Heuristic approach for dynamics of wave-induced currents over an along- shore beach

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1.1 Introduction

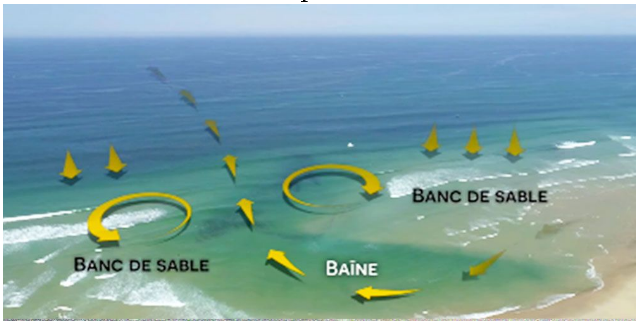
A wave-induced current is a strong return current that draws the waters offshore while these waters are brought by the big waves that break on the beaches.

The purpose will be to determine what is the minimal model to simulate a baine with a very simplified approach. We won't consider waves.

We will consider a 2D approach with a section of the beach. Let's define the steps that we will follow:

1. determinee the main scales of the problem to obtain an incompressible and laminar flow with Blasius boundary conditions *i.e.* the viscosity, viscous boundary layer width and the average velocity field.

2. determinee numerically which coast shape "optimizes" wall-normal velocity with a 2D approach when we impose a Blasius as boundary conditions.



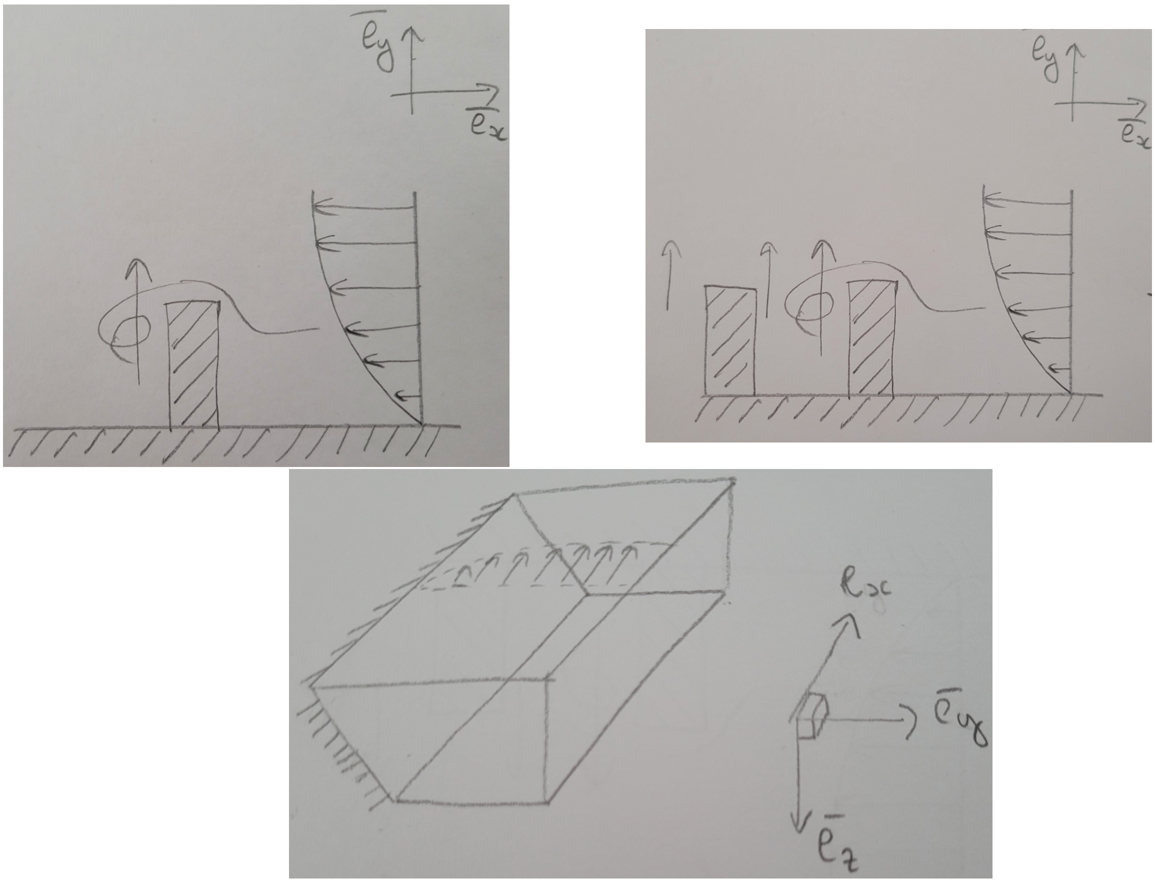


Figure 1: We will try several geometric configurations both in 2D and 3D in order to obtain a flow that has a component according to ey.
On the bottom scheme the beach is on the left

The article written by Bruno Castelle [1] shows that rip currents can be obtained by the following wave model where $\bar{Q}(x,y,t) = \int_Z^{\eta} v_i(x,y,z,t)dz$ is water volume flux for coordinate i.

$$\frac{\partial \bar{Q}_i}{\partial t} + \frac{\partial}{\partial x_j}(\frac{\bar{Q}_i \bar{Q}_j}{\bar{h}}) = -g\bar{h}\frac{\partial \eta}{\partial x_j} - \frac{1}{\rho}\frac{S_{ij}}{\partial x_j} - \frac{1}{\rho}\frac{R_{ij}}{\partial x_j} + \frac{1}{\rho}\frac{\partial T_{ij}}{\partial x_j} - \frac{\tau_i^b}{\rho} \tag{1}$$

$$\frac{\partial \eta}{\partial x_j} + \frac{\partial Q_{ij}}{\partial x_{ij}} = 0. \tag{2}$$

(1) and (2) are the conservation equations of momentum and mass. The variables g, T, S_{ij}, R_{ij}, and T_{ij} represent the gravitational acceleration, mass density of water, radiation stress components, time-averaged bed stress, and excess momentum flux caused by the wave roller, respectively.

2 Numerical approach

In this section we aim to determine geometrical conditions that must be fulfilled to obtain significant rip currents. We will work on a 2D case. In order to study a monophasic phenomena we will study the impact of simple obstacles on a inlet Blasius flow parallel to the alongshore beach.

We will observe whether a normal component of the velocity field (to the beach) is created depending on the ratio between the thickness of the viscous boundary layer and the characteristic length shape of the obstacle.

2.1 Order analysis

From Navier-Stockes equations we can give a simple estimate of the thickness of the viscous boundary layer. Let's consider a no slip condition on the beach side. Far from the beach the velocity field is U_0

$$\begin{cases} u(x, y = \delta) = U_0 \\ u(x, y = 0) = 0 \end{cases} \tag{3}$$

Under assumptions that $\frac{\partial p}{\partial y} << 1$ and with the pressure evaluated to infinity of the beach p_∞, we obtain the following result:

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{dp_\infty}{dx} \end{cases} \tag{4}$$

The conservation equation of mass implies $v = \frac{\delta}{x}U_\infty$, and thus from the conservation equation of momentum we obtain:
$\frac{U_\infty^2}{x} \sim \nu\frac{U_\infty}{\delta^2} \Rightarrow \delta \sim \frac{x}{{R_{ex}}^{\frac{1}{2}}}$ with $R_{ex} = \frac{U_\infty x}{\nu}$

It can be shown that the Blasius solution implies that the friction coefficient is $Cf_x = 0.664{R_{ex}}^{-\frac{1}{2}}$ and by definition $Cf_x = 2\frac{\nu}{U_\infty \delta} = 2{R_{ex}}^{-1}\frac{x}{\delta}$
Thus $0.664{R_{ex}}^{-\frac{1}{2}} = 2{R_{ex}}^{-1}\frac{x}{\delta}$ and $\delta = 2\frac{x}{0.664{R_x}^{\frac{1}{2}}} = \frac{x}{0.332R_{ex}} = \frac{\sqrt{x}\sqrt{\nu}}{0.332U_\infty}$

2.2 Blasius profile on the inlet

On the inlet side we generate a velocity flow from a polynomial function of order 2 with $\eta = \frac{y}{\delta}$. We will therefore have to choose the value of $\delta = \delta^\star$ at the entrance of the domain, this will allow us to master the evolution of the viscous boundary layer.

$$\frac{u(\eta)}{U_\infty} = \begin{cases} a\eta^2 + b\eta + c, \forall 0 \leq \eta \leq 1 \\ 1, \forall \eta \geq 1 \end{cases} \tag{5}$$

(3) $\Rightarrow u(\eta = 0) = 0 \Rightarrow c = 0$ and $u(\eta = 1) = 1 \Rightarrow a + b = 1$.
Moreover, the free surface condition involved $\frac{du}{d\eta}(\eta = 1) = 1 \iff 2a + b = 1$.

Thus, $\begin{cases} a = -1 \\ b = 2 \end{cases}$ and $u(\eta) = 2\eta - \eta^2, \forall 0 \leq \eta \leq 1$

2.3 Nondimensionalization

Field measurements have been gathered from drifter deyployments from Boscombe beach [2]. It is a beach with several groynes. The maximum longshore velocity is about $U_\infty = 0.5m \cdot s^{-1}$. The mean surf zone width is $X_b = 50m$. Thus as Reynolds number can be chose as $R_e = \frac{.5\times 50}{1} = 25$. By choosing the scales of references as $U^\star_\infty = 1, X^\star_g = 1$ we can work with $\nu^\star = 0.04$.

One choose the following benchmark for the numerical simulation. Further we will take $\delta^\star = 5$ at the entrance of the domain. For the velocity field and the pressure field we choose the following conditions:

Boundary	Value for the velocity field	Value for the pressure field
inlet	$u^\star(\eta) = 2\eta - \eta^2$	zeroGradient
outlet	zeroGradient	uniform 0
Bottom	noSlip	zeroGradient
top	zeroGradient	zeroGradient

For every simulation we will choose $\Delta x = 0.8, \Delta t = 0.2$ and a rectangular domain of size $L \times l$ with $0 \leq L \leq 200$ and $l = 40$.

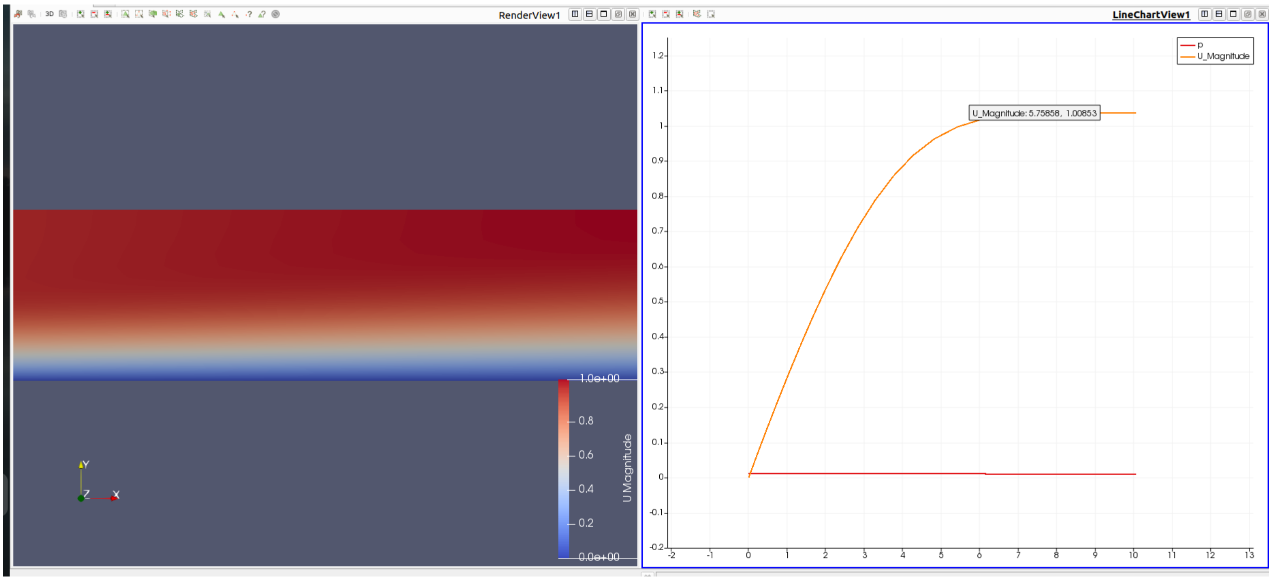


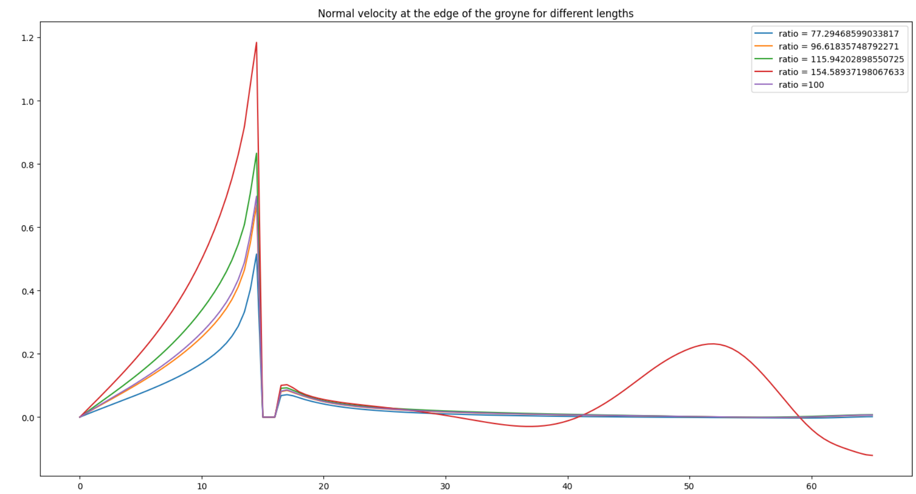
Figure 2: 2D Blasius flow, the flow calculated represented the currents over an alongshore beach. On the right side we represent the velocity field profil, it shows as expected, as Blasius courbure

We know that the thickness of the boundary layer is $\delta(x) = \frac{\sqrt{x}\sqrt{\nu}}{0.332 U_\infty}$, and then $x = \frac{(0.332 U_\infty \delta)^2}{\nu}$. Therefore at the entrance $x^\star = \frac{(0.332 U^\star_\infty \delta^\star)^2}{\nu^\star} = 68.89$. The flow is coming from the left and the first groyne is at position 15 in the domain and the thickness of the boundary layer should then be $\delta^\star(68.89 + 15) = 5.5175$.

Figure 3: Norm of the velocity field when the length of the groynes is 7

2.4 One obstacle

Let us oppose an obstacle to the flow of Blasius. We will try to determine what length of the groyne can induce the strongest rip current. On figure 3 we observe a boundary layer detachment.



Lenght of the groynes	pourcentage of the thickness boundary layer (%)
4	72.49
5	90.61
5.175	100
6	108.74
8	144.99

Figure 4: Normal component of the velocity field at the edges of the groynes. We observe that the bigger $\frac{L_g}{\delta}$ is, the bigger is the normal velocity. Indeed the section is reduced as the groyne is bigger and the velocity is expected to grow according to Bernoulli equation.

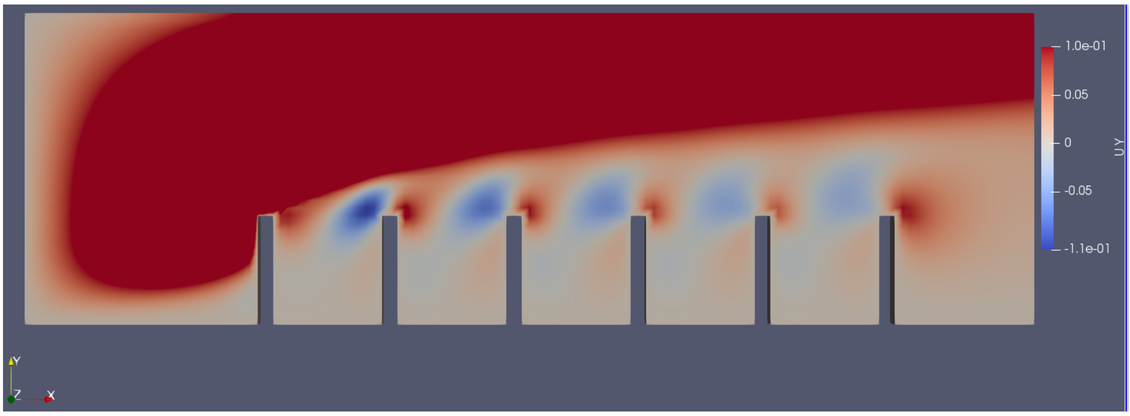
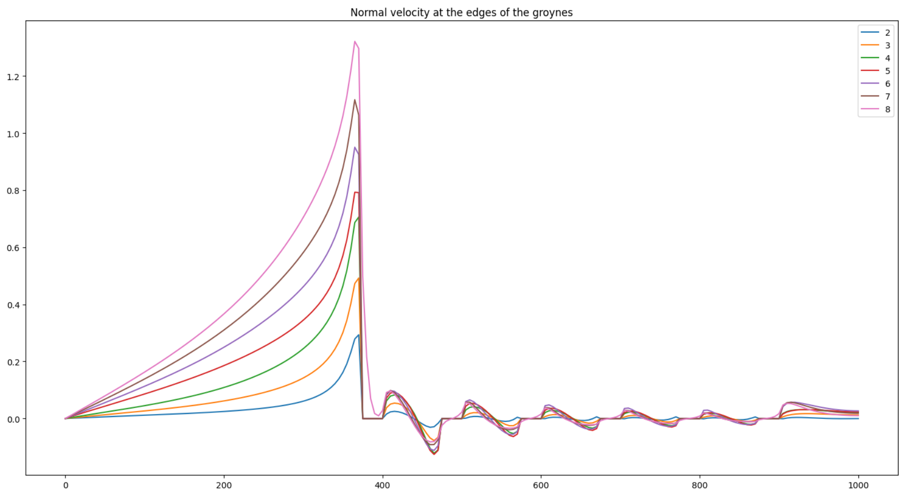


Figure 6: Normal component of the velocity with groyne length fixed at 7.

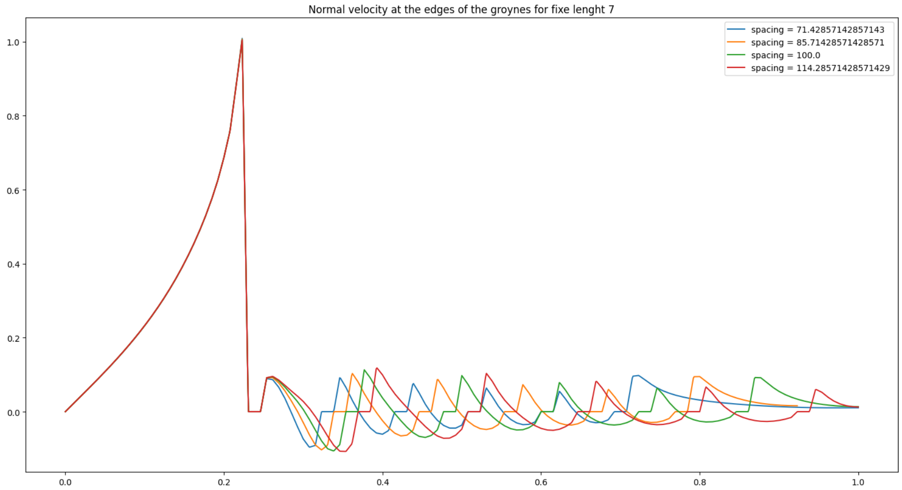
2.5 Several obstacles

We set several groynes with regular spacing and observe the normal velocity over a line at the edge for several choices of length. The flow is coming from the left and the first groyne is at position 15.



Spacing	pourcentage of the thickness boundary layer (%)
2	36.24
3	54.37
4	72.59
5	90.619
6	108.74
7	126.86
8	144.99

Figure 5: Normal component of at the edges of the groynes. (meanwhile we keep values between -0.1 et 0.1). We observe that between the groynes the velocity is positive on the left part and negative on the rigth part. The bigger the groynes are compared to the thickness of the boundary layer , the bigger is the normal component of the velocity.



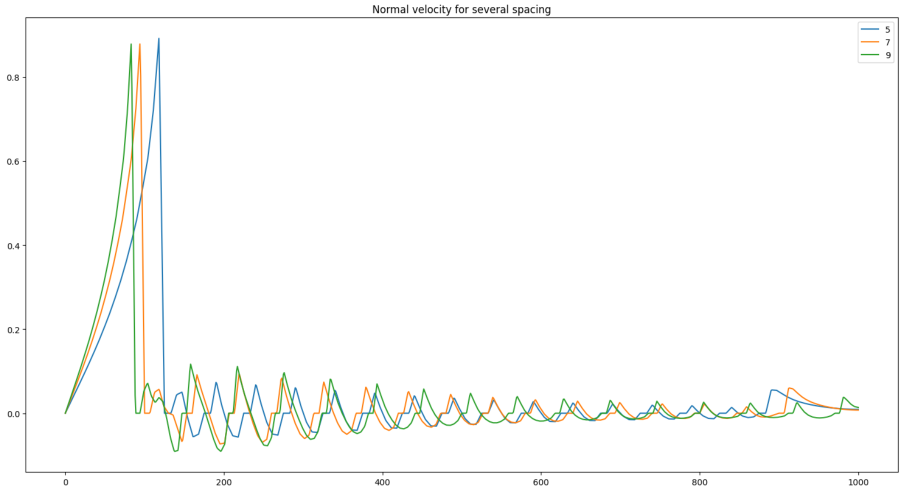
Spacing X_g	ratio $\frac{X_g}{L_g}$ %
5	71.4285
6	85.714
7	100.0
8	114.285

Figure 7: Normal component of the velocity at the edges of the groynes for a fixed length of 7 and different values of the ratio between the spacing and the length of the groynes. Indeed, the bigger is the

Impact of the ratio between the length of the groynes and the spacing
The works that have been done in [2] show that two distinct flow regimes depending on the ratio between the groynes length and the spacing however this phenomena comes with an upcoming velocity flow with an angle, (not parallel to the beach). Nevertheless we tried to spot different regimes by running several simulations with a fixed length groyne of 7 and different values for the spacing. Therefore we first tried with 5 groynes and then with 16 groynes. We have not tried a many possibilities, nevertheless it seems that the rip current increases with the spacing between the groynes.

3 Conclusion

We have shown empirically that rip currents phenomena depends on the ratio between the thickness of the boundary layer (because the higher is bigger is the groyne, the bigger is the normal component of the velocity) and the length of the groynes, as well as the spacing between the groynes.



Spacing X_g	ratio $\frac{X_g}{L_g}$ %
5	71.4285
7	100.0
8	128.571

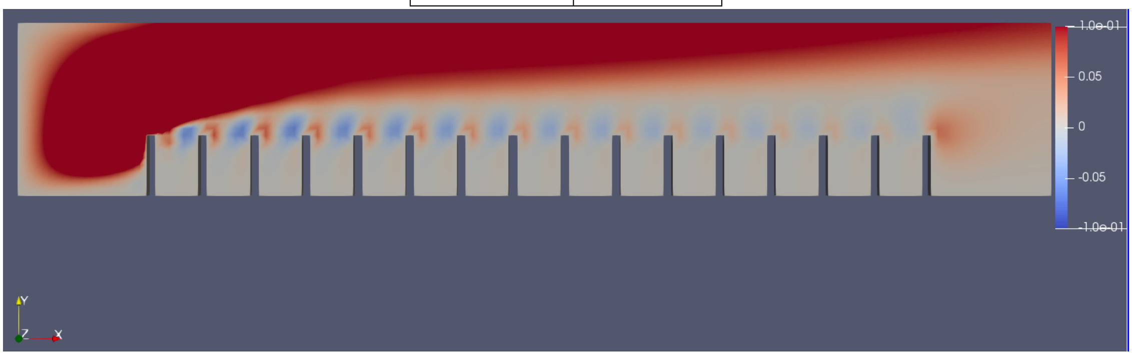


Figure 8: On top we see Normal component of the velocity at the edges of the groynes for a fixed length of 7 and different values of the ratio between the spacing and the length of the 16 groynes

References

[1] Bruno Castelle, Philippe Bonneton, Nadia Sénéchal, Hélène Dupuis, Rémi Butel, Denis Michel «Dynamics of wave-induced currents over an alongshore non-uniform multiple-barred sandy beach on the Aquitanian Coast, France»

[2] Castelle Bruno Modelling the alongshore variability of optimum rip current escape strategies on a multiple rip-channelled beach

[3] Thèse de Arthur Mouragues co-dirigée par Bruno Castelle «Étude de la dynamique instationnaire des vagues et des circulations associées en milieu littoral»