# The More Irresistible Hi($\mathcal{SROIQ}$) for Meta-modeling and Meta-query Answering (Supplementary File for ESWC2019)

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**Proof of Proposition 1.**

*Proof.* (1) $\mathcal{K}$ is $v$-satisfiable, so it has a $v$-model $\mathcal{V}$. By $\mathcal{V}$, an interpretation $\mathcal{I}$ of $\mathsf{Dl}(\mathcal{K})$ can be constructed by setting (i) $a^{\mathcal{I}} = a^{\mathcal{V}}$ for each $a \in \mathbf{N}$; (ii) $\{o\}^{\mathcal{I}} = \{o^{\mathcal{I}}\}$ for $o \in \mathbf{N}$ and $A^{\mathcal{I}} = \mathfrak{C}^{\mathcal{V}}(\boldsymbol{v}_c^-(A))$ for $A \in \mathbf{C}$; and (iii) $P^{\mathcal{I}} = \mathfrak{R}^{\mathcal{V}}(\boldsymbol{v}_r^-(P))$ for $P \in \mathbf{R}$ [3]. $\mathcal{V}$ and $\mathcal{I}$ follow the same way to interpret class and role constructors, so it holds trivially that (♠) $\mathfrak{C}^{\mathcal{V}}(C) = \tau_c(C)^{\mathcal{I}}$ for each Hi($\mathcal{SROIQ}$) class $C$ and $\mathfrak{R}^{\mathcal{V}}(R) = \tau_r(R)^{\mathcal{I}}$ for each Hi($\mathcal{SROIQ}$) role $R$. By (♠), it holds trivially that $\mathcal{I}$ satisfies all the axioms and assertions in $\mathsf{Dl}(\mathcal{K})$, i.e., $\mathcal{I}$ is a model of $\mathsf{Dl}(\mathcal{K})$. So $\mathsf{Dl}(\mathcal{K})$ is satisfiable.

(2) If $\mathcal{K}$ is not $v$-satisfiable then this conclusion holds trivially. Now we assume $\mathcal{K}$ is $v$-satisfiable, so $\mathsf{Dl}(\mathcal{K})$ is satisfiable. Let $\boldsymbol{u} \in \mathsf{Ans}(\mathsf{Dl}(Q), \mathsf{Dl}(\mathcal{K}))$. Let $\mathcal{V}$ be an arbitrary $v$-model of $\mathcal{K}$. According to (1), a model $\mathcal{I}$ of $\mathsf{Dl}(\mathcal{K})$ can be constructed. Then there exists a binding $\pi$ of $\mathsf{Dl}(Q(\boldsymbol{u}))$ over $\mathcal{I}$ such that $\mathcal{I}, \pi \models \mathsf{Dl}(Q(\boldsymbol{u}))$. From $\pi$, we can construct a binding $\pi'$ of $Q(\boldsymbol{u})$ over $\mathcal{V}$ by setting $\pi'(x) = \pi(x)$ for each variable $x$ in $Q(\boldsymbol{u})$ and $\pi'(a) = a^{\mathcal{V}}$ for each name $a$ in $Q(\boldsymbol{u})$. By (♠), it holds that $\mathcal{V}, \pi' \models_v Q(\boldsymbol{u})$. So $\boldsymbol{u} \in \mathsf{Ans}_v(Q, \mathcal{K})$ holds. Therefor $\mathsf{Ans}(\mathsf{Dl}(Q), \mathsf{Dl}(\mathcal{K})) \subseteq \mathsf{Ans}_v(Q, \mathcal{K})$ holds.

**Proof of Lemma 1.**

*Proof.* By Proposition 1, we just need to prove the ($\Leftarrow$) direction of (1) and the ($\subseteq$) direction of (2).

($1.\Leftarrow$) $\mathsf{Dl}(\mathcal{K})$ is satisfiable, so it has a model $\mathcal{I}$. We assume that (♠) for every two different names $a$ and $b$ in $\mathbf{N}$, $a^{\mathcal{I}} \neq b^{\mathcal{I}}$ holds. If $\mathcal{I}$ does not satisfy (♠), then for every two different names $c$ and $d$ satisfying $c^{\mathcal{I}} = d^{\mathcal{I}}$, by the condition in this lemma, we can get that $c$ or $d$ is not used as individual in $\mathcal{K}$. Suppose $d$ is not used as individual in $\mathcal{K}$. Let $o$ be a new element that does not occur in $\Delta^{\mathcal{I}}$. Then add $o$ to $\Delta^{\mathcal{I}}$ and set $d^{\mathcal{I}} = o$. Because $d$ is not used as individual in

[3] We use $f^-$ to denote the inverse function of a bijective function $f$.

$\mathcal{K}$, we can easily get that $\mathcal{I}$ is still a model of $\mathsf{DI}(\mathcal{K})$. From $\mathcal{I}$, we can construct a $v$-interpretation $\mathcal{V}$ by setting (i) $\Delta^{\mathcal{V}} = \Delta^{\mathcal{I}}$ and (ii) $a^{\mathcal{V}} = a^{\mathcal{I}}$, $\mathfrak{C}^{\mathcal{V}}(a^{\mathcal{V}}) = \boldsymbol{v}_c(a)^{\mathcal{I}}$ and $\mathfrak{R}^{\mathcal{V}}(a^{\mathcal{V}}) = \boldsymbol{v}_r(a)^{\mathcal{I}}$ for each name $a \in \mathbf{N}$. By (♠), we can get that $\mathcal{V}$ is correctly defined, i.e., for every two different names $a$ and $b$, if $a^{\mathcal{V}} = b^{\mathcal{V}}$ then $\mathfrak{C}^{\mathcal{V}}(a^{\mathcal{V}}) = \mathfrak{C}^{\mathcal{V}}(b^{\mathcal{V}})$ and $\mathfrak{R}^{\mathcal{V}}(a^{\mathcal{V}}) = \mathfrak{R}^{\mathcal{V}}(b^{\mathcal{V}})$. $\mathcal{V}$ and $\mathcal{I}$ take the same way to interpret class and role constructors. So (♣) $\mathfrak{C}^{\mathcal{V}}(C^{\mathcal{V}}) = \tau_c(C)^{\mathcal{I}}$ holds for each Hi($\mathcal{SROIQ}$) class $C$ and $\mathfrak{R}^{\mathcal{V}}(R^{\mathcal{V}}) = \tau_r(R)^{\mathcal{I}}$ holds for each Hi($\mathcal{SROIQ}$) role $R$. Hence by (♣), we can get that $\mathcal{V}$ satisfies all the axioms and assertions in $\mathcal{K}$, i.e., $\mathcal{V}$ is a $v$-model of $\mathcal{K}$. Thus $\mathcal{K}$ is $v$-satisfiable.

(2. $\subseteq$) If $\mathcal{K}$ is not $v$-satisfiable, this direction holds trivially. We assume $\mathcal{K}$ is $v$-satisfiable, so $\mathsf{DI}(\mathcal{K})$ is satisfiable. Let $\boldsymbol{u} \in \mathsf{Ans}_v(Q, \mathcal{K})$. We prove $\boldsymbol{u} \in \mathsf{Ans}(\mathsf{DI}(Q), \mathsf{DI}(\mathcal{K}))$. Let $\mathcal{I}$ be an arbitrary model of $\mathsf{DI}(\mathcal{K})$. From $\mathcal{I}$, using the way in (1.$\Leftarrow$), a $v$-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed. Hence, there exists a binding $\pi$ of $Q(\boldsymbol{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v Q(\boldsymbol{u})$. From $\pi$, we can construct a binding $\pi$ of $\mathsf{DI}(Q(\boldsymbol{u}))$ over $\mathcal{I}$ by setting $\pi'(x) = \pi(x)$ for each variable $x$ in $\mathsf{DI}(Q(\boldsymbol{u}))$ and $\pi'(a) = a^{\mathcal{I}}$ for each individual $a$ in $\mathsf{DI}(Q(\boldsymbol{u}))$. By the construction of $\mathcal{V}$, it holds trivially that $\mathcal{I}, \pi' \models \mathsf{DI}(Q(\boldsymbol{u}))$. So $\boldsymbol{u} \in \mathsf{Ans}(\mathsf{DI}(Q), \mathsf{DI}(\mathcal{K}))$. This direction holds.

**Proof of Theorem 3.**

*Proof.* For a CIERF $\mathfrak{E}$ of $\mathcal{K}$, $\mathsf{DI}([\mathcal{K}\mathfrak{E}])$ can be obtained in liner time w.r.t. the size of $\mathcal{K}$. Each CIERF of $\mathcal{K}$ is a subset of $(\mathsf{ind}(\mathcal{K}))^2$. Thus $\mathcal{K}$ has no more than $2^{|\mathsf{ind}(\mathcal{K})| \times |\mathsf{ind}(\mathcal{K})|}$ CIERFs. As stated in [7] (Theorem 11), satisfiability checking of a $\mathcal{SROIQ}$ KB can be done in N2ExpTime. Hence by Theorem 1, we can get that $v$-satisfiability checking in Hi($\mathcal{SROIQ}$) can be done in N2ExpTime. On the other hand, answering CQs without non-distinguished variables over $\mathcal{SROIQ}$ KBs can be reduced to individual assertion entailment checking which can further be reduced to KB satisfiability checking. Then by Theorem 2, we can obtain that CQ answering in Hi($\mathcal{SROIQ}$) can be done in N2ExpTime.

**Proof of Theorem 4**

*Proof.* By Definition 7, the ($\supseteq$) direction holds trivially. Next, we show the ($\subseteq$) direction. Let $\boldsymbol{u} \in \mathsf{Ans}_v(Q, \mathcal{K})$. Let $\xi$ be the CP-binding of $Q$ over $\mathcal{K}$ such that for each class (role) variable $x$ of $Q$, $\xi(x) = u[i]$ holds, where $i$ is the position of $x$ in $\mathsf{Hd}(Q)$. Let $\mathcal{V}$ be an arbitrary $v$-model of $\mathcal{K}$. For $\boldsymbol{u}$, there exists a binding $\pi$ of $Q(\boldsymbol{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v Q(\boldsymbol{u})$. From $\pi$, we construct a binding $\pi'$ of $Q\xi(\boldsymbol{u})$ over $\mathcal{V}$ such that $\pi'(x) = \pi(x)$ for each variable $x$ in $Q\xi(\boldsymbol{u})$ and $\pi'(a) = a^{\mathcal{V}}$ for each name $a$ in $Q\xi(\boldsymbol{u})$. By the construction of $\xi$, it holds trivially that $\mathcal{V}, \pi' \models_v Q\xi(\boldsymbol{u})$. Thus $\boldsymbol{u} \in \mathsf{Ans}_v(Q\xi, \mathcal{K})$.