Project 2: Root Locating Methods

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CS 301 w/ Amar Raheja

Program Output

(a) f(x) = 2(x^3) - 11.7(x^2) + 17.7(x) - 5

=====Bisection Method=====

n	x	f(x)	approx. error
0	0.5000	1.1750	----
1	0.2500	-1.2750	1.0000
2	0.3750	0.0977	0.3333
3	0.3125	-0.5503	0.2000
4	0.3438	-0.2169	0.0909
5	0.3594	-0.0573	0.0435
6	0.3672	0.0208	0.0213
7	0.3633	-0.0181	0.0108
8	0.3652	0.0014	0.0053
9	0.3643	-0.0084	0.0027
10	0.3647	-0.0035	0.0013
11	0.3650	-0.0011	0.0007
12	0.3651	0.0001	0.0003
13	0.3651	-0.0005	0.0002
14	0.3651	-0.0002	0.0001
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0	2.0000	-0.4000	----
1	1.5000	1.9750	0.3333
2	1.7500	0.8625	0.1429
3	1.8750	0.2383	0.0667
4	1.9375	-0.0806	0.0323
5	1.9062	0.0791	0.0164
6	1.9219	-0.0007	0.0081
7	1.9141	0.0392	0.0041
8	1.9180	0.0193	0.0020
9	1.9199	0.0093	0.0010
10	1.9209	0.0043	0.0005
11	1.9214	0.0018	0.0003
12	1.9216	0.0006	0.0001
13	1.9218	-0.0001	0.0001
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0	3.5000	-0.6250	----
1	3.7500	2.3125	0.0667
2	3.6250	0.6867	0.0345
3	3.5625	-0.0069	0.0175

4	3.5938	0.3303	0.0087
5	3.5781	0.1593	0.0044
6	3.5703	0.0756	0.0022
7	3.5664	0.0342	0.0011
8	3.5645	0.0136	0.0005
9	3.5635	0.0033	0.0003
10	3.5630	-0.0018	0.0001
11	3.5632	0.0008	0.0001

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=====Newton-Raphson Method=====

n	x	f(x)	approx. error
0	0.7500	2.5375	----
1	0.0301	-4.4771	23.8824
2	0.2935	-0.7624	0.8973
3	0.3607	-0.0442	0.1863
4	0.3651	-0.0002	0.0121
5	0.3651	-0.0000	0.0001

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0	1.5000	1.9750	----
1	2.0064	-0.4327	0.2524
2	1.9215	0.0012	0.0442
3	1.9217	-0.0000	0.0001
4	1.9217	0.0000	0.0000

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0	3.0000	-3.2000	----
1	5.1333	48.0901	0.4156
2	4.2698	12.9562	0.2023
3	3.7929	2.9476	0.1257
4	3.5998	0.3980	0.0536
5	3.5643	0.0124	0.0100
6	3.5632	0.0000	0.0003
7	3.5632	0.0000	0.0000

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=====Secant Method=====

n	x	f(x)	approx. error
0	1.0000	3.0000	----
1	0.6250	1.9805	0.6000
2	-0.1034	-6.9585	7.0417
3	0.4636	0.8904	0.3481

4	0.3318	-0.3425	0.3974
5	0.3684	0.0327	0.0994
6	0.3652	0.0010	0.0087
7	0.3651	-0.0000	0.0003
8	0.3651	0.0000	0.0000

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0	1.0000	3.0000	----
1	1.9677	-0.2352	0.4918
2	1.8974	0.1244	0.0371
3	1.9217	0.0001	0.0127
4	1.9217	-0.0000	0.0000

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0	3.0000	-3.2000	----
1	3.3265	-1.9689	0.0982
2	3.8487	3.8338	0.1357
3	3.5037	-0.5904	0.0506
4	3.5796	0.1751	0.0212
5	3.5622	-0.0097	0.0049
6	3.5631	-0.0001	0.0003
7	3.5632	0.0000	0.0000

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=====False Position Method=====

n	x	f(x)	approx. error
0	0.6250	1.9805	----
1	0.4477	0.7585	0.3961
2	0.3887	0.2298	0.1517
3	0.3716	0.0646	0.0460
4	0.3669	0.0178	0.0129
5	0.3656	0.0049	0.0036
6	0.3652	0.0013	0.0010
7	0.3651	0.0004	0.0003
8	0.3651	0.0001	0.0001

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0	1.9677	-0.2352	----
1	1.8974	0.1244	0.0371
2	1.9217	0.0001	0.0127
3	1.9217	0.0000	0.0000

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0	3.3265	-1.9689	----

1	3.4813	-0.7959	0.0444
2	3.5371	-0.2671	0.0158
3	3.5551	-0.0840	0.0051
4	3.5607	-0.0259	0.0016
5	3.5624	-0.0079	0.0005
6	3.5629	-0.0024	0.0001
7	3.5631	-0.0007	0.0000

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=====Modified Secant Method=====

n	x	f(x)	approx. error
0	0.7500	2.5375	----
1	0.0190	-4.6685	38.5437
2	0.2895	-0.8078	0.9345
3	0.3604	-0.0473	0.1966
4	0.3651	-0.0001	0.0129
5	0.3651	0.0000	0.0000

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0	1.5000	1.9750	----
1	2.0013	-0.4064	0.2505
2	1.9214	0.0015	0.0415
3	1.9217	0.0000	0.0002
4	1.9217	0.0000	0.0000

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0	3.0000	-3.2000	----
1	4.8926	35.7632	0.3868
2	4.1429	9.7305	0.1809
3	3.7423	2.2031	0.1071
4	3.5911	0.3004	0.0421
5	3.5647	0.0162	0.0074
6	3.5632	0.0005	0.0004
7	3.5632	0.0000	0.0000

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(b) f(x) = x + 10 - xcosh(50/x)

=====Bisection Method=====

n	x	f(x)	approx. error
0	125.0000	-0.1340	----
1	127.5000	0.0698	0.0196
2	126.2500	-0.0311	0.0099
3	126.8750	0.0196	0.0049
4	126.5625	-0.0057	0.0025
5	126.7188	0.0070	0.0012
6	126.6406	0.0007	0.0006
7	126.6016	-0.0025	0.0003
8	126.6211	-0.0009	0.0002
9	126.6309	-0.0001	0.0001

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=====Newton-Raphson Method=====

n	x	f(x)	approx. error
0	130.0000	0.2655	----
1	129.8620	0.2549	0.0011
2	129.7294	0.2447	0.0010
3	129.6022	0.2349	0.0010
4	129.4800	0.2254	0.0009
5	129.3628	0.2164	0.0009
6	129.2502	0.2076	0.0009
7	129.1422	0.1992	0.0008
8	129.0386	0.1912	0.0008
9	128.9391	0.1834	0.0008
10	128.8437	0.1760	0.0007
11	128.7521	0.1688	0.0007
12	128.6643	0.1619	0.0007
13	128.5800	0.1553	0.0007
14	128.4992	0.1490	0.0006
15	128.4217	0.1429	0.0006
16	128.3473	0.1370	0.0006
17	128.2760	0.1314	0.0006
18	128.2076	0.1260	0.0005
19	128.1420	0.1208	0.0005
20	128.0792	0.1158	0.0005
21	128.0189	0.1111	0.0005
22	127.9610	0.1065	0.0005

23	127.9056	0.1021	0.0004
24	127.8524	0.0979	0.0004
25	127.8015	0.0938	0.0004
26	127.7526	0.0899	0.0004
27	127.7058	0.0862	0.0004
28	127.6609	0.0826	0.0004
29	127.6179	0.0792	0.0003
30	127.5767	0.0759	0.0003
31	127.5371	0.0728	0.0003
32	127.4992	0.0697	0.0003
33	127.4629	0.0668	0.0003
34	127.4281	0.0640	0.0003
35	127.3948	0.0614	0.0003
36	127.3628	0.0588	0.0003
37	127.3322	0.0564	0.0002
38	127.3028	0.0540	0.0002
39	127.2747	0.0518	0.0002
40	127.2477	0.0496	0.0002
41	127.2219	0.0475	0.0002
42	127.1971	0.0455	0.0002
43	127.1734	0.0436	0.0002
44	127.1507	0.0418	0.0002
45	127.1289	0.0401	0.0002
46	127.1080	0.0384	0.0002
47	127.0880	0.0368	0.0002
48	127.0689	0.0352	0.0002
49	127.0505	0.0338	0.0001
50	127.0329	0.0323	0.0001
51	127.0161	0.0310	0.0001
52	126.9999	0.0297	0.0001
53	126.9845	0.0284	0.0001
54	126.9697	0.0272	0.0001
55	126.9555	0.0261	0.0001
56	126.9419	0.0250	0.0001
57	126.9288	0.0240	0.0001
58	126.9164	0.0229	0.0001

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=====Secant Method=====

n	x	f(x)	approx. error

0	130.0000	0.2655	----
1	126.8156	0.0148	0.0251
2	126.6274	-0.0004	0.0015
3	126.6324	0.0000	0.0000

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=====False Position Method=====

n	x	f(x)	approx. error
0	126.8156	0.0148	----
1	126.6424	0.0008	0.0014
2	126.6330	0.0000	0.0001

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=====Modified Secant Method=====

n	x	f(x)	approx. error
0	130.0000	0.2655	----
1	126.5037	-0.0104	0.0276
2	126.6336	0.0001	0.0010
3	126.6324	-0.0000	0.0000

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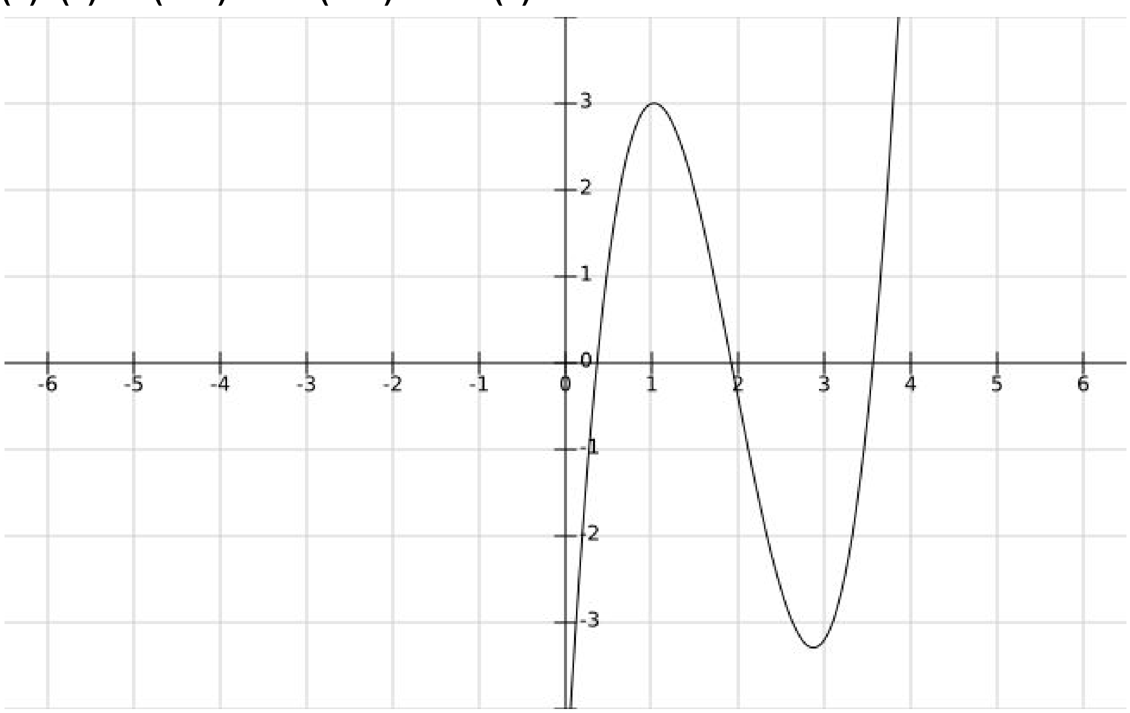
PLOTS

(a) f(x) = 2(x^3) - 11.7(x^2) + 17.7(x) - 5



zero @ ~0.3651

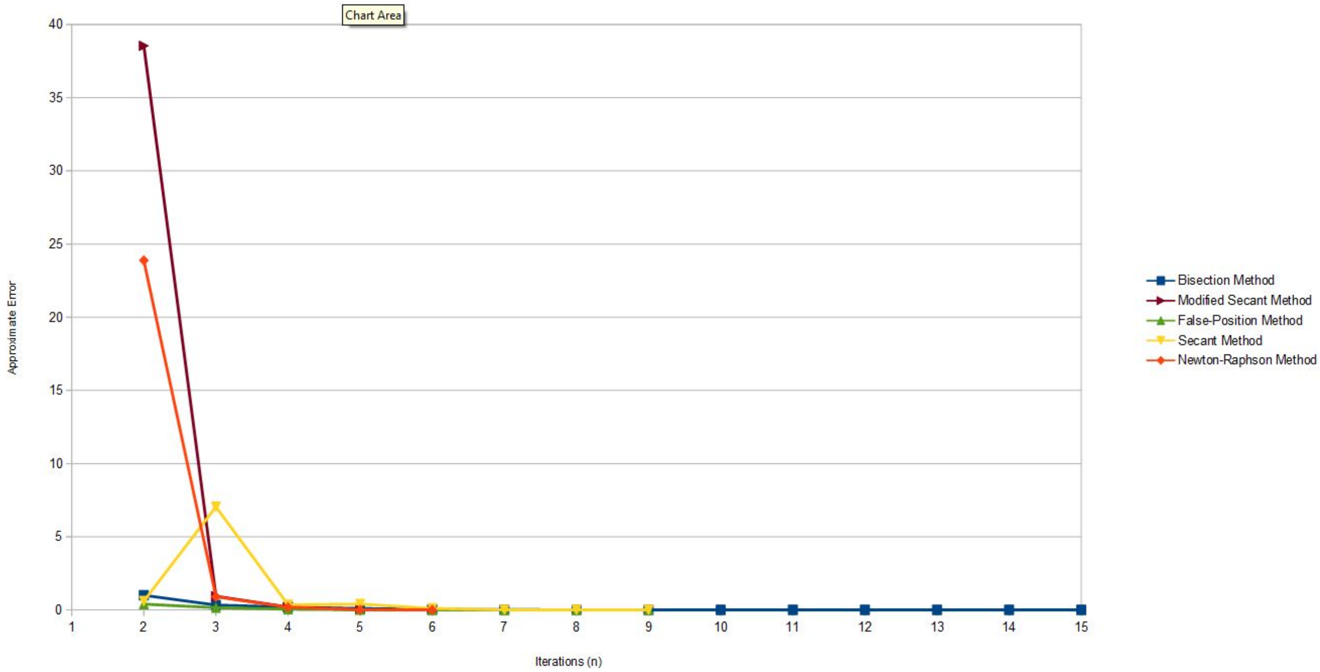


Chart Area
Approximate Error
Iterations (n)
Bisection Method
Modified Secant Method
False-Position Method
Secant Method
Newton-Raphson Method

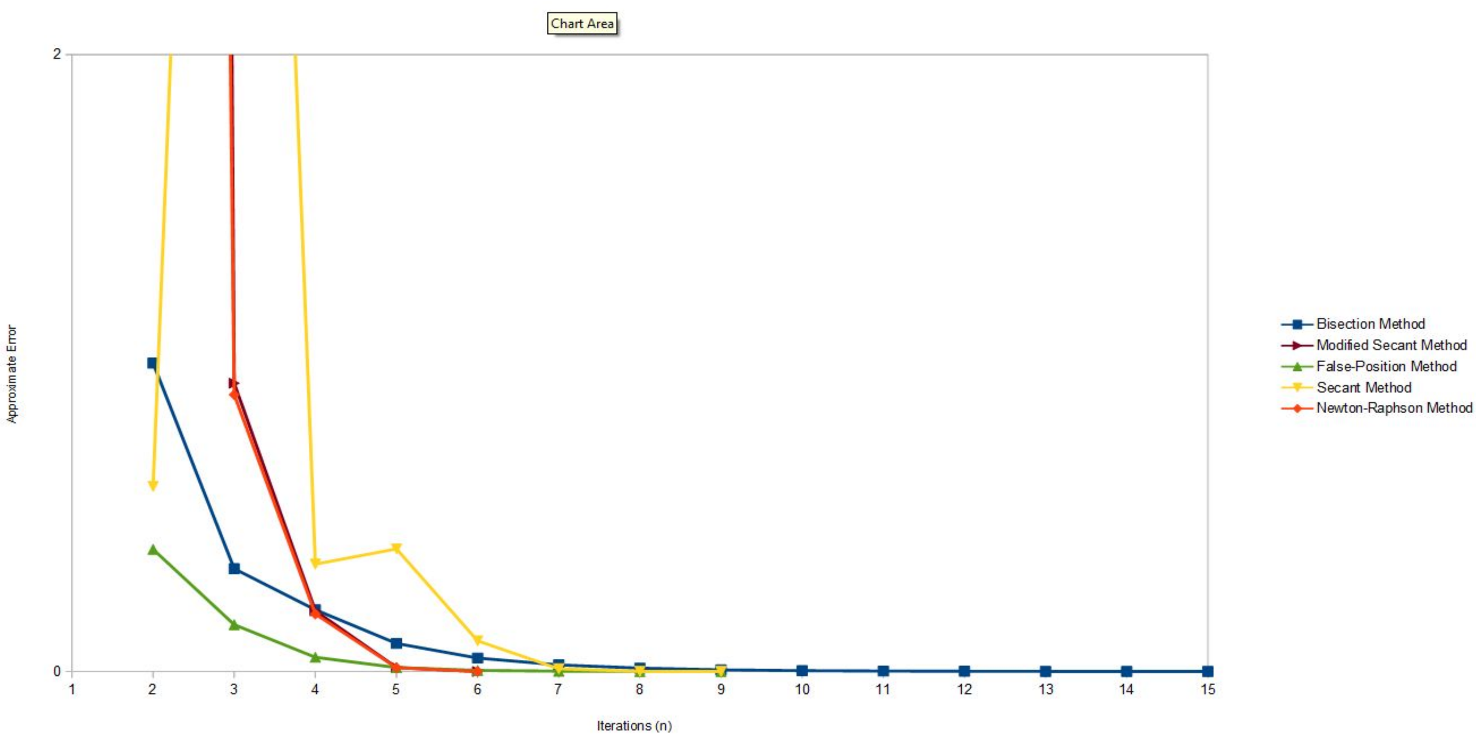
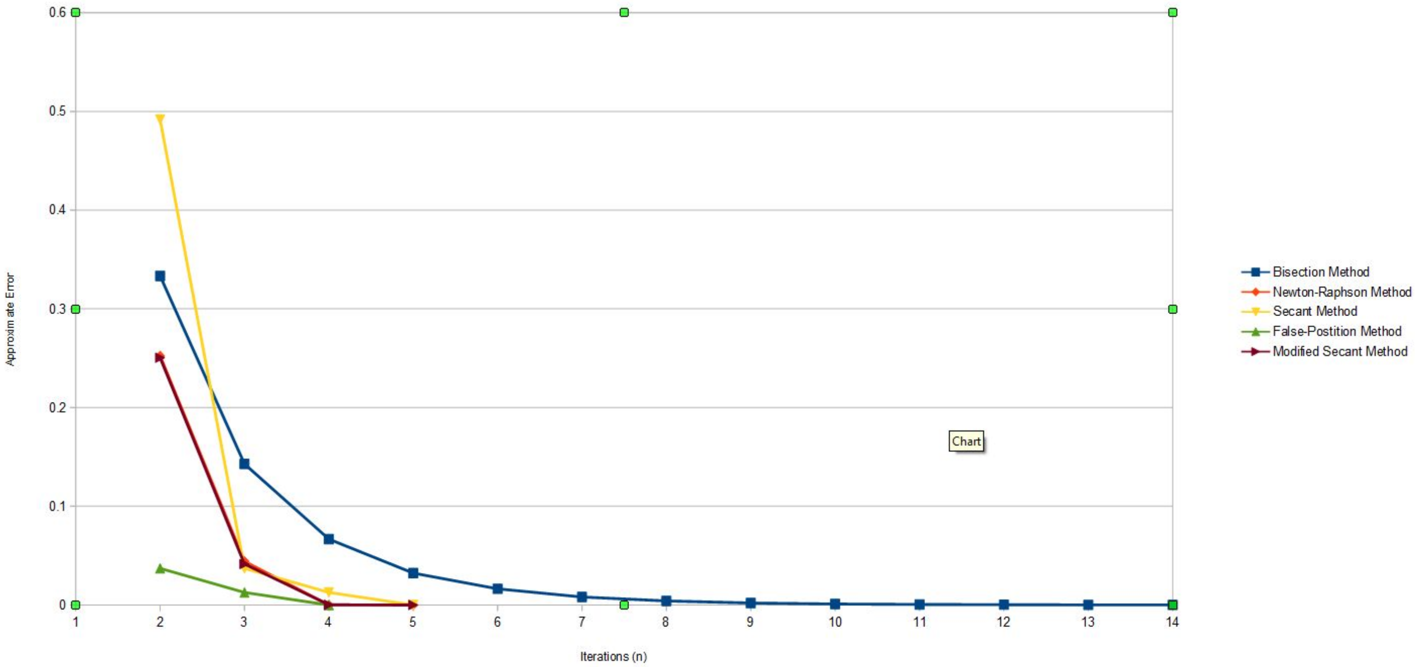
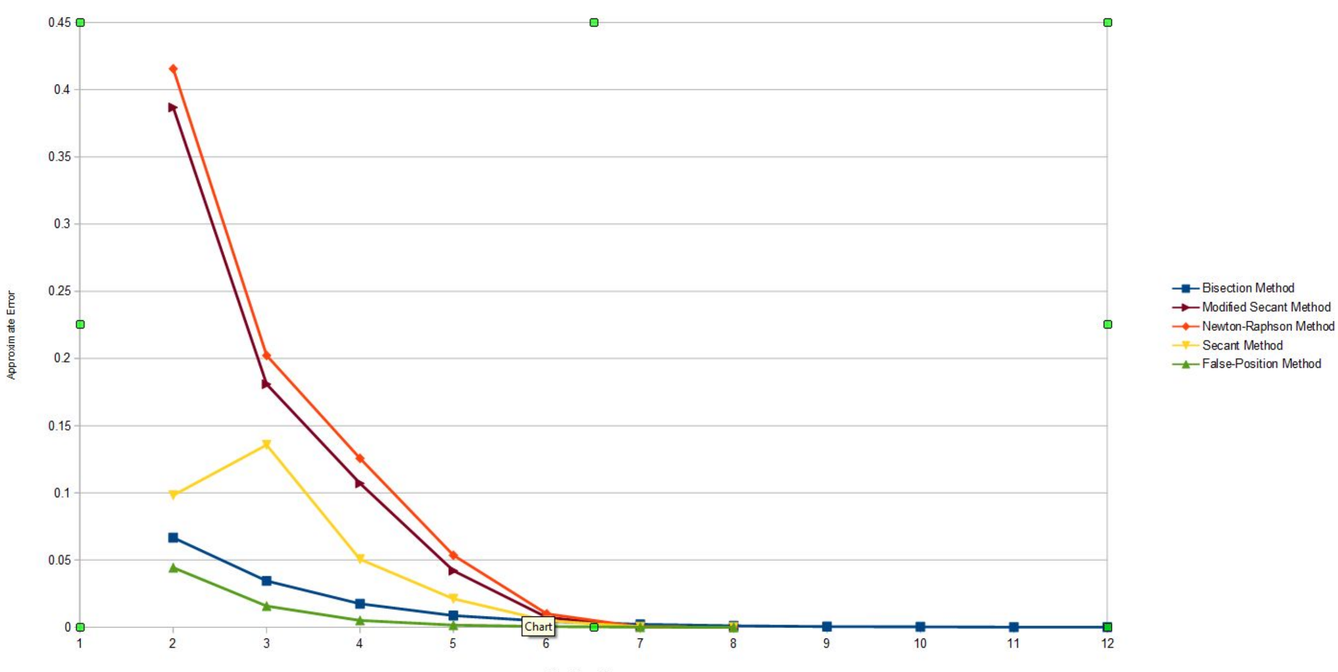


Chart Area
Approximate Error
Iterations (n)
Bisection Method
Modified Secant Method
False-Position Method
Secant Method
Newton-Raphson Method

zero @ 1.9218

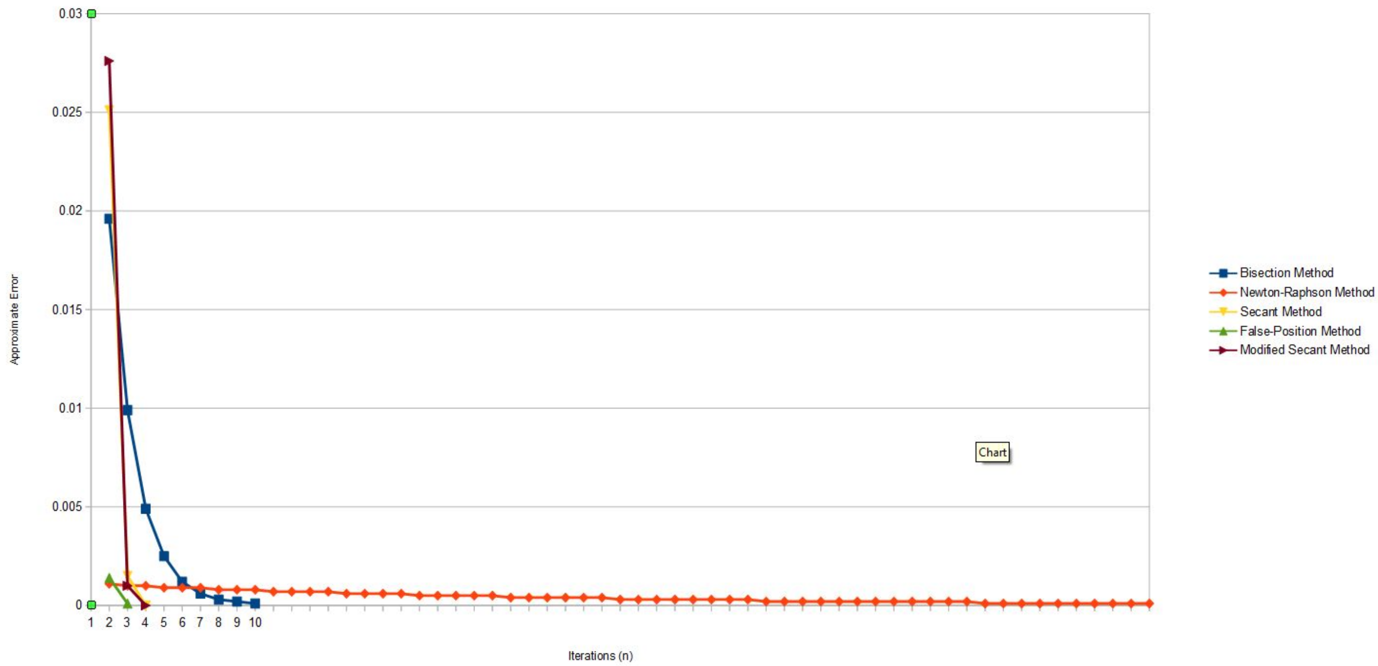


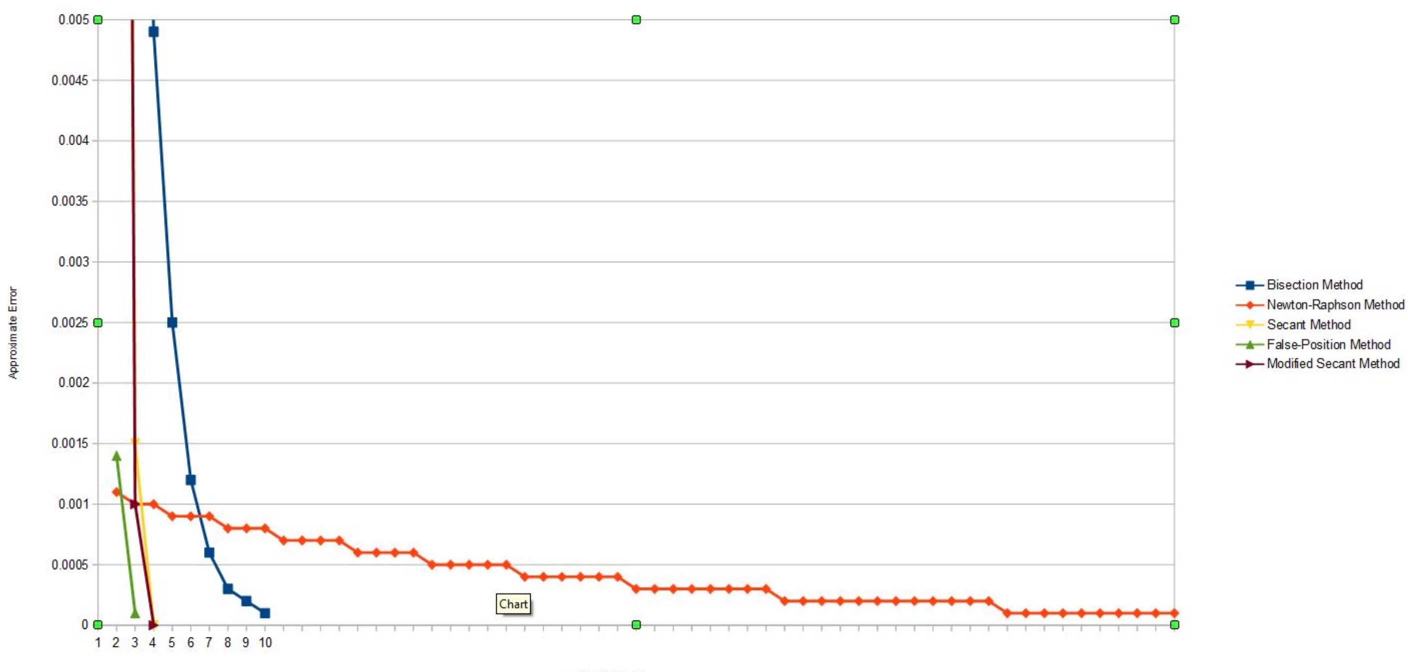
zero @ 3.5625



(b) f(x) = x + 10 - xcosh(50/x)

<u>zero @ ~126.6324</u>





<u>**Conclusions**</u>

The data that was pulled was created using very specific starting point values that were re-used wherever necessary and were chosen to give each method a “best-case scenario.” These points were chosen simply by graphing the function with an external program and picking points that looked like they would work well.

For problem (a), the x-values of (0.0, 1.0), (3.0, 1.0), and (3.0, 4.0) were used for the first, second, and third zero, respectively, when using the Bisection, Secant, and False-Position methods. These were all nice round numbers that happened to bracket all three roots. The values of 0.75, 1.5, and 3.0 were used for the x-values for the first, second and third zero, respectively, when using the Newton-Raphson and Modified Secant methods. These numbers were chosen because they are all located on slopes that were headed towards their respective zero.

Using these methods with these starting points produced the average of 38, 16, 19, 18, and 16 iterations to find all zeros at 0.01% approximate error for Bisection, Newton-Raphson, Secant, False-Position, and Modified Secant methods, respectively.

Bisection method always converges at the same rate, no matter the problem. So I won’t discuss its convergence further. Newton’s Method converges quickly and produces an approximate to the 100th at the 5th iteration or earlier. The Secant, False-Position, and Modified Secant share very similar results. Although the Secant and Modified Secant methods sometimes start with pretty sporadic results in the first few iterations.

For problem (b), the x-values of (120, 130) were used for locating the zero when using the Bisection, Secant, and False-Position methods, respectively. These numbers were chosen because they were presented in the problem and they bracket the root. The x-value of 130 was used for the x-value when finding the zero using the Newton-Raphson and Modified Secant methods. This numbers was chosen because its slope points towards the zero and it was presented in the problem.

Using these methods with these starting points, the roots were found in 9, 58, 3, 2, and 3 iterations at 0.01% approximate error for Bisection, Newton-Raphson, Secant, False-Position, and Modified Secant methods, respectively.

As one might be able to guess from the information presented before, the Newton-Raphson took an incredibly long time to converge to the solution. The ones place was not even accurate until the 52nd iteration. The remaining methods converged extremely quickly.

Other values were used as starting values for locating roots. Some did not give quality results. This mainly had to do with methods that depended on the slope of the graph, like the Newton-Raphson or Modified Secant methods. The approximate values would either quickly diverge or discover the wrong zero.

Doubles were used for calculations and allow up to 15 decimal digits of accuracy.