An Expressive Sub-language of OWL 2 Full for Domain Meta-modeling (Supplementary File)

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Appendix

Proof of Proposition 1.

Proof. $\mathcal{K}$ is ν-satisfiable, so it has a ν-model $\mathcal{V}$. By $\mathcal{V}$, an interpretation $\mathcal{I}$ of $\tau_{\mathsf{dl}}(\mathcal{K})$ can be constructed by setting (1) $\Delta^{\mathcal{I}} = \Delta^{\mathcal{V}}$; (2) $a^{\mathcal{I}} = a^{\mathcal{V}}$ for each $a \in \mathbf{N}$; (3) $\{o\}^{\mathcal{I}} = \{o^{\mathcal{I}}\}$ for each $o \in \mathbf{N}$ and $A^{\mathcal{I}} = \mathfrak{C}^{\mathcal{V}}(\boldsymbol{v}_c^-(A))$ for each $A \in \mathbf{C}$; and (4) $P^{\mathcal{I}} = \mathfrak{R}^{\mathcal{V}}(\boldsymbol{v}_r^-(P))$ for each $P \in \mathbf{R}$ [3]. $\mathcal{V}$ and $\mathcal{I}$ follow the same way to interpret class and role constructors, so it holds trivially that (♠) $\mathfrak{C}^{\mathcal{V}}(C) = \tau_c(C)^{\mathcal{I}}$ for each Hi($\mathcal{SROIQ}$) class C and $\mathfrak{R}^{\mathcal{V}}(R) = \tau_r(R)^{\mathcal{I}}$ for each Hi($\mathcal{SROIQ}$) role R. By (♠), it holds trivially that $\mathcal{I}$ satisfies all the axioms and assertions in $\tau_{\mathsf{dl}}(\mathcal{K})$. So $\mathcal{I}$ is a model of $\tau_{\mathsf{dl}}(\mathcal{K})$. Thus $\tau_{\mathsf{dl}}(\mathcal{K})$ is satisfiable. □

Proof of Lemma 1.

Proof. By Proposition 1, we just need to show that if $\tau_{\mathsf{dl}}(\mathcal{K})$ is satisfiable then $\mathcal{K}$ is ν-satisfiable. Suppose $\tau_{\mathsf{dl}}(\mathcal{K})$ is satisfiable. So it has a model $\mathcal{I}$. We assume that (♠) for every two different names a and b occurring in $\mathcal{K}$, $a^{\mathcal{I}} \neq b^{\mathcal{I}}$ holds. If $\mathcal{I}$ does not satisfy (♠), then for every two different names c and d occurring in $\mathcal{K}$ such that $c^{\mathcal{I}} = d^{\mathcal{I}}$, by the condition in this lemma, we can get that c or d is not used as an individual in $\tau_{\mathsf{dl}}(\mathcal{K})$. Suppose d is not used as an individual in $\tau_{\mathsf{dl}}(\mathcal{K})$. Let e be an element in $\Delta^{\mathcal{I}}$ such that there does not exist a name a in $\mathcal{K}$ satisfying that $e = a^{\mathcal{I}}$. Then set $d^{\mathcal{I}} = e$. d is not used as an individual in $\tau_{\mathsf{dl}}(\mathcal{K})$. Thus $\mathcal{I}$ is still a model of $\tau_{\mathsf{dl}}(\mathcal{K})$. From $\mathcal{I}$, we construct a ν-interpretation $\mathcal{V}$ by setting (1) $\Delta^{\mathcal{V}} = \Delta^{\mathcal{I}}$; (2) $a^{\mathcal{V}} = a^{\mathcal{I}}$ for each $a \in \mathbf{N}$; (3) for each $e \in \Delta^{\mathcal{V}}$, if there exists a name a in $\mathcal{K}$ such that $a^{\mathcal{V}} = e$ then set $\mathfrak{C}^{\mathcal{V}}(e) = v_c(a)^{\mathcal{I}}$, else set $\mathfrak{C}^{\mathcal{V}}(e) = \emptyset$; and (4) for each $e \in \Delta^{\mathcal{V}}$, if there exists a name a in $\mathcal{K}$ such that $a^{\mathcal{V}} = e$ then set $\mathfrak{R}^{\mathcal{V}}(e) = v_r(a)^{\mathcal{I}}$, else set $\mathfrak{R}^{\mathcal{V}}(e) = \emptyset$. By (♠), we can get that $\mathcal{V}$ is correctly defined, i.e., for every two different names a and b, if $a^{\mathcal{V}} = b^{\mathcal{V}}$ then $\mathfrak{C}^{\mathcal{V}}(a^{\mathcal{V}}) = \mathfrak{C}^{\mathcal{V}}(b^{\mathcal{V}})$ and $\mathfrak{R}^{\mathcal{V}}(a^{\mathcal{V}}) = \mathfrak{R}^{\mathcal{V}}(b^{\mathcal{V}})$ hold. $\mathcal{V}$ and $\mathcal{I}$ take the same way to interpret class and role constructors. So, we can further obtain that $\mathcal{V}$ satisfies all the axioms and assertions in $\mathcal{K}$. Thus $\mathcal{V}$ is a ν-model of $\mathcal{K}$. Hence $\mathcal{K}$ is ν-satisfiable. □

[3] We use f^- to denote the inverse function of a bijective function f.

Proof of Theorem 1.

Proof. $\mathcal{K}$ adopts the UNSRA, then based on the construction of $[\mathcal{K}E]$, we can get that for each $(a,b) \in \mathsf{ind}([\mathcal{K}E])^2$ and $a \neq b$, $[\mathcal{K}E]$ contains the assertion $a \not\approx b$. Then based on Lemma 1, we just need to show that $\mathcal{K}$ is ν-satisfiable iff there exists a CIERF E of $\mathcal{K}$ such that $[\mathcal{K}E]$ is ν-satisfiable.

($\Rightarrow$) $\mathcal{K}$ is ν-satisfiable. So it has a ν-model $\mathcal{V}$. Let $\mathcal{P}$ be a partition of $\mathsf{ind}(\mathcal{K}) - \mathsf{nSR}(\mathcal{K})$ such that for each $(a,b) \in (\mathsf{ind}(\mathcal{K}) - \mathsf{nSR}(\mathcal{K}))^2$ and $a \neq b$, a and b occur in a same set in $\mathcal{P}$ iff $a^{\mathcal{V}} = b^{\mathcal{V}}$. Let E be a CIERF of $\mathcal{K}$ satisfying that for each set $\mathrm{U} \in \mathcal{P}$, there exists $a \in \mathrm{U}$ such that $E(b) = a$ holds for each $b \in \mathrm{U}$. Next, we show that $\mathcal{V}$ is also a ν-model of $[\mathcal{K}E]$. From the construction of E, we can get that ($\spadesuit$) $a^{\mathcal{V}} = (E(a))^{\mathcal{V}}$ holds for each $a \in \mathsf{dom}(E)$. For each axiom (assertion) α in $\mathcal{K}$, $\mathcal{V} \models_\nu \alpha$ holds, then by ($\spadesuit$), $\mathcal{V} \models_\nu \alpha E$ holds. For each assertion $a \not\approx b$ occurring in $[\mathcal{K}E]$, by the construction of E, $a^{\mathcal{V}} \neq b^{\mathcal{V}}$ holds. So $\mathcal{V}$ satisfies all the axioms and assertions in $[\mathcal{K}E]$. Thus $\mathcal{V}$ is a ν-model of $[\mathcal{K}E]$. Hence $[\mathcal{K}E]$ is ν-satisfiable.

($\Leftarrow$) $[\mathcal{K}E]$ is ν-satisfiable. So it has a ν-model $\mathcal{V}$. From $\mathcal{V}$, we construct a ν-interpretation $\mathcal{V}'$ by just setting $a^{\mathcal{V}} = (E(a))^{\mathcal{V}}$ for each $a \in \mathsf{dom}(E) - \mathsf{ran}(E)$ while keeping the others unchanged. For each $a \in \mathsf{ran}(E) \cap \mathsf{ran}(E)$, $E(a) = a$ holds. Thus ($\blacklozenge$) $a^{\mathcal{V}'} = (E(a))^{\mathcal{V}'}$ holds for each $a \in \mathsf{dom}(E)$. All the names in $\mathsf{dom}(E) - \mathsf{ran}(E)$ do not occur in $[\mathcal{K}E]$. So we can easily obtain that $\mathcal{V}'$ is still a ν-model of $[\mathcal{K}E]$. Thus for each axiom or assertion α in $\mathcal{K}$, $\mathcal{V}' \models_\nu \alpha E$ holds. Then by ($\blacklozenge$), we can further obtain that $\mathcal{V}' \models_\nu \alpha$ holds. Thus $\mathcal{V}'$ is a ν-model of $\mathcal{K}$. Hence $\mathcal{K}$ is ν-satisfiable. $\square$

Proof of Lemma 2.

Proof. By Lemma 1, we just need to show that $[\mathcal{K}E_1]$ is ν-satisfiable iff $[\mathcal{K}E_2]$ is satisfiable.

($\Rightarrow$) $\mathcal{K}E_1$ is ν-satisfiable, so it has a ν-model $\mathcal{V}_1$. From $\mathcal{V}_1$, next we construct a ν-model of $\mathcal{K}E_2$. By the condition in this lemma, we can get that E_1 and E_2 are constructed from a same partition $\{\mathrm{U}_1, \cdots, \mathrm{U}_n\}$ of $\mathsf{ind}(\mathcal{K})$, i.e., ($\blacklozenge$) for each U_i, there exist $a_i \in \mathrm{U}_i$ and $b_i \in \mathrm{U}_i$ such that $E_1(o) = a_i$ and $E_2(o) = b_i$ hold for each $o \in \mathrm{U}_i$. Let $\mathcal{V}_2$ be a ν-interpretation constructed from $\mathcal{V}_1$ by just setting ($\spadesuit$) $b_i^{\mathcal{V}_1} = a_i^{\mathcal{V}_1}$ for each $1 \leq i \leq n$ while keeping the others unchanged. By the construction of $[\mathcal{K}E_1]$, we can get that for each $1 \leq i \leq n$, either b_i is a name not occurring in $[\mathcal{K}E_1]$ or $b_i = a_i$ holds. Thus, $\mathcal{V}_2$ is still a ν-model of $[\mathcal{K}E_1]$. Let α be an arbitrary axiom (assertion) in $[\mathcal{K}E_2]$. If α has the form $b_i \not\approx b_j$ where $i \neq j$, then $\mathcal{V}_2 \models_\nu \alpha$ holds since $a_i^{\mathcal{V}_1} \neq a_j^{\mathcal{V}_1}$ holds. Otherwise, there exists an axiom (assertion) α' in $\mathcal{K}$ such that $\alpha = \alpha' E_2$. $\alpha' E_1$ occurs in $[\mathcal{K}E_1]$. So $\mathcal{V}_2 \models_\nu \alpha' E_1$ holds. Then by ($\spadesuit$) and ($\blacklozenge$), $\mathcal{V}_2 \models_\nu \alpha E_2$ holds. Thus $\mathcal{V}_2$ is a ν-model of $[\mathcal{K}E_2]$. Hence $[\mathcal{K}E_2]$ is ν-satisfiable. The ($\Leftarrow$) direction can be proved similarly. Therefore this lemma holds. $\square$

Proof of Theorem 2.

Proof. For a CIERF E of $\mathcal{K}$, $\tau_{\mathsf{dl}}([\mathcal{K}E])$ can be obtained in liner time w.r.t. the size of $\mathcal{K}$. $\mathcal{K}$ has no more than $2^{|\mathsf{ind}(\mathcal{K})-\mathsf{nSR}(\mathcal{K})|^2}$ CIERFs. As stated in [18], satisfiability checking of a $\mathcal{SROIQ}$ KB can be done in N2ExpTime w.r.t. the size of the KB. Hence by Theorem 1 and Lemma 2, this theorem holds. □

Proof of Lemma 3.

Proof. Suppose there exists $(a,b) \in (\mathsf{ind}(\mathcal{K}) - \mathsf{nSR}(\mathcal{K}))^2$ such that $E(a) = E(b)$ and $\tau_{\mathsf{dl}}(\mathcal{K}) \models a \not\approx b$. By Definition 7, we can get that $\tau_{\mathsf{dl}}(\mathcal{K})$ contains the assertion $E(a) \not\approx E(b)$. Obviously, $\tau_{\mathsf{dl}}(\mathcal{K})$ is not satisfiable. On the other hand, suppose there exists $(a,b) \in (\mathsf{ind}(\mathcal{K}) - \mathsf{nSR}(\mathcal{K}))^2$ such that $E(a) \neq E(b)$ and $\tau_{\mathsf{dl}}(\mathcal{K}) \models a \approx b$. Again by Definition 7, we know that $\tau_{\mathsf{dl}}(\mathcal{K})$ contains the assertions $E(a) \approx E(b)$ and $E(a) \not\approx E(b)$ at the same time. Obviously, $\tau_{\mathsf{dl}}(\mathcal{K})$ is not satisfiable. □

Proof of Proposition 2.

Proof. Let $\boldsymbol{u} \in \mathsf{answer}(\tau_{\mathsf{dl}}(Q), \tau_{\mathsf{dl}}(\mathcal{K}))$. Let $\mathcal{V}$ be an arbitrary ν-model of $\mathcal{K}$. According to the proof of Proposition 1, a model $\mathcal{I}$ of $\tau_{\mathsf{dl}}(\mathcal{K})$ can be constructed. Then there exists a binding π of $\tau_{\mathsf{dl}}(Q(\boldsymbol{u}))$ over $\mathcal{I}$ such that $\mathcal{I}, \pi \models \tau_{\mathsf{dl}}(Q(\boldsymbol{u}))$ holds. From π, a binding π' of $Q(\boldsymbol{u})$ over $\mathcal{V}$ can be constructed by setting $\pi'(x) = \pi(x)$ for each variable x in $Q(\boldsymbol{u})$ and $\pi'(a) = a^{\mathcal{V}}$ for each name a in $Q(\boldsymbol{u})$. By ($\spadesuit$) in the proof of Proposition 1, it holds that $\mathcal{V}, \pi' \models_{\nu} Q(\boldsymbol{u})$. So $\boldsymbol{u} \in \mathsf{answer}_{\nu}(Q, \mathcal{K})$ holds. Thus $\mathsf{answer}(\tau_{\mathsf{dl}}(Q), \tau_{\mathsf{dl}}(\mathcal{K})) \subseteq \mathsf{answer}_{\nu}(Q, \mathcal{K})$ holds. □

Proof of Lemma 4.

Proof. ($\subseteq$) Let $\boldsymbol{u} \in \mathsf{answer}_{\nu}(Q, \mathcal{K})$. Let $\mathcal{I}$ be an arbitrary model of $\tau_{\mathsf{dl}}(\mathcal{K})$. By the proof of Lemma 1, a ν-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed. Thus there exists a binding π of $Q(\boldsymbol{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_{\nu} Q(\boldsymbol{u})$ holds. From π, we can construct a binding π of $\tau_{\mathsf{dl}}(Q(\boldsymbol{u}))$ over $\mathcal{I}$ by setting $\pi'(x) = \pi(x)$ for each variable x in $\tau_{\mathsf{dl}}(Q(\boldsymbol{u}))$ and $\pi'(a) = a^{\mathcal{I}}$ for each individual a in $\tau_{\mathsf{dl}}(Q(\boldsymbol{u}))$. By the construction of $\mathcal{V}$, we can get that for each name n occurring in $\mathcal{K}$, $\mathfrak{C}^{\mathcal{V}}(a) = v_c(a)^{\mathcal{I}}$ and $\mathfrak{R}^{\mathcal{V}}(a) = v_r(a)^{\mathcal{I}}$ hold. So $\mathcal{I}, \pi' \models \tau_{\mathsf{dl}}(Q(\boldsymbol{u}))$ holds. Thus $\boldsymbol{u} \in \mathsf{answer}(\tau_{\mathsf{dl}}(Q), \tau_{\mathsf{dl}}(\mathcal{K}))$ holds. Hence $\mathsf{answer}_{\nu}(Q, \mathcal{K}) \subseteq \mathsf{answer}(\tau_{\mathsf{dl}}(Q), \tau_{\mathsf{dl}}(\mathcal{K}))$ holds. By Proposition 2, the ($\supseteq$) direction holds. Thus this lemma holds. □

Proof of Theorem 3.

Proof. ($\Rightarrow$) Let $\boldsymbol{u} \in \mathsf{answer}_{\nu}(Q, \mathcal{K})$. Let E be an arbitrary function in $\mathcal{E}$. Next, we show $\boldsymbol{u}E \in \mathsf{answer}_{\nu}(QE, [\mathcal{K}E])$. Let $\mathcal{V}$ be an arbitrary ν-model of $[\mathcal{K}E]$. Based on $\mathcal{V}$, a ν-model $\mathcal{V}'$ of $\mathcal{K}$ can be constructed using the way illustrated in ($\Leftarrow$) in the proof of Theorem 1. So there exists a binding π of $Q(\boldsymbol{u})$ over $\mathcal{V}'$ such that $\mathcal{V}', \pi \models_{\nu} Q(\boldsymbol{u})$ holds. From π, we construct a binding π' of $Q(\boldsymbol{u})E$ over $\mathcal{V}$ by setting $\pi'(x) = \pi(x)$ for each variable x in $Q(\boldsymbol{u})E$ and $\pi'(a) = a^{\mathcal{V}}$ for each name

a in $Q(\boldsymbol{u})E$. By ($\blacklozenge$) in ($\Leftarrow$) in the proof of Theorem 1 and $\mathcal{V}', \pi \models_\nu Q(\boldsymbol{u})$, it holds trivially that $\mathcal{V}, \pi' \models_\nu Q(\boldsymbol{u})E$. $Q(\boldsymbol{u})E = (QE)(\boldsymbol{u}E)$ holds. Thus $\mathcal{V}, \pi' \models_\nu (QE)(\boldsymbol{u}E)$ holds. By $\mathcal{V}$ being arbitrary, we can get that $\boldsymbol{u}E \in \mathsf{answer}_\nu(QE, [\mathcal{K}E])$. By Lemma 4, $\boldsymbol{u}E \in \mathsf{answer}(\tau_{\mathsf{dl}}(QE), \tau_{\mathsf{dl}}([\mathcal{K}E]))$ holds. Then by E being arbitrary, we can further obtain that $\boldsymbol{u}$ belongs to the set in the right hand side of the equation in this theorem.

($\Leftarrow$) Let $\boldsymbol{u} \in \cap_{E\in\mathcal{E}}\{\boldsymbol{u}' \mid \boldsymbol{u}'E \in \mathsf{answer}(\tau_{\mathsf{dl}}(QE), \tau_{\mathsf{dl}}([\mathcal{K}E]))\}$. By Lemma 4, we can get $\boldsymbol{u} \in \cap_{E\in\mathcal{E}}\{\boldsymbol{u}' \mid \boldsymbol{u}'E \in \mathsf{answer}_\nu(QE, [\mathcal{K}E])\}$. Next, we show $\boldsymbol{u} \in \mathsf{answer}_\nu(Q, \mathcal{K})$. Let $\mathcal{V}$ be an arbitrary ν-model of $\mathcal{K}$. By the proof of Theorem 1, we know that there exists a CIERF E of $\mathcal{K}$ such that $\mathcal{V}$ is also a ν-model of $[\mathcal{K}E]$ and ($\spadesuit$) $a^{\mathcal{V}} = E(a)^{\mathcal{V}}$ holds for each $a \in \mathsf{dom}(E)$. For $\boldsymbol{u}$, $\boldsymbol{u}E\mathsf{answer}_\nu(QE, [\mathcal{K}E])$ holds. Thus there exists a binding π of $(QE)(\boldsymbol{u}E)$ over $\mathcal{V}$ such that ($\blacklozenge$) $\mathcal{V}, \pi \models_\nu (QE)(\boldsymbol{u}E)$ holds. Based on π, we construct a binding π' of $Q(\boldsymbol{u})$ over $\mathcal{V}$ by setting $\pi'(x) = \pi(x)$ for each x in $Q(\boldsymbol{u})$ and $\pi'(a) = a^{\mathcal{V}}$ for each name a in $Q(\boldsymbol{u})$. The equation $(QE)(\boldsymbol{u}E) = Q(\boldsymbol{u})E$ holds. Then by ($\spadesuit$) and ($\blacklozenge$), it holds trivially that $\mathcal{V}, \pi \models_\nu Q(\boldsymbol{u})$. By $\mathcal{V}$ being arbitrary, $\boldsymbol{u} \in \mathsf{answer}_\nu(Q, \mathcal{K})$ holds. □

Proof of Lemma 5.

Proof. By Lemma 4, we just need to prove the equation $\mathsf{answer}_\nu(QE_1, [\mathcal{K}E_1]) = \mathsf{answer}_\nu(QE_2, [\mathcal{K}E_2])$ holds.

($\subseteq$) Let $\boldsymbol{u} \in \mathsf{answer}_\nu(QE_1, [\mathcal{K}E_1])$. Next, we show $\boldsymbol{u} \in \mathsf{answer}_\nu(QE_2, [\mathcal{K}E_2])$. By the condition in this lemma, we can get that there exists a partition $\{\mathrm{U}_1, \cdots, \mathrm{U}_n\}$ of $\mathsf{ind}(\mathcal{K}) - \mathsf{nSR}(\mathcal{K})$ satisfying that ($\spadesuit$) for each U_i, there exist $a_i, b_i \in \mathrm{U}_i$ such that $E_1(o) = a_i$ and $E_2(o) = b_i$ hold for each $o \in \mathrm{U}_i$. Let $\mathcal{V}_2$ be an arbitrary model of $[\mathcal{K}E_2]$. Based on the proof of Lemma 2, a ν-model $\mathcal{V}_1$ of $[\mathcal{K}E_1]$ can be constructed from $\mathcal{V}_2$ by just setting ($\blacklozenge$) $a_i^{\mathcal{V}_1} = b_i^{\mathcal{V}_1}$ while keeping the others unchanged. Thus there exists a binding π_1 of $(QE_1)(\boldsymbol{u})$ over $\mathcal{V}_1$ such that $\mathcal{V}_1, \pi_1 \models_\nu (QE_1)(\boldsymbol{u})$. Let π_2 be a binding of QE_2 over $\mathcal{V}_2$ such that $\pi_2(x) = \pi_1(x)$ for each variable x in $(QE_2)(\boldsymbol{u})$ and $\pi_2(a) = a^{\mathcal{V}_2}$ for each name a in $(QE_2)(\boldsymbol{u})$. Then it holds that $((QE_1)(\boldsymbol{u}))\pi_1 = ((QE_2)(\boldsymbol{u}))\pi_2$. So $\mathcal{V}_2, \pi_2 \models_\nu QE_2(\boldsymbol{u})$ holds. So $\boldsymbol{u} \in \mathsf{answer}_\nu(QE_2, [\mathcal{K}E_2])$ holds. The ($\supseteq$) direction can be proved similarly. Thus this lemma holds. □

Proof of Theorem 4.

Proof. Without non-distinguished variables, conjunctive query answering can be reduced to individual assertion entailment checking which can be eventually reduced to satisfiability checking. Then by Theorem 2, this theorem holds. □

Proof of Theorem 5.

Proof. The ($\supseteq$) direction holds trivially. Next, we show the ($\subseteq$) direction. Let $\boldsymbol{u} \in \mathsf{answer}_\nu(Q, \mathcal{K})$. Each class (resp. role) variable of Q occurs in $\mathsf{head}(Q)$. Thus, from $\boldsymbol{u}$, a CRV-Binding ξ of Q over $\mathcal{K}$ can be constructed by setting $\xi(x) =$

$\mathsf{head}(Q)[i]$, where i is the position of x in $\mathsf{head}(Q)$, for each class (role) variable x of Q. Let $\mathcal{V}$ be an arbitrary ν-model of $\mathcal{K}$. For $\boldsymbol{u}$, there exists a binding π of $Q(\boldsymbol{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_\nu Q(\boldsymbol{u})$. From π, a binding π' of $Q\xi(\boldsymbol{u})$ over $\mathcal{V}$ can be constructed by setting $\pi'(x) = \pi(x)$ for each variable x in $Q\xi(\boldsymbol{u})$ and $\pi'(a) = a^{\mathcal{V}}$ for each name a in $Q\xi(\boldsymbol{u})$. By the construction of ξ, it holds trivially that $\mathcal{V}, \pi' \models_\nu Q\xi(\boldsymbol{u})$. Thus $\boldsymbol{u} \in \mathsf{answer}_\nu(Q\xi, \mathcal{K})$. $\square$

Proof of Theorem 6.

Proof. Suppose $\mathcal{K}$ has n different names. Then Q has no more than $n^{|Q|}$ CRV-bindings. n is linear with the size of $\mathcal{K}$. By Theorems 4–5, this theorem holds. $\square$