NLO QCD Corrections to Scattering Processes Using θ-Parameters in the NSC Subtraction Scheme

Bachelor's Degree in Physics

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24 October 2025

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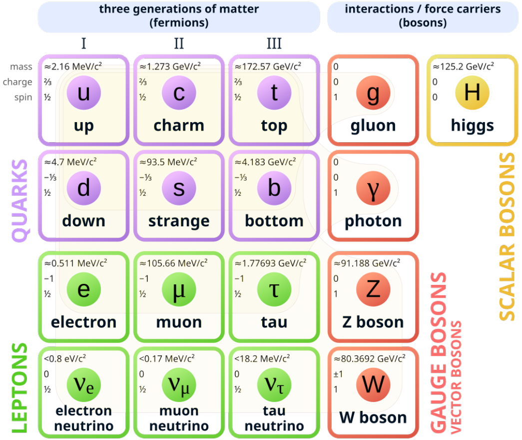
The Standard Model of Particle Physics

State of the art

Standard Model of Elementary Particles



High energy $\rightarrow$ Special Relativity
Small particles $\rightarrow$ Quantum Mechanics
$\downarrow$
Quantum Field Theory
(mathematical framework)

Evidences for physics beyond the SM

- Dark matter and dark energy
- Matter-antimatter asymmetry
- Origin of neutrino masses

Collider Physics

One of the principal methods of research

Large **H**adron **C**ollider at CERN

Proton beams accelerated to ~ 13.6 TeV

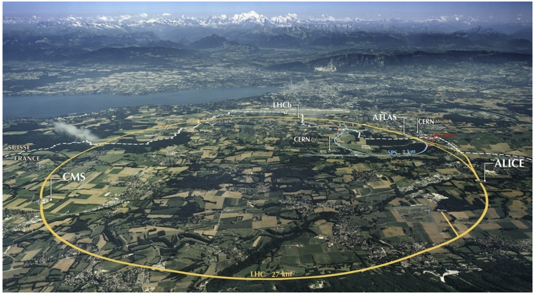


Figure: Maximilien Brice/CERN

Not sufficient to the discovery of new particles.

Solutions:

- Increase energy
  $\rightarrow$ not feasible or extremely expensive with existing technology
- Increase precision
  $\rightarrow$ both experimental and **theoretical**

Quantum Chromodynamics

Hard scattering processes

- Strong interactions are described by Quantum Chromodynamics (QCD).
- Non-Abelian gauge theory, $SU(3)$ symmetry group.
- The QCD Lagrangian is not analytically solvable.

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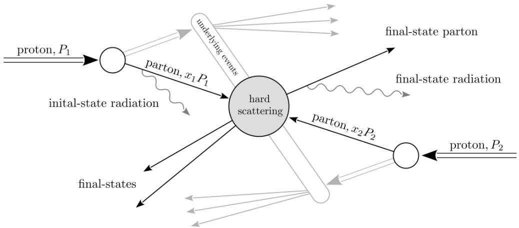


Figure: Konstantin Asteriadis/KIT

Hadron collisions

- Elastic scattering
- Diffractive dissociation
- **Hard scattering** (momentum exchange ~ 100 GeV)

Asymptotic freedom

Interacting partons can be approximated as being nearly free

$\rightarrow$ **Perturbative description**

The collinear factorization theorem

A framework for describing hard scattering processes

Colliding hadrons are treated as **beams of partons**, carrying a fraction of the hadron's total momentum

Separation of energy scales

- SM interactions $Q \sim 100\mathrm{GeV} - 1\mathrm{TeV}$
- Hadronic structure $\Lambda_{\mathrm{QCD}} \sim 100\mathrm{MeV}$

$\rightarrow$ **Decoupling** the motions of partons from proton's dynamics

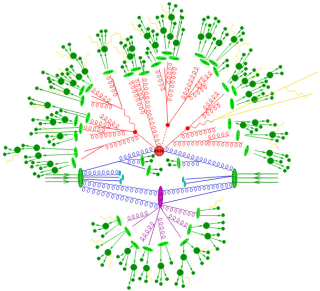


Figure: Stefan Höche/SLAC National Accelerator Laboratory

$$d\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, d\hat{\sigma}_{a,b}(x_1, x_2, \mu_F, \mu_R; \mathcal{O}) \left(1 + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^n\right),\ n \geq 1$$

The partonic cross section

A perturbative description

The partonic cross section can be expanded in powers of the strong and the electroweak coupling constants, α_S and α

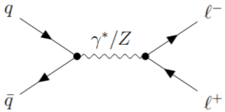
$$\mathrm{d}\hat{\sigma}_{a,b} = \mathrm{d}\hat{\sigma}_{a,b}^{(0,0)} + \alpha_s \mathrm{d}\hat{\sigma}_{a,b}^{(1,0)} + \alpha_s^2 \mathrm{d}\hat{\sigma}_{a,b}^{(2,0)} + \alpha_s^3 \mathrm{d}\hat{\sigma}_{a,b}^{(3,0)} + \alpha \mathrm{d}\hat{\sigma}_{a,b}^{(0,1)} + \alpha\alpha_s \mathrm{d}\hat{\sigma}_{a,b}^{(1,1)} + \ldots$$

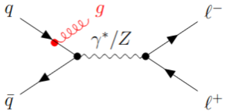
We focus on NLO QCD corrections

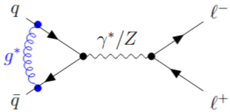
Account for short distance, high-energy effects

They consist of three terms

$$\mathrm{d}\hat{\sigma}_{a,b}^{\mathrm{NLO}} = \mathrm{d}\hat{\sigma}_{a,b}^{\mathrm{R}} + \mathrm{d}\hat{\sigma}_{a,b}^{\mathrm{V}} + \mathrm{d}\hat{\sigma}_{a,b}^{\mathrm{pdf}}$$







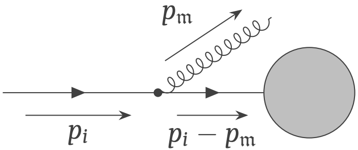
UV and IR divergences

A considerable obstacle

The treatment of real and virtual corrections is **non-trivial**

① **Ultraviolet** (UV) singularities in virtual contributions
$\longrightarrow$ Renormalization

② **Infrared** (IR) singularities in both real and virtual contributions
$\longrightarrow$ Low-momentum (soft) and small-angle (collinear) kinematic regions



$$\sim \frac{1}{(p_i - p_{\mathrm{m}})^2} = \frac{1}{2E_iE_{\mathrm{m}}\,(1-\cos\theta_{i\mathrm{m}})} \xrightarrow[E_{\mathrm{m}},\,\theta_{i\mathrm{m}}\to 0]{} \infty$$

UV and IR divergences

Dimensional regularization and pole cancellation

Dimensional regularization

$$d = 4 - 2\epsilon \qquad \epsilon \in \mathbb{C}\,, \mathrm{Re}(\epsilon) < 0$$

Divergences appear as poles in $1/\epsilon$

- Virtual $\rightarrow$ explicit
- Real $\rightarrow$ phase space integration needed ①

- Singularities signal the presence of long-distance (low-energy) effects
- The division into real and virtual terms is therefore a computational tool

Bloch-Nordsieck and Kinoshita-Lee-Nauenberg theorems

Infrared divergences are **guaranteed** to cancel when we sum real and virtual corrections

- **Mismatch** in the dimensionality of the integration domains ②

IR divergences

Subtraction scheme

Insights

- **Factorization** of the amplitudes in the soft and collinear limits
- Real emissions are **unresolved** in singular kinematics regions

We adopt a **subtraction method**

$$2s_{a,b}\mathrm{d}\hat{\sigma}^{\mathrm{R}}_{a,b} = \int [\mathrm{d}p_{\mathfrak{m}}] F_{\mathrm{LM}}(\mathfrak{m}) = {\color{red}\int [\mathrm{d}p_{\mathfrak{m}}](F_{\mathrm{LM}}(\mathfrak{m}) - \mathcal{S})} + {\color{blue}\int [\mathrm{d}p_{\mathfrak{m}}]\mathcal{S}}$$

${\color{blue}\longrightarrow}$ Describes the singular behaviour of the amplitude

${\color{red}\longrightarrow}$ Singular behaviour removed: numerical integration with Monte Carlo methods

Restricting the integration region to the **minimal necessary volume** is crucial for improving the efficiency of the computation

Current calculations require ~ 106 CPUh[1]
$\longrightarrow$ Further refinements needed for NNLO calculations

[1] Czakon et al., SciPost Phys. 11, 001 (2021), Tour de force in Quantum Chromodynamics

NSC Subtraction Scheme

Sequential extraction of singularities

- FKS (Frixione, Kunszt, Signer) constructs S from the soft and collinear limits

$$S_i A = \lim_{E_i \to 0} A \,, \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A$$

- The singularities are isolated and extracted **sequentially** in a nested manner

$$\langle \Delta^{(\mathfrak{m})} F_{\mathrm{LM}}(\mathfrak{m}) \rangle = \langle S_{\mathfrak{m}} F_{\mathrm{LM}}(\mathfrak{m}) \rangle + \sum_{i=1}^{N_p} \langle \bar{S}_{\mathfrak{m}} C_{i\mathfrak{m}} \Delta^{(\mathfrak{m})} F_{\mathrm{LM}}(\mathfrak{m}) \rangle + \langle \bar{S}_{\mathfrak{m}} \bar{C}_{i\mathfrak{m}} \Delta^{(\mathfrak{m})} \omega^{\mathfrak{m}i} F_{\mathrm{LM}}(\mathfrak{m}) \rangle$$

Computational cost
Subtraction scheme

$$\langle S_{\mathfrak{m}} F_{\mathrm{LM}}(\mathfrak{m}) \rangle = \int \mathrm{d}E_{\mathfrak{m}} E_{\mathfrak{m}}^{d-5}\, \theta(E_{\max} - E_{\mathfrak{m}}) \int \frac{\mathrm{d}\Omega_{d-1}}{2(2\pi)^{d-1}} (-g_{s,b}^2) \sum_{(i \neq j)}^{N_p} \frac{\rho_{ij}}{\rho_{i\mathfrak{m}} \rho_{j\mathfrak{m}}} (\vec{T}_i \cdot \vec{T}_j) F_{\mathrm{LM}}\,,$$

Computational cost

Subtraction scheme

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