Thermal Conduction and Partial Differential Equations

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1 Introduction

Thermal conduction is a time and space varying phenomena. In physics, this system is describe as a partial differential equation(PDE):

$$\frac{\partial u}{\partial t} = \lambda(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2})$$

Here $\lambda = \frac{K}{c\varrho}$, u is temperature, t is time, and (x,y,z) is the space.

To solve the PDE, we need to provide the *initial conditions* and *boundary conditions*[1]. One must specify the spatial domain($x \in [0, 1]$) over which the solution should be defined. For equations with one spatial dimension, we can take the solution domain as an interval [a,b]. The initial condition must then be specified over the entire spatial domain, as a function of position:

$$u(x, 0) = \varphi(x) = 4x(1 - x),\ x \in [0, 1]$$

Boundary conditions specify how the solution behaves at the ends of the special interval. These conditions must be specified for all time. There are three kinds of conditions[1]:

1. *Dirichlet boundary conditions.* Claim the end-popint values of the solution to given functions. $u(0, t) = 0; u(1, t) = 0$
2. *Neumann boundary conditions.* Specify the solution gradient at the end-points.
3. *Robin conditions.* Specify both the value and gradient at the end-points.

Now, we restrict ourselves to one spacial dimension(of the thermal conduction problem). To solve this PDE numerically, we need to construct a mesh of time-points and space-points. We first get this difference Equation:

$$u_{i,k+1} = \alpha u_{i+1,k} + (1 - 2\alpha)u_{i,k} + \alpha u_{i-1,k}$$

Here, i is the special step, ranging from 0 to [l/h](l is the length of the object, and h is the length of one step); k is the temporal step, ranging from 0 to $[t_max/\tau]$, and τ is the length of one temporal step. And $\alpha = \frac{\tau\lambda}{h^2}$.

If $\alpha = \frac{\tau\lambda}{h^2} \leqslant \frac{1}{2}$, the numeric method used here can get stable solution.

2 Result

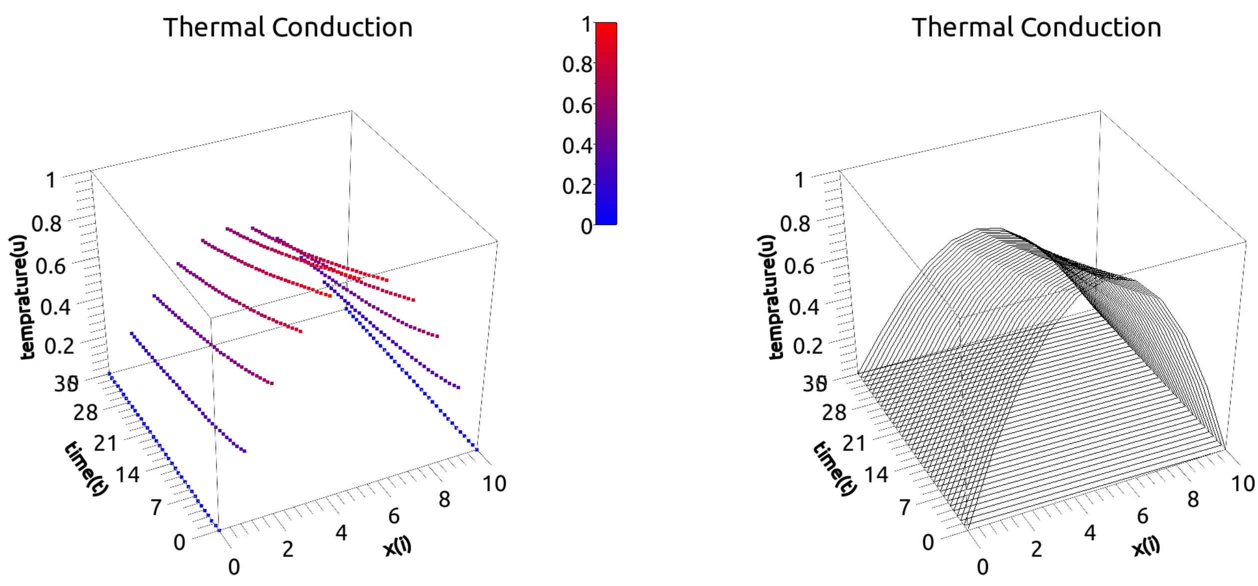

Figure 1: Thermal Conduction. $\alpha = 1/6$, $t_max = 0.06(s)$, $x \in [0, 1](m)$, $\lambda = 1$, mesh of time points: $\tau = 1/600(s)$, mesh of space points: $h = 0.01(m)$. In figure, x axis shows the special mesh, time axis shows the temporal mesh, and the temperature is the last axis.

References

[1] Brian P. Ingalls (2013). Mathematical Modelling in Systems Biology: An Introduction (Cambridge, Massachusetts: The MIT Press).

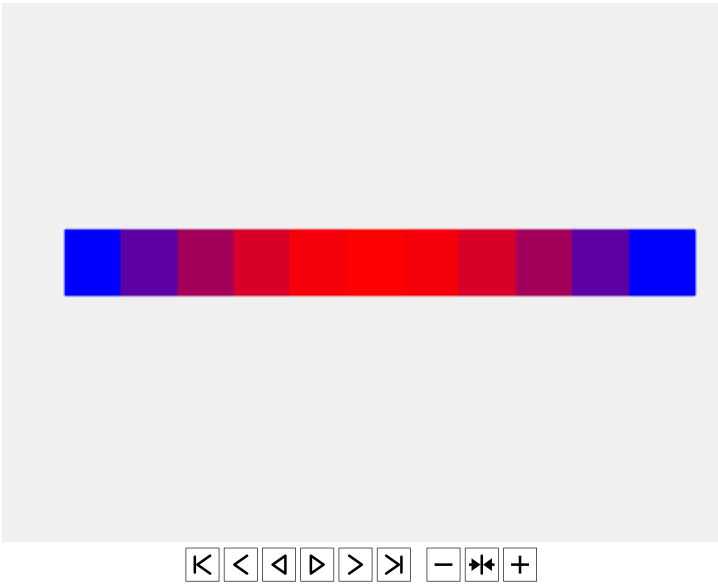

Figure 2: Thermal Conduction dynamic graph. You can see this in Adobe Readers.