Constraint Modelling

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1 Proposed model

An elastic beam is divided into two rigid bodies which relative motion among each other is constrained at all six dof with a compliance joint. The constraint on the translational movement is formulated similair to a ball joint [1] and the constraint on the roational movement is formulated as the "angular constraint" in [2].

1.1 Translational

Consider two local reference frame attached to body 1 and 2, $\vec{e^1_x}, \vec{e^1_y}, \vec{e^1_z}$ and $\vec{e^2}_x, \vec{e^2_y}, \vec{e^2_z}$, in a global reference frame $\vec{e_x}, \vec{e_y}, \vec{e_z}$. The distance between body 1 and body 2 is a vector in the global reference frame which is initially a constant length in the local reference system of body 1

$$d^0_{1,2} = \begin{pmatrix} \Delta x^0 \\ \Delta y^0 \\ \Delta z^0. \end{pmatrix} \tag{1}$$

Where Δx^0 is the intial length in e^1_x and so on. At time $t > 0$ the distance is expressed as a vector in the global reference, $d_{1,2}$ system projected onto the local reference system of body 1

$$d^1_{1,2} = R(e_1)d_{1,2} \tag{2}$$

where $R(e_1)$ is the translation matrix of local reference system 1 expressed in terms of euler angles e_1. Since we want the two bodies to act as one the three translational constraint is then[1]

$$d^1_{1,2} - d^0_{1,2} = 0. \tag{3}$$

1.2 Rotational

We want to constrain the bending and twisting between body 1 and 2 to do this we again consider the two local reference frames of body 1 and 2, $\vec{e^1_x}, \vec{e^1_y}, \vec{e^1_z}$ and $\vec{e^2}_x, \vec{e^2_y}, \vec{e^2_z}$. Say that the bending occurs around $\vec{e^1_x}, \vec{e^2}_x$, the cross-product of

$$\vec{e^1_y} \times \vec{e^2_y} = |\vec{e^1_y}||\vec{e^2_y}|sin(\theta)\vec{n} \tag{4}$$

[1] Might as well use ball-joint?

and the dot product of

$$\vec{e_y^1} \cdot \vec{e_y^2} = |\vec{e_y^1}||\vec{e_y^2}|cos(\theta) \tag{5}$$

where θ is the angle in the plane of $\vec{e_y^1}, \vec{e_z^1}$ and $\vec{e_y^2}, \vec{e_z^2}$[2]. Combining equation (4) and (5) we find that

$$\frac{|\vec{e_y^1} \times \vec{e_y^2}|}{\vec{e_y^1} \cdot \vec{e_y^2}} = tan(\theta) \tag{6}$$

which is valid for angles in between $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and the constraint on bending motion is $\theta = 0$.

Say that the twisting occurs around $\vec{e_y^1}, \vec{e_y^2}$ then with the same approach as for the bending constraint, we find the angle ϕ in the plane of $\vec{e_x^1}, \vec{e_z^1}$ and $\vec{e_x^2}, \vec{e_z^2}$[3] from

$$\frac{|\vec{e_z^1} \times \vec{e_z^2}|}{\vec{e_z^1} \cdot \vec{e_z^2}} = tan(\phi) \tag{7}$$

which is valid for angles in between $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and the constraint on twisting motion is $\phi = 0$.

References

[1] Erleben, Sporring, Henriksen and Dohlman *PHYSICS-BASED ANIMATION, Chapter 7*, (THOMOSON, 2005).

[2] Martin Servin and Claude Lacoursiére *Rigid Body Cable for Virtual Environments, the angular constraint*, (IEEE TRANSACTIONS ON VISUALIZATION AND COMPUTER GRAPHICS, 2007).

[2]Is this correct?

[3]Is this correct?