Appendix 1 - Black body spectral radiance

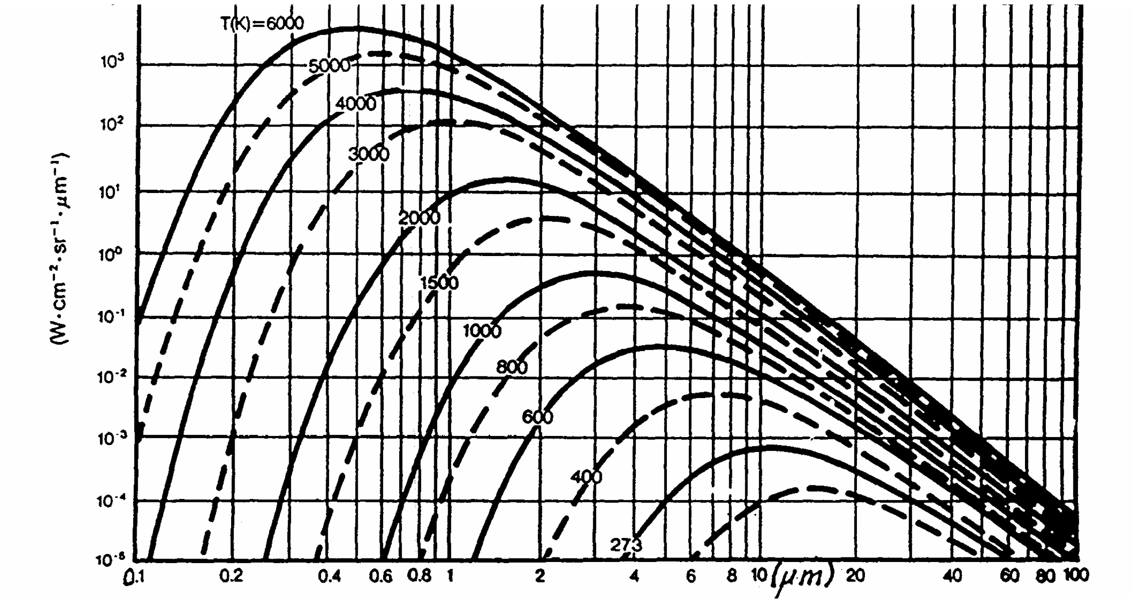


Figure 10.5: Black Body spectral radiance

Appendix 2 - Atmospheric transmittance

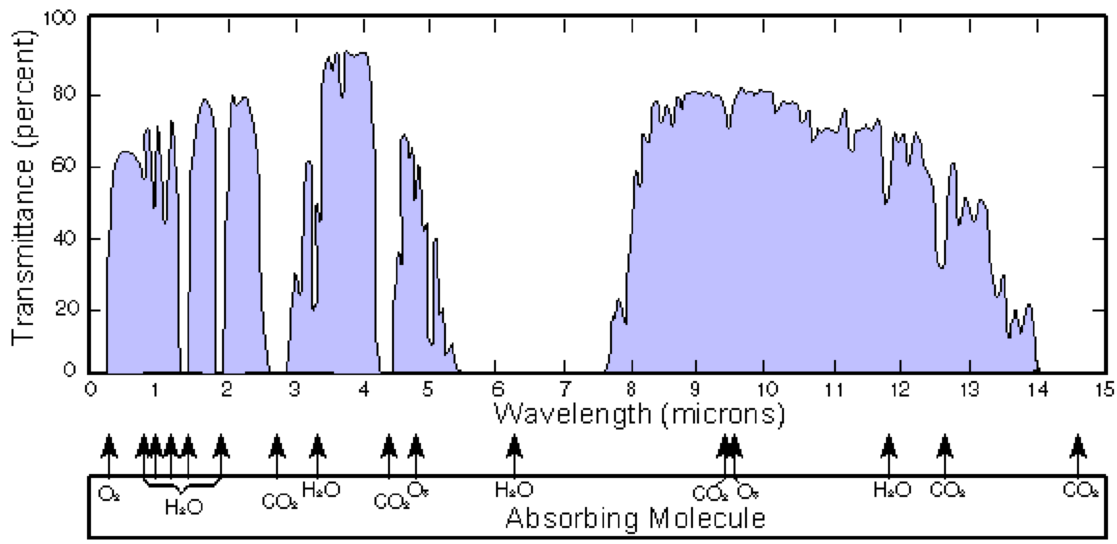


Figure 10.6: Atmospheric transmittance

Appendix 3 - Spectral detectivity

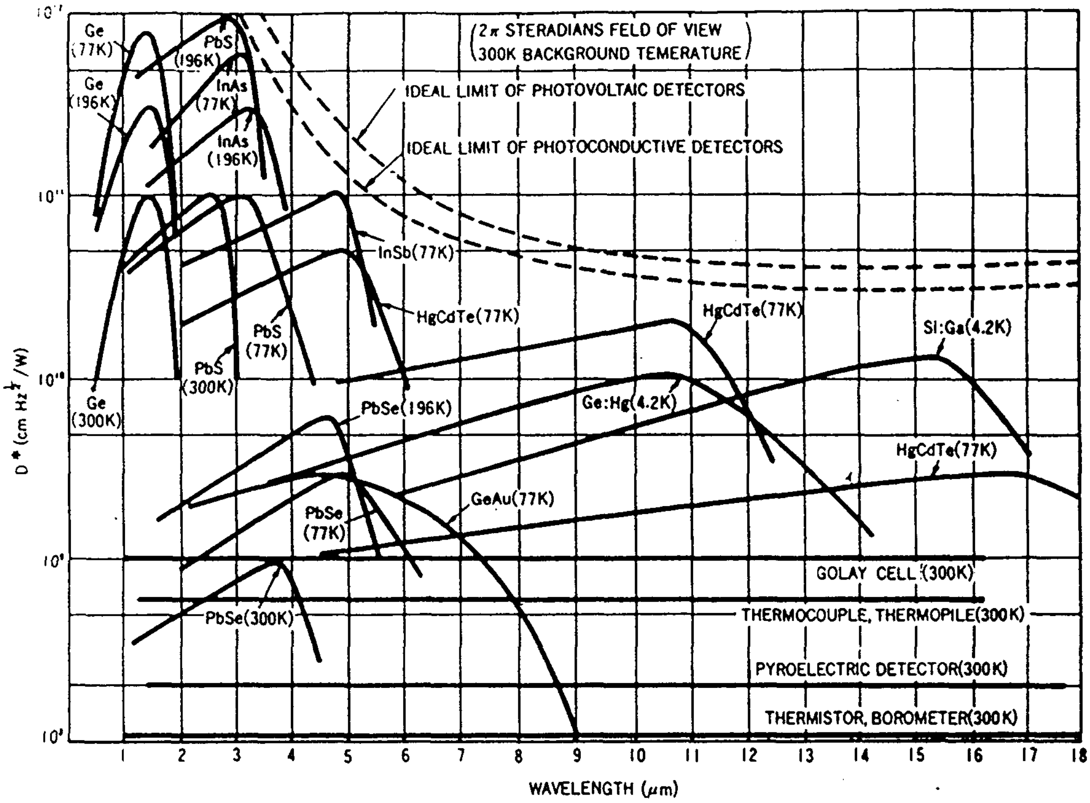
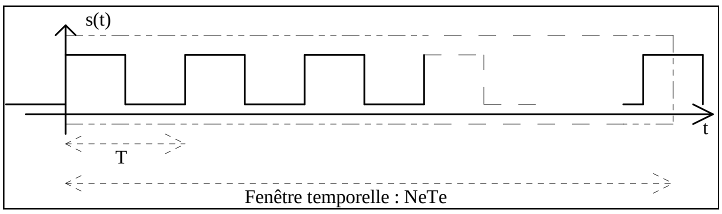


Figure 10.7: Spectral detectivities of IR detectors

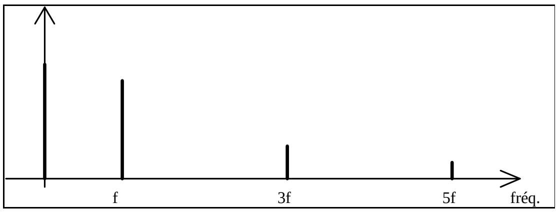
Appendix 4 - Fast Fourier Transform and windowing

Let's take the example of a square signal, $s(t)$, which frequency is $f = 100\,\mathrm{Hz}$ ($T = 10\,\mathrm{ms}$). This signal is digitized at a sampling frequency, $F_e = 1\,\mathrm{kHz}$ ($T_e = 1\,\mathrm{ms}$). We use N_e acquired points .



square wave signal $s(t)$

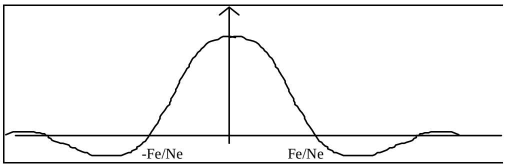
The signal, $s(t)$, is periodic, so its spectrum is represented by :



Spectrum of $s(t)$

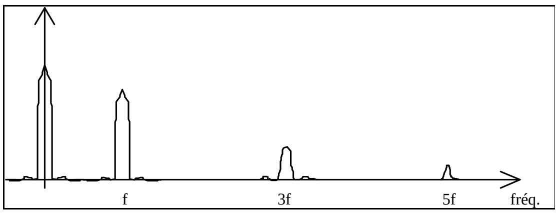
Windowing

The temporal window is rectangular, its length is N_eT_e, and its Fourier Transform is a sinc function:



Spectrum of a rectangular window of length N_eT_e

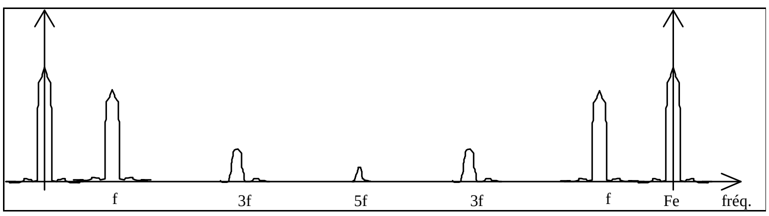
So, the spectrum of the signal limited by this rectangular window is given by the spectrum of the periodic signal $s(t)$ convoluted by this sinc function :



Spectrum of $s(t)$ in a rectangular window.

Sampling

The spectrum of the sampled signal is also a periodic spectrum with a period $F_e = 1/Te$. This may produce aliasing. This is the reason why a low pass filter is needed with a cut-off frequency $fc < F_e/2$.

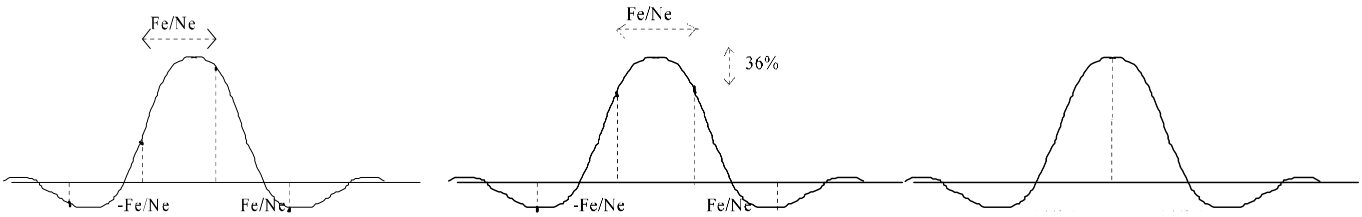


Sampled signal spectrum

FFT Algorithm

Fast Fourier Transform algorithm uses exactly the same number, N_e , in the frequency domain as in the time domain. A FFT calculate N_e points of the spectrum between 0 and F_e . So the spacing between 2 points is $\Delta f = \frac{F_e}{N_e}$.

Knowing that the width of the sinc function for a rectangular window is $2/N_eT_e$, you realize that the FFT algorithm calculates only 2 points on the sinc function main lobe. This will produce a very important measurement error on the calculation of a periodic signal spectral component amplitudes. And this error will not depend on the number of sample points Ne you choose. This error may be 36% of the real amplitude (case b). The amplitude is well calculated only in case c, when the signal is a multiple of Δf :



cas a/b/c

To avoid this error, the usual solution is to take a different temporal window. We should choose a window which is at least larger than $\frac{F_e}{N_e}$. This is the case for the "Flat Top" window (see windows and corresponding spectrum at the end of this appendix). So the sampled periodic signal is multiplied by a "Flat Top" window before the FFT is calculated..

ENBW : Equivalent Noise Band Width

For noise analysis any window will be suited, but the ENBW is

Temporal Window	Leakage	Max error on the amplitude	ENBW
Rectangular	-13dB : 0.224	36%	$\frac{F_e}{N_e}$
Von Hann	-32dB : 0.025	15%	$1.5 \times \frac{F_e}{N_e}$
Hamming	-43dB : 0.0070	18.5%	$1.37 \times \frac{F_e}{N_e}$
Blackmann-Harris	-67dB : 0.00045	12%	$1.71 \times \frac{F_e}{N_e}$
Flat-Top	-44dB : 0.0063	0.1%	$2.91 \times \frac{F_e}{N_e}$

Temporal windows are calculated by :

$$w_k = a_0 + a_1 \cos\left(\frac{2\pi k}{N_e}\right) + a_2 \cos\left(\frac{4\pi k}{N_e}\right) \text{ with } k = 0, ..., N_e - 1$$

Coefficients a_0, a_1, a_2

Temporal window	a_0	a_1	a_2
Rectangular	1.0	0.0	0.0
Von Hann	0.5	-0.5	0.0
Hamming	0.54	-0.46	0.0
Blackmann-Harris	0.423	-0.497	0.079
Flat-Top	0.281	-0.521	0.198