Mathematics, Horror and Disease: A Primer

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Presenter Disclosures

- The presenter has no conflicts of interest to disclose.
- This applies to all presentations given by E.T. L. during this Learning Institute.

Acknowledgements

- This work was done in collaboration with Dr. Nina H. Fefferman, Rutgers University
- Films used in this Learning Institute include:
  - *28 Days Later* (2002) Fox Searchlight Pictures
  - *28 Weeks Later* (2007) 20th Century Fox
  - *Dawn of the Dead* (2004) Universal Pictures
  - *I Am Legend* (2007) Warner Bros.
  - *Shaun of the Dead* (2004) Universal Pictures
  - *Zombieland* (2009) Columbia Pictures

Mathematical Models in Epidemiology

- Modeling is an established discipline in public health
  - Bernoulli in 1760 with Smallpox
  - Modern incarnation in the early 1900's: Ross, Reed, Frost, etc.
- An invaluable tool in modern public health
  - Can illustrate difficult concepts
    - Herd immunity
    - Competing/complementary interventions
  - Quantitative research on difficult, expensive or otherwise infeasible questions
    - Vaccines, pandemic planning, etc.

Motivation: How Modeling is Taught

- Public health: Emphasis on interpretation and composition of models
  - Complex math left as a “black box”
- Math: Emphasis on mathematical analysis of models as dynamical systems problems
- Problems for both:
  - Public health students have only a loose understanding of model implementation and analysis
  - Math students are focused on abstract or mathematically interesting points rather than public health needs

Comparison to Observational Epidemiology

Observational Epi

- Disease-agnostic general toolkit adapted to specific questions
- Very little subject matter expertise needed to use basic methods
- Solid foundation to extend to more complex approaches

	Exposed	Unexposed
Disease	A	B
No Disease	C	D

Mathematical Epi

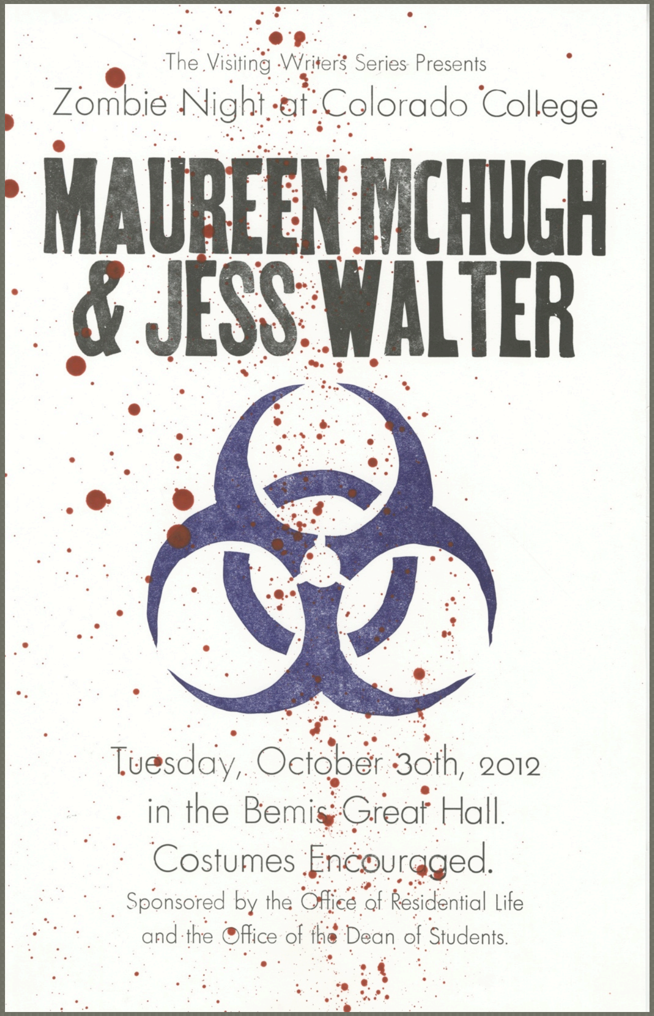
- Single-use disease specific tools
- Very high levels of subject matter expertise required to design a useful model
- Models are frequently borrowed, adapted or modified, often incorrectly
- "Malaria is like Dengue is like Yellow Fever is like hospital-acquired MRSA"

Solution… Zombies

- Approachable math for public health students
  - Difference equations that can be implemented in familiar software
- Familiar, flexible example disease upon which a number of different models can be based

Why Zombies?

- Zombie outbreaks are a common cultural reference point
- Large library of media with zombie outbreaks
  - Differing interpretations
- One can become a "subject matter expert" by watching a few films



An Outline of Today

- Brief overview of common types of models seen in mathematical epidemiology
- More in-depth treatment of each type in turn
- "Grey box" mathematics
- Some discussion of software implementation
- Discussion, questions, etc. throughout

Compartmental Models

- By far the most commonly used model in mathematical epidemiology

$$S \longrightarrow I \longrightarrow R$$

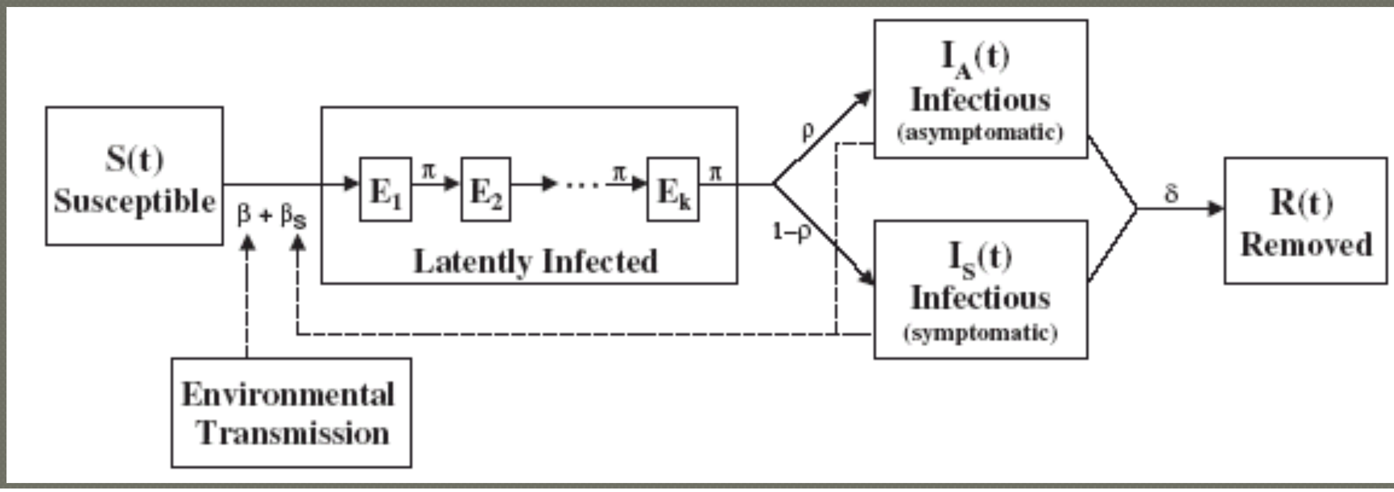
- Population grouped into health states (compartments)
- Transitions between compartments usually expressed as a series of differential equations
- Near infinite variety

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Infinite Variety and Arbitrarily Complexity



$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = -\beta(t)S(t) - \beta_S S(t)(I_A(t) + I_S(t))$$

$$\frac{\mathrm{d}E_1(t)}{\mathrm{d}t} = \beta(t)S(t) + \beta_S S(t)(I_A(t) + I_S(t)) - \pi E_1(t)$$

$$\beta(t) = I\{t \leqslant 23\}\beta_0 + I\{23 < t \leqslant 38\}\beta_1 + I\{t > 38\}\beta_2$$

$$\frac{\mathrm{d}E_2(t)}{\mathrm{d}t} = \pi E_1(t) - \pi E_2(t)$$

$$\cdots$$

$$\frac{\mathrm{d}E_k(t)}{\mathrm{d}t} = \pi E_{k-1}(t) - \pi E_k(t)$$

$$\frac{\mathrm{d}I_A(t)}{\mathrm{d}t} = \rho\pi E_k(t) - \delta I_A(t)$$

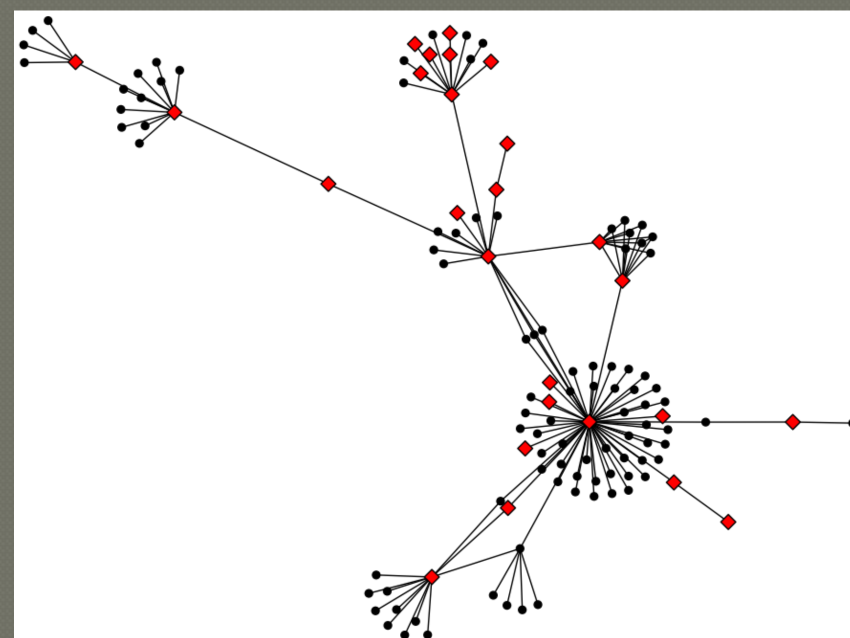
$$\frac{\mathrm{d}I_S(t)}{\mathrm{d}t} = (1-\rho)\pi E_k(t) - \delta I_S(t)$$

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} = \delta(I_A(t) + I_S(t))$$

Brookhart et al. *Statist. Med.* 2002; 21:3627–3638

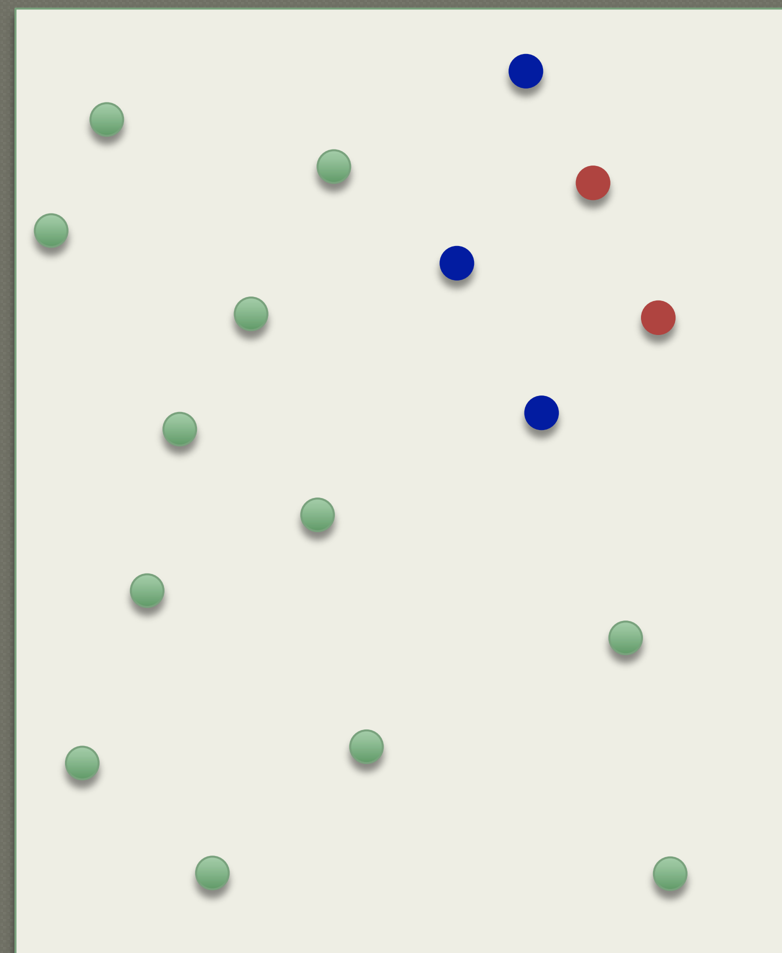
Network Models

- Connections between people modeled explicitly
- Disease passes from one individual to another along their connections
- Again, can be vastly complex networks



Agent-Based Models

- Computer simulations of many individuals in the same environment
- Can program rules that govern movement, disease susceptibility, interaction, etc.
- Have a type of randomness the other models don't have
- Depending on how you program them, results may resemble the previous models



Internet Resources for This Course

- The code, handout and PDF versions of the slides (no video) have been uploaded to GitHub
- https://github.com/elofgren/zombies
- GitHub is an online site for code storage, collaboration and sharing