Module 4

Time Value of Money

A FINANCIAL TIMES COMPANY

TIME VALUE OF MONEY

The time value of money deals with the equivalence of cash flows received or paid at different points in time. It reflects the valuation of time preference.

For example:

- Would you rather have $1,000 now or $1,000 a year from now?
- By what amount would you have to "discount" the $1,000 where you wouldn't care which you received?

TIME VALUE OF MONEY

- The rate that ties the $1,000 received a year from now with the amount that you could receive now is the discount rate
- Your required rate or the minimum rate you must receive to accept the "investment"
- Opportunity cost is what is forgone by selecting one of the two choices

COMPONENTS OF THE NOMINAL INTEREST RATE

Nominal Rate of Interest equals:

i = Real Risk-Free Rate

+ Expected Inflation

+ Default Risk Premium

+ Liquidity Premium

+ Maturity Premium

THE NOMINAL INTEREST RATE

Nominal Rate of Interest equals the sum of:

- **Real Risk-Free Rate** - This assumes no credit risk or uncertainty and simply reflects differences the preference to spend now and pay back later versus lend now and collect later.
- **Expected Inflation** – If market expects prices to rise then the currency's purchasing power is reduced by the inflation rate. Inflation makes currency less valuable in the future and is factored into determining the nominal interest rate.

THE NOMINAL INTEREST RATE

- **Default-Risk Premium** - What is the chance that the borrower won't make payments on time, or will be unable to pay what is owed? This component will be high or low depending on the creditworthiness of borrower.
- **Liquidity Premium**- Some investments are highly liquid, meaning they are easily exchanged for cash (i.e. U.S. Treasury debt). Other securities are less liquid and trade infrequently. Holding other factors equal, a less liquid security must compensate the holder by offering a higher interest rate.
- **Maturity Premium** - A bond obligation will be more sensitive to interest rate fluctuations the longer the time to maturity.

TIME VALUE OF MONEY

Future Value (FV)

- Looks forward: takes today's cash flows and projects them to a single point in the future
- Known as the **compounding** of interest

Present Value (PV)

- Looks backward: takes future cash flows and returns them to today (also a single point)
- Known as the **discounting** of interest

VALUATION OF TIME PREFERENCE

Which would you rather have?

- $100 today?
- Or the promise of $100 in one year's time?

Answer: $100 today because of the valuation of time preference.

VALUATION OF TIME PREFERENCE

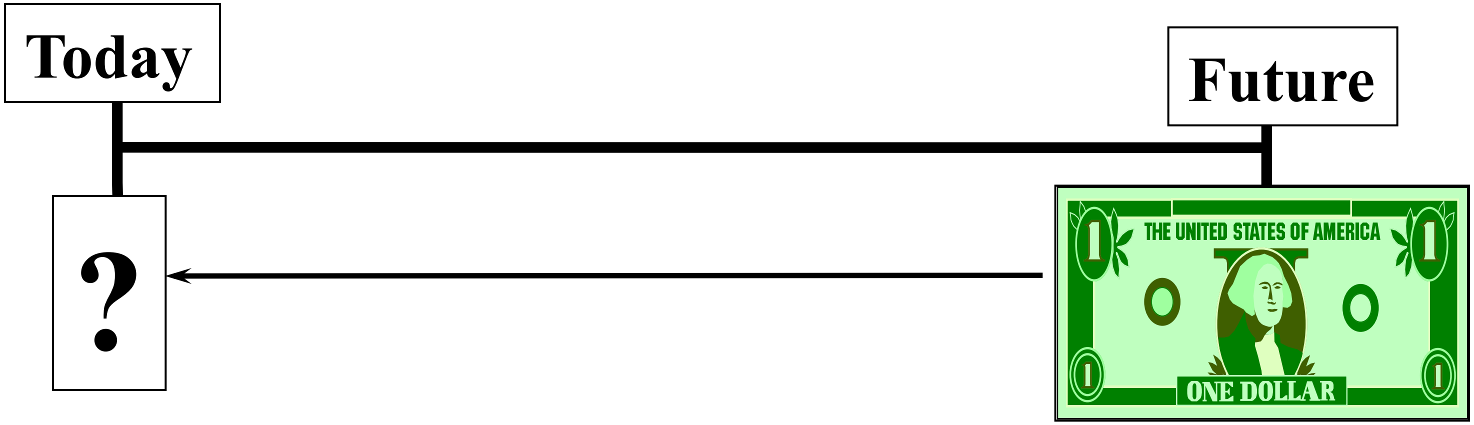
- This valuation is wired into all human beings. One prefers present consumption to future consumption *ceteris paribus*.
- Interpret consumption broadly to mean not only consumptive activity but any activity undertaken in the present to include using the $100 to set-up a rainy day fund or using the $100 in making an investment.
- In order to be successful in borrowing the $100 a would-be borrower must persuade the would-be lender to prefer future consumption to present consumption.

VALUATION OF TIME PREFERENCE

- Thus interest arises serving as an incentive to reverse the time-preference valuation. The would-be lender reverses direction and now prefers future consumption to present consumption.
- A $100 interest-bearing loan is made to the would-be borrower in this example. Everyone has a personal rate of time preference.

TIME VALUE OF MONEY

- What is the value today of a sum promised in the future?
- This process is known as ***discounting***.



TIME VALUE OF MONEY

$$PV = \frac{FV}{(1+r)^N}$$

PV	=	Present Value of a single sum of money
FV	=	Future Value of a single sum of money
r	=	Interest rate (expressed as a decimal)
N	=	Number of annual compounding periods

TIME VALUE OF MONEY - PROBLEM #1

We will receive $1,000 from an investment in ten year's time. If the interest rate is expected to be 8 percent annually over the ten years, then today's value of the investment is closest to:

A $463

B $467

C $2,144

D $2,159

TIME VALUE OF MONEY - ANSWER IS 'A'

$$PV = \frac{FV}{(1+r)^N}$$

$PV = FV / (1 + r)^N$

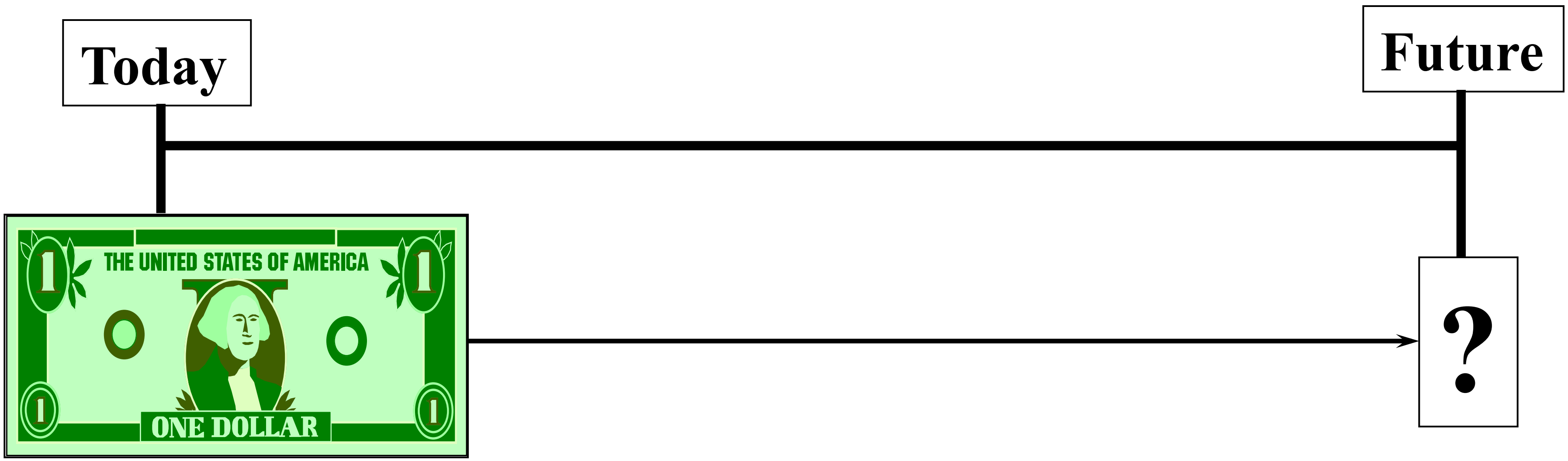
$PV = \$1{,}000 / (1 + 0.08)^{10}$

$PV = \$1{,}000 / 2.15892$

$PV = \$463.19$ (or \$463 rounded down)

TIME VALUE OF MONEY

- You invest a single sum of money over a period of time and receive interest
  - How large does the sum grow?
  - This is process is known as **compounding**



TIME VALUE OF MONEY

$$FV = PV \times (1+r)^N$$

FV	=	Future Value of a single sum of money
PV	=	Present Value of a single sum of money
r	=	Interest rate (expressed as a decimal)
N	=	Number of annual compounding periods

TIME VALUE OF MONEY - PROBLEM #2

We have $1,000 to invest today. If the interest rate is expected to be 8 percent annually over the ten years, then the value of the investment in ten year's time is closest to:

A $463

B $467

C $2,144

D $2,159

TIME VALUE OF MONEY - ANSWER IS 'D'

$$FV = PV \times (1+r)^N$$

$$FV = \$1{,}000 \times (1 + r)^N$$

$$FV = \$1{,}000 \times (1 + 0.08)^{10}$$

$$FV = \$1{,}000 \times 2.15892$$

$FV = \$2{,}158.92$ (or rounded up to $2,159)

TIME VALUE OF MONEY: ANNUITIES

- So far, we've looked at single sums of money. We can also calculate PV and FV of an **annuity**

- An annuity is a series of **equal cash flows** that occur at **equal intervals** over a **set period of time**

- **CAUTION:** there are two types of annuity:

TIME VALUE OF MONEY: ANNUITIES

- Ordinary annuity: first cash flow will be received at Time 1
  - An example of an annuity with payments in arrears would be the payments on a coupon-bearing bond
- Annuity due: first cash flow will be received at Time 0 (today)
  - An example of an annuity with payments up front would be the payments on an insurance policy

TIME VALUE OF MONEY: ANNUITIES

$$PV = A\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$$

Where:

PV = Present value of an ordinary annuity (with first payment beginning **next year**)

A = Annuity amount (payment)

r = Annual interest rate

N = Number of years over which annuity payments are made

TIME VALUE OF MONEY - PROBLEM #3

We will receive payments of $1,000 per year for the next ten years, with the first payment paid *next year*. If the interest rate is expected to be 8 percent annually over the ten years, then today's value of the payments is closest to:

A $6,710

B $7,247

C $14,487

D $15,645

TIME VALUE OF MONEY - ANSWER IS 'A'

$$PV = A\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$$

PV = \$1,000 {[1 – (1/(1.08)^{10})] / 0.08}

PV = \$1,000 (1 – 0.46319) / 0.08

PV = \$1,000 × 6.71008

PV = \$6,710.08 (rounded down to \$6,710)

TIME VALUE OF MONEY: ANNUITIES

$$PV_{AD} = A\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right](1+r)$$

Where:

PV_{AD}= Present value of an annuity due (with first payment beginning **today**)

A = Annuity amount (payment)

r = Annual interest rate

N = Number of years over which annuity payments are made

TIME VALUE OF MONEY - PROBLEM #4

We will receive payments of $1,000 per year for the next ten years, with the first payment paid *today*. If the interest rate is expected to be 8 percent annually over the ten years, then today's value of the payments is *closest* to:

A $6,710

B $7,247

C $14,487

D $15,645

TIME VALUE OF MONEY - ANSWER IS 'B'

$$PV_{AD} = A\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right](1+r)$$

PV = \$1,000 {[1 – (1/(1.08)^{10})] / 0.08} ×1.08

PV = \$1,000 (1 – 0.46319) / 0.08 ×1.08

PV = \$1,000 × 6.71008 ×1.08

PV = \$7,246.89 (rounded up to \$7,247)

TIME VALUE OF MONEY: ANNUITIES

$$FV = A \left[\frac{(1+r)^N - 1}{r} \right]$$

Where:

FV = Future value of an ordinary annuity

A = Annuity amount (payment)

r = Annual interest rate

N = Number of years over which annuity payments are made

TIME VALUE OF MONEY - PROBLEM #5

We will receive payments of $1,000 per year for the next ten years, with the first payment paid *next year*. If the interest rate is expected to be 8 percent annually over the next ten years, then the cumulative value of the payments at the end of ten years is *closest* to:

A $6,710

B $7,247

C $14,487

D $15,645

TIME VALUE OF MONEY - ANSWER IS 'C'

$$FV = A \left[\frac{(1+r)^N - 1}{r} \right]$$

FV = $\$1,000\ [(1.08)^{10} - 1]\ /\ 0.08$

FV = $\$1,000\ (2.15892 - 1)\ /\ 0.08$

FV = $\$1,000 \times 14.48656$

FV = $14,486.56 (rounded up to $14,487)

TIME VALUE OF MONEY: ANNUITIES

$$FV_{AD} = A \left[\frac{(1+r)^N - 1}{r} \right] (1+r)$$

Where:

FV = Future value of an annuity due (first payment beginning **today**)

A = Annuity amount (payment)

r = Annual interest rate

N = Number of years over which annuity payments are made

TIME VALUE OF MONEY - PROBLEM #6

We will receive payments of $1,000 per year for the next ten years, with the first payment paid *today*. If the interest rate is expected to be 8 percent annually over the next ten years, then the cumulative value of the payments at the end of ten years is *closest* to:

A $6,710.

B $7,247

C $14,487

D $15,645

TIME VALUE OF MONEY - ANSWER IS 'D'

$$FV_{AD} = A \left[\frac{(1+r)^N - 1}{r} \right] (1+r)$$

FV = \$1,000 [(1.08)^{10} – 1] / 0.08 × 1.08

FV = \$1,000 (2.15892 – 1) / 0.08 × 1.08

FV = \$1,000 × 14.48656 × 1.08

FV = \$15,645.48 (rounded down to \$15,645)

TIME VALUE OF MONEY

- PV of annuity due > PV of ordinary annuity
  - For an annuity due, we receive all cash flows one period early
  - For example, first cash flow received at Time 0 gets one fewer discounting period

- FV of annuity due > FV of ordinary annuity
  - For an annuity due, we can invest all cash flows for one extra period
  - For example, first cash flow received at Time 0 gets an extra compounding period

U.S. TREASURY BILL CALCULATIONS AND THE BOND EQUIVALENT YIELD (BEY)

We can compared the yields of Treasury Bills (which are usually stated in terms of bank discount yield) and the Bond Equivalent Yield (BEY).

We can calculate and interpret the bank discount yield, holding period yield, effective annual yield, and money market yield for a U.S. Treasury bill.

Then we can interpret and convert among holding period yields, money market yields, effective annual yields and the bond equivalent yields.

r_{BD} = (D/F) * (360 / t) where

r_{BD} = annualized yield on bank discount basis

F = face value of the T-bill

D = dollar discount = F – P (P is the purchase price)

t = days to maturity

T-BILL YIELDS

- Bank discount yield is widely used, but not a good measure of return earned by investors.

- **Holding Period Yield (HPY)** – the return earned by an investor if the T-Bill is held to maturity

$$HPY = (P_1 - P_0 + D_1) / P_0,$$

where P_0 = purchase price; P_1 = price at maturity; D_1 = interest

T-BILL YIELDS

- **Effective Annual Yield (EAY):** Compounds the HPY to convert it to an equivalent annual yield

$$EAY = (1 + HPY)^{(365 / t)} - 1$$

- **Money market yield** – CD equivalent yield. Makes yield comparable to money-market securities that pay interest on 360-day basis. There are two ways to calculate r_{MM}:

$$r_{MM} = HPY * (360 / t)$$
$$r_{MM} = (360 * r_{BD}) / (360 - t * r_{BD})$$

T-BILL YIELD EXERCISES

Calculate the bank discount yield of a $100,000 T-bill with a purchase price of $98,800 with 183 days to maturity:

r_{BD} = (D/F) * (360 / t), so

r_{BD} = ($1,200 / $100,000) * (360 / 183) = 2.36%

Calculate the holding period return and effective annual yield of T-bill with a purchase price of $98,800 with 183 days to maturity

HPY = (P_1 – P_0 + D_1) / P_0, so

HPY = (100 – 98.8 + 0) / 98.8 = 1.215%

EAY = $(1 + HPY)^{(365\,/\,t)} - 1$, so

EAY = $(1 + 0.01215)^{(365\,/183)} - 1$ = 2.438%

T-BILL YIELD EXERCISE

Calculate the money market yield of T-bill with a purchase price of $98,800 with 183 days to maturity:

r_{MM} = HPY * (360 / t), so

r_{MM} = 1.215% * (360 / 183) = 2.39%

r_{MM} = (360 * r_{BD}) / (360 – (t * r_{BD}))

r_{MM} = (360 * 2.36%) / (360 – (183 * 2.36%)) = 2.39%

Note: Both methods for money market yield shown.