Hyperbolic Functions

$$\sinh x = \frac{e^x - e^x}{2} \quad \mathbb{R} \to \mathbb{R} \qquad \operatorname{csch} x = \frac{2}{e^x - e^x} \quad \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$$

$$\cosh x = \frac{e^x + e^x}{2} \quad \mathbb{R} \to (1, +\infty) \qquad \operatorname{sech} x = \frac{2}{e^x + e^x} \quad \mathbb{R} \to (0, 1)$$

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1} \quad \mathbb{R} \to (-1, 1) \qquad \coth x = \frac{e^{2x} + 1}{e^{2x} - 1} \quad \mathbb{R} - \{0\} \to (-\infty, -1) \cup (1, +\infty)$$

$$\operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right) \quad \mathbb{R} \to \mathbb{R} \qquad \operatorname{arccsch} x = \ln\left(\frac{1 \pm \sqrt{1 + x^2}}{x}\right) \quad \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$$

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2 - 1}\right) \quad [1, +\infty) \to [0, +\infty) \qquad \operatorname{arcsech} x = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right) \quad (0, 1] \to [0, +\infty)$$

$$\operatorname{arctanh} x = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right) \quad (-1, 1) \to \mathbb{R} \qquad \operatorname{arccoth} x = \frac{1}{2} \ln\left(\frac{x + 1}{x - 1}\right) \quad (-\infty, -1) \cup (1, +\infty) \to \mathbb{R} - \{0\}$$

Derivaives

$$\frac{d}{dx}\, x^\alpha = \alpha x^{\alpha - 1} \qquad \frac{d}{dx}\, \alpha^x = \alpha^x \ln \alpha \qquad \frac{d}{dx}\, \log_\alpha x = \frac{1}{x \ln \alpha}$$

$$\frac{d}{dx}\, \sin x = \cos x \qquad \frac{d}{dx}\, \cos x = -\sin x \qquad \frac{d}{dx}\, \tan x = \sec^2 x$$

$$\frac{d}{dx}\, \csc x = -\csc x \cot x \qquad \frac{d}{dx}\, \sec x = \sec x \tan x \qquad \frac{d}{dx}\, \cot x = -\csc^2 x$$

$$\frac{d}{dx}\, \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}\, \arccos x = -\frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}\, \arctan x = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}\, \operatorname{arccsc} x = -\frac{1}{|x| \sqrt{1 - x^2}} \qquad \frac{d}{dx}\, \operatorname{arcsec} x = \frac{1}{|x| \sqrt{1 - x^2}} \qquad \frac{d}{dx}\, \operatorname{arccot} x = -\frac{1}{1 + x^2}$$

Taylor Series

$$(1+x)^{\alpha} = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n = 1 + \alpha x + \frac{\alpha\left(\alpha-1\right)}{2}x^2 + \frac{\alpha\left(\alpha-1\right)\left(\alpha-2\right)}{6}x^3 + \frac{\alpha\left(\alpha-1\right)\left(\alpha-2\right)\left(\alpha-3\right)}{24}x^4 + \ldots \quad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \ldots \quad |x| < \infty$$

$$\ln\left(1+x\right) = \sum_{n=1}^{\infty} \left(-1\right)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \ldots \quad |x| \leq 1$$

$$\sin x = \sum_{n=0}^{\infty} \left(-1\right)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} + \ldots \quad |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \left(-1\right)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \ldots \quad |x| < \infty$$

$$\tan x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \frac{62x^9}{2835} - \frac{1382x^{11}}{155925} + \ldots \quad |x| < \frac{\pi}{2}$$

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}\left(n!\right)^2\left(2n+1\right)} x^{2n+1} = x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + \ldots = \frac{\pi}{2} - \arccos x \quad |x| \leq 1$$

$$\arctan x = \sum_{n=0}^{\infty} \left(-1\right)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \quad |x| \leq 1$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{6} + \frac{x^5}{120} + \ldots \quad |x| < \infty$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^6}{720} + \ldots \quad |x| < \infty$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \frac{62x^9}{2835} - \frac{1382x^{11}}{155925} + \ldots \quad |x| < \frac{\pi}{2}$$

$$\arcsin x = \sum_{n=0}^{\infty} \left(-1\right)^n \frac{(2n)!}{2^{2n}\left(n!\right)^2\left(2n+1\right)} x^{2n+1} = x - \frac{x^3}{6} + \frac{3x^5}{40} - \frac{5x^7}{112} + \ldots \quad |x| \leq 1$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots \quad |x| \leq 1$$