Multiple View Geometry: Exercise 1



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Wednesdays 16:00-18:15 at Hörsaal 2, "Interims I" (5620.01.102), and on RBG Live

Exercise: May 03, 2023

Math Background

1. Show for each of the following sets (1) whether they are linearly independent, (2) whether they span \mathbb{R}^3 and (3) whether they form a basis of \mathbb{R}^3 :

(a)
$$B_1 = \left\{ \begin{pmatrix} 1\\1\\1 \end{pmatrix}, \begin{pmatrix} 0\\1\\1 \end{pmatrix}, \begin{pmatrix} 0\\0\\1 \end{pmatrix} \right\}$$

(b)
$$B_2 = \left\{ \begin{pmatrix} 2\\1\\0 \end{pmatrix}, \begin{pmatrix} 1\\1\\0 \end{pmatrix} \right\}$$

(c)
$$B_3 = \left\{ \begin{pmatrix} 2\\1\\0 \end{pmatrix}, \begin{pmatrix} 3\\1\\0 \end{pmatrix}, \begin{pmatrix} 0\\0\\1 \end{pmatrix}, \begin{pmatrix} 1\\0\\1 \end{pmatrix} \right\}$$

2. Which of the following sets forms a group (with matrix-multiplication)? Prove or disprove!

(a)
$$G_1 := \left\{ A \in \mathbb{R}^{n \times n} | \det(A) \neq 0 \land A^\top = A \right\}$$

(b)
$$G_2 := \{ A \in \mathbb{R}^{n \times n} | \det(A) = -1 \}$$

(c)
$$G_3 := \{ A \in \mathbb{R}^{n \times n} | \det(A) > 0 \}$$

3. Prove or disprove: There exist vectors $\mathbf{v}_1,...,\mathbf{v}_5\in\mathbb{R}^3\setminus\{\mathbf{0}\}$, which are pairwise orthogonal, i.e.

$$\forall i, j = 1, ..., 5: i \neq j \implies \langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0$$

4. Which groups have you seen in the lecture? Write down the names and the correct inclusions! (e.g.: group $A \subset \text{group } B$)

5. Let A be a symmetric matrix, and λ_a , λ_b eigenvalues with eigenvectors v_a and v_b . Prove: if v_a and v_b are not orthogonal, it follows: $\lambda_a = \lambda_b$.

Hint: What can you say about $\langle Av_a, v_b \rangle$?

6. Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix with the orthonormal basis of eigenvectors v_1, \ldots, v_n and eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$. Find all vectors x, that minimize the following term:

1

$$\min_{||x||=1} x^{\top} A x$$

1. Show for each of the following sets (1) whether they are linearly independent, (2) whether they span \mathbb{R}^3 and (3) whether they form a basis of \mathbb{R}^3 :

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(b)
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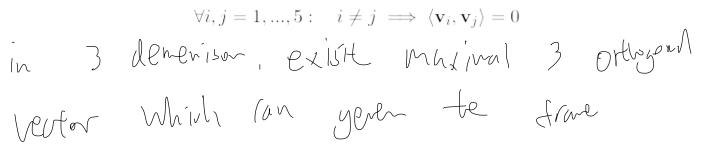
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(a) not closed under a group (AB) 1 = BTAT = BA + AB

(b) consist no newtral element

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$$Z = Va^{T} A V_{b} = Z^{T} = V_{b}^{T} A^{T} Va = V_{b}^{T} A V_{0}$$

$$V_{a}^{T} \lambda_{b} V_{b} = V_{b}^{T} \lambda_{a} V_{a}$$

$$V_{a}^{T} \lambda_{b} V_{b} = V_{b}^{T} \lambda_{a} V_{a}$$

$$(\lambda_{b} - \lambda_{a}) (V_{b}^{T} V_{a}) = 0$$

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$$\mathcal{Z} = \min_{||x||=1} x^{\mathsf{T}} A x$$

How many solutions exist? How can the term be maximized?

Hint: Use the expression $x = \sum_{i=1}^{n} \alpha_i v_i$ with coefficients $\alpha_i \in \mathbb{R}$ and compute appropriate coefficients!

$$\frac{2}{2} = \sum_{i=1}^{N} \frac{2}{2} 2 x_i x_j \qquad \frac{4}{3} y_i = 0$$

7. Let $A \in \mathbb{R}^{m \times n}$. Prove that $kernel(A) = kernel(A^{\top}A)$.

Hint: Consider a) $x \in \text{kernel}(A)$ $\Rightarrow x \in \text{kernel}(A^{\top}A)$ and b) $x \in \text{kernel}(A^{\top}A)$ $\Rightarrow x \in \text{kernel}(A)$.

$$\frac{1}{X \in \text{kenel}(A)} = \frac{1}{X} \times \frac{1}{X} \times$$

8. Singular Value Decomposition (SVD)

Let $A = USV^{\top}$ be the SVD of A.

- (a) Write down possible dimensions for A, U, S and V.
- (b) What are the similarities and differences between the SVD and the eigenvalue decomposition?
- (c) What do you know about the relationship between U, S, V and the eigenvalues and eigenvectors of $A^{\top}A$ and AA^{\top} ?
- (d) What is the interpretation of the entries in S and what do the entries of S tell us about A?

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$$\begin{array}{ll} \textit{Hint:} \; \mathsf{Consider} & \mathsf{a}) \; \; x \in \mathsf{kernel}(A) & \Rightarrow x \in \mathsf{kernel}(A^\top A) \\ & \mathsf{and} & \mathsf{b}) \; \; x \in \mathsf{kernel}(A^\top A) & \Rightarrow x \in \mathsf{kernel}(A). \end{array}$$

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