Multiple View Geometry: Exercise 3

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Image Formation

We are looking at the formation of an image in camera coordinates $\mathbf{X} = (X\ Y\ Z\ 1)^\top$. The following relation of homogeneous pixel coordinates $\mathbf{x}'$ and $\mathbf{X}$ holds:

$$\lambda \mathbf{x}' = K\Pi_0\mathbf{X} \tag{1}$$

with the intrinsic camera matrix K.

Extra Infos on intrinsic camera matrix:

If the camera is not centered at the optical center, we have an additional translation o_x , o_y and if pixel coordinates do not have unit scale, we need to introduce an additional scaling in x- and y -direction by s_x and s_y . If the pixels are not rectangular, we have a skew factor s_θ . You can assume that focal lengths along the u and v axes are identical. Accordingly, they are both denoted by f. To clearly differentiate between camera coordinates and pixel coordinates, call the pixel coordinates u and v: $\mathbf{x}' = (u\ v\ 1)^\top$. The pixel coordinates $(u, v, 1)$ as a function of homogeneous camera coordinates $\mathbf{X}$ are then given by

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} s_x & s_\theta & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_s} \underbrace{\begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_f} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\equiv \Pi_0} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \tag{2}$$

After the perspective projection Π_0 (with focal length 1), we have an additional transformation which depends on the (intrinsic) camera parameters. This can be expressed by the intrinsic parameter matrix $K = K_s K_f$.

Furthermore, let the non-homogeneous camera coordinates be $\tilde{\mathbf{X}} := \Pi_0\mathbf{X} = (X\ Y\ Z)^\top$. (1) is then equivalent to

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K\tilde{\mathbf{X}}\,. \tag{3}$$

Let $s_x = s_y = 1$ and $s_\theta = 0$ in the intrinsic camera matrix.

1. Compute λ and show that (3) is equivalent to

$$u = \frac{fX}{Z} + o_x\,, \quad v = \frac{fY}{Z} + o_y\,. \tag{4}$$

2. A classic ambiguity of the perspective projection is that one cannot tell an object from another object that is exactly *twice as big but twice as far*. Explain why this is true.

透视投影的一个典型的模糊性是，人们无法区分一个物体和另一个物体，而这个物体正好是两倍大，但两倍远。解释一下为什么这是真的。

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} s_x & s_\theta & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_s} \underbrace{\begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv K_f} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\equiv \Pi_0} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

① $s_x = s_y = 1 \quad s_\theta = 0$

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & o_x \\ 0 & 1 & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & & 0 \\ & f & \\ 0 & & 1 \end{pmatrix} \begin{pmatrix} I & | & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} f & 0 & o_x \\ 0 & f & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} I & | & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

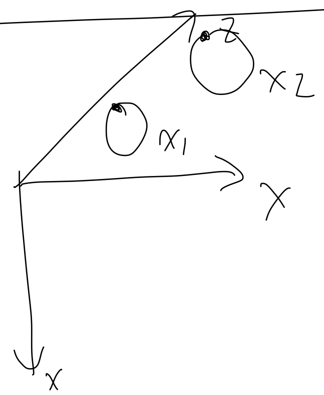
$$= \begin{pmatrix} f & 0 & o_x & 0 \\ 0 & f & o_y & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

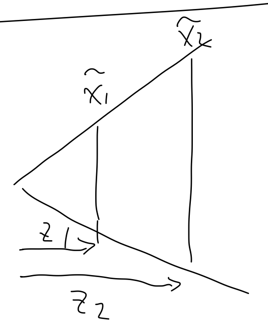
$$= \begin{pmatrix} fx + z \cdot o_x \\ fy + z \cdot o_y \\ z \end{pmatrix} = \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

$$\lambda = z \qquad u = \frac{fx + z \cdot o_x}{z} = \frac{fx}{z} + o_x$$

$$v = \frac{fy + z\, o_y}{z} = \frac{fy}{z} + o_y$$

(b)





$\tilde{x}_1 = (x_1 \quad y_1 \quad z_1)^T$

$\tilde{x}_2 = (x_2 \quad y_2 \quad z_2)^T$

$z_2 = 2z_1 \qquad x_2 = 2x_1$

$y_2 = 2y_1$

$$u_1 = \frac{fx_1}{z_1} + o_x$$

$$v_1 = \frac{fy_1}{z_1} + o_y$$

$$u_2 = \frac{fx_2}{z_2} + o_x = \frac{fx_1}{z_1} + o_x$$

$$v_2 = \frac{fy_2}{z_2} + o_y = \frac{fy_1}{z_1} + o_y$$

$u_2 = u_1$

$v_2 = v_1$

3. For a camera with $f = 540$, $o_x = 320$ and $o_y = 240$, compute the pixel coordinates u and v of a point $\tilde{\mathbf{X}} = (60\ 100\ 180)^\top$. Explain with the help of (b) why the units of $\tilde{\mathbf{X}}$ are not needed for this task. Will the projected point be in the image if it has dimensions 640×480?

We define the generic projection π of $\tilde{\mathbf{X}}$ to 2D coordinates as follows:

$$\pi(\tilde{\mathbf{X}}) := \begin{pmatrix} X/Z \\ Y/Z \end{pmatrix} \tag{5}$$

4. Using the generic projection π, show that (4) — and therefore also (1) and (3) — is equivalent to

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} . \tag{6}$$

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$$u = \frac{fx}{z} + o_x \qquad u = \frac{540 \cdot 60}{180} + 320 = 500$$

$$v = \frac{fy}{z} + o_y \qquad v = \frac{540 \cdot 100}{180} + 240 = 540$$

outside the image

scaling

$$\tilde{x} = \begin{pmatrix} 60 \\ 100 \\ 180 \end{pmatrix} = \begin{pmatrix} 60 \cdot 100 \\ 100 \cdot 100 \\ 180 \cdot 100 \end{pmatrix}$$

scaling factor

$$\tilde{x}_{scale} = \begin{pmatrix} \alpha \cdot 60 \\ \alpha \cdot 100 \\ \alpha \cdot 180 \end{pmatrix} \qquad \tilde{x}_{unscale} = \begin{pmatrix} 60 \\ 100 \\ 180 \end{pmatrix}$$

$$u_{scaled} = \frac{f \cdot \alpha \cdot 60}{\alpha 180} + o_x = \frac{f 60}{180} + o_x = u_{unscale}$$

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(d) $\pi(\tilde{x}) = \begin{pmatrix} X/Z \\ Y/Z \end{pmatrix}$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & o_x \\ 0 & 1 & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X/Z \\ Y/Z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} f & 0 & o_x \\ 0 & f & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X/Z \\ Y/Z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} f(X/Z) + o_x \\ f(Y/Z) + o_y \\ 1 \end{pmatrix} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$