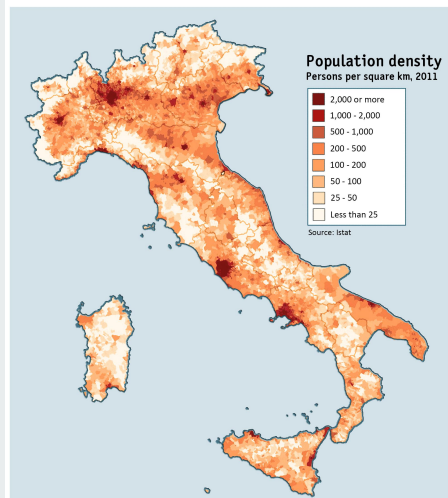


Simulating a SEIR Model in a Commuting Mobility Network



Application in Italian Commuting Networks:
Lazio
Puglia
North Italy

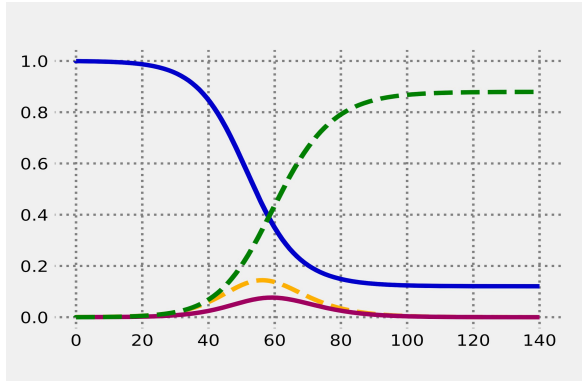


How it works: SEIR

1. Variant of the SIR Epidemic Model

Divides a population in

- S -> Susceptible
- E -> Exposed
- I -> Infected
- R -> Recovered



2. Models the evolution of SEIR values in time

$$\frac{dS}{dt} = -\beta \frac{SI}{N}$$

$$\frac{dE}{dt} = \beta \frac{SI}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

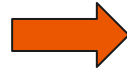
$$N = S + E + I + R,$$

3. Parameters:

- β Transition rate between Susceptible and Exposed.
- σ Rate of becoming infected
- γ Gamma is inverse of Recovery time.

How it works: SEIR Network

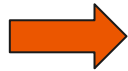
SEIR Model assumption: each element of the population is in contact with each other.



SEIR Network Model:

- Divide in subpopulations connected by a network.
- Treat each subpopulation as SEIR
- **Adds spreading through a directed network**

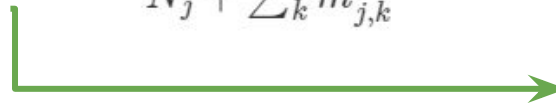
Number of new Exposed at location j



$$E_{j,t+1} = E_{j,t} + \frac{\beta_{j,t} S_{j,t} I_{j,t}}{N_j} + \frac{S_{j,t} \sum_k m_{j,k}^t x_{k,t} \beta_{k,t}}{N_j + \sum_k m_{j,k}^t},$$



Infected arriving from other locations



Infected at the location j

Data and Implementation

Handled data to get a main dataframe with:

- Residence of the commuters
- Destination of the commuters
- Flux weight

Precision: Municipality

Resid	Dest	Flux
1005	96004	1
1006	1006	1186
1006	1272	661
1006	1013	254
1006	1219	229
1006	1303	85
1006	1115	72
1006	1189	61
1006	1120	56
1006	1045	54
1006	1270	51
1006	1008	48

Commuting mobility matrix from Istat

<https://www.istat.it/it/archivio/139381>

- Municipalities = Nodes
- Edges represent people traveling between two municipalities

- Edges are directed
- Edges are weighted according to the number of people traveling

SEIR Network implementation:

- Using OD Matrix derived from df
- Cycle of SEIR evolution for every municipality

Used some code from:

https://www.databentobox.com/2020/03/28/covid19_city_seir/

First applications

Lazio Region

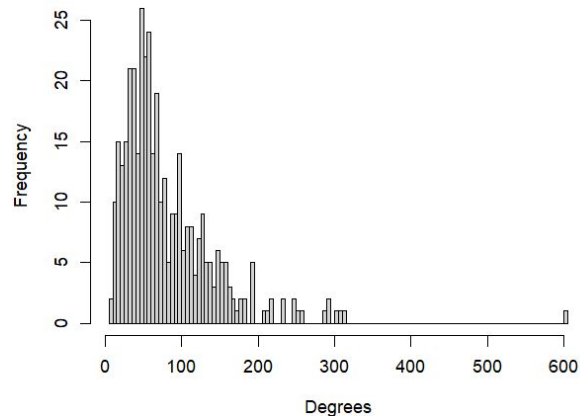
Region of Lazio . Global efficiency: 0.519

Graph Structure

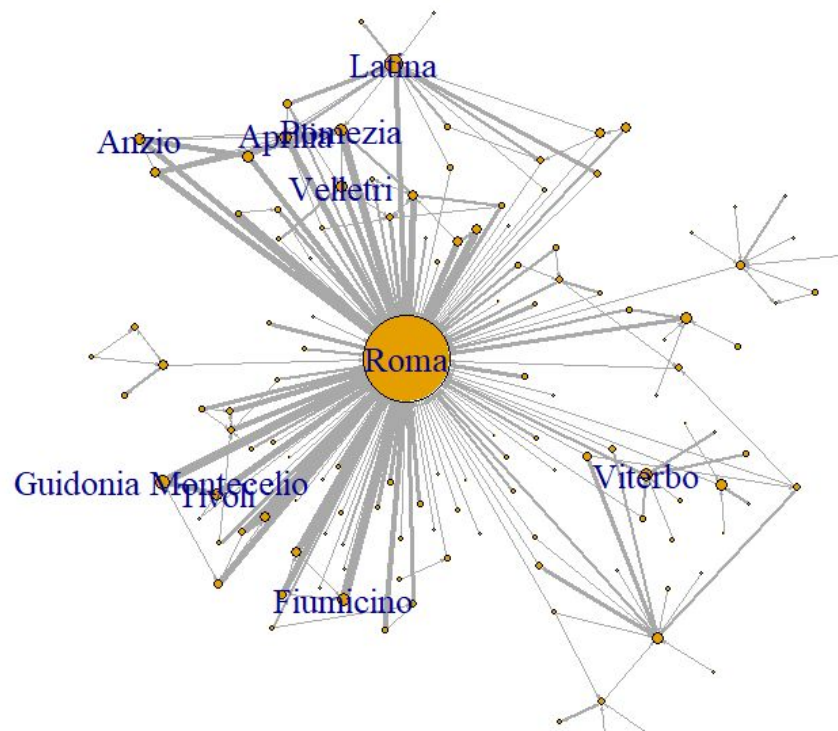
378 Nodes

15411 Edges

Degrees of Nodes (Municipality): Lazio.



- Approaches a small-world network.
- There are not many nodes with a degree particularly small.
- There is one big hub, few intermediate.



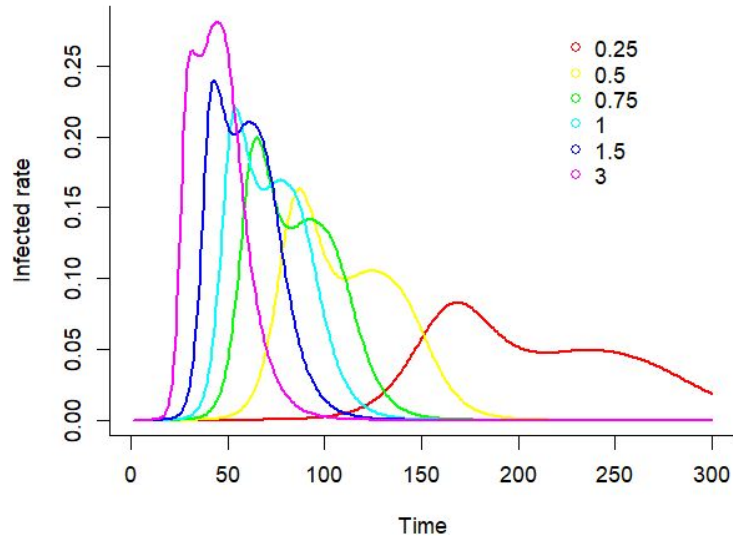
First results

Parameter testing

Using as baseline Lazio network with Rome as start of infection.

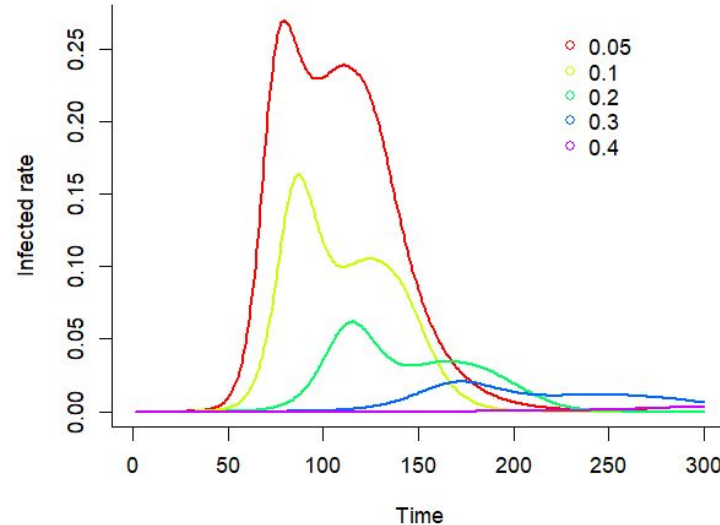
Beta: the parameter controlling how often a susceptible-infected contact results in a new exposure

Infected rate with different Beta values



Gamma: the rate an infected recovers and moves into the resistant phase ($1/\text{Recovery_Time}$)

Infected rate with different gamma values

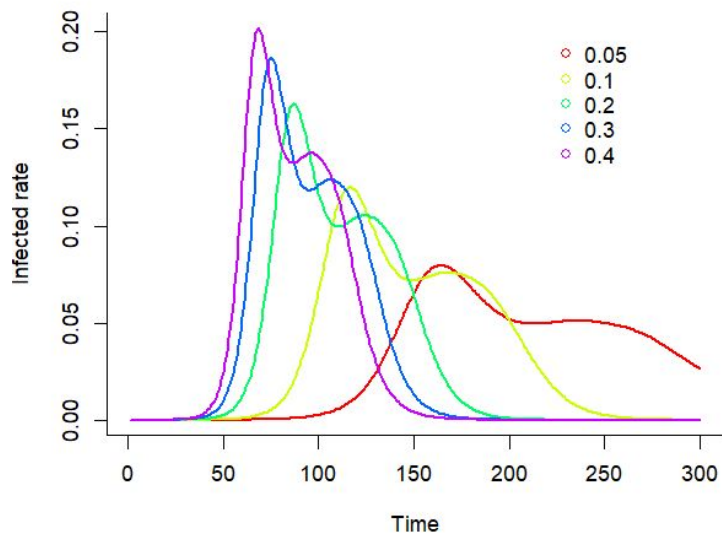


First results

Parameter testing

Sigma: the rate at which an exposed person becomes infective ($1/\text{Incubation_Time}$)

Infected rate with different sigma values



SEIR Network parameters testing confirms that:

1. Beta controls how **quick** an epidemic can outbreak.
2. Gamma, as connected to the Recovery time, can model the **intensity**.
3. Sigma represents the **dilation** in time of the infected peak.

First applications

Lazio Region

Observations:

Starting at the location with max degree **speeds up** the Infection curve.

Starting at the location with min degree **is not always the most effective way to slow down** the infection curve.

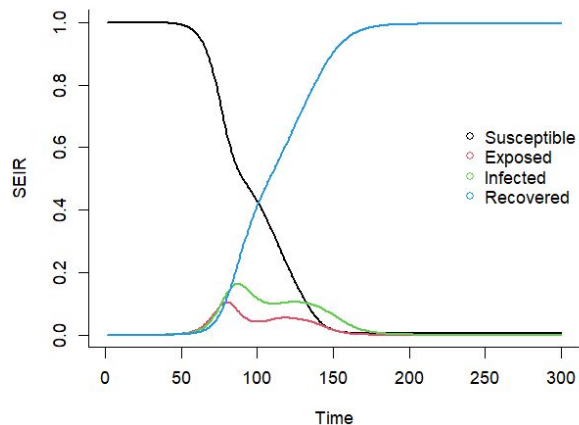
Why?

Start at max node degree:
Roma.

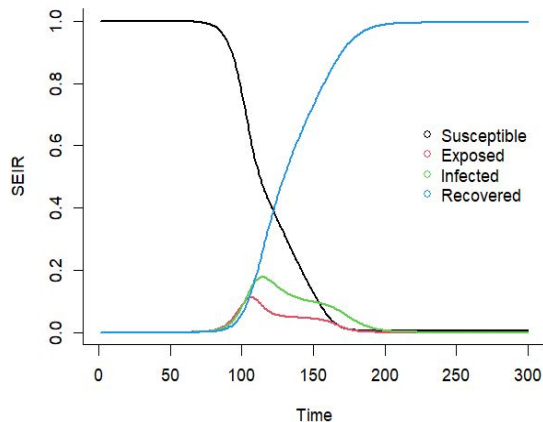
Start at min node degree:
Marcella

Start at other location

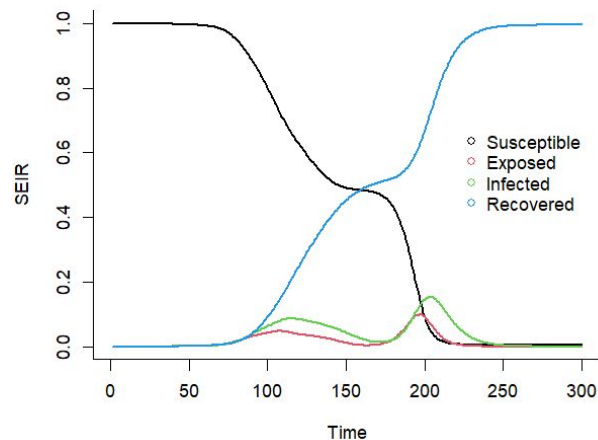
SEIR Model, Lazio . Start at: Roma



SEIR Model, Lazio . Start at: Marcella



SEIR Model, Lazio . Start at: Castel Madama



Why other locations:

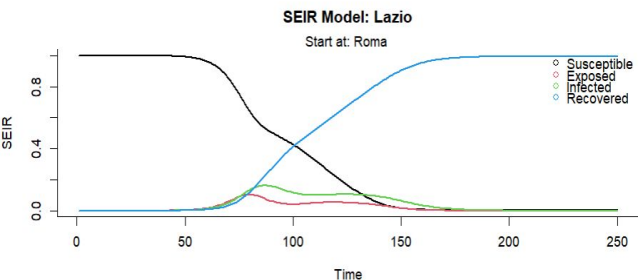
Lazio Region

There are other locations that **'hold' better** the contact with the hubs and slow more the infection.

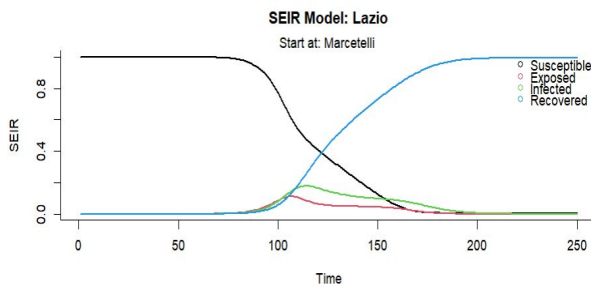
In this case starting **Castel Madama** spread the disease in other smaller hubs and later in Rome.

Add City visualization

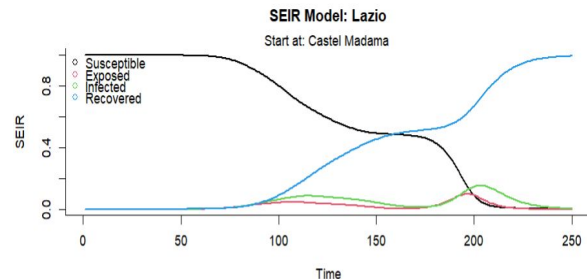
Start at max node degree:
Roma



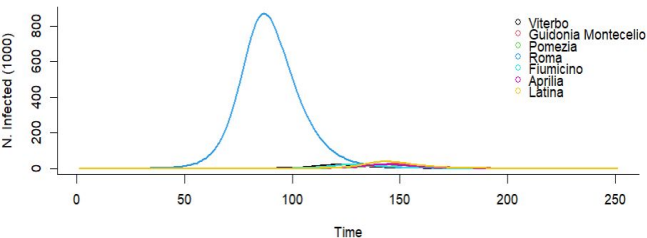
Start at min node degree:
Marcetelli



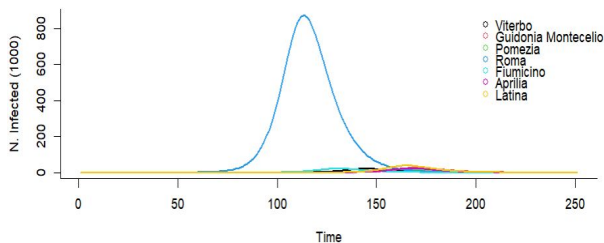
Start at other location



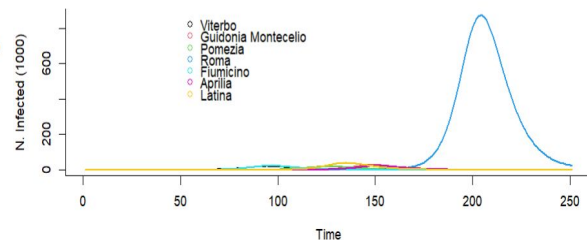
Infection rate at different big cities



Infection rate at different big cities



Infection rate at different big cities



Second application

Puglia Region

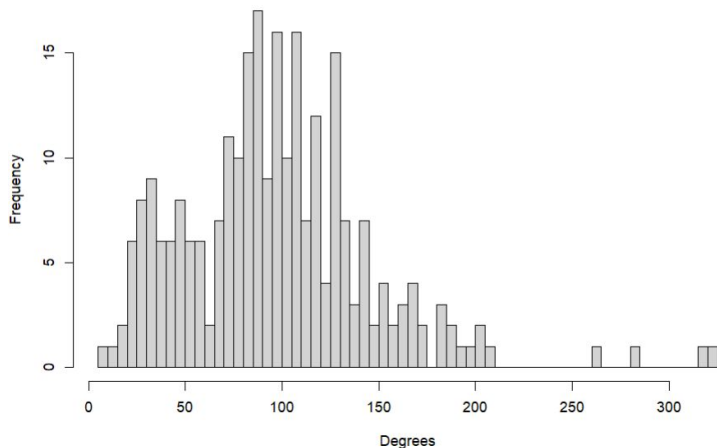
Graph Structure

258 Nodes

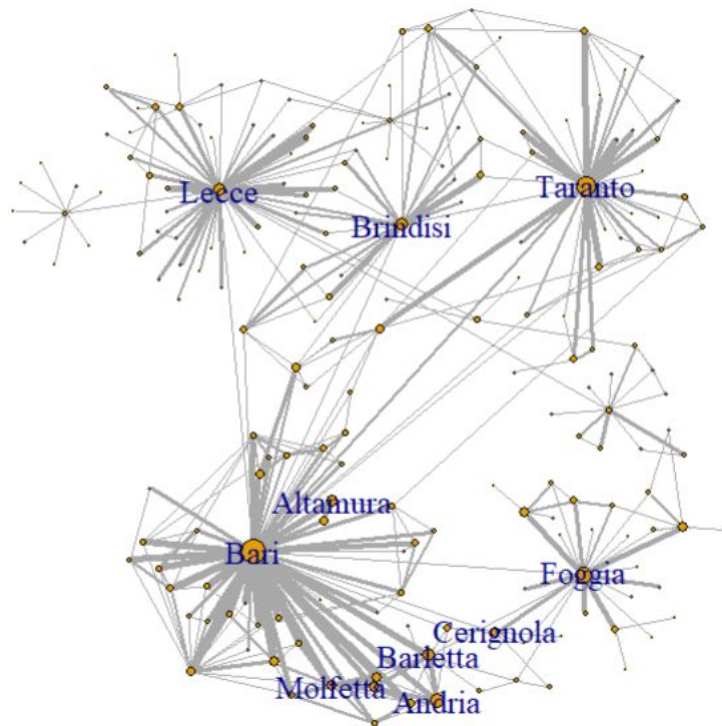
12559 Edges

- No single Hub like Rome, but multiples.
- More intermediate nodes.
- More clustered

Degrees of Nodes (Municipality): Puglia.



Region of Puglia . Global efficiency: 0.554

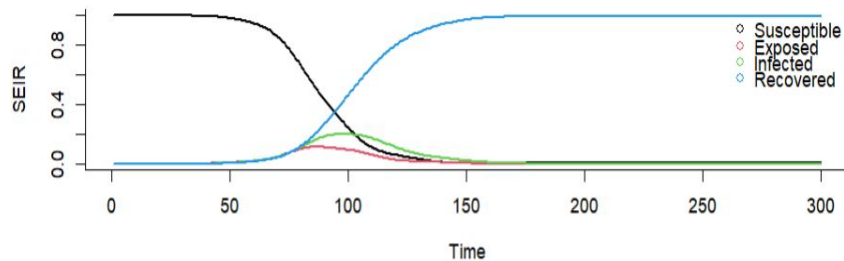


Second application

Puglia Region

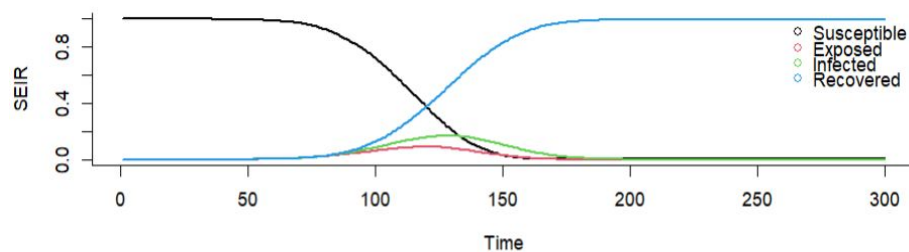
Start at max node degree: Lecce

SEIR Model, Puglia . Start at: Lecce

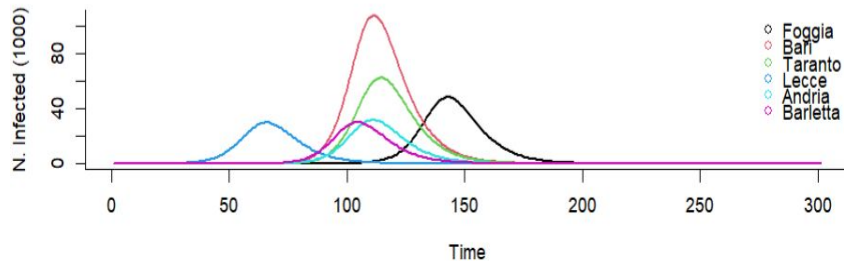


Start at min node degree: Isole Tremiti

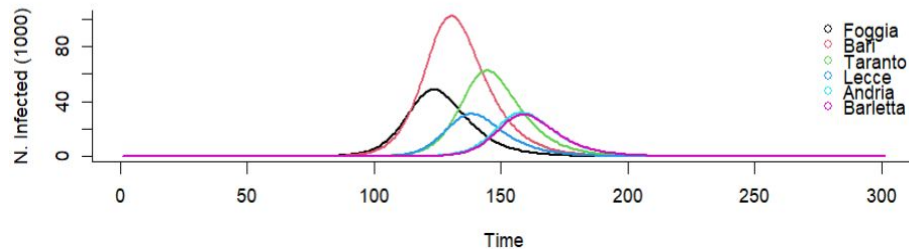
SEIR Model, Puglia . Start at: Isole Tremiti



Infection rate at different big cities



Infection rate at different big cities



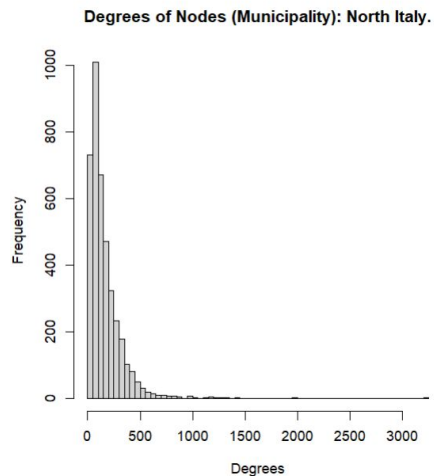
Bigger area

North Italy

Graph Structure

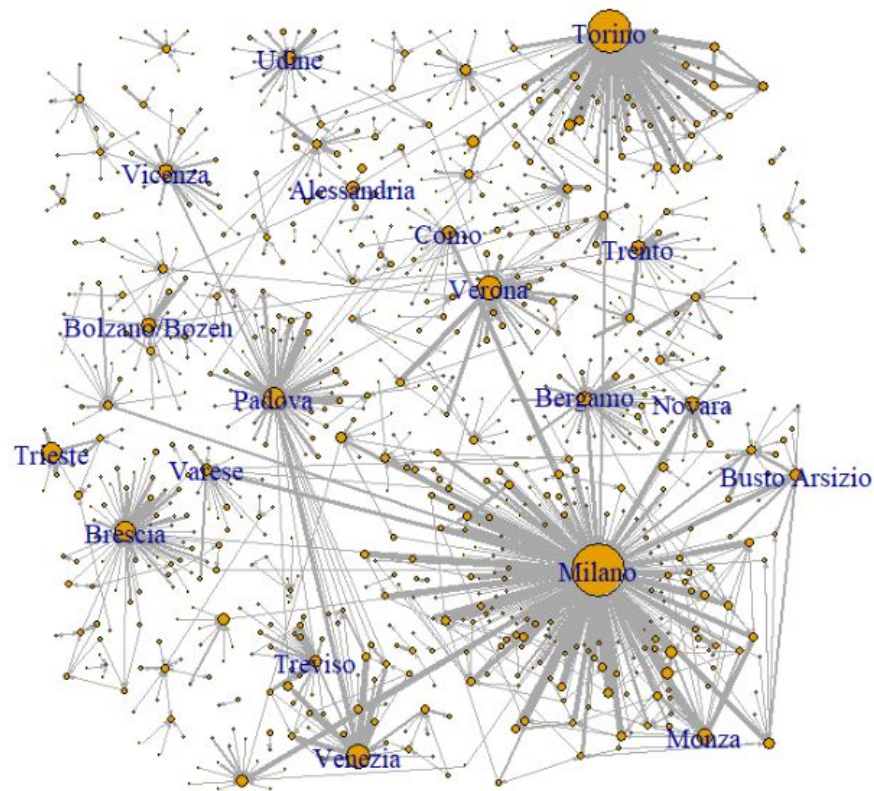
3956 Nodes

318380 Edges



Regions:
Val d'Aosta, Piemonte,
Lombardia, Veneto,
Trentino, Friuli

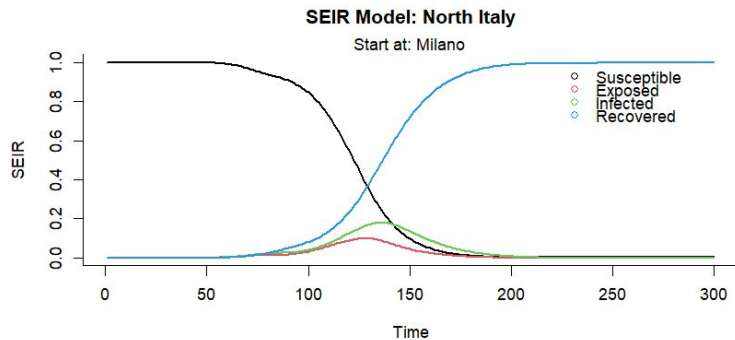
North Italy. Global efficiency 0.39



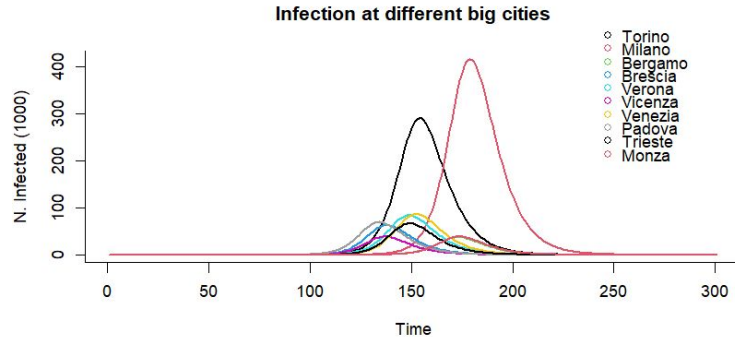
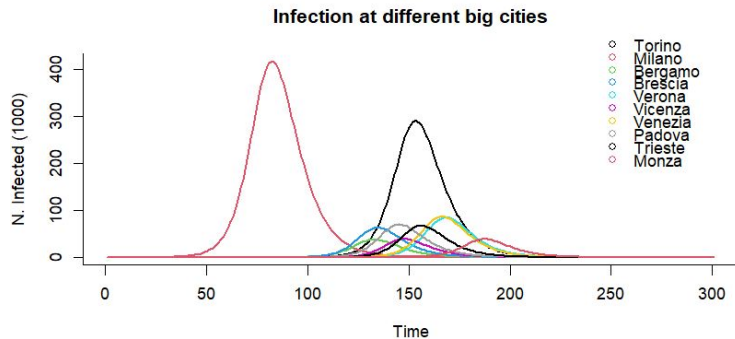
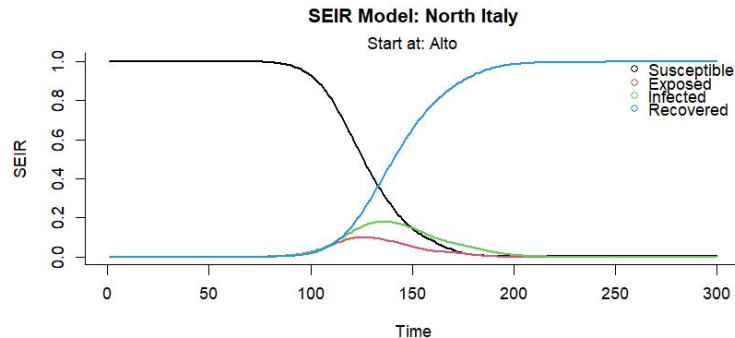
Bigger area

North Italy

Start at max node degree: Milano



Start at min node degree: Isole Tremiti

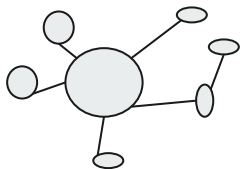


Conclusions

Parameter Testing confirms the Beta, Gamma and Sigma function in the SEIR Model.

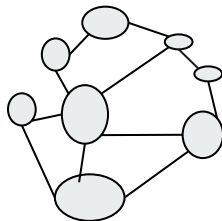
Lazio

In a region with only a big Hub and some intermediate hubs, there is diverse spreading of the disease depending on where and when the infection reach the big Hub (Rome).



North Italy

In a bigger area and with more diverse important hubs and good connection, there is no difference in the peak infection overall, around 150's day, even when started in lower degree nodes.



Puglia

Due to medium hubs of smaller dimension, the disease spreads slowly if starting in a low degree node.

