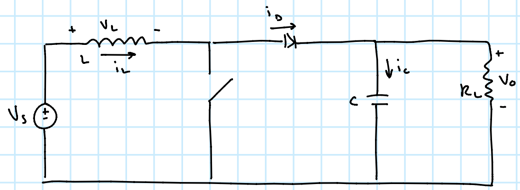
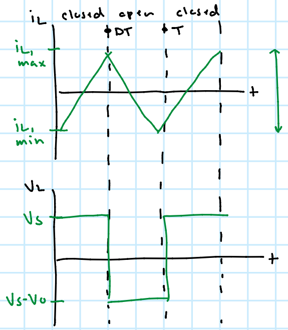
Boost Converter Analysis

Wednesday, September 9, 2020 11:00 AM



Switch closed:

- diode reverse biased
- $V_L = V_s = L\frac{di_L}{dt} \rightarrow \frac{di_L}{dt} = \frac{V_s}{L}$ (constant)
- $\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L} \rightarrow \Delta i_L = \frac{V_s DT}{L}$ current increases linearly



Switch open:

- B/C inductor current cannot change instantaneously, diode becomes forward biased to provide a path for the inductor current
- $V_L = V_s - V_o = L\frac{di_L}{dt} \rightarrow \frac{di_L}{dt} = \frac{V_s - V_o}{L}$ (constant)
- $\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L} \rightarrow \Delta i_L = \frac{(V_s - V_o)(1-D)T}{L}$

current decreases linearly

Steady-state operation,

net change in i_L must be zero:

$\Delta i_{L,closed} + \Delta i_{L,open} = 0$

$\frac{V_s DT}{L} + \frac{(V_s - V_o)(1-D)T}{L} = 0$

$V_s(D + 1 - D) - V_o(1-D) = 0$

$$\boxed{V_o = \frac{V_s}{1-D}}$$

average inductor voltage must be zero:

$V_L = V_s D + (V_s - V_o)(1-D) = 0$

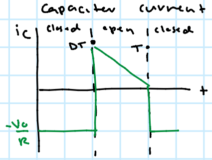
$V_o = \frac{V_s}{1-D}$

verify ✓

this would allow for V_o to approach ∞, however non-ideal components have losses which prevent this

Output Voltage Ripple

- finite capacitance yields fluctuation in output voltage (ripple)
- Peak-to-peak output voltage ripple can be calculated from capacitor current waveform



change in capacitor charge:

$|\Delta Q_c| = \frac{V_o}{R} DT = C\Delta V_o$

ripple voltage:

$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf} \rightarrow \frac{\Delta V_o}{V_o} = \frac{D}{RCf}$

capacitance in terms of output voltage:

$$\boxed{C = \frac{D}{R\left(\frac{\Delta V_o}{V_o}\right)f}}$$

Voltage ripple due to equivalent series resistance (ESR)

$$\boxed{\Delta V_{o,ESR} = \Delta i_c \cdot r_C = I_{L,max} \cdot r_C}$$

Power

- average power supplied by source = average power absorbed by R_L

$P_o = \frac{V_o^2}{R} = V_o I_o$

$P_i = V_s I_s = V_s I_L$

$V_s I_L = \frac{V_o^2}{R} = \frac{\left[\frac{V_s}{(1-D)}\right]^2}{R} = \frac{V_s^2}{(1-D)^2 R}$

- I_L can be expressed as

$I_L = \frac{V_s}{(1-D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s}$

$i_{L,min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$

$i_{L,max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$

- Continuous inductor current denotes > 0,

$I_{min} = 0 = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \rightarrow \frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} \rightarrow \frac{V_s D}{2Lf}$

- minimum inductance value/operating frequency for continuous operation:

$(Lf)_{min} = \frac{D(1-D)^2 R}{2} \rightarrow L_{min} = \frac{D(1-D)^2 R}{2f} \rightarrow \frac{V_s DT}{\Delta i_L} = \frac{V_s D}{\Delta i_L f}$

Example

$V_o = 8V$, $I_o = 1A$, $V_s = 2.7 - 4.2V$, ripple < 2%, Δi_L < 40% $i_{L,av}$

- $D = 1 - \frac{V_s}{V_o} = 0.663, 0.475$
- $I_L = \frac{V_o I_o}{V_s} = 2.96A, 1.9A$
- $\Delta i_L = 0.4(I_L) = 1.19A, 0.762A$
- $L = \frac{V_s D}{\Delta i_L f} = 7.5\mu H, 13.1\mu H$
- $C = \frac{D}{R(\Delta V_o / V_o) f} = \frac{D}{(V_o/I_o)(\Delta V_o / V_o) f} = ?$
- $\Delta i_L = \frac{V_s D}{Lf} = 0.683A$
- $I_{L,max,2.7V} = I_L + \frac{\Delta i_L}{2} = 3.3A$
- $I_{L,max,4.2} = I_L + \frac{\Delta i_L}{2} = 2.28A$
- $\Delta V_{o,ESR} = \Delta i_C r_C = I_{L,max} r_C = 3.3$

$r_C = \frac{0.16V}{3.3A} = 48m\Omega$

Example

$V_o = 30V$, $V_s = 12V$, ripple < 1%, $R = 50\Omega$, $f = 25kHz$

- $D = 1 - \frac{V_s}{V_o} = 0.6$
- $L_{min} = \frac{D(1-D)^2 R}{2(25kHz)} = 96\mu H$, $L = 120\mu H$
- $I_L = \frac{V_s}{(1-D)^2 R} = 1.5A$
- $\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = 1.2A$

$i_{L,min} = 0.3A$

$i_{L,max} = 2.7A$

- $C \geq \frac{D}{R\left(\frac{\Delta V_o}{V_o}\right)f} = \frac{0.6}{50(0.01)25e3} = 48\mu F$

Inductor Resistance (non-ideal inductor)

$P_s = P_o + P_{R_L}$

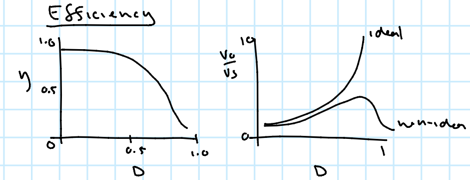
$V_s I_L = V_o I_o + I_L^2 R_L$ → series resistance of inductor

$I_D = I_L(1-D)$ diode current = I_L or zero

steps

$$\boxed{V_o = \left(\frac{V_s}{1-D}\right)\left(\frac{1}{1 + R_L/[R(1-D)^2]}\right)}$$

Efficiency



$\eta = \frac{P_o}{P_o + P_{loss}} = \frac{V_o^2/R}{V_o^2/R + I_L^2 r_L} = \frac{1}{1 + r_L[R(1-D)^2]}$