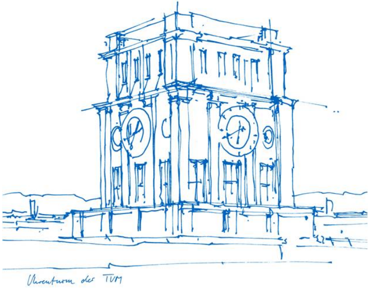
Computer Vision II: Multiple View Geometry (IN2228)

Chapter 06 2D-2D Geometry
(Part 3 3D Reconstruction)

Dr. Haoang Li

14 June 2023 12:00-13:30



Announcement Before Class

- Updated Lecture Schedule

For updates, slides, and additional materials:
https://cvg.cit.tum.de/teaching/ss2023/cv2

90-minute course; 45-minute course

Foundation

Wed 19.04.2023 Chapter 00: Introduction
Thu 20.04.2023 Chapter 01: Mathematical Backgrounds

Wed 26.04.2023 Chapter 02: Motion and Scene Representation (Part 1)
Thu 27.04.2023 Chapter 02: Motion and Scene Representation (Part 2)

Wed 03.05.2023 Chapter 03: Image Formation (Part 1)
Thu 04.05.2023 Chapter 03: Image Formation (Part 2)

Wed 10.05.2023 Chapter 04: Camera Calibration
Thu 11.05.2023 Chapter 05: Correspondence Estimation (Part 1)

Wed 17.05.2023 Chapter 05: Correspondence Estimation (Part 2)

Thu 18.05.2023 No lecture (Public Holiday)

Wed 24.05.2023 No lecture (Conference)
Thu 25.05.2023 No lecture (Conference)

Videos and reading materials about the combination of deep learning and multi-view geometry

Core part

Wed 31.05.2023 Chapter 05: Correspondence Estimation (Part 3)
Thu 01.06.2023 Chapter 06: 2D-2D Geometry (Part 1)

Wed 07.06.2023 Chapter 06: 2D-2D Geometry (Part 2)
Thu 08.06.2023 No lecture (Public Holiday)

Wed 14.06.2023 Chapter 06: 2D-2D Geometry (Part 3)
Thu 15.06.2023 Chapter 06: 2D-2D Geometry (Part 4)

Wed 21.06.2023 Chapter 07: 3D-2D Geometry
Thu 22.06.2023 Chapter 08: 3D-3D Geometry

Wed 28.06.2023 Chapter 09: Single-view Geometry
Thu 29.06.2023 Chapter 10: Combination of Different Configurations

Advanced topics and high-level tasks

Wed 05.07.2023 Chapter 11: Photometric Error (Direct Method)
Thu 06.07.2023 Chapter 12: Bundle Adjustment and Optimization

Wed 12.07.2023 Chapter 13: Robust Estimation
Thu 13.07.2023 Question Explanation and Knowledge Review

Wed 19.07.2023 Chapter 14: SLAM and SFM (Part 2)
Thu 20.07.2023 Chapter 14: SLAM and SFM (Part 1)

Today's Outline

- Overview of 3D Reconstruction
- Triangulation (General Case)
- Stereo Vision (Simplified Case)

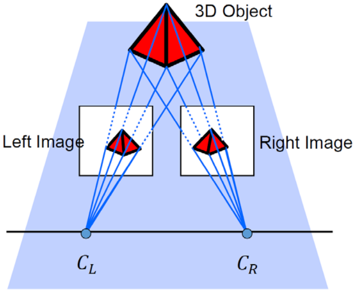
Overview

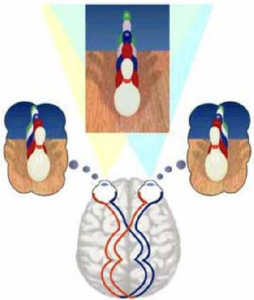
➢ Intuitive Illustration

目标：通过计算相应射线的交点来恢复三维结构。? 人眼的工作原理：投射在我们视网膜上的物体是上下颠倒的，但我们的大脑让我们把它们看作是直立的物体。

✓ Goal: recover the 3D structure by computing the intersection of corresponding rays.

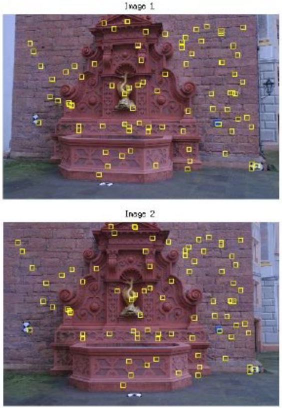
✓ Working principle of human eye: Objects projected on our retinas are up-side-down, but our brain makes us perceive them as upright objects.



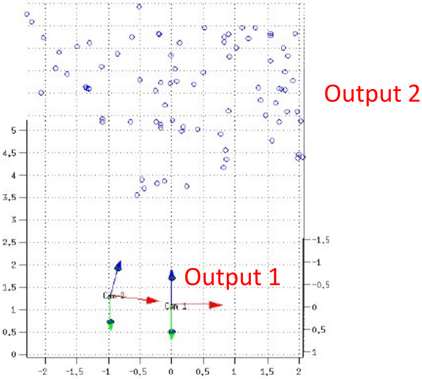


Overview

- Input and Output



Input: 2D-2D point correspondence



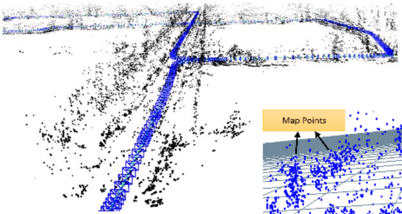
Estimated poses and 3D structure

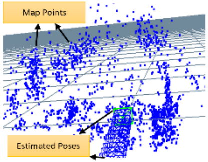
Overview

➢ Classification

✓ General case (for sparse reconstruction)

- Triangulation



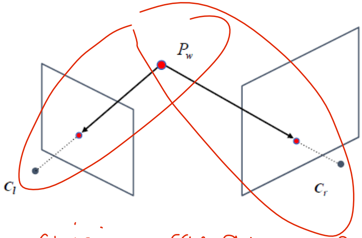


(a)


(b)

General case

(non identical cameras and not aligned)



origin camera pose

Overview

➢ Classification

✓ Simplified case (for dense reconstruction)

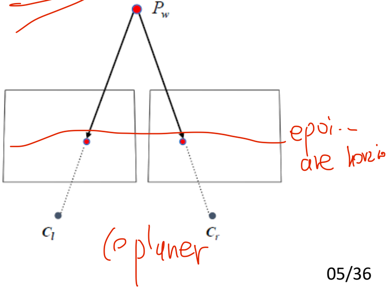
- Depth from disparity

Input Stereo Sequence

Simplified case

(identical cameras and aligned)

virtual



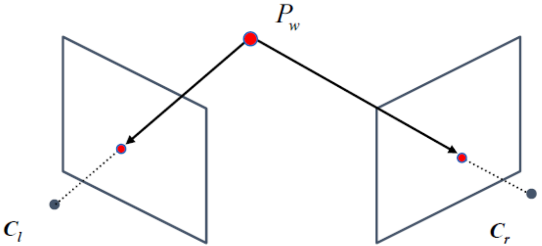
Triangulation

➢ Overview

✓ Prior information

[R|T]

- Extrinsic parameters (relative rotation and translation) obtained by epipolar constraint (or the other methods, e.g., PnP and ICP).
- Intrinsic parameters (focal length , principal point of each camera). We can obtain them by using a calibration method e.g., Tsai's method or Zhang's method.

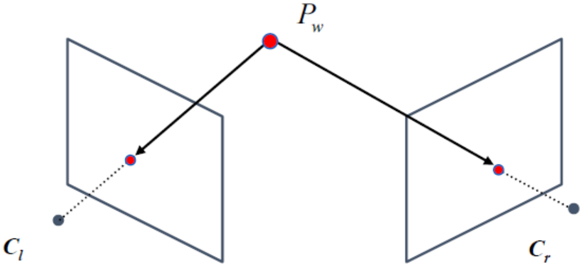


Triangulation

➢ Overview

✓ Definition

Triangulation is the problem of determining the **3D position of a point** given a set of **corresponding 2D points** and known **camera poses**.

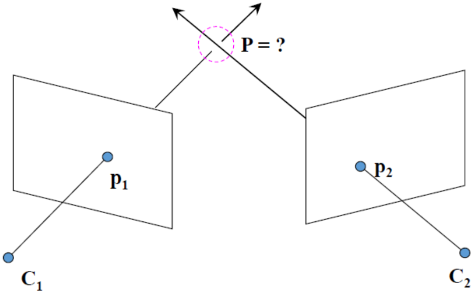


Triangulation

➢ Overview

✓ Definition

We want to **intersect** the two projection rays corresponding to p_1 and p_2.
Because of noise and numerical errors, two rays won't meet exactly, so we can only compute an approximation.



Triangulation

➢ Basic Constraints

$X_c = R X_w + t$

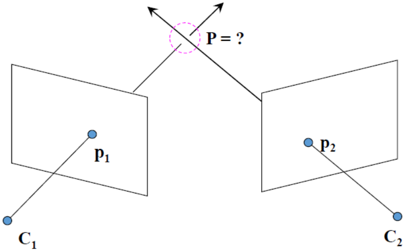
In the left camera frame, we have the perspective projection constraints:

Left camera:

$$\lambda_1 \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = K_1 \boxed{[I|0]} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Right camera:

$$\lambda_2 \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = K_2 \boxed{[R|T]} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$



We express 3D point in the **left camera frame**.

Triangulation

➢ Basic Constraints

We generate the system of equations of the left and right cameras:

Left camera:

$$\lambda_1 \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \boxed{K_1[I|0]} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \Rightarrow \lambda_1 p_1 = \boxed{M_1} \cdot P \quad \Rightarrow 0 = \underline{p_1 \times M_1 \cdot P}$$

Known

Collinearity (up-to-scale)

Cross product

Right camera:

$$\lambda_2 \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \boxed{K_2[R|T]} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \Rightarrow \lambda_2 p_2 = \boxed{M_2} \cdot P \quad \Rightarrow 0 = \underline{p_2 \times M_2 \cdot P}$$

Triangulation

➢ Least Square Approximation

$$[\mathbf{a}_\times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

We get a homogeneous system of equations

Left camera: $0 = p_1 \times M_1 \cdot P \quad \Rightarrow [p_{1\times}] \cdot M_1 \cdot \boxed{P} = 0$ Two independent linear constraints

Unknown

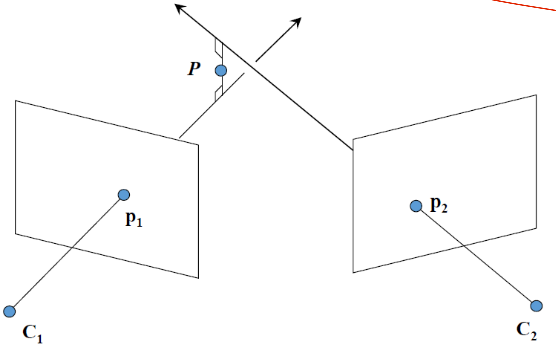
Right camera: $0 = p_2 \times M_2 \cdot P \quad \Rightarrow [p_{2\times}] \cdot M_2 \cdot \boxed{P} = 0$ Two independent linear constraints

We get a homogeneous system of equations.
Mathematically, 3D point $\boldsymbol{P}$ can be determined using SVD.

Triangulation

- Least Square Approximation

Geometrically, ***P*** is computed as the midpoint of **the shortest 3D line segment** connecting the two lines.

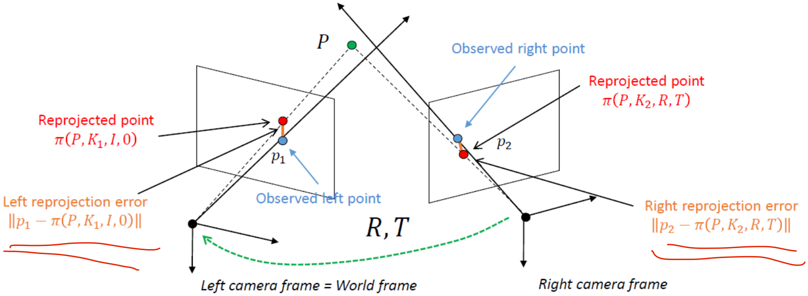


Triangulation

➢ Follow-up Non-linear Optimization (Optional)

✓ Initialize ***P*** using the least square approximation introduced before
✓ Refine $\boldsymbol{P}$ by minimizing the sum of left and right squared re-projection errors:
✓ We can only optimize 3D point, or jointly optimize pose and 3D point.

$$P = argmin_P \ \|p_1 - \pi(P, K_1, I, 0)\|^2 + \|p_2 - \pi(P, K_2, R, T)\|^2$$



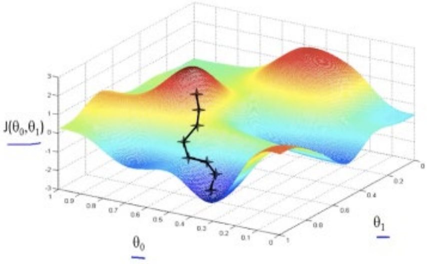
Triangulation

$$P = argmin_P \ \ \|p_1 - \pi(P, K_1, I, 0)\|^2 + \|p_2 - \pi(P, K_2, R, T)\|^2$$

find optimal K R T

- Non-linear Optimization

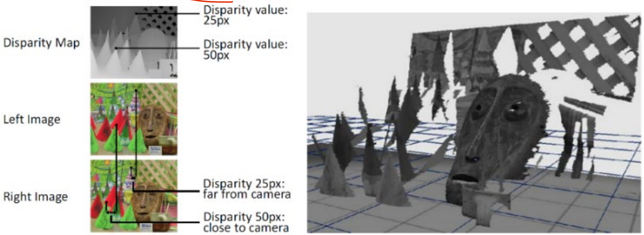
The reprojection error can be minimized using Levenberg-Marquardt (more robust than Gauss-Newton method to local minima)
The gradient-descent algorithms will be introduced in the future.



Stereo Vision

➢ Overview

✓ Input: known extrinsic camera parameters measured/calibrated beforehand
✓ Main knowledge
- Disparity and Depth
- Stereo Rectification
- Dense Correspondence Establishment (introduced in the next class)



Disparity and depth

Stereo rectification

Stereo Vision

➢ Disparity and Depth

✓ Intuitive illustration

- Our brain allows us to perceive **disparity (displacement vector of a point)** from the left and right images.
- **Depth** is inversely proportional to the **disparity**.

Image from the left eye

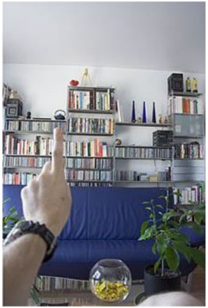
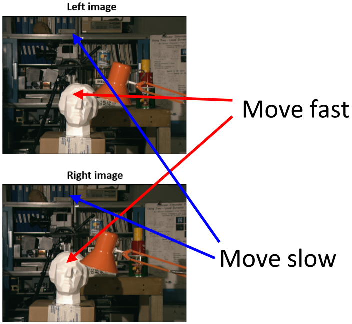


Image from the right eye

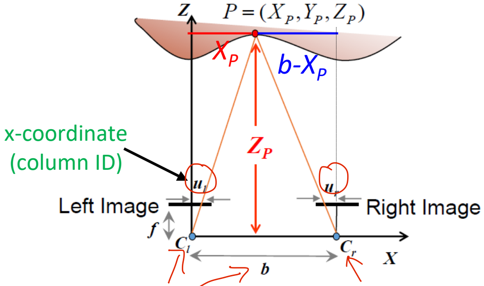


Stereo Vision

➢ Disparity and Depth

✓ Mathematical computation

Assume that both cameras are identical (i.e., have the same intrinsic parameters) and are aligned to the x-axis.



Baseline = distance between the optical centers of the two cameras

From Similar Triangles:

$$\frac{f}{Z_P}=\frac{u_l}{X_P}$$

$$\frac{f}{Z_P}=\frac{-u_r}{b-X_P}$$

$$\Rightarrow \quad Z_P=\frac{bf}{\boxed{u_l-u_r}}$$

Disparity

difference in image location of the projection of a 3D point on two image planes

Stereo Vision

➢ Disparity and Depth

✓ Mathematical computation

一旦立体对被矫正，每个点的差异和深度就可以被计算出来。

Once the stereo pair is rectified, the **disparity and depth** of each point can be computed.

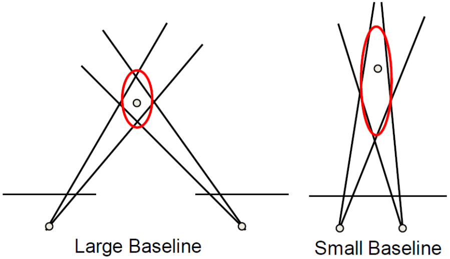
$$\underbrace{Z_P}_{\text{Depth}} = \frac{\overbrace{b}^{\text{Baseline}}\overbrace{f}^{\text{Focal length}}}{\underbrace{u_l - u_r}_{\text{Disparity}}}$$

Stereo Vision

➢ Disparity and Depth

✓ What's the optimal baseline?

$$Z_P = \frac{\boxed{b}f}{u_l - u_r}$$



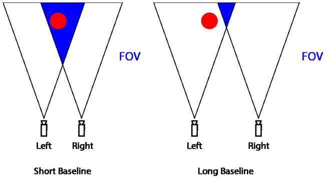
Large baseline:

- Advantage: Small depth error
- Disadvantage: Difficult search problem for close objects (projection may be outside the right image)

Small baseline:

- Advantage: Large depth error
- Disadvantage: Cons: Easier search problem for close objects

Outside the FOV of right camera



激励

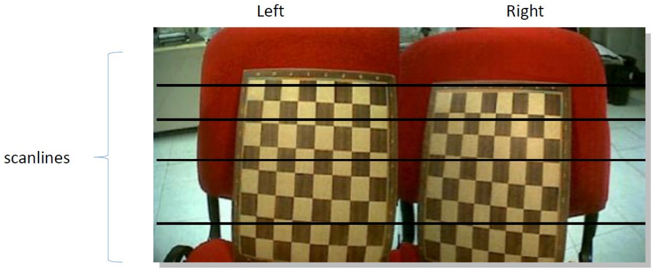
Stereo Vision

- 在我们之前的推导中，我们假设图像对已经被整顿。
- 即使是商用立体相机，左右两幅图像也不可能完全对齐。
- 在实践中，如果表极线与水平扫描线对齐是很方便的，因为对应搜索可以非常有效（只沿同一扫描线搜索点）。

➢ Stereo Rectification

✓ Motivation

- In our previous derivations, we assume that the image pairs have been rectified.
- Even for a commercial stereo camera, the left and right images are never perfectly aligned.
- In practice, **it is convenient if the epipolar lines are aligned to the horizontal scanlines** because the correspondence search can be very efficient (only search the point along the same scanlines).



Raw stereo pair (**unrectified**): scanlines do not coincide with epipolar lines

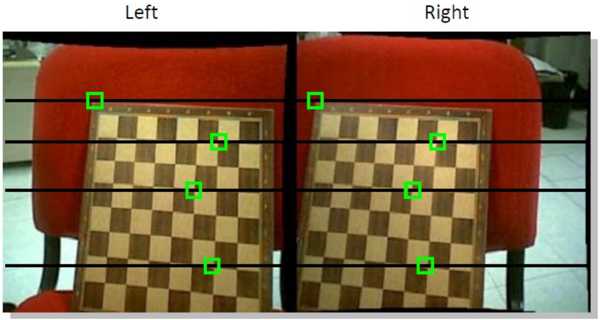
Stereo Vision

➢ Stereo Rectification

立体矫正将左、右图像扭曲成新的 "矫正 "图像，使上极线与水平扫描线重合。

✓ Definition

- Stereo rectification warps the left and right images into new “rectified” images such that the **epipolar lines coincide with the horizontal scanlines.**



Rectified stereo pair: **scanlines coincide with epipolar lines**

概述

Stereo Vision

- 将原始图像平面扭曲到与基线平行的（虚拟）平面上。

➢ Stereo Rectification

- 它通过计算两个变换来工作，每个图像一个变换

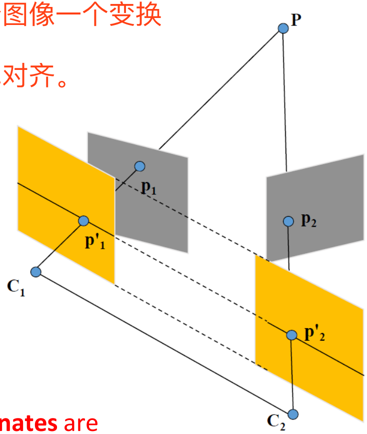
- 结果是，新的表极线与水平扫描线对齐。

✓ Overview

- Warps the original image planes onto a (virtual) planes **parallel to the baseline**

- It works by computing two transformations, one for each image

- As a result, the new epipolar lines are aligned to the horizontal scanlines.



Such a transformation describes **2D coordinate transformation**.
If an image plane is transformed in 3D, a **2D projection point's coordinates** are changed accordingly.

这样的变换描述了二维坐标变换。
如果一个图像平面进行了三维变换，一个二维投影点的坐标也会相应改变。

Stereo Vision

- 我们定义了两个新的矩阵，分别围绕其光学中心旋转旧的图像平面。新的图像平面成为共面，并且都与基线平行。

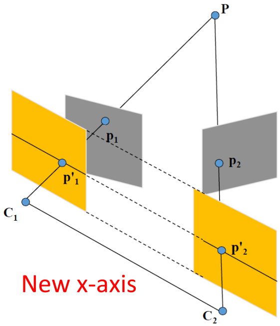
- 这确保了表极线是平行的。

- 为了获得水平的（而不仅仅是平行的）外极线，基线必须平行于两个新相机帧的新X轴。

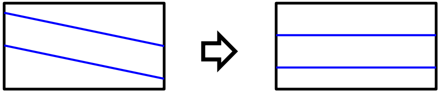
➢ Stereo Rectification

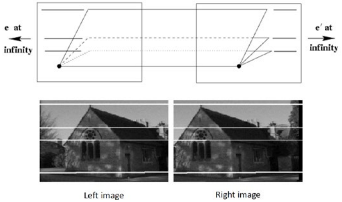
✓ Pipeline

- We define two new matrices to **rotate** the old image planes **respectively** around their optical centers. The new image planes become coplanar, and are both parallel to the baseline.
- This ensures that **epipolar lines are parallel**.



- To have **horizontal** (not just parallel) epipolar lines, the baseline must be parallel to the new X axis of both new camera frames.





Stereo Vision

➢ Overview

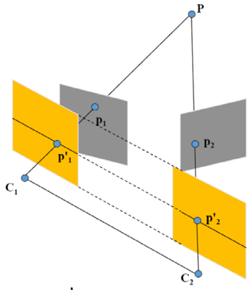
✓ Pipeline

- In addition, to have a proper rectification, corresponding points must **have the same y-coordinate (row ID)**. This is obtained by requiring that the new cameras **have the same intrinsic parameters**.

- In other words, the **displacement of a 2D point** in the image is **only** caused by extrinsic parameters (relative pose of camera).

- 此外，为了进行适当的矫正，相应的点必须有相同的y坐标（行ID）。这是通过要求新的相机具有相同的内在参数而得到的。

- 换句话说，图像中二维点的位移只由外在参数（相机的相对姿势）引起。



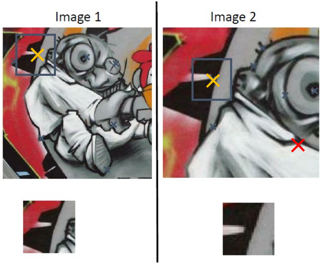


Image pairs obtained by different **focal lengths**

Stereo Vision

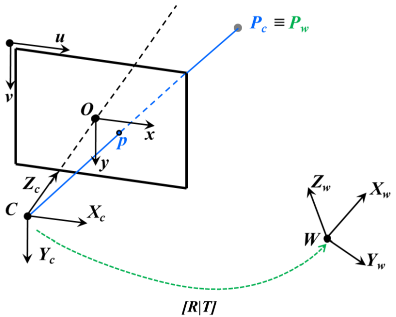
➢ Detailed Procedures (Step 1)

✓ Recap on perspective projection

The perspective equation for a point P_w in the world frame is defined by the following equation, where R=R_{cw} and T=T_{cw} transform points from the **World frame** to the **Camera frame**.

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \left(R \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + T \right)$$

$[R|T]$



Stereo Vision

$$Y = RX + t \quad \Rightarrow \quad X = R^T(Y - t) = \boxed{R^T}Y \boxed{- R^T t}$$

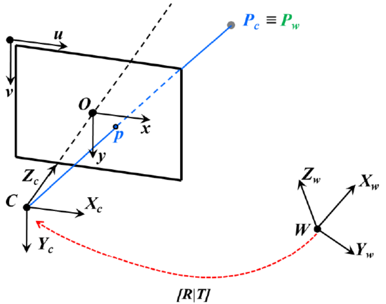
Inverse transformation introduced before

➢ Detailed Procedures (Step 1)

- ✓ For Stereo Vision, however, it is more common to use $\boldsymbol{R} \equiv \boldsymbol{R}_{wc}$ and $\boldsymbol{T} \equiv \boldsymbol{T}_{wc}$, where now $\boldsymbol{R}$, and $\boldsymbol{T}$ transform points from the **Camera frame** to the **World frame**.
- ✓ This is more convenient because $\boldsymbol{T}$=$\boldsymbol{C}$ directly represents the **coordinates** of the camera center **in the world frame** (see page 17/57 of Chapter02 Part1).
- ✓ The projection equation can be re written as:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K\left(R\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + T\right) \longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = KR^{-1}\left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - T\right)$$

$$\rightarrow \boxed{\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = KR^{-1}\left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C\right)}$$



Stereo Vision

➢ Detailed Procedures (Step 2)

$$\rightarrow \boxed{\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = KR^{-1}\left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C\right)}$$

We can now write the Perspective Equation for the **Left** and **Right** cameras, respectively. Here, we assume that Left and Right cameras have different intrinsic parameter matrices, K

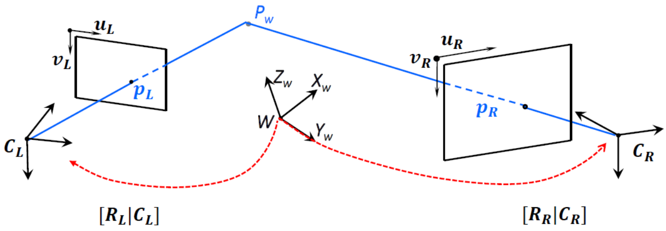
Left camera

$$\lambda_L \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix} = \boxed{K_L {R_L}^{-1}}\left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L\right)$$

If matrices $\{K_L, K_R\}$ are the same, and $\{R_L, R_R\}$ are the same, image planes are aligned.

Right camera

$$\lambda_R \begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix} = K_R {R_R}^{-1}\left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_R\right)$$



Stereo Vision

- ➤ Detailed Procedures (Step 3)

立体矫正的目标是对左右摄像机图像进行扭曲，使其图像平面对齐（通过引入相同的新旋转和新的内在参数）。

The goal of stereo rectification is to warp the left and right camera images such that their image planes are aligned (by introducing the same **new** rotation $\hat{\boldsymbol{R}}$ and **new** intrinsic parameters $\hat{\boldsymbol{K}}$).

If an image plane is transformed in 3D, a **2D projection's coordinates** are changed accordingly.

$$\lambda_L \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix} = K_L {R_L}^{-1} \left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L \right)$$

Old Left camera

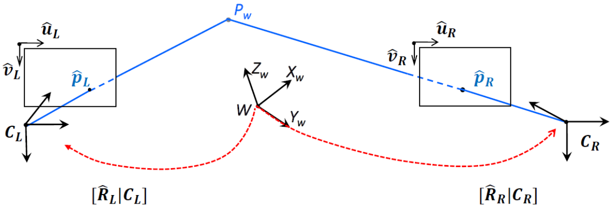
Relation expression in 2D?

$$\rightarrow \hat{\lambda}_L \begin{bmatrix} \hat{u}_L \\ \hat{v}_L \\ 1 \end{bmatrix} = \boxed{\hat{\boldsymbol{K}} \hat{\boldsymbol{R}}^{-1}} \left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L \right)$$

New Left camera

Computed image coordinates

Known matrices that are **manually assigned**.



Stereo Vision

$$\lambda_L \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix} = K_L {R_L}^{-1} \left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L \right)$$ **Old Left camera**

$$\rightarrow \hat{\lambda}_L \begin{bmatrix} \hat{u}_L \\ \hat{v}_L \\ 1 \end{bmatrix} = \widehat{\boldsymbol{K}} \widehat{\boldsymbol{R}}^{-1} \left(\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L \right)$$ **New Left camera**

- ➢ Detailed Procedures (Step 4)

✓ Coordinate change in 2D can be expressed by a 3*3 transformation.

✓ We first compute the transformation, and then use it to compute the warped coordinates:

$$\hat{\lambda}_L \begin{bmatrix} \hat{u}_L \\ \hat{v}_L \\ 1 \end{bmatrix} = \underbrace{\boxed{\lambda_L} \widehat{\boldsymbol{K}} \widehat{\boldsymbol{R}}^{-1} R_L {K_L}^{-1}}_{\text{Transformation of Left Camera}} \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix}$$

$$\hat{\lambda}_R \begin{bmatrix} \hat{u}_R \\ \hat{v}_R \\ 1 \end{bmatrix} = \underbrace{\lambda_R \widehat{\boldsymbol{K}} \widehat{\boldsymbol{R}}^{-1} R_R {K_R}^{-1}}_{\text{Transformation of Right Camera}} \begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix}$$

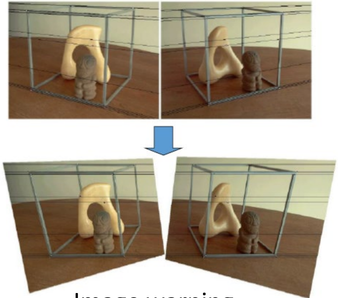
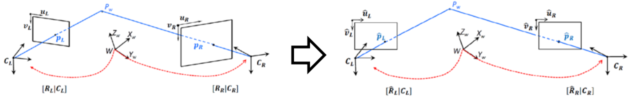


Image warping

Stereo Vision

➢ Intrinsic and Rotation Matrices



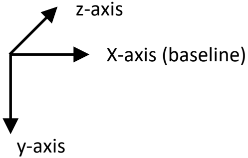
Origins of cameras remain unchanged

✓ How do we choose the new $\widehat{\boldsymbol{K}}$? A common choice is to take the arithmetic average of K_L and K_R

$$\boxed{\widehat{\boldsymbol{K}} = \frac{K_L + K_R}{2}}$$

✓ How do we choose the new $\widehat{\boldsymbol{R}} = [\widehat{r_1}, \widehat{r_2}, \widehat{r_3}]$ with $\widehat{r_1}, \widehat{r_2}, \widehat{r_3}$ being the column vectors of $\widehat{R}$?

A common choice is as follows:



$$\widehat{r_1} = \frac{C_R - C_L}{\|C_R - C_L\|}$$ This makes the new image planes parallel to the baseline

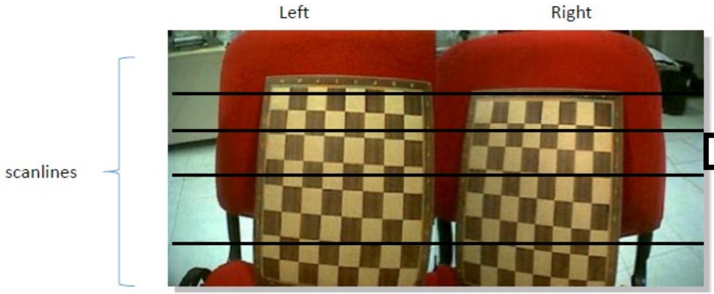
$$\widehat{r_2} = \underbrace{r_{3L}}_{\text{Old } r_3} \times \underbrace{\widehat{r_1}}_{\text{New } r_1}$$ where r_{3L} is the 3rd column of the rotation matrix of the left camera, i.e., R_L

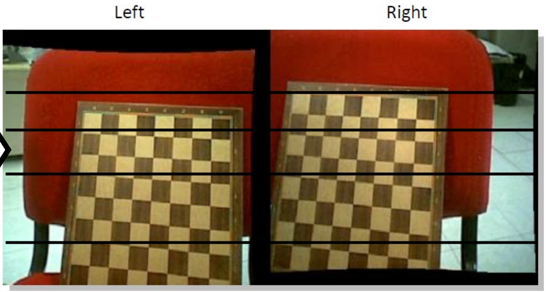
$$\widehat{r_3} = \widehat{r_1} \times \widehat{r_2}$$

Stereo Vision

- Example

✓ Preprocessing step: image undistortion (introduced before)





Input image pair

Compute lens distortion

[?] 然后，计算变换，以整顿/扭曲图像。

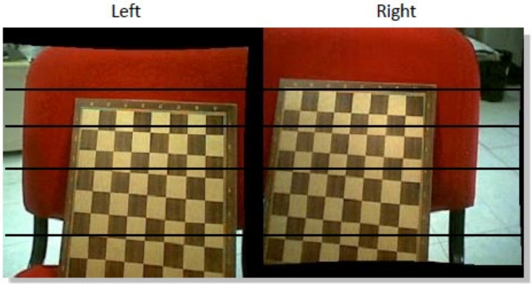
Stereo Vision

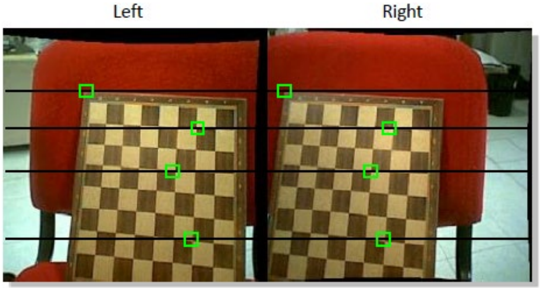
[?] 使用插值法来生成扭曲的图像。变换后的坐标为浮点数字，但像素坐标是典型的整数。(如果有必要，我将介绍如何在未来解决这个问题）。

➢ Example

✓ Then, compute transformation to rectify/warp images.

✓ Use **interpolation** to generate the warped image. The transformed coordinates are float numbers, but pixel coordinates are typical integer numbers. (if necessary, I will introduce how to solve this problem in the future).





Stereo Vision

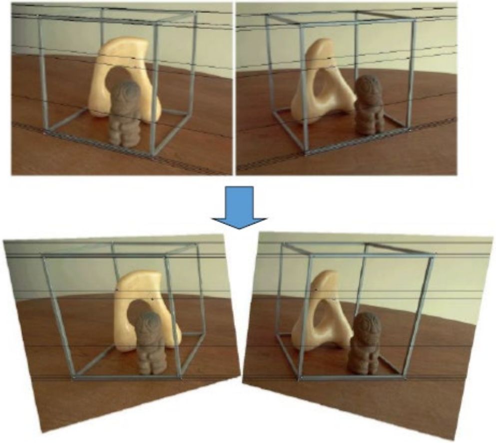
➢ Follow-up Task of 3D Reconstruction

✓ Result of stereo rectification: Corresponding epipolar lines are horizontal and collinear.

✓ We can conduct the 1D dense correspondence search.

[?] 立体整顿的结果：相应的表极线是水平的和相邻的。

[?] 我们可以进行一维密集的对应搜索。



Stereo Vision

➢ Follow-up Task of 3D Reconstruction

✓ From Rectified Image Pair to Disparity Map

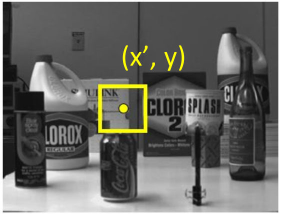
- For every pixel in the left image, find its corresponding point in the right image based on descriptor similarity (introduced in the next class).
- Compute the **disparity** for each found pair of correspondences, i.e., x’-x

[?] 从校正后的图像对到差异图

- 对于左边图像中的每个像素，根据描述符的相似性（在下一课中介绍）在右边图像中找到其对应点。

- 计算每一对找到的对应点的悬殊，即x'-x

Left image



Right image

Close objects experience bigger disparity
→ appear brighter in disparity map

Stereo Vision

➢ Follow-up Task of 3D Reconstruction

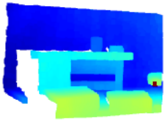
✓ From Disparity Map to 3D Point Cloud

- Once the disparity is obtained, the depth of each point can be computed recalling that:

$$\underbrace{Z_P}_{\text{Depth}} = \frac{\overbrace{b}^{\text{Baseline}}\overbrace{f}^{\text{Focal length}}}{\underbrace{u_l - u_r}_{\text{Disparity}}}$$

- From depth to 3D (introduced before)

$$\begin{cases} z = depth(i, j) \\ x = \dfrac{(j - c_x) \times z}{f_x} \\ y = \dfrac{(i - c_y) \times z}{f_y} \end{cases}$$



Summary

- Overview of 3D Reconstruction
- Triangulation (General Case)
- Stereo Vision (Simplified Case)

Thank you for your listening!

If you have any questions, please come to me :-)