Assignment 8: Cluster Analysis

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Goals & Data Examination

We will be using principal components analysis to reduce dimensionality of a data set as a preprocessor for cluster analysis. Our data set is employment reporting for various industry segments, as a percentage measurement, for thirty European nations.

The Variables in our data set are:

Variable	Type	Length	Format	Informat
AGR	Num	8	8.1	F10.1
CON	Num	8	8.1	F10.1
COUNTRY	Char	20	35.	-
FIN	Num	8	8.1	F10.1
GROUP	Char	8	10.	-
MAN	Num	8	8.1	F10.1
MIN	Num	8	8.1	F10.1
PS	Num	8	8.1	F10.1
SER	Num	8	8.1	F10.1
SPS	Num	8	8.1	F10.1
TC	Num	8	8.1	F10.1

Table 1: Alphabetic List of Variables and Attributes

Variable	Industrial Sector
AGR	Agriculture
MIN	Mining
MAN	Manufacturing
PS	Power and Water Supply
CON	Construction
SER	Services
FIN	Finance
SPS	Social and Personal Services
TC	Transport and Communications

Table 2: Variable and Industrial Sector

We observe that this data set has a variable (group) that provides subdivision into classes. Examining the contents of group tell us that the subdivision appears to be by trade bloc. Trade bloc is a type of intergovernmental agreement where regional barriers to trade are reduced or eliminated amongst the participating nation-states. The tutorial requires us to examine the data set in an unsupervised fashion. This group classification would provide us a basis to perform our exploratory data analysis in a supervised fashion. We'd imagine that if groupings we're known for a data set, it would be typical to begin exploratory data analysis from a supervised perspective.

We observe that four countries (Cyprus, Gibralta, Matla, and Turkey) are all within the ‘Other’ group. If we were to assume that the group assignment was purely a basis of local, then we’d consider re-naming the ‘Other’ category to something more contextually appropriate, such as ‘Mediterranean’. This however can be misleading as several nation-states from the EU would be considered to be in the Mediterranean region.

Correlation and Visualization

We’ll begin by examining the simple Pearson correlation for the variables within our data set, this produces the scatter-plot matrix:

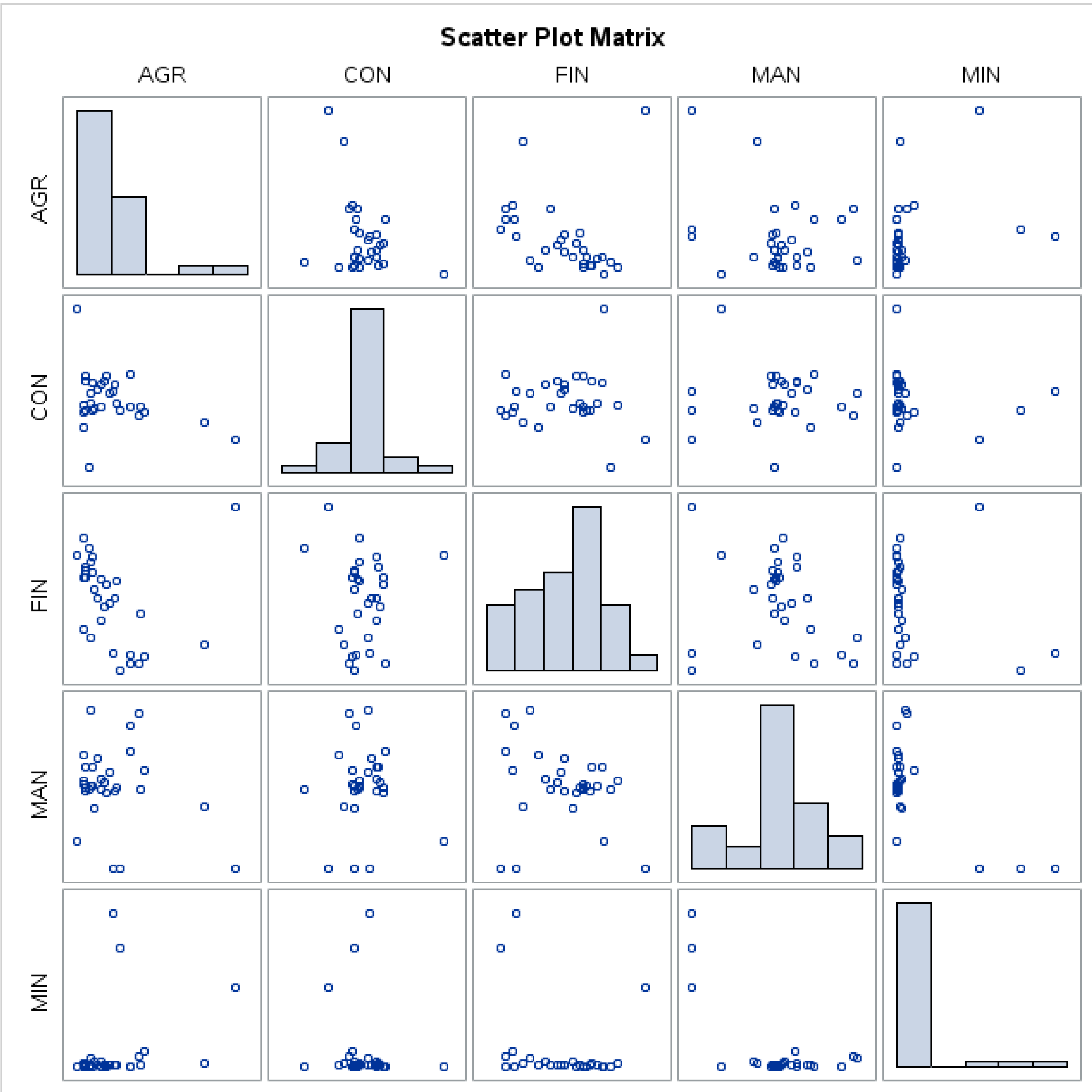


Figure 1: Pearson Correlation Scatter-Plot Matrix

This graphic doesn’t fully encompass the correlation matrix. The strongest correlation, with a statistically significant test, was between AGR and SPS. The correlation being 0.81148 with a probability $> |r|$ under $H_0 : \rho = 0$ test statistic of < 0.0001. Well continue to examine this correlation by producing a scatter-plot:

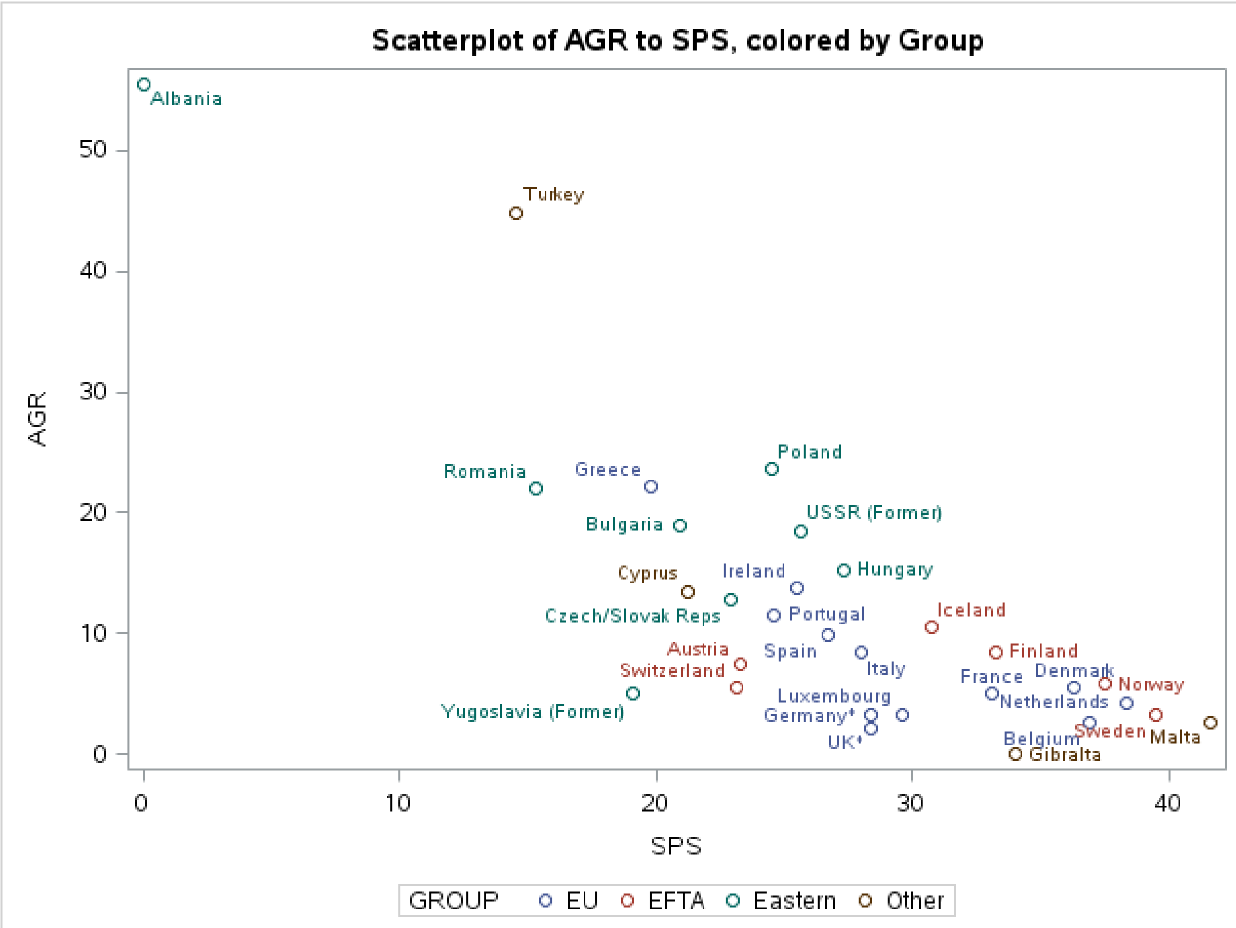


Figure 2: Scatterplot of AGR to SPS, colored by Group

Principal Components, Dimensionality Reduction

As we have nine variables within the data set, we'll use PCA as a dimensionality reduction method. We'll use the variability table, or scree plot to examine how many components we require to account for 90% of the data variability.

Observation	Eigenvalue	Difference	Proportion	Cumulative
1	3.11225795	1.30302071	0.3458	0.3458
2	1.80923724	0.31301704	0.2010	0.5468
3	1.49622020	0.43277636	0.1662	0.7131
4	1.06344384	0.35318631	0.1182	0.8312
5	0.71025753	0.39891874	0.0789	0.9102
6	0.31133879	0.01791787	0.0346	0.9448
7	0.29342091	0.08960446	0.0326	0.9774
8	0.20381645	0.20380935	0.0226	1.0000
9	0.00000710	0.0000	1.0000	-

Table 3: Eigenvalues of the Correlation Matrix

	Prin1	Prin2	Prin3	Prin4	Prin5	Prin6	Prin7	Prin8	Prin9
AGR	-.511492	0.023475	-.278591	0.016492	-.024038	0.042397	-.163574	0.540409	0.582036
MIN	-.374983	-.000491	0.515052	0.113606	0.346313	-.198574	0.212590	-.448592	0.418818
MAN	0.246161	-.431752	-.502056	0.058270	-.233622	0.030917	0.236015	-.431757	0.447086
PS	0.316120	-.109144	-.293695	0.023245	0.854448	-.206471	-.060565	0.155122	0.030251
CON	0.221599	0.242471	0.071531	0.782666	0.062151	0.502636	-.020285	0.030823	0.128656
SER	0.381536	0.408256	0.065149	0.169038	-.266673	-.672694	0.174839	0.201753	0.245021
FIN	0.131088	0.552939	-.095654	-.489218	0.131288	0.405935	0.457645	-.027264	0.190758
SPS	0.428162	-.054706	0.360159	-.317243	-.045718	0.158453	-.621330	-.041476	0.410315
TC	0.205071	-.516650	0.412996	-.042063	-.022901	0.141898	0.492145	0.502124	0.060743

Table 4: Eigenvectors

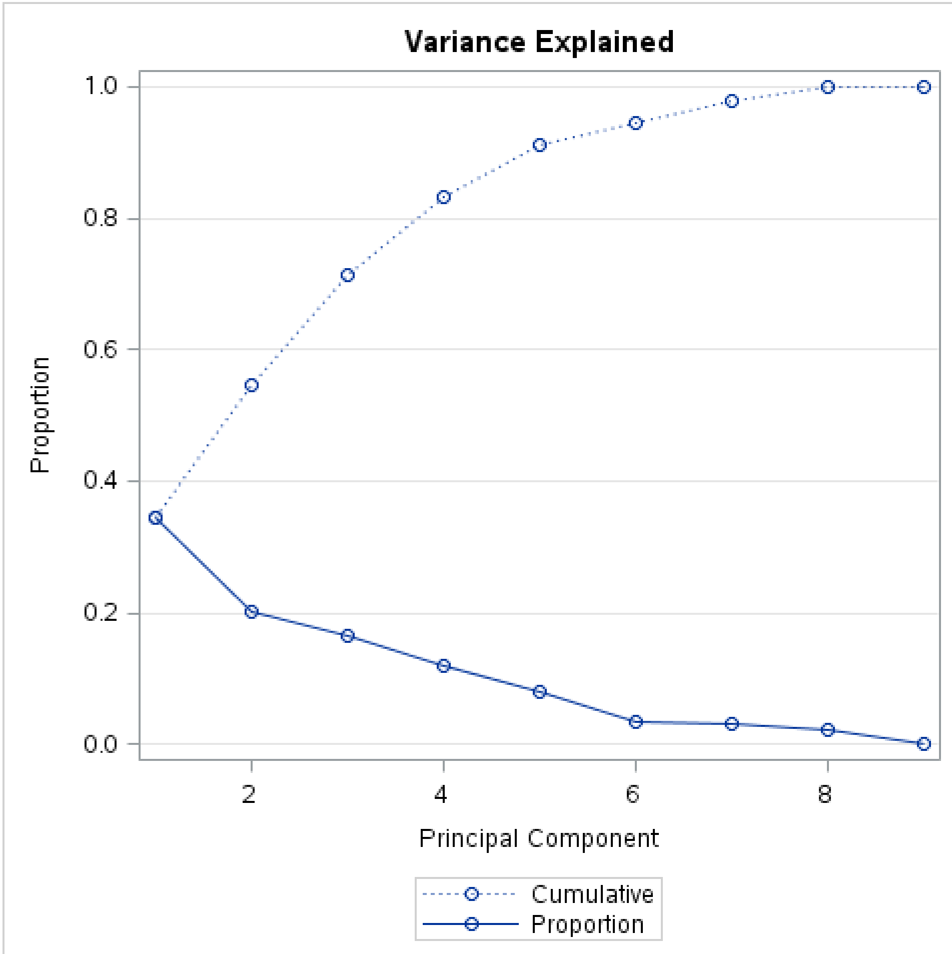
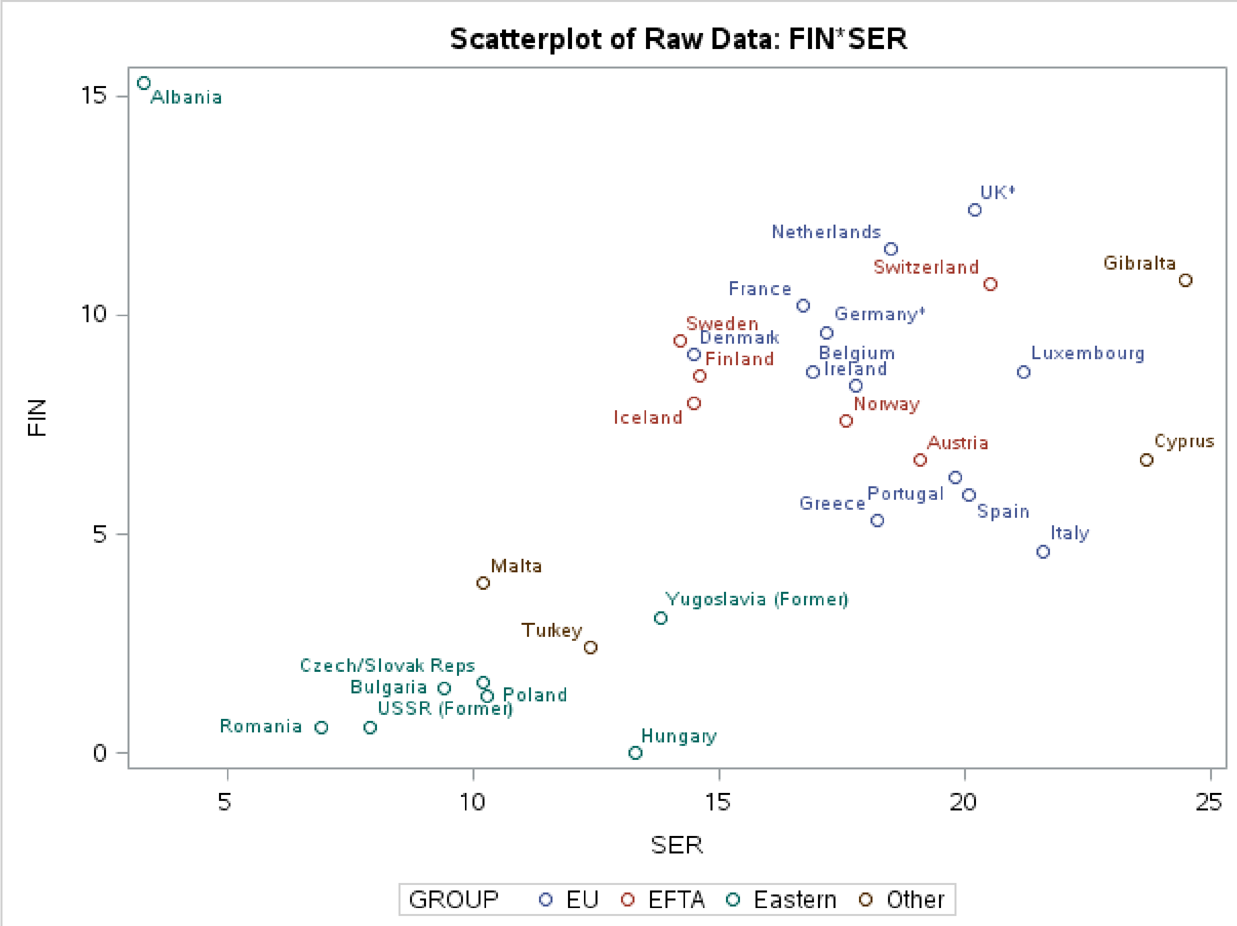


Figure 3: Scree Cumulative Variability

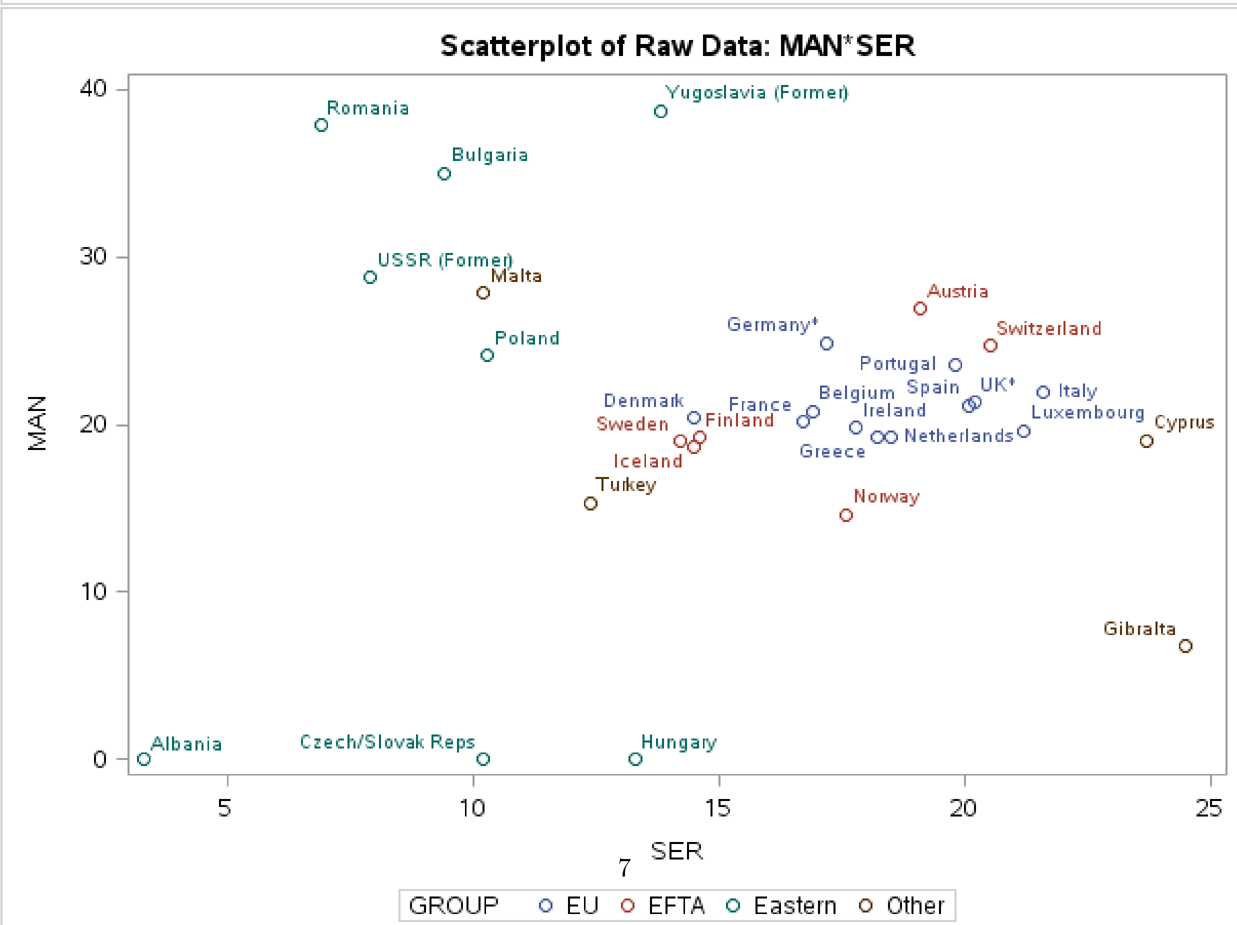
From the diagnostic output of the PCA procedure we see that we'll have to use the first five principal components to explain greater than 90% of the variability in our data. We reluctantly accept this as we made the initial decision, prior to examining output, to take forward 90% of the explained variability. It would be more ideal from a ease-of-modeling perspective to take forward fewer principal components, and in turn less of the explained variability.

Cluster Analysis

We'll begin by making some of our own scatter plots, selecting the FIN and SER, as well as the MAN and SER variables:



Scatterplot of Raw Data: FIN*SER
FIN
SER
0
5
10
15
5
10
15
20
25
Albania
UK*
Netherlands
Switzerland
Gibralta
France
Germany*
Sweden
Denmark
Finland
Belgium
Ireland
Luxembourg
Iceland
Norway
Austria
Cyprus
Greece
Portugal
Spain
Italy
Malta
Yugoslavia (Former)
Turkey
Czech/Slovak Reps
Bulgaria
Poland
USSR (Former)
Romania
Hungary
GROUP EU EFTA Eastern Other



Scatterplot of Raw Data: MAN*SER
MAN
SER
0
10
20
30
40
5
10
15
20
25
Romania
Yugoslavia (Former)
Bulgaria
USSR (Former)
Malta
Poland
Austria
Germany*
Switzerland
Portugal
Denmark
France
Belgium
Spain
UK*
Italy
Sweden
Finland
Ireland
Luxembourg
Cyprus
Netherlands
Greece
Iceland
Turkey
Norway
Gibralta
Albania
Czech/Slovak Reps
Hungary
GROUP EU EFTA Eastern Other

With the naked eye we observer two clusters within each of the graphs. There are some outlier countriers, but in both graphs the Eastern group seems to cluster, where as the EU and EFTA groups seem to cluster. Albania and Gibralta both seem to be outliers in their own right. We will use the cluster procedure within SAS to automatically create clusters with a hierarchical approach. As this is a hierarchal approach we, as the analyst, do not have to specify the amount of desired clusters. Instead we can examine diagnostic output for different criteria and make our decision. There are no completely satisfactory methods that can be used for determining the number of population clusters for any type of cluster analysis [1]. We'll examine the diagnostic output of the cluster procedure, and look for the Cubic Clustering Criterion (CCC), Pseudo F, and the Pseudo T-Squared:

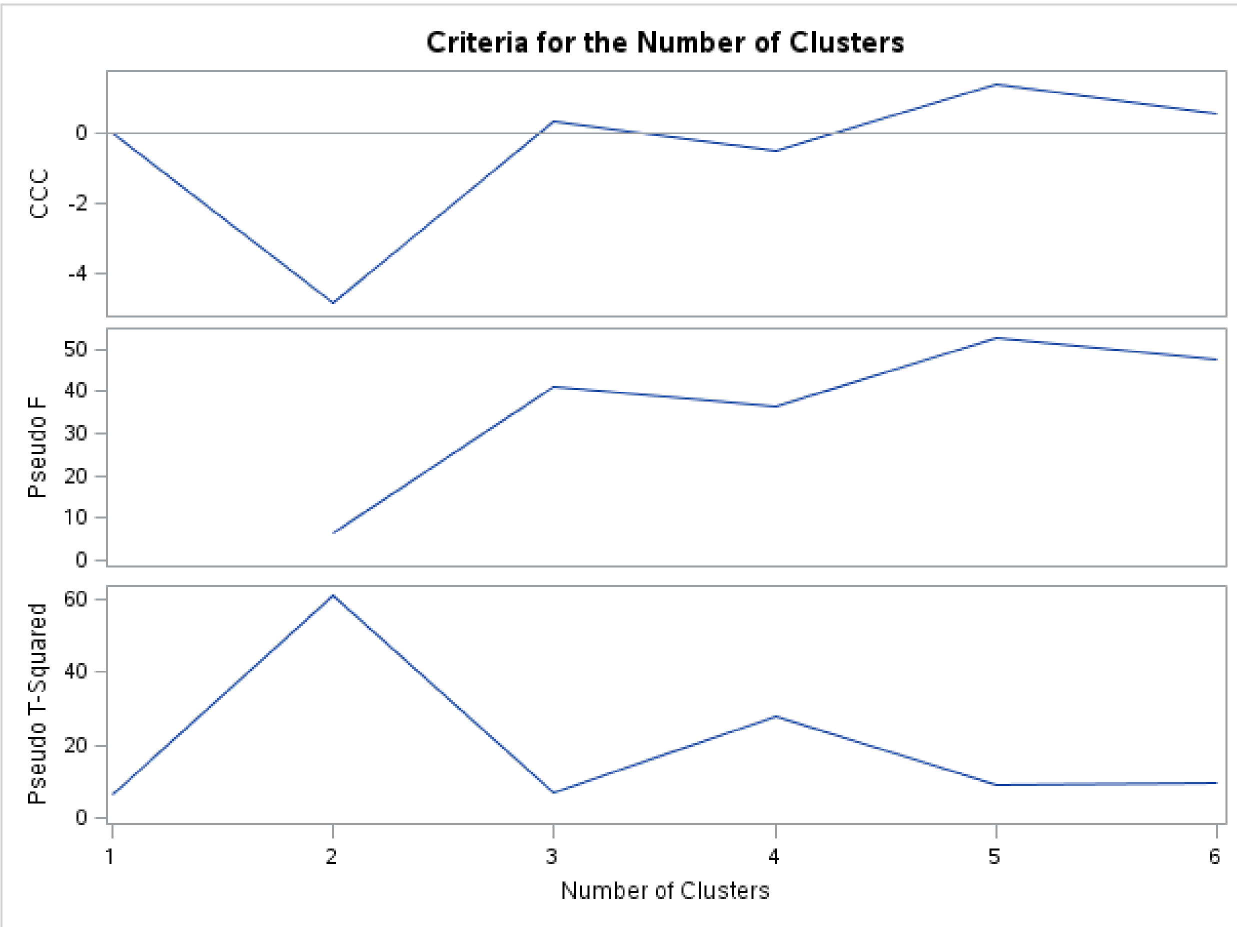


Figure 4: Criteria for Number of Clusters

We will interpret the measurements with the following assumptions, assuming that the criterion are all graphed in relation the number of clusters (as it is above):

- Cubic Clustering Criterion
  - Peaks on the plot with the CCC greater than 2 or 3 indicate good clusterings.
  - Peaks with the CCC between 0 and 2 indicate possible clusters but should be interpreted cautiously.
  - There may be several peaks if the data has a hierarchical structure.
  - Very distinct non-hierarchical spherical clusters usually show a sharp rise before the peak followed by a gradual decline.
  - Very distinct non-hierarchical elliptical clusters often show a sharp rise to the correct number of clusters followed by a further gradual increase and eventually a gradual decline.

  - If all values of the CCC are negative and decreasing for two or more clusters, the distribution is probably unimodal or long-tailed.
  - Very negative values of the CCC, say, -30, may be due to outliers. Outliers generally should be removed before clustering.

- Pseudo F
  - Look for a relatively large value.

From this we conclude (between the CCC and Pseudo F) that we would be likely happy with at-least three clusters, maybe four.

We use the tree procedure to assign observations to a specified number of clusters after the hierarchal clustering. We'll examine the tabular output between the three cluster tree and four cluster tree:

Group	Albania	CL3	CL6	Total
EFTA	0	6	0	6
EU	0	12	0	12
Eastern	1	0	7	8
Other	0	2	2	4
Total	1	20	9	30

Table 5: Frequency of Group to Cluster with Three Clusters

Group	Albania	CL4	CL5	CL6	Total
EFTA	0	5	1	0	6
EU	0	10	2	0	12
Eastern	1	0	0	7	8
Other	0	1	1	2	4
Total	1	16	4	9	30

Table 6: Frequency of Group to Cluster with Four Clusters

We observe that membership group, for this data set, is a fairly coherent guide for where classification into clusters will occur. The three cluster table shows that the existing groups distribute almost solely into a single cluster. Within the four cluster table we see that EFTA and EU give up some of their members to be distributed amongst other clusters. For simplicity, and to reinforce the contextual information within the data set, we'd prefer three clusters.

We'll now perform a hierarchical clustering with the principal components data set.

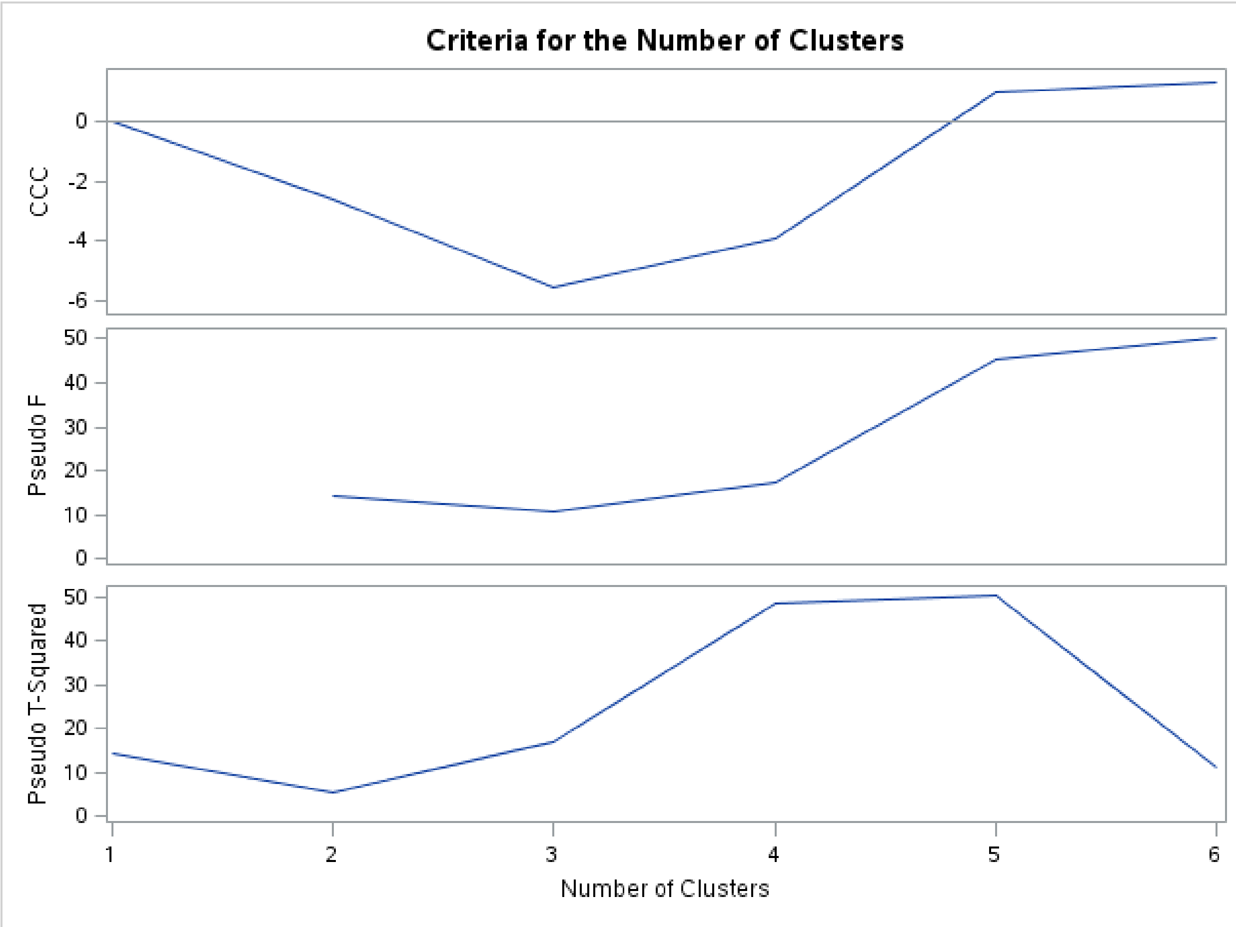


Figure 5: Criteria for Number of Clusters

Using our above assumptions for interpreting the criteria, we conclude (between the CCC and Pseudo F) that we would be likely happy with at-least five, maybe six clusters.

We use the tree procedure to assign observations to a specified number of clusters after the hierarchal clustering. We'll examine the tabular output between the three cluster tree and four cluster tree:

Group	Albania	CL3	Gibralta	Total
EFTA	0	6	0	6
EU	0	12	0	12
Eastern	1	7	0	8
Other	0	3	1	4
Total	1	28	1	30

Table 7: Frequency of Group to Cluster with Three Clusters

Group	Albania	CL4	CL6	Gibralta	Total
EFTA	0	6	0	0	6
EU	0	12	0	0	12
Eastern	1	4	3	0	8
Other	0	2	1	1	4
Total	1	24	4	1	30

Table 8: Frequency of Group to Cluster with Four Clusters

We now see that membership group breaks down a bit. Using the principal components data set seems to have pushed our clustering towards accentuating the outlier members within the data. We would prefer using the raw data for clustering. If the goal of cluster analysis is to group objects that are more similar (based on some distance methodology), it would seem to be more useful to have a better distribution of entities into your respective clusters. With the principal components data set we see that the clusters have a more skewed amount of membership, as opposed to the raw data set.

We do fear an assumed bias when working with data that already has an indication of group membership. In the case of this analysis we saw some reinforcement of that group membership in our initial clustering. This may have been enough to reinforce analyst bias to an undoubted level.

Procedures

```
title 'Assignment 8';
libname mydata '/scs/crb519/PREDICT_410/SAS_Data/' access=readonly;

* create a temporary variable (data source is read only);
title 'Examination of Initial Data Set';
data employ;
  set mydata.european_employment;

proc contents data=employ;

ods graphics on;
proc corr data=employ nomiss plots=matrix(histogram);
   var AGR CON FIN MAN MIN PS SER SPS TC;

title 'Scatterplot of AGR to SPS, colored by Group';
proc sgplot data=employ;
  scatter y=AGR x=SPS / datalabel=country group=group;

title 'Modeling the Data, Dimensionality Reduction';

proc princomp data=employ out=employ_prin outstat=eigenvectors plots=scree(unpackpanel);

*proc print data=eigenvectors(where=(_TYPE_='SCORE'));


title 'Cluster Analysis: Scatter Plots';

proc sgplot data=employ;
  title 'Scatterplot of Raw Data: FIN*SER';
  scatter y=fin x=ser / datalabel=country group=group;

proc sgplot data=employ;
  title 'Scatterplot of Raw Data: MAN*SER';
  scatter y=man x=ser / datalabel=country group=group;

title 'Cluster Analysis: Automated Cluster Selection';
proc cluster data=employ method=average outtree=tree1 pseudo ccc plots=all;
  var fin ser;
  id country;


proc tree data=tree1 ncl=3 out=_3_clusters;
  title 'Three Cluster Tree';
  copy fin ser;

proc tree data=tree1 ncl=4 out=_4_clusters;
  title 'Four Cluster Tree';
  copy fin ser;


* macro function for displaying displaying the assignment of the observations to the determined cluster
%macro makeTable(treeout,group,outdata);
```

```
  data tree_data;
    set &treeout.(rename=(_name_=country));

  proc sort data=tree_data; by country;

  data group_affiliation;
    set &group.(keep=group country);

  proc sort data=group_affiliation;
    by country;

  data &outdata.;
    merge tree_data group_affiliation;
    by country;

  proc freq data=&outdata.;
    table group*clusname / nopercent norow nocol;

%mend makeTable;

* Call macro function;
%makeTable(treeout=_3_clusters,group=employ,outdata=_3_clusters_with_labels);
%makeTable(treeout=_4_clusters,group=employ,outdata=_4_clusters_with_labels);

proc sgplot data=_3_clusters_with_labels;
  title 'Three Clusters with Labels';
  scatter y=fin x=ser / datalabel=country group=clusname;

proc sgplot data=_4_clusters_with_labels;
  title 'Four Clusters with Labels';
  scatter y=fin x=ser / datalabel=country group=clusname;

proc cluster data=employ_prin method=average outtree=tree3 pseudo ccc plots=all;
  title 'Cluster with Prin1 and Prin2';
  var prin1 prin2;
  id country;

proc tree data=tree3 ncl=3 out=_3_clusters;
  title 'Three Cluster Tree';
  copy prin1 prin2;

proc tree data=tree3 ncl=4 out=_4_clusters;
  title 'Foud Cluster Tree';
  copy prin1 prin2;

%makeTable(treeout=_3_clusters,group=employ,outdata=_3_clusters_with_labels);
%makeTable(treeout=_4_clusters,group=employ,outdata=_4_clusters_with_labels);

proc sgplot data=_3_clusters_with_labels;
  title 'Three Clusters with Labels';
  scatter y=prin2 x=prin1 / datalabel=country group=clusname;

proc sgplot data=_4_clusters_with_labels;
  title 'Four Clusters with Labels';
```

```
  scatter y=prin2 x=prin1 / datalabel=country group=clusname;

%makeTable(treeout=_3_clusters,group=employ,outdata=_3_clusters_with_labels);
%makeTable(treeout=_4_clusters,group=employ,outdata=_4_clusters_with_labels);

run;
ods graphics off;
```

References

[1]J. A. Hartigan, “Statistical theory in clustering,” *Journal of classification*, vol. 2, no. 1, pp. 63–76, 1985.