NLO QCD Corrections to Scattering Processes Using θ -Parameters in the NSC Subtraction Scheme

Bachelor's Degree in Physics

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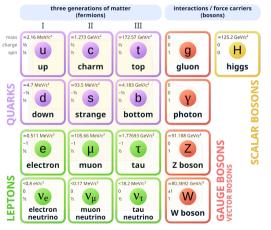




The Standard Model of Particle Physics

State of the art

Standard Model of Elementary Particles



 $\begin{array}{l} \text{High energy} \rightarrow \text{Special Relativity} \\ \text{Small particles} \rightarrow \text{Quantum Mechanics} \end{array}$

Quantum Field Theory (mathematical framework)

Evidences for physics beyond the SM

- Dark matter and dark energy
- Matter-antimatter asymmetry
- Origin of neutrino masses



Collider PhysicsOne of the principal methods of research

Large Hadron Collider at CERN Proton beams accelerated to $\sim13.6\,\mathrm{TeV}$



Figure: Maximilien Brice/CERN

Not sufficient to the discovery of new particles.

Solutions:

- Increase energy
 - ightarrow not feasible with existing technology
- Increase precision
 - ightarrow both experimental and theoretical



Quantum Chromodynamics

Hard scattering processes

- Strong interactions are described by Quantum Chromodynamics (QCD).
- Non-Abelian gauge theory, SU(3) symmetry group.
- The QCD Lagrangian is not analytically solvable.

Hadron collisions

- Elastic scattering
- Diffractive dissociation
- Hard scattering (momentum exchange $\sim 100\,\mathrm{GeV}$)

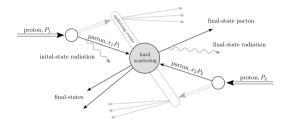


Figure: Konstantin Asteriadis/KIT

Asymptotic freedom

Interacting partons can be approximated as being nearly free

ightarrow Perturbative description



The collinear factorization theorem

A framework for describing hard scattering processes

Colliding hadrons are treated as **beams of partons**, carrying a fraction of the hadron's total momentum

Separation of energy scales

- SM interactions $\emph{Q} \sim 100 {\rm GeV} 1 {\rm TeV}$
- Hadronic structure $\Lambda_{\rm QCD} \sim 100 {\rm MeV}$
- \rightarrow **Decoupling** the motions of partons from proton's dynamics

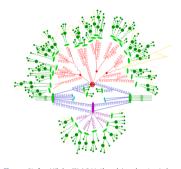


Figure: Stefan Höche/SLAC National Accelerator Laboratory

$$\mathrm{d}\sigma = \sum_{a,b} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_a(x_1,\mu_\mathsf{F}) f_b(x_2,\mu_\mathsf{F}) \, \mathrm{d}\hat{\sigma}_{a,b}(x_1,x_2,\mu_\mathsf{F},\mu_\mathsf{R};\mathcal{O}) \left(1 + \mathcal{O}\left(\frac{\Lambda_\mathsf{QCD}}{Q}\right)^n\right), \, n \geq 1$$



The partonic cross section

A perturbative description

The partonic cross section can be expanded in powers of the strong and the electroweak coupling constants, $\alpha_{\rm S}$ and α

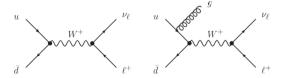
$$\mathrm{d}\hat{\sigma}_{a,b} = \mathrm{d}\hat{\sigma}_{a,b}^{(0,0)} + \alpha_s \mathrm{d}\hat{\sigma}_{a,b}^{(1,0)} + \alpha_s^2 \mathrm{d}\hat{\sigma}_{a,b}^{(2,0)} + \alpha_s^3 \mathrm{d}\hat{\sigma}_{a,b}^{(3,0)} + \alpha \mathrm{d}\hat{\sigma}_{a,b}^{(0,1)} + \alpha \alpha_s \mathrm{d}\hat{\sigma}_{a,b}^{(1,1)} + \dots$$

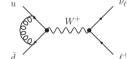
We focus on NLO QCD corrections

Account for short distance, high-energy effects

They consist of three terms

$$\mathbf{d}\hat{\sigma}_{a,b}^{\text{NLO}} = \mathbf{d}\hat{\sigma}_{a,b}^{\text{R}} + \mathbf{d}\hat{\sigma}_{a,b}^{\text{V}} + \mathbf{d}\hat{\sigma}_{a,b}^{\text{pdf}}$$







UV and IR divergences

A considerable obstacle

The treatment of real and virtual corrections is **non-trivial**

- 1 Ultraviolet (UV) singularities in virtual contributions
 - \rightarrow Renormalization
- (IR) singularities in both real and virtual contributions
 - → Low-momentum (soft) and small-angle (collinear) kinematic regions

$$\begin{array}{cccc}
& p_{\text{m}} & & \\
& p_{i} & p_{i} & p_{\text{m}} & \\
\end{array}
\sim \frac{1}{(p_{i} - p_{\text{m}})^{2}} = \frac{1}{2E_{i}E_{\text{m}}\left(1 - \cos\theta_{i\text{m}}\right)} & \xrightarrow{E_{\text{m}},\theta_{i\text{m}} \to 0} & \infty$$



UV and IR divergences

Dimensional regularization and pole cancellation

Dimensional regularization

$$d = 4 - 2\epsilon$$
 $\epsilon \in \mathbb{C}$, $\operatorname{Re}(\epsilon) < 0$

Divergences appear as poles in $1/\epsilon$

- Virtual \rightarrow explicit
- Real → integration needed (1)
- Singularities signal the presence of non-perturbative contributions
- We lack methods to treat non-perturbative QCD effects from first principles

Bloch-Nordsieck and Kinoshita-Lee-Nauenberg theorems

Infrared divergences are guaranteed to cancel when we sum real and virtual corrections

• Mismatch in the dimensionality of the integration domains 2



IR divergences

Subtraction scheme

Insights

- Factorization of the amplitudes in the soft and collinear limits
- Real emissions are unresolved in singular kinematics regions

We adopt a subtraction method

$$2s_{a,b}\mathrm{d}\hat{\sigma}_{a,b}^{\mathrm{R}}=\int[\mathrm{d}p_{\mathfrak{m}}]F_{\mathrm{LM}}(\mathfrak{m})=\int[\mathrm{d}p_{\mathfrak{m}}](F_{\mathrm{LM}}(\mathfrak{m})-\mathcal{S})+\int[\mathrm{d}p_{\mathfrak{m}}]\mathcal{S}$$

- → Describes the singular behaviour of the amplitude
- → Singular behaviour removed: numerical integration with Monte Carlo methods

Restricting the integration region to the **minimal necessary volume** is crucial for improving the efficiency of the computation

Prova