Chair of Artificial Intelligence, Robotics, and Embedded Systems
Department of Informatics
Technical University of Munich

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- Cross your Registration number(with leading zero). It will be evaluated automatically.
- Sign in the corresponding signature field.

Robotics

Exam: IN2067 / Endterm
Examiner: Prof. Dr.-Ing. Darius Burschka
Date: Monday 13th February, 2023
Time: 08:00 – 09:30

Working instructions

- This exam consists of **12 pages** with a total of **3 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 128 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from __________ to __________ / Early submission at __________

Problem 1 Kinematics (42 credits)

0 1 2 3 4 5 6

a)* Write the rotation matrix ${}^1_2R$ (as defined in the lecture) between the coordinate frames from Figure 1.1. Write the general constraints on the elements of the rotation matrix.

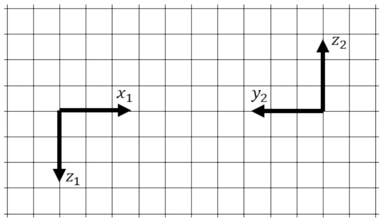


Figure 1.1: Coordinate frames {1} and {2}

1 ✓ per correct entries R_{01}, R_{10}, R_{22} of the rotation matrix. 1 ✓ for the written rotation matrix having determinant = 1, 1 ✓ for writing that rotation matrix is orthonormal (or that its determinant is 1) and 1 ✓ for writing that the transpose is its inverse.

$${}^1_2R = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

b)* Figure 1.2 shows a robot having its z_{EE} axis pointing outside the paper plane. Write the Denavit-Hartenberg table of the robot. Ensure maximal number of zeros in the table. Write the values for the rotational joint parameters as seen in the drawn configuration.

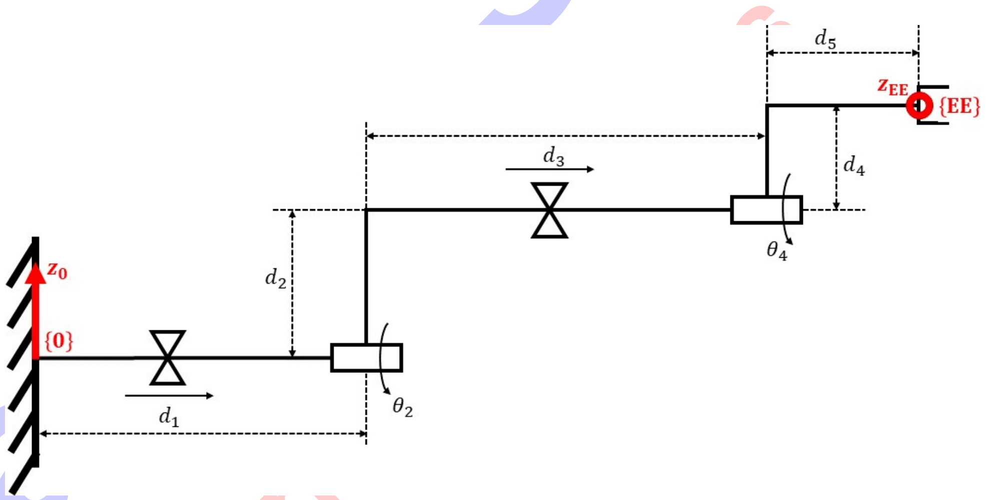


Figure 1.2: Schematic of a PRPR robot

1 ✓ per correct line (excl. value) of the DH table. 1 ✓ for writing a final 5th line of the table. 2 ✓ for the correct explicicly written values of the rotational parameters.

CF	α_{i-1}	a_{i-1}	d_i	θ_i	value
1	90°	0	$d_1 + d_3 + d_5$	0°	-
2	0°	0	0	θ_2	90°
3	0°	d_2	0	0°	-
4	0°	0	0	θ_4	0°
EE	−90°	d_4	0	0°	-

c) If the coordinate frame of the end-effector was not specified in the drawing, then you would have been able to select the orientation of the coordinate frame. Could you have chosen an orientation in which z_{EE} is parallel to z_0 at the same position of the origin of $\{EE\}$ as in the drawing? Justify your answer.

0 1 2 3 4 5

In the current drawing, the axis that is parallel to z_0 is x_{EE} $\checkmark$. To have z_{EE} in place of x_{EE}, a rotation around the y_{EE} axis must be performed $\checkmark$ $\checkmark$. However, this is not possible in the DH-convention $\checkmark$. Answer: at this position of the end-effector, z_{EE} can not be parallel to z_0 $\checkmark$.

d) Determine the position of the end effector relative to the origin. Show your work. What is the shape of the outer form of the robot's workspace?

0 1 2 3 4 5 6 7 8 9

$\checkmark$ for the correct x-component; $\checkmark$ for the correct y-component; $\checkmark$ for the correct z-component; 4 $\checkmark$ for explanation. 2 $\checkmark$ for the shape of the workspace which is a cylinder.

One can see that the only transformation happening along the y_0 axis is a translation with $-d_1 - d_3 - d_5$. The rotational joints only rotate in the x_0 and z_0 plane. From the planar robots investigated in the tutorials, we know the result of the position when the rotational axes are parallel.

$$^0p_{EE} = \begin{pmatrix} d_2c_2 + d_4c_{24} \\ -d_1 - d_3 - d_5 \\ d_2s_2 + d_4s_{24} \end{pmatrix}$$

The shape of the robot's workspace is a cylinder.

0 1 2 3 4 5

e) How many degrees of freedom does this robot have and what are its redundancies? How would you (mathematically, computationally or geometrically) check if a robot has redundancies?

The robot has three degrees of freedom ✓ because the two prismatic joints can be condensed into one joint ✓. Mathematically, this amounts to checking if the jacobian (or part of it) has full rank or not ✓. Geometrically, if there are two prismatic joints with the same ${}^0z_i$-axis direction, then the joints can be reduced ✓. Also, two rotational joints with origins lying on the same z-axis direction which also have their z-axes parallel can be condensed into one joint ✓.

0 1 2 3 4 5 6 7

f)* When deriving the Jacobian for a different robot with 4 joints, you arrive at the result

$$J = \begin{pmatrix} -c_1 + d_2 c_{13} & 0 & d_2 c_{13} & 0 \\ -s_1 + d_2 s_{13} & 0 & d_2 s_{13} & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & c_1 & 0 \\ 0 & 0 & s_1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

How many rotational joints does the robot have? And how many prismatic joints? Determine how many singular configurations this robot has when considering the position jacobian with respect to its first three joints.

Looking at the orientation part of the jacobian, we see that, out of the 4 columns, two are non-zero and two are zero ✓. Thus, there are 2 ✓ rotational joints and 2 ✓ prismatic joints.

The Jacobian that we investigate is $J = \begin{pmatrix} -c_1 + d_2 c_{13} & 0 & d_2 c_{13} \\ -s_1 + d_2 s_{13} & 0 & d_2 s_{13} \\ 0 & 1 & 0 \end{pmatrix}$. Its determinant is $d_2 s_3$ ✓ ✓, which is 0, when $\theta_3 = 0$ ✓. We conclude, there are an infinite number ✓ of singular configurations. For instance, all configurations $\begin{pmatrix} \theta_1 \\ d_2 \\ 0 \end{pmatrix}$ lead to a singularity.

Problem 2 Dynamics (41 credits)

a)* Convert the following joint torques equation to M-B-C-G form. Show your work and clearly mark M, B, C and G. Under which circumstances is it advantageous to use this form of the joint torques equation? 0 1 2 3 4 5 6 7

$$\tau = \begin{pmatrix} m_1(l_1^2 c_1^2 + 2) & c_1 & 0 \\ m_2(1+c_2) & m_2 l_2^2 & m_2+m_3 \\ (l_3+1/2)^2(m_1+m_2) & m_2 & m_3 l_3^2 \end{pmatrix} \begin{pmatrix} \ddot\theta_1 \\ \ddot\theta_2 \\ \ddot d_3 \end{pmatrix} + \begin{pmatrix} \dot\theta_1\dot\theta_2 m_1(l_1-2) - d_3 m_3 g + \dot\theta_1 \dot d_3 d_3 c_2 + m_3 g l_3 c_1 c_2 \\ -m_3 g l_3 s_1 c_2 + \dot\theta_2^2(2m_2+3) - \dot\theta_1(\dot d_3 + l_2 c_2 \dot\theta_1) \\ \dot d_3^2 - \ddot\theta_2 + \dot\theta_1\dot\theta_2 m_1 l_2 + m_2 g l_2 s_2 - \dot\theta_2^2 d_3 c_{12} \end{pmatrix}$$

✓ for correct M-B-C-G equation WITH θs expanded!

$$\tau = M(\theta)\begin{pmatrix} \ddot\theta_1 \\ \ddot\theta_2 \\ \ddot d_3 \end{pmatrix} + B(\theta)\begin{pmatrix} \dot\theta_1\dot\theta_2 \\ \dot\theta_1 \dot d_3 \\ \dot\theta_2 \dot d_3 \end{pmatrix} + C(\theta)\begin{pmatrix} \dot\theta_1^2 \\ \dot\theta_2^2 \\ \dot d_3^2 \end{pmatrix} + G(\theta)$$

$$M = \begin{pmatrix} m_1(l_1^2 c_1^2 + 2) & c_1 & 0 \\ m_2(1+c_2) & m_2 l_2^2 & m_2+m_3 \\ (l_3+1/2)^2(m_1+m_2) & m_2 - 1 & m_3 l_3^2 \end{pmatrix}, B = \begin{pmatrix} m_1(l_1-2) & d_3 c_2 & 0 \\ 0 & -1 & 0 \\ m_1 l_2 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 & 0 \\ -l_2 c_2 & 2m_2+3 & 0 \\ 0 & -d_3 c_{12} & 1 \end{pmatrix},$$

$$G = \begin{pmatrix} m_3 g(l_3 c_1 c_2 - d_3) \\ -m_3 g l_3 s_1 c_2 \\ m_2 g l_2 s_2 \end{pmatrix}$$

1 ✓ per completely correct term (M, B, C and G).

In the case where we lack computing power ✓ , we could further split the M-V-G equation in this M-B-C-G form, where the matrices only depend on the joint values θ, not also on the joint velocities $\dot\theta$ ✓ .

b)* "Because the potential energy depends on the position of each robot joint, the robot will have more potential energy at 500m altitude than at 0m altitude". Explain why this statement is or is not correct. Does it have an effect on the Lagrange analysis of robot dynamics? Explain your two answers. 0 1 2 3 4

First answer: The statement is correct, because the formula for potential energy is $mg^T p$, where g is the gravity vector and p the position of the body's center of mass. ✓ for correct formula, accept also one-dimensional variant of the formula.
Second answer: This does not have an effect on the Lagrange analysis. We compute the potential energy of the robot's joint relative to the global (static) coordinate frame 0, which has a constant potential energy ✓ . And because the potential energy is differentiated with respect to time in the Lagrange analysis ✓ , this constant term vanishes, thus having no influence on the joint torques expression ✓ .

c)* When determining the M-B-C-G equation for a robot's joint torques, you get a 28x28-dimensional B-matrix. How many joints does this robot have? 0 1 2 3 4

For an n-jointed robot, the formula for the size of the B-matrix is $\frac{n(n-1)}{2}$x$\frac{n(n-1)}{2}$ ✓ . Which means, we have to solve $n^2 - n - 56 = 0$ ✓ , leading to the solutions $x_1 = 8$ and $x_2 = -7$ ✓ . Because the number of joints of a robot has to be non-negative, the answer is $n = 8$ ✓ .

Figure 2.1 presents a robot to be analysed with the Lagrange method. The centers of mass for each link are located at $\frac{1}{3}$ and $\frac{1}{2}$ of the links' length respectively, with masses m_1 and m_2. The robot's DH-table is given.

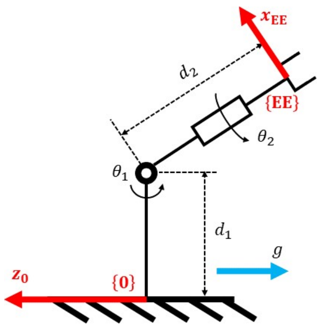


Figure 2.1: Schematic of an RR robot

CF	α_{i-1}	a_{i-1}	d_i	θ_i	value
1	90°	d_1	0	θ_1	30°
2	90°	0	0	θ_2	0°
EE	0°	0	d_2	0°	–

The inertial matrices are ${}^{c_1}I_1 = \begin{pmatrix} I_{1xx} & 0 & 0 \\ 0 & I_{1yy} & 0 \\ 0 & 0 & I_{1zz} \end{pmatrix}$

and ${}^{c_2}I_2 = \begin{pmatrix} I_{2xx} & 0 & 0 \\ 0 & I_{2yy} & 0 \\ 0 & 0 & I_{2zz} \end{pmatrix}$

0 1 2 3 4 5 6 7 8

d)* Write the values of all velocities that you need for the Lagrange analysis.

✓ per correct value of v_{c1}, v_{c2}, ${}^1\omega_1$, and ${}^2\omega_2$. 1 ✓ also for writing out the correct ${}^2_1R$ matrix. ✓ for both formulae of the velocity of the center of mass and for the iterative formula to compute the angular velocity. ✓ per correct writing of the coordinates of the link mass points.

$${}^0P_{c1} = \begin{pmatrix} \frac{d_1}{3} \\ 0 \\ 0 \end{pmatrix}, {}^0P_{c2} = \begin{pmatrix} d_1 + \frac{d_2 s_1}{2} \\ 0 \\ \frac{-d_2 c_1}{2} \end{pmatrix}$$

$$v_{c1} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, v_{c2} = \frac{1}{2}\begin{pmatrix} d_2 c_1 \dot{\theta}_1 \\ 0 \\ d_2 s_1 \dot{\theta}_1 \end{pmatrix}$$

$${}^1\omega_1 = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{pmatrix}, {}^2\omega_2 = \begin{pmatrix} \dot{\theta}_1 s_2 \\ \dot{\theta}_1 c_2 \\ \dot{\theta}_2 \end{pmatrix}, {}^2_1R = \begin{pmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{pmatrix}$$

$${}^0v_{ci} = \frac{d}{dt}{}^0P_{ci} \text{ and } {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \cdot {}^i\omega_i + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{pmatrix}$$

0 1 2 3 4 5 6 7

e) Compute the kinetic and potential energies for each link.

1 ✓ per correct k_1, k_2, u_1, and u_2. ✓ for formula of k_i and u_i. ✓ for writing explicitly the gravity vector G.

$$k_i = \frac{1}{2}\left(m_i(v_{ci}^T v_{ci}) + {}^i\omega_i^{T\,ci} I_i {}^i\omega_i\right)$$

$$u_i = -m_i G^{T\,0} P_{ci}$$

$$G = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$

$$k_1 = \frac{1}{2} I_{1zz}\dot{\theta}_1^2$$

$$k_2 = \frac{1}{2}\left(I_{2xx}\dot{\theta}_1^2 s_2^2 + I_{2yy}\dot{\theta}_1^2 c_2^2 + I_{2zz}\dot{\theta}_2^2 + \frac{m_2}{4} d_2^2 \dot{\theta}_1^2\right)$$

$$u_1 = 0$$

$$u_2 = -\frac{m_2}{2} g d_2 c_1$$

f) Compute the needed energy derivatives to complete the Lagrange analysis and write the joint torque equation.

0 1 2 3 4 5 6 7 8 9 10 11

1 ✓ per correct k and u terms. 1 ✓ for each correct derivative $\frac{\partial u}{\partial \theta_1}$, $\frac{\partial u}{\partial \theta_2}$, $\frac{\partial k}{\partial \theta_1}$, $\frac{\partial k}{\partial \theta_2}$, $\frac{d}{dt}\frac{\partial k}{\partial \theta_1}$, and $\frac{d}{dt}\frac{\partial k}{\partial \theta_2}$.
1 ✓ for the general joint torques formula, and 1 ✓ per correct row of the joint torque equation.

$k = I_{1zz}\dot{\theta}_1^2/2 + I_{2xx}\dot{\theta}_1^2 s_2^2/2 + I_{2yy}\dot{\theta}_1^2 c_2^2/2 + I_{2zz}\dot{\theta}_2^2/2 + d_2^2 m_2 \dot{\theta}_1^2/8$

$u = -d_2 g m_2 c_1/2$

$\frac{\partial u}{\partial \theta_1} = \frac{m_2}{2} g d_2 s_1$

$\frac{\partial u}{\partial \theta_2} = 0$

$\frac{\partial k}{\partial \theta_1} = 0$

$\frac{\partial k}{\partial \theta_2} = \dot{\theta}_1^2 s_2 c_2 (I_{2xx} - I_{2yy})$

$\frac{d}{dt}\frac{\partial k}{\partial \theta_1} = 2\dot{\theta}_2\dot{\theta}_1 s_2 c_2 (I_{2xx} - I_{2yy}) + \ddot{\theta}_1 (I_{1zz} + I_{2xx} s_2^2 + I_{2yy} c_2^2 + d_2^2 m_2/4)$

$\frac{d}{dt}\frac{\partial k}{\partial \theta_2} = I_{2zz}\ddot{\theta}_2$

$\tau_i = \frac{d}{dt}\frac{\partial k}{\partial \dot{\theta}_i} - \frac{\partial k}{\partial \theta_i} + \frac{\partial u}{\partial \theta_i}$

$$\tau = \begin{pmatrix} \frac{d_2 g m_2 s_1}{2} + \dot{\theta}_2 (2 I_{2xx}\dot{\theta}_1 s_2 c_2 - 2 I_{2yy}\dot{\theta}_1 s_2 c_2) + \ddot{\theta}_1 (I_{1zz} + I_{2xx} s_2^2 + I_{2yy} c_2^2 + \frac{d_2^2 m_2}{4}) \\ -I_{2xx}\dot{\theta}_1^{\ 2} s_2 c_2 + I_{2yy}\dot{\theta}_1^2 s_2 c_2 + I_{2zz}\ddot{\theta}_2 \end{pmatrix}$$

Problem 3 Control (45 credits)

A car, m_1, actuated by a force $\vec{f}$ is pulling a trailer, m_2, connected to it by a spring with the spring constant $k = 4$. The spring force is $f_s = k \cdot \Delta x = k \cdot (x_2 - x_1)$. The velocity dependent damping force can be calculated by $f_d = b \cdot \dot{x}_i$, where $b = 4$ and x_i is the position of the car or trailer $i = \{1, 2\}$ (Fig. 3.1). For simplicity, we assume that $x_1 = 0, x_2 = 0$ if no force is acting on the spring (even if the 0 positions of x_i do not coincide). The masses of the car and the trailer are $m_i = 4$. The resonance frequency of the trailer is $\omega_{res} = 1$.

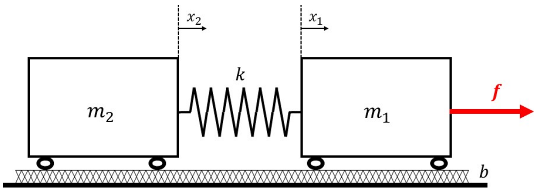
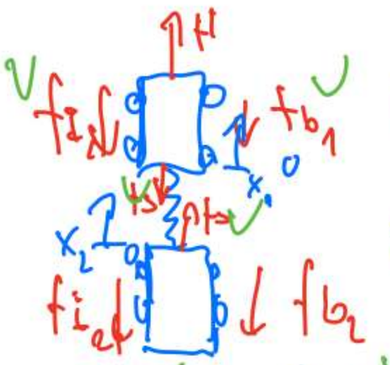


Figure 3.1: SMD system

0 1 2 3 4 5 6 7 8 9 10 11 12

a)* Draw the forces acting on the trailer and the car after "cutting" (seperating) the car from trailer (including $\vec{f}$) and write the two equation balancing all forces on these two systems. Re-write them to a form, where all internal force parameters (i.e. the elements dependent on the x_i parameter for a given system) are on the left and other forces on the right of the system.



$m_1\ddot{x}_1 + b\dot{x}_1 + k(x_1 - x_2) = f$

$m_2\ddot{x}_2 + b\dot{x}_2 = k(x_1 - x_2)$

$b = 4 \quad k = 4$
$m = 4$

$m_1\ddot{x}_1 + b\dot{x}_1 + k_1 x_1 = f + kx_2$

$m_2\ddot{x}_2 + b\dot{x}_2 + kx_2 = kx_1$

0 1 2 3

b) Check the left side of the above equation for the trailer for its response to disturbances using the characteristic equation from the lecture. Which type of the response will you see?

$s_{1/2} = -\frac{b}{2m} \pm \frac{\sqrt{b^2 - 4mk}}{24} = -\frac{1}{2} \pm i\frac{\sqrt{48}}{8}$

oscillation

$= -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

c)* Assume that the car and trailer are driving with a constant speed $\dot{x}_1 = \dot{x}_2 = 3$. The force in the spring is used to compensate the damping force in the trailer. Estimate the stretch of the spring $(x_2 - x_1)$. Give the form and parameters of the solution for $x_2(t)$ for the case that the front car halts instantaneously at t=0 and remains static. Assume an oscillating solution here. What is the trailer's natural and oscillation frequency?

0 1 2 3 4 5 6 7 8 9 10 11

$k(x_2 - x_1) = b\dot{x}_2$ ✓ ✓

$\rightarrow \Delta x = \frac{4 \cdot 3}{4} = 3$ ✓

$x_2 = A \cdot \cos(\omega t - \varphi)$ ✓

$x_2(0) = \Delta x \quad \dot{x}_2(0) = 3$ ✓

$\dot{x}_2 = -A\omega \sin(\omega t - \varphi)$ ✓

$3 = A \cdot \cos(-\varphi) \quad (x)$ ✓

$3 = A/4 \sin(-\varphi) \quad (\dot{x})$ ✓

$\tan(-\varphi) = 4 \Rightarrow A \text{ from } (x)$ ✓ ✓

$\omega_n = \sqrt{\frac{k}{m}} = 1$ ✓ $\quad \omega = \frac{\sqrt{3}}{2}$

d)* Which simple control law (no control law partitioning) needs to be used to calculate the value of f_2 to get the ideal response of the trailer without oscillations? Calculate the values of the control law given the resonance frequency of the trailer above.

0 1 2 3 4 5 6 7 8 9

$f = -k_v \dot{x}_2 - k_p x_2 =$ ✓ ✓

$m\ddot{x}_2 + b\dot{x}_2 + kx_2$

$b' = (b + k_v)$ ✓ $\quad k' = (k + k_p)$ ✓

$\omega_n \neq \frac{1}{2}\omega_{res} = \frac{1}{2} = \sqrt{\frac{k'}{m}}$ ✓

$\rightarrow k' = 1 \rightarrow k_p = -3$ ✓ ✓

$b' - 4mk' = 0 \rightarrow b' = 4 \cdot 4 = 16$ ✓

$k_v = 12$ ✓

0 1 2 3 4 5 6 7 8 9 10

e)* Explain briefly giving the corresponding equation the control law partitioning and draw the structure of the corresponding controller for a simple spring-mass-damper (SMD) system. Which parameters need to be adjusted for the asymptotic solution of the motion equation and how are the actual parameter values calculated.

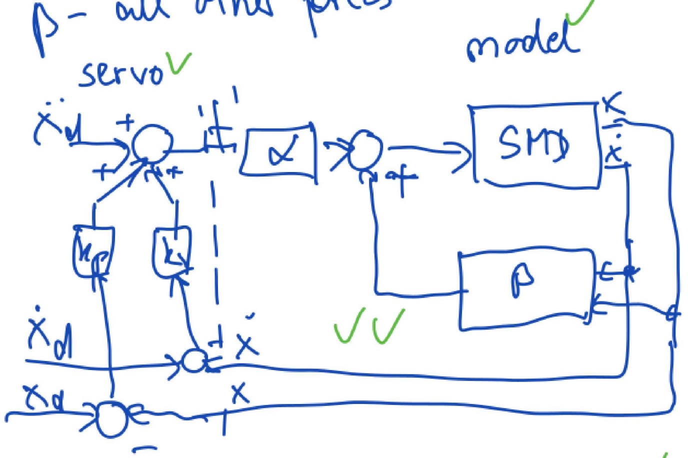
control law eq 2 explanation inertia in alpha (1) and (other forces in beta (1) making ideal unit mass (1) drawing (model and servo part) (1) structure model (content of beta (1) and content alpha (1)) kv=sqrt kp (1) kp=sqrt (1/2 omegar-res) (1)

$$f = \alpha f' + \beta$$

inertie

$f' \rightarrow$ control force for free floating unit mass

β - all other forces



$$k_v = 2\sqrt{k_p} \qquad \frac{\omega_{res}}{2} = \sqrt{k_p}$$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.