Chair of Artificial Intelligence, Robotics, and Embedded Systems
Department of Informatics
Technical University of Munich

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Note:
- Cross your Registration number(with leading zero). It will be evaluated automatically.
- Sign in the corresponding signature field.

Robotics

Exam:	IN2067 / Endterm	**Date:**	Monday 13th February, 2023
Examiner:	Prof. Dr.-Ing. Darius Burschka	**Time:**	08:00 – 09:30

Working instructions

- This exam consists of **12 pages** with a total of **3 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 128 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

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Problem 1 Kinematics (42 credits)

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a)* Write the rotation matrix ${}^1_2R$ (as defined in the lecture) between the coordinate frames from Figure 1.1. Write the general constraints on the elements of the rotation matrix.

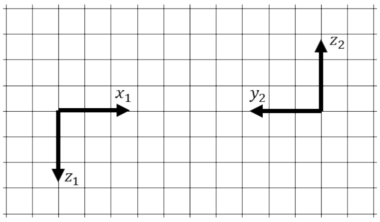

Figure 1.1: Coordinate frames {1} and {2}

0 1 2 3 4 5 6 7 8 9 10

b)* Figure 1.2 shows a robot having its z_{EE} axis pointing outside the paper plane. Write the Denavit-Hartenberg table of the robot. Ensure maximal number of zeros in the table. Write the values for the rotational joint parameters as seen in the drawn configuration.

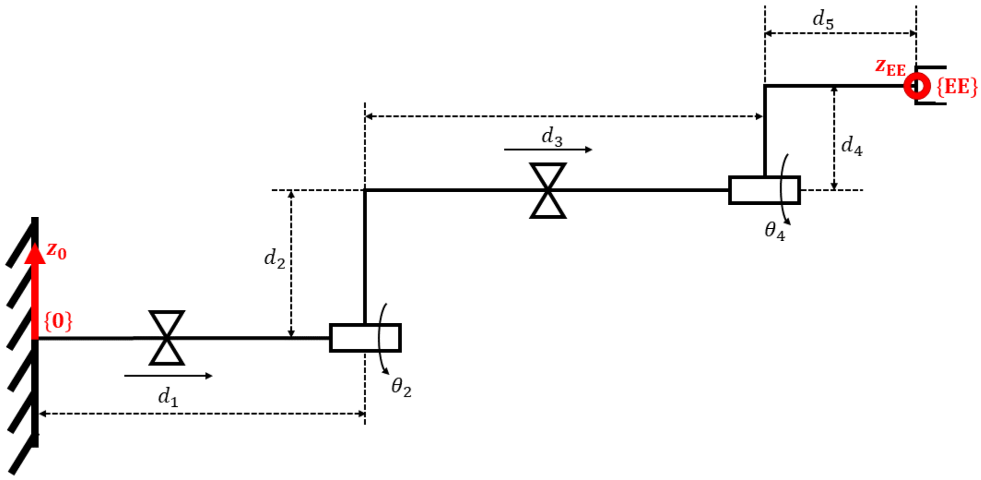

Figure 1.2: Schematic of a PRPR robot

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c) If the coordinate frame of the end-effector was not specified in the drawing, then you would have been able to select the orientation of the coordinate frame. Could you have chosen an orientation in which z_{EE} is parallel to z_0 at the same position of the origin of $\{EE\}$ as in the drawing? Justify your answer.

0 1 2 3 4 5

d) Determine the position of the end effector relative to the origin. Show your work. What is the shape of the outer form of the robot's workspace?

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e) How many degrees of freedom does this robot have and what are its redundancies? How would you (mathematically, computationally or geometrically) check if a robot has redundancies?

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f)* When deriving the Jacobian for a different robot with 4 joints, you arrive at the result

$$J = \begin{pmatrix} -c_1 + d_2 c_{13} & 0 & d_2 c_{13} & 0 \\ -s_1 + d_2 s_{13} & 0 & d_2 s_{13} & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & c_1 & 0 \\ 0 & 0 & s_1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

How many rotational joints does the robot have? And how many prismatic joints? Determine how many singular configurations this robot has when considering the position jacobian with respect to its first three joints.

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Problem 2 **Dynamics (41 credits)**

a)* Convert the following joint torques equation to M-B-C-G form. Show your work and clearly mark M, B, C and G. Under which circumstances is it advantageous to use this form of the joint torques equation?

0 1 2 3 4 5 6 7

$$\tau = \begin{pmatrix} m_1(l_1^2 c_1^2 + 2) & c_1 & 0 \\ m_2(1 + c_2) & m_2 l_2^2 & m_2 + m_3 \\ (l_3 + 1/2)^2(m_1 + m_2) & m_2 & m_3 l_3^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{d}_3 \end{pmatrix} + \begin{pmatrix} \dot{\theta}_1\dot{\theta}_2 m_1(l_1 - 2) - d_3 m_3 g + \dot{\theta}_1 \dot{d}_3 d_3 c_2 + m_3 g l_3 c_1 c_2 \\ -m_3 g l_3 s_1 c_2 + \dot{\theta}^2(2m_2 + 3) - \dot{\theta}_1(\dot{d}_3 + l_2 c_2 \dot{\theta}_1) \\ \dot{d}_3^2 - \ddot{\theta}_2 + \dot{\theta}_1\dot{\theta}_2 m_1 l_2 + m_2 g l_2 s_2 - \dot{\theta}_2^2 d_3 c_{12} \end{pmatrix}$$

b)* "Because the potential energy depends on the position of each robot joint, the robot will have more potential energy at 500m altitude than at 0m altitude". Explain why this statement is or is not correct. Does it have an effect on the Lagrange analysis of robot dynamics? Explain your two answers.

0 1 2 3 4

c)* When determining the M-B-C-G equation for a robot's joint torques, you get a 28x28-dimensional B-matrix. How many joints does this robot have?

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Figure 2.1 presents a robot to be analysed with the Lagrange method. The centers of mass for each link are located at $\frac{1}{3}$ and $\frac{1}{2}$ of the links' length respectively, with masses m_1 and m_2. The robot's DH-table is given.

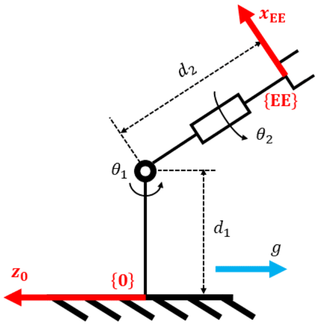

Figure 2.1: Schematic of an RR robot

CF	α_{i-1}	a_{i-1}	d_i	θ_i	value
1	$90°$	d_1	0	θ_1	$30°$
2	$90°$	0	0	θ_2	$0°$
EE	$0°$	0	d_2	$0°$	–

The inertial matrices are ${}^{C_1}I_1 = \begin{pmatrix} I_{1xx} & 0 & 0 \\ 0 & I_{1yy} & 0 \\ 0 & 0 & I_{1zz} \end{pmatrix}$

and ${}^{C_2}I_2 = \begin{pmatrix} I_{2xx} & 0 & 0 \\ 0 & I_{2yy} & 0 \\ 0 & 0 & I_{2zz} \end{pmatrix}$

0 1 2 3 4 5 6 7 8

d)* Write the values of all velocities that you need for the Lagrange analysis.

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e) Compute the kinetic and potential energies for each link.

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f) Compute the needed energy derivatives to complete the Lagrange analysis and write the joint torque equation.

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Problem 3 Control (45 credits)

A car, m_1, actuated by a force $\vec{f}$ is pulling a trailer, m_2, connected to it by a spring with the spring constant $k = 4$. The spring force is $f_s = k \cdot \Delta x = k \cdot (x_2 - x_1)$. The velocity dependent damping force can be calculated by $f_d = b \cdot \dot{x}_i$, where $b = 4$ and x_i is the position of the car or trailer $i = \{1, 2\}$ (Fig. 3.1). For simplicity, we assume that $x_1 = 0, x_2 = 0$ if no force is acting on the spring (even if the 0 positions of x_i do not coincide). The masses of the car and the trailer are $m_i = 4$. The resonance frequency of the trailer is $\omega_{res} = 1$.

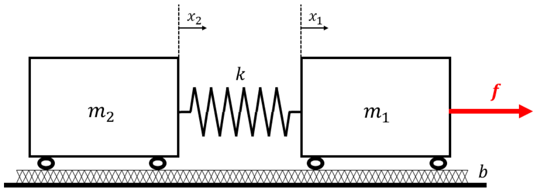

Figure 3.1: SMD system

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a)* Draw the forces acting on the trailer and the car after "cutting" (seperating) the car from trailer (including $\vec{f}$) and write the two equation balancing all forces on these two systems. Re-write them to a form, where all internal force parameters (i.e. the elements dependent on the x_i parameter for a given system) are on the left and other forces on the right of the system.

0 1 2 3

b) Check the left side of the above equation for the trailer for its response to disturbances using the characteristic equation from the lecture. Which type of the response will you see?

c)* Assume that the car and trailer are driving with a constant speed $\dot{x}_1 = \dot{x}_2 = 3$, The force in the spring is used to compensate the damping force in the trailer. Estimate the stretch of the spring $(x_2 - x_1)$. Give the form and parameters of the solution for $x_2(t)$ for the case that the front car halts instantaneously at t=0 and remains static. Assume an oscillating solution here. What is the trailer's natural and oscillation frequency?

0 1 2 3 4 5 6 7 8 9 10 11

d)* Which simple control law (no control law partitioning) needs to be used to calculate the value of f_2 to get the ideal response of the trailer without oscillations? Calculate the values of the control law given the resonance frequency of the trailer above.

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e)* Explain briefly giving the corresponding equation the control law partitioning and draw the structure of the corresponding controller for a simple spring-mass-damper (SMD) system. Which parameters need to be adjusted for the asymptotic solution of the motion equation and how are the actual parameter values calculated.

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.

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