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**MOD 6 PROJECT:** ALY6050-Optimization

**CPS WINTER QUATER**

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INTRODUCTION

**Project Objectives**

In this project, mathematical formulas are built to solve for two optimization problem. The first one is the transportation problem of Rockhill Shipping & Transport Company, which requires an optimization for the transportation routes to minimize its cost. In this problem, wastes are produced by the plant and should be transported to the destination, the disposal sites. The second problem requires an optimization solution for a portfolio to achieve minimize risk. Six assets are chosen to be evaluate through multiple mathematical functions and inequalities. Excel solver is used to calculate the variables and objectives in these two problems with corresponding mathematical equations implanted. To solve for the optimization solution, we need to follow several steps including defining the decision variables (parameters), the objective, the constraints, and mathematical function to implement all these requirements.

**Linear Optimization Model**

Linear Optimization Model is the mathematical model that helps to solve the optimization problems. It is a type of linear programming problem which can be solved by using simple algebraic techniques of solving linear equations and inequalities. The basic idea behind Linear Optimization Model is that we have a set of variables, which are called as objective function and they can take any values. In this case, we will use some value to represent each variable in our problem. For example, if we want to find out how many cars should be produced per month in order to maximize profit for our company then our objective function will be Profit = (Cars Produced) x (Price Per Car). We also need some constraints or rules on these variables so that we don’t get an infinite number of solutions for our problem like having more than one car produced at once or producing more than what you sell.

**Transportation optimization**

Transportation optimization is an efficient, sustainable and cost-effective way to move people and goods. It’s about making the most of existing transportation resources by reducing congestion, improving efficiency and increasing access. Transportation Optimization can be achieved through a variety of means including reducing vehicle miles traveled (VMT) on roads; improving fuel economy; building new or expanding existing transit systems

**Risk Minimization**

Risk minimization is the process of identifying and reducing risks to an acceptable level. It is a systematic approach that involves evaluating the potential impact of risk, assigning priorities to those risks, and taking action to reduce or eliminate them. Risk minimization may be used as a tool for controlling costs.

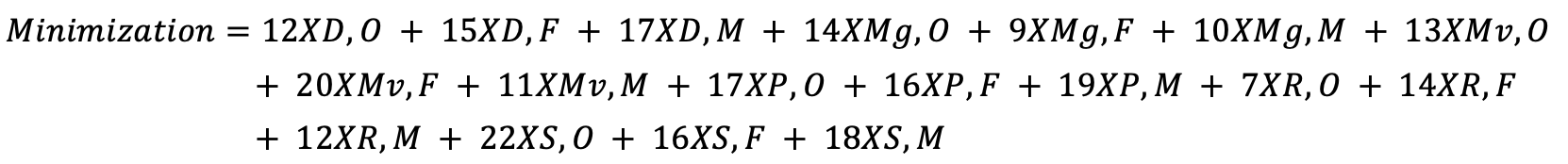
ANALYSIS

**Part I: Transportation Optimization**

In this section, we will plan the optimal solution for a transportation company to obtain the lowest transportation cost. The waste is produced at six factories and eventually sent to a recycling site. We propose two solutions for this: the first is to send the waste directly to the recycling station, but in this case because the transportation process of some wastes is very careful and time-consuming; Another option is to transport the waste to nearby factories or recycling stations, which will be used as transfer stations, where the waste will be trucked to the final recycling station. The next step is to use a mathematical model to calculate the cost of these two options and determine which is the best option. We will be performing our analysis in the following sections.

**Plan 1: Transportation from plants to disposal sites.**

Suppose that the 6 plants are denoted by D, Mg, Mv, P, R, S. And the 3 waste disposal sites are denoted by O, F, M.Th. decision variables, are defined as the number of barrels of waste shipped from plant i to waste proposal site j, where , and . The objective function is expressed below:



The constraints are expressed by mathematical equations below, which indicate the waste produced by each plant.

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The maximized capacity of each disposal site is limited and is expressed by the following equations:

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We should note that for the constraints of wastes produced, equations are used because the exact amount of waste produced are specified and for the constraints of maximum capacity of disposal sites, we use inequalities.

The final mathematical model is written below:

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By building the decision variable matrix, we are able to use Excel SUMPRODUCT () function to multiply the variables with corresponding unit costs. The SUM () function in Excel is used to calculate the total amount of wastes in both the plants and disposal sites. Then we can use the Solver to solve for our minimization solution by the SIMPLE LP method, the Excel results are shown below:

Graphical user interface, application, table

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Table 1: Plan 1 solutions

From the table shown above, we are informed that the optimization solution for the first plan is $2988. The constraints we set for each variable are met and allocations of the wastes are shown in the green area in this table. Now we move on to the second plan.

**Plan 2: Transporting wastes through intermediate sites.**

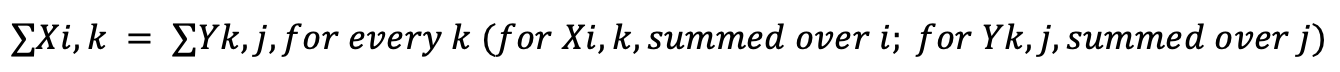
As we introduced our variables in the previous part, we are going to use the same decision variables in this part. However, some changes are made to accommodate for the second solution. Xi,k is still defined as the barrels of wastes from the plants, Yk,j represents the barrels of waste picked up from the intermediate sites. The objective of this mathematical model is like that from Plan 1, that is, the minimization of total cost. The objective functions are shown below:







Like in the previous part, we need some constraints for the solution, we need the same maximum capacity for the disposal sites and the number of wastes produced by the plants. In addition, we need to assure that the total pickup from the plants to the intermediate sites equals the total drop off from the intermediate sites to the disposal sites.



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Next, we implement our mathematical model into Excel. SUMPRODUCT () function is used to multiplying the variable cells with the coefficient cells (unit cost tables). SUM () function is used to calculate the total barrels of waste in each plant or intermediate site. SIMPLEX LP is used for our mathematical functions. The cost from plants to intermediate sites and the cost from intermediate sites to the disposal sites are calculated and sum up to get the final solution. Then we use the Excel solver to solve for the total cost. Three constraints are set.

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Table 2: Decision variables from plants to disposal sites

Graphical user interface, application, table, Excel

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Table 4: Solutions for the second plan

From the solution table above, we are informed that the optimization solution for the minimum cost is $2674 while the cost from plants to intermediate sites is $1485 and the cost from intermediate sites to disposal sites is $1189.

In conclusion, if we compare our solution from Plan 2 to Plan 1, we can say that setting the intermediate sites is a good solution to minimize the cost since **$2674** is less than $2988. The allocations of wastes barrels are demonstrated below:

Diagram

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Figure 1: Intermediate transportation

From the graph provided above, we are informed that the three plants Morganton, Morrisville, and Rockhill are used as intermediate sites while the other three plants are not used.

**Part II: Risk Optimization**

In this part, we will be analyzing the six assets in the portfolio each is denoted by a, b, c, d, and e,f. The decision variable is denoted by Xi, while i equals a, b, c, d, e, and f. In this part, the risk of the portfolio is determined by the variance provided. In the case of constant returns, we need to choose the portfolio with less risk, so we need to choose the scheme with the least variance. We can write our objective function as below:

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We also have several constraints that need to add to our mathematical model. First, we need to consider the total budget for this investment which equals $10000, and we can write the equation like this:



The expected return is 11%, and we can write the constraint as below:



The final step is to add non negativity, and our completed model is show below:

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From the plot, we find a nonlinear relationship of the risk and expected return. Therefore, we can say that our model is nonlinear and is similar to a quadratic function following the trendline we draw above. Next, we use the Excel solver and the GRG nonlinear method to solve for the optimization solution.

Table

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Table 5: Decision variables

Table

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Table 6 : Variance table

From the table of solution above, we can observe that the optimization solution for the variance is 73564.47. The optimal solution for investment allocation is also shown in the table above with all the constraints satisfied (total budget of $10000 and return of 11%).

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In this part, we will plot the solutions of minimized risk and the expected return. The results are shown below:

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Table 7: Return and risk table

Chart, line chart

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Table 8: Plot of risk and expected return

From the table and the plot above, we are informed that the relationship is quadratic not linear in this case. And as the expected return increases, the minimized risk increases as well. And as the minimized risk is denoted by r and the expected return is denoted by e, we can say that the mathematical relationship between r and e is quadratic.

CONCLUSION

In this project, we solve the linear optimization and the nonlinear optimization problem related to transshipment and risk investment. Excel Solver is used with SIMPLEX LP and GRG nonlinear to solve for the mathematical function we build. The results in the first part shows that the optimization solution for transshipment of waste is to set intermediate sites in order to save time and money. For this problem, it is important to know that the constraints for the pick-up and drop off in the intermediates site should be consistent. The optimization solution for the first part is **$2674**, which is less than the cost if wastes are just transported directly to the disposal sites. In the second part related to the risk investment, we have the optimal solution for investment allocation of $1890.85, 1084.06, $2709.22, $481.71, $2540.51, and $1293.65 in bonds, high tech stocks, foreign stocks, call options, put options, and in gold, respectively. The return at 11% and total budget of $10000 is satisfied. And by analyzing the variance, we are informed that risk is a quadratic function of expected return.

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Evans, J. (2012). Statistics, Data Analysis, and Decision Modeling (5th ed.). Pearson.