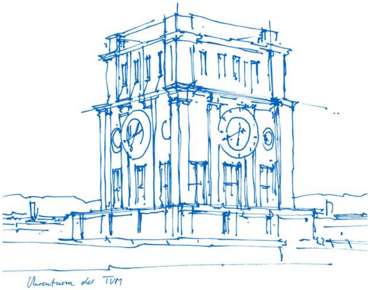
Computer Vision II: Multiple View Geometry (IN2228)

Chapter 12 Bundle Adjustment
(Part 1 Fundamentals)

Dr. Haoang Li

06 July 2023 11:00-11:45



Announcement Before Class

➢ Updated Lecture Schedule

For updates, slides, and additional materials:
https://cvg.cit.tum.de/teaching/ss2023/cv2

90-minute course; 45-minute course

Foundation

Wed 19.04.2023 Chapter 00: Introduction
Thu 20.04.2023 Chapter 01: Mathematical Backgrounds

Wed 26.04.2023 Chapter 02: Motion and Scene Representation (Part 1)
Thu 27.04.2023 Chapter 02: Motion and Scene Representation (Part 2)

Wed 03.05.2023 Chapter 03: Image Formation (Part 1)
Thu 04.05.2023 Chapter 03: Image Formation (Part 2)

Wed 10.05.2023 Chapter 04: Camera Calibration
Thu 11.05.2023 Chapter 05: Correspondence Estimation (Part 1)

Wed 17.05.2023 Chapter 05: Correspondence Estimation (Part 2)
Thu 18.05.2023 No lecture (Public Holiday)

Wed 24.05.2023 No lecture (Conference)
Thu 25.05.2023 No lecture (Conference)
} Videos and reading materials about the combination of deep learning and multi-view geometry

Wed 31.05.2023 Chapter 05: Correspondence Estimation (Part 3)

Core part

Thu 01.06.2023 Chapter 06: 2D-2D Geometry (Part 1)

Wed 07.06.2023 Chapter 06: 2D-2D Geometry (Part 2)
Thu 08.06.2023 No lecture (Public Holiday)

Wed 14.06.2023 Chapter 06: 2D-2D Geometry (Part 3)
Thu 15.06.2023 Chapter 06: 2D-2D Geometry (Part 4)

Wed 21.06.2023 Chapter 07: 3D-2D Geometry
Thu 22.06.2023 Chapter 08: 3D-3D Geometry

Wed 28.06.2023 Chapter 09: Single-view Geometry
Thu 29.06.2023 Chapter 10: Combination of Different Configurations

Advanced topics and high-level tasks

Wed 05.07.2023 Chapter 11: Photometric Error and Direct Method
Thu 06.07.2023 Chapter 12: Bundle Adjustment (Part 1)

Wed 12.07.2023 Chapter 12: Bundle Adjustment (Part 2)
Chapter 13: Robust Estimation
Thu 13.07.2023 Exam Information and Knowledge Review

Wed 19.07.2023 Chapter 14: SLAM and SFM

Thu 20.07.2023 No Onsite Lecture. Alternative: Online Meeting for Question Answering

Today's Outline

- Error Metrics
- Definition of Bundle Adjustment
- Basic Knowledge of Non-linear Optimization

- Application of Non-linear Optimization to Bundle Adjustment Based on Lie Algebra (next class)

Error Metrics

➢ Overview

✓ The quality of the estimated camera pose can be measured using different error metrics:

- Algebraic error
- Epipolar Line Distance (only for 2D-2D)
- Reprojection Error

✓ By minimizing any of the above error, we can optimize the camera pose.

✓ The above metrics are not limited to 2D-2D. We can also use them to evaluate 3D-2D case. In our class, let us take 2D-2D for example.

Error Metrics

➢ Algebraic Error

✓ We consider 8-point algorithm for illustration. It seeks to minimize the **algebraic error:**

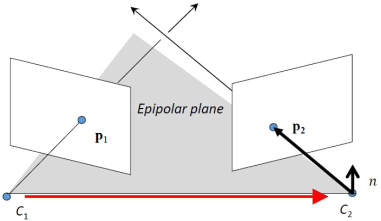
Essential

$$err = \|QE\|^2 = \sum_{i=1}^{N} (\bar{p}^{i\,\mathrm{T}}_{2} \boldsymbol{E} \bar{p}^{i}_{1})^2$$

✓ From the derivation of the epipolar constraint and the property of dot product, we can observe:

$$\|\bar{\boldsymbol{p}}_2^{\mathrm{T}} \boldsymbol{E} \bar{\boldsymbol{p}}_1\| = \underbrace{\| \bar{\boldsymbol{p}}_2^{\mathrm{T}} \cdot (\boldsymbol{E} \bar{\boldsymbol{p}}_1)\|}_{\text{Associative law}} \underbrace{= \|\bar{\boldsymbol{p}}_2\| \|\boldsymbol{E} \bar{\boldsymbol{p}}_1\| \cos(\theta)}_{\text{Property of dot product}}$$

$$\Rightarrow\ = \underbrace{\|\bar{\boldsymbol{p}}_2\| \|[\mathrm{T}_\times] R\, \bar{\boldsymbol{p}}_1\| \cos(\theta)}_{\text{Definition of essential matrix (in right camera frame)}}$$



Error Metrics

$$= \|\bar{\boldsymbol{p}}_2\| \, \| [T_\times] R \, \bar{\boldsymbol{p}}_1 \| \cos(\theta)$$

first -> second

Normal in the right camera frame

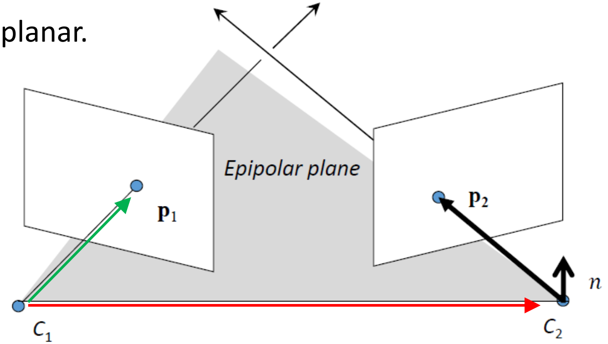
cross product

in right frame

- Algebraic Error

✓ We can see that this product depends on the angle θ between $\bar{\boldsymbol{p}}_2$ and the normal to the epipolar plane.

✓ It is nonzero when $\bar{\boldsymbol{p}}_1, \bar{\boldsymbol{p}}_2$, and $\boldsymbol{T}$ are not coplanar.



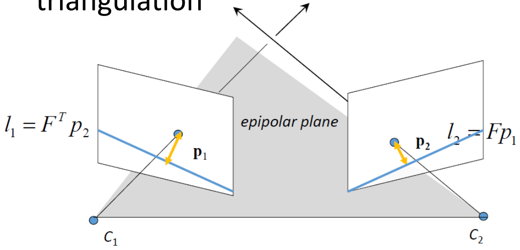
Error Metrics

➢ Epipolar Line Distance (only for 2D-2D configuration)

✓ Sum of squared epipolar-line-to-point distances:

$$err = \sum_{i=1}^{N} \left(d(p_1^i, l_1^i)\right)^2 + \left(d(p_2^i, l_2^i)\right)^2$$

✓ Cheaper than reprojection error (introduced later) because does not require point triangulation



im age line

Left point

$$\begin{bmatrix} u_2^i \\ v_2^i \\ 1 \end{bmatrix}^T \boxed{\boxed{F} \begin{bmatrix} u_1^i \\ v_1^i \\ 1 \end{bmatrix}} = 0$$

Fundamental Matrix $F = K_2^{-T}\, E\, K_1^{-1}$

Point lies on a line: dot(p, l)=0

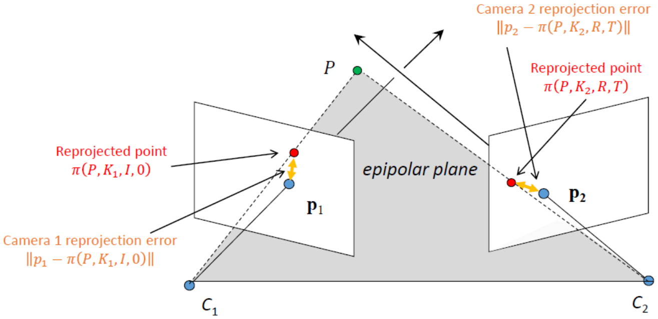
Epipolar line computed by the right point

Error Metrics

➢ Reprojection Error

✓ Sum of the Squared Reprojection Errors $$err = \sum_{i=1}^{N} \|p_1^i - \pi(P^i, K_1, I, 0)\|^2 + \|p_2^i - \pi(P^i, K_2, R, T)\|^2$$

✓ More expensive than the previous errors because it requires to first **triangulate** the 3D points.



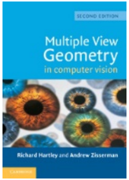
Error Metrics

➢ Reprojection Error

✓ It is the most popular because more accurate. The reason is that the error is computed directly with respect to the original input data, i.e., the image points. It is point-to-point distance.

✓ Previous algebraic error is with respect to 3D direction; Epipolar line distance is a point-to-line distance.

✓ Reprojection error is commonly called "golden standard" in our society. For a systematic analysis, please refer to [1].



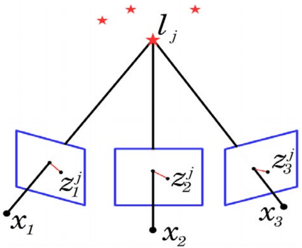
[1] "Multiple View Geometry in Computer Vision": R. Hartley and A. Zisserman
Link: https://www.robots.ox.ac.uk/~vgg/hzbook/

Error Metrics

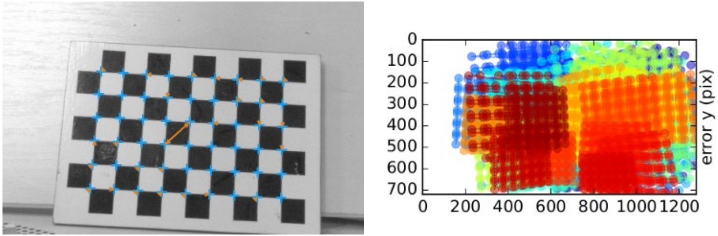
➢ Reprojection Error

✓ We often use reprojection error to perform two tasks:

- Pose and 3D point optimization
- Accuracy evaluation



Bundle adjustment for optimization



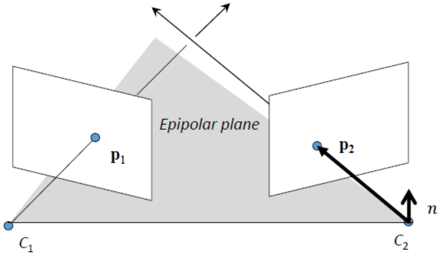
Calibration evaluation

Error Metrics

➢ Error Minimization

✓ Let us consider 8-point method. For **more than 8 points**, error will only be 0 if there is **no noise** in the data (if there is image noise, the linear system becomes overdetermined)

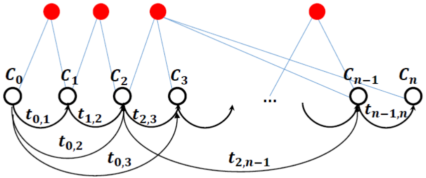
✓ We aim to find the optimal camera pose to minimize the least-squares error.

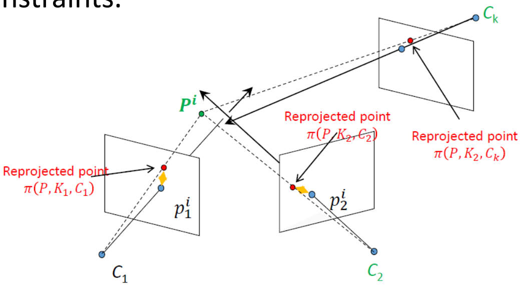


Bundle Adjustment

➢ Definition

- ✓ We extend two-view reprojection minimization to multi-view case, which is called "bundle adjustment".
- ✓ We typically treat the first camera as the world frame.
- ✓ We can reformulate the problem as a "graph optimization problem". Nodes are parameters to optimize, and edges are constraints.





Bundle Adjustment

➢ Definition

Multiple camera

Multiple point

✓ We jointly optimize camera poses of all the cameras and 3D points:

$$P^i, C_1, \ldots, C_n = argmin_{X^i, C_1, \ldots, C_n} \boxed{\sum_{k=1}^{n}} \boxed{\sum_{i=1}^{N}} \rho\left(p_k^i - \pi(P^i, K_k, C_k)\right)$$

deal noise

deal outliers

where $\rho()$ is the Huber norm for robust estimation (introduced next week)

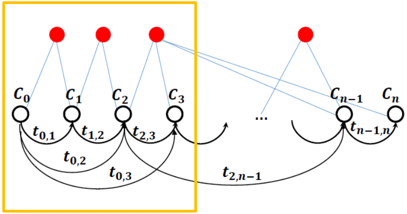
✓ We often use non-linear optimization, e.g., Gauss-Newton algorithm to minimize the error. Details will be introduced later.

Bundle Adjustment

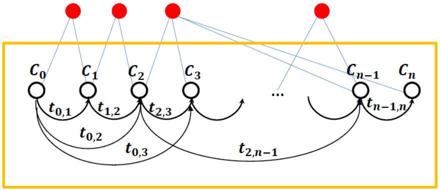
➢ Strategies for acceleration

✓ A small window size limits the number of parameters for the optimization and thus makes real time bundle adjustment possible.

✓ It is possible to reduce the computational complexity by just optimizing over the camera parameters and keeping the 3D landmarks fixed, e.g., motion-only BA.



A small window



Motion-only BA

Bundle Adjustment

➢ Photometric Bundle Adjustment

We can extend the photometric error between 1-by-1 frames to 1-by-N frames.

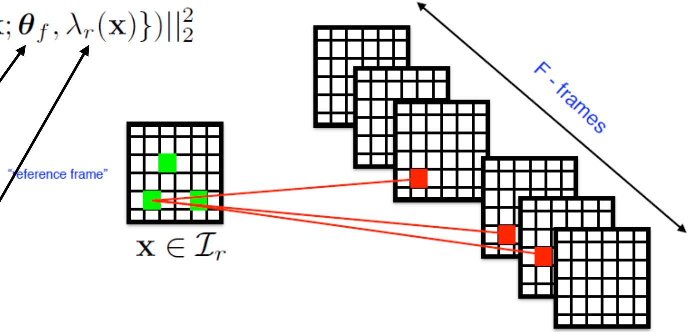
$$\arg\min_{\boldsymbol{\lambda},\boldsymbol{\theta}} \sum_{r=1}^{F} \sum_{\mathbf{x}\in\mathcal{I}_r} \sum_{f\in \mathrm{obs}(\mathbf{x})} \|\mathcal{I}_r(\mathbf{x}) - \mathcal{I}_f(\mathcal{W}\{\mathbf{x};\boldsymbol{\theta}_f,\lambda_r(\mathbf{x})\})\|_2^2$$

Perspective projection

Pixel

Pose

Depth



Non-linear Optimization

➢ Problem Formulation

✓ A teaser of curve fitting

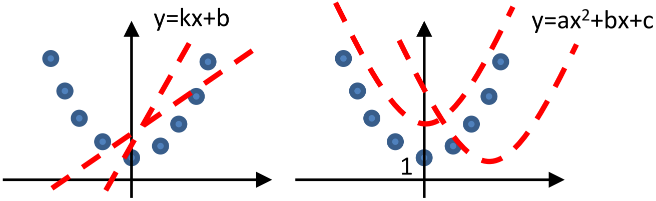
give

Input: A set of observed discrete points (no outliers here)

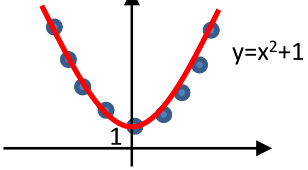
fit

Step 1: Select a suitable model/function with unknown parameters

Step 2: Estimate the parameters by the least-squares method: We define an objective function, i.e., the sum of squared distances.



Model/function selection



Parameter estimation

Non-linear Optimization

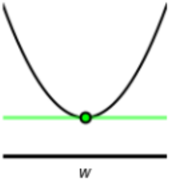
➢ Motivation of Gradient Descent Algorithm

To minimize the function, we can employ first-order optimality condition

$$\min_{\mathbf{x}} F(\mathbf{x}) = \frac{1}{2}\|f\left(\mathbf{x}\right)\|_2^2 \qquad \frac{\mathrm{d}F}{\mathrm{d}\mathbf{x}} = \mathbf{0}$$

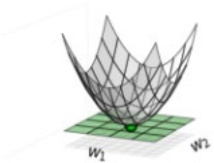
saddle point → local / global Minimum

If the derivative is simple, we can directly obtain the global minimum of objective function. However, what if the objective function is more complex?



$$\frac{\mathrm{d}}{\mathrm{d}w} g\left(w\right) = 0$$

1D function



$$\frac{\partial}{\partial w_1} g(\mathbf{v}) = 0$$
$$\frac{\partial}{\partial w_2} g(\mathbf{v}) = 0$$
$$\vdots$$
$$\frac{\partial}{\partial w_N} g(\mathbf{v}) = 0$$

2D function

Non-linear Optimization

➢ Motivation of Gradient Descent Algorithm

Instead of directly obtaining the global minimum, we iteratively minimize the function.

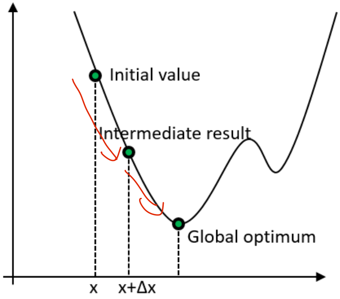
Initial

x_k is a temporary value. It is known.

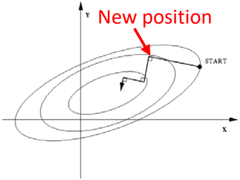
Δx_k is the adjustment of the above temporary value. It is unknown.

Aim to find Δx_k

1. Give an initial value $\mathbf{x}_0$.
2. For k-th iteration, we find an incremental value of $\Delta\mathbf{x}_k$, such that the object function $\boxed{\|f(\mathbf{x}_k + \Delta\mathbf{x}_k)\|_2^2}$ reaches a smaller value.
3. If $\Delta\mathbf{x}_k$ is small enough, stop the algorithm.
4. Otherwise, let $\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta\mathbf{x}_k$ and return to step 2.



Non-linear Optimization



➢ Steepest method

Now consider the *k*-th iteration. Suppose the current solution is at $\mathbf{x}_k$ and we want to find the increment $\Delta\mathbf{x}_k$. For problem simplification, we use the first-order Taylor expansion to re-write the objective function:

$$F(\mathbf{x}_k + \Delta\mathbf{x}_k) \approx F(\mathbf{x}_k) + \mathbf{J}(\mathbf{x}_k)^T \Delta\mathbf{x}_k$$

Gradient (Jacobi Matrix)

Known Known Unknown

Along the minus gradient **direction**, we can ensure that the function decreases:

$$\boxed{\Delta\mathbf{x}^* = -\mathbf{J}(\mathbf{x}_k)}$$

We do not explicitly compute Δx

Δx is only a direction. We also manually select another step length parameter (learning rate), say, λ. The smaller function value is $F(\mathbf{x}_k) - \mathbf{J}(\mathbf{x}_k)\,\lambda$ step

Non-linear Optimization

- Newton's method

$$\boxed{F(\mathbf{x}_k + \Delta\mathbf{x}_k) \approx F(\mathbf{x}_k) + \mathbf{J}(\mathbf{x}_k)^T \Delta\mathbf{x}_k + \frac{1}{2}\Delta\mathbf{x}_k^T \mathbf{H}(\mathbf{x}_k)\Delta\mathbf{x}_k}$$

more accurate/reliable decent

We can also use the second-order Taylor expansion to re-write the objective function:

$$\Delta\mathbf{x}^* = \arg\min \left(F(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \Delta\mathbf{x} + \frac{1}{2}\Delta\mathbf{x}^T \mathbf{H}\Delta\mathbf{x} \right)$$

Known Unknown

update

We leverage the first-order optimality condition, i.e., computing the derivative with respect to Δx and setting the result to zero. We thus can obtain

$$\mathbf{J} + \mathbf{H}\Delta\mathbf{x} = \mathbf{0} \Rightarrow \mathbf{H}\Delta\mathbf{x} = -\mathbf{J}$$

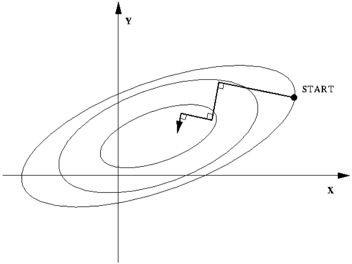
goal

Hessian matrix

Non-linear Optimization

➢ Gauss-Newton Method

✓ Motivation



Steepest method results in the zig-zag descending trajectory

Newton's method is time consuming due to the computation of Hessian matrix

$$\mathbf{H}_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1\,\partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1\,\partial x_n} \\ \frac{\partial^2 f}{\partial x_2\,\partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2\,\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n\,\partial x_1} & \frac{\partial^2 f}{\partial x_n\,\partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

We need a more effective method: We will introduce a representative method "**Gauss-Newton algorithm**".

Non-linear Optimization

$$\min_{\mathbf{x}} F(\mathbf{x}) = \frac{1}{2}\left\|\boxed{f(\mathbf{x})}\right\|_2^2$$

- Gauss-Newton Method

Similar to the steepest method, we begin with first-order Taylor expansion

$$\boxed{f(\mathbf{x}+\Delta\mathbf{x})} \approx \boxed{f(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \Delta\mathbf{x}}$$

We aim to find the optimal Δx to minimize this function

$$\Delta\mathbf{x}^* = \arg\min_{\Delta\mathbf{x}} \frac{1}{2}\left\|\boxed{f(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \Delta\mathbf{x}}\right\|^2$$

square

Let us first expand this function:

$$\begin{aligned}\frac{1}{2}\left\|f(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \Delta\mathbf{x}\right\|^2 &= \frac{1}{2}\left(f(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \Delta\mathbf{x}\right)^T\left(f(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \Delta\mathbf{x}\right)\\ &= \frac{1}{2}\left(\|f(\mathbf{x})\|_2^2 + 2f(\mathbf{x})\mathbf{J}(\mathbf{x})^T\Delta\mathbf{x} + \Delta\mathbf{x}^T\mathbf{J}(\mathbf{x})\mathbf{J}(\mathbf{x})^T\Delta\mathbf{x}\right)\end{aligned}$$

Non-linear Optimization

$$\boxed{\frac{1}{2}\left(\|f(\mathbf{x})\|_2^2 + 2f(\mathbf{x})\,\mathbf{J}(\mathbf{x})^T\Delta\mathbf{x} + \Delta\mathbf{x}^T\mathbf{J}(\mathbf{x})\mathbf{J}(\mathbf{x})^T\Delta\mathbf{x}\right)}$$

- Gauss-Newton Method

We compute the derivative of the above function with respect to Δx, and then set the derivate to zero:

$$\mathbf{J}(\mathbf{x})f(\mathbf{x}) + \mathbf{J}(\mathbf{x})\mathbf{J}^T(\mathbf{x})\,\Delta\mathbf{x} = \mathbf{0}$$

We transform the above into

$$\boxed{\underbrace{\mathbf{J}(\mathbf{x})\mathbf{J}^T}_{\mathbf{H}(\mathbf{x})}(\mathbf{x})}\,\Delta\mathbf{x} = \underbrace{-\mathbf{J}(\mathbf{x})f(\mathbf{x})}_{\mathbf{g}(\mathbf{x})}$$

We obtain a linear system to compute Δx

$$\boxed{\mathbf{H}\Delta\mathbf{x} = \mathbf{g}}$$

An approximation to Hessian matrix

Non-linear Optimization

➢ Application to Bundle Adjustment (A Teaser)

Jacobian matrix w.r.t. pose and point

✓ General objective function simplification by Gauss-Newton

$$\boldsymbol{e}(\boldsymbol{x}+\Delta\boldsymbol{x}) \approx \boldsymbol{e}(\boldsymbol{x}) + \boldsymbol{J}\Delta\boldsymbol{x}.$$

Adjustment of camera pose and 3D point

✓ We have to compute derivative w.r.t. SO3/SE3. It evolves addition and subtraction operation.

✓ Intuitively, R1 is in SO3 and R2 is in SO3, but we cannot guarantee that R1 + R2 is in SO3.

✓ To solve this problem, we first map Lie Group to Lie Algebra, and compute the derivative by Lie Algebra. More details will be introduced next week.

Summary

- Error Metrics
- Bundle Adjustment
- Non-linear Optimization

Thank you for your listening!
If you have any questions, please come to me :-)