DL-Lite Full: a sub-Language of OWL 2 Full for Powerful Meta-modeling (Appendixes)

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Appendix A: Proofs of Section 3

Proof of Lemma 1

Proof. (1) $\mathcal{K}$ is ν-satisfiable, thus it has a ν-model $\mathcal{V}$. From $\mathcal{V}$, we construct an interpretation $\mathcal{I}$ of $\tau(\mathcal{K})$ by setting (a) $\Delta^{\mathcal{I}} = \Delta^{\mathcal{V}}$; (b) for each $a \in \mathsf{N}$, $a^{\mathcal{I}} = a^{\mathcal{V}}$; (c) for each $A \in \mathsf{C}$, $A^{\mathcal{I}} = \mathcal{C}^{\mathcal{V}}(v_c^-(A)^{\mathcal{V}})$; and (d) for each $P \in \mathsf{R}$, $P^{\mathcal{I}} = \mathcal{R}^{\mathcal{V}}(v_r^-(A)^{\mathcal{V}})$, where v_c^- and v_r^- denote the inverse functions of v_c and v_r respectively. DL-Lite Full and DL-Lite$_{\mathcal{R}}$ follow the same principles to interpret the constructors of classes and roles. Thus we can get that (♠) $\mathcal{C}^{\mathcal{V}}(C) = \tau_c(C)^{\mathcal{I}}$ for each DL-Lite Full class C and $\mathcal{R}^{\mathcal{V}}(R) = \tau_r(R)^{\mathcal{I}}$ for each DL-Lite Full role R. Then we can further obtain that $\mathcal{I}$ satisfies all the axioms and assertions in $\tau(\mathcal{K})$. Thus $\tau(\mathcal{K})$ is satisfiable.

(2) If $\mathcal{K}$ is not ν-satisfiable then this conclusion holds directly. Next, we assume $\mathcal{K}$ is ν-satisfiable. Thus $\tau(\mathcal{K})$ is satisfiable. Let $\vec{u} \in \mathsf{ans}(\tau(Q), \tau(\mathcal{K}))$. Let $\mathcal{V}$ be an arbitrary ν-model of $\mathcal{K}$. By the approach described in (1), a model $\mathcal{I}$ of $\tau(\mathcal{K})$ can be constructed. Thus there exists a binding π of $\tau(Q)$ over $\mathcal{I}$ such that $\mathcal{I}, \pi \models \tau(Q(\vec{u}))$ holds. From π, a binding π' of $Q(\vec{u})$ over $\mathcal{V}$ can be constructed by setting $\pi'(x) = \pi(x)$ for each variable x of $Q(\vec{u})$ and $\pi'(a) = \pi(a)$ for each name a in $Q(\vec{u})$. Then by (♠), we can further obtain that $\mathcal{V}, \pi' \models Q(\vec{u})$. Thus $\vec{u} \in \mathsf{ans}_\nu(Q, \mathcal{K})$ holds. Therefore, this direction holds. □

Proof of Theorem 1

Proof. Let $\tau^{\mathsf{m}}(\mathcal{K}) = (\tau(\mathcal{T}), \tau(\mathcal{A}) \cup \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3)$ where $\mathcal{N}_1 - \mathcal{N}_3$ are the sets of assertions obtained from the Step 1-3 respectively. Let $\mathcal{A}_1 - \mathcal{A}_3$ be the sets of assertions such that $\tau(\mathcal{A}_i) = \mathcal{N}_i$ for $i \in \{1, 2, 3\}$. Suppose:

$$\begin{aligned} \mathcal{A}_2 &= \{A_1(a_1), \cdots, A_k(a_k)\} \\ \mathcal{A}_3 &= \{P_1(b_1, c_1), c_1(b_1), \cdots, P_m(b_m, c_m), c_m(b_m)\} \quad \text{then:} \end{aligned}$$

$$\begin{aligned} \mathcal{N}_2 &= \{v_c(A_1)(a_1), \cdots, v_c(A_k)(a_k)\} \\ \mathcal{N}_3 &= \{v_r(P_1)(b_1, c_1), v_c(c_1)(b_1), \cdots, v_r(P_m)(b_m, c_m), v_c(c_m)(b_m)\} \end{aligned}$$

And let $\mathcal{K}' = (\mathcal{T}, \mathcal{A} \cup \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3)$. Then $\tau^{\mathsf{m}}(\mathcal{K}) = \tau(\mathcal{K}')$ holds. And for each $1 \le i \le k$, $\tau(\mathcal{K}) \models \exists v_r(\mathsf{type})^-(A_i)$, and for each $1 \le j \le m$, $\tau(\mathcal{K})$ entails $\exists v_r(P_j)(c_j)^-$ and $v_r(P_j) \sqsubseteq v_r(\mathsf{type})$. The corresponding individual assertions in $\mathcal{N}_1$ can also be entailed by $\mathcal{K}$. For example, if $A(a) \in \mathcal{N}_1$ then $\mathcal{K}$ ν-entails $v_c^-(A)(a)$, since $\tau(\mathcal{K})$ entails $v_r(\mathsf{type})(a, v_c^-(A))$. Thus here we do not give the concrete assertions in $\mathcal{N}_1$.

(1. ⇒) $\mathcal{K}$ is ν-satisfiable, thus it has a ν-model $\mathcal{V}$. The satisfiability of $\tau^m(\mathcal{K})$ can be proved by showing that $\mathcal{K}'$ is satisfiable. Next, based on $\mathcal{V}$, we construct a model for $\mathcal{K}'$. $a_1, \cdots, a_k$ and $b_1, \cdots, b_m$ do not occur in $\mathcal{K}$. So we can assume that the class and role extensions of the interpretations of $a_1, \cdots, a_k$ and $b_1, \cdots, b_m$ are empty set, and the interpretations of $a_1, \cdots, a_k$ and $b_1, \cdots, b_m$ do not occur in the class and role extensions of other elements. Otherwise, we can make the following operation. Let $\{a'_1, \cdots, a'_k, b'_1, \cdots, b'_m\}$ be $n + m$ new elements not occurring in $\Delta^{\mathcal{V}} \cup \mathsf{N}$.

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In $\mathcal{V}$, set $\Delta^{\mathcal{V}} = \Delta^{\mathcal{V}} \cup \{a'_1, \cdots, a'_k, b'_1, \cdots, b'_m\}$, $a_i^{\mathcal{V}} = a'_i$ and $b_j^{\mathcal{V}} = b'_j$ as well as $C^{\mathcal{V}}(o) = \mathcal{R}^{\mathcal{V}}(o) = \emptyset$ for $1 \leq i \leq k$, $1 \leq j \leq m$ and $o \in \{a'_1, \cdots, a'_k, b'_1, \cdots, b'_m\}$, while keeping the others unchanged. Then $\mathcal{V}$ is still a ν-model of $\mathcal{K}$.

Next, based on $\mathcal{V}$, we construct a ν-model of $\mathcal{K}'$ by making a copy of $\mathcal{V}$ for each a_i and b_j and then merging these copies. For each $\gamma \in \{a_1, \cdots, a_k, b_1, \cdots, b_m\}$, we make a copy $\mathcal{V}_\gamma = (\Delta^{\mathcal{V}_\gamma}, \cdot^{\mathcal{V}_\gamma}, C^{\mathcal{V}_\gamma}, \mathcal{R}^{\mathcal{V}_\gamma})$ of $\mathcal{V}$ where (a1) $|\Delta^{\mathcal{V}_\gamma}| = |\Delta^{\mathcal{V}}|$, $\Delta^{\mathcal{V}_\gamma} \cap \Delta^{\mathcal{V}} = \emptyset$ and there exists a bijective function $\delta_\gamma : \Delta^{\mathcal{V}_\gamma} \to \Delta^{\mathcal{V}}$; (b1) for each $o \in \mathsf{N}$, $a^{\mathcal{V}_\gamma} = \delta_\gamma^-(a^{\mathcal{V}})$; and (c1) for each $o \in \Delta^{\mathcal{V}_\gamma}$, $C^{\mathcal{V}_\gamma}(o) = C^{\mathcal{V}}(\delta_\gamma(o))$ and $\mathcal{R}^{\mathcal{V}_\gamma}(o) = \mathcal{R}^{\mathcal{V}}(\delta_\gamma(o))$. Then $\mathcal{V}_\gamma$ is also a ν-model of $\mathcal{K}$. For every different elements γ and γ' in $\{a_1, \cdots, a_k, b_1, \cdots, b_m\}$, we assume $\Delta^{\mathcal{V}_\gamma} \cap \Delta^{\mathcal{V}_{\gamma'}} = \emptyset$. For each $A_i(a_i)$, we know $\mathcal{K}$ entails $\exists\mathsf{type}^-(A_i)$. Thus there exists $(e_i, A_i^{\mathcal{V}}) \in \mathcal{R}^{\mathcal{V}}(\mathsf{type}^{\mathcal{V}})$. Similarly, for each $P_j(b_j, c_j)$ and $c_j(b_j)$, there exists $o_j \in \Delta^{\mathcal{V}}$ such that $(o_j, b_j^{\mathcal{V}}) \in \mathcal{R}^{\mathcal{V}}(P_j^{\mathcal{V}})$ and $(o_j, b_j^{\mathcal{V}}) \in \mathcal{R}^{\mathcal{V}}(\mathsf{type}^{\mathcal{V}})$. From $\mathcal{V}, \mathcal{V}_{a_1}, \cdots, \mathcal{V}_{a_k}, \mathcal{V}_{b_1}, \cdots, \mathcal{V}_{b_m}$, we construct a new ν-interpretation $\mathcal{V}'$ by setting:

(a2) $\Delta^{\mathcal{V}'} = \Delta^{\mathcal{V}} \cup (\bigcup_{i=1}^k \Delta^{\mathcal{V}_{a_i}}) \cup (\bigcup_{j=1}^m \Delta^{\mathcal{V}_{b_j}})$;

(b2) For each $a \in \mathsf{N} - \{a_1, \cdots, a_k, b_1, \cdots, b_m\}$, $a^{\mathcal{V}'} = a^{\mathcal{V}}$; for each a_i, $a_i^{\mathcal{V}'} = \delta_{a_i}(e_i)$; and for each b_j, $b_j^{\mathcal{V}'} = \delta_{b_j}(o_j)$ for $1 \leq i \leq k$ and $1 \leq j \leq m$

(c2) For each $o \in \Delta^{\mathcal{V}'}$, if there exists $a \in \mathsf{N}$ such that $o = a^{\mathcal{V}'}$ then $C^{\mathcal{V}'}(o) = C^{\mathcal{V}}(a^{\mathcal{V}}) \cup (\bigcup_{i=1}^k C^{\mathcal{V}_{a_i}}(a^{\mathcal{V}_{a_i}})) \cup (\bigcup_{j=1}^m C^{\mathcal{V}_{b_j}}(a^{\mathcal{V}_{b_j}}))$ and $\mathcal{R}^{\mathcal{V}'}(o) = \mathcal{R}^{\mathcal{V}}(a^{\mathcal{V}}) \cup (\bigcup_{i=1}^k \mathcal{R}^{\mathcal{V}_{a_i}}(a^{\mathcal{V}_{a_i}})) \cup (\bigcup_{j=1}^m \mathcal{R}^{\mathcal{V}_{b_j}}(a^{\mathcal{V}_{b_j}}))$; otherwise, there exists $V \in \{\mathcal{V}, \mathcal{V}_{a_1}, \cdots, \mathcal{V}_{a_k}, \mathcal{V}_{b_1}, \cdots, \mathcal{V}_{b_m}\}$ such that $o \in \Delta^V$, then set $C^{\mathcal{V}'}(o) = C^V(o)$ and $\mathcal{R}^{\mathcal{V}'}(o) = C^V(o)$.

Then it holds trivially that $\mathcal{V}'$ is a ν-model of $\mathcal{K}'$. From $\mathcal{V}'$, a model $\mathcal{I}$ can be constructed for $\tau^{\mathsf{m}}(\mathcal{K})$ using the approach in the proof of Lemma 1. Thus $\tau^{\mathsf{m}}(\mathcal{K})$ is satisfiable.

(1. $\Leftarrow$) $\tau^{\mathsf{m}}(\mathcal{K})$ is satisfiable. Thus it has a canonical model $\mathcal{I}$. Let $(e_1, a_1), \cdots, (e_k, a_k)$ be all the elements in $\nu_r(\mathsf{type})^{\mathcal{I}}$ such that $e_i \notin \mathsf{N}$ and $a_i \in \mathsf{N}$. Let $\mathcal{I}_{e_i}$ be a canonical model of $(\tau(\mathcal{T}), \{\nu_c(a_i)(e_i)\})$ (here e_i is treated as a name) for $1 \leq i \leq k$, where $(\Delta^{\mathcal{I}} \cap \Delta^{\mathcal{I}_{e_i}}) - (\mathsf{N} - \{e_i\}) = \emptyset$ and $(\Delta^{\mathcal{I}_{e_i}} \cap \Delta^{\mathcal{I}_{e_j}}) - \mathsf{N} = \emptyset$ when $i \neq j$. From $\mathcal{I}$ and $\mathcal{I}_{e_1} - \mathcal{I}_{e_k}$, we construct another interpretation $\mathcal{I}'$ by setting (a1) $\Delta^{\mathcal{I}'} = \Delta^{\mathcal{I}} \cup (\bigcup_{i=1}^m \Delta^{\mathcal{I}_{e_i}})$; (b1) for each $a \in \mathsf{N}$, $a^{\mathcal{I}'} = a$; and (c1) for each $A \in \mathsf{C}$, $A^{\mathcal{I}'} = A^{\mathcal{I}} \cup (\bigcup_{i=1}^m A^{\mathcal{I}_{e_i}})$, and for each $P \in \mathsf{R}$, $P^{\mathcal{I}'} = P^{\mathcal{I}} \cup (\bigcup_{i=1}^m P^{\mathcal{I}_{e_i}})$. Obviously, $\mathcal{I}'$ satisfies the individual assertions as well as positive inclusion axioms in $\tau^{\mathsf{m}}(\mathcal{K})$. Next, we show $\mathcal{I}'$ also satisfies the negative axioms in $\tau(\mathcal{T})$. Actually, we just need to check class disjoint axioms. For each $C \sqsubseteq \neg D \in \tau(\mathcal{T})$, suppose (♠) there exists $o \in C^{\mathcal{I}'} \cap D^{\mathcal{I}'}$. Then by the construction of $\mathcal{I}'$, we can get that there exists e_i such that $o = e_i$, and $e_i \in C^{\mathcal{I}}$ and $e_i \in D^{\mathcal{I}_{e_i}}$ holds, or $e_i \in C^{\mathcal{I}_{e_i}}$ and $e_i \in D^{\mathcal{I}}$ holds, since $C^{\mathcal{I}} \cap D^{\mathcal{I}} = C^{\mathcal{I}_{e_i}} \cap D^{\mathcal{I}_{e_i}} = \emptyset$. Suppose $e_i \in C^{\mathcal{I}} \cap D^{\mathcal{I}_{e_i}}$. Then we can get that $\tau(\mathcal{K})$ entails $\nu_c(c_i) \sqsubseteq D$ and there exists role S such that $(e_i, c_i) \in S^{\mathcal{I}}$ and $\tau(\mathcal{T})$ entails $\exists S \sqsubseteq C$ and $S \sqsubseteq \nu_r(\mathsf{type})$. Then there exists $S(b_i, c_i), \nu_c(c_i)(b_i)$ in $\mathcal{A}_2$. And this contradicts with the fact that $\mathcal{K}_o = (\tau(\mathcal{T}), \{S(b_i, c_i), \nu_c(c_i)(b_i)\})$ is satisfiable, since $\mathcal{K}_o$ entails $C(b_i)$ and $D(b_i)$. Thus assuption (♠) does not hold. Thus $\mathcal{I}'$ satisfies all the negative axioms in $\tau(\mathcal{T})$. Thus $\mathcal{I}'$ is a model of $\tau^m(\mathcal{K})$. From $\mathcal{I}'$, we construct a ν-interpretation $\mathcal{V}$ for $\mathcal{K}$ by setting $(a2)$ $\Delta^{\mathcal{V}} = \Delta^{\mathcal{I}'}$; $(b2)$ for each $a \in \mathsf{N}$, $a^{\mathcal{V}} = a$; $(c2)$ for each $o \in \Delta^{\mathcal{V}}$, if $o \in \mathsf{N}$, then $C^{\mathcal{V}}(o) = o^{\mathcal{I}'}$, otherwise $C^{\mathcal{V}}(o) = \{e | (e, o) \in \mathsf{type}^{\mathcal{I}'}\}$; and $(d2)$ for each $o \in \Delta^{\mathcal{V}}$, if $o \in \mathsf{N} - \{\mathsf{type}\}$, then $\mathcal{R}^{\mathcal{V}}(o) = o^{\mathcal{I}'}$, otherwise if $o = \mathsf{type}$ then $\mathcal{R}^{\mathcal{V}}(o) = o^{\mathcal{I}'} \cup \{(e, e') | e' \in \Delta^{\mathcal{V}} \wedge e \in C^{\mathcal{V}}(e')\}$, otherwise $\mathcal{R}^{\mathcal{V}}(o) = \emptyset$. Next, we show $\mathcal{V}$ is correctly defined by checking the interpretation of type, i.e., (♣) $\mathcal{R}^{\mathcal{V}}(\mathsf{type}) = \{(e, o) | o \in \Delta^{\mathcal{V}}, e \in C^{\mathcal{V}}(o)\}$ holds. For each $(e, o) \in \mathcal{R}^{\mathcal{V}}(\mathsf{type})$, if $o \in \mathsf{N}$ then $e \in \nu_c(o)^{\mathcal{I}'}$ holds, thus $(e, o) \in \{(e, o) | o \in \Delta^{\mathcal{V}}, e \in C^{\mathcal{V}}(o)\}$ holds; otherwise $(e, o) \in \{(e, o) | o \in \Delta^{\mathcal{V}}, e \in C^{\mathcal{V}}(o)\}$ holds directly. The other direction of (♣) holds directly. Thus $\mathcal{V}$ is a ν-model of $\mathcal{K}$. $\mathcal{V}$ and $\mathcal{I}'$ follow the same principles to interpret class and role constructors. Then by the construction of $\mathcal{V}$, we can get that $\mathcal{V}$ satisfies all the axioms and assertions in $\mathcal{K}$. Thus $\mathcal{K}$ is ν-satisfiable.

(2. $\subseteq$) Let $\vec{u} \in \mathsf{ans}_\nu(Q, \mathcal{K})$. Obviously $\vec{u}$ does not contain the extra names in $\mathcal{N}_2 \cup \mathcal{N}_3$. Let $\mathcal{I}$ be a canonical model of $\tau^{\mathsf{m}}(\mathcal{K})$. From $\mathcal{I}$, a ν-model $\mathcal{V}$ can be constructed from an extension $\mathcal{I}'$ of $\mathcal{I}$ by the approach described in (1. $\Leftarrow$). Thus there exists a binding π of Q over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_\nu Q(\vec{u})$. From π, a binding π' of $\tau(Q(\vec{u}))$ over $\mathcal{I}$ can be constructed by setting $\pi'(x) = \pi(x)$ for each variable x in $\tau(Q)$ and $\pi'(a) = \pi(a)$ for each individual a in $\tau(Q)$. Then, $\mathcal{I}', \pi' \models \tau(Q(\vec{u}))$ holds. Then from the construction of $\mathcal{I}'$ and the obtain of $\mathcal{N}_2$ and $\mathcal{N}_3$, we can construct a binding π'' of $\tau(Q(\vec{u}))$ over $\mathcal{I}$ such that $\mathcal{I}, \pi'' \models \tau(Q(\vec{u}))$ holds. Thus $\vec{u} \in \mathsf{ans}(Q, \tau^{\mathsf{m}}(\mathcal{K}))^-$ holds.

(2. $\supseteq$) Let $\vec{u} \in \mathsf{ans}(\tau(Q), \tau^{\mathsf{m}}(\mathcal{K}))^-$. Next, we show that $\vec{u} \in \mathsf{ans}_\nu(Q, \mathcal{K})$. Let $\mathcal{V}$ be an arbitrary model of $\mathcal{K}$. From $\mathcal{V}$, we can construct a model $\mathcal{V}'$ of $\mathcal{K}'$ using the approach in (1. $\Rightarrow$). Then a model $\mathcal{I}$ of $\tau^{\mathsf{m}}(\mathcal{K})$ can be constructed from $\mathcal{V}'$ using the approach described in (1. $\Rightarrow$). Thus there exists a binding π of $\tau(Q(\vec{u}))$ over $\mathcal{I}$ such that $\mathcal{I}, \pi \models \tau(Q\vec{u})$. From π, a binding π' of $Q(\vec{u})$ over $\mathcal{V}'$ can be constructed by setting $\pi'(a) = a^{\mathcal{V}'}$ for each name a in $Q(\vec{u})$ and $\pi'(x) = \pi(x)$ for each variable x in $Q(\vec{u})$. Then by the construction of $\mathcal{I}$, $\mathcal{V}', \pi' \models Q(\vec{u})$ holds. $\vec{u}$ does not

contain the names not occurring in $\mathcal{K}$. Then by the construction of $\mathcal{V}'$, we can easily obtain that there exists a binding π'' of $Q(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi'' \models Q(\vec{u})$ holds. From the arbitrary feature of $\mathcal{V}$, $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$ holds. $\square$

Proof of Theorem 2

Proof. Suppose $\tau^{\mathsf{m}}(\mathcal{T}) = (\tau(\mathcal{T}), \tau(\mathcal{A}) \cup \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3)$. Similar to the proof of Theorem 1, Let $\mathcal{A}_1 - \mathcal{A}_3$ be the sets of assertions such that $\tau(\mathcal{A}_i) = \mathcal{N}_i$ for $i \in \{1, 2, 3\}$. Let $\mathcal{K}' = (\mathcal{T}, \mathcal{A} \cup \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3)$. Then $\tau^{\mathsf{m}}(\mathcal{K}) = \tau(\mathcal{K}')$. It holds trivially that for each conjunctive query Q' and DL-Lite Full ABox $\mathcal{A}$, (C) $\mathsf{ans}_v(Q', \mathcal{A}) = \mathsf{ans}(\tau(Q'), \tau(\mathcal{A}))$ holds.

(1. $\Rightarrow$) For each $q \in \mathsf{Violates}_v(\mathcal{T})$ such that $\mathsf{ans}_v(q, \mathcal{A}) \neq \emptyset$. Then there exists a disjoint axiom $\gamma \sqsubseteq_l \neg\gamma'$ ($l \in \{r, c\}$), such that $q \in \mathsf{PerfectRef}_v(Q, \mathcal{T})$, where Q is the query corresponding to $\gamma \sqsubseteq_l \neg\gamma'$. For example, if $\gamma \sqsubseteq_l \neg\gamma'$ is $A \sqsubseteq_c \neg B$, then Q is $A(?x) \wedge B(?x) \rightarrow q()$ where A and B are names. According to the algorithm $\mathsf{PerfectRef}_v$, we can get that $\mathsf{ans}_v(q, \mathcal{A}) \subseteq \mathsf{ans}_v(Q, \mathcal{K})$. Thus $\mathsf{ans}_v(Q, \mathcal{K}) \neq \emptyset$. This means $\mathcal{K}$ entails that γ and γ' share some individuals. This contradicts with that $\mathcal{K}$ is v-satisfiable. Thus for each $q \in \mathsf{Violate}_v(\mathcal{T})$, $\mathsf{ans}_v(q, \mathcal{A}) = \emptyset$.

(1. $\Leftarrow$) Suppose ($\spadesuit$) $\mathcal{K}$ is not v-satisfiable. Then $\tau^{\mathsf{m}}(\mathcal{K})$ is not satisfiable. Then there exists a disjoint axiom in $\mathsf{cl}(\tau(\mathcal{T}))$, i.e., in the negative closure of $\tau(\mathcal{T})$, suppose it is $A \sqsubseteq \neg B$, such that ($\clubsuit$) $\mathsf{ans}(q, \tau(\mathcal{A}) \cup \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3)$, where q is the query $A(?x) \wedge B(?x) \rightarrow q()$. From the algorithm $\mathsf{Violates}_v$, we can get that there exists $Q \in \mathsf{Violates}_v(\mathcal{T})$ such that $\tau(Q) = q$. From ($\clubsuit$), we can further get that (a) $\mathsf{ans}(q, \tau(\mathcal{A})) \neq \emptyset$; (bi) $\mathsf{ans}(q, \mathcal{N}_i) \neq \emptyset$ where $i = 1, 2, 3$, since $\mathcal{N}_1$, $\mathcal{N}_2$ and $\mathcal{N}_3$ do not share any individuals; (c) $\mathsf{ans}(q, \tau(\mathcal{A}) \cup \mathcal{N}_1) \neq \emptyset$. If (a) holds, then obviously $\mathsf{ans}_v(Q, \mathcal{A}) \neq \emptyset$. This contradicts with the condition that (C) for each $Q_o \in \mathsf{Violates}_v(\mathcal{T})$, $\mathsf{ans}_v(Q_o, \mathcal{A}) \neq \emptyset$. If (b1) holds, i.e., $\mathsf{ans}(q, \mathcal{N}_1) \neq \emptyset$. Then there exists individual a such that $A(a)$ and $B(a)$ in $\mathcal{N}_1$. Then from the construction of $\mathcal{N}_1$, $\tau(\mathcal{K}) \models v_r(\mathsf{type})(a, v_c^-(A))$ and $\tau(\mathcal{K}) \models v_r(\mathsf{type})(a, v_c^-(B))$ hold. Thus there exists $\mathsf{I}(P_1, a, v_c^-(A))$ and $\mathsf{I}(P_2, a, v_c^-(B))$ in $\tau(\mathcal{A})$ such that $\tau(\mathcal{T}) \models P_1 \sqsubseteq v_r(\mathsf{type})$ and $\tau(\mathcal{T}) \models P_2 \sqsubseteq v_r(\mathsf{type})$. According to $\mathsf{PerfectRef}_v$, we can get that $q : P_1(?x, v_c^-(A)) \wedge P_2^-(?x, v_c^-(B)) \rightarrow q()$ in $\mathsf{Violates}_v(\mathcal{T})$. Obviously, $\mathsf{ans}_v(q, \mathcal{A}) \neq \emptyset$. This also contradicts with (C), thus (b1) does not hold. Condition (b2), (b3) and (c) as well as other forms of the disjoint axioms can be validated similarly. Thus the assumption ($\spadesuit$) does not holds. Thus $\mathcal{K}$ is satisfiable.

(2. $\supseteq$) This direction holds directly according to the following two observations. (O_1) For two conjunctive queries Q' and Q'', if Q'' is obtained from Q' by applying an inclusion axiom over an atom of Q' via the rule described in Algorithm 1, then it holds trivially that $\mathsf{ans}_v(Q'', \mathcal{K}) \subseteq \mathsf{ans}_v(Q', \mathcal{K})$ holds. (O_2) for each conjunctive query Q', $\mathsf{ans}_v(Q', \mathcal{A}) \subseteq \mathsf{ans}_v(Q', \mathcal{K})$. Combing ($\mathsf{O}_1$) and ($\mathsf{O}_2$), this direction holds trivially.

(2. $\subseteq$) Let $\vec{u} \in \mathsf{ans}_v(Q, \mathcal{K})$. Next, we show that ($\clubsuit$) there exists $Q' \in \mathsf{PerfectRef}_v(Q, \mathcal{T})$ such that $\vec{u} \in \mathsf{ans}_v(Q', \mathcal{A})$. By Theorem 1, $\vec{u} \in \mathsf{ans}(\tau(Q), \tau^{\mathsf{m}}(\mathcal{K}))$ holds. Thus there exists $q \in \mathsf{PerfectRef}(\tau(Q), \tau(\mathcal{T}))$ such that $\vec{u} \in \mathsf{ans}(q, \tau(\mathcal{A}) \cup \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3)$. Let Q' be the query such that $\tau(Q') = q$. Then $Q' \in \mathsf{PerfectRef}_v(Q, \mathcal{T})$. If $\vec{u} \in \mathsf{ans}(q, \tau(\mathcal{A}))$ holds, then $\vec{u} \in \mathsf{ans}_v(Q', \mathcal{A})$ holds directly. Otherwise let $\mathcal{N}_1'$, $\mathcal{N}_2'$ and $\mathcal{N}_3'$ be the smallest sets of $\mathcal{N}_1$, $\mathcal{N}_2$ and $\mathcal{N}_3$ respectively such that $\vec{u} \in \mathsf{ans}(q, \tau(\mathcal{A}) \cup \mathcal{N}_1' \cup \mathcal{N}_2' \cup \mathcal{N}_3')$. Suppose:

$$\begin{aligned}
\mathcal{N}_1' &= \{A_1(a_1), \cdots, A_k(a_k)\} \\
\mathcal{N}_2' &= \{B_1(b_1), \cdots, B_m(b_m)\} \\
\mathcal{N}_3' &= \{P_1(c_1, d_1), v_c(d_1)(c_1), \cdots, P_l(c_l, d_l), v_c(d_l)(c_l)\} \cup \{C_1(o_1), \cdots, C_n(o_n)\} \cup \{S_1(e_1, f_1), \cdots, S_h(e_h, f_h)\}
\end{aligned}$$

Then q is the query:

$$(\bigwedge_{i=1}^{j} \alpha_i) \wedge (\bigwedge_{i=1}^{k} A_i(x_i^1)) \wedge (\bigwedge_{i=1}^{m} B_i(x_i^2)) \wedge (\bigwedge_{i=1}^{l} (P_i(x_i^3, d_i) \wedge v_c(d_i)(x_i^3))) \wedge (\bigwedge_{i=1}^{n} C_i(x_i^4)) \wedge (\bigwedge_{i=1}^{l} S_i(x_i^5, y_i^5)) \rightarrow q(\vec{x})$$

Let $Q'' = Q'$. Then do the following operations. (a) For each $A_i(a_i)$, $\tau(\mathcal{K}) \models v_r(\mathsf{type})(a_i, v_c^-(A_i))$. Thus, there exists $R_i(a_i, v_c^-(A_i)) \in \tau(\mathcal{A})$ such that $\tau(\mathcal{T}) \models R_i \sqsubseteq v_r(\mathsf{type})$. Then replace $v_c^-(A_i)(x_i^1)$ in Q'' with $v_r^-(R_i)(x_i^1, v_c^-(A_i))$. (b) For each $P_i(c_i, d_i)$, $v_c(d_i)(c_i)$, there exists $P(e, d_i)$ (or $A(d_i)$) in $\tau(\mathcal{A})$ such that $\tau(\mathcal{T})$ entails $\exists P \sqsubseteq \exists P_i$ (or $A \sqsubseteq \exists P_i$) and $P_i^- \sqsubseteq v_r(\mathsf{type})$. Then replace $P_i(x_i^3, d_i) \wedge v_c(d_i)(x_i^3)$ in Q' with $P(x_i^3, d_i)$ (or $A(d_i)$). For each $C_i(o_i)$, there also exists $P(e, v_c^-(C_i))$ (or $A(v_c^-(C_i))$) in $\tau(\mathcal{A})$ and role S such that $\tau(\mathcal{T})$ entails $\exists P \sqsubseteq \exists S$ (or $A \sqsubseteq \exists S$) and $S^- \sqsubseteq v_r(\mathsf{type})$. Then replace $C_i(x_i^4)$ in Q' with $P(x_i^4, v_c^-(C_i))$ or $A(v_c^-(C_i))$; (d) For each $S_i(e_i, f_i)$, there also exists $P(e, f_i)$ (or $A(f_i)$) in $\tau(\mathcal{A})$ such that $\tau(\mathcal{T})$ entails $\exists P \sqsubseteq \exists S_i$ (or $A \sqsubseteq \exists S_i$) and $S_i^- \sqsubseteq v_r(\mathsf{type})$. Then replace $S_i(x_i^5, y_i^5)$ with $P(x_i^5, y_i^5)$ (or $A(y_i^5)$). Then Q'' is the query obtained from Q' by applying axioms over Q' and $Q'' \in \mathsf{PerfectRef}_v(Q, \mathcal{T})$ and $\vec{u} \in \mathsf{ans}_v(Q'', \mathcal{A})$ holds. Thus this direction holds. $\square$

Proof of Theorem 3

Proof. By Theorem 2, this theorem holds. □

Proof of Theorem 4

Proof. Let $\tau^{\mathsf{m}}(\mathcal{K}) = (\tau(\mathcal{T}), \tau(\mathcal{A}) \cup \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3)$ where $\mathcal{N}_1$, $\mathcal{N}_2$ and $\mathcal{N}_3$ are the sets of assertions obtained from Steps 1-3, respectively.

($\subseteq$) Let $\vec{u} \in \mathsf{ans}_\nu(Q, \mathcal{K})$. Next, we show there exists $\theta \in \mathsf{MVB}(Q, \mathcal{K})$ and $Q' \in \mathsf{PerfectRef}_\nu(Q\theta, \mathcal{T})$ without class variables such that $\vec{u} \in \mathsf{ans}_\nu(Q', \mathcal{A})$ holds. Let $\mathcal{V}$ be a ν-model of $\mathcal{K}$ constructed from a canonical model $\mathcal{I}$ of $\tau^{\mathsf{m}}(\mathcal{K})$ using the approach in the (1. $\Leftarrow$) direction of the proof of Theorem 1. Then there exists a binding π of $Q(\vec{u})$ over $\mathcal{V}$ such that $V, \pi \models Q(\vec{u})$. Based on π, we construct a MV-Binding $\theta = (\theta_r, \theta_c)$ of Q over $\mathcal{K}$. For each role variable $?x$ of Q, if $?x$ occurs in the i-th position of $\vec{u}$, then set $\theta_r(?x) = \vec{u}[i]$, otherwise, we know $\pi(?x)$ is a name, then set $\theta_r = \pi(?x)$. For each class variable $?y$ of $[Q\theta_r]$, if $?y$ occurs in the i-th position of $\vec{u}$ then set $\theta_c(?y) = \vec{u}[i]$, otherwise if $\pi(?y)$ is a name occurring in $\mathcal{K}$ then set $\theta_c(?y) = \pi(?y)$. Let Q' be the query obtained by replacing each atom $P(x, y)$ in $Q\theta$ with $\nu_r(P)(x, y)$, $A(x)$ in $Q\theta$ with $\nu_c(A)(x)$, and

$?c(x)$ in $Q\theta$ with $\nu_r(\mathsf{type})(x, y)$ where A and B are name; x and y are names or variables; and $?c$ is a variable. Then $\vec{u} \in \mathsf{ans}(Q', \tau^{\mathsf{m}}(\mathcal{K}))$ holds. Then there exists $Q'' \in \mathsf{PerfectRef}(Q', \tau(\mathcal{T}))$ such that $\vec{u} \in \mathsf{ans}(Q'', \tau(\mathcal{A}) \cup \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3)$.

($\supseteq$) Obviously, for each MV-Binding ϑ of Q over $\mathcal{K}$, $\mathsf{ans}_\nu(Q\vartheta, \mathcal{K}) \subseteq \mathsf{ans}_\nu(Q, \mathcal{K})$. Besides, if a query Q'' is obtained from a query Q' according to Algorithm 1, then $\mathsf{ans}_\nu(Q'', \mathcal{K}) \subseteq \mathsf{ans}_\nu(Q', \mathcal{K})$ holds. Moreover, for a query Q', $\mathsf{ans}_\nu(Q', \mathcal{A}) \subseteq \mathsf{ans}_\nu(Q', \mathcal{K})$ holds. Then combing these three aspects, this direction of the equation in this theorem holds directly. □

Proof of Lemma 2

Proof. Under ν-semantics, it holds trivially that $\vec{u} \in \mathsf{ans}(Q, (\emptyset, \mathcal{A}))$ iff there exists a function f such that f maps each variable in $Q(\vec{u})$ to a name in $\mathcal{A}$ and all the atoms in $Q(\vec{u})f$ occur in $\mathcal{A}$. Then this lemma holds. □

Proof of Lemma 3

Proof. (1) ($\supseteq$) By Definition 10, i.e., the definition of extensions of partial MV-Bindings, this direction of the equation in this lemma holds. ($\subseteq$) Let θ be an arbitrary MV-Binding of Q over $\mathcal{K}$. Next, we construct a partial MV-Binding ϑ of Q over $\mathcal{T}$ such that $\theta \in \mathsf{ePMVB}(\vartheta, Q, \mathcal{K})$. For each role variable x of Q, set $\vartheta(x) = \theta(x)$ *iff* $\theta(x) \in \mathsf{N}^{rr}_{\mathcal{T}} - \{\mathsf{type}\}$, and for each class variable x of Q, set $\vartheta(x) = \theta(x)$ *iff* $\theta(x) \in \mathsf{N}^{rc}_{\mathcal{T}}$. Then by Definition 10, $\theta \in \mathsf{ePMVB}(\vartheta, Q, \mathcal{K})$ holds. Thus the direction ($\subseteq$) holds.

(2) According to the algorithm $\mathsf{PerfectRef}_\nu$, we can get that (♠) for a conjunctive query q and atom $A(x)$ (resp. $P(x, y)$), if A (resp. P) does not occur in the right hands of any inclusion axioms in a DL-Lite Full TBox $\mathcal{T}$, then this query atom will not be extended by the inclusion axioms in $\mathcal{T}$ to generate new queries, i.e., it will be occur in each query in $\mathsf{PerfectRef}(q, \mathcal{T})$.

($\subseteq$ .1) Let $\theta \in \mathsf{ePMVB}(\vartheta, Q, \mathcal{K})$ and $\vec{u} \in \mathsf{ans}_\nu(Q\theta, \mathcal{K})$. Next, we show there exists $Q' \in \mathsf{PerfectRef}_\nu(Q\vartheta, \mathcal{T})$ such that $\vec{u} \in \mathsf{ans}_\nu(Q', (\emptyset, \mathcal{A}))$. By Theorem 2, there exists $q \in \mathsf{PerfectRef}_\nu(Q\theta, \mathcal{T})$ such that $\vec{u} \in \mathsf{ans}_\nu(Q, (\emptyset, \mathcal{A}))$. Let θ' be a function satisfying that (a) $\mathsf{dom}(\theta') = \mathsf{dom}(\theta) - \mathsf{dom}(\vartheta)$ and for each $x \in \mathsf{dom}(\theta)$, $\theta'(x) = \theta(x)$ holds. Obviously θ' is a MV-Binding of $Q\vartheta$ over $\mathcal{K}$ that maps the class (resp. role) variables of $Q\vartheta$ to the names not occurring in $\mathsf{N}^{\mathsf{rc}}_{\mathcal{T}}$ (resp. $\mathsf{N}^{\mathsf{rr}}_{\mathcal{T}}$). For q, by (♠) and the Definitions of $\mathsf{PerfectRef}_\nu$ and $\mathsf{PerfectRef}_\nu$, we can get that there exists $Q' \in \mathsf{PerfectRef}_\nu(Q\vartheta, \mathcal{T})$ such that $q = Q'\theta'$. Thus $\vec{u} \in \mathsf{ans}_\nu(Q'\theta', (\emptyset, \mathcal{A}))$. Then by Theorem 3, $\vec{u} \in \mathsf{ans}_\nu(Q', (\emptyset, \mathcal{A}))$ holds. Thus the first inclusion holds.

($\subseteq$ .2) Obviously, $\mathsf{ans}_\nu(Q\vartheta, \mathcal{K}) \subseteq \mathsf{ans}_\nu(Q, \mathcal{K})$ holds. Besides, for each query Q'' obtained from query Q' by applying an inclusion axiom over one atom of Q' or by unifying two atoms of Q', then $\mathsf{ans}_\nu(Q'', \mathcal{K}) \subseteq \mathsf{ans}_\nu(Q', \mathcal{K})$ holds, and for each meta-query Q', $\mathsf{ans}_\nu(Q', \mathcal{A}) \subseteq \mathsf{ans}_\nu(Q', \mathcal{K})$ holds. Then this direction holds directly. □

Proof of Theorem 5

Proof. For the MQ Q, KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ and partial MV-binding ϑ_o of Q over $\mathcal{T}$, Lemma 3 indicates the following relations:

$$\mathsf{MVB}(Q, \mathcal{K}) = \bigcup_{\vartheta \in \mathsf{PMVB}(Q,\mathcal{T})} \mathsf{ePMVB}(\vartheta, Q, \mathcal{K})$$

$$\bigcup_{\theta \in \mathsf{ePMVB}(\vartheta_o,Q,\mathcal{K})} \mathsf{ans}_\nu(Q\theta, \mathcal{K}) \subseteq \bigcup_{Q' \in \mathsf{PerfectRef}_\nu(Q\vartheta_o,\mathcal{T})} \mathsf{ans}_\nu(Q', (\emptyset, \mathcal{A})) \subseteq \mathsf{ans}_\nu(Q, \mathcal{K})$$

Obviously, $\mathsf{ans}_\nu(Q, \mathcal{K}) = \bigcup_{\theta \in \mathsf{MVB}(Q,\mathcal{K})} \mathsf{ans}_\nu(Q\theta, \mathcal{K})$ holds. Then by the above equations and inclusions, we can get:

$$\begin{array}{rcl}
\mathsf{ans}_\nu(Q, \mathcal{K}) & = & \bigcup_{\vartheta \in \mathsf{PMVB}(Q,\mathcal{T})} \bigcup_{\theta \in \mathsf{ePMVB}(\vartheta,Q,\mathcal{K})} \mathsf{ans}_\nu(Q\theta, \mathcal{K}) \\
\bigcup_{\vartheta \in \mathsf{PMVB}(Q,\mathcal{T})} \bigcup_{\theta \in \mathsf{ePMVB}(\vartheta,Q,\mathcal{K})} \mathsf{ans}_\nu(Q\theta, \mathcal{K}) & \subseteq & \bigcup_{\vartheta \in \mathsf{PMVB}(Q,\mathcal{T})} \bigcup_{Q' \in \mathsf{PerfectRef}_\nu(Q\vartheta,\mathcal{T})} \mathsf{ans}_\nu(Q', (\emptyset, \mathcal{A})) \\
\bigcup_{\vartheta \in \mathsf{PMVB}(Q,\mathcal{T})} \bigcup_{Q' \in \mathsf{PerfectRef}_\nu(Q\vartheta,\mathcal{T})} \mathsf{ans}_\nu(Q', (\emptyset, \mathcal{A})) & \subseteq & \mathsf{ans}_\nu(Q, \mathcal{K})
\end{array}$$

Combing the above inclusion and equation relationships, the following equation holds:

$$\mathsf{ans}_\nu(Q, \mathcal{K}) = \bigcup_{\vartheta \in \mathsf{PMVB}(Q,\mathcal{T})} \bigcup_{Q' \in \mathsf{PerfectRef}_\nu(Q\vartheta,\mathcal{T})} \mathsf{ans}_\nu(Q', (\emptyset, \mathcal{A}))$$

Thus, the equation in this theorem holds. □

Proof of Lemma 4

Proof. Conjunctive query answering over databases has AC_0 data complexity. By Lemma 2, this theorem holds. □

Proof of Theorem 6

Proof. We first show that MQ answering over DL-Lite Full ABox has AC_0 data complexity. CQ answering over databases has AC_0 data complexity. By Lemma 2, we can get that evaluating MQs over DL-Lite Full ABox has AC_0 data complexity.

If Q has meta-variables, then it has no more than $2^{2|Q|}(2|\mathcal{T}|+2)^{2|Q|}$ partial MV-Bindings over $\mathcal{T}$. Then by Theorem 5, Definitions 4 and Lemma 4, the complexity results for meta-query answering holds. □