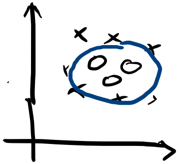
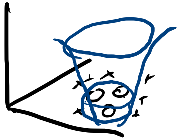
SVM

Support vector machines

- Grundidé: hyperplan för att separera klasser





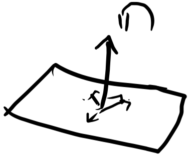
$\mathcal{O}(n^3) \sim O(n^2)$

design matrisens storlek

- skulle kunna göra feature expansion
- kerneltrick

· Hyperplan har vi redan sett men inte namngivit:

$$\beta_0 + \beta_1 x_1 + \dots + \beta_p x_p = 0$$



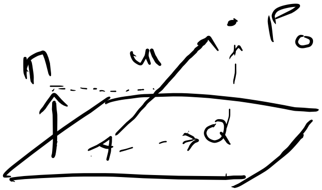
(Sparr 5.75) $(p = 3)$

$$ax + by + cz + d = 0$$

$n = (a, b, c)$, om $p > 3$ $n = (\beta_1, \dots, \beta_p)$

$\beta_0 = 0$ så ingår origo, annars inte.

Avstånd punkt/plan



$P_0 = (x_0, y_0, z_0)$

$A = (p, q, r)$

$$|\vec{QP}| = \frac{|(x_0-p)a + (y_0-q)b + (z_0-r)c|}{\sqrt{a^2+b^2+c^2}}$$

$\Leftrightarrow$

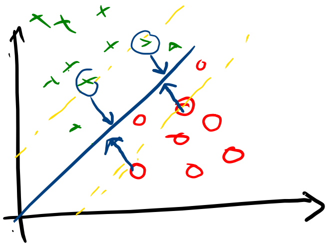
$$|\vec{QP}| = \left|\frac{u \cdot n}{n \cdot n} n\right|$$

villkor: $|n| = 1 = \sqrt{\beta_1^2 + \dots + \beta_p^2}$

$$\sum_{i=1}^{p} \beta_i^2 = 1$$

$n \cdot n = |n|^2$

$$\left|\frac{u \cdot n}{|n|^2}\right| |n| \rightarrow \underline{u \cdot n}$$



Maximal margin

Om marginalen måste vara tom blir metoden känslig för brus.

→ Soft Margin

$$\max_{\beta_0, \ldots, \beta_p} M$$

sådant att $\sum_{j=1}^{p} \beta_j^2 = 1$

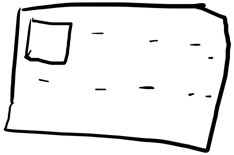
(i → varje rad i indatan)

$$\beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip} \geq M(1 - \varepsilon_i)$$

$\varepsilon_i \geq 0$, $\sum_{i=1}^{n} \varepsilon_i \leq C$ (budget)

- Kernel

- Matris med "vikter"



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$u_j \cdot v_i = \sum_{j=1}^{p} u_i v_i$$

$$\langle u, v \rangle = u \cdot v \quad \text{(euclidiskt)}$$

Klassificeringen kan skrivas som funktion (svår härledning):

$$f(\underline{x}) = \beta_0 + \sum_{i=1}^{n} \alpha_i \langle \underline{x}, \underline{x}_i \rangle$$

$$\Rightarrow \beta_0 + \sum_{i \in S} \hat{\alpha}_i \langle x, x_i \rangle$$

S = support set
stödvektorerna vi valt

Kernel funktioner

$$K(\mathbb{u}, \mathbb{v}) = \left(1 + \sum_{j=1}^{p} u_j v_j\right)^d \Leftrightarrow (1 + \mathbb{u} \cdot \mathbb{v})^d$$

beräknar inre produkter för d-dimensionella polynom.

$\binom{p+d}{d}$ bas, motsvarar polynom expansion

RBF (Radial basis funktion)

$$K(\mathbb{u}, \mathbb{v}) = e^{-\gamma \sum_{j=1}^{p} (u_j - v_j)^2}$$

$\gamma \gg 0$, liten σ^2

$\gamma \ll 1$, stor σ^2

(mjuka linjer)

$\gamma \Rightarrow$ hyperparameter

Multinomiella klasser

Ova : one-versus-all

ovo : one-versus-one

kan bli dyrt för stora datamängden (dvs gigabyte+)

Gradient descent

Grundidé: ta ett "litet" steg i en "bra" riktning för att nå minimum

- Iterativa funktioner

Iteration

$$x_{n+1} = x_n$$

Basfall

$$x_0 = a$$

- Differens ekvation:
  itererar över tid

- Differential ekvation
  "iteration" (integration) över kontinuerlig tid

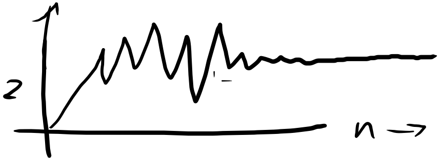
Logistisk ekvation (iterativ)

$$x_{n+1} = r(1 - x_n)$$

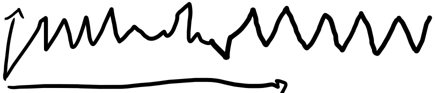
$$\boxed{x_0 = 0.5}$$

$r = 3.58$

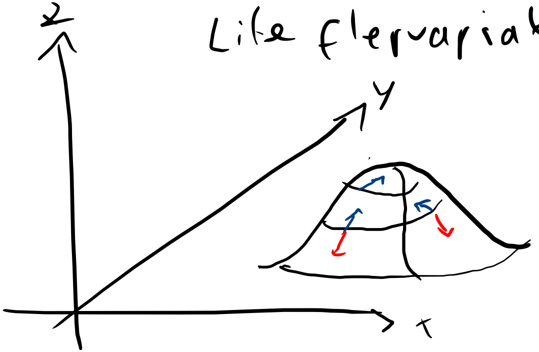
$r \approx 2$

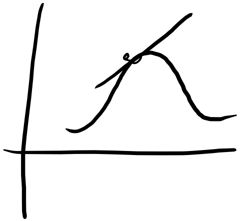


$r \approx 2$



Lite flervariabel:





$$\left(\frac{\partial RSS}{\partial b_0}, \frac{\partial RSS}{\partial b_1} \right) = -\nabla$$

· pekar ~~alltid~~ i den rikting som har störst förändringshastighet

jacobi matris
$$\begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial b} \\ \vdots & \vdots \end{bmatrix}$$

OLS:

$$C(\beta) = MSE$$

Ridge

$$C(\beta) = MSE + \lambda \sum_{i=1}^{p} \beta_i^2$$

GD

$$C(\theta) = \frac{1}{n-k-1} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$= \frac{1}{n-k-1} \sum_{i=1}^{n} (y_i - \theta \cdot x_i)^2$$

som iterativ (rekurrent) funktion:

som iterativ funktion:

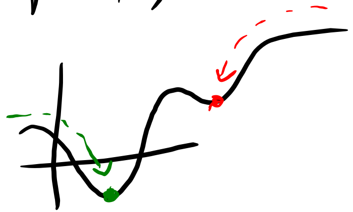
$$\theta_{i+1} = \theta_i - \eta \nabla C(\theta)$$

$$\theta_{j+1} = \theta_j - \eta \left(\frac{2}{n-k-1} X^T (X\theta_j - y) \right)$$

<u>Batch</u> $\longleftrightarrow$ stickprov (stat. sample)

<u>Sample</u> $\longleftrightarrow$ datapunkt (data point)

<u>Batch Gradient Descent</u>



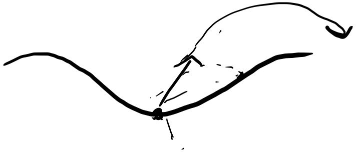
tenderar att fastna i lokala min.

Stochastic Gradient Descent

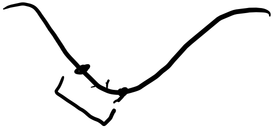
Välj en slumpmässig datapunkt.
Räkna ∇ på den punkten.
upprepa givet antal gånger.

→ gör en "Epok" (Epoch)

η = hur stora steg tar vi



konvergerar ej!



Minibatch

Välj delmängd av punkter och räkna gradient annars som SGD.

Gradient Descent with Momentum

$$x_{k+1} = x_k - s z_k \qquad x_{k+1} = x_k - \eta \cdot \nabla f_k$$

$$z_k = \underline{\nabla f_k} + \underline{\beta z_{k-1}} \qquad x_{k+1} \quad \dots \quad x_k, \dots x_{k-1}$$