# Discrete probability densities

$$f(x) = P[X = x]$$

necessary and sufficient conditions for a discrete density:

1. $f(x) \geq 0$
2. $\sum_x f(x) = 1$

(probability function)
sv. täthetsfunction

Cumulative distribution function:

$$F(x) = P[X \leq x]$$

Expected value (väntevärde):

$$E[X] = \sum_{x} x f(x)$$

Standard deviation:

$$\sigma = \sqrt{\operatorname{Var} X} = \sqrt{\sigma^2}$$

Ex: tärning:

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \ldots + 6 \cdot \frac{1}{6} = 3.5$$

Variance

$$\operatorname{Var} X = \sigma^2 = E[(X-\mu)^2]$$

$$\sigma^2 = \operatorname{Var} X = E[X^2] - (E[X])^2$$

# Geometric distribution.

$X$ = the number of trials until first success

$S = \{s, fs, ffs, fffs, \ldots\}$

pf: $f(x) = (1-p)^{x-1} p \Rightarrow q^{x-1} p$

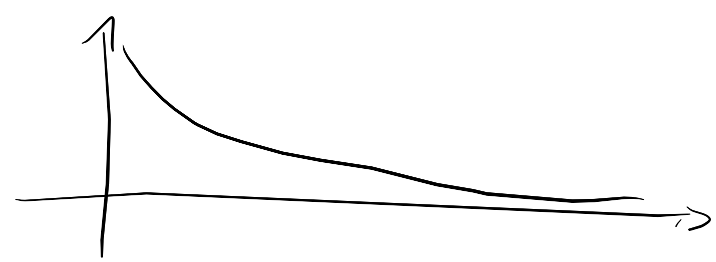
cdf: $F(x) = 1 - q^{\lfloor x \rfloor}$ $\quad q = 1-p$ $\quad \lfloor x \rfloor$ = largest integer smaller than $x$

(eg $1.9 = 1$
$0.9 = 1$
$1.2 = 1$)

moments:

$E[X] = 1/p$

$\text{Var}\, X = q/p^2$



## Binomal distribution

- A fixed number of trials $(n)$

$X$ = how many successes over the $n$ trials.

p.f. $$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$q = 1-p$$

cdf $$F(t) = \sum_{x=0}^{\lfloor t \rfloor} \binom{n}{x} p^x (1-p)^{n-x}$$

$$E[X] = \mu = np$$

$$\mathrm{Var}\, X = \sigma^2 = npq$$

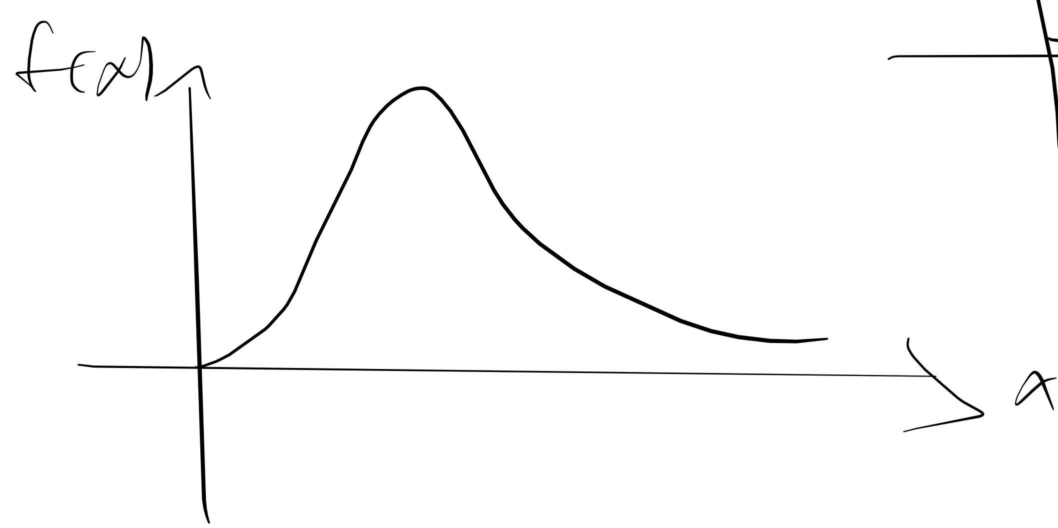
# Negative binomial distribution

$X$ = number of trials until $r$ successes.

$$f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r \qquad x = r, r+1, r+2, \ldots$$

$$E[X] = r/p \qquad q = 1-p$$

$$Var(X) = rq/p^2$$



# Hypergeometric distribution

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

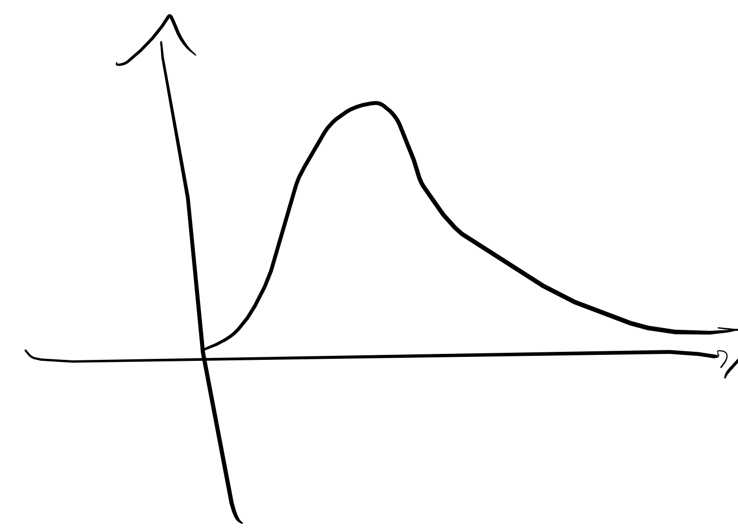
$N$ – number of objects

$n$ – sample size

$r$ – number of objects with a particular trait.

$$E[X] = n\left(\frac{r}{N}\right)$$

$$\operatorname{Var} X = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$$



# Poisson Distribution

$$f(x) = \frac{e^{-k} k^{x}}{x!}$$

$k > 0$, $x = 1, 2, 3, \ldots$

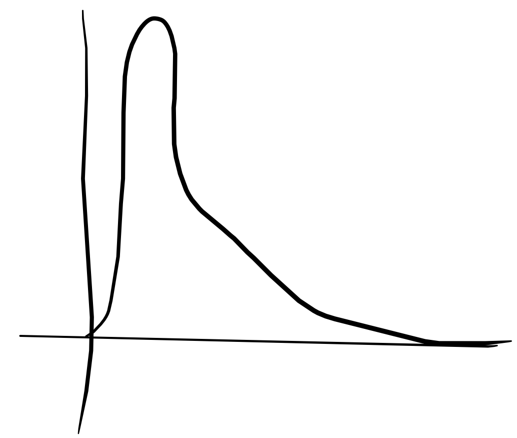
$k = \lambda s$

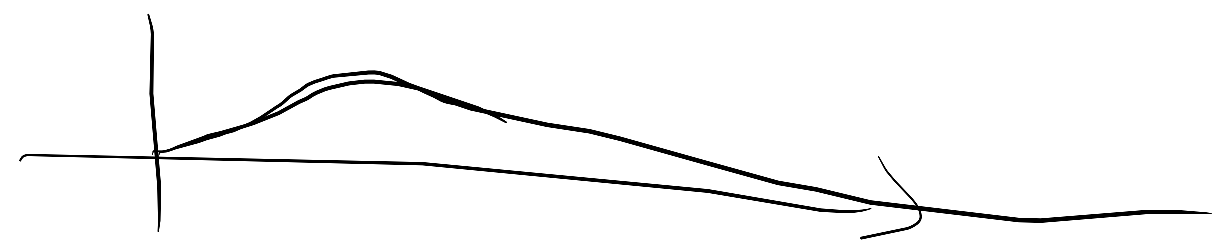
$\lambda =$ average occurences per time unit

$E[X] = k$

$\operatorname{Var} X = k$

eg. radioative decay





# Normal distribution (continous)

Def:

A random variable $X$ is <u>continous</u> if it can assume any real value in some interval and the probability that it assumes any specific value is $\underline{0}$.

$$f(x) = F'(x)$$

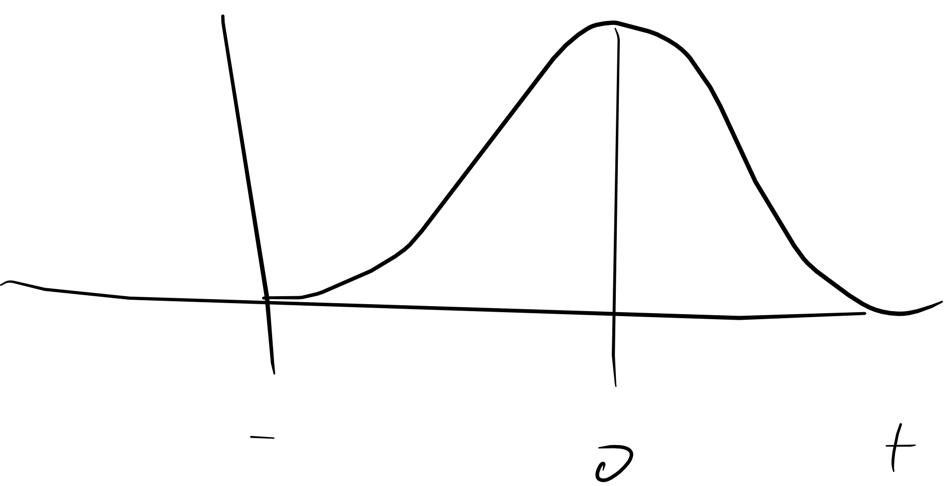
$$f(x) = P[X = x] \Rightarrow 0 \text{ everywhere?}$$

$$\boxed{\begin{array}{ll} 1.\ f(x) \geq 0 & 3.\ P[a \leq X \leq b] = \int_a^b f(x)\,dx \\ 2.\ \int_{-\infty}^{\infty} f(x)\,dx = 1 & \end{array}}$$

# Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}((x-\mu)/\sigma)^2}$$

$\mu, \sigma \Rightarrow$ mean, standard deviation




$$X \sim N(\mu, \sigma)$$

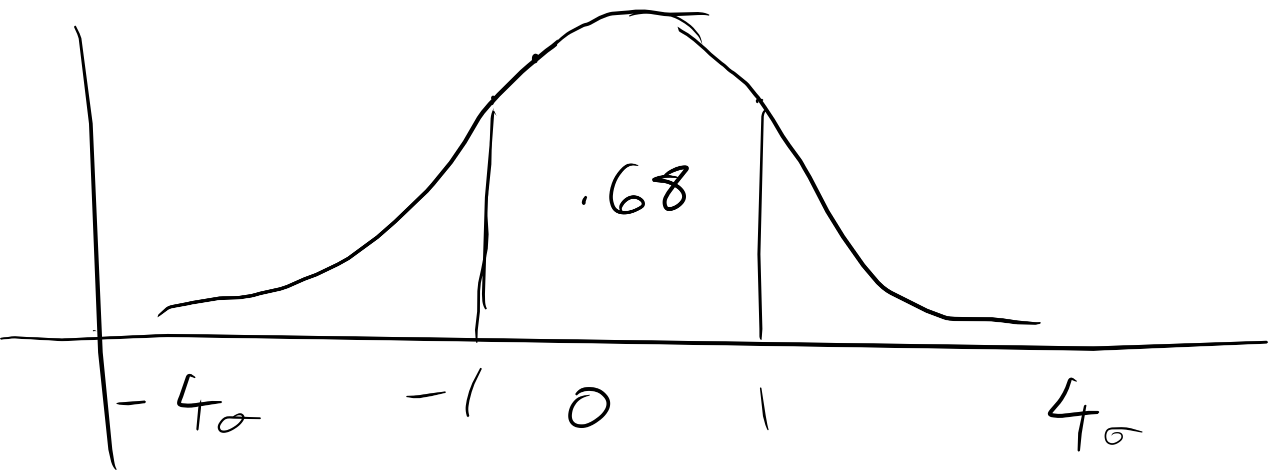
$$\boxed{\frac{X-\mu}{\sigma} \sim N(0, \sigma)}$$

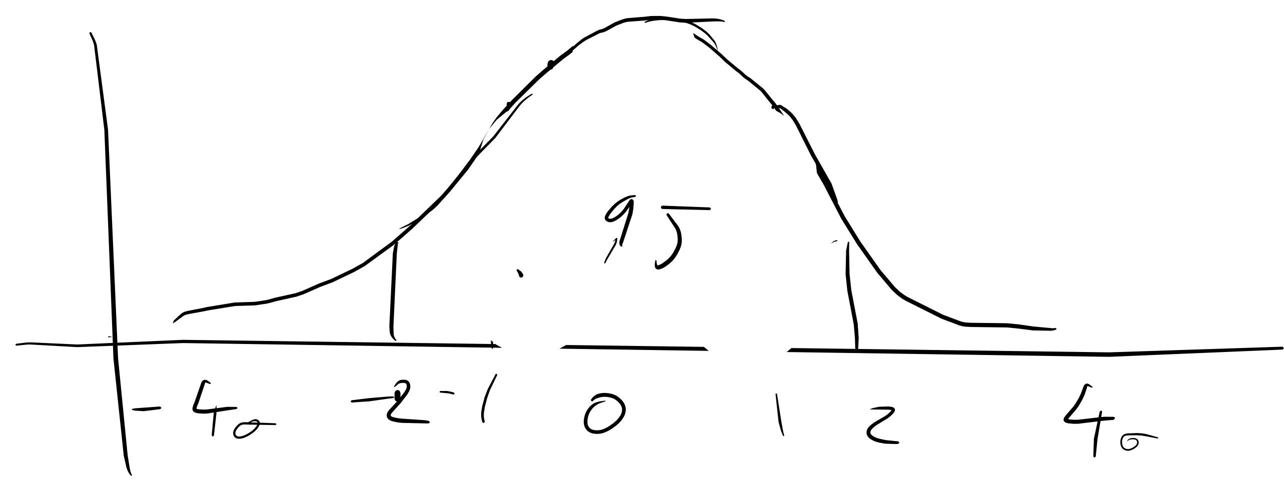
Normal probability rule (68-95-99)

$$P[-\sigma < X-\mu < \sigma] = .68$$

$$P[-2\sigma < X-\mu < 2\sigma] = .95$$

$$P[-3\sigma < X-\mu < 3\sigma] = .997$$



A $100(1-\alpha)\%$ confidence interval for a parameter $\theta$ is a random interval $[L_1, L_2]$ such that

$$P[L_1 \leq \theta \leq {}_2] = 1-\alpha$$

$$\boxed{\bar{X} \pm z_{\alpha/2}\, \sigma/\sqrt{n}}$$

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0, \sigma)$$

$$\bar{X} \pm 1.96(\sigma/\sqrt{n})$$

$$\bar{X} \pm 1.96(.3/19)$$

$\rightarrow$ standardization

$$\boxed{\hat{\beta}_i \pm t_{\alpha/2}\, S\sqrt{c_{ii}}}$$