Jump-adapted Euler scheme for jump-diffusion SDEs

We wish to simulate the following SDE with jumps:

$$X(t_0) = x_0, \quad dX(t) = a(t, X(t))\, dt + b(t, X(t))\, dW(t) + c(t, X(t^-))\, dJ(t).$$

Let $\{t_1^* < t_2^* < \cdots < t_{N_1}^* = T\}$ be a regular grid of N_1 points, and let $\{\tau_1 < \tau_2 < \cdots < \tau_{N_2}\}$ be the set of jump times simulated in $[t_0, t_0 + T]$, each with intensity $Y_j \sim f_Y(y)$. We consider the following jump-adapted (non-regular) grid,

$$\{t_1, \dots, t_N\} = \{t_1^*, \dots t_{N_1}^*\} \cup \{\tau_1, \dots, \tau_{N_2}\},$$

and we define $\Delta T_n = t_{n+1} - t_n$ for all n. For convenience, let ξ_n denote the approximation of $X(t)$ at time t_n. For a single simulation, we proceed as follows:

1. Set $\xi_0 = x_0$.
2. For a generic approximation ξ_{n+1}, we perform the following calculations:
   - Set $\xi_{n+1}^- = \xi_n + a(t_n, \xi_n)\Delta T_n + b(t_n, Y_n)\sqrt{\Delta T_n} Z_n$, where $Z_n \sim \mathcal{N}(0, 1)$ iid.
   - Take the possible jump into account by setting the definitive approximation as

     $$\xi_{n+1} = \xi_{n+1}^- + c(t_{n+1}, \xi_{n+1}^-)\Delta J_n,$$

     where ΔJ_n is defined as:

     $$\Delta J_n = \begin{cases} Y_j & \text{if } \exists j : t_{n+1} = \tau_j, \\ 0 & \text{otherwise.} \end{cases}$$

Note that in each step there can be at most one jump (since the jump times are included in the grid). Now there are two options for the return values:

- **Option 1.** Return the complete set of approximations $\{\xi_0, \dots, \xi_N\}$ at non-regular time steps.
- **Option 2.** Return only those approximations corresponding to the original regular grid of N_1 points.

Option 2 seems to be the most reasonable choice, since in Option 1 we would get different grid sizes for each simulation.