Hyperbolic Functions

$$\sinh x = \frac{e^x - e^x}{2} \quad \mathbb{R} \to \mathbb{R} \qquad \operatorname{csch} x = \frac{2}{e^x - e^x} \quad \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$$

$$\cosh x = \frac{e^x + e^x}{2} \quad \mathbb{R} \to (1, +\infty) \qquad \operatorname{sech} x = \frac{2}{e^x + e^x} \quad \mathbb{R} \to (0, 1)$$

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1} \quad \mathbb{R} \to (-1, 1) \qquad \coth x = \frac{e^{2x} + 1}{e^{2x} - 1} \quad \mathbb{R} - \{0\} \to (-\infty, -1) \cup (1, +\infty)$$

$$\operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right) \quad \mathbb{R} \to \mathbb{R} \qquad \operatorname{arccsch} x = \ln\left(\frac{1 \pm \sqrt{1 + x^2}}{x}\right) \quad \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$$

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2 - 1}\right) \quad [1, +\infty) \to [0, +\infty) \qquad \operatorname{arcsech} x = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right) \quad (0, 1] \to [0, +\infty)$$

$$\operatorname{arctanh} x = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right) \quad (-1, 1) \to \mathbb{R} \qquad \operatorname{arccoth} x = \frac{1}{2} \ln\left(\frac{x + 1}{x - 1}\right) \quad (-\infty, -1) \cup (1, +\infty) \to \mathbb{R} - \{0\}$$