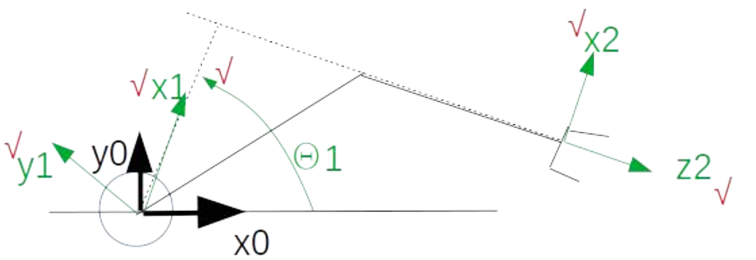


Figure 2.1: 2-DoF robot.

a)* Draw the coordinates frames (origins and directions) for the two links including the end-effector for the robot above using Denavit-Hartenberg convention as a line sketch into the box below. Define the angle Θ_1 that should point from the horizontal to the x_1 axis of your system.

0 1 2 3 4 5 6 7



credit for correct direction,extra two credits for putting the second coordinate frame at the endeffector. Alternatively, third coordinate frame with same directions as x2,z2 somewhere along the last link with no points ✓ ✓

b) Estimate the distance a_1 between the origin of the {0} frame and the link l_2 considering the value of $\phi = 45°$ and $l_1 = \sqrt{2}$. Estimate the distance x from the intersection point of your x_1 axis to the point where the link l_1 joins the link 2. Draw the DH table for the robot from Fig. 2.1.

0 1 2 3 4 5 6 7

$a_1 = 1$,✓ x=1✓ isosceles triangle

i	α_{i-1}	a_i	d_i	Θ_i
1	0	0	0	Θ_1 ✓
2	90° ✓	1 ✓	1 ✓ +z ✓ $+l_2/2$	0°

there can be alternative different 2 and additional 3 of the type

2	90° ✓	1 ✓	1 +z ✓	0°
3	0° ✓	0	$l_2/2$	0°

P_2

$${}^1P_{c_1} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$$

$$2a_1^2 = L_1^2 = 2$$
$$a_1^2 = 1$$
$$a_1 = 1$$

$$a_1 + \frac{1}{2}L_2 + Z = 1 + Z + \frac{1}{2}L_2$$

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	1	$90°$	$1+Z+\frac{1}{2}L_2$	0

Lkhu

$${}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1-Z-\frac{1}{2}L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_1 $\quad Z$ d_2

$${}^0P_{c_1} = \begin{pmatrix} \frac{L_1}{2}\cdot\cos(\theta_1 - 45°) \\ \frac{L_1}{2}\cdot\sin(\theta_1 - 45°) \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt2}{2}\cdot c_1\cdot\frac{\sqrt2}{2} + \frac{\sqrt2}{2}\cdot s_1\cdot\frac{\sqrt2}{2} \\ \frac{\sqrt2}{2}\, s_1\cdot\frac{\sqrt2}{2} - \frac{\sqrt2}{2}\, c_1\cdot\frac{\sqrt2}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(c_1+s_1) \\ \frac{1}{2}(s_1-c_1) \\ 0 \end{pmatrix}$$

$${}^0V_{c_1} = \frac{d\,{}^0P_{c_1}}{dt} = \begin{pmatrix} -\frac{1}{2}s_1\cdot\dot\theta_1 + \frac{1}{2}c_1\cdot\dot\theta_1 \\ \frac{1}{2}c_1\cdot\dot\theta_1 + \frac{1}{2}s_1\dot\theta_1 \\ 0 \end{pmatrix}$$

$${}^0_2T = \begin{pmatrix} -c_1 & 0 & s_1 & c_1 + (1+Z+\frac{1}{2}L_2)s_1 \\ s_1 & 0 & -c_1 & s_1 - (1+Z+\frac{1}{2}L_2)c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0P_{c_2} = {}^0_2T\cdot{}^2P_{c_2} = \begin{pmatrix} -c_1 & 0 & s_1 & c_1 + (1+Z+\frac{1}{2}L_2)s_1 \\ s_1 & 0 & -c_1 & s_1 - (1+Z+\frac{1}{2}L_2)c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}L_2 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 + (1+Z)s_1 \\ s_1 - (1+Z)c_1 \\ 0 \end{pmatrix}$$

$${}^0V_{c_2} = \frac{d\,{}^0P_{c_2}}{dt} = \begin{pmatrix} -s_1\cdot\dot\theta_1 + c_1\cdot\dot\theta_1 + \dot Z s_1 + Z c_1\dot\theta_1 \\ c_1\cdot\dot\theta_1 + s_1\cdot\dot\theta_1 - \dot Z c_1 + Z s_1\dot\theta_1 \\ 0 \end{pmatrix}$$

(d)

$$ {}^0V_{C1} = \begin{pmatrix} \frac{1}{2}(C_1+S_1) \\ \frac{1}{2}(S_1-C_1) \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{pmatrix} $$

$$ = \begin{pmatrix} \frac{1}{2}\dot{\theta}_1 (S_1-C_1) \\ -\frac{1}{2}\dot{\theta}_1 (C_1+S_1) \\ 0 \end{pmatrix} $$

$$ K_i = \frac{1}{2} m_i \cdot {}^0V_{C_i}^T \cdot {}^0V_{C_i} + \frac{1}{2}\, {}^i w_i^T \cdot {}^{C_i}I_1 \cdot {}^i w_i $$

$$ = \frac{1}{2} m_1 \cdot \left[\frac{1}{4}\dot{\theta}_1^2 \left(S_1^2 - 2\cancel{S_1C_1} + C_1^2 + C_1^2 + 2\cancel{C_1S_1} + S_1^2 \right) \right] + \frac{1}{2} \begin{pmatrix} 0 & 0 & \dot{\theta}_1 \end{pmatrix} \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_1 & 0 \\ 0 & 0 & I_1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{pmatrix} $$

$$ = \frac{1}{4} m_1 \cdot \dot{\theta}_1^2 + \frac{1}{2} L_1 \cdot \dot{\theta}_1^2 $$

$$ {}^0V_{C2} = \begin{pmatrix} C_1 + (1+Z)S_1 \\ S_1 - (1+Z)C_1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{pmatrix} $$

$$ = \begin{pmatrix} S_1\dot{\theta}_1 - (1+Z)\dot{\theta}_1 C_1 \\ -C_1\dot{\theta}_1 - (1+Z)\dot{\theta}_1 S_1 \\ 0 \end{pmatrix} $$

$$ K_2 = \frac{1}{2} m_2 \left[\dot{\theta}_1^2 \left(S_1^2 - 2\cancel{(1+Z)C_1S_1} + (1+Z)^2 C_1^2 + C_1^2 + 2\cancel{(1+Z)S_1C_1} + (1+Z)^2 S_1^2 \right) \right] + \frac{1}{2} L_1 \dot{\theta}_1^2 $$

$$ = \dots \left[\dot{\theta}_1^2 \left((2+Z)^2 S_1^2 + (2+Z)^2 C_1^2 \right) \right] \dots $$

$$ = \frac{1}{2} m_2 \dot{\theta}_1^2 (2+Z)^2 + \frac{1}{2} L_1 \dot{\theta}_1^2 $$

$$ K = \frac{1}{4} m_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 \dot{\theta}_1^2 (2+Z)^2 + L_1 \dot{\theta}_1^2 $$

0 1 2 3 4 5 6 7 8 9 10 11 12

c) Calculate the velocities (${}^{i}\omega_i, v_{Ci}$) of the centers of mass

$$L = 1 + z + l_2/2 \checkmark$$

$${}^{0}\omega_0 = 0, \quad {}^{1}\omega_1 = (0,\ 0,\ \dot{\Theta}_1)^T \checkmark, \quad {}^{2}\omega_2 = (0,\ \dot{\Theta}_1,\ 0)^T \checkmark$$

$${}^{0}P_{C1} = (l_1/2 \cdot c1,\ l_1/2 \cdot s1,\ 0)^T \checkmark, \quad v_{C1} = \frac{d}{dt}{}^{0}P_{C1} \checkmark = (-s1 \cdot \dot{\Theta}_1 \cdot l_1/2 \checkmark,\ c1 \cdot \dot{\Theta}_1 \cdot l_1/2 \checkmark,\ 0)^T$$

$${}^{0}P_{C2} = (1 \checkmark \cdot c1 + L \checkmark \cdot s1, 1 \cdot s1 - L \cdot c1, 0)^T, \quad v_{C2} = \frac{d}{dt}{}^{0}P_{C2} = \begin{pmatrix} -s1 \cdot \dot{\Theta}_1 + L \cdot \dot{\Theta}_1 \cdot c1 \checkmark + \dot{z} \checkmark \cdot s1 \\ c1 \cdot \dot{\Theta}_1 + L \cdot \dot{\Theta}_1 \cdot s1 - \dot{z} \cdot c1 \checkmark \\ 0 \end{pmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

d) Calculate torque τ in the joint for the case when the second joint is not moving with (z=0,$\dot{z} = 0$). Use here for velocity v_{ci} estimation the equation lever cross angular velocity ($l \times \dot{\Theta}$)

$$v_{c1} = l_1/2 \cdot \dot{\Theta}_1 \checkmark, \quad v_{c2} = \sqrt{1 + L^2} \cdot \Theta_1 \checkmark, \quad k = \sum_i k_i = \frac{1}{2} m_1 ||v_{c1}||^2 + \frac{1}{2} m_1 ||v_{c2}||^2 + \frac{1}{2} I_1 ||\omega_1||^2 + \frac{1}{2} I_1 ||\omega_2||^2 \checkmark$$

$$k = \frac{1}{2} m_1 \cdot (l_1/2 \cdot \dot{\Theta}_i)^2 \checkmark + \frac{1}{2} m_2 (\sqrt{1 + L^2} \cdot \dot{\Theta}_i)^2 \checkmark + I_1 \cdot \dot{\Theta}_1^2 \checkmark,$$

$$u = \sum_i u_i = -m_1 \cdot g^T \cdot {}^{0} P_{C1} - m_2 \cdot g^T \cdot {}^{0} P_{c2} \checkmark = m_1 \cdot g \cdot l_1/2 \cdot s1 \checkmark + m_2 \cdot g \cdot (1 \cdot s1 - L \cdot c1) \checkmark$$

$$\frac{d}{dt} \frac{\delta}{\delta \dot{\Theta}_1} k = m_1 \cdot (l_1/2)^2 \cdot \ddot{\Theta}_1 \checkmark + m_2 \cdot \sqrt{1 + L^2} \cdot \ddot{\Theta}_1 \checkmark + 2 \cdot I_1 \ddot{\Theta}_1 \checkmark$$

$$\frac{\delta}{\delta \Theta_1} u = m \cdot g \cdot c1 \checkmark + m2 \cdot g(c1 + L \cdot s1) \checkmark, \quad \tau = \frac{d}{dt} \frac{\delta}{\delta \dot{\Theta}_1} - \frac{\delta}{\delta \Theta_1} u \checkmark \checkmark \checkmark$$

e) Identify the M,V,G parameter in the equation for τ above:

$$M = m_1 \cdot l_1/2 + m_2 \cdot \sqrt{1+L^2} + 2 \cdot l_1 \checkmark, \quad V = 0 \checkmark, \quad G = m_1 \cdot g \cdot c1 + m_2 \cdot g(c1 + L \cdot s1) \checkmark$$

0 1 2 3

f) How can we calculate the inertia matrix I of an arbitrary structure? Give equations for the 9 elements of the matrix. How does it changes if we move the point of rotation by a vector $\vec{t}$?

0 1 2 3 4

$$I = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & -I_{zz} \end{pmatrix} \checkmark \; I_{xx} = \int (y^2 + z^2) dm, \quad I_{yy} = \int (x^2 + z^2) dm, \quad I_{zz} = \int (x^2 + y^2) dm \checkmark$$

$$I_{xy} = \int xydm, \quad I_{xz} = \int xzdm, \quad I_{yz} = \int yzdm \checkmark$$

$$I' = I + m \cdot \hat{t}^T \cdot \hat{t} \checkmark$$

Problem 3 **PID Control (27 credits)**

For the pole-cart system shown in Figure 3.1, f_1 is the external force on the cart, τ_2 is the torque to drive the pole, m_1 and m_2 are the masses of the cart and the pole, x and q are the displacements of the cart and the pole, and L is the length of the pole. Assume that the system is in the vertical plane and there is no friction and the rotational energy of the wheels can be neglected.

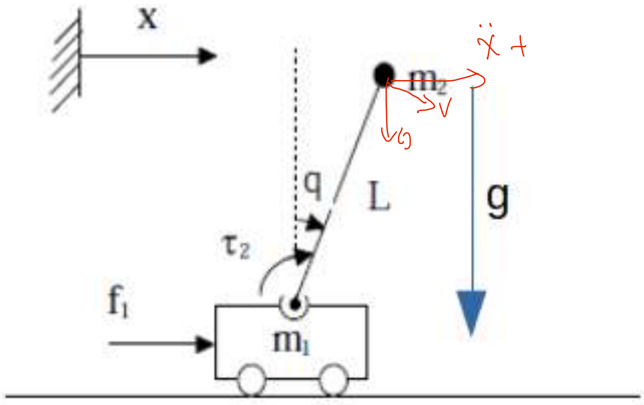


Figure 3.1: Cart balancing a pole.

The system is supposed to follow a trajectory while swinging the pole. The trajectories are specified independently for the cart as $x_d, \dot{x}_d, \ddot{x}_d$ and for the pole as $q_d, \dot{q}_d, \ddot{q}_d$.

0 1 2 3 4 5 6 7 8 9 10

a)*
Estimate the torque τ_2 and the force f_1 acting similar to a prismatic joint on the cart using Lagrangian method for manipulator analysis. The height of the cart can be idealized to 0.

velocity of m2 $v = \begin{pmatrix} L\cdot\dot{q}\cdot\cos q + \dot{x} \\ L\cdot\dot{q}\cdot\sin q \end{pmatrix}$

$$k = \sum_i k_i = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\cdot v^T v, \quad u = m_2\cdot g\cdot L\cdot\cos q$$

$$\frac{d}{dt}\frac{\delta}{\delta\dot{x}}k = m_1\cdot\ddot{x} + m_2\cdot(L\ddot{q}\cos q - L\dot{q}^2\sin q + \ddot{x}),$$

$$\frac{d}{dt}\frac{\delta k}{\delta\dot{q}}k = m_2\cdot(L^2\ddot{q} + L\dot{x}\cos q - L\dot{[}q]\sin q),$$

$$\frac{\delta}{\delta}u = 0, \quad \frac{\delta}{\delta q}u = -m_2\cdot g\cdot L\cdot\sin q$$

$$f_1 = \frac{d}{dt}\frac{\delta}{\delta\dot{x}}k.$$

$$\tau_2 = \frac{\delta}{\delta\dot{q}}k - \frac{\delta}{\delta q}u$$

$$V = L \cdot \dot{q}$$

$$V_x = L \cdot \cos(q) \cdot \dot{q}$$

$$V_y = L \cdot \sin(q) \cdot \dot{q}$$

$$V_2 = \begin{pmatrix} L \cdot \cos(q) \cdot \dot{q} + \dot{x} \\ L \cdot \sin(q) \cdot \dot{q} \end{pmatrix}$$

$$K = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left[L^2 \cos^2(q) \cdot \dot{q}^2 + 2L\cos(q) \cdot \dot{q} \cdot \dot{x} + \dot{x}^2 + L^2 \sin^2(q) \cdot \dot{q}^2 \right]$$

$$K = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left[L^2 \dot{q}^2 + 2L\cos(q) \cdot \dot{q}\,\dot{x} + \dot{x}^2 \right]$$

$$U = m_2 g L \cos(q)$$

$$\frac{\partial K}{\partial \dot{x}} = m_1 \dot{x} + m_2 \cdot L \cdot \cos(q) \cdot \dot{q} + m_2 \cdot \dot{x}$$

$$\frac{\partial K}{\partial \dot{q}} = m_2 L^2 \cdot \dot{q} + m_2 L \cos(q) \cdot \dot{x}$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{x}} = m_1 \ddot{x} + \left[-m_2 L \cdot \sin(q) \cdot \dot{q}^2 + m_2 L \cdot \cos(q) \cdot \ddot{q} \right] + m_2 \ddot{x}$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{q}} = m_2 L^2 \ddot{q} + \left[-m_2 L \cdot \sin(q) \cdot \dot{q} \cdot \dot{x} + m_2 L \cdot \cos(q) \cdot \ddot{x} \right]$$

$$\frac{\partial K}{\partial x} = 0 \qquad \tau_2 = m_2 L^2 \ddot{q} + m_2 L \cos(q) \ddot{x} - m_2 g L \sin(q)$$

$$\frac{\partial K}{\partial q} = -m_2 L \sin(q) \cdot \dot{q} \cdot \dot{x} \qquad f_1 = \tau_1 = \ldots$$

$$\frac{\partial U}{\partial x} = 0$$

$$\frac{\partial U}{\partial q} = -m_2 g L \sin(q)$$

$$m_2 L^2 \ddot{q} + m_2 L \cos(q) \ddot{x} - m_2 g L \sin(q) = \tau = \alpha \tau' + \beta$$

$$\tau' = \ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e$$

$$f' = \ddot{q} = \ddot{q}_d + k_v \dot{e}' + k_p e'$$

$$e = x_d - x$$

$$e' = q_d - q$$

$$\ddot{e} + k_v \dot{e} + k_p e = 0$$

$$s^2 + 2\zeta \omega_n s + \omega_n^2 = 0$$

$\omega_n = 7\ \text{rad/s}$ [illegible] $k_v = 2\sqrt{k_p}$ $\omega_n = \sqrt{k_p}$

$$\sqrt{k_p} = \omega_n.$$

b)*
Explain the principle of control law partitioning at the above example. What are the control laws for the two driving parameters f_1 and τ_2 to allow the above system to follow a specified trajectory?

0 1 2 3 4 5 6 7

Control law partitioning seperates model dependent parameters, like mass, friction, gravitational influence from ideal unit inertia system τ'.

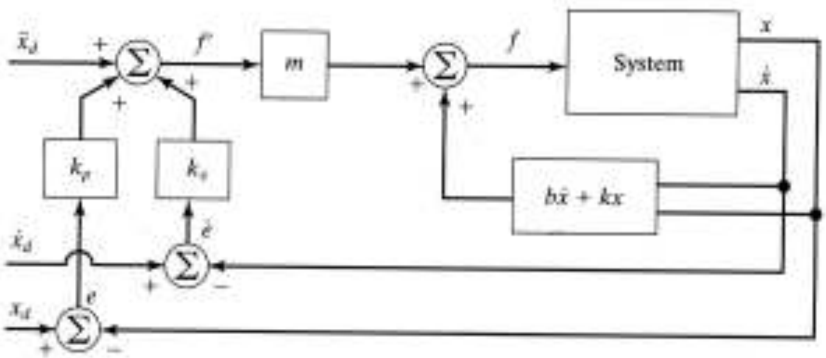
$$\tau = \alpha \cdot \tau' + \beta$$

Control law

$$\tau_2' = \ddot{q}_d + k_{v2} \cdot \dot{e} + k_{p2} \cdot e \quad , \quad f_1' = \ddot{x}_d + k_{v1} \cdot \dot{e} + k_{p1} \cdot e$$

c)* Draw the block diagram of the control scheme.

0 1 2 3



d) Choose the parameters for your cart controller such that the resulting closed-loop system is decoupled, critically damped, and natural frequency $\omega = 5 rad/s$.

0 1 2 3

$$\omega_n = \sqrt{k_p} = 5 \quad , \quad k_p = 25 \quad , \quad k_v = 2\sqrt{k_p} = 10$$

0
1
2
3
4

e)*

Explain, how the characteristic equation is created for a Spring-Mass-Damper system and explain, when the system is critically damped

$$m\ddot{x} + b\dot{x} + kx = 0 \checkmark, \quad ms^2 + bs + k = 0 \checkmark, s_{1/2} = -b/2m + -\sqrt{b^2 - 4mk} \checkmark$$

$$b^2 = 4mk \checkmark$$

Critically damped