Stock Price Prediction using RNNs

Applied Masters Project

M.Sc. in Financial Engineering
Session 2

ACADEMIC YEAR 2024-2025

Dr. Dominic O'Kane

Version: January 2025

Overview

- You all have the datafile and should have started calculating momentum and other technical indicators
- You have also been able to do some ML prediction analysis
- I was hoping to talk to you about the sentiment analysis
- However, I will instead replace this with another approach
- Recurrent neural networks

RNNs versus Time Series Models

- Question is if a traditional time series model would do better
- A 2020 paper (Muncharaz) tested this using the S&P 500 found

Abstract

In the financial literature, there is great interest in the prediction of stock prices. Stock prediction is necessary for the creation of different investment strategies, both speculative and hedging ones. The application of neural networks has involved a change in the creation of predictive models. In this paper, we analyze the capacity of recurrent neural networks, in particular the long short-term recurrent neural network (LSTM) as opposed to classic time series models such as the Exponential Smooth Time Series (ETS) and the Arima model (ARIMA). These models have been estimated for 284 stocks from the S&P 500 stock market index, comparing the MAE obtained from their predictions. The results obtained confirm a significant reduction in prediction errors when LSTM is applied. These results are consistent with other similar studies applied to stocks included in other stock market indices, as well as other financial assets such as exchange rates.

- https://hal.archives-ouvertes.fr/hal-03149342/document
- I think this is possible – ML goes beyond the simple linear relationships of traditional linear regression-based models
- You will only know if you do careful cross-validation

TensorFlow

Tensor Flow 2.0

- TensorFlow is a low-level maths API like numpy but tailored for deep learning, and specifically for building multilayer NNs
- Designed by the Google Brain team
- If specialises in differential programming (needed to implement back propagation) and manipulating N-dim matrices (tensors)
- TensorFlow1 was low-level and not so easy to learn!
- TensorFlow2 has changed the library in several important ways and has integrated a high-level interface called Keras
- In this course I have designed most things to run on the CPU
- Tensorflow and the OS will assign jobs to run on different cores

Keras

- Created by Francois Chollet, a google engineer, in 2015
- It used to sit on top of TensorFlow, Theano and CNTK
- Since 2019 Keras is **fully integrated** into TensorFlow 2.0
- It comes as part of the TensorFlow installation
- Supports a broad range of neural network architectures
- Allows use of distributed training on GPUs
- There are two flavours of Keras
  - Standalone Keras which you install and call separately
  - tf.keras which is the Keras API integrated into TensorFlow2
- We will be using tf.Keras which is part of TensorFlow2

Accessing Keras

- To install Keras install the TensorFlow library from your notebook as follows

```
! pip install tensorflow
```

- To check the version

```
import tensorflow
print(tensorflow.__version__)
```

- To access Keras

```
# example of tf.keras python idiom
import tensorflow as tf
# use keras API
model = tf.keras.Sequential()
```

Defining a Neural Network Model

- Keras provides several Neural Network model types
- **Sequential** is the basic feedforward neural network model
- A sequential model is a stack of layers where each layer has one input tensor and one output tensor
- To begin, we need to pull in the Sequential model type and the input layer type **Input** and the hidden layer which here is **Dense**

```
# example of a model defined with the sequential api
from tensorflow.keras import Sequential
from tensorflow.keras.layers import Input
from tensorflow.keras.layers import Dense
```

- Dense layers means that every node in a layer is connected to every node in the next layer

Creating a Sequential Model

- This is the simplest way to build a feedforward neural network

```
model = Sequential()
model.add(Input(shape=(8,)))
model.add(Dense(2, activation="relu"))
model.add(Dense(3, activation="relu"))
model.add(Dense(1))
```

- Input layer has 8 inputs should correspond to size of training data
- Each layer is a set of neurons with defined connectivity (Dense) and a defined activation function (ReLu)
- This architecture defines 2 hidden layers – with 2, 3 neurons
- There is a single output neuron

Alternative ways to Build an ANN: Function Approach

- Call the Sequential function with the list of layers
- You haven't seen this form before, but you will see it here and in books, so it is good to be aware of it

```
model = Sequential([
    Input(shape=(5, 5)),  # Explicitly define the input shape
    Flatten(),            # Flatten the 2D input
    Dense(10, activation='relu'),
    Dropout(0.2),
    BatchNormalization(),
    Dense(3, activation='softmax')  # Output layer with 3 classes
])
```

- Adding is my preferred approach - fewer brackets to worry about
- Simpler to debug as we can step

We can see the parameters in the Model Summary

- Use the function **model.summary()**

Layer (type)	Output Shape	Param #
dense (Dense)	(None, 2)	18
dense_1 (Dense)	(None, 3)	9
dense_2 (Dense)	(None, 1)	4

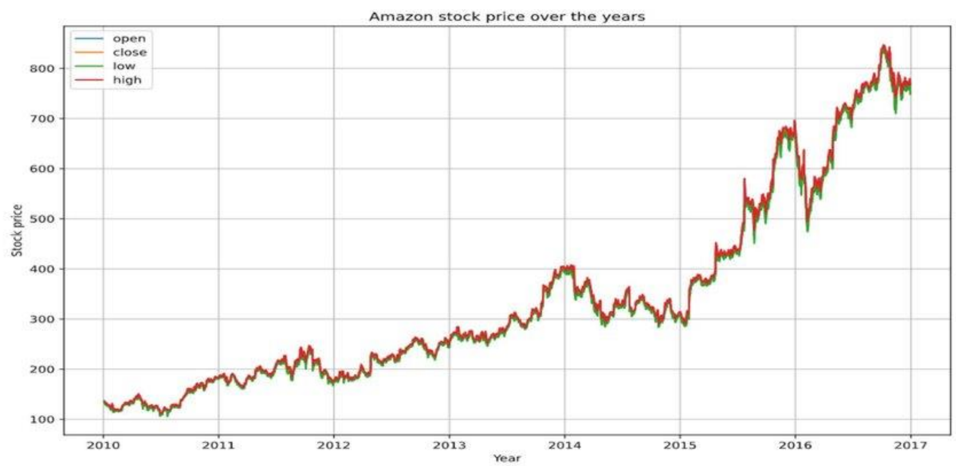
```
Total params: 31 (124.00 B)
Trainable params: 31 (124.00 B)
Non-trainable params: 0 (0.00 B)
```

- Layer 1: 8 inputs plus 1 bias to two layers = (8+1) x 2 = 18 params
- Layer 2: 2->3 neurons to 1 output = (2+1) x 3 = 9 params
- Layer 3: 3 neurons to 1 output = (3+1) = 4 params

The Basics:
Using an AR Model

Forecasting using an ANN

- We start with a time series, and we want to predict the next (unknown) values – multiple values out to some horizon



- We might want to go out e.g. 5 days ahead – this will be more challenging than just predicting one day ahead
- Before we start looking at RNNs, let us consider what we can do with non-RNN linear models

Using Linear Regression

- What is the size of our data ?
- Consider an example where we have a time series of length 10
- We want to predict the next value using the last 3 values
- N is the number of training examples
- Input matrix will be size N x 3, target vector will be of length N
- What is N ? It is 10 – 3 + 1 = 8, let us think
  - Series 1 will be x1, x2, x3 -> predict x4
  - Series 2 will be x2, x3, x4 -> predict x5
  - …
  - Series 7 will be x7, x8, x9 -> predict x10
- We only have N - 3 = 7 training examples

Linear Forecasting

- The basic equation for this problem is

$$\hat{x}_t = w_0 + w_1 x_{t-1} + w_1 x_{t-2} + w_2 x_{t-3}$$

- This is the well-known **linear autoregressive model**
- Now consider how we do our forecasting on our validation set that extends the time series beyond x10 – one approach is
  - Series 8 will be x8, x9, x10 -> predict x11
  - Series 9 will be x9, x10, x11 -> predict x12
  - Series 10 will be x10, x11, x12 -> predict x13
- But this is incorrect as we are only doing one period predictions and not 3 period predictions !

Predicting Multiple Time Steps

- To do it correctly we need to use our predictions as inputs to later predictions

$$\hat{x}_{11} = w_0 + w_1 x_{10} + w_1 x_9 + w_2 x_8$$

$$\hat{x}_{12} = w_0 + w_1 \hat{x}_{11} + w_1 x_{10} + w_2 x_9$$

$$\hat{x}_{13} = w_0 + w_1 \hat{x}_{12} + w_1 \hat{x}_{11} + w_2 x_{10}$$

- This will give us worse results, but it will be the correct method
- Results will be worse in general **as we are predicting based on predictions which are unlikely to be exact**
- We need to be forecast as we would in reality

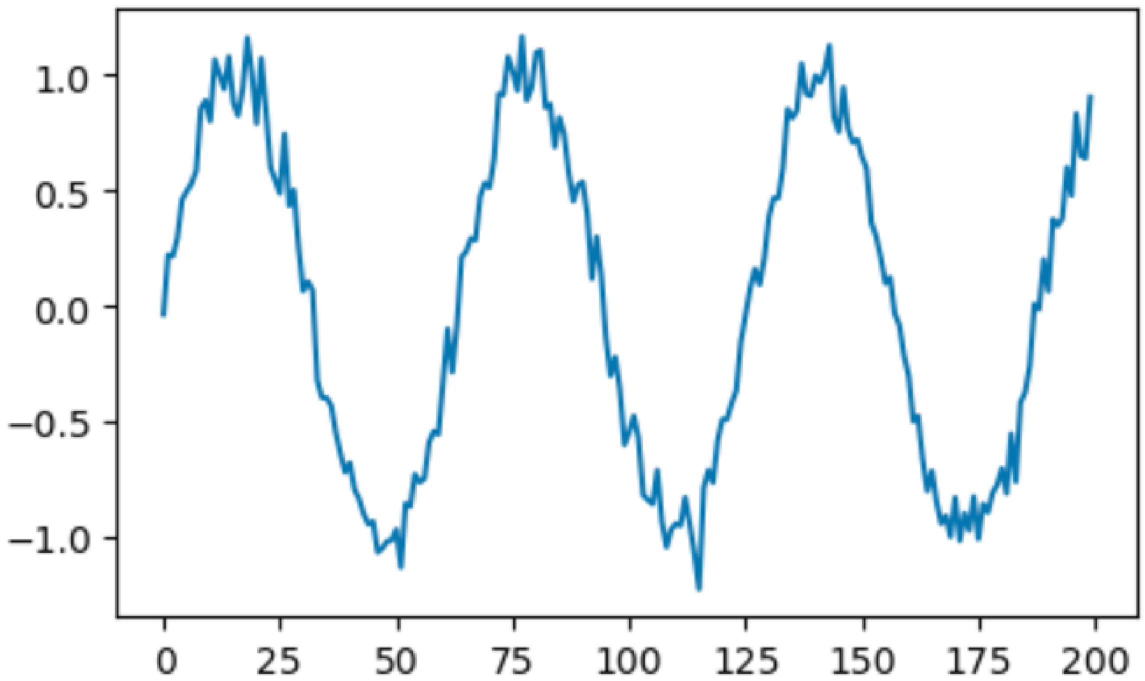
We can implement this in code

- To create a time series of data – we choose a noisy sine function

```
num_points = 200
```

```
time_series = np.sin(0.1*np.arange(num_points)) + np.random.randn(num_points)*0.1
```

```
plt.figure(figsize=(5,3))
plt.plot(time_series);
```



Create the training data

- We want to use 10 data points to predict the next point

```
T = 10
X = []
Y = []
for t in range(num_points - T):
    x = time_series[t:t+T]
    X.append(x)
    y = time_series[t+T]
    Y.append(y)

X = np.array(X).reshape(-1,T)
Y = np.array(Y)
print("X.shape", X.shape, "Y.shape", Y.shape)
```

```
X.shape (190, 10) Y.shape (190,)
```

- We have 200 data points but as T=10 we only have 190 series

Create our linear AR model

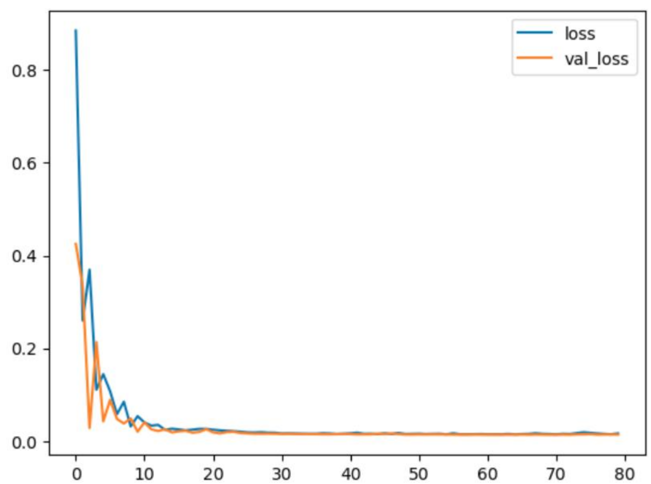
- We use a simple perceptron with T=10 inputs

```
model = Sequential()
model.add(Input(shape=(T,)))
model.add(Dense(1))
```

```
model.compile(loss="mse", optimizer=Adam(learning_rate=0.2))
```

```
N = len(X) # length of training data
```

```
r = model.fit(X[:-N//2], Y[:-N//2], epochs=80,
              validation_data=(X[-N//2:], Y[-N//2:]),)
```



Wrong Way to Do Multi-Period Forecasting

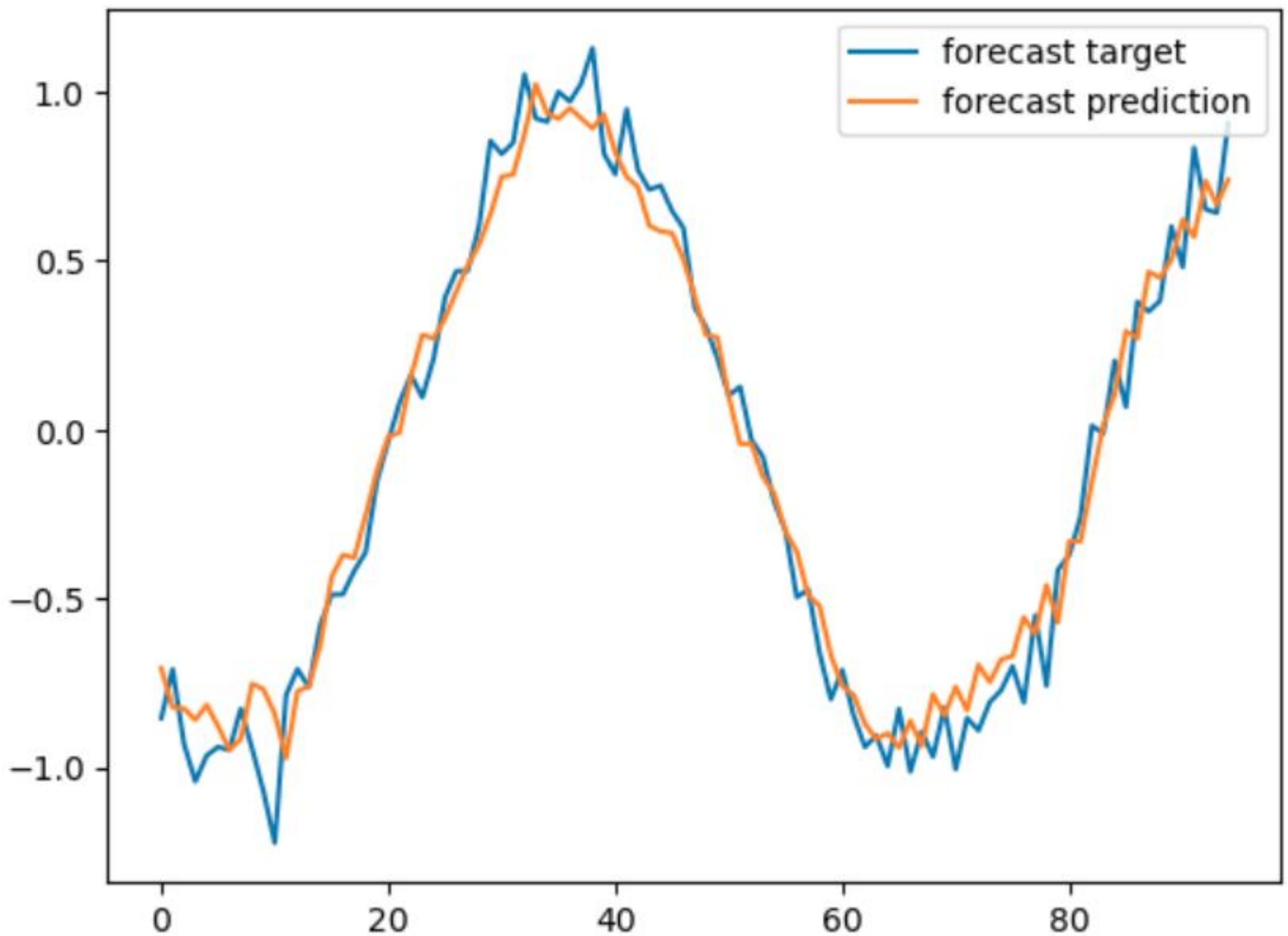
- In the code below we validate the model on the time series in the second half of the data set – the second 95 series

```
validation_targets = Y[-N//2:]
validation_predictions = []
small = 1e-2

# index of first validation input
i = -N//2

while len(validation_predictions) < len(validation_targets):
    x_series = X[i].reshape(1,-1)
    p = model.predict(x_series, verbose=0)[0,0]
    i = i + 1
    validation_predictions.append(p+small)
```

- For each we pull out the 10-period time series and just predict the next value and append that to the prediction list

Wrong-Way Forecasting Looks Good !

- Every prediction looks good, no divergence as we go forward



Correct way to Forecast

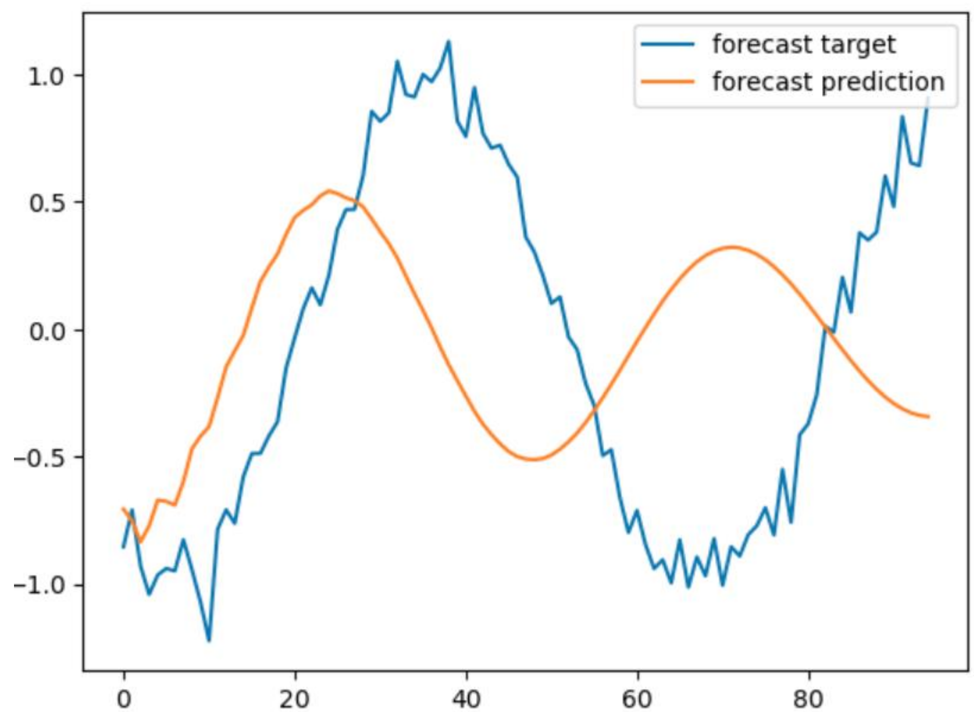
- We take first validation time series and extend that using the prediction in the next time series

```
validation_targets = Y[-N//2:]
validation_predictions = []

# index of first validation input
last_x = X[-N//2] # 1D array of length T

while len(validation_predictions) < len(validation_targets):
    x_series = last_x.reshape(1,-1)
    p = model.predict(x_series, verbose=0)[0,0]
    validation_predictions.append(p+small)
    # update time series for next prediction
    last_x = np.roll(last_x, -1)
    last_x[-1] = p # insert our prediction at the end
```

- We only use one of the time series here, but it shows you how to test the forecasting ability of a model correctly

Correct Forecasting Approach

- This is much worse – we do get a sinusoidal shape, but the phase is out, the frequency is ok, but the amplitude is also not correct



- But this is the correct test for multi-period forecasting
- How can we improve the model ? More complex ANN ?

Motivation for RNNs

- We've used neural networks to learn from labelled training sets
- But in many cases, the order of the data matters
- We want to be able to model Sequence data
- For example - word ordering matters - it affects our understanding of the sentence and as we process the meaning (output) that affects our understanding of what follows
- The aim of Recurrent Neural Networks is to capture order-dependent information
- The hope is that it will improve the accuracy of our predictions
- What sort of problems are RNNs used for ?

Simple RNN

The Idea of RNNs

- Make the output of a neuron not a single calculation but have a loop over the input time series
- Make the output at each time step a function of the past inputs
- That function should be like a standard neural network function that takes in all of the inputs and using a weights matrix **W** it outputs a value that is received at the output neuron
- Then an activation function is applied to the final value to give the final output / prediction

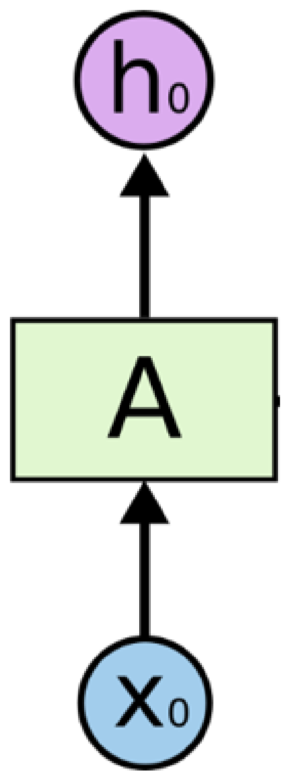
Basics: The Feedforward Neuron we already know

- So far our NNs are feedforward - activation flows in one direction
- An input X_0 –a number or a vector - passes through weights, is aggregated and passed via an activation function **A** to give h_0

<u>The</u>

<u>Feedforward</u>

<u>Neuron</u>

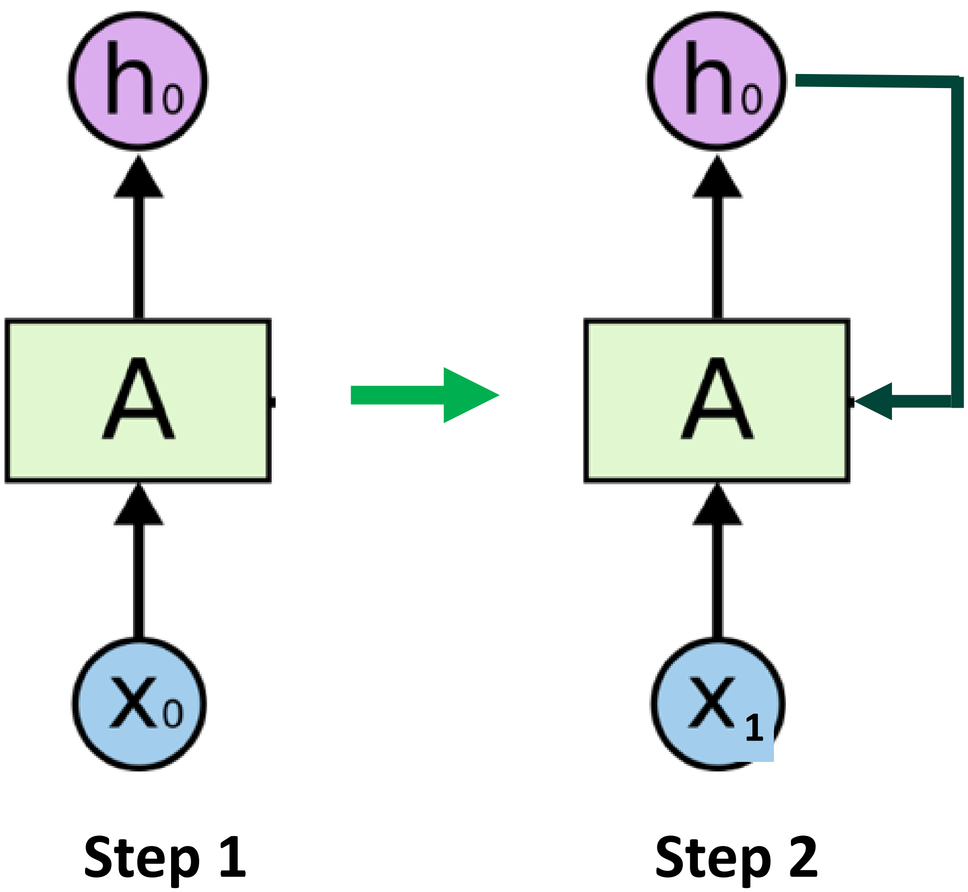


Initial input

X_0 combines with a weight vector and the aggregate z is passed into activation function A to output h_0

The Recurrent Neuron has an Output going Back

- Input X_0 passes through a set of weights, is aggregated and passed via an activation function to give h_0 which then goes back

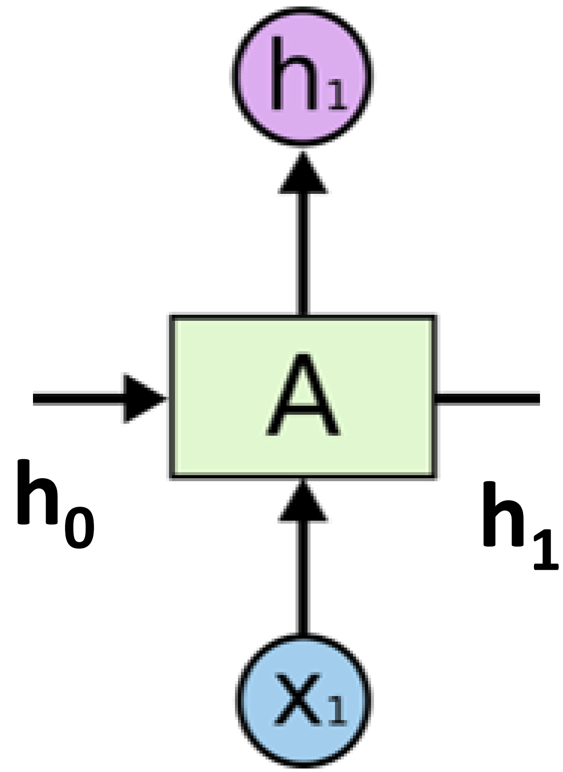


h_0 is the next output calculated by multiplying X_0 by a weight and adding a bias and it then goes back in along with the next input X_1

Inside the Single RNN cell after Each Time Step

- Any time after time zero there are **two inputs** to an RNN cell
  - First is x_t - the next sequence member
  - The other is h_{t-1}
  - This is the hidden state of the RNN cell
  - This was set on the previous iteration
- There is one output:
  - This is the value of h_t
  - It is calculated using weights w_x and w_h

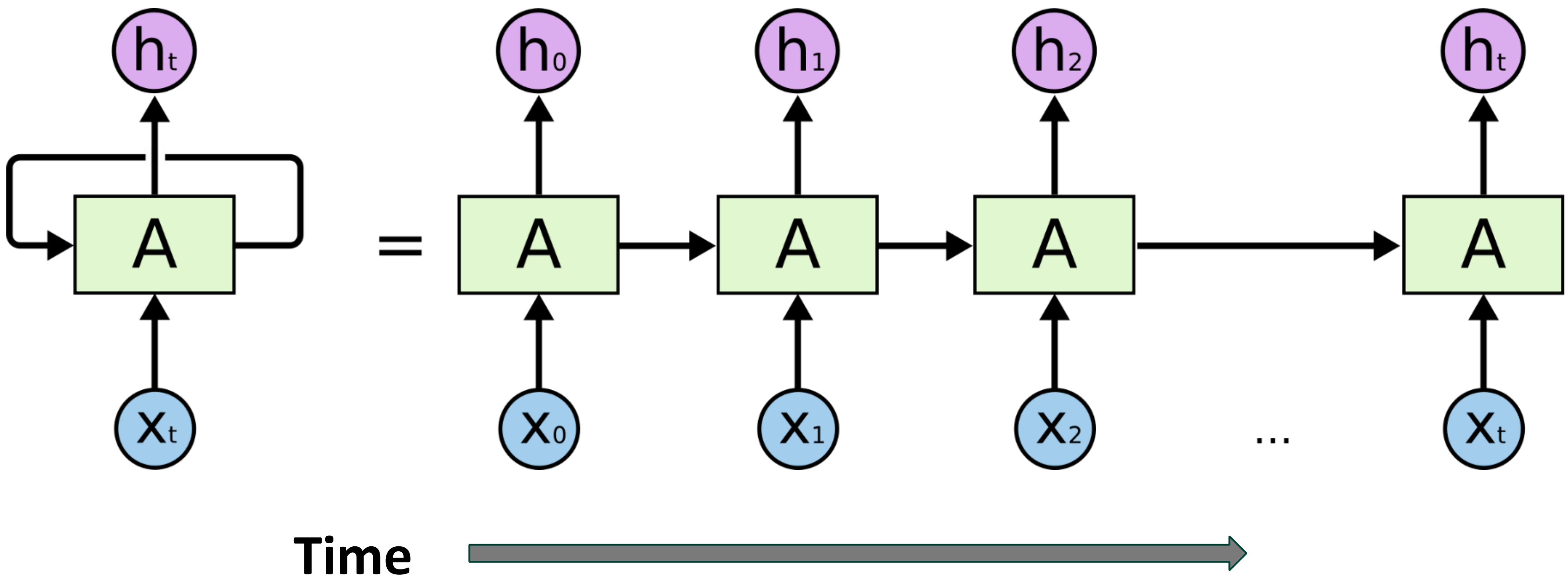
$$h_t = f\left(w_x x_t + w_h h_{t-1} + b\right)$$



- Here f is the activation function – usually a **tanh**
- Weights learn the importance of past versus the present

A Simple RNN – A cell iterates through time

- An RNN cell iterates the inputs through time – we draw this as



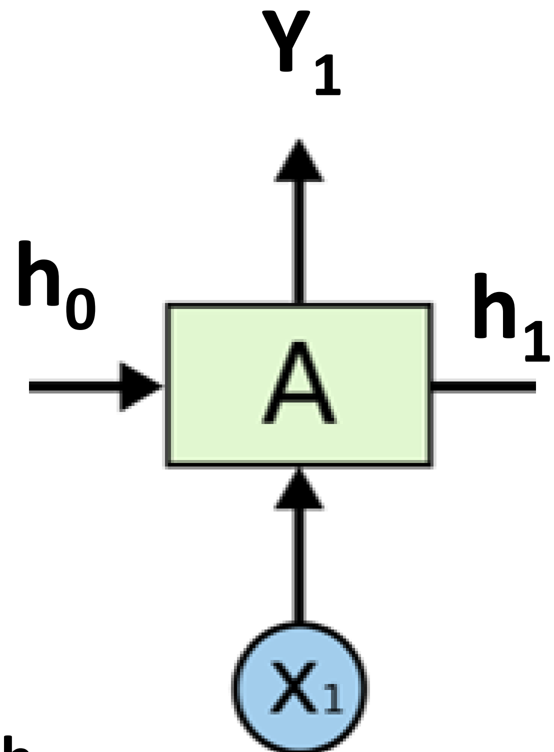
- This is the **same cell being self-iterated** or "unrolled"
- At the end it has generated a sequence h_0, h_1, ..., h_t
- We may use the entire sequence or just the final value

Variations on the Single RNN cell

- In some RNN cells the output is not the same as the value of h - h is an "internal" state
- In this case the output is called y_{t+1}
- It is a function of h_{t+1} and weight w_y
- Often it passes through another function
- In this case we would have

$$h_t = f\left(w_x x_t + w_h h_{t-1} + b_h\right)$$

$$y_t = g\left(w_y h_t + b_y\right)$$



- **For simplicity I will only consider an RNN with**

$$y_t = h_t$$

The Single Layer Simple RNN – How it works …

- We can generalise the RNN architecture to a layer of h cells
- Each takes in a sequence of data **X** and outputs a sequence **y**
- $\mathbf{W}_{xh}$ is the matrix of connections from input to hidden cells
- It has a size n x h where n is the number of input features
- $\mathbf{W}_{hh}$ is the matrix of connections from hidden-to-hidden cells
- It has a size h x h where n is the number of input features
- **b** is the h x 1 vector of bias terms for each hidden cell
- The output vector across all hidden units is given by

$$\underset{h \times 1}{\mathbf{y}_t} = f\left(\underset{h \times n}{\mathbf{W}_x^T} \cdot \underset{n \times 1}{\mathbf{X}_t} \cdot + \underset{h \times h}{\mathbf{W}_y^T} \cdot \underset{h \times 1}{\mathbf{h}_{t-1}} + \underset{h \times 1}{\mathbf{b}}\right)$$

The Single Layer RNN

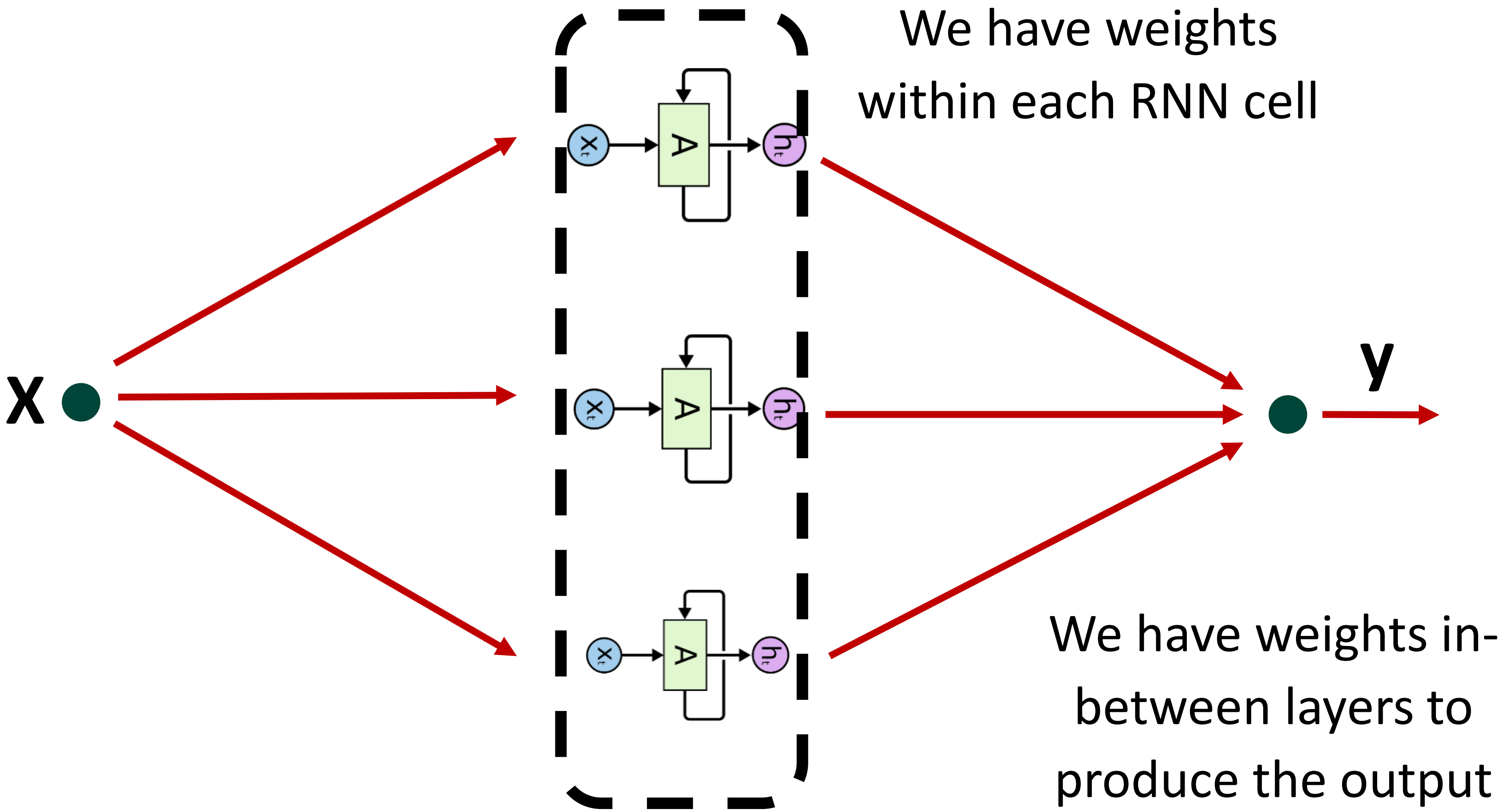
- This equation is repeatedly "unrolled" over the rows (number of time steps T) of input matrix **X**

$$\mathbf{y}_t = f\left(\mathbf{W}_x^T \cdot \mathbf{X}_t \cdot + \mathbf{W}_y^T \cdot \mathbf{h}_{t-1} + \mathbf{b}\right)$$

- T has no impact on the number of weights
- At the end we can (for each RNN unit) either
  - Output the final output value of the sequence y_T
  - Output the full sequence vector **y**
- We will only use the last value if for example this is the final prediction
- We will use the full sequence if we are to pass the output to another RNN layer or we are doing multi-period prediction

A Layer of RNNs

- X is the input - The RNN layer unrolls – Y is the weighted output



Return Sequences - The Multiple Layer RNN

- When we have multiple layers of RNN cells then we let each layer process the sequence and then output the resulting sequence
- This is then passed on to the next hidden RNN layer
- You might just want to output the final value to the next layer
- In TensorFlow you use the flag

return_sequences = True

to control this

Parameters in a Single Layer RNN

- How many parameters are there in a single layer RN ?

$$(\text{Input size} \times \text{Num Units}) + (\text{Units} \times \text{Units}) + \text{Units}$$

- Breakdown as follows:
  - **Input to Hidden Weights (WW):**
    - Number of parameters: Input size × Units
  - **Hidden to Hidden Weights (UU):**
    - Number of parameters: Units × Units
  - **Biases (bb):**
    - Number of parameters: Units

Consider this example

```
# we just pass last value of sequence to next layer
model = keras.models.Sequential()
model.add(Input(shape=[None, 1]))
model.add(SimpleRNN(20, return_sequences=True))
model.add(keras.layers.Dense(1))
```

```
model.summary()
```

```
Model: "sequential_9"
```

Layer (type)	Output Shape	Param #
simple_rnn_7 (SimpleRNN)	(None, None, 20)	440
dense_8 (Dense)	(None, None, 1)	21

```
Total params: 461 (1.80 KB)
Trainable params: 461 (1.80 KB)
Non-trainable params: 0 (0.00 B)
```

1 input to 20 RNNs Weights = 20

20 RNNs to 20 RNNs Weights = 400

20 RNN biases = 20

Total = 440

Implementing our own RNN Layer in Python

- Consider the underlying Python (untested pseudo) code for an **RNNLayer** class with n RNN cells in the layer

```
class RNNLayer:

    def __init__(self, n, wx, wh): # wx and wh are vectors of length n
        self.h = np.zeros(n)
        self.wx = wx
        self.wh = wh

    def step(self, x):

        # update the hidden state using a tanh activation function
        self.h = np.tanh(np.dot(self.wh, self.h) + np.dot(self.wx, x))
        # compute the output vector – one for each neuron
        return self.h
```

Unrolling the RNN Layer over T time steps

- Consider unrolling the RNNLayer
- We have input vector x, and two weight vectors W_{hh} and W_{xh}
- We iterate the T time steps

```
h1 = rnn.step(x1)
h2 = rnn.step(x2)
…
hT = rnn.step(xT)
```

- Because the hidden state is held internally and used in each step to compute the next value of h, we have a "memory" cell
- It remembers the sequence and is aware that the input series X has a specific order
- This architecture is fundamentally different from feedforward NN

Training an RNN

- Training RNNs is simple
- Just think of each iteration of the RNN layer as a new layer of the neural network
- We can therefore use Backpropagation to train the connections

Batches and Time Series Data

- When we want to predict the final value of a time series

[5, 7, 9, 12, 18, 28]

- We need to split this and pass in the input sequence

[5, 7, 9, 12, 18]

- And then pass in the label for this which is **[28]**
- We will then train the RNN to predict this next value
- In practice we only train the RNN on a fixed length sequence
- If we choose a sequence length of 3 – our training data of X, y is

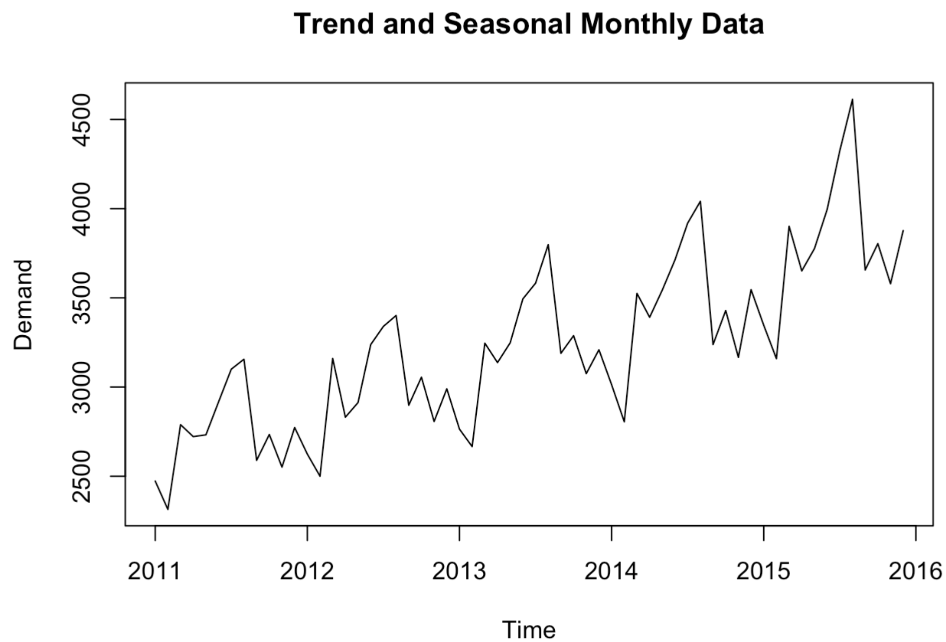
[5, 7, 9] -> [12]

[7, 9, 12] -> [18]

[9, 12, 18] -> [28]

Choice of Sequence Length

- The **sequence length** should be determined by the data series
- It should be long enough to identify any patterns in the data



- If the data is seasonal – i.e. repeats annually, **then the sequence length should be longer than a year**
- You can make this a hyper-parameter

Predicting One Period into the Future

- We start by predicting one time step into the future
- Suppose our time series is

[5, 7, 9, 12, 18, 28]

- Use a sequence length of 3 periods to **predict one period ahead**
- Our RNN predicts

[5, 7, 9] -> [13]

- Then we predict the next period

[7, 9, 12] -> [19]

- Going out again one more period

[9, 12, 18] -> [27]

- We train the RNN network to close the prediction error

Predicting Multiple Periods into the Future

- We may decide to predict multiple periods into the future

[5, 7, 9, 12, 18, 28]

- Let's use a sequence length of 3 to predict 3 time periods ahead

[5, 7, 9] -> [13]

- Then we predict the next period using the previous prediction

[7, 9, 13] -> [20]

- Once again the prediction is not correct – made worse by the fact that we **are predicting based on a prediction!**
- Going out again one more period

[9, 13, 20] -> [30]

- We have predicted three periods ahead – a tougher task!
- We train the network to close the prediction error

Batch Sizes

- When supplying data to the network we need to organise the sequence data in batches
- **Backpropagation** is done for one batch at a time
- We need an input sequence and an output label for each training example in the batch
- Suppose our data series is **[5, 7, 9, 12, 18, 28, 40, 63, …..]**
- A sequence length of 3 and a batch size of 4 would give the following as a single batch

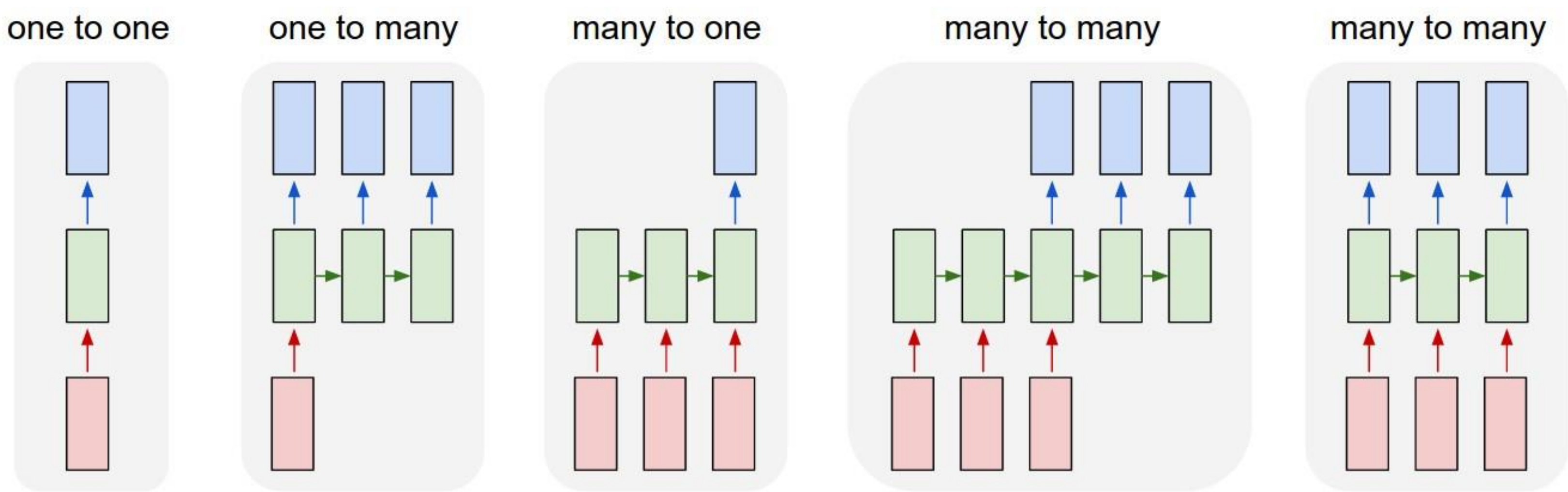
[5, 7, 9], [12]

[7, 9, 12], [18]

[9, 12, 18], [28]

[12, 18, 28], [40]

The RNN can take several Possible Structures



- Sequence to Sequence
  - Pass in a time series and get the shifted time series
- Sequence to Value or Vector
  - Pass in a series of words from sentence - get sentiment score
- Vector to Sequence
  - Pass in an image matrix or vector and get back a sequence

Case Study: Predicting a Noisy Sine Wave

Generating a Noisy Sine Wave

- I want to generate a single mini-batch of **batch_size** examples
- **n_steps** is the length of the sequence of each batch
- The NN will want an input matrix of size

n_batches x batch_size x n_steps

```
def generate_time_series(batch_size, n_steps):

            … see on next slide

    return …
```

- This function will return one batch
- So it returns a matrix of size **1 x batch_size x n_steps**

How to Generate a Noisy Sine Wave

```
def generate_time_series(batch_size, n_steps):
    freq1, freq2, offsets1, offsets2 = np.random.rand(4, batch_size, 1)
    time = np.linspace(0, 1, n_steps)
    series = 0.5 * np.sin((time - offsets1) * (freq1 * 10 + 10))  #   wave 1
    series += 0.2 * np.sin((time - offsets2) * (freq2 * 20 + 20)) # + wave 2
    series += 0.1 * (np.random.rand(batch_size, n_steps) - 0.5)   # + noise
    return series[..., np.newaxis].astype(np.float32)
```

- First generate 4 uniform randoms for frequencies and offsets
- Generate time vector with n_steps out to time 1 year
- Compute 2 sine functions using the different frequencies and offsets which we add using different weights
- We add on a noise using random uniforms zeroed at 0
- We add a dimension and convert the resulting series to float32

Calling the Noisy Sine Wave Generator

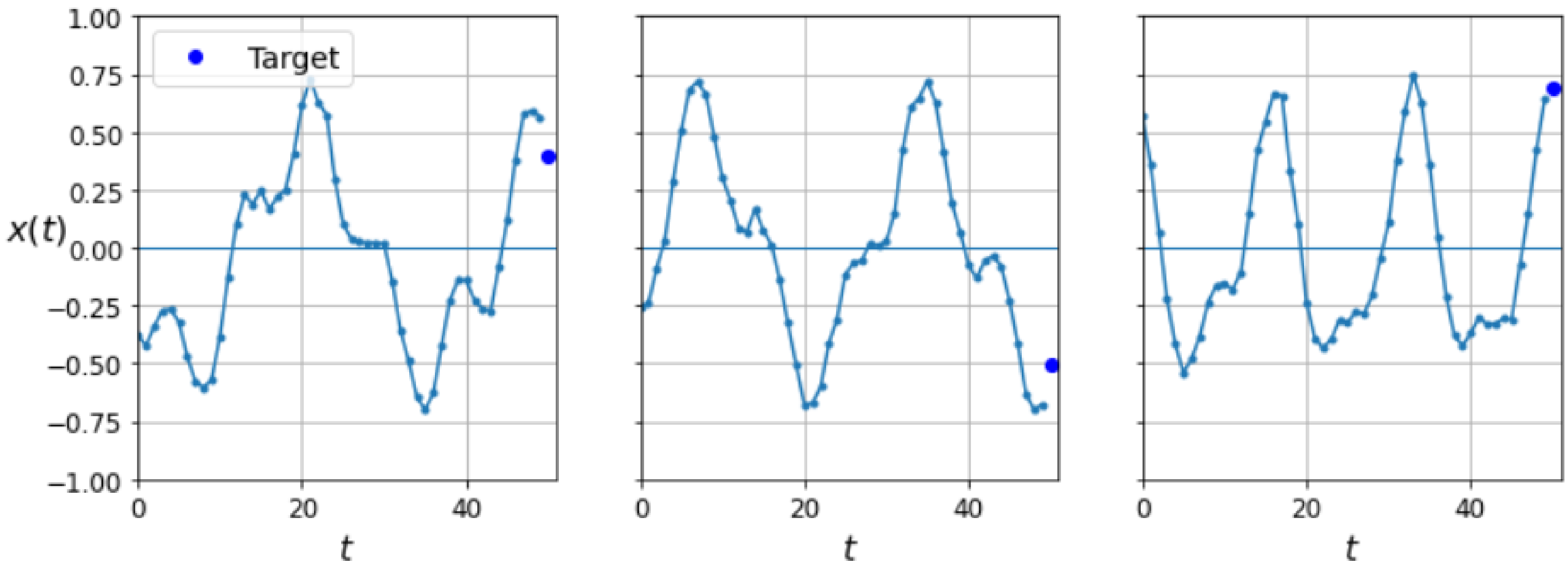
- We call the noisy sine wave single batch generator as follows

```
n_steps = 50
batch_size  = 10000
series = generate_time_series(batch_size, n_steps + 1)
X_train, y_train = series[:7000, :n_steps], series[:7000, -1]
X_valid, y_valid = series[7000:9000, :n_steps], series[7000:9000, -1]
X_test, y_test = series[9000:, :n_steps], series[9000:, -1]
```

- I generate **one** batch of 10K samples, each of 51 time-steps
- X_train has the first 50 elements of the sequence
- Y_train is the **final element** of the sequence – to be predicted
- I then split this into a training, validation and a test set
- The training, validation and test sets have 7K, 2K and 1K samples

The Time Series and the Prediction

- I am going to predict a noisy Sine Wave – see 3 examples below
- **The blue dot is the next value – the target to be predicted by the RNN**



- There is periodicity at different frequencies and randomness
- Not an easy task !

I predict we will do the following:

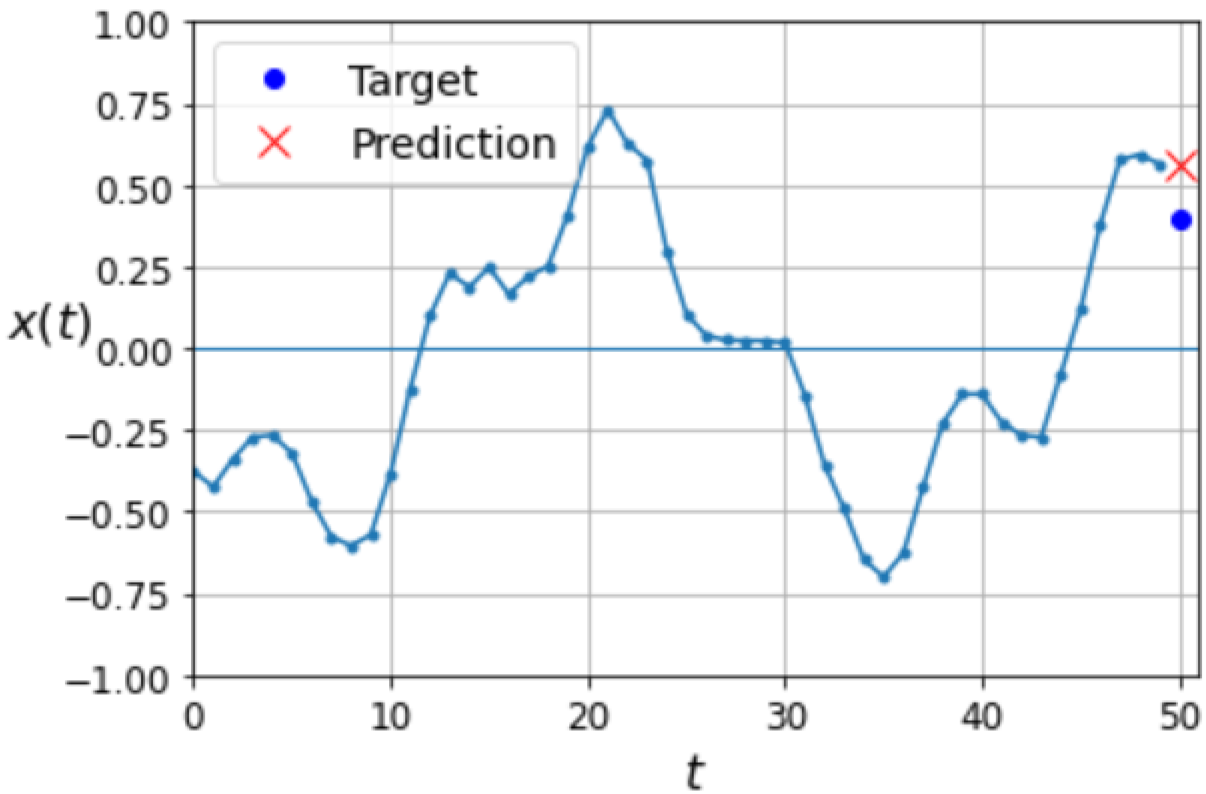
- Prediction of the next value
  - Use the previous value
  - Using linear deep learning feedforward ANN
  - Use a simple RNN to predict a sequence one value at a time
- Predict the entire sequence in one pass using an RNN
- Do this using an LSTM and then a GRU

First Approach – Predict using the Previous Value

- Calculate the mean squared error predicting using last value of x

```
from tensorflow import keras
y_pred = X_valid[:, -1]
np.mean(keras.losses.mean_squared_error(y_valid, y_pred))
```

```
0.020211367
```



Basic Feedforward NN as a Baseline

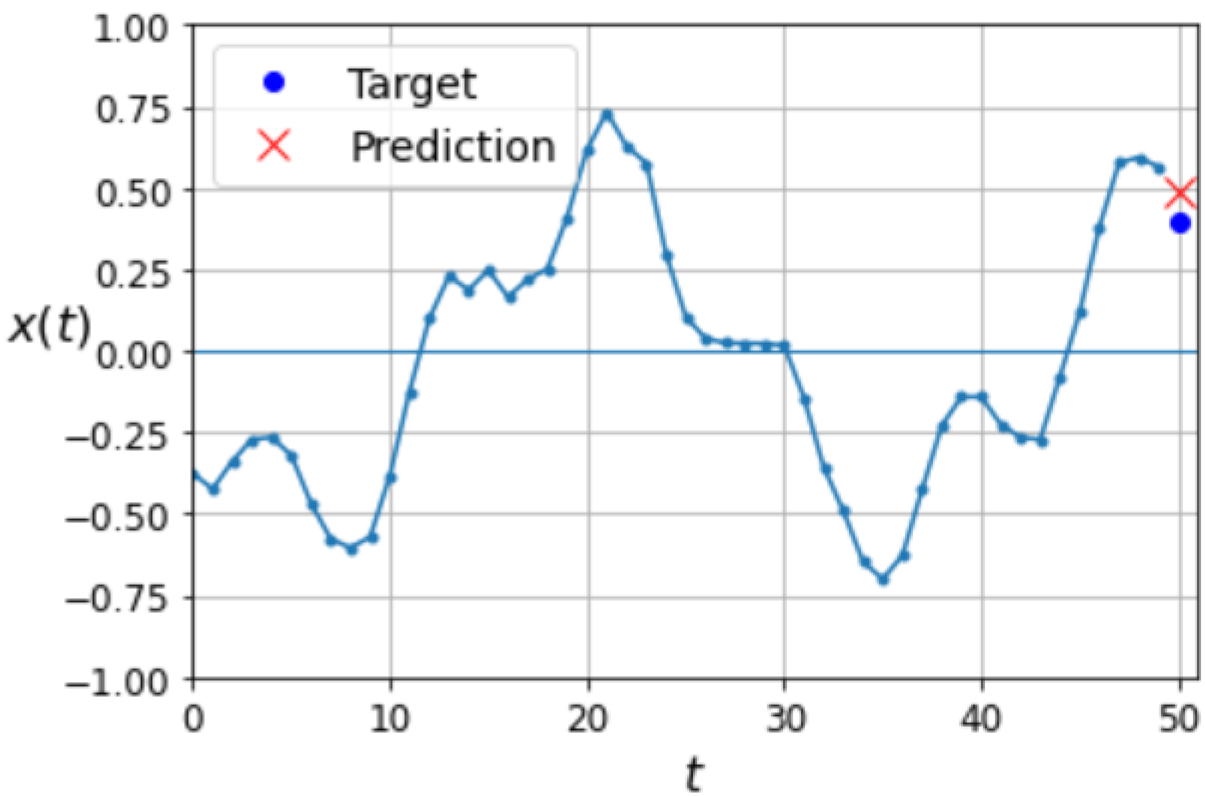
- We would like a reference NN to give us a sense of what is good
- We build a simple ANN with an input layer of 50 neurons
- This takes in the 50 historical prices
- I add in two layers of 50 neurons and a linear output neuron
- As it's a regression, I minimise the mean squared error

```
np.random.seed(42)
tf.random.set_seed(42)
model = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[50, 1]),
    keras.layers.Dense(50),
    keras.layers.Dense(50),
    keras.layers.Dense(1)
])
model.summary()
model.compile(loss="mse", optimizer="adam")
```

- This model has 5,151 parameters

Simple Fully Connected NN

- The Fully Connected NN trains quickly
- After 20 epochs the prediction loss is 0.00326
- The prediction error for this example looks OK
- It fails to anticipate the steepness of the drop in the time series



What does this NN know about a sequence ?

- A feedforward NN cannot know that **X** is an ordered sequence
- What it sees is an input set of 50 features
- We know that these consists of the function values **x**
- It always sees x[0] at input 1 and … x[49] at input 50
- It can "learn" that the prediction is usually close to x[49]
- It also learns about the different shapes of this curve
- As the shape keeps changing, this is much harder than deterministic function fitting
- Making the architecture "understand" about sequences and order may give the ANN an extra edge
- **In an RNN the idea is fundamentally different – we have 1 feature which is a sequence of length 50**

We repeat this with a Deep RNN

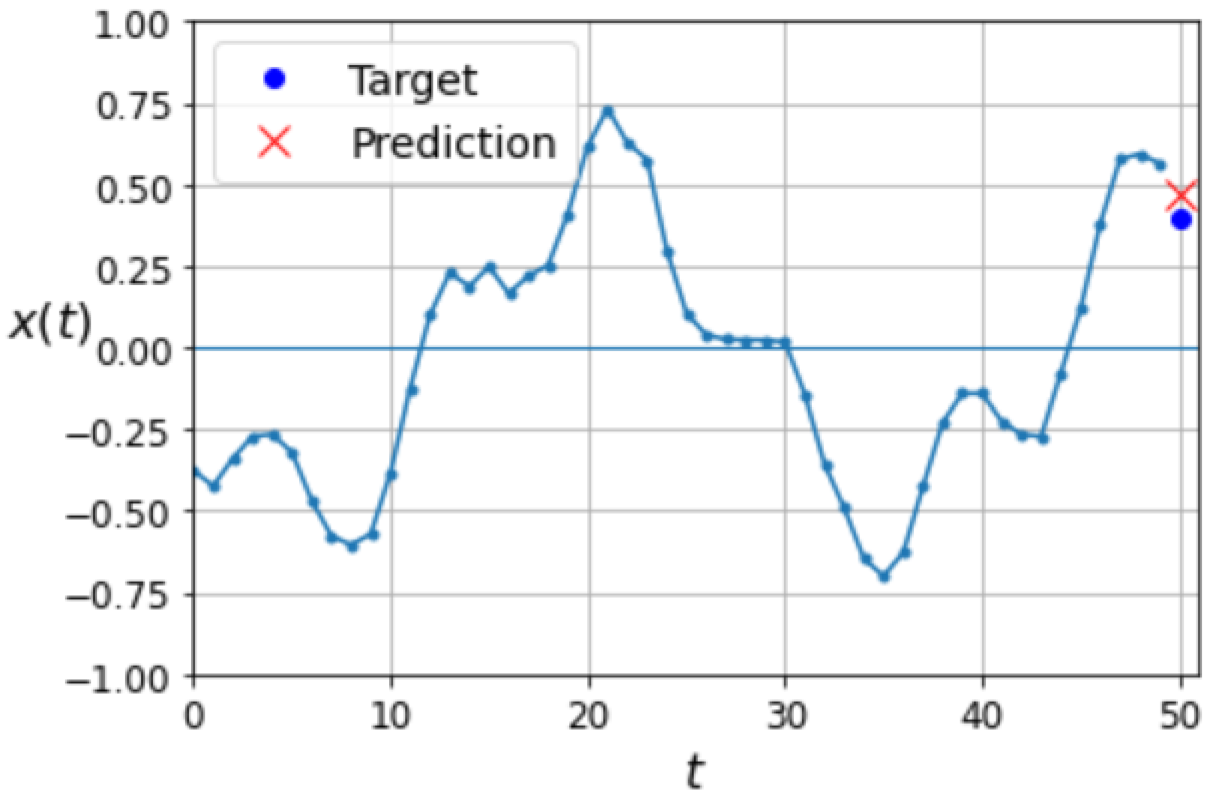
- We build a simple RNN with 2 hidden layers and 20 cells in each
- The input shape is a 2D array with an undefined number of rows and one column

```
model = keras.models.Sequential([
    keras.layers.SimpleRNN(20, return_sequences=True,
                           input_shape=[None, 1]),
    keras.layers.SimpleRNN(20),
    keras.layers.Dense(1)
])
```

- It is trained over the 7K series in the training set
- Set **return_sequences = True** to pass time series to next layer
- The output is a simple neuron – we're only interested in 1 value
- It is the last output of the sequence
- This model has 1,281 parameters – fewer than the feedforward

Deep RNN Performance

- After 20 epochs the prediction loss is 0.0026 – it's even better!
- The prediction error for this example looks much better too



- We did this with fewer parameters – fewer neurons in each layer

Predicting Multiple Periods Ahead

Predicting Several Periods Ahead

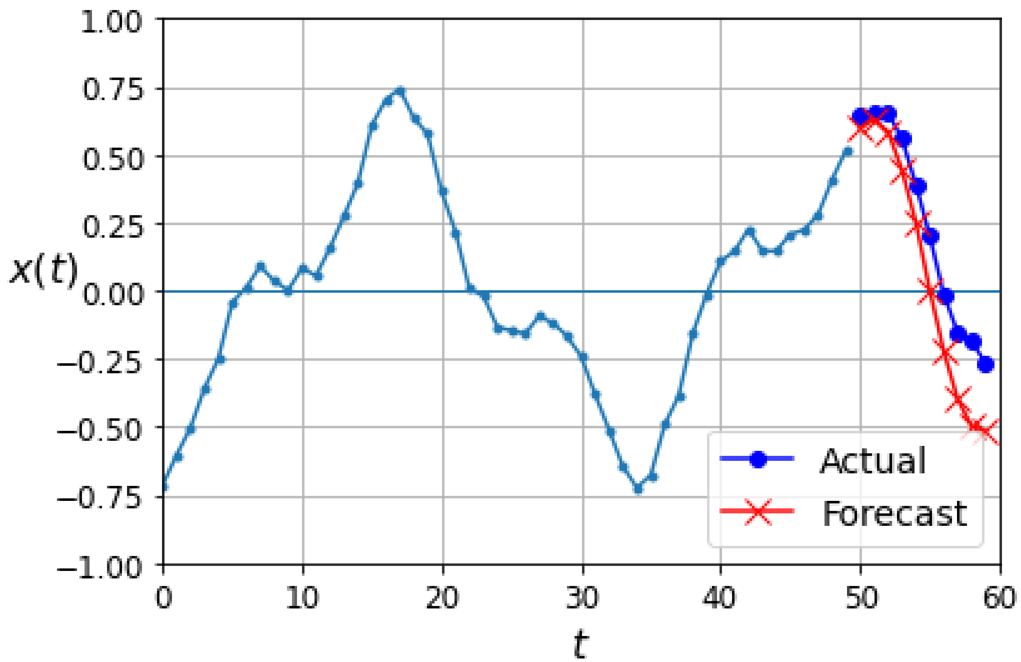
- Up until now we only look one period ahead
- Now we want to see if we can use the memory of the RNN **to predict several periods ahead**
- We can always take the models we have trained and ask them to do it. The algorithm would be:
  1. We predict the next value
  2. We add the next value onto the series
  3. We use the model to compute the next value
  4. Go to 2 until we have gone far enough forward
- This might work but it might not be as good as actually training it to do this specifically

One-Period Model Predicting Multiple Periods

- See how well our simple deep RNN predicts multiple periods
- First, we generate a series **with 10 extra values on the end**
- We loop over 10 time-steps and use the existing one period model to predict the next value
- We then add the prediction to the time series and use that for the next period

```
series = generate_time_series(1, n_steps + 10)
X_new, Y_new = series[:, :n_steps], series[:, n_steps:]
X = X_new
for step_ahead in range(10):
    y_pred_one = model.predict(X[:, step_ahead:])[:, np.newaxis, :]
    X = np.concatenate([X, y_pred_one], axis=1)

Y_pred = X[:, n_steps:]
```

One Period Model – Multi-Period Prediction Results



- The result looks OK – the downward gradient is monotonic
- It overshoots a bit and then slows down correctly
- The error increases with time
- **Expected as we only trained it to learn one time step !!**

Idea: Have the Neural Network Output 10 Values!

- The input X_train data has 7,000 series from period 1:50
- And similar for the validation and test set

```
n_steps = 50
series = generate_time_series(10000, n_steps + 10)
X_train, Y_train = series[:7000, :n_steps], series[:7000, -10:, 0]
X_valid, Y_valid = series[7000:9000, :n_steps], series[7000:9000, -10:, 0]
X_test, Y_test = series[9000:, :n_steps], series[9000:, -10:, 0]
```

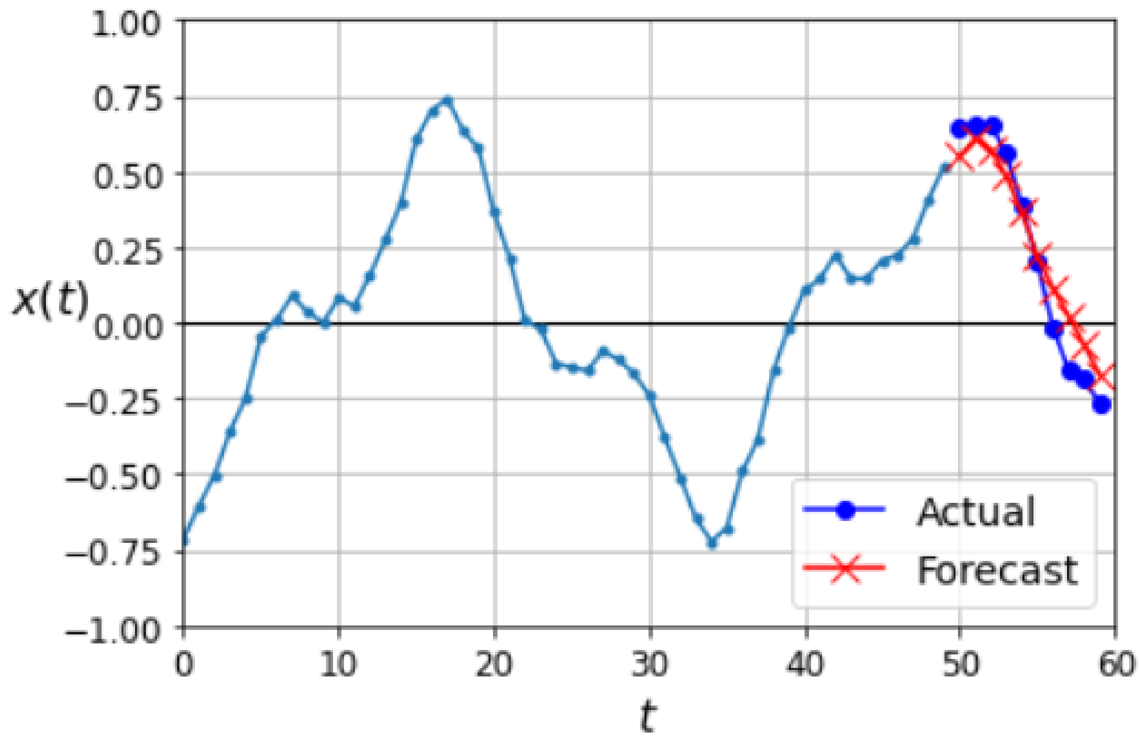
- We build a deep RNN with 10 outputs

```
model = keras.models.Sequential([
    keras.layers.SimpleRNN(20, return_sequences=True,
                           input_shape=[None, 1]),
    keras.layers.SimpleRNN(20),
    keras.layers.Dense(10)
])
```

- The model is trained to match the 10 neurons in the output layer to the next 10 sequences values and **return_sequences** is true from first to second layer

Multi-Period Prediction Results

- The quality of the prediction **looks better than single-period**
- It has the turning point and a similar slope and the error does not noticeably diverge with the number of periods



- But this is still not yet a sequence-to-sequence model

Sequence to Sequence Modelling

Sequence to Sequence Model

- Want a model that predicts next 10 time periods from the knowledge of what preceded it
- There will be an error (loss) for every time step not just the final time step and this should stabilize and speed up training
- For example, it forecasts periods 51-60 from period 1:50
- Need to set up the initial data sets as before with 10 extra times
- The input X_train data has 7000 series from period 1:50
- And similar for the validation and test set

```
n_steps = 50
series = generate_time_series(10000, n_steps + 10)
X_train = series[:7000, :n_steps]
X_valid = series[7000:9000, :n_steps]
X_test = series[9000:, :n_steps]
```

Generating the Label Data

- The Y data is what we need to predict – a list of next 10 values
- We need to generate this by iterating through the data series
- To manage the training data more easily than before, we create a Y array for the 10K time series, each with 50 vectors (for each time period) with each of length 10 (the prediction horizon)

```
Y = np.empty((10000, n_steps, 10))
for step_ahead in range(1, 10 + 1):
    Y[..., step_ahead - 1] = series[..., step_ahead:step_ahead + n_steps, 0]
Y_train = Y[:7000]
Y_valid = Y[7000:9000]
Y_test = Y[9000:]
```

- We loop over the prediction horizon (out to 10 time-periods)
- At each step_ahead we assign the corresponding series of values which start at the next time period and end 50 periods later

The Sequence-Sequence Deep RNN

- NN must output a sequence so set **return_sequences = True** into the final layer which must be able to manage a sequence
- The output layer needs to handle a sequence of values passing through a Dense layer - use a special Keras **TimeDistributed** layer

```
model = keras.models.Sequential([
    keras.layers.SimpleRNN(20, return_sequences=True, input_shape=[None, 1]),
    keras.layers.SimpleRNN(20, return_sequences=True),
    keras.layers.TimeDistributed(keras.layers.Dense(10))
])

model.summary()
```

```
Model: "sequential_14"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 simple_rnn_27 (SimpleRNN)   (None, None, 20)          440

 simple_rnn_28 (SimpleRNN)   (None, None, 20)          820

 time_distributed_2 (TimeDis  (None, None, 10)         210
 tributed)
```

Time Distributed Layer

- We have an RNN outputting the entire sequence of the sequence h after it has iterated through all the inputs it has received
- The output of each layer is the entire sequence – when **return_sequence = False**, only the final value is returned
- We then want to pass each element of the series through a Dense layer before outputting a single value
- In the previous example we see that it has 210 parameters
- There are 20 RNN neurons inputting to this layer
- Each is passing to 10 output neurons
- Hence the number of parameters is (20 + 1) x 10 = 210
- The same weights are applied to each element of the sequence - **TimeDistributed** is simply adding a loop over the time series

Evaluation Cost Function

- In the model the cost function used to perform back propagation is the mean squared error
- It looks across the sequence of predictions and the sequence of known values and calculates their MSE
- However, for prediction purposes, the number that interests us most is the **error of the final value of the 10-step prediction**
- We therefore define a custom metric that is reported during training **even if it is not the objective function**

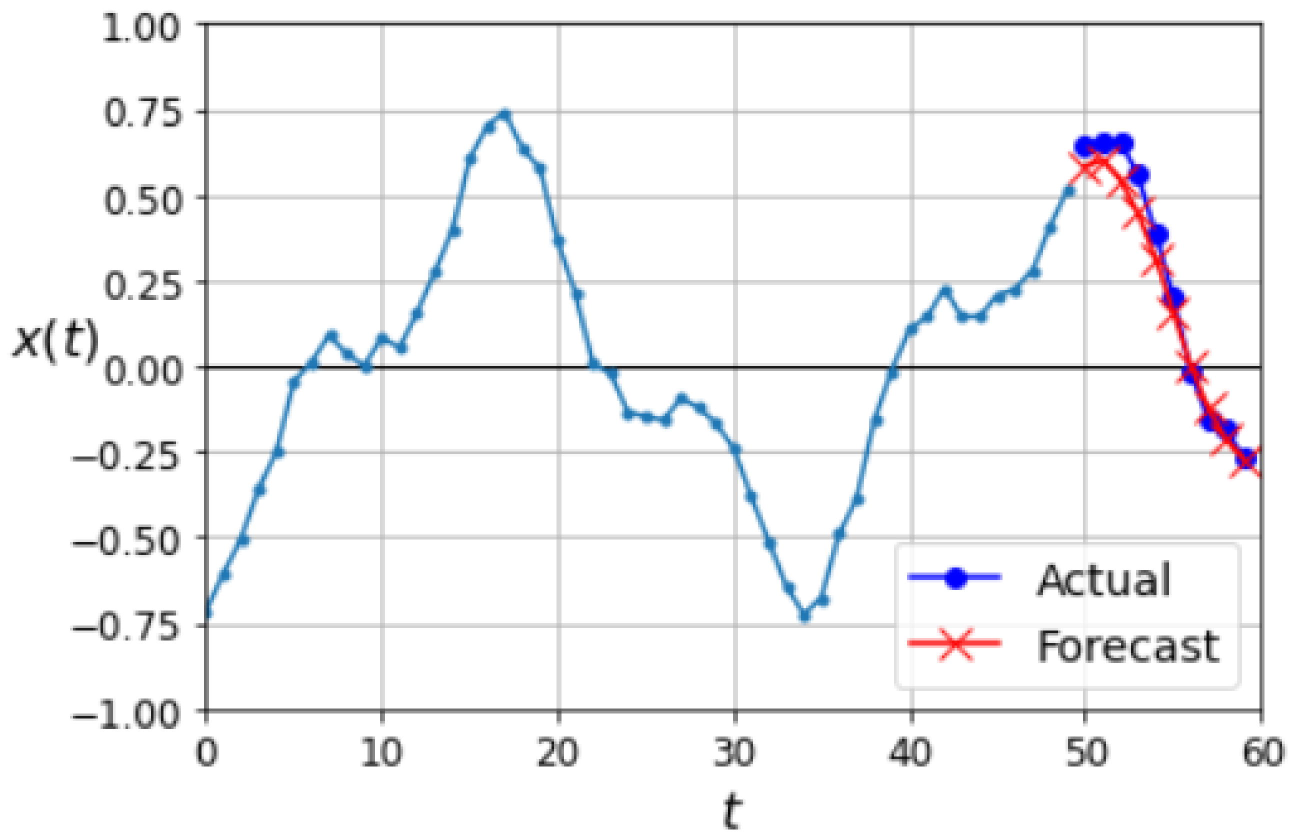
```
model.compile(loss="mse", optimizer=keras.optimizers.Adam(learning_rate=0.01),
              metrics=[last_time_step_mse])
```

- It is defined as

```
def last_time_step_mse(Y_true, Y_pred):
    return keras.metrics.mean_squared_error(Y_true[:, -1], Y_pred[:, -1])
```

Sequence to Sequence Results

- The resulting prediction is as follows

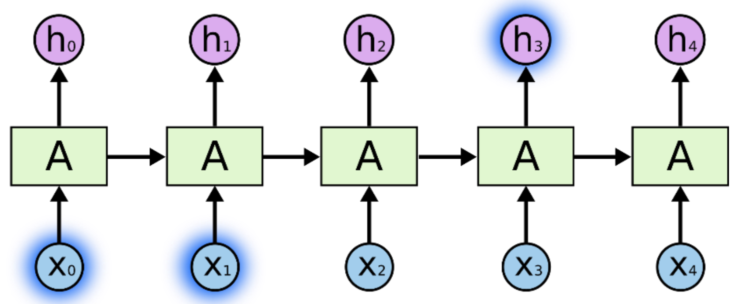


- Looks good ! The final period MSE was found to be 0.0069 and for the validation set it is 0.0090

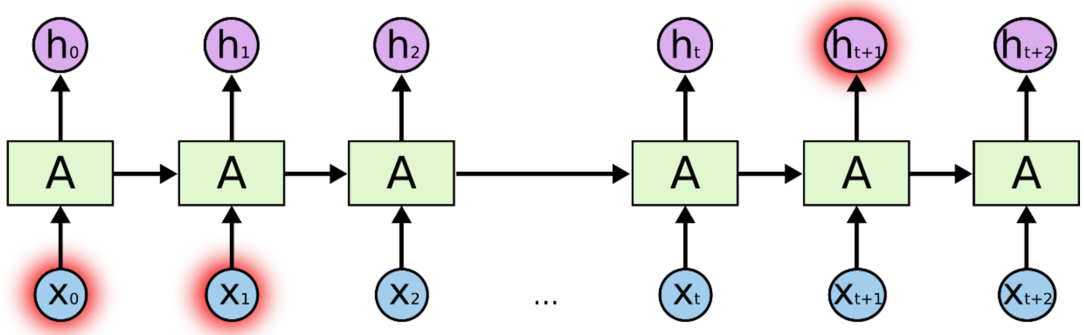
Long Short Term Memory Cells (LSTMs)

Long-Term Dependencies

- The point of RNN is for the sequence history to be remembered
- For example, if we are predicting words, then a word earlier in the same sentence may help us to predict the next word



- But if the word was 2-3 sentences ago then RNNs find it difficult to retain that information



Some problems with RNNs

- The vanishing gradients problems means that many iterations of the RNN will make it more likely that the neurons saturate
- This makes training hard when using backpropagation
- This limits the length of the memory
- A related issue is simply that early memories are overwritten by more recent memories
- This is where the LSTM comes in

Why we need LSTMs

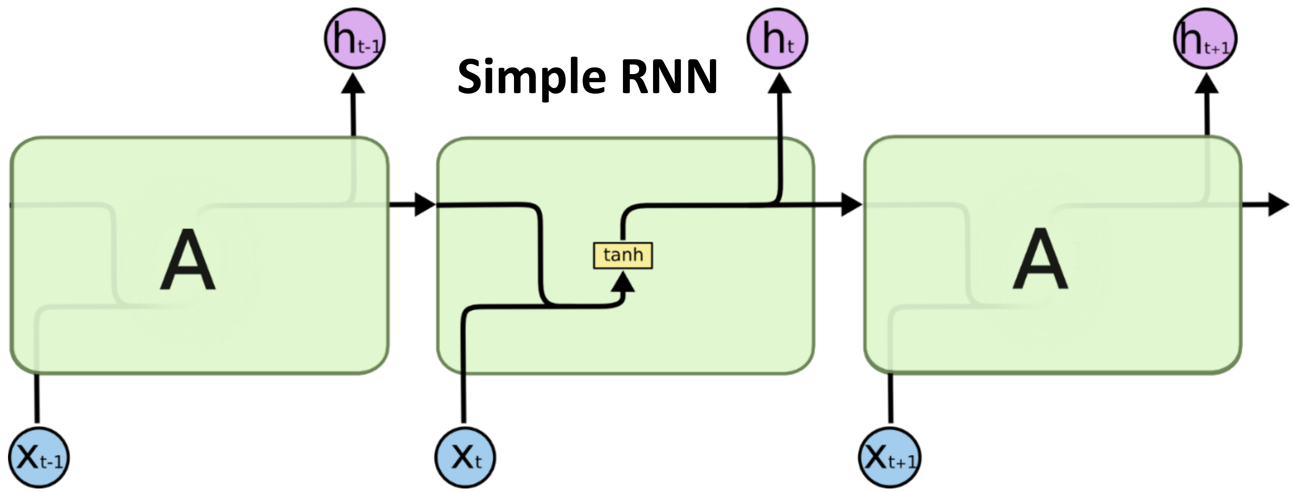
- If we have a long sequence and need a long chain of RNNs, we start to get problems with back propagation
- For sigmoidal activations we get derivatives tending to zero
- The accumulation of the chain rule of many products of small derivatives means that we end up with vanishing gradients
- We can also end up with exploding gradients – if the gradient is large and we multiply many of them, the result can explode
- Both problems prevent the NN from learning
- We can overcome this problem using
  - ReLu Activation functions
  - Gradient clipping
  - Long Short-Term Memory units (LSTMs)

Long Short-Term Memory (LSTM)

- An LSTM is an artificial recurrent neural network (RNN)
- As the name suggests, it has a long and a short-term memory
- LSTMs are good at making predictions based on lengthy time-series, e.g., market prices, sequences of words or game actions
- We find that they train and converge faster than basic RNNs
- LSTMs are quite complex to understand as they are composed of a cell, an input gate, an output gate and a forget gate
- They were developed to solve the vanishing gradients problem
- LSTMs were used in the core of the NN developed by DeepMind's AlphaStar to play Starcraft II in late 2018, beating a top pro.

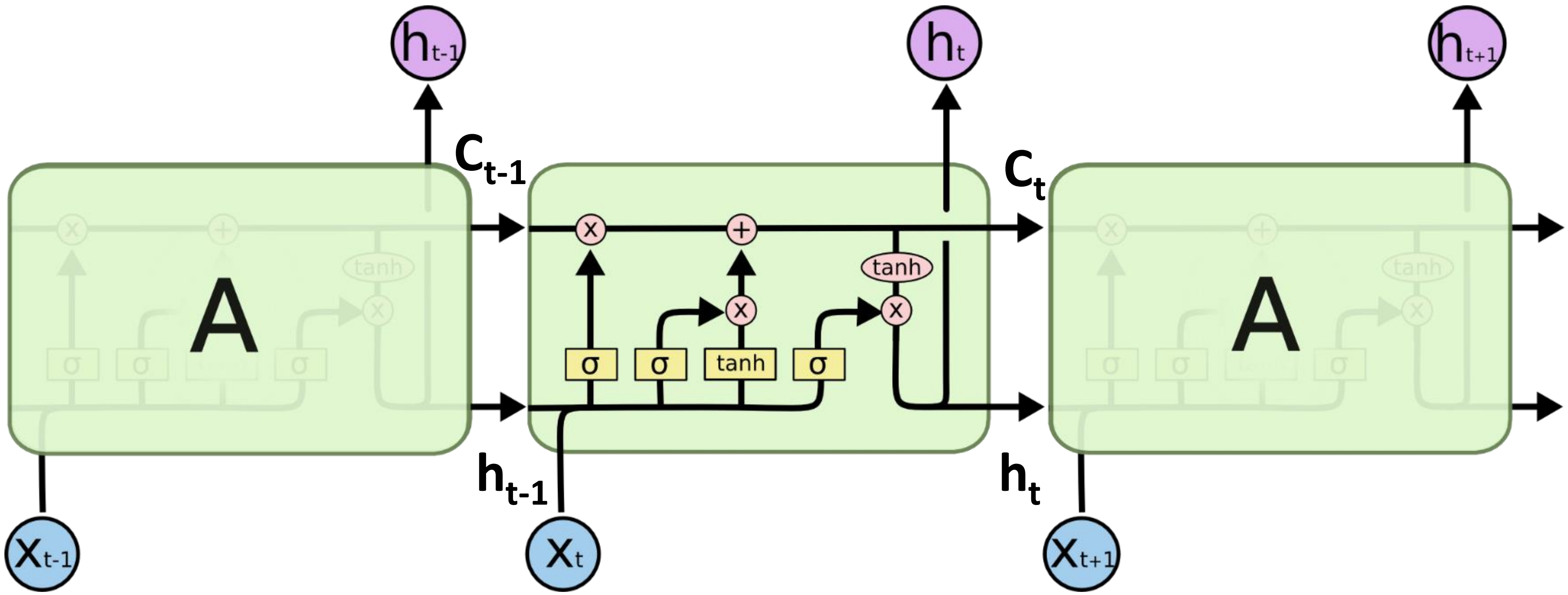
The LSTM Chain Solves this Problem

- From outside the LSTM looks just the same as an RNN unroll
- As before an input x enters and an output h appears



- However, internally the LSTM is far more complicated
- It maintains an internal "short-term" and "long-term" memory
- The long term memory carries information from before
- Gates (multiplying values by a number 0-1) control what is kept

Inside an LSTM

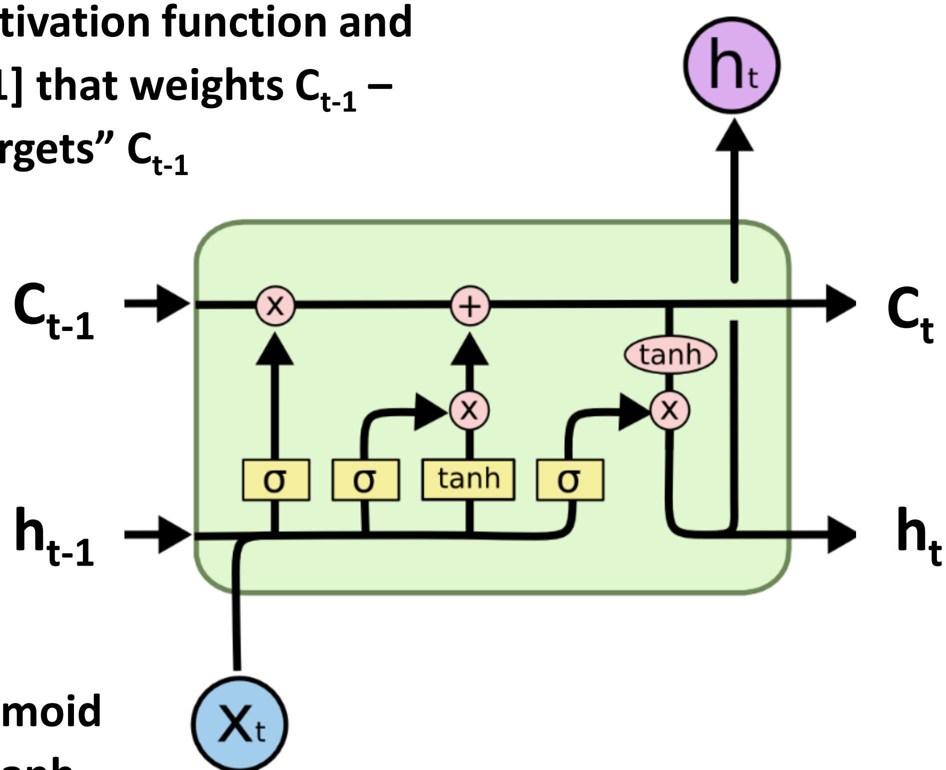


- At time step t, an LSTM cell has 1 input, 1 output and 2 states
  - **Input:** X_t
  - **Output**: h_t
  - **States:** LT memory C_{t-1}, ST memory h_{t-1}
- Gates control how much of the LT memory is kept/forgotten

The LSTM Mechanics

Step 1: FORGET GATE

h_{t-1} and x_t combine via weights, pass through a sigmoid activation function and output a number [0,1] that weights C_{t-1} – a value close to 0 "forgets" C_{t-1}



Step 3: OUTPUT GATE

h_{t-1} goes through a sigmoid (with weights) and multiplies the new LT state value after passing through a tanh (via weights) and this is the new ST output (hidden) state h_t

Step 2: UPDATE GATE

h_{t-1} goes through a sigmoid (with weights) and a tanh (via weights) and they multiply and update the new LT cell state C_t

Thankfully we don't have to code this up ourselves – it's all done in TensorFlow ;-)

We Repeat Sine Wave but with an LSTM

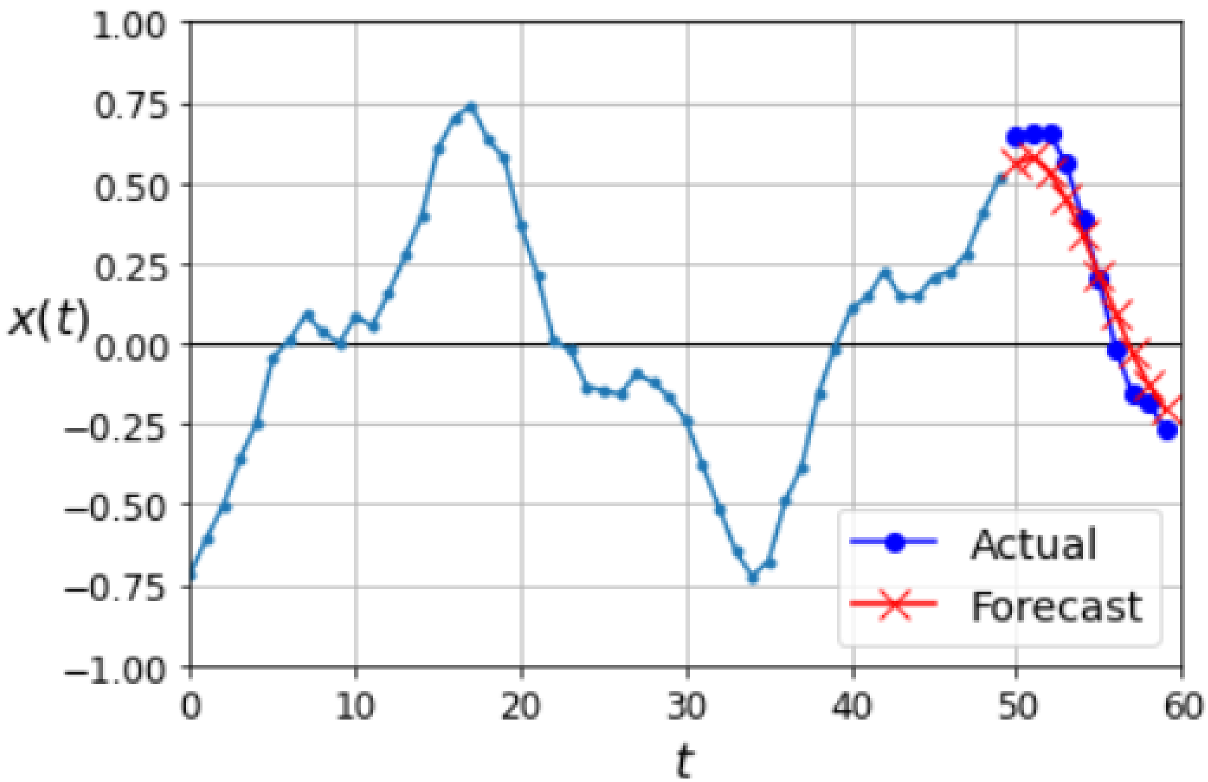
- We build a deep LSTM network
- You should be very familiar with the syntax now !

```
model = keras.models.Sequential([
    keras.layers.LSTM(20, return_sequences=True, input_shape=[None, 1]),
    keras.layers.LSTM(20, return_sequences=True),
    keras.layers.TimeDistributed(keras.layers.Dense(10))
])
```

- It has 20 LSTM cells in 2 hidden layers
- Once again, we are doing sequence-to-sequence, so each layer has **return_sequences** = True
- The output layer is a **TimeDistributed** layer wrapping a Dense layer with 10 outputs

Deep LSTM

- The Deep LSTM does well and has almost identical results to the Deep Sequence to Sequence RNN
- Its final period MSE is 0.0083 – not bad !

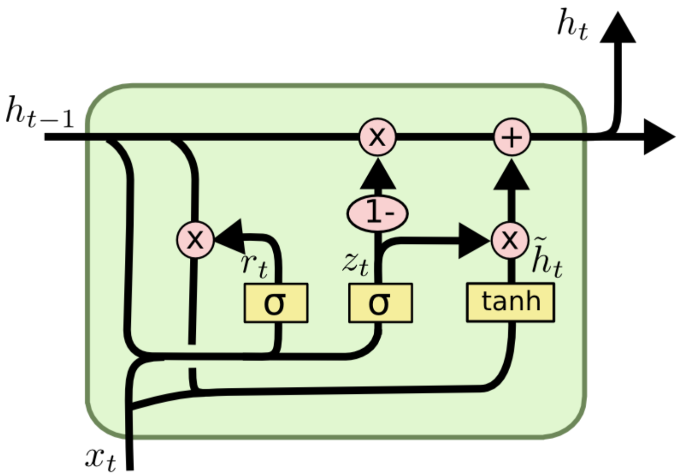


Number of Parameters

- An LSTM layer has trainable parameters for:
  - Input weights: Connecting inputs to LSTM units.
  - Hidden weights: Connecting LSTM units to each other (recurrent connections).
  - Biases: One bias per unit for each gate.
- Formula for LSTM parameters:
  - $\text{Parameters} = 4 \times ((\text{Input size} \times \text{Units}) + (\text{Units} \times \text{Units}) + \text{Units})$
- Explanation:
  - 4 = # of gates (input, forget, cell, and output gates).
  - Input size: Number of features in the input sequence
  - Units: Number of LSTM units.

The Architecture of the Gated Recurrent Unit (GRU)

- The Gated Recurrent Unit (GRU) was introduced by Cho et al (2014)



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

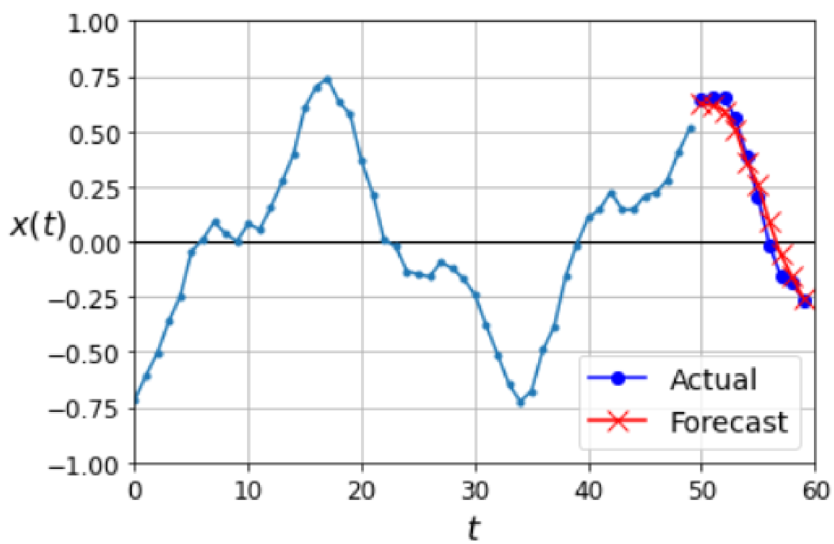
- Combines the input and forget gates into a single update gate
- This makes it simpler than the LSTM
- Growing in popularity as it's simpler but similar results to LSTM

The Special Layer

- At the end of my notebook, I showed a new layer
- It is a **Convolution Layer**, and you will learn about it next week

```
model = keras.models.Sequential([
    keras.layers.Conv1D(filters=20, kernel_size=4, strides=2,
                        padding="valid", input_shape=[None,1]),
    keras.layers.GRU(20, return_sequences=True),
    keras.layers.GRU(20, return_sequences=True),
    keras.layers.TimeDistributed(keras.layers.Dense(10))
])
```

- It works well ! I get a last time step MSE of 0.0081



Test Case: Predicting Seasonal Data

Predict Clothing and Clothing Accessory Stores

- We predict a seasonal time series from FRED

We Load the Data from FRED and Remove 2020

- The data can be easily imported

```
import pandas_datareader.data as web
df = web.DataReader("MRTSSM448USN", 'fred')
```

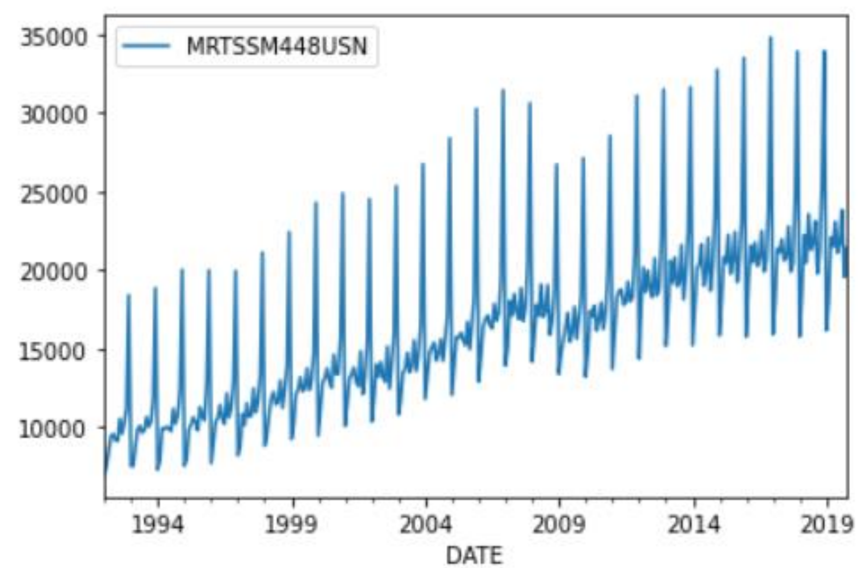
```
df.head(5)
```

DATE	MRTSSM448USN
2017-03-01	21332
2017-04-01	21143
2017-05-01	21945
2017-06-01	20618
2017-07-01	20795

```
df = df[0:334]
```

Let's look at the data series we want to predict

```
df.plot();
```



- It has monthly sales values starting in March 2017
- We will cut off 2020 – predicting Covid is too complicated !

Preparing the Data

- We use the last 18 months as test data
- We also do a min-max scaler – we calculate the scaling on the training data and apply it to both training and test data

```
n_train = df.shape[0] - 18
```

```
df_train = df[0:n_train]
df_test  = df[n_train:]
```

```
len(df_train), len(df_test)
```

```
(39, 18)
```

```
from sklearn.preprocessing import MinMaxScaler
scaler = MinMaxScaler()
scaler.fit(df_train)
```

```
MinMaxScaler()
```

```
df_train_scaled = scaler.transform(df_train)
df_test_scaled = scaler.transform(df_test)
```

Time Series Generator

- To manage the time series and batches, I use the Keras TimeseriesGenerator object
- Look in the Keras documentation to see how this works
- I also set a batch length of 12 – fine as it's less than the test set of 18 months

```
from tensorflow.keras.preprocessing.sequence import TimeseriesGenerator
```

```
batch_length = 12 # Must be less than the size of the test set
batch_size = 1 # Number of batches in each training iteration

generator = TimeseriesGenerator(df_train_scaled,
                                df_train_scaled,
                                length=batch_length,
                                batch_size=batch_size)
```

Build an LSTM Model

- The architecture is very familiar by now – one neuron in the output layer

```
n_features = 1 # it's a time series
```

```
model = Sequential()
model.add(LSTM(100, activation="relu", input_shape=(batch_length, n_features)))
model.add(Dense(1))
model.compile(optimizer='adam', loss='mse')
```

```
model.summary()
```

```
Model: "sequential_8"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 lstm_7 (LSTM)               (None, 100)               40800

 dense_7 (Dense)             (None, 1)                 101

=================================================================
Total params: 40,901
Trainable params: 40,901
Non-trainable params: 0
```

Early Stopping

- We would like the training to stop if it is not improving
- We can do this using an EarlyStopping callback
- This stops if a defined metric does not improve

```
from tensorflow.keras.callbacks import EarlyStopping
early_stop = EarlyStopping(monitor="val_loss", patience = 2)
```

- Here the training stops if the validation loss does not improve after 2 epochs of training, and we apply it as follows

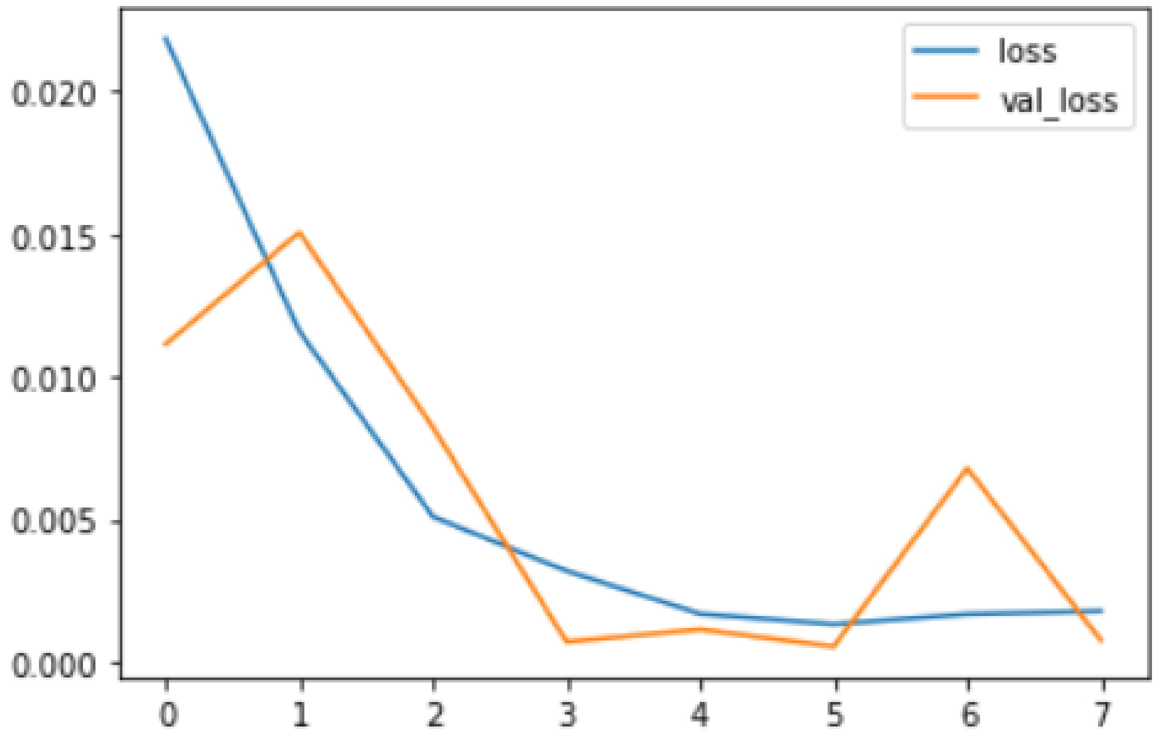
```
model.fit(generator, epochs=20,
          validation_data = validation_generator,
          callbacks = [early_stop])
```

```
Epoch 1/20
304/304 [==============================] - 2s 3ms/step - loss: 0.0249 - val_loss: 0.0130
Epoch 2/20
304/304 [==============================] - 1s 2ms/step - loss: 0.0167 - val_loss: 0.0049
Epoch 3/20
304/304 [==============================] - 1s 2ms/step - loss: 0.0081 - val_loss: 0.0027
```

Training Results

```
losses = pd.DataFrame(model.history.history)
```

```
losses.plot();
```



Prediction Results

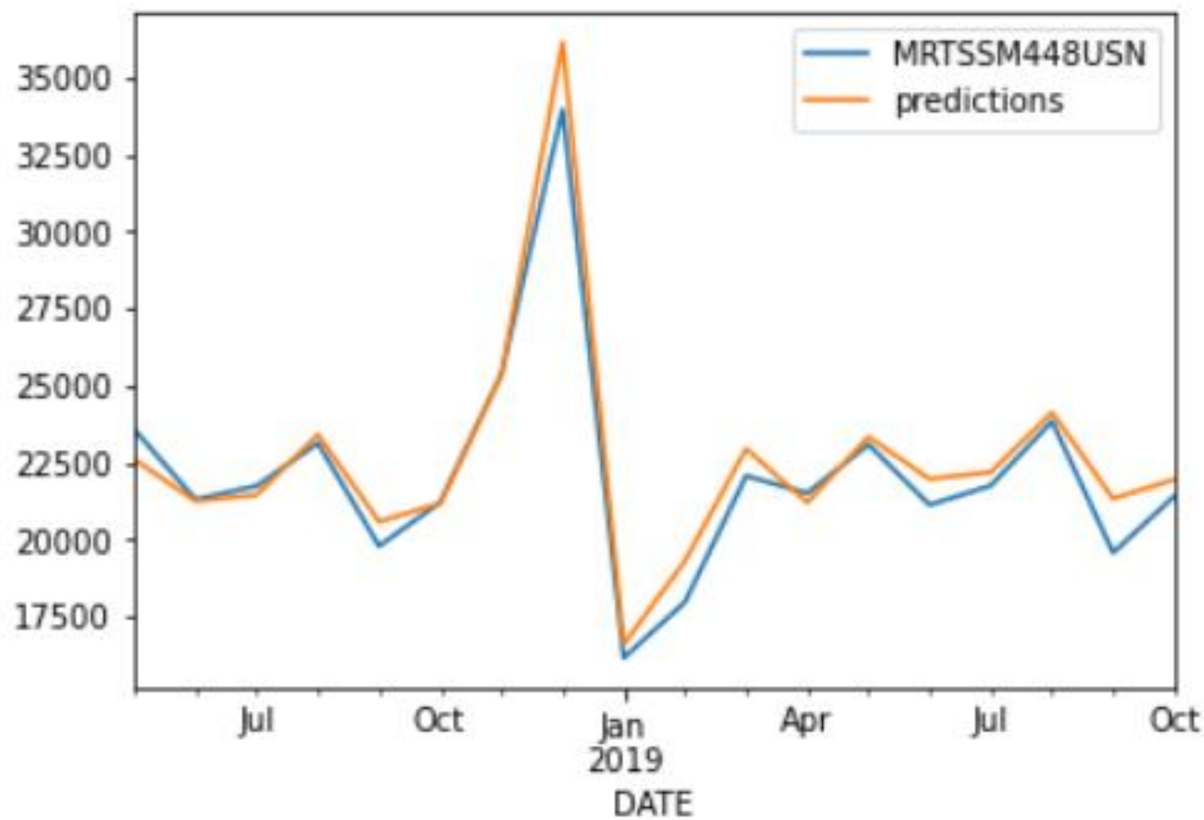
- Loop over the test set of 18 months
- Predict the next value
- Add it to the test predictions vector
- Update the next batch with this next prediction and drop first

```
test_predictions = []
first_eval_batch = df_train_scaled[-batch_length:]
current_batch = first_eval_batch.reshape((1, batch_length, n_features))

for i in range(len(df_test)):

    # predict one time step ahead
    current_pred = model.predict(current_batch)[0]

    # store the prediction
    test_predictions.append(current_pred)

    # update batch to now include prediction and drop first value
    current_batch = np.append(current_batch[:,1:,:],[[current_pred]],axis=1)
```

Looks Good

- The prediction looks good over the 18-month period
- Is it good ? Did we train it correctly ? What about sequences ?

```
df_test.plot();
```



Stock Price Prediction

- Some questions to consider
- Do you want to predict more than one period ahead ?
- Are you using only prices to predict prices ?
- What other features can we use to predict prices ?
- Predicting prices looks easy – if we use yesterday's price as the prediction then the model already looks good !!
- Better to provide a time series and predict return ?
- Or to make it simpler, make it a binary problem to see if the stock goes up or down ?
- Lots of possible approaches …

Detailed Guidelines

- **Chapter 1**: Discussion of time series forecasting, issues, challenges, literature review, intro to Artificial Neural Nets (ANN)
- **Chapter 2**: Calculation of technical indicators. Focus on S&P 500 stocks. Use technicals in a simple NN to do single-period prediction. Examine price prediction versus return prediction. Make the problem a binary (up or down) model. Does this work better ? Play with the shape of the ANN (artificial neural network) – does it help to have more layers ? Do cross-validation.
- **Chapter 3**: Using TensorFlow construct a neural network with recursive neurons. Describe the simple RNN but focus on the more powerful LSTM. Consider multi-period forecasting. Analyse same questions as Chapter 2.
- **Chapter 4**: I will tell you what to do when we next meet.