DL-Lite Full: a sub-Language of OWL 2 Full for the web-scale Open Data for Powerful Meta-modeling and Query Answering

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Appendix A: Semantic conditions of meta-names w.r.t. a u-model $\mathcal{U}$

Figure 1: Interpretation of roles, classes, axioms and assertions w.r.t a v-interpretation $\mathcal{V}$, where $P, A, a, b \in \mathbf{N}$.

Syntax	Semantics
P	$\mathcal{R}^{\mathcal{V}}(P^{\mathcal{V}})$
P^-	$\{(x,y) \mid (y,x) \in \mathcal{R}^{\mathcal{V}}(P^{\mathcal{V}})\}$
$\neg S$	$\Delta^{\mathcal{V}} \times \Delta^{\mathcal{V}} - \mathcal{R}^{\mathcal{V}}(S)$
A	$C^{\mathcal{V}}(A^{\mathcal{V}})$
$\exists S$	$\{x \mid \exists y.(x,y) \in \mathcal{R}^{\mathcal{V}}(S)\}$
$\exists S.B$	$\{x \mid \exists y.(x,y) \in \mathcal{R}^{\mathcal{V}}(S) \wedge y \in C^{\mathcal{V}}(B)\}$
$\neg B$	$\Delta^{\mathcal{V}} - C^{\mathcal{V}}(B)$
$B \sqsubseteq_c C$	$C^{\mathcal{V}}(B) \subseteq C^{\mathcal{V}}(C)$
$S \sqsubseteq_r R$	$\mathcal{R}^{\mathcal{V}}(S) \subseteq \mathcal{R}^{\mathcal{V}}(R)$
$P(a,b)$	$(a^{\mathcal{V}}, b^{\mathcal{V}}) \in \mathcal{R}^{\mathcal{V}}(P^{\mathcal{V}})$

Figure 2: Interpretation of roles, classes, axioms and assertions w.r.t a u-interpretation $\mathcal{U}$, where $\mathsf{C}(B)$ (resp. $\mathsf{R}(S)$) denotes the set of names occurring in the class (resp. role) positions of DL-Lite Full class B (resp. role S), and $P, A, a, b \in \mathbf{N}$.

Syntax	Semantics
$B \sqsubseteq_c C$	$c^{\mathcal{U}} \in \Delta^{C}$ for each $c \in \mathsf{C}(B) \cup \mathsf{C}(C)$, $p^{\mathcal{U}} \in \Delta^{\mathcal{R}}$ for each $p \in \mathsf{R}(B) \cup \mathsf{R}(C)$, and $C^{\mathcal{U}}(B) \subseteq C^{\mathcal{U}}(C)$
$S \sqsubseteq_r R$	$p^{\mathcal{U}} \in \Delta^{\mathcal{R}}$ for each $p \in \mathsf{R}(S) \cup \mathsf{R}(R)$, $\mathcal{R}^{\mathcal{U}}(S) \subseteq \mathcal{R}^{\mathcal{U}}(R)$
$P(a,b)$	$P^{\mathcal{P}} \in \Delta^{\mathcal{R}}$, $(a^{\mathcal{U}}, b^{\mathcal{U}}) \in \mathcal{R}^{\mathcal{U}}(P^{\mathcal{U}})$

Inductive definition of $C^{\mathcal{U}}(B)$, $C^{\mathcal{U}}(B)$, $\mathcal{R}^{\mathcal{U}}(S)$ and $\mathcal{R}^{\mathcal{U}}(R)$	
P	$\mathcal{R}^{\mathcal{U}}(P^{\mathcal{U}})$
P^-	$\{(x,y) \mid (y,x) \in \mathcal{R}^{\mathcal{U}}(P^{\mathcal{U}})\}$
$\neg S$	$\Delta^{\mathcal{U}} \times \Delta^{\mathcal{U}} - \mathcal{R}^{\mathcal{U}}(S)$
A	$C^{\mathcal{U}}(A^{\mathcal{U}})$
$\exists S$	$\{x \mid \exists y.(x,y) \in \mathcal{R}^{\mathcal{U}}(S)\}$
$\exists S.B$	$\{x \mid \exists y.(x,y) \in \mathcal{R}^{\mathcal{U}}(S) \wedge y \in C^{\mathcal{U}}(B)\}$
$\neg B$	$\Delta^{\mathcal{U}} - C^{\mathcal{U}}(B)$

Figure 3: Interpretation and semantic conditions of preserved-names w.r.t a u-interpretation.

class A	$A^{\mathcal{U}}$	$C^{\mathcal{U}}(A^{\mathcal{U}})$
Class	$\in \Delta^{C}$	$= \Delta^{C}$
Property	$\in \Delta^{C}$	$= \Delta^{\mathcal{R}}_{u}$
SymmetricProperty	$\in \Delta^{C}$	$\subseteq \Delta^{\mathcal{R}}$
AsymmetricProperty	$\in \Delta^{C}$	$\subseteq \Delta^{\mathcal{R}}$

role P	$P^{\mathcal{U}}$	$\mathcal{R}^{\mathcal{U}}(P^{\mathcal{U}})$
type	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{\mathcal{U}} \times \Delta^{C}$
subClassOf	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{C} \times \Delta^{C}$
equivalentProperty	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{C} \times \Delta^{C}$
disjointWith	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{C} \times \Delta^{C}$
domain	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{\mathcal{R}} \times \Delta^{C}$
range	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{\mathcal{R}} \times \Delta^{C}$
subPropertyOf	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{\mathcal{R}} \times \Delta^{\mathcal{R}}$
equivalentProperty	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{\mathcal{R}} \times \Delta^{\mathcal{R}}$
inverseOf	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{\mathcal{R}} \times \Delta^{\mathcal{R}}$
propertyDisjointWith	$\in \Delta^{\mathcal{R}}$	$\subseteq \Delta^{\mathcal{R}} \times \Delta^{\mathcal{R}}$

$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{subClassOf}^{\mathcal{U}})$	*iff*	$x, y \in \Delta^{C}, C^{\mathcal{U}}(x) \subseteq C^{\mathcal{U}}(y)$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{equivalentProperty}^{\mathcal{U}})$	*iff*	$x, y \in \Delta^{C}, C^{\mathcal{U}}(x) = C^{\mathcal{U}}(y)$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{disjointWith}^{\mathcal{U}})$	*iff*	$x, y \in \Delta^{C}, C^{\mathcal{U}}(x) \cap C^{\mathcal{U}}(y) = \emptyset$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{domain}^{\mathcal{U}})$	*iff*	$x \in \Delta^{\mathcal{R}}, y \in \Delta^{C}$, $\{o \mid (o,e) \in \mathcal{R}^{\mathcal{U}}(x)\} \subseteq C^{\mathcal{U}}(y)$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{range}^{\mathcal{I}})$	*iff*	$x \in \Delta^{\mathcal{R}}, y \in \Delta^{C}$, $\{e \mid (o,e) \in \mathcal{R}^{\mathcal{U}}(x)\} \subseteq C^{\mathcal{U}}(y)$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{subPropertyOf}^{\mathcal{U}})$	*iff*	$x, y \in \Delta^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x) \subseteq \mathcal{R}^{\mathcal{U}}(y)$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{equivalentProperty}^{\mathcal{U}})$	*iff*	$x, y \in \Delta^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x) = \mathcal{R}^{\mathcal{U}}(y)$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{propertyDisjointWith}^{\mathcal{U}})$	*iff*	$x, y \in \Delta^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x) \cap \mathcal{R}^{\mathcal{U}}(y) = \emptyset$
$(x,y) \in \mathcal{R}^{\mathcal{U}}(\mathsf{inverseOf}^{\mathcal{U}})$	*iff*	$x, y \in \Delta^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x) = \mathcal{R}^{\mathcal{U}}(y)^-$
$s \in C^{\mathcal{U}}(\mathsf{SymmetricProperty}^{\mathcal{U}})$	*iff*	$\mathcal{R}^{\mathcal{U}}(x) = \mathcal{R}^{\mathcal{U}}(x)^-$
$x \in C^{\mathcal{U}}(\mathsf{AsymmetricProperty}^{\mathcal{U}})$	*iff*	$\mathcal{R}^{\mathcal{U}}(x) \cap \mathcal{R}^{\mathcal{U}}(x)^- = \emptyset$

Appendix B: Proofs of the results in the paper

Proof of Theorem 1.

Proof. (1. $\Rightarrow$) If $\mathcal{K}$ is v-satisfiable then it has a v-model $\mathcal{V}$. Next, we show $\tau_{dl}(\mathcal{K})$ is satisfiable. From $\mathcal{V}$, an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ for $\tau_{dl}(\mathcal{K})$ can be constructed by setting (a) $\Delta^{\mathcal{I}} = \Delta^{\mathcal{V}}$; (b) for each $a \in \mathsf{N}$, $a^{\mathcal{I}} = a^{\mathcal{V}}$; (c) for each $P \in \mathsf{R}$, $P^{\mathcal{I}} = \mathcal{R}^{\mathcal{V}}((v_r^-(P))^{\mathcal{V}})$; and (d) for each $A \in \mathsf{C}$, $A^{\mathcal{I}} = C^{\mathcal{V}}((v_c^-(A))^{\mathcal{V}})$. $\mathcal{I}$ and $\mathcal{V}$ obey the same principles to interpret class and role constructors. Thus it holds that (♠) $(\tau_r(R))^{\mathcal{I}} = \mathcal{R}^{\mathcal{V}}(R)$ for each DL-Lite Full role R and $(\tau_c(C))^{\mathcal{I}} = C^{\mathcal{V}}(C)$ for each DL-Lite Full class C. For each axiom or assertion α in $\tau_{dl}(\mathcal{K})$, if there exists α' in $\mathcal{K}$ such that $\alpha = \tau(\alpha')$ then by (♠) and $\mathcal{V} \models \alpha'$, $\mathcal{I} \models \alpha$ holds.

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Otherwise, α is an individual assertion $A(a)$ satisfying that there exists $\mathsf{gr}(P,a,A)$ in $\mathcal{K}$ such that $P \sqsubseteq_r^* \mathsf{type}$ holds. $a^{\mathcal{I}} \in C^{\mathcal{V}}(A^{\mathcal{V}})$ holds. Then by (♠), $a^{\mathcal{I}} \in v_c(A)^{\mathcal{I}}$ holds, i.e., $\mathcal{I} \models A(a)$ holds. So $\mathcal{I}$ satisfies all the axioms and assertions in $\tau_{dl}(\mathcal{K})$. Therefore, $\tau_{dl}(\mathcal{K})$ is satisfiable.

(1. $\Leftarrow$) If $\tau_{dl}(\mathcal{K})$ is satisfiable. Then it has a canonical model $\mathcal{I}$ [1]. From $\mathcal{I}$, a v-interpretation $\mathcal{V} = (\Delta^{\mathcal{V}}, \cdot^{\mathcal{V}}, C^{\mathcal{V}}, \mathcal{R}^{\mathcal{V}})$ can be constructed by setting (a) $\Delta^{\mathcal{V}} = \Delta^{\mathcal{I}}$; (b) for each $a \in \mathsf{N}$, $a^{\mathcal{V}} = a$; (c) for each $o \in \Delta^{\mathcal{V}}$, if $o \in \mathsf{N} - \{\mathsf{type}\}$ then $\mathcal{R}^{\mathcal{V}}(o) = v_r(o)^{\mathcal{I}}$, else if $o = \mathsf{type}$ then $\mathcal{R}^{\mathcal{V}}(\mathsf{type}) = v_r(\mathsf{type})^{\mathcal{I}} \cup \{(o,e) | e \in \mathsf{N} \wedge o \in v_c(e)^{\mathcal{I}}\}$, else $\mathcal{R}^{\mathcal{V}}(o) = \emptyset$; and (d) for each $o \in \Delta^{\mathcal{V}}$, $C^{\mathcal{V}}(o) = \{e | (o,e) \in \mathcal{R}^{\mathcal{V}}(\mathsf{type})\}$. $\mathcal{V}$ and $\mathcal{I}$ obey the same principles to interpret class and role constructors. Then from the construction of $\mathcal{V}$, we can get that (♣) for each DL-Lite Full class C, $C^{\mathcal{V}}(C) = (\tau_c(C))^{\mathcal{I}}$ holds, and for each DL-Lite Full role R, if $R \neq \mathsf{type}$ and $R \neq \mathsf{type}^-$ then $\mathcal{R}^{\mathcal{V}}(R) = (\tau_r(R))^{\mathcal{I}}$ otherwise $(\tau_r(R))^{\mathcal{I}} \subseteq \mathcal{R}^{\mathcal{V}}(R)$. The meta-role type just occurs in the right-hands of role inclusion axioms. Thus, for each axiom or individual assertion α in $\mathcal{K}$, from $\mathcal{I} \models \tau(\alpha)$ and (♣), $\mathcal{V} \models_v \alpha$ holds. So $\mathcal{V}$ is a v-model of $\mathcal{K}$. Therefore $\mathcal{K}$ is v-satisfiable.

(2) If $\mathcal{K}$ is not v-satisfiable, then by (1), $\tau_{dl}(\mathcal{K})$ is not satisfiable and this conclusion holds trivially. Next, we assume that $\mathcal{K}$ is v-satisfiable. And by (1), $\tau_{dl}(\mathcal{K})$ is also satisfiable.

Let $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$. Next, we show $\vec{u} \in \mathsf{ans}(\tau_{dl}(Q), \tau_{dl}(\mathcal{K}))$. Let $\mathcal{I}$ be a canonical model of $\tau_{dl}(\mathcal{K})$. Then from $\mathcal{I}$, a v-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed using the way presented in (1. $\Leftarrow$). So there exists a binding π of $Q(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v Q(\vec{u})$. From π, a binding π' of $\tau_{dl}(Q(\vec{u}))$ over $\mathcal{I}$ can be constructed by setting $\pi'(x) = \pi(x)$ for each variable x in $\tau_{dl}(Q(\vec{u}))$ and $\pi'(a) = a^{\mathcal{I}}$ for each individual a in $\tau_{dl}(Q(\vec{u}))$. Then by (♣) and the construction of $\mathcal{V}$, $\mathcal{I}, \pi' \models \tau_{dl}(Q(\vec{u}))$ holds. $\tau_{dl}(Q(\vec{u})) = \tau_{dl}(Q)(\mathsf{head}(\tau_{dl}(Q))/\vec{u})$ holds. Thus $\vec{u} \in \mathsf{ans}(\tau_{dl}(Q), \tau_{dl}(\mathcal{K}))$ holds. Hence the inclusion $\mathsf{ans}_v(Q, \mathcal{K}) \subseteq \mathsf{ans}(\tau_{dl}(Q), \tau_{dl}(\mathcal{K}))$ holds. $\mathsf{ans}(\tau_{dl}(Q), \tau_{dl}(\mathcal{K})) \subseteq \mathsf{ans}_v(Q, \mathcal{K})$ can be proved analogously. Thus $\mathsf{ans}_v(Q, \mathcal{K}) = \mathsf{ans}(\tau(Q), \tau(\mathcal{K}))$ holds. □

Proof of Lemma 1.

Proof. Let $\mathcal{A}_o = \mathcal{A} \cup \{\mathsf{type}(a,A) | \mathsf{gr}(P,a,A) \in \mathcal{A} \wedge P \sqsubseteq_r^* \mathsf{type}\}$, i.e., the ABox obtained by materializing the non-standard use of type in the original KB. Then for each CQ Q, $\mathsf{ans}_v(Q, (\emptyset, \mathcal{A}_o)) = \mathsf{ans}(\tau_{dl}(Q), (\emptyset, \tau_{dl}(\mathcal{A})))$ holds trivially, since:

$$\tau_{dl}(\mathcal{A}) = \{v_c(A)(a) | \mathsf{type}(a,A) \in \mathcal{A}_o\} \cup \{v_r(P)(a,b) | P(a,b) \in \mathcal{A}_o \wedge P \neq \mathsf{type}\}$$

Thus, the equation in the lemma can be proved by showing the following equation holds:

$$\textstyle\bigcup_{Q \in \mathsf{RefType}(Q,\mathcal{T})} \mathsf{ans}_v(Q, (\emptyset, \mathcal{A})) = \bigcup_{Q \in \mathcal{Q}} \mathsf{ans}_v(Q, (\emptyset, \mathcal{A}_o))$$

($\subseteq$) Let $Q \in \mathsf{RefType}(Q, \mathcal{T})$. And let $\vec{u} \in \mathsf{ans}_v(Q, (\emptyset, \mathcal{A}))$. Next, we show that (♠) there exists $Q' \in \mathcal{Q}$ such that $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}_o))$. If $Q \in \mathcal{Q}$ then (♠) holds trivially, since $\mathcal{A} \subseteq \mathcal{A}_o$. Otherwise, there exists a query Q':

$$\textstyle\bigwedge_{l=1}^{n} \alpha_l \wedge \bigwedge_{i=1}^{m} \mathsf{type}(x_i, A_i) \rightarrow q(\vec{x})$$

in $\mathcal{Q}$ such that $P_k \sqsubseteq_r^* \mathsf{type}$ for each $1 \leq k \leq m$ and Q is the query:

$$\textstyle\bigwedge_{l=1}^{n} \alpha_l \wedge \bigwedge_{i=1}^{m} P_i(x_i, A_i) \rightarrow q(\vec{x})$$

Next we prove $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}))$. For $\vec{u}$, there exists a function f that maps all the variables in $Q(\vec{u})$ to names and all the atoms in $(Q(\vec{u}))f$ occur in $\mathcal{A}$. For each $P_i(a_i, A_i)$ in $(Q(\vec{u}))f$, $type(a_i, A_i)$ occurs in $\mathcal{A}_o$. Thus all the atoms in $(Q'(\vec{u}))f$ occur in $\mathcal{A}_o$. So $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}_o))$ holds. Hence (♠) holds. Therefore the ($\subseteq$) direction holds.

($\supseteq$) Let $Q \in \mathcal{Q}$. And let $\vec{u} \in \mathsf{ans}_v(Q, (\emptyset, \mathcal{A}_o))$. In the following, we show that (♣) there exists $Q' \in \mathsf{RefType}(Q, \mathcal{T})$ such that $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}))$. If $\vec{u} \in \mathsf{ans}_v(Q, (\emptyset, \mathcal{A}))$, then (♣) holds trivially, since $\mathcal{A} \subseteq \mathcal{A}_o$ and $Q \in \mathsf{RefType}(Q, \mathcal{T})$. Otherwise, let $\mathcal{S} \subseteq \mathcal{A}_o - \mathcal{A}$ such that $\vec{u} \in \mathsf{ans}_v(Q, (\emptyset, \mathcal{A} \cup \mathcal{S}))$ and there does not exist $\mathcal{S}' \subsetneq \mathcal{S}$ satisfying $\vec{u} \in \mathsf{ans}_v(Q, (\emptyset, \mathcal{A} \cup \mathcal{S}'))$. Suppose $\mathcal{S} = \bigcup_{i=1}^{n} \{\mathsf{type}(a_i, A_i)\}$. Then for each $1 \leq i \leq n$, there exists $\mathsf{gr}(P_i, a, A_i) \in \mathcal{A}$ such that $P_i \sqsubseteq_r^* \mathsf{type}$. For $\vec{u}$, there exists a function f that maps all the variables in $Q(\vec{u})$ to names and all the atoms in $(Q(\vec{u}))f$ occur in $\mathcal{A} \cup \mathcal{S}$. Let Q' be the query obtained by replacing the atom $\mathsf{type}(x, A_i)$ in Q with $\mathsf{gr}(P_i, x, A_i)$ if $x = a_i$, $f(x) = a_i$, or x occurs in the k-th position of $\mathsf{head}(Q)$ and $\vec{u}[k] = a_i$, for $1 \leq i \leq n$. Then all the atoms in $(Q'(\vec{u}))f$ occur in $\mathcal{A}$. So $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}))$ holds. Thus the ($\supseteq$) direction holds. □

Proof of Theorem 2.

Proof. (1) By Lemma 1 and Definition 6, the following equation holds trivially:

$$\bigcup_{Q \in \mathsf{Violates}_v(\mathcal{T})} \mathsf{ans}_v(Q, (\emptyset, \mathcal{A})) = \bigcup_{q \in \mathsf{Violates}(\tau_{dl}(\mathcal{T}))} \mathsf{ans}(q, (\emptyset, \tau_{dl}(\mathcal{A})))$$

By Theorem 1, $\mathcal{K}$ is v-satisfiable iff $\tau_{dl}(\mathcal{K})$ is satisfiable. And $\tau_{dl}(\mathcal{K})$ is satisfiable iff $\bigcup_{q \in \mathsf{Violates}(\tau_{dl}(\mathcal{T}))} \mathsf{ans}(q, (\emptyset, \tau_{dl}(\mathcal{A}))) = \emptyset$. Thus this conclusion holds. (2) For each CQ Q, the corresponding equation can be proved analogous to (1). □

Proof of Theorem 3.

Proof. ($\subseteq$) Let $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$. Next, we show there exists $\theta \in \mathsf{fullBind}(Q, \mathcal{K})$ such that $\vec{u} \in \mathsf{ans}_v(Q\theta, \mathcal{K})$. Let $\mathcal{V}$ be a v-model of $\mathcal{K}$ constructed from a canonical model of $\tau_{dl}(\mathcal{K})$ by the approach presented in the (1. $\Leftarrow$) direction of the proof of Theorem 1. Then we can get that (♠) for each $o \in \Delta^{\mathcal{V}} - \mathsf{N}$, $C^{\mathcal{V}}(o) = \mathcal{R}^{\mathcal{V}}(o) = \emptyset$. For $\vec{u}$, there exists a binding π of $Q(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v Q(\vec{u})$. From π and $\vec{u}$, we can construct a full MV-Binding θ of Q over $\mathcal{K}$. For each role variable x of Q, if x occurs in the i-th position of $\mathsf{head}(Q)$ then set $\theta_r(x) = \vec{u}[i]$, otherwise from (♠), we know there exists $n \in \mathsf{N}$ such that $\pi(x) = n$, and then set $\theta_r(x) = n$. And for each class variable x of $Q\theta_r$, if x occurs in the i-th position of $\mathsf{head}(Q)$ then set $\theta_c(x) = \vec{u}[i]$, otherwise by (♠), we know there exists $n \in \mathsf{N}$ such that $\pi(x) = n$,

[1] A DL-Lite$_{\mathcal{R}}$ KB O has a canonical interpretation $\mathcal{I}$ satisfying that (1) $a^{\mathcal{I}} = a$ for each $a \in \mathsf{N}$; (2) O is satisfiable iff $\mathcal{I}$ satisfies O; and (3) if O is satisfiable then for each conjunctive query q, $\vec{u} \in \mathsf{ans}(q, O)$ iff $\mathcal{I}$ satisfies $q(\vec{u})$. If O is satisfiable then $\mathcal{I}$ is called a canonical model of O.

and then set $\theta_c(x) = n$. Next, we prove $\vec{u} \in \mathsf{ans}_v(Q\theta, \mathcal{K})$. Let π' be a binding of $Q\theta(\vec{u})$ over $\mathcal{V}$ such that $\pi'(x) = \pi(x)$ for each variable x in $Q\theta(\vec{u})$ and $\pi'(a) = a$ for each name a in $Q\theta(\vec{u})$. Then $\mathcal{V}, \pi' \models_v Q\theta(\vec{u})$ holds, since $(Q\theta(\vec{u}))\pi' = (Q(\vec{u}))\pi$ holds. Thus $\vec{u} \in \mathsf{ans}_v(Q\theta, \mathcal{K})$ holds. Therefore the relation $\mathsf{ans}_v(Q, \mathcal{K}) \subseteq \bigcup_{\theta \in \mathsf{fullBind}(Q,\mathcal{K})} \mathsf{ans}_v(Q\theta, \mathcal{K})$ holds.

($\supseteq$) Let $\theta \in \mathsf{fullBind}(Q, \mathcal{K})$ and $\vec{u} \in \mathsf{ans}_v(Q\theta, \mathcal{K})$. Next, we show $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$. Let $\mathcal{V}$ be an arbitrary v-model of $\mathcal{K}$. Then there exists a binding π of $(Q\theta)(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v (Q\theta)(\vec{u})$. From π, a binding π' of $Q(\vec{u})$ over $\mathcal{K}$ can be constructed by the settings that (a) for each variable x occurring in $Q(\vec{u})$, if $x \in \mathsf{dom}(\theta_r)$ (resp. $x \in \mathsf{dom}(\theta_c)$) then set $\pi'(x) = (\theta_r(x))^{\mathcal{V}}$ (resp. $\pi'(x) = (\theta_c(x))^{\mathcal{V}}$); and (b) for each name a occurring in $Q(\vec{u})$, set $\pi'(a) = a^{\mathcal{V}}$. Obviously $\mathcal{V}, \pi' \models_v Q(\vec{u})$ holds. Thus $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$ holds. Therefore $\mathsf{ans}_v(Q, \mathcal{K}) \supseteq \bigcup_{\theta \in \mathsf{fullBind}(Q,\mathcal{K})} \mathsf{ans}_v(Q\theta, \mathcal{K})$ holds. □

Proof of Lemma 2.

Proof. Under v-semantics, it holds trivially that $\vec{u} \in \mathsf{ans}(Q, (\emptyset, \mathcal{A}))$ iff there exists a function f such that f maps each variable in $Q(\vec{u})$ to a name in $\mathcal{A}$ and all the atoms in $Q(\vec{u})f$ occur in $\mathcal{A}$. Then this lemma holds. □

Proof of Lemma 3.

Proof. (1) ($\supseteq$) By Definition 10, i.e., the definition of extensions of partial MV-Bindings, this direction of the equation in this lemma holds. ($\subseteq$) Let θ be an arbitrary full MV-Binding of Q over $\mathcal{K}$. Next, we construct a partial MV-Binding ϑ of Q over $\mathcal{T}$ such that $\theta \in \mathsf{extPBind}(\vartheta, Q, \mathcal{K})$ holds. For each role variable x of Q, set $\vartheta_r(x) = \theta_r(x)$ *iff* $\theta_r(x) \in \mathsf{N}^{rr}_{\mathcal{T}} \cup \{\mathsf{type}\}$. Then $Q\theta_r$ and $Q\vartheta_r$ have the same class variables. For each class variable x of $Q\vartheta_r$, set $\vartheta_c(x) = \theta_c(x)$ *iff* $\theta_c(x) \in \mathsf{N}^{rc}_{\mathcal{T}}$. Then by Definition 10, $\theta \in \mathsf{extPBind}(\vartheta, Q, \mathcal{K})$ holds. Thus the direction ($\subseteq$) holds.

(2) According to the algorithm $\mathsf{PerfectRef}$, we can get that (♠) for a CQ q and atom $A(x)$ (resp. $P(x, y)$), if A (resp. P) does not occur in the right hands of any inclusion axioms in a DL-Lite$_{\mathcal{A}}$ TBox $\mathcal{T}$, then this query atom will not be extended by the inclusion axioms in $\mathcal{T}$ to generate new queries, i.e., it will be occur in each query in $\mathsf{PerfectRef}(q, \mathcal{T})$.

($\subseteq$ .1) Let $\theta \in \mathsf{extPBind}(\vartheta, Q, \mathcal{K})$ and $\vec{u} \in \mathsf{ans}_v(Q\theta, \mathcal{K})$. Next, we show there exists $Q' \in \mathsf{PerfectRef}^{mq}_v(Q\vartheta, \mathcal{T})$ such that $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}))$. By Theorem 2, there exists $q \in \mathsf{PerfectRef}^{cq}_v(Q\theta, \mathcal{T})$ such that $\vec{u} \in \mathsf{ans}_v(Q, (\emptyset, \mathcal{A}))$. Let $\theta' = (\theta'_r, \theta'_c)$ be a tuple of functions satisfying that (a) $\mathsf{dom}(\theta'_r) = \mathsf{dom}(\theta_r) - \mathsf{dom}(\vartheta_r)$ and for each $x \in \mathsf{dom}(\theta'_r)$, $\theta'_r(x) = \theta_r(x)$ holds and (b) $\mathsf{dom}(\theta'_c) = \mathsf{dom}(\theta_c) - \mathsf{dom}(\vartheta_c)$ and for each $x \in \mathsf{dom}(\theta'_c)$, $\theta'_c(x) = \theta_c(x)$ holds. Obviously θ' is a full MV-Binding of $Q\vartheta$ over $\mathcal{K}$ that maps the class (resp. role) variables of $Q\vartheta$ to the names not occurring in $\mathsf{N}^{rc}_{\mathcal{T}}$ (resp. $\mathsf{N}^{rr}_{\mathcal{T}}$). For q, by (♠) and Definition 6 and 9, i.e., the definition of $\mathsf{PerfectRef}^{cq}_v$ and $\mathsf{PerfectRef}^{mq}_v$, we can get that there exists $Q' \in \mathsf{PerfectRef}^{mq}_v(Q\vartheta, \mathcal{T})$ such that $q = Q'\theta'$. Thus $\vec{u} \in \mathsf{ans}_v(Q'\theta', (\emptyset, \mathcal{A}))$. Then by Theorem 3, $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}))$ holds. Thus the first inclusion holds.

($\subseteq$ .2) Let $Q' \in \mathsf{PerfectRef}^{mq}_v(Q\vartheta, \mathcal{T})$. And let $\vec{u} \in \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}))$. Next, we show $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$. By Theorem 3, there exists a full MV-Binding θ of Q' over $(\emptyset, \mathcal{A})$ such that $\vec{u} \in \mathsf{ans}_v(Q'\theta, (\emptyset, \mathcal{A}))$ holds. Let θ' be a full MV-Binding of Q over $\mathcal{K}$ such that (a) $\mathsf{dom}(\theta'_r) = \mathsf{dom}(\vartheta_r) \cup \mathsf{dom}(\theta_r)$ and for each $x \in \mathsf{dom}(\theta'_r)$, if $x \in \mathsf{dom}(\vartheta_r)$ then $\theta'_r(x) = \vartheta_r(x)$, otherwise $\theta'_r(x) = \theta_r(x)$; and (2) $\mathsf{dom}(\theta'_c) = \mathsf{dom}(\vartheta_c) \cup \mathsf{dom}(\theta_c)$ and for each $x \in \mathsf{dom}(\theta'_c)$, if $x \in \mathsf{dom}(\vartheta_c)$ then $\theta'_c(x) = \vartheta_c(x)$ otherwise $\theta'_c(x) = \theta_c(x)$. Then by Definition 9, i.e., the definition of $\mathsf{PerfectRef}^{mq}_v$, $Q'\theta \in \mathsf{PerfectRef}^{cq}_v(Q\theta', \mathcal{T})$ holds. Then by Theorem 2 and $\vec{u} \in \mathsf{ans}_v(Q'\theta, (\emptyset, \mathcal{A}))$, $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$ holds. Thus the second inclusion relation holds. □

Proof of Theorem 4.

Proof. By Theorem 3 and Lemma 3, the following equation and inclusions hold:

$$\begin{aligned}
\mathsf{ans}_v(Q, \mathcal{K}) &= \textstyle\bigcup_{\theta \in \mathsf{fullBind}(Q,\mathcal{K})} \mathsf{ans}_v(Q\theta, \mathcal{K}) \\
&= \textstyle\bigcup_{\vartheta \in \mathsf{partBind}(Q,\mathcal{T})} \bigcup_{\theta \in \mathsf{extPBind}(\vartheta,Q,\mathcal{K})} \mathsf{ans}_v(Q\theta, \mathcal{K}) \\
&\subseteq \textstyle\bigcup_{\vartheta \in \mathsf{partBind}(Q,\mathcal{T})} \bigcup_{Q' \in \mathsf{PerfectRef}^{mq}_{\mu}(Q\vartheta,\mathcal{T})} \mathsf{ans}_v(Q', (\emptyset, \mathcal{A})) \\
&\subseteq \textstyle\bigcup_{\vartheta \in \mathsf{partBind}(Q,\mathcal{T})} \mathsf{ans}_v(Q, \mathcal{K}) \\
&\subseteq \mathsf{ans}_v(Q, \mathcal{K})
\end{aligned}$$

Thus the following equation holds, i.e., this theorem holds:

$$\mathsf{ans}_v(Q, \mathcal{K}) = \textstyle\bigcup_{\vartheta \in \mathsf{partBind}(Q,\mathcal{T})} \bigcup_{Q' \in \mathsf{PerfectRef}^{mq}_{\mu}(Q\vartheta,\mathcal{T})} \mathsf{ans}_v(Q', (\emptyset, \mathcal{A}))$$

□

Proof of Lemma 4.

Proof. Conjunctive query answering over databases has AC^0 data complexity. By Lemma 2, this theorem holds. □

Proof of Theorem 5.

Proof. By Definition 6 and Theorem 2, the complexity results of v-satisfiability checking and CQ answering hold trivially. If Q has meta-variables, then it has no more than $2^{2|Q|}(2|\mathcal{T}| + 2)^{2|Q|}$ partial MV-Bindings over $\mathcal{T}$. Then by Theorem 4, Definitions 9 and 6 and Lemma 4, the complexity results for meta-query answering holds. □

Proof of Lemma 5.

Proof. (1) $\mathcal{K}$ is u-satisfiable, so it has a u-model $\mathcal{U} = (\Delta^{\mathcal{U}}, \Delta^{\mathcal{R}}, \Delta^{\mathcal{C}}, \cdot^{\mathcal{U}}, \mathcal{R}^{\mathcal{U}}, \mathcal{C}^{\mathcal{U}})$. From $\mathcal{U}$, a v-interpretation $\mathcal{V} = (\Delta^{\mathcal{V}}, \cdot^{\mathcal{V}}, \mathcal{R}^{\mathcal{V}}, \mathcal{C}^{\mathcal{V}})$ can be constructed by setting (a) $\Delta^{\mathcal{V}} = \Delta^{\mathcal{U}}$; (b) for each $a \in \mathsf{N}$, $a^{\mathcal{V}} = a^{\mathcal{U}}$; (c) for each $o \in \Delta^{\mathcal{V}}$, if $o \in \Delta^{\mathcal{R}}$ then $\mathcal{R}^{\mathcal{V}}(o) = \mathcal{R}^{\mathcal{U}}(o)$, otherwise $\mathcal{R}^{\mathcal{V}}(o) = \emptyset$, and if $o \in \Delta^{\mathcal{C}}$ then $\mathcal{C}^{\mathcal{V}}(o) = \mathcal{C}^{\mathcal{U}}(o)$, otherwise $\mathcal{C}^{\mathcal{V}}(o) = \emptyset$. $\mathcal{U}$ and $\mathcal{V}$ obey the same principles to interpret class and role constructors. Thus (♠) $\mathcal{R}^{\mathcal{V}}(R) = \mathcal{R}^{\mathcal{U}}(R)$ holds for each DL-Lite Full role R and $\mathcal{C}^{\mathcal{V}}(C) = \mathcal{C}^{\mathcal{U}}(C)$ holds for each DL-Lite Full class C. Then we can further obtain that $\mathcal{V}$ satisfies all the axioms and assertions in $\mathcal{K}$. So $\mathcal{V}$ is a v-model of $\mathcal{K}$. Thus $\mathcal{K}$ is v-satisfiable.

(2) If $\mathcal{K}$ is not u-satisfiable then this conclusion holds directly. Suppose $\mathcal{K}$ is u-satisfiable. Let $\mathcal{U}$ be an arbitrary u-models of $\mathcal{K}$. From $\mathcal{U}$, a v-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed by the approach presented in (1). Then $\mathcal{V} \models_v \alpha$ holds. Then by (♠) in (1), $\mathcal{U} \models_u \alpha$ holds. Based on the arbitrary feature of $\mathcal{U}$, $\mathcal{K} \models_u \alpha$ holds.

(3) If $\mathcal{K}$ is not u-satisfiable then this conclusion holds trivially. We assume $\mathcal{K}$ is u-satisfiable. Let $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$. And let $\mathcal{U}$ be an arbitrary u-models of $\mathcal{K}$. Then from $\mathcal{U}$, a v-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed by the way in (1). Then there exists a binding π of $Q(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v Q(\vec{u})$ holds. Obviously, π is also a binding of $Q(\vec{u})$ over $\mathcal{U}$. Then by (♠) in (1), $\mathcal{U} \models_u Q(\vec{u})$ holds. So $\vec{u} \in \mathsf{ans}_u(Q, \mathcal{K})$ holds. Thus, we can further obtain that $\mathsf{ans}_v(Q, \mathcal{K}) \subseteq \mathsf{ans}_u(Q, \mathcal{K})$. □

Proof of Lemma 6.

Proof. (1) Let $\mathcal{K} = (\mathcal{T}_m \cup \mathcal{T}_o, \mathcal{A}_m \cup \mathcal{A}_o)$ and $\mathcal{K}_{u\to v} = (\mathcal{T}, \mathcal{A})$. If $\mathcal{K}$ is v-satisfiable, the following conclusion holds trivially. For a preserved-class C_p, $\mathcal{K} \models_v$ *type*(a, C_p) *iff* there exists $\mathsf{gr}(P, a, b) \in \mathcal{A}_p$ such that $P \sqsubseteq_r^* \mathsf{type}$ and $\mathcal{K} \models_v b \sqsubseteq_c C_p$ or $\mathcal{K} \models_v \exists P \sqsubseteq_c C_p$. And for a preserved-role P_p, $\mathcal{K} \models_v P_p(a, b)$ *iff* there exists $\mathsf{gr}(S, a, b) \in \mathcal{A}_m$ such that $\mathcal{K} \models_v S \sqsubseteq_r P_p$. The names occurring in $\mathcal{T}_m$ do not used as individuals in $\mathcal{A}_m$. Thus we can get that $\mathcal{T}'_o = \mathcal{T} - \mathcal{T}_m - \mathcal{T}_o$, i.e., the set of axioms added to $\mathcal{K}$ according to the individual assertions of preserved-names implied by $\mathcal{K}$, do not contain the names occurring in $\mathcal{T}_m$. Thus $\mathcal{K}_{u\to v}$ is still a DL-Lite Full KB, since its TBox is $\mathcal{T}_m \uplus (\mathcal{T}_o \cup \mathcal{T}'_o)$ which satisfies the conditions in Definition 2.

(2) This conclusion holds by the following facts. Let $\mathcal{K}$ be a DL-Lite Full KB. (a) For each axiom or assertion α, if $\mathcal{K} \models_v \alpha$ then $\mathcal{K} \models_u \alpha$; (b) For an axiom or assertion α, if $\mathcal{K} \models_u \alpha$, then for each axiom or assertion α', if $\mathcal{K} \cup \{\alpha\} \models_u \alpha'$ then $\mathcal{K} \models_u \alpha'$; (c) For the axioms (assertions) with the forms in Figure 3 and u-entailed by $\mathcal{K}$, then the corresponding assertions (axioms) are u-entailed by $\mathcal{K}$. For example, $\mathcal{K} \models_u \mathsf{subClassOf}(A, B)$ iff $\mathcal{K} \models_u A \sqsubseteq_c B$. □

Proof of Theorem 6.

Proof. We first claim that for a v-satisfiable DL-Lite Full KB $\mathcal{K}$, let $\mathcal{V}$ be a v-model of $\mathcal{K}$ constructed from a canonical model of the DL-Lite$_{\mathcal{A}}$ KB $\tau_{dl}(\mathcal{K})$ by the way presented in (1. ⇐) in the Proof of Theorem 1. Then $\mathcal{V}$ satisfies that (a) $a^{\mathcal{V}} = a$ for each $a \in \mathsf{N}$; and (b) by Theorem 1 and 3, it can be easily proved that for each meta-query Q, $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$ iff $\mathcal{V} \models_v Q(\vec{u})$. In the following, we call $\mathcal{V}$ ***a canonical v-model*** of $\mathcal{K}$.

Lemma 6 indicates that $\mathcal{K}$ and $\mathcal{K}_{u\to v}$ are u-semantic equivalent, i.e., they have the same u-models. By Lemma 5, we just need to prove that (1) if $\mathcal{K}_{u\to v}$ is u-satisfiable then $\mathcal{K}_{u\to v}$ is v-satisfiable; and (2) if Q is a MQ without non-distinguished meta-variables, $\mathsf{ans}_v(Q, \mathcal{K}_u) \subseteq \mathsf{ans}_u(Q, \mathcal{K}_{u\to v})$ holds.

(1) $\mathcal{K}_{u\to v}$ is v-satisfiable, thus it has a canonical v-model $\mathcal{V} = (\Delta^{\mathcal{V}}, \cdot^{\mathcal{V}}, \mathcal{R}^{\mathcal{V}}, \mathcal{C}^{\mathcal{V}})$. From $\mathcal{V}$, we construct a u-interpretation $\mathcal{U} = (\Delta^{\mathcal{U}}, \Delta^{\mathcal{R}}, \Delta^{C}, \cdot^{\mathcal{U}}, \mathcal{R}^{\mathcal{U}}, \mathcal{C}^{\mathcal{U}})$ by setting (a) $\Delta^{\mathcal{U}} = \Delta^{\mathcal{V}}$; (b) set $\Delta^{\mathcal{R}} = \mathcal{C}^{\mathcal{V}}(\mathsf{Property})$, and for each name a used as role in $\mathcal{K}_{u\to v}$, add a to $\Delta^{\mathcal{R}}$; (c) set $\Delta^{C}_u = \mathcal{C}^{\mathcal{V}}(\mathsf{Class})$, and for each name a used as class in $\mathcal{K}_{u\to v}$, add a to $\Delta^{\mathcal{R}}$; (d) set $a^{\mathcal{U}} = a$ for each $a \in \mathsf{N}$; (e) for each $o \in \Delta^{\mathcal{R}}$, set $\mathcal{R}^{\mathcal{U}}(o) = \mathcal{R}^{\mathcal{V}}(o)$; and (f) for each $o \in \Delta^{C}$, if $o = \mathsf{Property}$ then set $\mathcal{C}^{\mathcal{U}}(o) = \Delta^{\mathcal{R}}_u$, else if $o = \mathsf{Class}$ then set $\mathcal{C}^{\mathcal{U}}(o) = \Delta^{C}_u$, else set $\mathcal{C}^{\mathcal{U}}(o) = \mathcal{C}^{\mathcal{V}}(o)$. In order to make $\mathcal{U}$ satisfy the semantic conditions of meta-names listed in Appendix A , we need to make the extra setting:

- For each $(o, e) \in \Delta^{C} \times \Delta^{C}$, if $\mathcal{C}^{\mathcal{U}}(o) \subseteq \mathcal{C}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{subClassOf}^{\mathcal{U}})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{subClassOf})$; if $\mathcal{C}^{\mathcal{U}}(o) = \mathcal{C}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{equivalentClass})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{equivalentClass})$; and if $\mathcal{C}^{\mathcal{U}}(o) \cap \mathcal{C}^{\mathcal{U}}(e) = \emptyset$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{disjointWith})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{disjointWith})$;
- For each $(o, e) \in \Delta^{\mathcal{R}} \times \Delta^{\mathcal{R}}$, if $\mathcal{R}^{\mathcal{U}}(o) \subseteq \mathcal{R}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{subPropertyOf})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{subPropertyOf})$; if $\mathcal{R}^{\mathcal{U}}(o) = \mathcal{R}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{equivalentProperty})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{equivalentProperty})$; if $\mathcal{R}^{\mathcal{U}}(o) \cap \mathcal{R}^{\mathcal{U}}(e) = \emptyset$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{propertyDisjoint})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{propertyDisjoint})$; and if $\mathcal{R}^{\mathcal{U}}(o) = \{(y, x) | (x, y) \in \mathcal{R}^{\mathcal{U}}(e)\}$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{inverseOf})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{inverseOf})$;
- For each $(o, e) \in \Delta^{\mathcal{R}} \times \Delta^{C}$, if $\{x | (x, y) \in \mathcal{R}^{\mathcal{U}}(o)\} \subseteq \mathcal{C}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathsf{domain})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{domain})$; and if $\{y | (x, y) \in \mathcal{R}^{\mathcal{U}}(o)\} \subseteq \mathcal{C}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(\mathit{range})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(\mathsf{range})$;
- For each $o \in \Delta^{\mathcal{R}}$, if $\mathcal{R}^{\mathcal{V}}(o) = \{(y, x) | (x, y) \in \mathcal{R}^{\mathcal{V}}(o)\}$ and $o \notin \mathcal{C}^{\mathcal{U}}(\mathsf{SymmetricProperty})$, then add o to $\mathcal{C}^{\mathcal{U}}(\mathsf{SymmetricProperty})$; and if $\mathcal{R}^{\mathcal{V}}(o) \cap \{(y, x) | (x, y) \in \mathcal{R}^{\mathcal{V}}(o)\} = \emptyset$ and $o \notin \mathcal{C}^{\mathcal{U}}(\mathsf{AsymmetricProperty})$, then add o to $\mathcal{C}^{\mathcal{U}}(\mathsf{AsymmetricProperty})$.

Then $\mathcal{U}$ satisfies the semantic conditions of preserved-names listed in Appendix A. So it is a u-interpretation. $\mathcal{U}$ and $\mathcal{V}$ obey the same rules to interpret the class and role constructors. Thus, we can get that (♣) for each DL-Lite Full class C, if C is not a preserved-class then $\mathcal{C}^{\mathcal{U}}(C) = \mathcal{C}^{\mathcal{V}}(C)$ otherwise $\mathcal{C}^{\mathcal{V}}(C) \subseteq \mathcal{C}^{\mathcal{U}}(C)$, and for each DL-Lite Full Role R, if R is not a preserved-role or inverse of a preserved-role then $\mathcal{R}^{\mathcal{U}}(R) = \mathcal{R}^{\mathcal{V}}(R)$ otherwise $\mathcal{R}^{\mathcal{V}}(R) \subseteq \mathcal{R}^{\mathcal{U}}(R)$. preserved-names do not occur in the left-hands of inclusion axioms. So for each axiom or individual assertion α, by $\mathcal{V} \models_v \alpha$ and (♣), $\mathcal{U} \models_u \alpha$ holds. So $\mathcal{U}$ is a u-model of $\mathcal{K}$. Thus $\mathcal{K}$ is u-satisfiable.

(2) We first prove that (♠) for each assertion α with the form $P_p(a, b)$ or $\mathsf{type}(a, A_p)$, where P_p is a preserved-role except type and A_p is a preserved-class, then if $\mathcal{K}_{u\to v} \models_u \alpha$ then $\mathcal{K}_{u\to v} \models_v \alpha$. Assume (A) $\mathcal{K}_{u\to v} \not\models_v \alpha$. Let $\mathcal{V}$ be a canonical v-model of $\mathcal{K}_{u\to v}$. Suppose α is an assertion $\mathsf{subClassOf}(A, B)$. Then $(A, B) \notin \mathcal{R}^{\mathcal{V}}(\mathsf{subClassOf})$. Let $\mathcal{T}$ be the TBox of $\mathcal{K}_{u\to v}$, and let $\mathcal{A}' = \{\mathsf{type}(o_A, A)\}$ where o_A is an ordinary name not occurring in $\mathcal{K}_{u\to v}$. Obviously $(\mathcal{T}, \mathcal{A}')$ is v-satisfiable, since $\mathcal{T} \not\models_v A \sqsubseteq_c B$ implies $\mathcal{T} \not\models_v A \sqsubseteq_c \neg A$. Let $\mathcal{V}'$ be a canonical v-model of $(\mathcal{T}, \mathcal{A}')$. We assume $(\Delta^{\mathcal{V}} - \mathsf{N}) \cap (\Delta^{\mathcal{V}'} - \mathsf{N}) = \emptyset$, i.e., $\mathcal{V}'$ and $\mathcal{V}$ do not share any anonymous element. From $\mathcal{V}$ and $\mathcal{V}'$, we construct another v-interpretation $\mathcal{V}''$ by setting (a) $\Delta^{\mathcal{V}''} = \Delta^{\mathcal{V}} \cup \Delta^{\mathcal{V}'}$; (b) $a^{\mathcal{V}''} = a$ for each $a \in \mathsf{N}$; and (c) $\mathcal{C}^{\mathcal{V}''}(o) = \mathcal{C}^{\mathcal{V}}(o) \cup \mathcal{C}^{\mathcal{V}'}(o)$ and $\mathcal{R}^{\mathcal{V}''}(o) = \mathcal{R}^{\mathcal{V}}(o) \cup \mathcal{R}^{\mathcal{V}'}(o)$ for each $o \in \Delta^{\mathcal{V}''}$. It can be trivially validate that $\mathcal{V}''$ is a v-model of $\mathcal{K}$. And $\mathcal{C}^{\mathcal{V}''}(A) \not\subseteq \mathcal{C}^{\mathcal{V}''}(B)$ holds. From $\mathcal{V}''$, a u-model $\mathcal{U}$ can be constructed using the way presented in (1). By the settings (§), we can get that $(A, B) \notin \mathcal{R}^{\mathcal{V}''}(\mathsf{subClassOf})$. This contradicts

with that $\mathcal{U}$ is a u-model of $\mathcal{K}_{u\to v}$. So assumption (A) does not hold. Thus $\mathcal{K}_{u\to v} \models_v \mathsf{subClassOf}(A, B)$ holds. The other forms of α can be proved analogously.

Let $\vec{u} \in \mathsf{ans}_u(Q, \mathcal{K}_{u\to v})$. We prove $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K}_{u\to v})$. Let $\mathcal{S}$ be the set of all the atoms in $Q(\vec{u})$ with the forms $P_p(a, b)$ or $\mathsf{type}(a, C_p)$, where P_p is a preserved-role except type and C_p is a preserved-class except Class and $\mathsf{Property}$. Then all the atoms in $\mathcal{S}$ do not contain variables and $\mathcal{K}_{u\to v} \models_u \alpha$ holds for each $\alpha \in \mathcal{S}$. Thus ($\bigstar$) $\mathcal{K}_{u\to v} \models_v \alpha$ holds for each $\alpha \in \mathcal{S}$. Let Q' be the query $\wedge_{\alpha\in \mathsf{body}(Q(\vec{u}))-\mathcal{S}} \to q()$. Then by ($\bigstar$), $\vec{u} \in \mathsf{ans}_u(Q, \mathcal{K}_{u\to v})$ can be proved by showing $() \in \mathsf{ans}_v(Q', \mathcal{K}_{u\to v})$, i.e., Q' is true over $\mathcal{K}_{u\to v}$. Let $\mathcal{V}$ be a canonical v-model of $\mathcal{K}_{u\to v}$. Then a u-model $\mathcal{U}$ of $K_{u\to v}$ can be constructed using the way presented in (1). Thus there exists a binding π of Q' over $\mathcal{U}$ such that $\mathcal{U}, \pi \models_u Q'$. Q' does not contain atoms with the forms $P_\mathsf{p}(a, b)$ or $\mathsf{type}(a, C_\mathsf{p})$, where P_p is a meta-role except type and C_p is a preserved-class except Class and $\mathsf{Property}$. From the construction of $\mathcal{U}$, we can get that π is also a binding of Q' over $\mathcal{V}$. Thus $() \in \mathsf{ans}_v(Q', \mathcal{K}_{u\to v})$ holds. Hence $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K}_{u\to v})$. Therefore $\mathsf{ans}_u(Q, \mathcal{K}_{u\to v}) \subseteq \mathsf{ans}_v(Q, \mathcal{K}_{u\to v})$ holds. $\square$

Proof of Lemma 7.

Proof. This lemma holds according to the following facts. Let $\mathcal{K} = (\mathcal{T}, \mathcal{A}_m \cup \mathcal{A}_o)$ be an asserted DL-Lite Full KB. (a) For a DL-Lite Full axiom α, $\mathcal{K} \models_v \alpha$ iff $\tau_{dl}(\mathcal{T}) \models \tau(\alpha)$; (b) For an assertion $P_p(a, b)$ such that P_p is a preserved-role except type, then $\mathcal{K} \models_v P_p(a, b)$ iff there exists $\mathsf{gr}(P, a, b) \in \mathcal{A}_m$ such that $P \sqsubseteq_r^* P_p$ w.r.t. $\mathcal{T}$; (c) For an assertion $\mathsf{type}(a, C_p)$ such that C_p is a preserved-class except, then $\mathcal{K} \models_v \mathsf{type}(a, C_p)$ iff there exists $\mathsf{gr}(P, a, b) \in \mathcal{A}_m$ such that $P \sqsubseteq_r^* \mathsf{type}$ and $\mathcal{K} \models_v b \sqsubseteq_c C_p$, or $\mathcal{K} \models_v \exists P \sqsubseteq_c C_p$. $\square$

Proof of Theorem 7.

Proof. By Lemma 7, Theorem 6 and Theorem 5, this theorem holds trivially. $\square$