

# VISION - TME 6 – Implementation of two optical flow methods

Lucrezia Tosato & Marie Diez

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Comparaison Horn-Schunck and Lucas-Kanade methods</b>	<b>2</b>
2.1	With reference .flo . . . . .	2
2.1.1	Mysine . . . . .	2
2.1.2	Rubberwhale . . . . .	6
2.1.3	Square . . . . .	10
2.1.4	Yosemite . . . . .	14
2.2	Without reference .flo . . . . .	18
2.2.1	Personal experiment . . . . .	18
2.2.2	Nasa . . . . .	19
2.2.3	Rubic . . . . .	20
2.2.4	Taxi . . . . .	21
<b>3</b>	<b>Conclusion</b>	<b>23</b>
<b>4</b>	<b>Bibliography</b>	<b>24</b>

# 1 Introduction

In this practicals we will implement and compare 2 methods of flow estimations : Horn-Schunck and Lucas-Kanade.

## 2 Comparaison Horn-Schunck and Lucas-Kanade methods

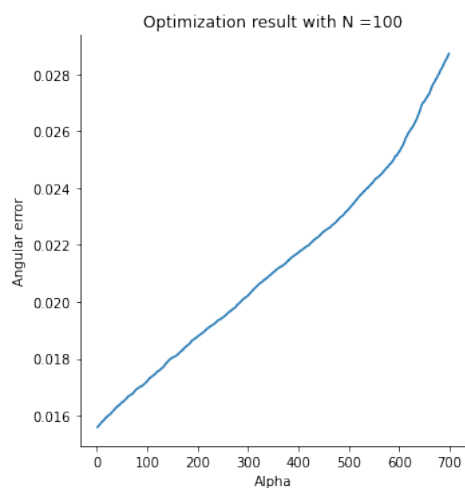
### 2.1 With reference .flo

#### 2.1.1 Mysine

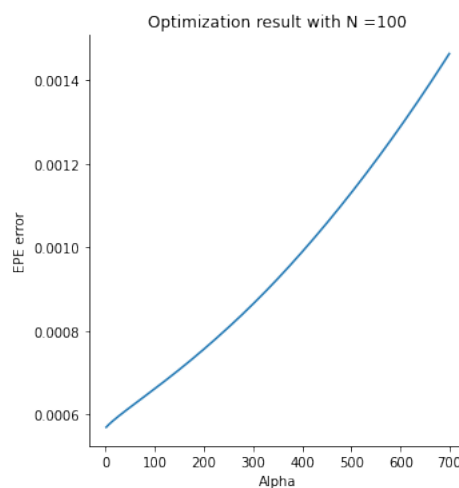
First, we will optimize the parameters of Horn-Schunck and lucas Kanade methods.

- Horn-Schunck  
The regularization parameter  $\alpha$
- Lucas-Kanade  
windows size  $ws$

#### Horn-Schunck optimisation



(a) Alpha in function of the angular error



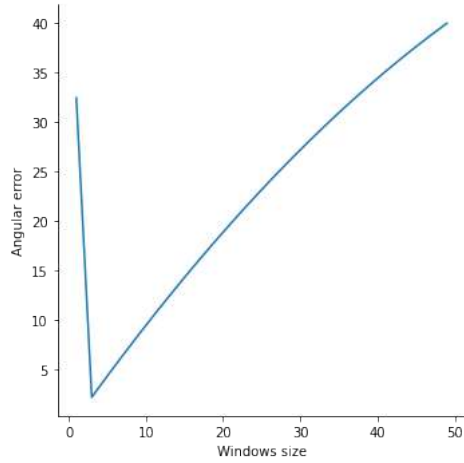
(b) Alpha in function of the EPE error

Figure 1: Alpha in function of different errors

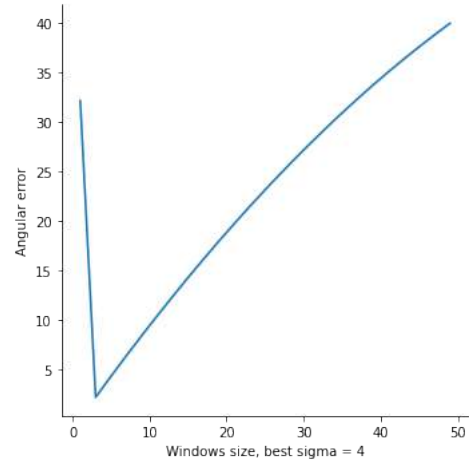
Optimal value  $\alpha^*$  with N=100 minimizing :

- Angular error :  $\alpha^* = 1$
- EPE error :  $\alpha^* = 1$

## Lucas optimisation

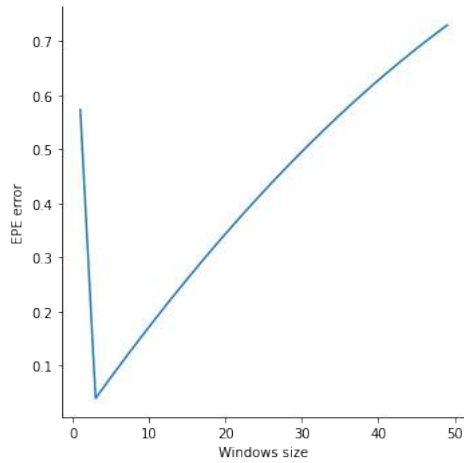


(a) Windows size in function with rectangular windows

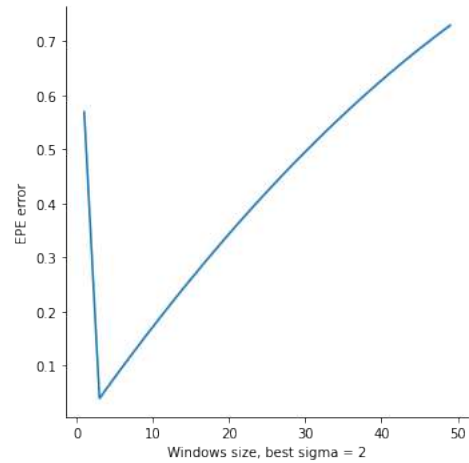


(b) Windows size in function with gaussian windows

Figure 2: Windows size optimal minimizing the angular error



(a) Windows size in function with rectangular windows



(b) Windows size in function with gaussian windows

Figure 3: Windows size optimal minimizing the EPE error

Remark : The best sigma for the gaussian windows is found between 0 and 5.

Optimal value  $*$  with  $N=100$  minimizing :

- Angular error :
  - Rectangular windows :  $*$  = 3
  - Gaussian windows :  $*$  = 3
- EPE error :
  - Rectangular windows :  $*$  = 3
  - Gaussian windows :  $*$  = 3

Now, we will compare the result of Horn-Schunck and Lucas-Kanade methods with the reference image and reference flow with :

- Horn-Schunck  
 $\alpha^* = 1$  optimizing with angular error
- Lucas-Kanade  
 $ws^* = 1$  optimizing with angular error and gaussian windows with  $\sigma^* = 2$

### Estimated flow comparaison

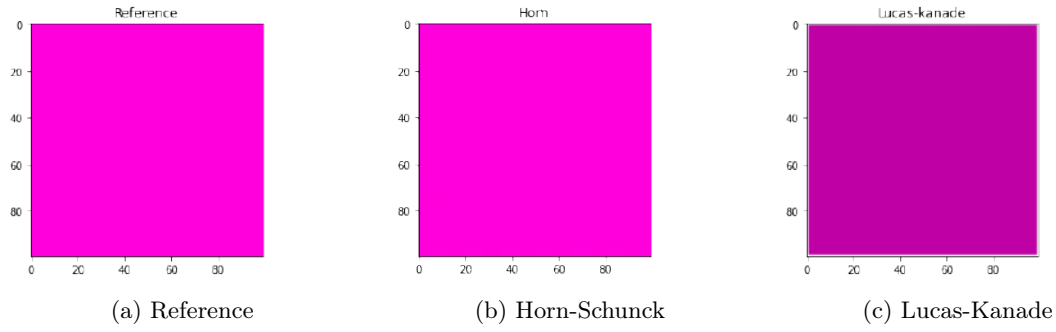


Figure 4: Mysine

### Quiver comparison

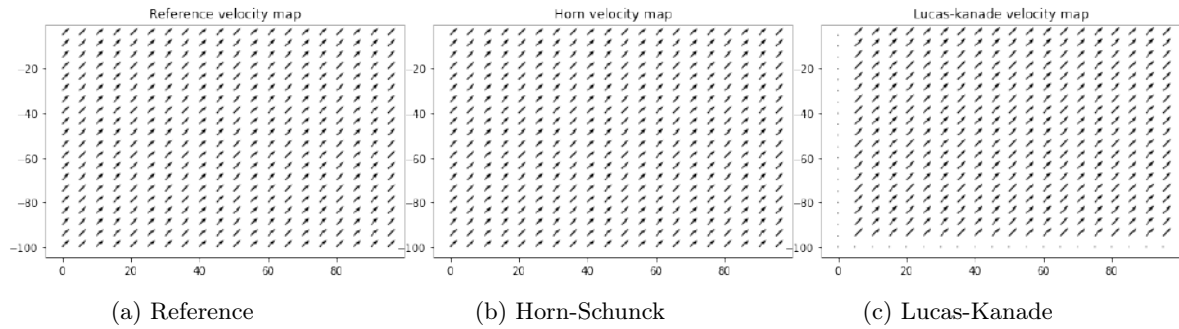


Figure 5: Mysine

### Statistiques comparaison

Table of statistiques for Horn-Schunck with  $\alpha^* = 1$  :

Table of statistiques	Mean	Std
Angular	0.01558626560247794	0.06054951924874972
Norm	0.00029941915529376996	0.0016641124324510573
EPE	0.0005700596780912053	0.0015918398103482523

Table of statistiques for Lucas-Kanade with  $ws^* = 1$  with gaussian windows  $\sigma = 2$ :

Table of statistiques	Mean	Std
Angular	2.167949104926676	10.6743329161505065
Norm	0.03960669669341888	0.19501676901389972
EPE	0.039611922814563806	0.19501570754818248

## Interpretation

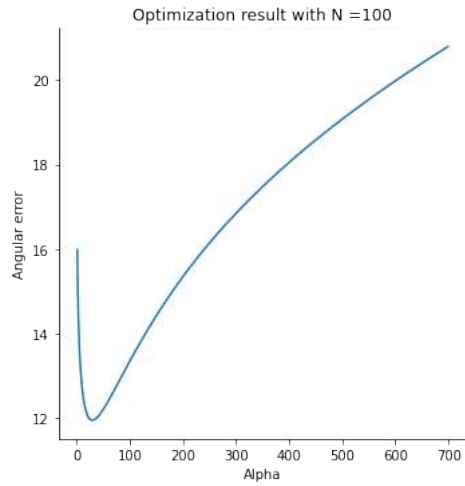
- We can see that the Horn-Schunck estimation (global method) is almost perfect for mysine, with very low error for the 3 tested (angular/EPE/norm), instead of Lucas-Kanade (local method) which has a less perfect estimation of the flow of the velocity map, indeed here as the image are very simple and without any noise or imperfections, Horn-Schunck can give a perfect solution, however Lucas use a windows and therefore we have some pixels that will be suppress and leads to a small higer error.
- Lucas-Kanade method have the same result with rectangular kernel or gaussian kernel, indeed there are no noise in the images so the windows type doesn't matter.
- We can also see that optimizing the angular error or the EPE error give approximatively the same best parameters, and therefore lead to approximatively same results.

### 2.1.2 Rubberwhale

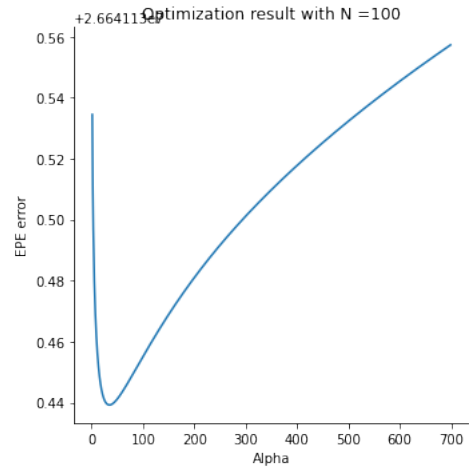
First, we will optimize the parameters of Horn-Schunck and Lucas Kanade methods.

- Horn-Schunck  
The regularization parameter  $\alpha$
- Lucas-Kanade  
windows size  $ws$

#### Horn-Schunck optimisation



(a) Alpha in function of the angular error



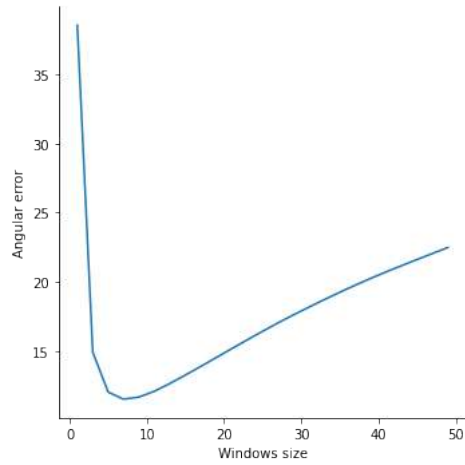
(b) Alpha in function of the EPE error

Figure 6: Alpha in function of different errors

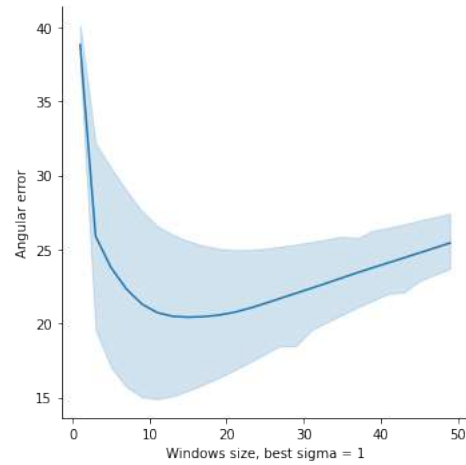
Optimal value  $\alpha^*$  with N=100 minimizing :

- Angular error :  $\alpha^* = 29$
- EPE error :  $\alpha^* = 34$

## Lucas optimisation

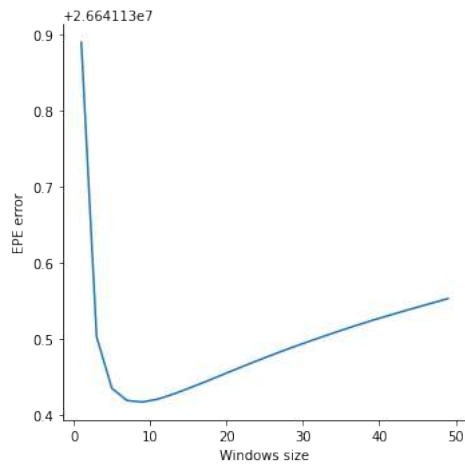


(a) Windows size in function with rectangular windows

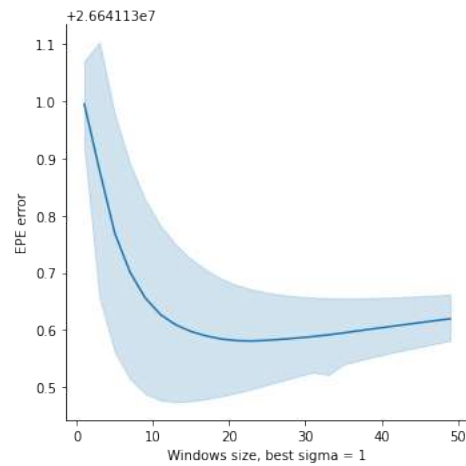


(b) Windows size in function with gaussian windows

Figure 7: Windows size optimal minimizing the angular error



(a) Windows size in function with rectangular windows



(b) Windows size in function with gaussian windows

Figure 8: Windows size optimal minimizing the EPE error

Remark : The best sigma for the gaussian windows is found between 0 and 5.

Optimal value \* with  $N=100$  minimizing :

- Angular error :
  - Rectangular windows : \* = 7
  - Gaussian windows : \* = 9
- EPE error :
  - Rectangular windows : \* = 9
  - Gaussian windows : \* = 11

Now, we will compare the result of Horn-Schunck and Lucas-Kanade methods with the reference image and reference flow with :

- Horn-Schunck  
with  $\alpha^* = 29$  is optimizing with angular error
- Lucas-Kanade  
with  $ws^* = 9$  is optimizing with angular error and gaussian windows with  $\sigma^* = 1$

### Estimated flow comparaison

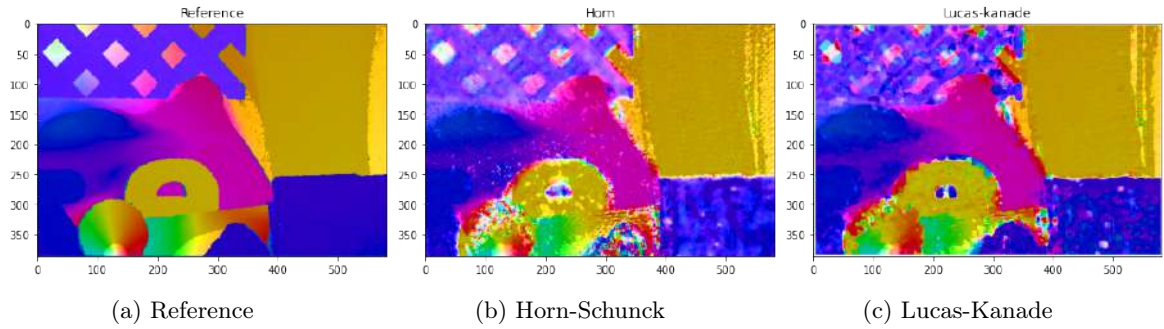


Figure 9: Rubberwhale

### Quiver comparison

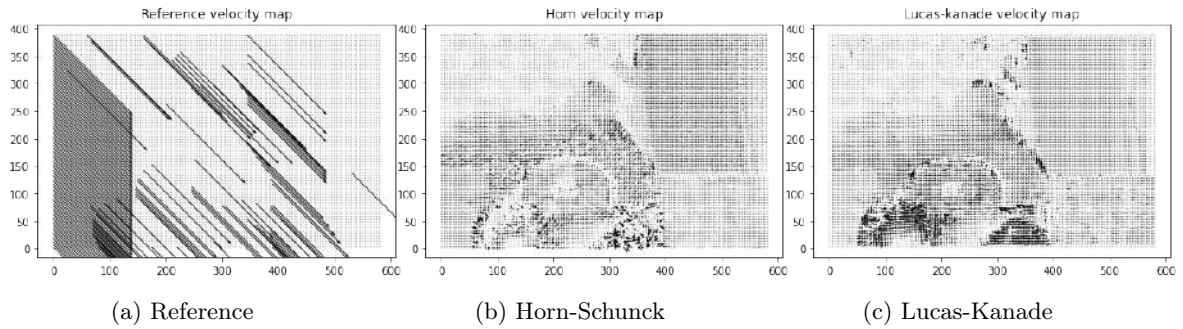


Figure 10: Rubberwhale

### Statistiques comparaison

Table of statistiques for Horn-Schunck with  $\alpha^* = 29$  :

Table of statistiques	Mean	Std
Angular	11.957872931368666	19.799655254654436
Norm	26641130.224221632	209026639.66150394
EPE	26641130.43920814	209026639.744343

Table of statistiques for Lucas-Kanade with  $ws^* = 9$  with gaussian windows with  $\sigma^* = 1$  :

Table of statistiques	Mean	Std
Angular	12.14709995727873	21.33014884511613
Norm	26641129.587678067	209026639.68244085
EPE	26641130.4184188	209026639.7588433



## Interpretation

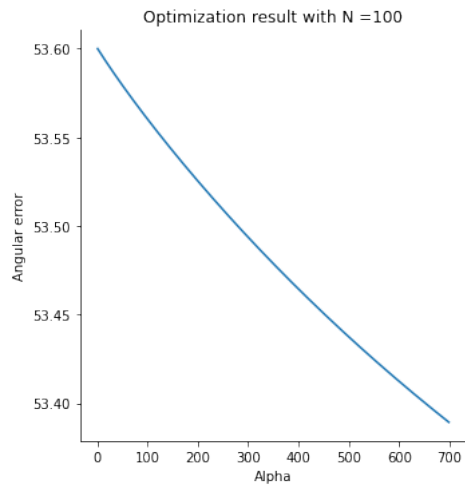
- We can see that the two algorithms do not give very interesting results on flow and velocity estimation, the images are indeed more complex than the previous one and the algorithm struggle to find a good solution.
- We can notice that the EPE error has very different order of magnitude than the angular error (Figure 7/8), this is due to the pixel wise comparaison with the EPE error that is very high in case of bad flow estimation.

### 2.1.3 Square

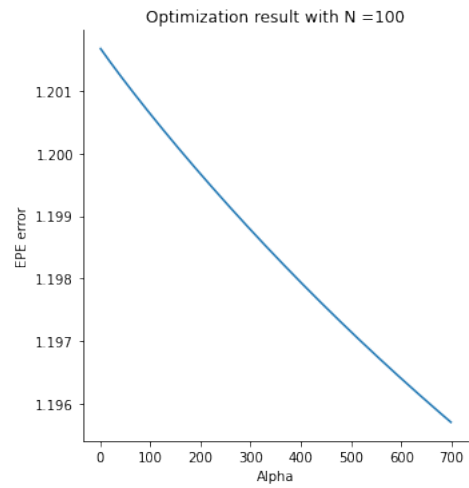
First, we will optimize the parameters of Horn-Schunck and Lucas Kanade methods.

- Horn-Schunck  
The regularization parameter  $\alpha$
- Lucas-Kanade  
windows size  $ws$

#### Horn-Schunck optimisation



(a) Alpha in function of the angular error



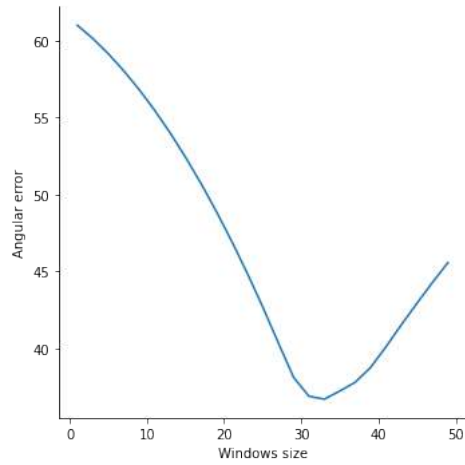
(b) Alpha in function of the EPE error

Figure 11: Alpha in function of different errors

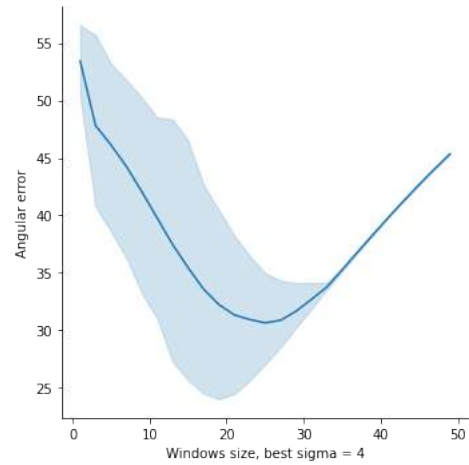
Optimal value  $\alpha^*$  with N=100 minimizing :

- Angular error :  $\alpha^* = 699$
- EPE error :  $\alpha^* = 699$

## Lucas optimisation

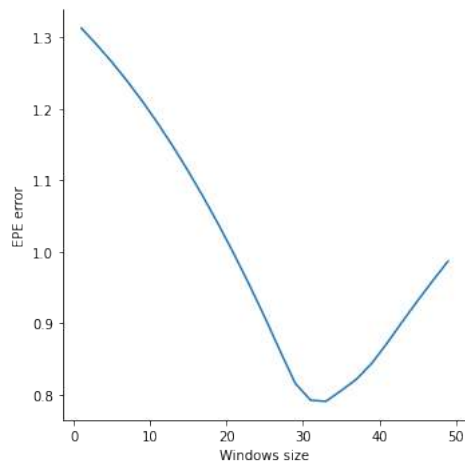


(a) Windows size in function with rectangular windows

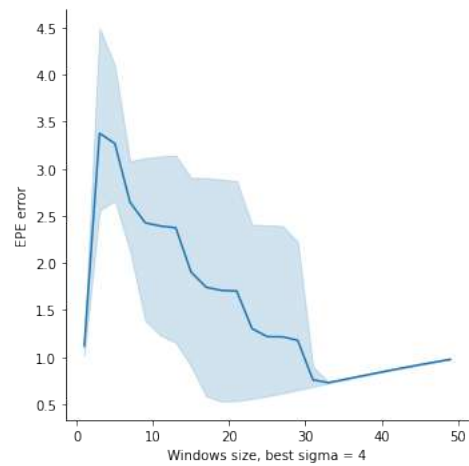


(b) Windows size in function with gaussian windows

Figure 12: Windows size optimal minimizing the angular error



(a) Windows size in function with rectangular windows



(b) Windows size in function with gaussian windows

Figure 13: Windows size optimal minimizing the EPE error

Remark : The best sigma for the gaussian windows is found between 0 and 5.

Optimal value \* with N=100 minimizing :

- Angular error :
  - Rectangular windows : \* = 33
  - Gaussian windows : \* = 15
- EPE error :
  - Rectangular windows : \* = 33
  - Gaussian windows : \* = 15

Now, we will compare the result of Horn-Schunck and Lucas-Kanade methods with the reference image and reference flow with :

- Horn-Schunck  
with  $\alpha^* = 699$  is optimizing with angular error
- Lucas-Kanade  
with  $ws^* = 15$  is optimizing with angular error and gaussian windows with  $\sigma^* = 4$

### Estimated flow comparaison

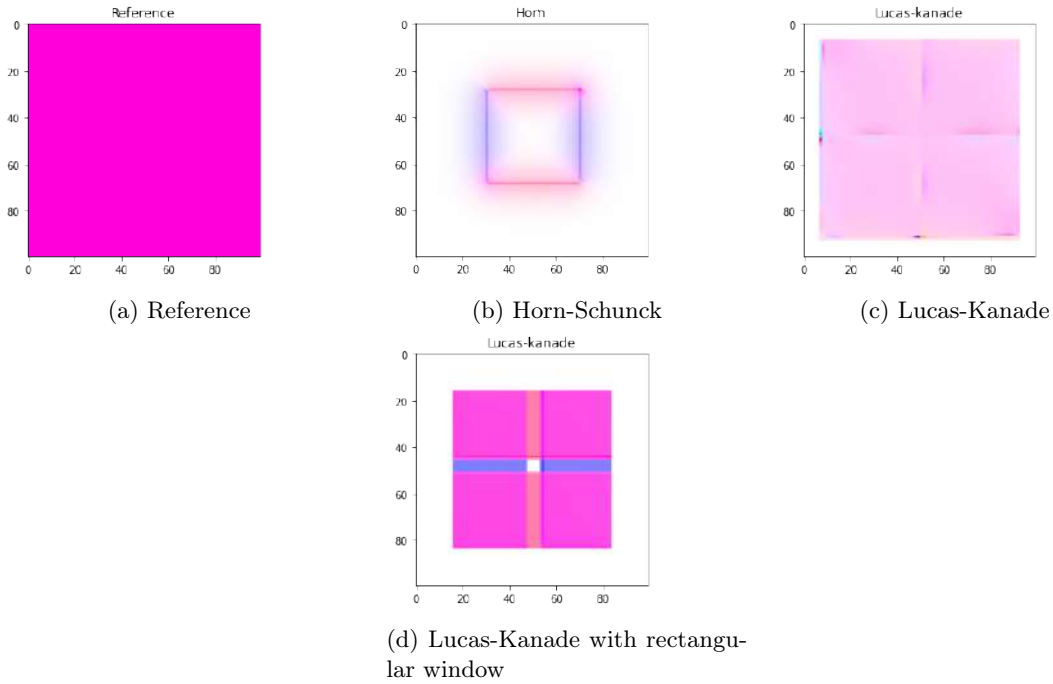


Figure 14: Square

### Quiver comparaison

### Statistiques comparaison

Table of statistiques for Horn-Schunck with  $\alpha^* = 29$  :

Table of statistiques	Mean	Std
Angular	20.450743481939718	26.031066744560995
Norm	-3.613403497526708	227.99599100464377
EPE	0.4570943953555843	0.5846665074047942

The negative value of Norm can be interpreted ad correct since we are normalizing the reference optical flow and the just calculated optical flow, so, when we are doing the subtraction it can happen to be negative.

Table of statistics for Lucas-Kanade with  $ws^* = 9$  with Gaussian windows  $\sigma^* = 4$  :

Table of statistiques	Mean	Std
Angular	12.14709995727873	21.33014884511613
Norm	>1000000	>1000000
EPE	>1000000	>1000000

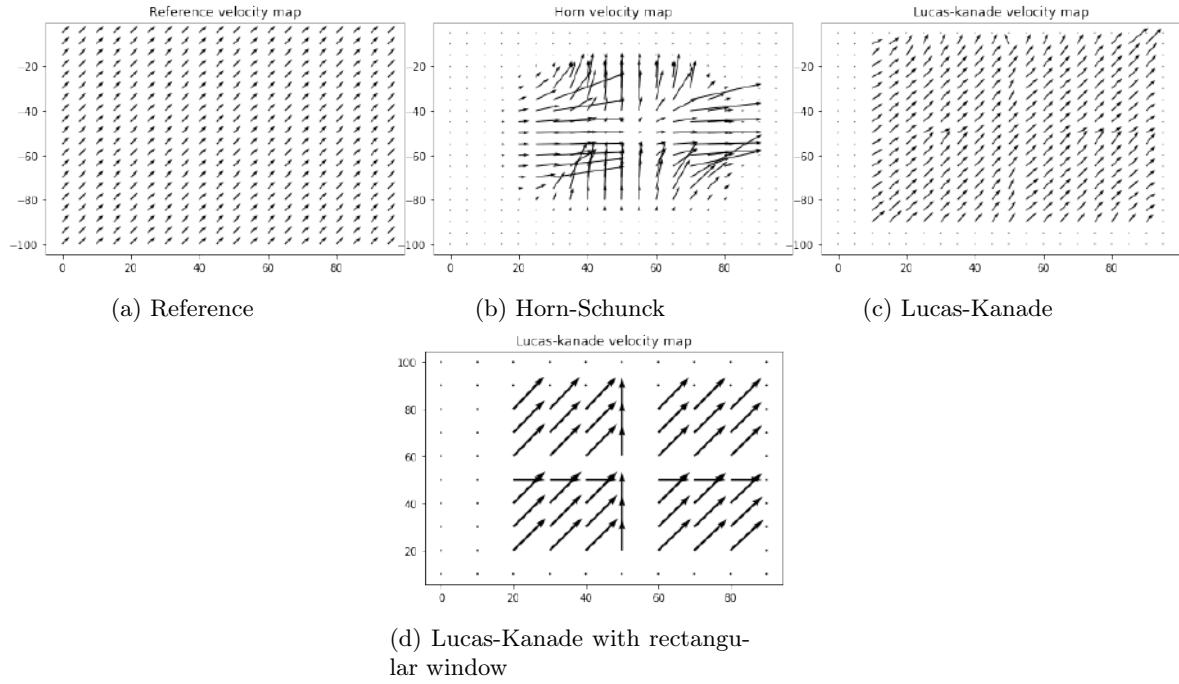


Figure 15: Square

## Interpretation

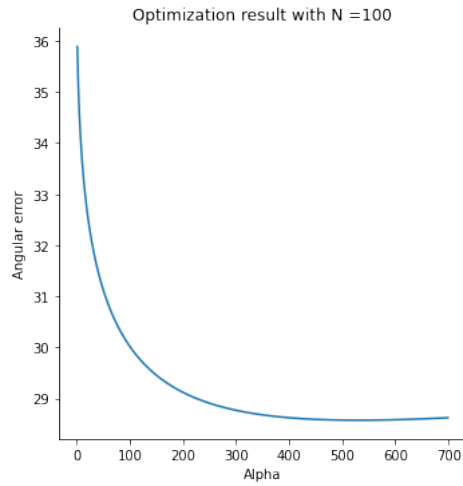
- In this situation we have much better estimation with the local estimation of Lucas-Kanade method, specially with the Gaussian smoothing windows. The local method seems to better understand the movement. Without the Gaussian windows, since it is a purely local method, Lucas-Kanade cannot provide flow information in the interior of uniform regions of the image. This is why here we have much better results with the Gaussian windows that help the propagation of the flow estimation.
- The Horn-Schunck method doesn't give good result, the advantages of the Horn-Schunck algorithm are that it produces a high density of flow vectors, the missing flow information in homogeneous objects is filled in from the motion boundaries. However it seems to struggle to find a good motion estimation with local motion of object.

### 2.1.4 Yosemite

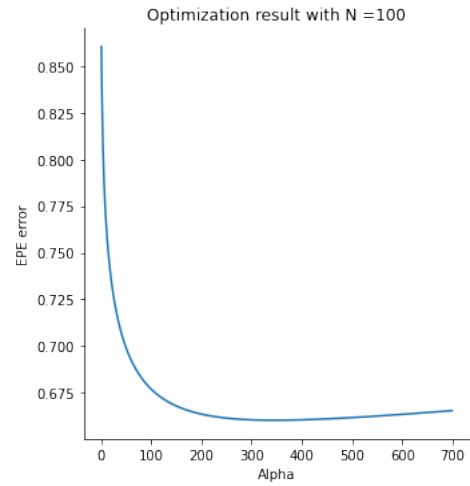
First, we will optimize the parameters of Horn-Schunck and Lucas Kanade methods.

- Horn-Schunck  
The regularization parameter  $\alpha$
- Lucas-Kanade  
windows size  $ws$

#### Horn-Schunck optimisation



(a) Alpha in function of the angular error



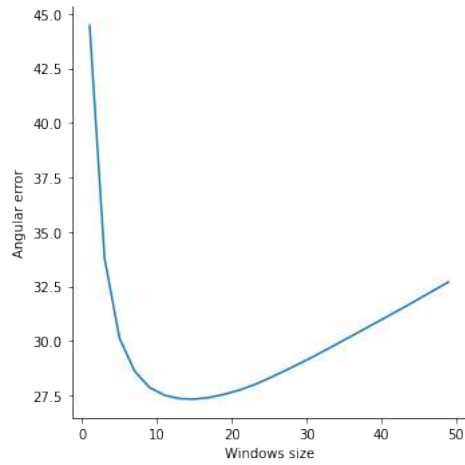
(b) Alpha in function of the EPE error

Figure 16: Alpha in function of different errors

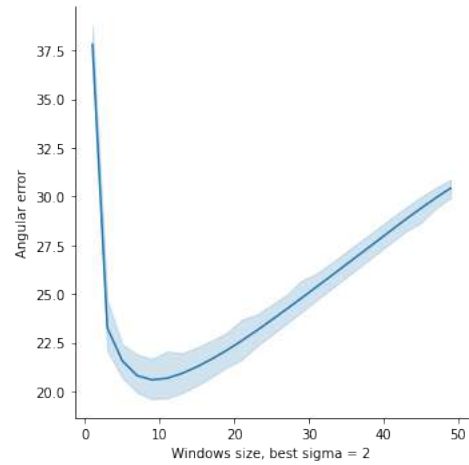
Optimal value  $\alpha^*$  with N=100 minimizing :

- Angular error :  $\alpha^* = 530$
- EPE error :  $\alpha^* = 346$

## Lucas optimisation

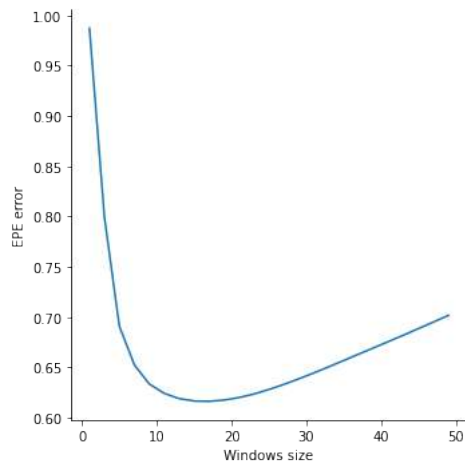


(a) Windows size in function with rectangular windows

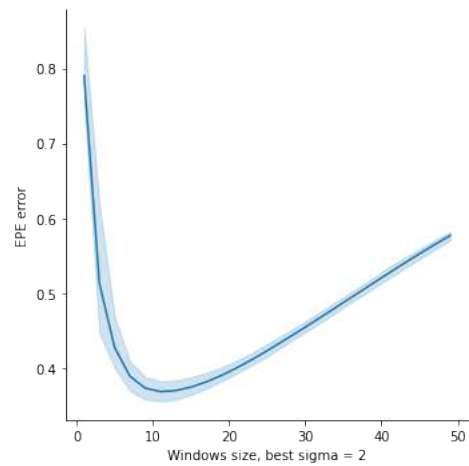


(b) Windows size in function with gaussian windows

Figure 17: Windows size optimal minimizing the angular error



(a) Windows size in function with rectangular windows



(b) Windows size in function with Gaussian windows

Figure 18: Windows size optimal minimizing the EPE error

Remark : The best sigma for the Gaussian windows is found between 0 and 5.

Optimal value  $*$  with  $N=100$  minimizing :

- Angular error :
  - Rectangular windows :  $*$  = 15
  - Gaussian windows :  $*$  = 9
- EPE error :
  - Rectangular windows :  $*$  = 17
  - Gaussian windows :  $*$  = 11

Now, we will compare the result of Horn-Schunck and Lucas-Kanade methods with the reference image and reference flow with :

- Horn-Schunck  
with  $\alpha^* = 530$  is optimizing with angular error
- Lucas-Kanade  
with  $ws^* = 9$  is optimizing with angular error and Gaussian windows with  $\sigma^* = 2$

### Estimated flow comparison

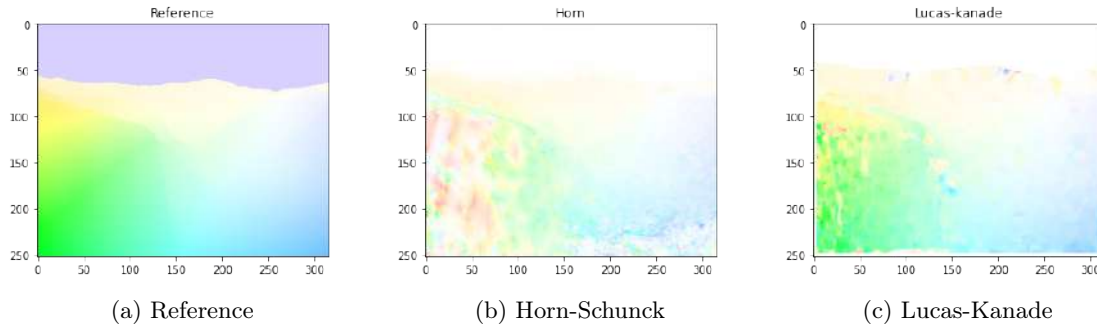


Figure 19: Yosemite

### Quiver comparison

#### Statistics comparison

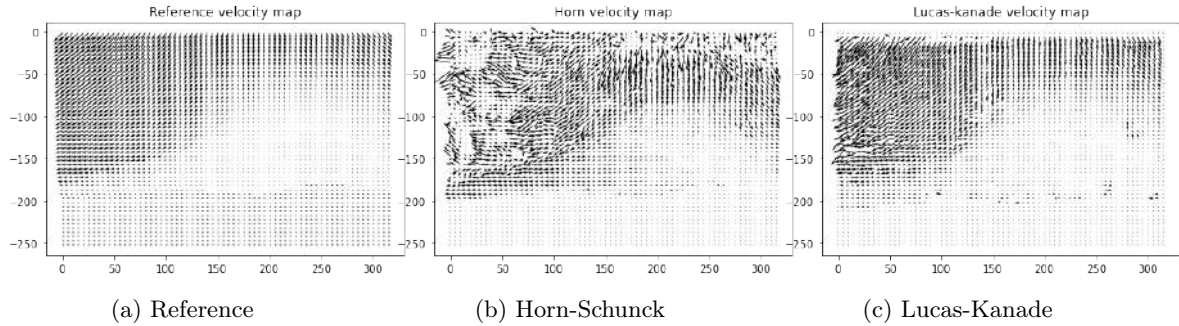


Figure 20: Yosemite

Table of statistics for Horn-Schunck with  $\alpha^* = 530$  :

Table of statistics	Mean	Std
Angular	28.576681091718545	22.482139766763922
Norm	0.6601982954478179	0.8439329764781277
EPE	0.6601982954478179	0.8439329764781277

Table of statistics for Lucas-Kanade with  $ws^* = 9$  with Gaussian windows  $\sigma^* = 2$  :

Table of statistics	Mean	Std
Angular	19.512621873991748	23.971098545984503
Norm	0.02970556899141261	1.352826923007677
EPE	0.3497124285593208	0.5934929765990018



## Interpretation

- We have here a more complex situation with noise and imperfections. We can see much better with Lucas-Kanade algorithm which is more robust to noise than the global Horn-Schunck method. The Gaussian windows give better results than the rectangular windows (Figure 18) because of the low-pass effect of Gaussian convolution that removes noise and other destabilising high frequencies.

## 2.2 Without reference .flo

### 2.2.1 Personal experiment

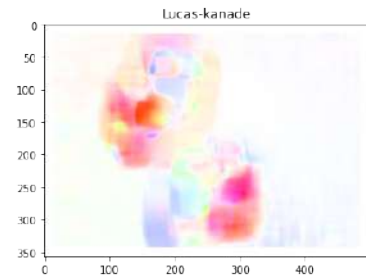


(a) image 1

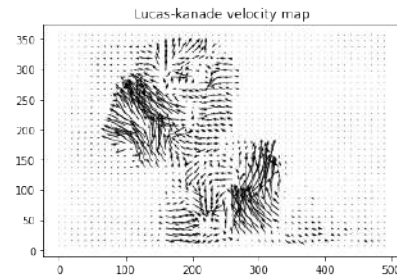


(b) image 2

Figure 21: Personal images

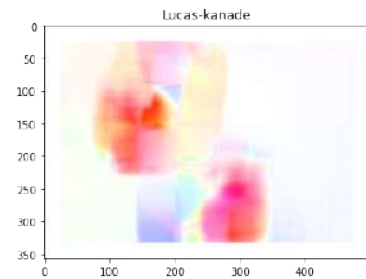


(a) Flow estimation with Lucas-kanade with gaussian windows,  $ws=30$ ,  $\sigma=2$

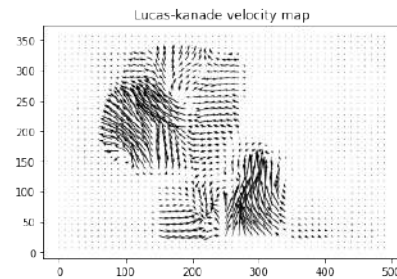


(b) Velocity map with Lucas-kanade with gaussian windows,  $ws=30$ ,  $\sigma=2$ ,  $step=10$

Figure 22: Personal images



(a) Flow estimation with Lucas-kanade with gaussian windows,  $ws=50$ ,  $\sigma=2$



(b) Velocity map with Lucas-kanade with gaussian windows,  $ws=50$ ,  $\sigma=2$ ,  $step=10$

Figure 23: Personal images

We can see here that the regularisation parameter  $\alpha$  of the Horn-Schunck method enable to control the smoothing and help to be less sensitive to noise, when  $\alpha$  is high we then have better result here. However Lucas-Kanade give better result, the method is much less sensitive to noise and imperfections when the window size is sufficiently big (we have to be careful to not choose a too big window to avoid loss of informations).

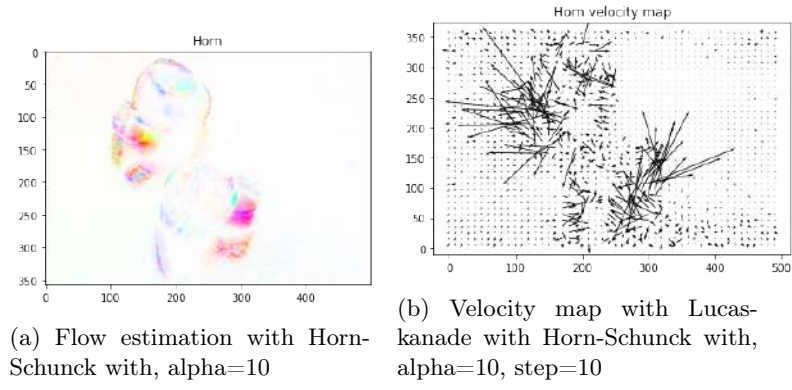


Figure 24: Personal images

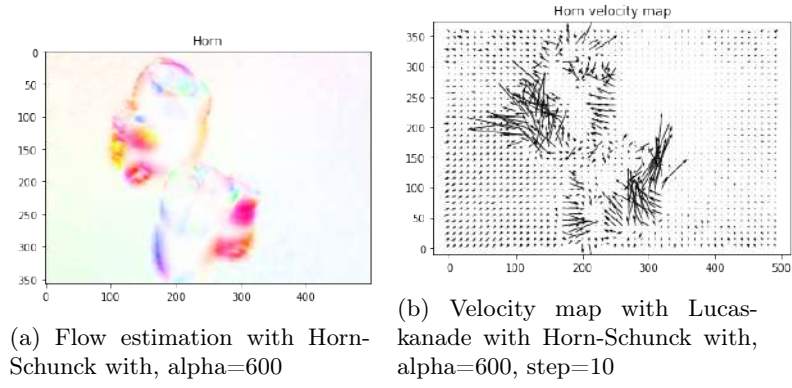


Figure 25: Personal images

## 2.2.2 Nasa

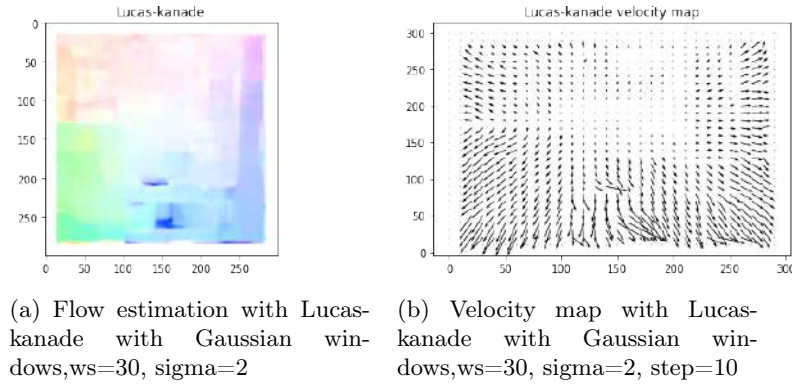


Figure 26: Nasa Lucas-kanade

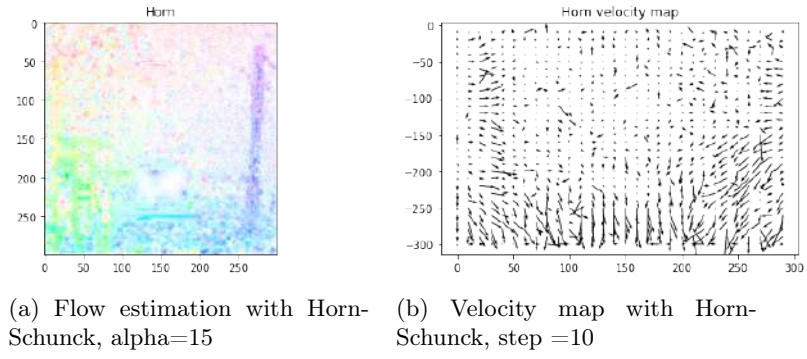


Figure 27: Nasa Horn-Schunck

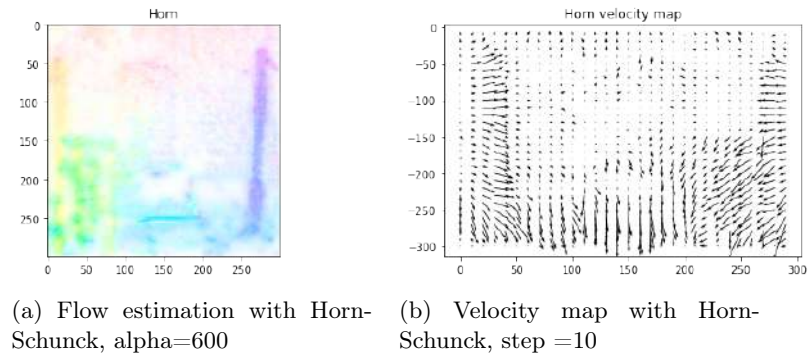


Figure 28: Nasa Horn-Schunck

### 2.2.3 Rubic

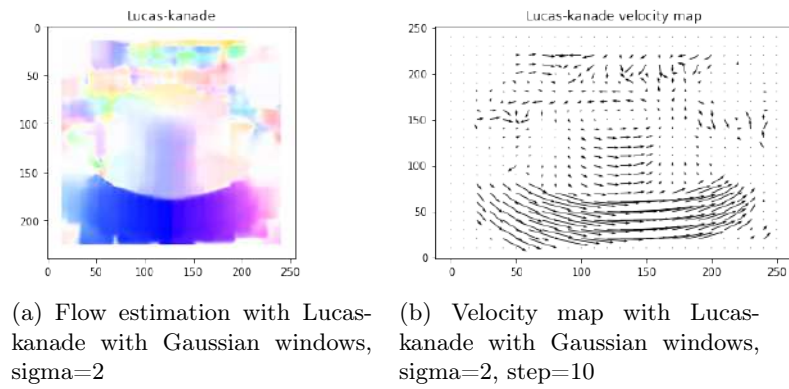


Figure 29: Rubic Lucas-kanade

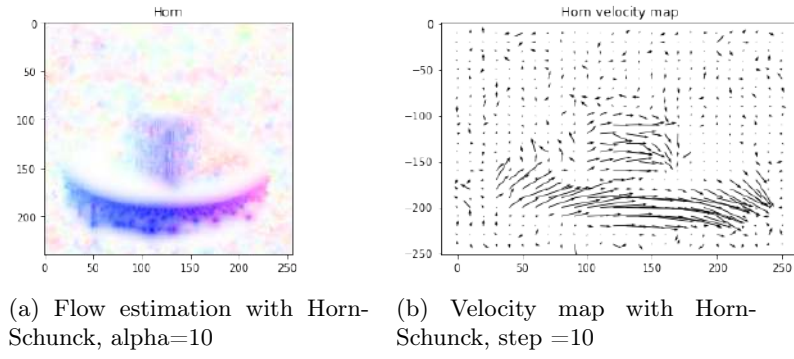


Figure 30: Rubic Horn-Schunck

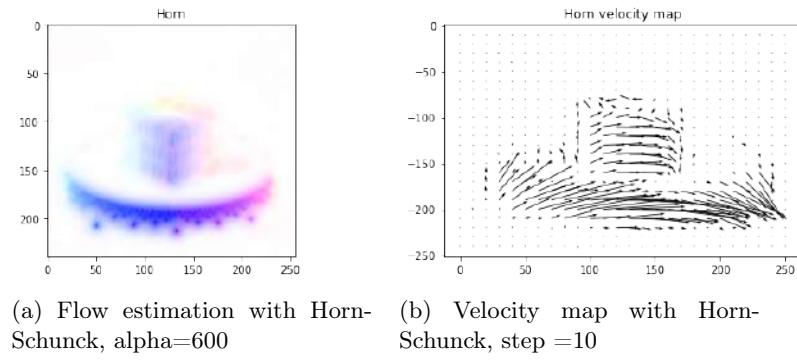


Figure 31: Rubic Horn-Schunck

## 2.2.4 Taxi

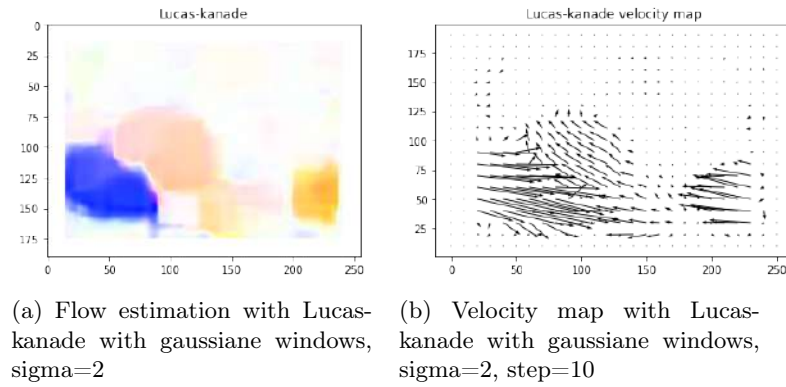


Figure 32: Taxi Lucas-kanade

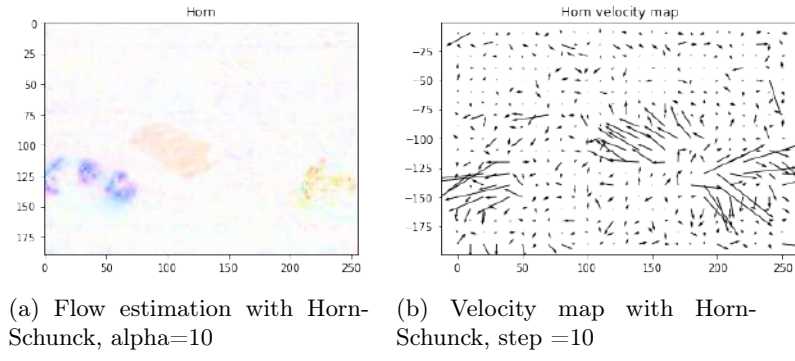


Figure 33: Taxi Horn-Schunck

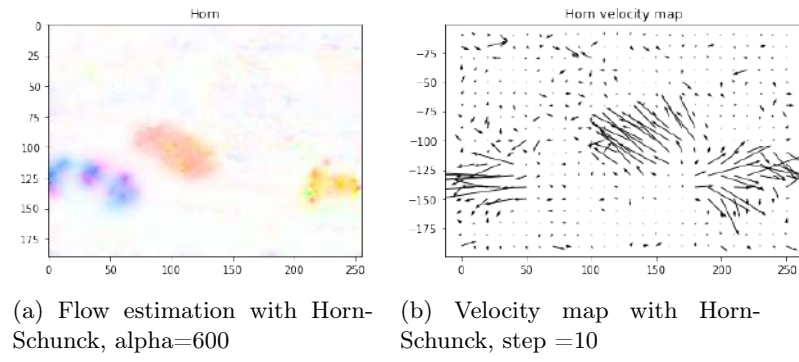


Figure 34: Taxi Horn-Schunck

### 3 Conclusion

To conclude on this practicals we saw 2 method of flow estimation : Horn-Schunck and Lucas-Kanade, the choice of this methods depends on the situation. In one hand we have a global method which more computational time processing and more sensitive to noise than the local method Lucas-Kanade but the regularization of Horn-Schunck improves the motion estimation in regions with little image structure compared to the Lucas-Kanade algorithm. And in the other hand the local faster Lucas-Kanade method more robust to noise and imperfection but that cannot provide flow information in the interior of uniform regions of the image, and for that and the noise reduction the Gaussian window can help.

## 4 Bibliography

[1]. S. D. T. Gudipudi Kanaka Sunanda Vemulapalli ,. Kartheek Chintalapati ,. Phanindra Sai Srinivas, "IJEET - Comparison Between The Optical Flow Computational Techniques," International Journal of Engineering Trends and Technology - IJETT, Accessed: Jan. 23, 2022. [Online]. Available: <http://ijettjournal.org/archive/ijett-v4i10p142>

R. Nieuwenhuizen, J. Dijk, and K. Schutte, "Dynamic turbulence mitigation for long-range imaging in the presence of large moving objects," EURASIP Journal on Image and Video Processing, vol. 2019, no. 1, p. 2, Jan. 2019, doi: 10.1186/s13640-018-0380-9.