DL-Lite Full: a sub-Language of OWL 2 Full for the web-scale Open Data for Powerful Meta-modeling and Query Answering

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Appendix A: Semantic conditions of meta-names w.r.t. a u-model $\mathcal{U}$

class A	$A^{\mathcal{U}}$	$C^{\mathcal{U}}(A^{\mathcal{U}})$
Class	$\in\Delta_u^C$	$=\Delta_u^C$
Property	$\in\Delta_u^C$	$=\Delta_u^{\mathcal{R}}$
SymmetricProperty	$\in\Delta_u^C$	$\subseteq\Delta_u^{\mathcal{R}}$
AsymmetricProperty	$\in\Delta_u^C$	$\subseteq\Delta_u^{\mathcal{R}}$

role P	$P^{\mathcal{U}}$	$\mathcal{R}^{\mathcal{U}}(P^{\mathcal{U}})$
type	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^{\mathcal{U}}\times\Delta_u^C$
subClassOf	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^C\times\Delta_u^C$
equivalentProperty	$\in\Delta^{\mathcal{R}}$	$\subseteq\Delta_u^C\times\Delta_u^C$
disjointWith	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^C\times\Delta_u^C$
domain	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^{\mathcal{R}}\times\Delta_u^C$
range	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^{\mathcal{R}}\times\Delta_u^C$
subPropertyOf	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^{\mathcal{R}}\times\Delta_u^{\mathcal{R}}$
equivalentProperty	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^{\mathcal{R}}\times\Delta_u^{\mathcal{R}}$
`inverseOf`	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^{\mathcal{R}}\times\Delta_u^{\mathcal{R}}$
propertyDisjointWith	$\in\Delta_u^{\mathcal{R}}$	$\subseteq\Delta_u^{\mathcal{R}}\times\Delta_u^{\mathcal{R}}$

$(x,y)\in\mathcal{R}^{\mathcal{U}}(subClassOf^{\mathcal{U}})$	*iff*	$x,y\in\Delta_u^C, C^{\mathcal{U}}(x)\subseteq C^{\mathcal{U}}(y)$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(equivalentProperty^{\mathcal{U}})$	*iff*	$x,y\in\Delta_u^C, C^{\mathcal{U}}(x)=C^{\mathcal{U}}(y)$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(disjointWith^{\mathcal{U}})$	*iff*	$x,y\in\Delta_u^C, C^{\mathcal{U}}(x)\cap C^{\mathcal{U}}(y)=\emptyset$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(domain^{\mathcal{U}})$	*iff*	$x\in\Delta_u^{\mathcal{R}}, y\in\Delta_u^C,$ $\{o\vert(o,e)\in\mathcal{R}^{\mathcal{U}}(x)\}\subseteq C^{\mathcal{U}}(y)$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(range^{\mathcal{I}})$	*iff*	$x\in\Delta_u^{\mathcal{R}}, y\in\Delta_u^C,$ $\{e\vert(o,e)\in\mathcal{R}^{\mathcal{U}}(x)\}\subseteq C^{\mathcal{U}}(y)$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(subPropertyOf^{\mathcal{U}})$	*iff*	$x,y\in\Delta_u^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x)\subseteq\mathcal{R}^{\mathcal{U}}(y)$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(equivalentProperty^{\mathcal{U}})$	*iff*	$x,y\in\Delta_u^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x)=\mathcal{R}^{\mathcal{U}}(y)$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(propertyDisjointWith^{\mathcal{U}})$	*iff*	$x,y\in\Delta_u^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x)\cap\mathcal{R}^{\mathcal{U}}(y)=\emptyset$
$(x,y)\in\mathcal{R}^{\mathcal{U}}(inverseOf^{\mathcal{U}})$	*iff*	$x,y\in\Delta_u^{\mathcal{R}}, \mathcal{R}^{\mathcal{U}}(x)=\mathcal{R}^{\mathcal{U}}(y)^-$
$s\in C^{\mathcal{U}}(SymmetricProperty^{\mathcal{U}})$	*iff*	$\mathcal{R}^{\mathcal{U}}(x)=\mathcal{R}^{\mathcal{U}}(x)^-$
$x\in C^{\mathcal{U}}(AsymmetricProperty^{\mathcal{U}})$	*iff*	$\mathcal{R}^{\mathcal{U}}(x)\cap\mathcal{R}^{\mathcal{U}}(x)^-=\emptyset$

Appendix B: Proofs of the results in the paper

Proof of Theorem 1.

Proof. (1. $\Rightarrow$) If $\mathcal{K}$ is v-satisfiable then it has a v-model $\mathcal{V}$. Next, we show $\tau_{dl}(\mathcal{K})$ is satisfiable. From $\mathcal{V}$, an interpretation $\mathcal{I}=(\Delta^{\mathcal{I}},\cdot^{\mathcal{I}})$ for $\tau_{dl}(\mathcal{K})$ can be constructed by setting (a) $\Delta^{\mathcal{I}}=\Delta^{\mathcal{V}}$; (b) $a^{\mathcal{I}}=a^{\mathcal{V}}$ for each $a\in\mathsf{N}$; (c) for each $P\in\mathsf{R}$, if $(v_r^-(P))^{\mathcal{V}}\in\Delta_v^{\mathcal{R}}$ then $P^{\mathcal{I}}=\mathcal{R}^{\mathcal{V}}((v_r^-(P))^{\mathcal{V}})$, otherwise $P^{\mathcal{I}}=\emptyset$; and (d) for each $A\in\mathsf{C}$, if $(v_c^-(A))^{\mathcal{V}}\in\Delta_v^C$ then $A^{\mathcal{I}}=C^{\mathcal{V}}((v_c^-(A))^{\mathcal{V}})$, otherwise $A^{\mathcal{I}}=\emptyset$. $\mathcal{I}$ and $\mathcal{V}$ obey the same principles to interpret class and role constructors. Thus it holds that ($\spadesuit$) $(\tau_r(R))^{\mathcal{I}}=\mathcal{R}^{\mathcal{V}}(R)$ for each DL-Lite Full role R and $(\tau_c(C))^{\mathcal{I}}=C^{\mathcal{V}}(C)$ for each DL-Lite Full class C. For each axiom or assertion α in $\tau_{dl}(\mathcal{K})$, if there exists α' in $\mathcal{K}$ such that $\alpha=\tau(\alpha')$ then by ($\spadesuit$) and $\mathcal{V}\models\alpha'$, $\mathcal{I}\models\alpha$ holds. Otherwise, α is an individual assertion $A(a)$ satisfying that there exists $\mathsf{gr}(P,a,A)$ in $\mathcal{K}$ such that $P\sqsubseteq_r^* type$ holds. $a^{\mathcal{I}}\in C^{\mathcal{V}}(A^{\mathcal{V}})$ holds. Then by ($\spadesuit$), $a^{\mathcal{I}}\in v_c(A)^{\mathcal{I}}$ holds, i.e., $\mathcal{I}\models A(a)$ holds. So $\mathcal{I}$ satisfies all the axioms and assertions in $\tau_{dl}(\mathcal{K})$. Thus $\tau_{dl}(\mathcal{K})$ is satisfiable.

(1. $\Leftarrow$) If $\tau_{dl}(\mathcal{K})$ is satisfiable. Then it has a canonical model $\mathcal{I}$ [1]. From $\mathcal{I}$, a v-interpretation $\mathcal{V}=(\Delta^{\mathcal{V}},\Delta_v^{\mathcal{R}},\Delta_v^C,\cdot^{\mathcal{V}},C^{\mathcal{V}},\mathcal{R}^{\mathcal{V}})$ can be constructed by setting (a) $\Delta^{\mathcal{V}}=\Delta^{\mathcal{I}}$; (b) $\Delta_v^{\mathcal{R}}$ consists of *type* and all the names $o\in\mathsf{N}$ such that $v_r(o)$ occurs in $\tau_{dl}(\mathcal{K})$; (c) Δ_v^C consists of all the names $o\in\mathsf{N}$ such that $v_c(o)$ occurs in $\tau_{dl}(\mathcal{K})$; (d) $a^{\mathcal{V}}=a$ for each $a\in\mathsf{N}$; (e) $\mathcal{R}^{\mathcal{V}}(o)=v_r(o)^{\mathcal{I}}$ for each $o\in\Delta_v^{\mathcal{R}}-\{type\}$ and $\mathcal{R}^{\mathcal{V}}(type)=\{(o,e)\vert e\in\Delta^C\wedge o\in v_c(e)^{\mathcal{I}}\}$; and (f) $C^{\mathcal{V}}(e)=\{o\vert(o,e)\in\mathcal{R}^{\mathcal{V}}(type)\}$ for each $o\in\Delta_v^C$. $\mathcal{V}$ and $\mathcal{I}$ obey the same principles to interpret class and role constructors. Thus we can get that ($\clubsuit$) for each DL-Lite Full class C, $C^{\mathcal{V}}(C)=(\tau_c(C))^{\mathcal{I}}$ holds, and for each DL-Lite Full role R, if $R\neq type$ and $R\neq type^-$ then $\mathcal{R}^{\mathcal{V}}(R)=(\tau_r(R))^{\mathcal{I}}$ otherwise $(\tau_r(R))^{\mathcal{I}}\subseteq\mathcal{R}^{\mathcal{V}}(R)$. The meta-role *type* just occurs in the right-hands of role inclusion axioms. Thus for each axiom or individual assertion α in $\mathcal{K}$, from $\mathcal{I}\models\tau(\alpha)$ and ($\clubsuit$), $\mathcal{V}\models_v\alpha$ holds. So $\mathcal{V}$ is a v-model of $\mathcal{K}$. Thus $\mathcal{K}$ is v-satisfiable.

(2) Let $\vec{u}$ be an arbitrary tuple in $\mathsf{answer}_v(Q,\mathcal{K})$. Next, we show $\vec{u}\in\mathsf{answer}(\tau_{dl}(Q),\tau_{dl}(\mathcal{K}))$. $\mathcal{K}$ is v-satisfiable. By (1), $\tau_{dl}(\mathcal{K})$ is satisfiable. Let $\mathcal{I}$ be a canonical model of $\tau_{dl}(\mathcal{K})$. Then from $\mathcal{I}$, a v-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed using the way presented in (1. $\Leftarrow$). So there exists a binding π of $Q(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V},\pi\models_v Q(\vec{u})$. From π, a binding π' of $\tau_{dl}(Q(\vec{u}))$ over $\mathcal{I}$ can be constructed by setting $\pi'(x)=\pi(x)$ for each variable x in $\tau_{dl}(Q(\vec{u}))$ and $\pi'(a)=a^{\mathcal{I}}$ for each individual a in $\tau_{dl}(Q(\vec{u}))$. Then by ($\clubsuit$) and the construction of $\mathcal{V}$, $\mathcal{I},\pi'\models\tau_{dl}(Q(\vec{u}))$ holds. $\tau_{dl}(Q(\vec{u}))=\tau_{dl}(Q)(\mathsf{head}(\tau_{dl}(Q))/\vec{u})$ holds. Thus $\vec{u}\in\mathsf{answer}(\tau_{dl}(Q),\tau_{dl}(\mathcal{K}))$ holds. Hence $\mathsf{answer}_v(Q,\mathcal{K})\subseteq\mathsf{answer}(\tau_{dl}(Q),\tau_{dl}(\mathcal{K}))$ holds.

[1] A DL-Lite$_{\mathcal{A}}$ KB O has a canonical interpretation $\mathcal{I}$ satisfying that (1) $a^{\mathcal{I}}=a$ for each $a\in\mathsf{N}$; (2) O is satisfiable iff $\mathcal{I}$ satisfies O; and (3) if O is satisfiable then for each conjunctive query q, $\vec{u}\in\mathsf{answer}(q,O)$ iff $\mathcal{I}$ satisfies $q(\vec{u})$. If O is satisfiable then $\mathcal{I}$ is called a canonical model of O.

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$\mathsf{answer}(\tau_{dl}(Q), \tau_{dl}(\mathcal{K})) \subseteq \mathsf{answer}_v(Q, \mathcal{K})$ can be proved analogously. Thus $\mathsf{answer}_v(Q, \mathcal{K}) = \mathsf{answer}(\tau(Q), \tau(\mathcal{K}))$ holds. □

Proof of Lemma 1.

Proof. Let $\mathcal{A}_o = \mathcal{A} \cup \{type(a, A) | \mathsf{gr}(P, a, A) \in \mathcal{A} \wedge P \sqsubseteq_r^* type\}$, i.e., the ABox obtained by materializing the non-standard use of *type* in the original KB. Then for each conjunctive query Q, $\mathsf{answer}_v(Q, (\emptyset, \mathcal{A}_o)) = \mathsf{answer}(\tau_{dl}(Q), (\emptyset, \tau_{dl}(\mathcal{A})))$ holds trivially, since:

$$\tau_{dl}(\mathcal{A}) = \{v_c(A)(a) | type(a, A) \in \mathcal{A}_o\} \cup \{v_r(P)(a, b) | P(a, b) \in \mathcal{A}_o \wedge P \neq type\}$$

Thus the equation in the lemma can be proved by showing the following equation holds:

$$\bigcup_{Q \in \mathsf{RefType}(Q, \mathcal{T})} \mathsf{answer}_v(Q, (\emptyset, \mathcal{A})) = \bigcup_{Q \in Q} \mathsf{answer}_v(Q, (\emptyset, \mathcal{A}_o))$$

($\subseteq$) Let $Q \in \mathsf{RefType}(Q, \mathcal{T})$. And let $\vec{u} \in \mathsf{answer}_v(Q, (\emptyset, \mathcal{A}))$. Next, we show that (♠) there exists $Q' \in Q$ such that $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}_o))$. If $Q \in Q$ then (♠) holds trivially, since $\mathcal{A} \subseteq \mathcal{A}_o$. Otherwise, there exists a query Q':

$$\bigwedge_{l=1}^{n} \alpha_l \wedge \bigwedge_{i=1}^{m} type(x_i, A_i) \rightarrow q(\vec{x})$$

in Q such that $P_k \sqsubseteq_r^* type$ for each $1 \leq k \leq m$ and Q is the query:

$$\bigwedge_{l=1}^{n} \alpha_l \wedge \bigwedge_{i=1}^{m} P_i(x_i, A_i) \rightarrow q(\vec{x})$$

Next we prove $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}))$. For $\vec{u}$, there exists a function f that maps all the variables in $Q(\vec{u})$ to names and all the atoms in $(Q(\vec{u}))f$ occur in $\mathcal{A}$. For each $P_i(a_i, A_i)$ in $(Q(\vec{u}))f$, $type(a_i, A_i)$ occurs in $\mathcal{A}_o$. Thus all the atoms in $(Q'(\vec{u}))f$ occur in $\mathcal{A}_o$. So $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}_o))$ holds. Hence (♠) holds. Therefore the ($\subseteq$) direction holds.

($\supseteq$) Let $Q \in Q$. And let $\vec{u} \in \mathsf{answer}_v(Q, (\emptyset, \mathcal{A}_o))$. Next, we show that (♣) there exists $Q' \in \mathsf{RefType}(Q, \mathcal{T})$ such that $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}))$. If $\vec{u} \in \mathsf{answer}_v(Q, (\emptyset, \mathcal{A}))$, then (♣) holds trivially, since $\mathcal{A} \subseteq \mathcal{A}_o$ and $Q \in \mathsf{RefType}(Q, \mathcal{T})$. Otherwise, let $\mathcal{S} \subseteq \mathcal{A}_o - \mathcal{A}$ such that $\vec{u} \in \mathsf{answer}_v(Q, (\emptyset, \mathcal{A} \cup \mathcal{S}))$ and there does not exist $\mathcal{S}' \subsetneq \mathcal{S}$ satisfying $\vec{u} \in \mathsf{answer}_v(Q, (\emptyset, \mathcal{A} \cup \mathcal{S}'))$. Suppose $\mathcal{S} = \bigcup_{i=1}^{n} \{type(a_i, A_i)\}$. Then for each $1 \leq i \leq n$, there exists $\mathsf{gr}(P_i, a, A_i) \in \mathcal{A}$ such that $P_i \sqsubseteq_r^* type$. For $\vec{u}$, there exists a function f that maps all the variables in $Q(\vec{u})$ to names and all the atoms in $(Q(\vec{u}))f$ occur in $\mathcal{A} \cup \mathcal{S}$. Let Q' be the query obtained by replacing the atom $type(x, A_i)$ in Q with $\mathsf{gr}(P_i, x, A_i)$ if $x = a_i$, $f(x) = a_i$, or x occurs in the k-th position of $\mathsf{head}(Q)$ and $\vec{u}[k] = a_i$, for $1 \leq i \leq n$. Then all the atoms in $(Q'(\vec{u}))f$ occur in $\mathcal{A}$. So $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}))$ holds. Thus the ($\supseteq$) direction holds. □

Proof of Theorem 3.

Proof. ($\subseteq$) Let $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K})$. Next, we show there exists $\theta \in \mathsf{fullBind}(Q, \mathcal{K})$ such that $\vec{u} \in \mathsf{answer}_v(Q\theta, \mathcal{K})$. Let $\mathcal{V}$ be a v-model of $\mathcal{K}$ constructed from a canonical model of $\tau_{dl}(\mathcal{K})$ by the approach presented in the (1. $\Leftarrow$) direction of the proof of Theorem 1. Then we can get that (♠) $\Delta_v^{\mathcal{R}} \subseteq \mathsf{N}$ and $\Delta_v^{\mathcal{C}} \subseteq \mathsf{N}$ hold. For $\vec{u}$, there exists a binding π of $Q(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v Q(\vec{u})$. From π and $\vec{u}$, we can construct a full MV-Binding θ of Q over $\mathcal{K}$. For each role variable x of Q, if x occurs in the i-th position of $\mathsf{head}(Q)$ then set $\theta_r(x) = \vec{u}[i]$, otherwise from (♠), we know there exists $n \in \mathsf{N}$ such that $\pi(x) = n$, then set $\theta_r(x) = n$. And for each class variable x of $Q\theta_r$, if x occurs in the i-th position of $\mathsf{head}(Q)$ then set $\theta_c(x) = \vec{u}[i]$, otherwise by (♠), we know there exists $n \in \mathsf{N}$ such that $\pi(x) = n$, then set $\theta_c(x) = n$. Next, we prove $\vec{u} \in \mathsf{answer}_v(Q\theta, \mathcal{K})$. Let π' be a binding of $Q\theta(\vec{u})$ over $\mathcal{V}$ such that $\pi'(x) = \pi(x)$ for each variable x in $Q\theta(\vec{u})$ and $\pi'(a) = a$ for each name a in $Q\theta(\vec{u})$. Then $\mathcal{V}, \pi' \models_v Q\theta(\vec{u})$ holds, since $(Q\theta(\vec{u}))\pi' = (Q(\vec{u}))\pi$ holds. Thus $\vec{u} \in \mathsf{answer}_v(Q\theta, \mathcal{K})$ holds. Therefore the relation $\mathsf{answer}_v(Q, \mathcal{K}) \subseteq \bigcup_{\theta \in \mathsf{fullBind}(Q, \mathcal{K})} \mathsf{answer}_v(Q\theta, \mathcal{K})$ holds.

($\supseteq$) Let $\theta \in \mathsf{fullBind}(Q, \mathcal{K})$ and $\vec{u} \in \mathsf{answer}_v(Q\theta, \mathcal{K})$. Next, we show $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K})$. Let $\mathcal{V}$ be an arbitrary v-model of $\mathcal{K}$. Then there exists a binding π of $(Q\theta)(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v (Q\theta)(\vec{u})$. From π, a binding π' of $Q(\vec{u})$ over $\mathcal{K}$ can be constructed by the settings that (a) for each variable x occurring in $Q(\vec{u})$, if $x \in \mathsf{domain}(\theta_r)$ (resp. $x \in \mathsf{domain}(\theta_c)$) then set $\pi'(x) = (\theta_r(x))^{\mathcal{V}}$ (resp. $\pi'(x) = (\theta_c(x))^{\mathcal{V}}$); and (b) for each name a occurring in $Q(\vec{u})$, set $\pi'(a) = a^{\mathcal{V}}$. Then obviously $\mathcal{V}, \pi' \models_v Q(\vec{u})$ holds. Thus $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K})$ holds. Therefore $\mathsf{answer}_v(Q, \mathcal{K}) \supseteq \bigcup_{\theta \in \mathsf{fullBind}(Q, \mathcal{K})} \mathsf{answer}_v(Q\theta, \mathcal{K})$ holds. □

Proof of Lemma 3.

Proof. (1) ($\supseteq$) By Definition 10, i.e., the definition of extensions of partial MV-Bindings, this direction of the equation in this lemma holds. ($\subseteq$) Let θ be an arbitrary full MV-Binding of Q over $\mathcal{K}$. Next, we construct a partial MV-Binding ϑ of Q over $\mathcal{T}$ such that $\theta \in \mathsf{extPBind}(\vartheta, Q, \mathcal{K})$ holds. For each role variable x of Q, set $\vartheta_r(x) = \theta_r(x)$ *iff* $\theta_r(x) \in \mathsf{N}_{\mathcal{T}}^{rr} \cup \{type\}$. Then $Q\theta_r$ and $Q\vartheta_r$ have the same class variables. For each class variable x of $Q\vartheta_r$, set $\vartheta_c(x) = \theta_c(x)$ *iff* $\theta_c(x) \in \mathsf{N}_{\mathcal{T}}^{rc}$. Then by Definition 10, $\theta \in \mathsf{extPBind}(\vartheta, Q, \mathcal{K})$ holds. Thus the direction ($\subseteq$) holds.

(2) According to the algorithm $\mathsf{PerfectRef}$, we can get that (♠) for a conjunctive query q and atom $A(x)$ (resp. $P(x, y)$), if A (resp. P) does not occur in the right hands of any inclusion axioms in a DL-Lite$_{\mathcal{A}}$ TBox $\mathcal{T}$, then this query atom will not be extended by the inclusion axioms in $\mathcal{T}$ to generate new queries, i.e., it will be occur in each query in $\mathsf{PerfectRef}(q, \mathcal{T})$.

($\subseteq$ .1) Let $\theta \in \mathsf{extPBind}(\vartheta, Q, \mathcal{K})$ and $\vec{u} \in \mathsf{answer}_v(Q\theta, \mathcal{K})$. Next, we show there exists $Q' \in \mathsf{PerfectRef}_v^{mq}(Q\vartheta, \mathcal{T})$ such that $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}))$. By Theorem 2, there exists $q \in \mathsf{PerfectRef}_v^{cq}(Q\theta, \mathcal{T})$ such that $\vec{u} \in \mathsf{answer}_v(Q, (\emptyset, \mathcal{A}))$. Let $\theta' = (\theta'_r, \theta'_c)$ be a tuple of functions satisfying that (a) $\mathsf{domain}(\theta'_r) = \mathsf{domain}(\theta_r) - \mathsf{domain}(\vartheta_r)$ and for each $x \in \mathsf{domain}(\theta'_r)$, $\theta'_r(x) = \theta_r(x)$ holds and (b) $\mathsf{domain}(\theta'_c) = \mathsf{domain}(\theta_c) - \mathsf{domain}(\vartheta_c)$ and for each $x \in \mathsf{domain}(\theta'_c)$, $\theta'_c(x) = \theta_c(x)$ holds. Obviously θ' is a full MV-Binding of $Q\vartheta$ over $\mathcal{K}$ that maps the class (resp. role) variables of $Q\vartheta$ to the names not occurring in $\mathsf{N}_{\mathcal{T}}^{rc}$ (resp. $\mathsf{N}_{\mathcal{T}}^{rr}$). For q, by (♠) and Definition 6 and 9, i.e., the definition of $\mathsf{PerfectRef}_v^{cq}$ and $\mathsf{PerfectRef}_v^{mq}$, we can get that there exists $Q' \in \mathsf{PerfectRef}_v^{mq}(Q\vartheta, \mathcal{T})$ such that $q = Q'\theta'$. Thus $\vec{u} \in \mathsf{answer}_v(Q'\theta', (\emptyset, \mathcal{A}))$. Then by Theorem 3, $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}))$ holds. Thus the first inclusion holds.

($\subseteq$ .2) Let $Q' \in \mathsf{PerfectRef}^{mq}_v(Q\vartheta, \mathcal{T})$. And let $\vec{u} \in \mathsf{answer}_v(Q', (\emptyset, \mathcal{A}))$. Next, we show $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K})$. By Theorem 3, there exists a full MV-Binding θ of Q' over $(\emptyset, \mathcal{A})$ such that $\vec{u} \in \mathsf{answer}_v(Q'\theta, (\emptyset, \mathcal{A}))$ holds. Let θ' be a full MV-Binding of Q over $\mathcal{K}$ such that (a) $\mathsf{domain}(\theta'_r) = \mathsf{domain}(\vartheta_r) \cup \mathsf{domain}(\theta_r)$ and for each $x \in \mathsf{domain}(\theta'_r)$, if $x \in \mathsf{domain}(\vartheta_r)$ then $\theta'_r(x) = \vartheta_r(x)$, otherwise $\theta'_r(x) = \theta_r(x)$; and (2) $\mathsf{domain}(\theta'_c) = \mathsf{domain}(\vartheta_c) \cup \mathsf{domain}(\theta_c)$ and for each $x \in \mathsf{domain}(\theta'_c)$, if $x \in \mathsf{domain}(\vartheta_c)$ then $\theta'_c(x) = \vartheta_c(x)$ otherwise $\theta'_c(x) = \theta_c(x)$. Then by Definition 9, i.e., the definition of $\mathsf{PerfectRef}^{mq}_v$, $Q'\theta \in \mathsf{PerfectRef}^{cq}_v(Q\theta', \mathcal{T})$ holds. Then by Theorem 2 and $\vec{u} \in \mathsf{answer}_v(Q'\theta, (\emptyset, \mathcal{A}))$, $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K})$ holds. Thus the second inclusion relation holds. □

Proof of Lemma 5.

Proof. (1) $\mathcal{K}$ is u-satisfiable, so it has a u-model $\mathcal{U} = (\Delta^{\mathcal{U}}, \Delta^{\mathcal{R}}_u, \Delta^{C}_u, \cdot^{\mathcal{U}}, \mathcal{R}^{\mathcal{U}}, C^{\mathcal{U}})$. From $\mathcal{U}$, a v-interpretation $\mathcal{V} = (\Delta^{\mathcal{V}}, \Delta^{\mathcal{R}}_v, \Delta^{C}_v, \cdot^{\mathcal{V}}, \mathcal{R}^{\mathcal{V}}, C^{\mathcal{V}})$ can be constructed by setting (a) $\Delta^{\mathcal{V}} = \Delta^{\mathcal{U}}$; (b) $\Delta^{\mathcal{R}}_v = \Delta^{\mathcal{R}}_u$; (c) $\Delta^{C}_v = \Delta^{C}_u$; (d) $a^{\mathcal{V}} = a^{\mathcal{U}}$ for each $a \in \mathsf{N}$; (e) $\mathcal{R}^{\mathcal{V}}(o) = \mathcal{R}^{\mathcal{U}}(o)$ for each $o \in \Delta^{\mathcal{R}}_v$; and (f) $C^{\mathcal{V}}(o) = C^{\mathcal{U}}(o)$ for each $o \in \Delta^{C}_v$. $\mathcal{U}$ and $\mathcal{V}$ obey the same principles to interpret class and role constructors. Thus (♠) $\mathcal{R}^{\mathcal{V}}(R) = \mathcal{R}^{\mathcal{U}}(R)$ holds for each DL-Lite Full role R and $C^{\mathcal{V}}(C) = C^{\mathcal{U}}(C)$ holds for each DL-Lite Full class C. Then we can further obtain that $\mathcal{V}$ satisfies all the axioms and assertions in $\mathcal{K}$. So $\mathcal{V}$ is a v-model of $\mathcal{K}$. Thus $\mathcal{K}$ is v-satisfiable.

(2) If $\mathcal{K}$ is not u-satisfiable then this conclusion holds directly. Suppose $\mathcal{K}$ is u-satisfiable. Let $\mathcal{U}$ be an arbitrary u-models of $\mathcal{K}$. From $\mathcal{U}$, a v-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed by the approach presented in (1). Then $\mathcal{V} \models_v \alpha$ holds. Then by (♠) in (1), $\mathcal{U} \models_u \alpha$ holds. Based on the arbitrary feature of $\mathcal{U}$, $\mathcal{K} \models_u \alpha$ holds.

(3) If $\mathcal{K}$ is not u-satisfiable then this conclusion holds trivially. We assume $\mathcal{K}$ is u-satisfiable. Let $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K})$. And let $\mathcal{U}$ be an arbitrary u-models of $\mathcal{K}$. Then from $\mathcal{U}$, a v-model $\mathcal{V}$ of $\mathcal{K}$ can be constructed by the way in (1). Then there exists a binding π of $Q(\vec{u})$ over $\mathcal{V}$ such that $\mathcal{V}, \pi \models_v Q(\vec{u})$ holds. Obviously, π is also a binding of $Q(\vec{u})$ over $\mathcal{U}$. Then by (♠) in (1), $\mathcal{U} \models_u Q(\vec{u})$ holds. So $\vec{u} \in \mathsf{answer}_u(Q, \mathcal{K})$ holds. Thus, we can further obtain that $\mathsf{answer}_v(Q, \mathcal{K}) \subseteq \mathsf{answer}_u(Q, \mathcal{K})$. □

Proof of Lemma 6.

Proof. (1) Let $\mathcal{K} = (\mathcal{T}_m \cup \mathcal{T}_o, \mathcal{A}_m \cup \mathcal{A}_o)$ and $\mathcal{K}_{u \to v} = (\mathcal{T}, \mathcal{A})$. If $\mathcal{K}$ is v-satisfiable, the following conclusion holds trivially. For a meta-class C_m, $\mathcal{K} \models_v$ *type*(a, C_m) *iff* there exists $\mathsf{gr}(P, a, b) \in \mathcal{A}_m$ such that $P \sqsubseteq^*_r$ *type* and $\mathcal{K} \models_v b \sqsubseteq_c C_m$ or $\mathcal{K} \models_v \exists P \sqsubseteq_c C_m$. And for a meta-role P_m, $\mathcal{K} \models_v P_m(a, b)$ *iff* there exists $\mathsf{gr}(S, a, b) \in \mathcal{A}_m$ such that $\mathcal{K} \models_v S \sqsubseteq_r P_m$. The names in $\mathcal{T}_m$ as well as the names occurring in the axioms $\mathsf{Fun}(S)$ do not used as individuals in $\mathcal{A}_m$. Thus we can get that $\mathcal{T}'_o = \mathcal{T} - \mathcal{T}_m - \mathcal{T}_o$, i.e., the set of axioms added to $\mathcal{K}$ according to the individual assertions of meta-names implied by $\mathcal{K}$, do not contain the names occurring in $\mathcal{T}_m$ or the axioms $\mathsf{Fun}(S)$ in $\mathcal{T}_o$. Thus $\mathcal{K}_{u \to v}$ is still a DL-Lite Full KB, since its TBox is $\mathcal{T}_m \uplus (\mathcal{T}_o \cup \mathcal{T}'_o)$ which satisfies the conditions in Definition 2.

(2) This conclusion holds by the following facts. Let $\mathcal{K}$ be a DL-Lite Full KB. (a) For each axiom or assertion α, if $\mathcal{K} \models_v \alpha$ then $\mathcal{K} \models_u \alpha$; (b) For an axiom or assertion α, if $\mathcal{K} \models_u \alpha$, then for each axiom or assertion α', if $\mathcal{K} \cup \{\alpha\} \models_u \alpha'$ then $\mathcal{K} \models_u \alpha'$; (c) For the axioms (assertions) with the forms in Figure 3 and u-entailed by $\mathcal{K}$, then the corresponding assertions (axioms) are u-entailed by $\mathcal{K}$. For example, $\mathcal{K} \models_u$ *subClassOf*(A, B) iff $\mathcal{K} \models_u A \sqsubseteq_c B$. □

Proof of Theorem 6.

Proof. We first claim that for a v-satisfiable DL-Lite Full KB $\mathcal{K}$, let $\mathcal{V}$ be a v-model of $\mathcal{K}$ constructed from a canonical model of the DL-Lite$_{\mathcal{A}}$ KB $\tau_{dl}(\mathcal{K})$ by the way presented in (1. $\Leftarrow$) in the Proof of Theorem 1. Then $\mathcal{V}$ satisfies that (a) $a^{\mathcal{V}} = a$ for each $a \in \mathsf{N}$; and (b) by Theorem 1 and 3, it can be easily proved that for each meta-query Q, $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K})$ iff $\mathcal{V} \models_v Q(\vec{u})$. In the following, we call $\mathcal{V}$ ***a canonical v-model*** of $\mathcal{K}$.

Lemma 6 indicates that $\mathcal{K}$ and $\mathcal{K}_{u \to v}$ are u-semantic equivalent, i.e., they have the same u-models. By Lemma 5, we just need to prove that (1) if $\mathcal{K}_{u \to v}$ is u-satisfiable then $\mathcal{K}_{u \to v}$ is v-satisfiable; and (2) if Q is a query without non-distinguished meta-variables, $\mathsf{answer}_v(Q, \mathcal{K}_u) \subseteq \mathsf{answer}_u(Q, \mathcal{K}_{u \to v})$ holds.

(1) $\mathcal{K}_{u \to v}$ is v-satisfiable, thus it has a canonical v-model $\mathcal{V} = (\Delta^{\mathcal{V}}, \Delta^{\mathcal{R}}_v, \Delta^{C}_v, \cdot^{\mathcal{V}}, \mathcal{R}^{\mathcal{V}}, C^{\mathcal{V}})$. From $\mathcal{V}$, we construct a u-interpretation $\mathcal{U} = (\Delta^{\mathcal{U}}, \Delta^{\mathcal{R}}_u, \Delta^{C}_u, \cdot^{\mathcal{U}}, \mathcal{R}^{\mathcal{U}}, C^{\mathcal{U}})$ by setting (a) $\Delta^{\mathcal{U}} = \Delta^{\mathcal{V}}$; (b) $\Delta^{\mathcal{R}}_u = \Delta^{\mathcal{R}}_v \cup C^{\mathcal{V}}(Property)$; (c) $\Delta^{C}_u = \Delta^{C}_v \cup C^{\mathcal{V}}(Class)$; (d) $a^{\mathcal{U}} = a^{\mathcal{V}}$ for each $a \in \mathsf{N}$; (e) for each $o \in \Delta^{\mathcal{R}}_u$, if $o \in \Delta^{\mathcal{R}}_v$ then $\mathcal{R}^{\mathcal{U}}(o) = \mathcal{R}^{\mathcal{V}}(o)$ otherwise $\mathcal{R}^{\mathcal{U}}(o) = \emptyset$; and (f) for each $o \in \Delta^{C}_u$, if $o = Class^{\mathcal{U}}$ then $C^{\mathcal{U}}(o) = \Delta^{C}_u$, else if $o = Property^{\mathcal{U}}$ then $C^{\mathcal{U}}(o) = \Delta^{\mathcal{R}}_u$, else if $o \in \Delta^{C}_v$ then $C^{\mathcal{U}}(o) = C^{\mathcal{V}}(o)$ else $C^{\mathcal{U}}(o) = \emptyset$. In order to make $\mathcal{U}$ satisfy the semantic conditions of meta-names listed in Appendix A , we need to make the extra setting:

§ For each $(o, e) \in \Delta^{C}_u \times \Delta^{C}_u$, if $C^{\mathcal{U}}(o) \subseteq C^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(subClassOf^{\mathcal{U}})$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(subClassOf)$; if $C^{\mathcal{U}}(o) = C^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(equivalentClass)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(equivalentClass)$; and if $C^{\mathcal{U}}(o) \cap C^{\mathcal{U}}(e) = \emptyset$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(disjointWith)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(disjointWith)$;

§ For each $(o, e) \in \Delta^{\mathcal{R}}_u \times \Delta^{\mathcal{R}}_u$, if $\mathcal{R}^{\mathcal{U}}(o) \subseteq \mathcal{R}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(subPropertyOf)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(subPropertyOf)$; if $\mathcal{R}^{\mathcal{U}}(o) = \mathcal{R}^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(equivalentProperty)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(equivalentProperty)$; if $\mathcal{R}^{\mathcal{U}}(o) \cap \mathcal{R}^{\mathcal{U}}(e) = \emptyset$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(propertyDisjoint)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(propertyDisjoint)$; and if $\mathcal{R}^{\mathcal{U}}(o) = \{(y, x) | (x, y) \in \mathcal{R}^{\mathcal{U}}(e)\}$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(inverseOf)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(inverseOf)$;

§ For each $(o, e) \in \Delta^{\mathcal{R}}_u \times \Delta^{C}_u$, if $\{x | (x, y) \in \mathcal{R}^{\mathcal{U}}(o)\} \subseteq C^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(domain)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(domain)$; and if $\{y | (x, y) \in \mathcal{R}^{\mathcal{U}}(o)\} \subseteq C^{\mathcal{U}}(e)$ and $(o, e) \notin \mathcal{R}^{\mathcal{U}}(range)$ then add (o, e) to $\mathcal{R}^{\mathcal{U}}(range)$;

§ For each $o \in \Delta^{\mathcal{R}}_u$, if $\mathcal{R}^{\mathcal{V}}(o) = \{(y, x) | (x, y) \in \mathcal{R}^{\mathcal{V}}(o)\}$ and $o \notin C^{\mathcal{U}}(SymmetricProperty)$, then add o to $C^{\mathcal{U}}(SymmetricProperty)$; and if $\mathcal{R}^{\mathcal{V}}(o) \cap \{(y, x) | (x, y) \in \mathcal{R}^{\mathcal{V}}(o)\} = \emptyset$ and $o \notin C^{\mathcal{U}}(AsymmetricProperty)$, then add o to $C^{\mathcal{U}}(AsymmetricProperty)$.

Then $\mathcal{U}$ satisfies the semantic conditions of meta-names listed in Appendix A. So it is a u-interpretation. $\mathcal{U}$ and $\mathcal{V}$ obey the same rules to interpret the class and role constructors. Thus, we can get that (♣) for each DL-Lite Full class C, if C is not a meta-class then $C^{\mathcal{U}}(C) = C^{\mathcal{V}}(C)$ otherwise $C^{\mathcal{V}}(C) \subseteq C^{\mathcal{U}}(C)$, and for each DL-Lite Full Role R, if R is not a meta-role or inverse of a meta-role then $\mathcal{R}^{\mathcal{U}}(R) = \mathcal{R}^{\mathcal{V}}(R)$ otherwise $\mathcal{R}^{\mathcal{V}}(R) \subseteq \mathcal{R}^{\mathcal{U}}(R)$. Meta-names do not occur in the left-hands of inclusion axioms or functional role assertions. So for each axiom or individual assertion α, by $\mathcal{V} \models_v \alpha$ and (♣), $\mathcal{U} \models_u \alpha$ holds. So $\mathcal{U}$ is a u-model of $\mathcal{K}$. Thus $\mathcal{K}$ is u-satisfiable.

(2) We first prove that (♠) for each assertion α with the form $P_m(a,b)$ or $type(a, A_m)$, where P_m is a meta-role except $type$ and A_m is a meta-class, then if $\mathcal{K}_{u\to v} \models_u \alpha$ then $\mathcal{K}_{u\to v} \models_v \alpha$. Assume (A) $\mathcal{K}_{u\to v} \not\models_v \alpha$. Let $\mathcal{V}$ be a canonical v-model of $\mathcal{K}_{u\to v}$. Suppose α is an assertion $subClassOf(A, B)$. Then $(A, B) \notin \mathcal{R}^{\mathcal{V}}(subClassOf)$. Let $\mathcal{T}$ be the TBox of $\mathcal{K}_{u\to v}$, and let $\mathcal{A}' = \{type(o_A, A)\}$ where o_A is an ordinary name not occurring in $\mathcal{K}_{u\to v}$. Obviously $(\mathcal{T}, \mathcal{A}')$ is v-satisfiable, since $\mathcal{T} \not\models_v A \sqsubseteq_c B$ implies $\mathcal{T} \not\models_v A \sqsubseteq_c \neg A$. Let $\mathcal{V}'$ be a canonical v-model of $(\mathcal{T}, \mathcal{A}')$. We assume $(\Delta^{\mathcal{V}} - \mathsf{N}) \cap (\Delta^{\mathcal{V}'} - \mathsf{N}) = \emptyset$, i.e., $\mathcal{V}'$ and $\mathcal{V}$ do not share any anonymous element. From $\mathcal{V}$ and $\mathcal{V}'$, we construct another v-interpretation $\mathcal{V}''$ by setting (a) $\Delta^{\mathcal{V}''} = \Delta^{\mathcal{V}} \cup \Delta^{\mathcal{V}'}$, $\Delta_v^{\mathcal{R}''} = \Delta_v^{\mathcal{R}} \cup \Delta_v^{\mathcal{R}'}$ and $\Delta_v^{\mathcal{C}''} = \Delta_v^{\mathcal{C}} \cup \Delta_v^{\mathcal{C}'}$; (b) $a^{\mathcal{V}''} = a$ for each $a \in \mathsf{N}$; and (c) $C^{\mathcal{V}''}(o) = C^{\mathcal{V}}(o) \cup C^{\mathcal{V}'}(o)$ for each $o \in \Delta_v^{\mathcal{C}''}$ and $\mathcal{R}^{\mathcal{V}''}(o) = \mathcal{R}^{\mathcal{V}}(o) \cup \mathcal{R}^{\mathcal{V}'}(o)$ for each $o \in \Delta_v^{\mathcal{R}''}$. It can be trivially validate that $\mathcal{V}''$ is a v-model of $\mathcal{K}$. And $C^{\mathcal{V}''}(A) \nsubseteq C^{\mathcal{V}''}(B)$ holds. From $\mathcal{V}''$, a u-model $\mathcal{U}$ can be constructed using the way presented in (1). By the settings (§), we can get that $(A, B) \notin \mathcal{R}^{\mathcal{V}''}(subClassOf)$. This contradicts with that $\mathcal{U}$ is a u-model of $\mathcal{K}_{u\to v}$. So assumption (A) does not hold. Thus $\mathcal{K}_{u\to v} \models_v subClassOf(A, B)$ holds. The other forms of α can be proved analogously.

Let $\vec{u} \in \mathsf{answer}_u(Q, \mathcal{K}_{u\to v})$. We prove $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K}_{u\to v})$. Let $\mathcal{S}$ be the set of all the atoms in $Q(\vec{u})$ with the forms $P_m(a,b)$ or $\mathsf{type}(a, C_m)$, where P_m is a meta-role except $type$ and C_m is a meta-class except $Class$ and $Property$. Then all the atoms in $\mathcal{S}$ do not contain variables and $\mathcal{K}_{u\to v} \models_u \alpha$ holds for each $\alpha \in \mathcal{S}$. Thus (★) $\mathcal{K}_{u\to v} \models_v \alpha$ holds for each $\alpha \in \mathcal{S}$. Let Q' be the query $\wedge_{\alpha \in \mathsf{body}(Q(\vec{u})) - \mathcal{S}} \to q()$. Then by (★), $\vec{u} \in \mathsf{answer}_u(Q, \mathcal{K}_{u\to v})$ can be proved by showing $() \in \mathsf{answer}_v(Q', \mathcal{K}_{u\to v})$, i.e., Q' is true over $\mathcal{K}_{u\to v}$. Let $\mathcal{V}$ be a canonical v-model of $\mathcal{K}_{u\to v}$. Then a u-model $\mathcal{U}$ of $K_{u\to v}$ can be constructed using the way presented in (1). Thus there exists a binding π of Q' over $\mathcal{U}$ such that $\mathcal{U}, \pi \models_u Q'$. Q' does not contain atoms with the forms $P_{\mathsf{m}}(a,b)$ or $type(a, C_{\mathsf{m}})$, where P_{m} is a meta-role except $type$ and C_{m} is a meta-class except $Class$ and $Property$. From the construction of $\mathcal{U}$, we can get that π is also a binding of Q' over $\mathcal{V}$. Thus $() \in \mathsf{answer}_v(Q', \mathcal{K}_{u\to v})$ holds. Hence $\vec{u} \in \mathsf{answer}_v(Q, \mathcal{K}_{u\to v})$. Therefore $\mathsf{answer}_u(Q, \mathcal{K}_{u\to v}) \subseteq \mathsf{answer}_v(Q, \mathcal{K}_{u\to v})$ holds. □

Proof of Theorem 7.

Proof. $\mathcal{K}$ and $\mathcal{K}_{u\to v}$ are u-semantic equivalent. So this theorem can be proved by showing the following equation holds.

$$\mathsf{answer}_u(Q, \mathcal{K}_{u\to v}) = \bigcap_{\sigma \subseteq \mathsf{NA}(\mathcal{K}_{u\to v})} \mathsf{answer}_v(Q, \mathcal{K}_{u\to v})$$

($\subseteq$) Let $\vec{u} \in \mathsf{answer}_u(Q, \mathcal{K}_{u\to v})$. Assume (A1) there exists $\sigma \subseteq \mathsf{NA}(\mathcal{K}_{u\to v})$ such that $\vec{u} \notin \mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma})$. Let $\mathcal{S}$ to be:

$$\{type(o_A, A) \mid \mathcal{K}_{u\to v} \models_v type(A, Class) \wedge \mathcal{K}_{u\to v} \not\models_v A \sqsubseteq_c \neg A\} \cup$$
$$\{P(o_P^1, o_P^2) \mid \mathcal{K}_{u\to v} \models_v type(P, Property) \wedge \mathcal{K}_{u\to v} \not\models_v P \sqsubseteq_r \neg P\}$$

where o_A and o_P^1 as well as o_P^2 are ordinary names not occurring in $\mathcal{K}_{u\to v}$ and uniquely chose for A and P, respectively. Then $\mathcal{S}$ is a set to falsify all subset relations that are not implied by $\mathcal{K}_{u\to v}$. By $\vec{u} \notin \mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma})$, we know $(\mathcal{K}_{u\to v})^{\sigma}$ is v-satisfiable. By the construction of $\mathcal{S}$, it can easily validated that $(\mathcal{K}_{u\to v})^{\sigma} \cup \mathcal{S}$ is v-satisfiable, and for Q, $\mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma}) = \mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma} \cup \mathcal{S})$, since Q is not a query with the form $P(x,y) \to q(\vec{x})$ or $type(x, A) \to q(\vec{x})$ where P and A are ordinary names. Let $\mathcal{V}$ be a canonical v-model of $(\mathcal{K}_{u\to v})^{\sigma} \cup \mathcal{S}$. Let $\mathcal{U}$ be a interpretation obtained from $\mathcal{V}$ by the settings (a)–(f) in the (1. $\Leftarrow$) direction of the proof of Theorem 6. Then $\mathcal{U}$ satisfies the semantic conditions of meta-names. Thus $\mathcal{U}$ is a u-model of $\mathcal{K}_{u\to v}$. And function π is a binding of $Q(\vec{u})$ over $\mathcal{V}$ iff π is a binding of $Q(\vec{u})$ over $\mathcal{V}$. Then from $\vec{u} \notin \mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma} \cup \mathcal{S})$, $\mathcal{V} \not\models_v Q(\vec{u})$ holds. Thus $\mathcal{U} \not\models_u Q(\vec{u})$ holds. Thus $\vec{u} \notin \mathsf{answer}_u(Q, \mathcal{K})$. This contradicts with $\vec{u} \in \mathsf{answer}_u(Q, \mathcal{K}_{u\to v})$. Thus assumption (A1) does not hold. So for each $\sigma \subseteq \mathsf{NA}(\mathcal{K}_{u\to v})$, $\vec{u} \in \mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma})$ holds. Thus $\mathsf{answer}_u(Q, \mathcal{K}_{u\to v}) \subseteq \bigcap_{\sigma \subseteq \mathsf{NA}(\mathcal{K}_{u\to v})} \mathsf{answer}_v(Q, \mathcal{K}_{u\to v})$ holds.

($\supseteq$) Let (♠) $\vec{u} \in \bigcap_{\sigma \subseteq \mathsf{NA}(\mathcal{K}_{u\to v})} \mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma})$. Assume (A2) $\vec{u} \notin \mathsf{answer}_u(Q, \mathcal{K}_{u\to v})$. Then there exists a u-model $\mathcal{U}$ of $\mathcal{K}_{u\to v}$ so that $\mathcal{U} \not\models_u Q(\vec{u})$. Let σ be the set consisting of all the axioms α in $\mathsf{NA}(\mathcal{K}_{u\to v})$ such that $\mathcal{U} \models_u \alpha$. Let $\mathcal{V}$ be a v-interpretation constructed from $\mathcal{U}$ by the way presented in the proof of Lemma 5. Then $\mathcal{V}$ satisfies all the axioms and assertions in $\mathcal{K}_{u\to v} \cup \sigma \cup \sigma'$ and $\mathcal{V} \not\models_v Q(\vec{u})$. Let $\mathcal{T}$ be the TBox of $\mathcal{K}_{u\to v} \cup \sigma \cup \sigma'$. Then $\mathcal{K}' = (\mathcal{T}, \bigcup_{\alpha \in \mathsf{NA}(\mathcal{K}_{u\to v}) - \sigma} \mathsf{VI}(\alpha, \mathcal{K}_{u\to v}))$ is v-satisfiable, since all the axioms in $\mathsf{NA}(\mathcal{K}_{u\to v}) - \sigma$ are violated by $\mathcal{V}$. For example, if $A \sqsubseteq_c \neg B \in \mathsf{NA}(\mathcal{K}_{u\to v}) - \sigma$, then $C^{\mathcal{V}}(A^{\mathcal{V}}) \cap C^{\mathcal{V}}(B^{\mathcal{V}}) \neq \emptyset$ which indicates that A and B can share some individuals. Let $\mathcal{V}'$ be a canonical model of $\mathcal{K}'$. Then from $\mathcal{V} \not\models_v Q(\vec{u})$, we can get that $\mathcal{V}' \not\models_v Q(\vec{u})$. We assume that $\mathcal{V}$ and $\mathcal{V}'$ do not share any anonymous element i.e., $(\Delta^{\mathcal{V}} - \mathsf{N}) \cap (\Delta^{\mathcal{V}'} - \mathsf{N}) = \emptyset$. From $\mathcal{V}$ and $\mathcal{V}'$, a v-interpretation $\mathcal{V}''$ can be constructed by the way presented in the (2. $\supseteq$) direction in the proof of Theorem 6. Then $\mathcal{V}''$ is a v-model of:

$$(\mathcal{K}_{u\to v})^{\sigma} = \mathcal{K}_{u\to v} \cup \sigma \cup \sigma' \cup \bigcup_{\alpha \in \mathsf{NA}(\mathcal{K}_{u\to v}) - \sigma} \mathsf{VI}(\alpha, \mathcal{K}_{u\to v})$$

and $\mathcal{V}'' \not\models_v Q(\vec{u})$. So $\vec{u} \notin \mathsf{answer}_v(Q, (\mathcal{K}_{u\to v})^{\sigma})$. This contradicts with (♠). So assumption ($\mathcal{A}$2) does not hold. Thus $\vec{u} \in \mathsf{answer}_u(Q, \mathcal{K}_{u\to v})$. So $\bigcap_{\sigma \subseteq \mathsf{NA}(\mathcal{K}_{u\to v})} \mathsf{answer}_v(Q, \mathcal{K}_{u\to v}) \subseteq \mathsf{answer}_u(Q, \mathcal{K}_{u\to v})$ holds. □

Proof of Lemma 7.

Proof. This lemma holds according to the following facts. Let $\mathcal{K} = (\mathcal{T}, \mathcal{A}_m \cup \mathcal{A}_o)$ be an asserted DL-Lite Full KB. (a) For a DL-Lite Full axiom α, $\mathcal{K} \models_v \alpha$ iff $\tau_{dl}(\mathcal{T}) \models \tau(\alpha)$; (b) For an assertion $P_m(a,b)$ such that P_m is a meta-role except $type$, then $\mathcal{K} \models_v P_m(a,b)$ iff there exists $\mathsf{gr}(P, a, b) \in \mathcal{A}_m$ such that $P \sqsubseteq_r^* P_m$ w.r.t. $\mathcal{T}$; (c) For an assertion $type(a, C_m)$ such that C_m is a meta-class except, then $\mathcal{K} \models_v type(a, C_m)$ iff there exists $\mathsf{gr}(P, a, b) \in \mathcal{A}_m$ such that $P \sqsubseteq_r^* type$ and $\mathcal{K} \models_v b \sqsubseteq_c C_m$, or $\mathcal{K} \models_v \exists P \sqsubseteq_c C_m$. □