NLO QCD Corrections to Scattering Processes Using θ-Parameters in the NSC Subtraction Scheme

Bachelor's Degree in Physics

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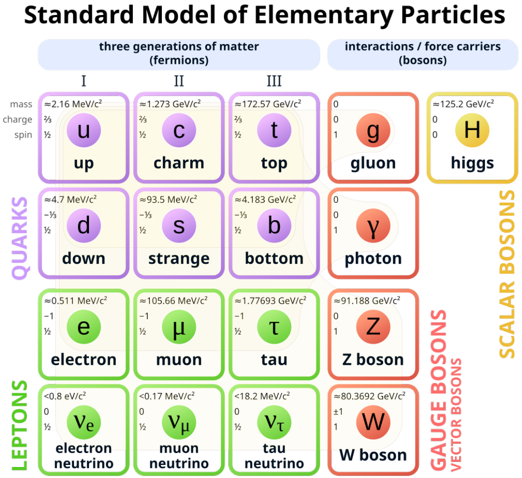
The Standard Model of Particle Physics

State of the art



High energy $\rightarrow$ Special Relativity

Small particles $\rightarrow$ Quantum Mechanics

$\downarrow$

Quantum Field Theory

(mathematical framework)

Evidences for physics beyond the SM

- Dark matter and dark energy
- Matter-antimatter asymmetry
- Origin of neutrino masses

Collider Physics

One of the principal methods of research

Large **H**adron **C**ollider at CERN

Proton beams accelerated to ~ 13.6 TeV

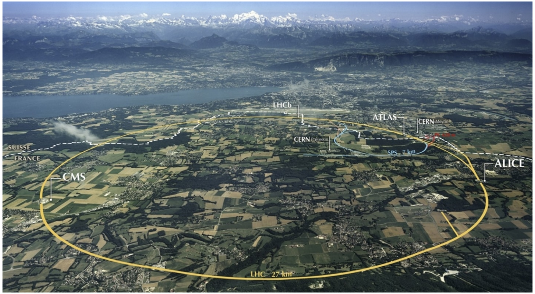


Figure: Maximilien Brice/CERN

Not sufficient to the discovery of new particles.

Solutions:

- Increase energy
  $\rightarrow$ not feasible with existing technology
- Increase precision
  $\rightarrow$ both experimental and **theoretical**

Quantum Chromodynamics
Hard scattering processes

- Strong interactions are described by Quantum Chromodynamics (QCD).
- Non-Abelian gauge theory, $SU(3)$ symmetry group.
- The QCD Lagrangian is not analytically solvable.

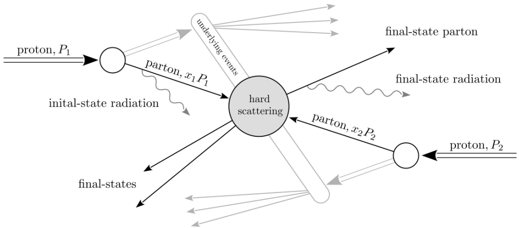


Figure: Konstantin Asteriadis/KIT

Hadron collisions

- Elastic scattering
- Diffractive dissociation
- Hard scattering (momentum exchange ~ 100 GeV)

Asymptotic freedom

Interacting partons can be approximated as being nearly free

$\rightarrow$ **Perturbative description**

The collinear factorization theorem

A framework for describing hard scattering processes

Colliding hadrons are treated as **beams of partons**, carrying a fraction of the hadron's total momentum

Separation of energy scales

- SM interactions $Q \sim 100\text{GeV} - 1\text{TeV}$
- Hadronic structure $\Lambda_{\text{QCD}} \sim 100\text{MeV}$

$\longrightarrow$ **Decoupling** the motions of partons from proton's dynamics

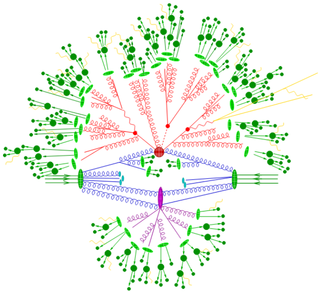


Figure: Stefan Höche/SLAC National Accelerator Laboratory

$$d\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_{\text{F}}) f_b(x_2, \mu_{\text{F}})\, d\hat{\sigma}_{a,b}(x_1, x_2, \mu_{\text{F}}, \mu_{\text{R}}; \mathcal{O}) \left(1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^n\right), \; n \geq 1$$

The partonic cross section

A perturbative description

The partonic cross section can be expanded in powers of the strong and the electroweak coupling constants, α_S and α

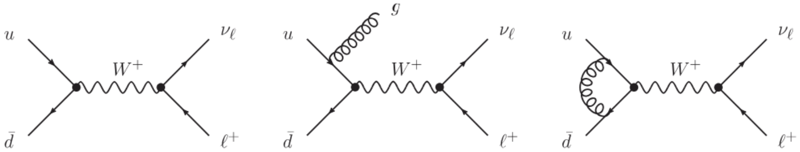
$$d\hat{\sigma}_{a,b} = d\hat{\sigma}_{a,b}^{(0,0)} + \alpha_s d\hat{\sigma}_{a,b}^{(1,0)} + \alpha_s^2 d\hat{\sigma}_{a,b}^{(2,0)} + \alpha_s^3 d\hat{\sigma}_{a,b}^{(3,0)} + \alpha d\hat{\sigma}_{a,b}^{(0,1)} + \alpha\alpha_s d\hat{\sigma}_{a,b}^{(1,1)} + \dots$$

We focus on NLO QCD corrections

Account for short distance, high-energy effects

They consist of three terms

$$d\hat{\sigma}_{a,b}^{\mathrm{NLO}} = d\hat{\sigma}_{a,b}^{\mathrm{R}} + d\hat{\sigma}_{a,b}^{\mathrm{V}} + d\hat{\sigma}_{a,b}^{\mathrm{pdf}}$$



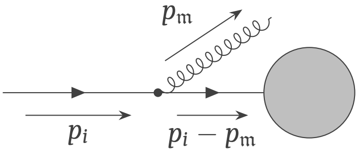
UV and IR divergences

A considerable obstacle

The treatment of real and virtual corrections is **non-trivial**

① **Ultraviolet** (UV) singularities in virtual contributions
$\longrightarrow$ Renormalization

② **Infrared** (IR) singularities in both real and virtual contributions
$\longrightarrow$ Low-momentum (soft) and small-angle (collinear) kinematic regions



$$\sim \frac{1}{(p_i - p_{\mathfrak{m}})^2} = \frac{1}{2E_iE_{\mathfrak{m}}\,(1 - \cos\theta_{i\mathfrak{m}})} \xrightarrow[E_{\mathfrak{m}},\theta_{i\mathfrak{m}}\to 0]{} \infty$$

UV and IR divergences

Dimensional regularization and pole cancellation

Dimensional regularization

$$d = 4 - 2\epsilon \qquad \epsilon \in \mathbb{C}\,, \mathrm{Re}(\epsilon) < 0$$

Divergences appear as poles in $1/\epsilon$

- Virtual $\to$ explicit
- Real $\to$ integration needed 1

- Singularities signal the presence of **non-perturbative contributions**
- We lack methods to treat non-perturbative QCD effects from first principles

Bloch-Nordsieck and Kinoshita-Lee-Nauenberg theorems

Infrared divergences are **guaranteed** to cancel when we sum real and virtual corrections

- **Mismatch** in the dimensionality of the integration domains 2

IR divergences

Subtraction scheme

Insights

- **Factorization** of the amplitudes in the soft and collinear limits
- Real emissions are **unresolved** in singular kinematics regions

We adopt a **subtraction method**

$$2s_{a,b}\mathrm{d}\hat{\sigma}^{\mathrm{R}}_{a,b} = \int [\mathrm{d}p_{\mathfrak{m}}] F^{ab}_{\mathrm{LM}} = \int [\mathrm{d}p_{\mathfrak{m}}] (F^{ab}_{\mathrm{LM}} - \mathcal{S}) + \int [\mathrm{d}p_{\mathfrak{m}}] \mathcal{S}$$

$\rightarrow$ Describes the singular behaviour of the amplitude

$\rightarrow$ Singular behaviour removed: numerical integration with Monte Carlo methods

Restricting the integration region to the **minimal necessary volume** is crucial for improving the efficiency of the computation