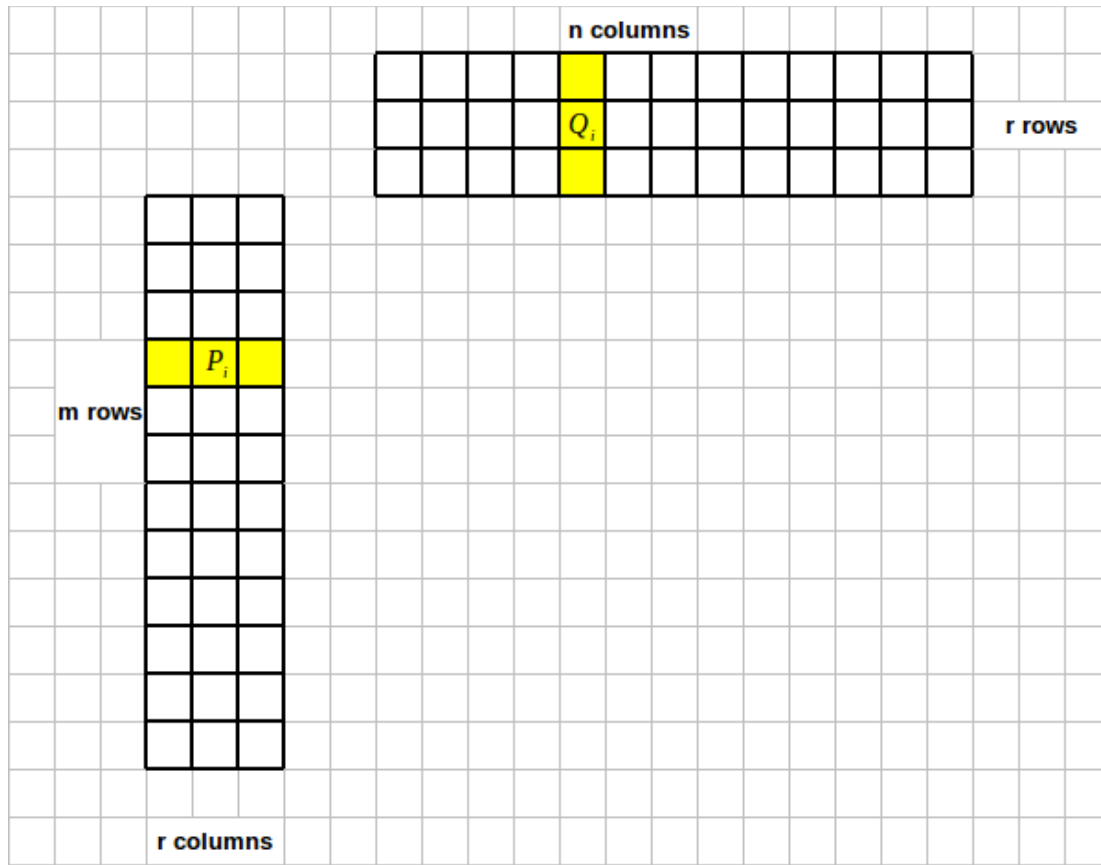


z5049624 Sida Zhang

The diagram illustrates a grid structure with 10 columns and 10 rows. The horizontal axis is labeled **items_n** and the vertical axis is labeled **users_n**. A red line is drawn across the grid, separating the top 5 rows from the bottom 5 rows.

it is expected that the matrix can be decomposed into two low-rank matrices that $P_i \cdot Q_j$ corresponds to user i 's rating on item j in the original matrix above, and r can be seen as the number of factors of user and item.



However, r cannot be guaranteed to achieve for both P and Q . As preferences of users must correspond to factors of items, the production of P and Q in such a manner can be an approximation to the user-item matrix, and we expect the error is minimized. If we use the traditional SVD, the matrix will be decomposed into three ones and this process will be computational expensive. More importantly, it does not suit the sparse matrix in our case.

Simon Funk introduced a method to break these barriers, that minimizing the cost function to achieve the decomposition rather than the inverse way. And it is easy to implement in computer program.

$$\operatorname{argmin}_{P_i, Q_j} \sum_{i,j} (m_{i,j} - P_i \cdot Q_j)^2,$$

P_i is a row vector, Q_j is a column vector, $m_{i,j}$ is rating of item j given by user i

In practice, we apply regularization to avoid overfitting.

$$J = \sum_{i,j} (m_{i,j} - P_i \cdot Q_j)^2 + \lambda (\|p_i\|_2^2 + \|q_j\|_2^2)$$

Take derivatives to obtain gradients:

$$\frac{\partial J}{\partial P_i} = -2(m_{ij} - P_i \cdot Q_j)Q_j + 2\lambda P_i,$$

$$\frac{\partial J}{\partial Q_j} = -2(m_{ij} - P_i \cdot Q_j)P_i + 2\lambda Q_j$$

Then the updating rules are:

$$P_i = P_i + \alpha((m_{ij} - P_i \cdot Q_j)Q_j - \lambda P_i),$$

$$Q_j = Q_j + \alpha((m_{ij} - P_i \cdot Q_j)P_i - \lambda Q_j),$$

α is learning rate

The above illustrates the Funk SVD algorithm. An improvement to it is adding a biased item $\mu + b_i + b_u$ where

b_u is the user bias, which measures the user's tendency to rate the items and b_i is the item bias, which measures the tendency of the item's ratings in the dataset, μ is average rating of all elements in the user-item matrix. Then the cost function accordingly becomes

$$J = \sum_{i,j} (m_{ij} - \mu - b_i - b_u - P_i \cdot Q_j)^2 + \lambda (\|P_i\|_2^2 + \|Q_j\|_2^2 + \|b_i\|_2^2 + \|b_j\|_2^2)$$

and the corresponding updating rules are

$$P_i = P_i + \alpha ((m_{ij} - \mu - b_i - b_u - P_i \cdot Q_j) Q_j - \lambda P_i),$$

$$Q_j = Q_j + \alpha ((m_{ij} - \mu - b_i - b_u - P_i \cdot Q_j) P_i - \lambda Q_j),$$

$$b_i = b_i + \alpha ((m_{ij} - \mu - b_i - b_u - P_i \cdot Q_j) - \lambda b_i),$$

$$b_u = b_u + \alpha ((m_{ij} - \mu - b_i - b_u - P_i \cdot Q_j) - \lambda b_u)$$

To predict a new item's rating, simply recover it from the trained model P,Q.

$$m_{ij} = \mu + b_i + b_u + P_i \cdot Q_j$$

3. Experiment

RMSE is used to indicate the number of iterations to stop the gradient descent in the experiment. As each iteration of the gradient descent takes around 15s which is fairly long for the whole process, a limit to the number of iterations (**MaxIter**), instead of the error change, is set to **20** according to average performance on all the 5 CV sets. Besides, to ensure the accuracy of converge, the learning rate will decline 10% after each iteration (**learningRate *= 0.9**). And the **number of latent factors**, which is the number of columns in P (and the row number in Q), is set to **10** which has the best performance in a range from 5 to 50. The last important parameter is **L2 penalty coefficient**, which gives the best results when it is **0.05**. (graphs to derive these values are in appendix)

4. Result

4.1 Running result

The recommender system can receive a user's id as input and predict the ratings that this user would give to not-yet-rated movies. And it is noticeable that the predicted ratings are float numbers, ranging from 0.0 to 5.0. Then, in such a manner, recommended movie list can be given, including movie titles and corresponding sorted ratings. Returning top-k recommendations is also supported.

4.2 Performance evaluation

Accuracy of the prediction is assessed using RMSE and MAE which are defined as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (predict_i - groundTruth_i)^2}$$

$$MAE = \frac{1}{n} \sum_{i=1}^n |predict_i - groundTruth_i|$$

For the 5-fold cross validation of the Movielens 100K dataset, the best result achieved in the experiment is shown as follow:

	u1	u2	u3	u4	u5	avg
RMSE	0.919698253396208	0.9132460483363246	0.9063472176375376	0.908711352078286	0.9095967927705968	0.9115318039252122
MAE	0.7204997928764971	0.7141578579506594	0.7103402739888629	0.7115254135143657	0.718575437861617	0.7150197552384004

This performance is close to mymedialite's benchmark (BiasedMatrixFactorization), and further works on adjusting Max Iteration, regularization coefficient, and number of factors could give a better result since they were optimized individually while the combination of them determines the performance.

MovieLens 100k

5-fold crossvalidation with --random-seed=1

Method	--recommender-options	RMSE	MAE
BipolarSlopeOne		0.96754	0.74462
UserItemBaseline	reg_u=5 reg_i=2	0.94192	0.74503
SlopeOne		0.93978	0.74038
UserKNNCosine	k=40 reg_u=12 reg_i=1	0.937	0.737
UserKNNPearson	k=60 shrinkage=25 reg_u=12 reg_i=1	0.92971	0.72805
ItemKNNCosine	k=40 reg_u=12 reg_i=1	0.924	0.727
FactorWiseMatrixFactorization	num_factors=5 num_iter=5 shrinkage=150	0.9212	0.7252
BiasedMatrixFactorization	num_factors=5 bias_reg=0.1 reg_u=0.1 reg_i=0.1 learn_rate=0.07 num_iter=100 bold_driver=true	0.91678	0.72289
BiasedMatrixFactorization	num_factors=10 bias_reg=0.1 reg_u=0.1 reg_i=0.12 learn_rate=0.07 num_iter=100 bold_driver=true	0.91496	0.72209
SVDPlusPlus	num_factors=4 regularization=0.1 bias_reg=0.005 learn_rate=0.01 bias_learn_rate=0.007 num_iter=50	0.9138	0.71836
ItemKNNPearson	k=40 shrinkage=2500 reg_u=12 reg_i=1	0.91327	0.7144
BiasedMatrixFactorization	num_factors=40 bias_reg=0.1 reg_u=1.0 reg_i=1.2 learn_rate=0.07 num_iter=100 frequency_regularization=true bold_driver=true	0.90764	0.71722
BiasedMatrixFactorization	num_factors=80 bias_reg=0.003 reg_u=0.09 reg_i=0.1 learn_rate=0.07 num_iter=100 bold_driver=true	0.91153	0.72013
SVDPlusPlus	num_factors=10 regularization=0.1 bias_reg=0.005 learn_rate=0.01 bias_learn_rate=0.007 num_iter=50	0.91096	0.7152
BiasedMatrixFactorization	num_factors=320 bias_reg=0.007 reg_u=0.1 reg_i=0.1 learn_rate=0.07 num_iter=500 bold_driver=true	0.91073	0.72053
BiasedMatrixFactorization	num_factors=160 bias_reg=0.003 reg_u=0.08 reg_i=0.1 learn_rate=0.07 num_iter=100 bold_driver=true	0.91047	0.71944

(<http://www.mymedialite.net/examples/datasets.html>)

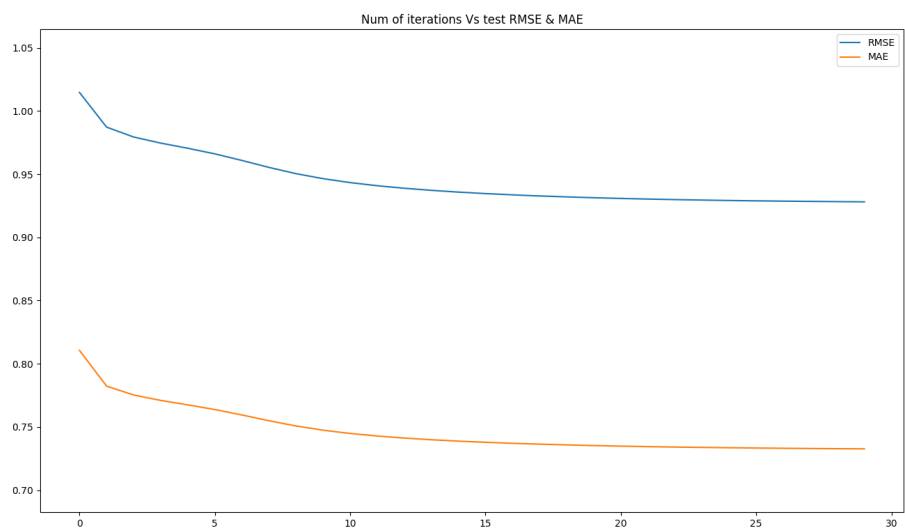
Name	Call Count	Time (ms)
SVD	1	46453 98.8%

(It takes 46s to train 80K records - e.g. u1.base)

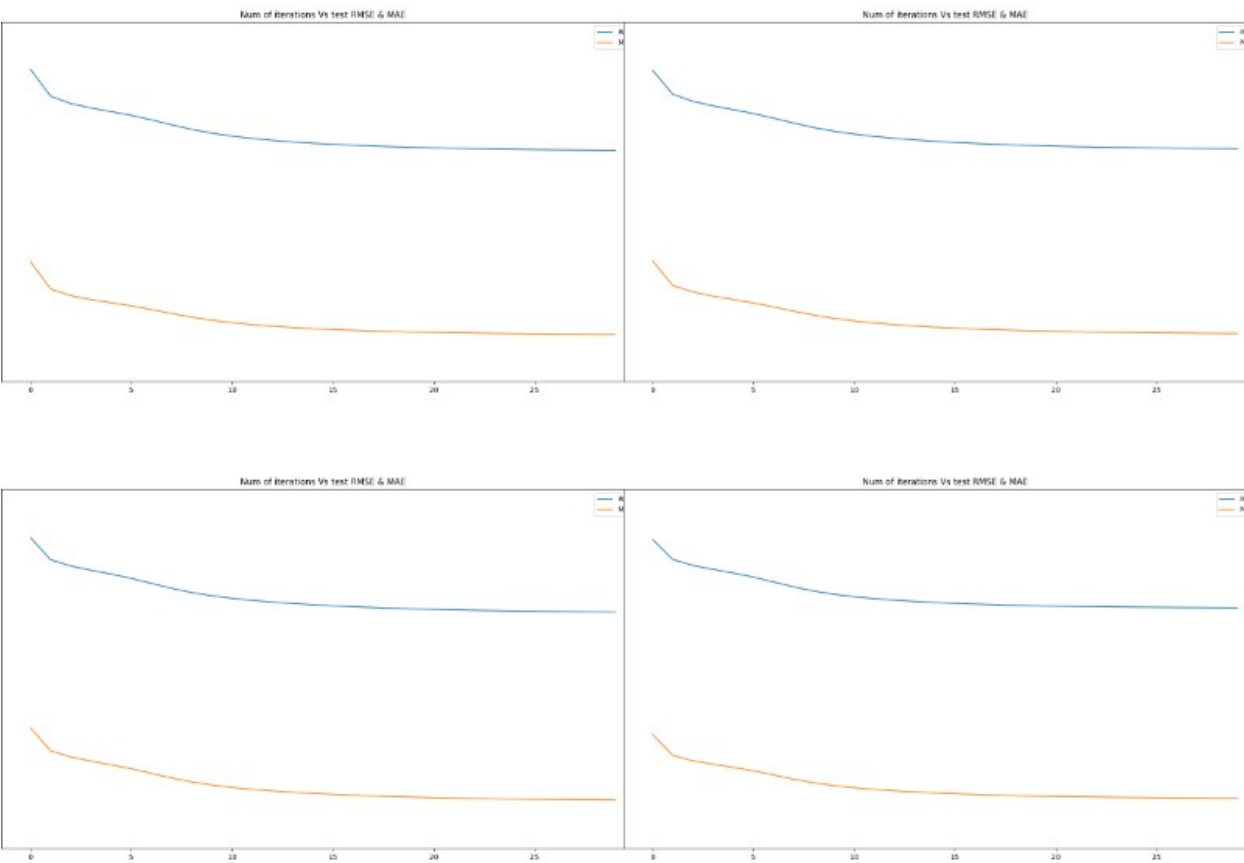
Predict	1	7046 91.4%
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(and 7s to predict - e.g. u1.test)

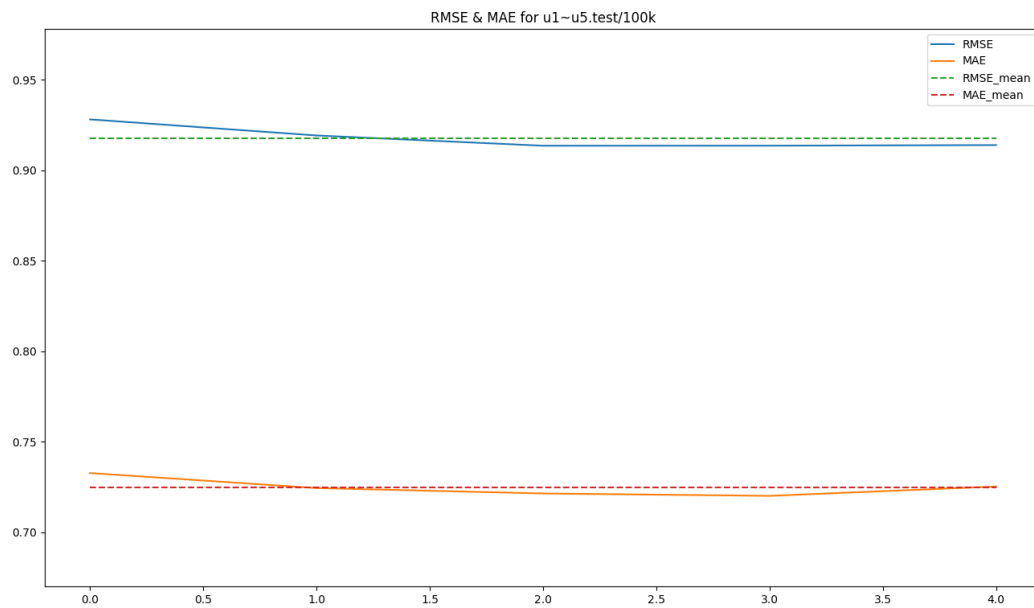
5. Appendix



(RMSE and MAE curves on u1.test with MaxIteration number from 1 to 29)



(Similar shapes on u2 ~ u5)



(RMSE and MAE on the 5 sets)

#Factors	RMSE	MAE
5	0.921054430731856	0.7276799351051431
6	0.9208890317628455	0.7274626719963752
7	0.9208870421543842	0.7275392068563215
8	0.9213644621154286	0.7278965132362223
9	0.9208666267703857	0.7275139317775654
10	0.9201111776456024	0.7269030420597229 ***
11	0.9205978624490095	0.727300737763004
12	0.9209702992430115	0.7276337914072017
13	0.9206935639687901	0.727340143186229
14	0.9207304799858461	0.727438374745595
15	0.9208728824591506	0.7275020525394659
16	0.9207296513773902	0.727428080365217
17	0.9205168451595235	0.7272292464484525
18	0.9205900982454656	0.7273032352573309
19	0.9205646991893468	0.727305973901702

(NumOfFactors=10, full list in 'FactorNumSelect.txt')

```

0.0
avgRMSE0.928577308084541
avgMAE0.7236033978471089
0.05 ***|
avgRMSE0.9110984802592551
avgMAE0.7145087848253888
0.1
avgRMSE0.9206017326657466
avgMAE0.7273212935628238
0.15
avgRMSE0.9351722299946081
avgMAE0.7418532755796182
0.2
avgRMSE0.9458032751748864
avgMAE0.7523944064480427
0.25
avgRMSE0.9482226401082048
avgMAE0.7559280778014481
0.3
avgRMSE0.9504830328266222
avgMAE0.759227749126499
0.35
avgRMSE0.9528895250605782
avgMAE0.7625441701280337
0.4
avgRMSE0.9554058876123751
avgMAE0.7658698310421078
0.45

```

(regularization coefficient = 0.05, full list in 'PenaltySelect.txt')

```

Configuring ...
Skip Conf ?(y\Enter)

Path to training data ?
ul.base
Path to testing data ?
ul.test
Path to save MODEL ? 'Enter' to ignore.

Check or Modify parameters for training? (y/n)
n
Prepare for training.
Skip Training ?(y\Enter)

Initializing...
Initialization completed.
Show Error of every loop? (y/n)

Training...
Total RMSE : 0.919407
Total MAE : 0.721160
=====
Skip Predicting ?(y\Enter)

Predicting ...
Predicting completed.
Results saved to DB.

Skip Recommending ?(y\Enter)

== Recommendation List ==
For which User? (id) 'Enter' to show All.
13
How many rows to view? From the top rating. "Enter" to show All.
13
sql? "Enter" to skip.
=====
=== Recommend to User 13 ===
RATING      TITLE
5.0 Fargo (1996)
5.0 Princess Bride, The (1987)
5.0 North by Northwest (1959)
4.96041 Silence of the Lambs, The (1991)
4.91276 Fish Called Wanda, A (1988)
4.74168 Monty Python and the Holy Grail (1974)
4.7329 Lone Star (1996)
4.72614 Arsenic and Old Lace (1944)
4.70033 Young Frankenstein (1974)
4.6905 Casablanca (1942)
=====
Configuring

```

(Interaction example)

References

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3. Lévy, P., 1997. *Collective intelligence*. New York: Plenum/Harper Collins.
4. Simon, F., 2006. Netflix Update: Try This at Home