Ordinary Differential Equations

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1 Introduction

The ordinary differential equation (ODE) is widely used in mathematics and science. In physics, the RLC circuit can be described as a system of ODEs. In the ecology, ODEs are used in modelling population growth, interspecific competition, enemy-victim interactions and so on.

An ODE is a mathematical descriptions of dynamically changing phenomena using differentials and derivatives[1]. It contains one or more functions of one independent variables and its derivatives. When one quantities can be defined as the rate of changes of another quantities(e.g. the electric current I=dQ/dt) or gradients of quantities, the ODEs can be used to describe the system.

1.1 RLC circuit

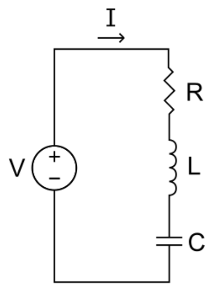

Figure 1: RCL circuit. From [2]

The RCL circuit has the following equation:

$$\begin{aligned} Va &= IT + L\frac{dI}{dt} + \frac{Q}{C} \\ I &= \frac{dQ}{dt} \end{aligned} \tag{1}$$

Combined with a initial state, the following ODEs can be solved numerically.

$$\begin{aligned} \frac{dQ}{dt} &= I \\ \frac{dI}{dt} &= \frac{1}{L}(Va - \frac{Q}{C} - IRQ(t_0) \quad = 0I(t_0) = 0 \end{aligned} \tag{2}$$

1.2 Runge Kutta methods

The 4th order Runge Kutta method is often used to solve the ODEs($dY/dt = f(Y, t)$):

$$Y_{n+1} = Y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{3}$$

Here,

$$
\begin{aligned}
k_1 &= f(Y_n, t_n) \\
k_2 &= f(Y_{n+\frac{\Delta t}{2}k1}, t_{n+\frac{1}{2}}) \\
k_3 &= f(Y_{n+\frac{\Delta t}{2}k2}, t_{n+\frac{1}{2}}) \\
k_4 &= f(Y_{n+\Delta tk3}, t_{n+1})
\end{aligned}
\tag{4}
$$

2 Design of generalized Runge Kutta ODE solver

A generalized ODE solver means that you can solve many ODEs system without modify my Runge_Kutta function.

2.1 Derivatives in RCL

```
double f1(double t, double x1,double x2){
    return x2;
}

double f2(double t, double x1,double x2){
    return 1/L*(Va-x1/C-x2*R);
}
```

2.2 Use array to store dependent variables

```
double f1(double t, double* x){
    return x[2];
}
double f2(double t, double* x){
    return 1/L(Va-x[1]/C-x[2]*R);
}
```

2.3 Common form of the two derivatives – using function pointer

```
double (*pf)(double t,double* x);
```

2.4 Array of function pointers

```
double (*pf[])(double t,double* x);
```

2.5 Use two array as parameters in the function 'void Runge_Kutta()'

```
void Runge_Kutta(double (*pf[])(double t,double* x),double* xn,int N_dependent_variable,double
    t_high,int N_t_steps,string file_name);
```

3 Results

Table 1: Parameters used in the system of ODEs(in RCL circuit)

Case	λ	Va	R	C	L
1	0.5	5	2	1	4
2	1	5	4	1	4
3	2	5	5	1	4

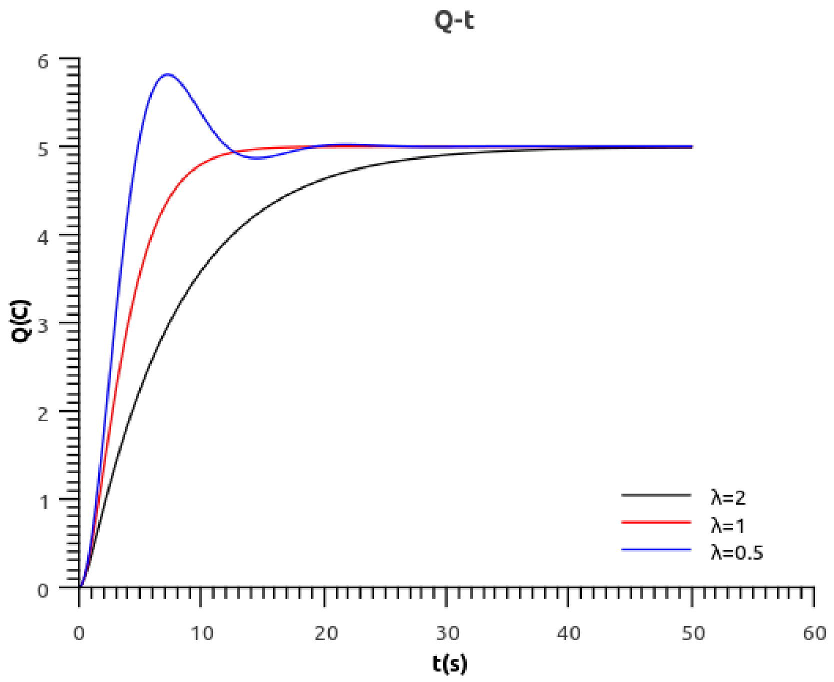

Figure 2: RCL Q-t curves in three cases. $\lambda = \frac{R}{2}\sqrt{\frac{C}{L}}$. $\lambda > 1$ is overdamped(black curve), $\lambda = 1$ is critically damped(red curve), and $\lambda < 1$ is underdamped(blue curve).

References

[1] Ordinary differential equation – Wikipedia
https://en.wikipedia.org/wiki/Ordinary_differential_equation

[2] RCL circuit – Wikipedia
https://en.wikipedia.org/wiki/RLC_circuit