Introduction to the SIR Model

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Basic Concepts

- Population modeled as being in one of a number of disease states
  - **S**usceptible: capable of being infected
  - **I**nfected/Infectious: infected, capable of spreading infection
  - **R**ecovered/Removed: recovered with immunity or dead

How the Model Works

- Population (usually) almost entirely susceptible
- Movement between compartments based on two rates:
  - β: Exposure and infection from exposure
  - γ: Duration of illness and probability of recovery

Difference Equations

$$S \xrightarrow{\beta} I \xrightarrow{\gamma} R$$

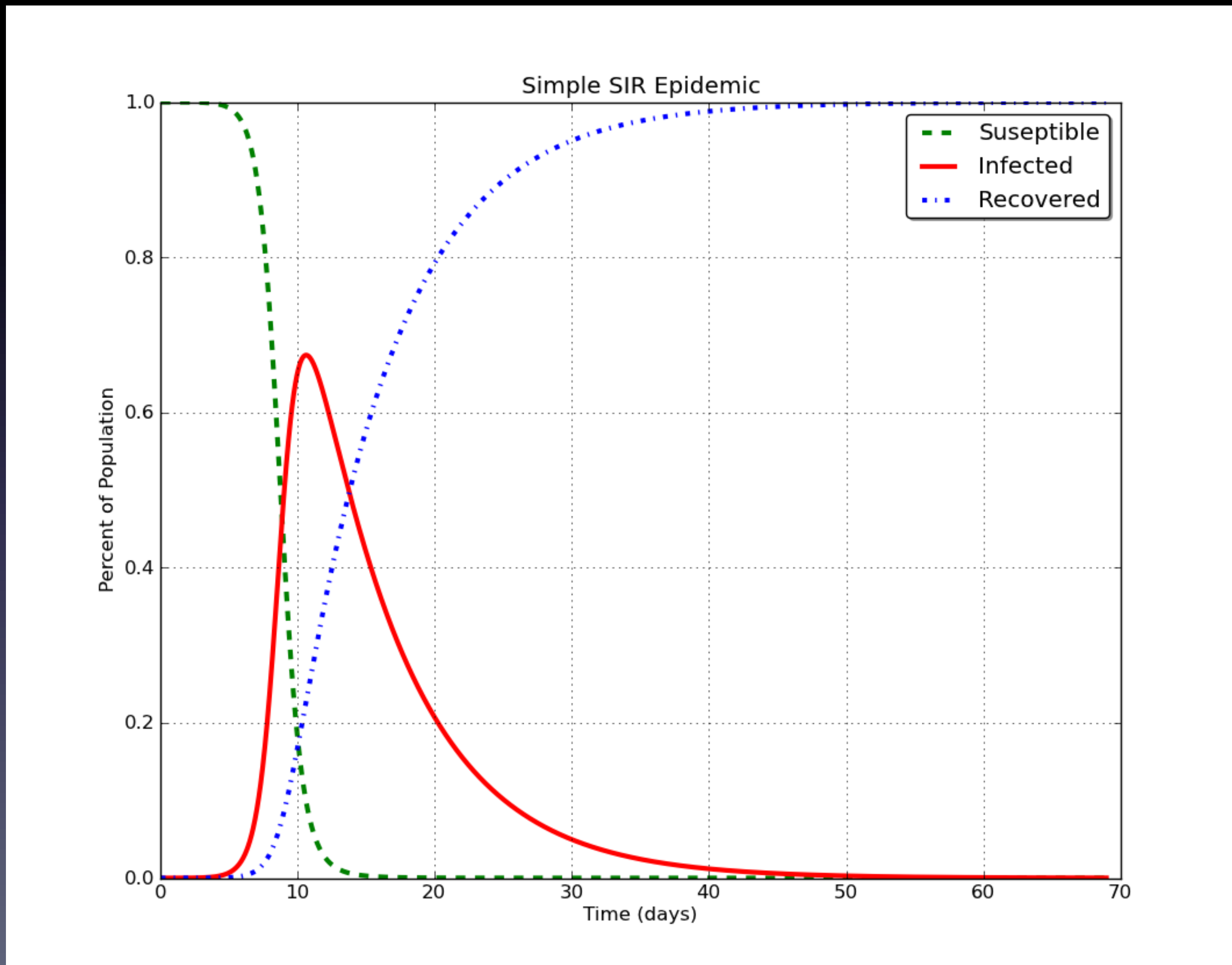
$$S_{t+1} = S_t - \beta I_t S_t$$

$$I_{t+1} = I_t + \beta I_t S_t - \gamma I_t$$

$$R_{t+1} = R_t + \gamma I_t$$

These equations are then run over many time points

What This Looks Like



Some Insights from SIR Models

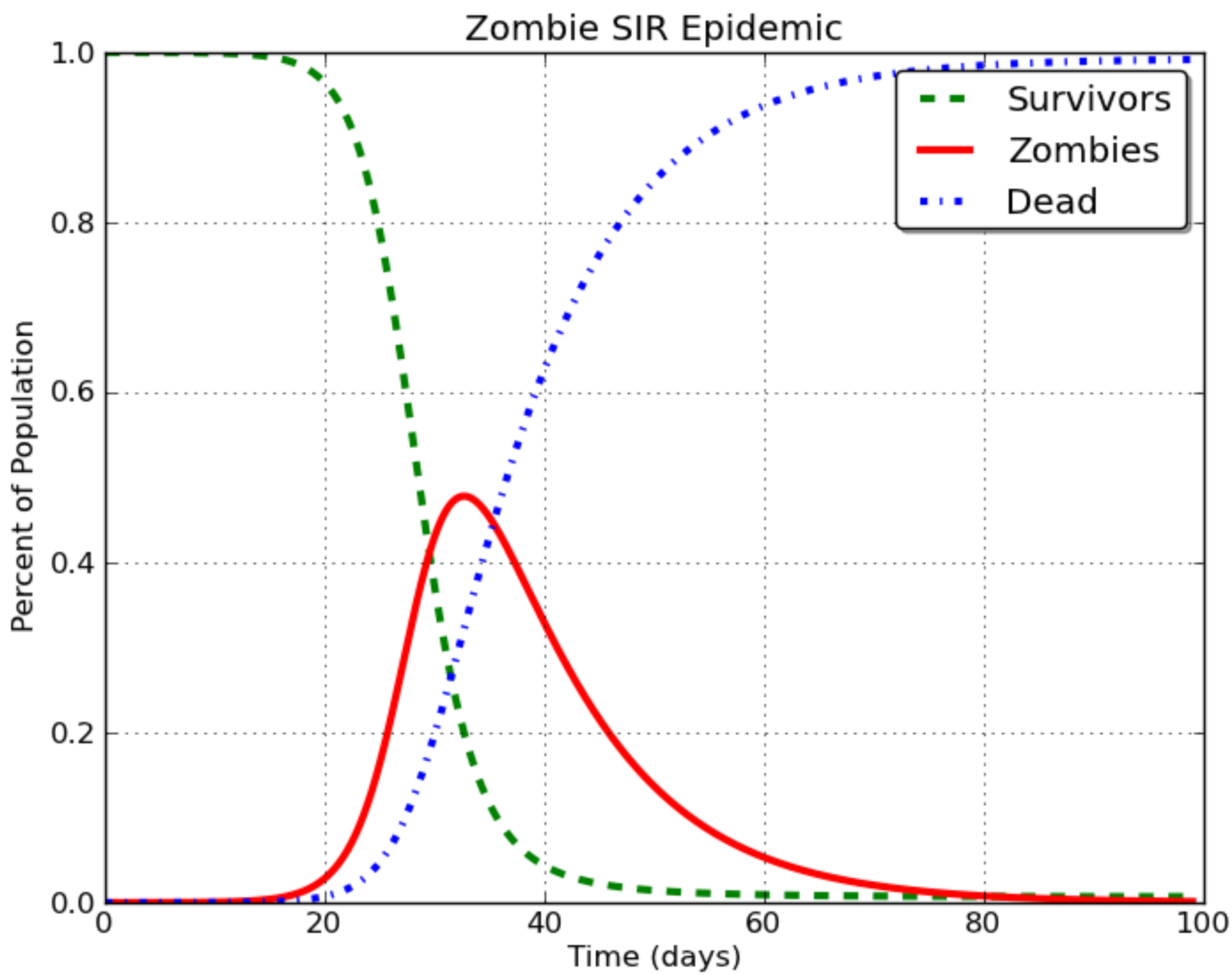
- Clear epidemic curves
- Social distancing
- R_0 : "Basic Reproductive Number"
  - Average number of cases a single infected individual will cause in an entirely susceptible population

$$R_0 = \frac{\beta}{\gamma}$$

- Must be greater than 1 for an epidemic to take off*
- If the fraction of the population susceptible to disease can be reduced to $1/R_0$ or lower the disease cannot invade*
- Altering γ or β can change R_0
- Real-world Examples:
  - Smallpox (3.5-6), Measles (16-18), Influenza (3-4), Rabies (2.44)

Zombie Parameters

- Assuming duration of infection = 10 days
  - $\gamma = 1/10$
- Fatality rate of 100%
- Assume an R_0 of 5.00 (~twice rabies)
- β=0.10*5.00=0.50

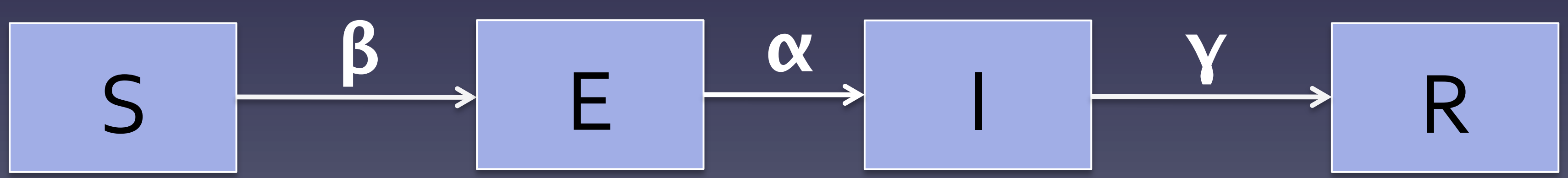

Zombie SIR Epidemic
Percent of Population
Time (days)
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
Survivors
Zombies
Dead

Are SIR Models Accurate?

- Yes and No
- Assumptions of the model:
  - The SIR process “accurately” reflects reality
    - We’ll get to complications and extensions of the process soon
  - Populations mix randomly
    - We’ll cover other types of models that don’t make this assumption later in the Learning Institute
  - Population is large

Latent/Incubation Period

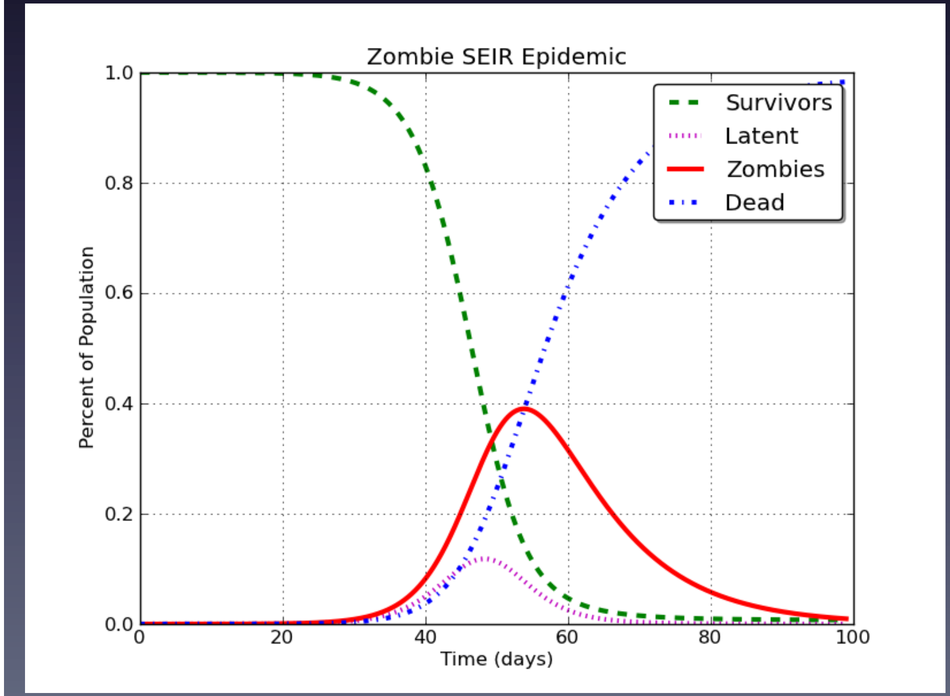
- Some time between the transition from being susceptible to being actively capable of transmitting disease
- Very common expansion of the SIR model
  - "SEIR"

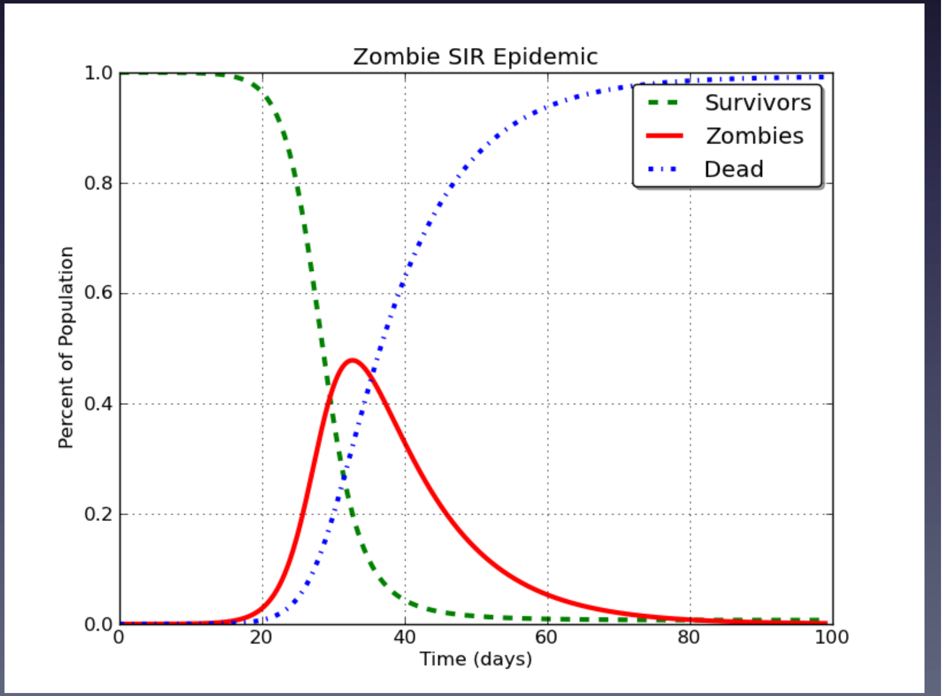


SEIR Equations and Results

$$S_{t+1} = S_t - \beta I_t S_t$$
$$E_{t+1} = E_t + \beta I_t S_t - \alpha E_t$$
$$I_{t+1} = I_t + \alpha E_t - \gamma I_t$$
$$R_{t+1} = R_t + \gamma I_t$$

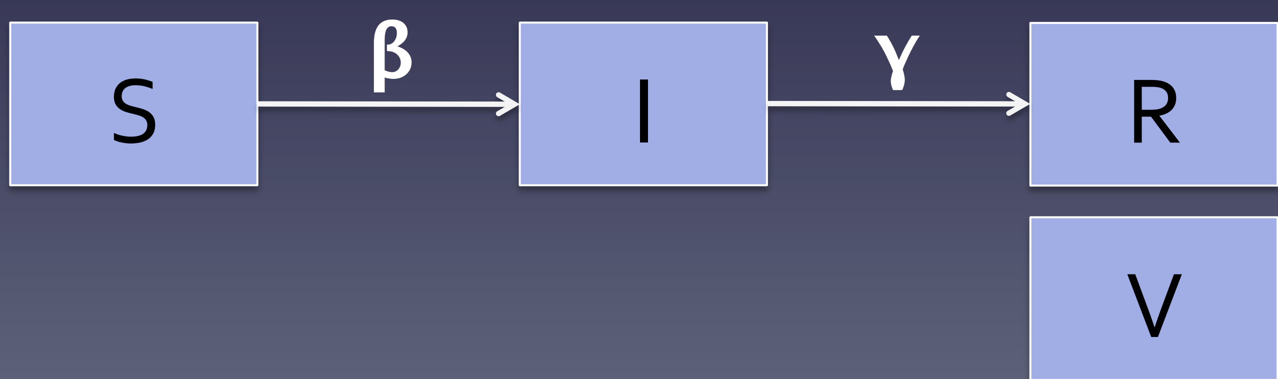
α =1/2





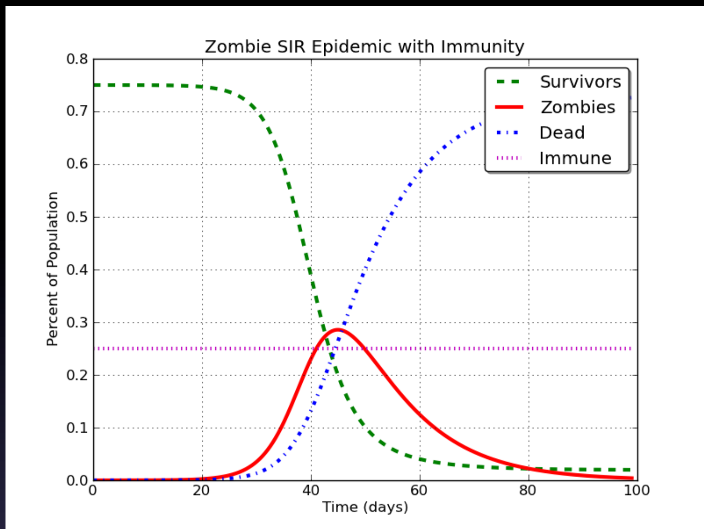
Immunity

- Some portion of the population is immune to infection
- Still mix randomly with everyone else
- Often move straight to R, in this case we have a new compartment V (Immune individuals)

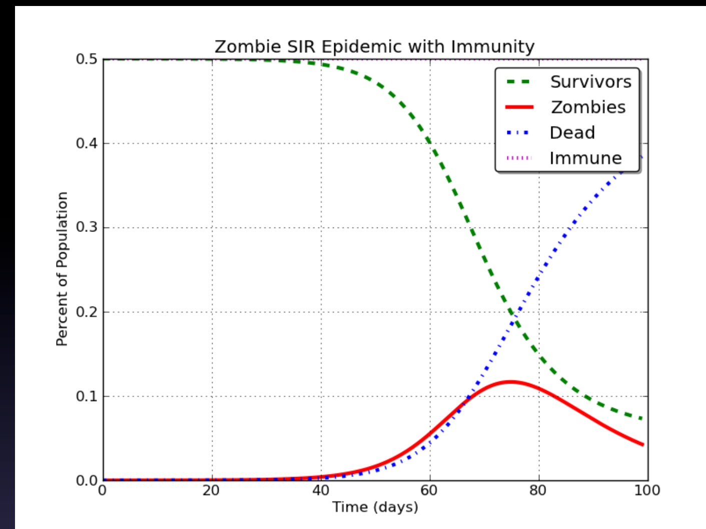


$$S_{t+1} = S_t - \beta I_t S_t$$
$$I_{t+1} = I_t + \beta I_t S_t - \gamma I_t$$
$$V_{t+1} = V_t$$
$$R_{t+1} = R_t + \gamma I_t$$

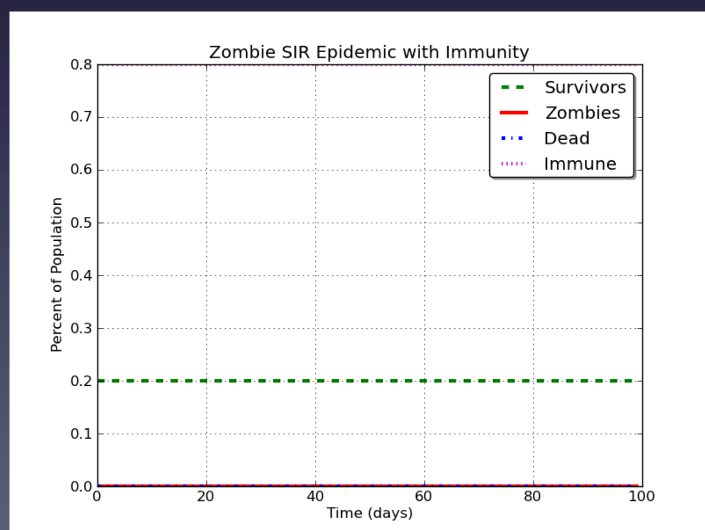
Immunity Results



25% Immune



50% Immune



80% Immune $1-(1/R_0)$

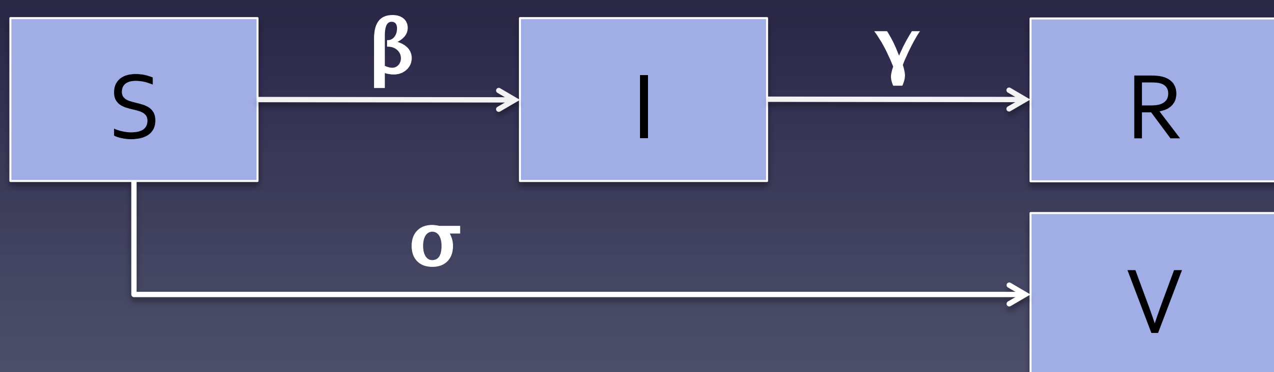
What About Vaccination?

$$S_{t+1} = S_t - \beta I_t S_t - \sigma S_t$$

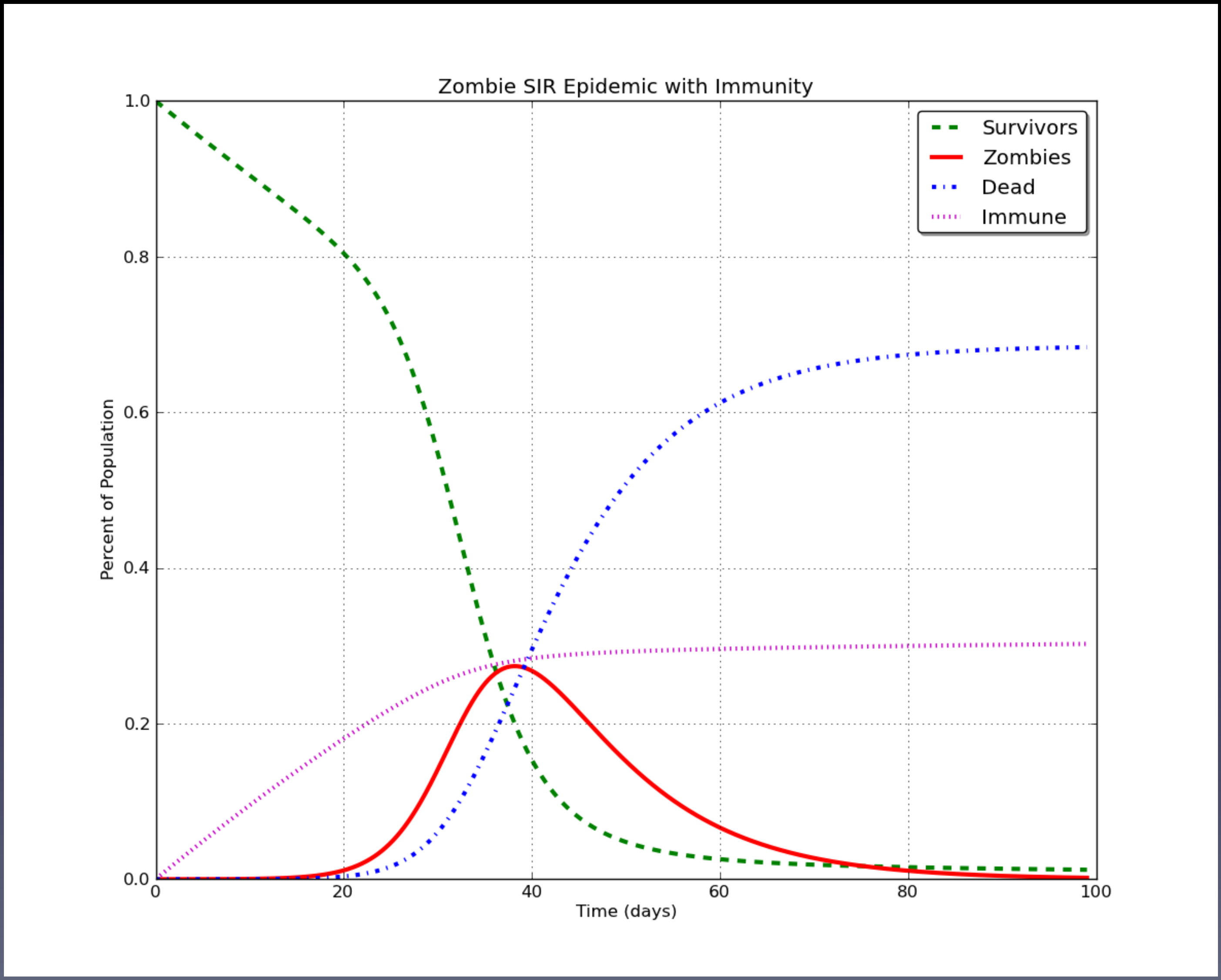
$$I_{t+1} = I_t + \beta I_t S_t - \gamma I_t$$

$$V_{t+1} = V_t + \sigma S_t$$

$$R_{t+1} = R_t + \gamma I_t$$



Vaccinate 1% of Susceptibles

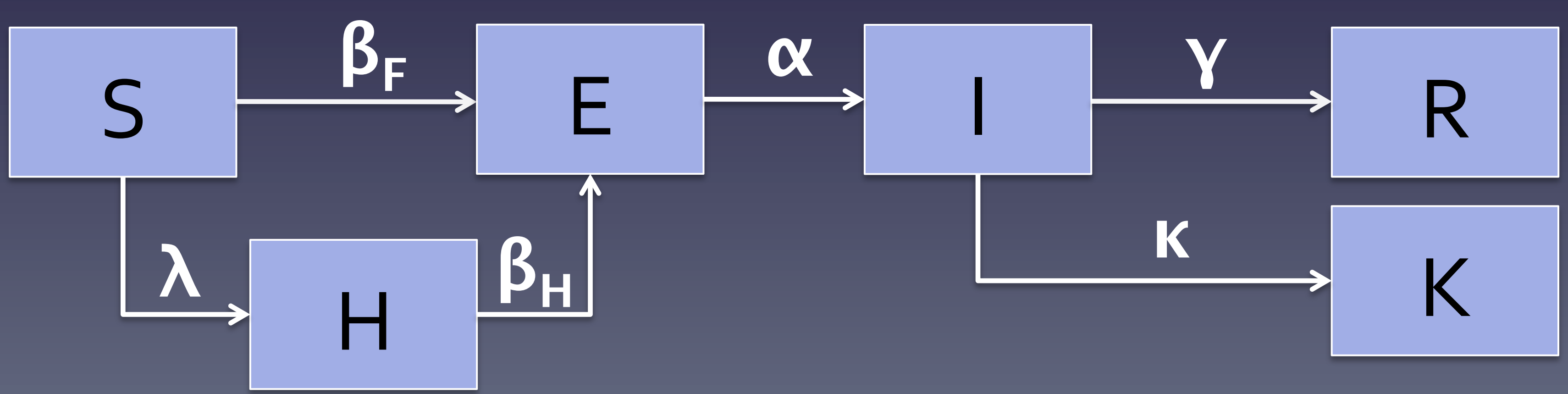


Infinite Variety

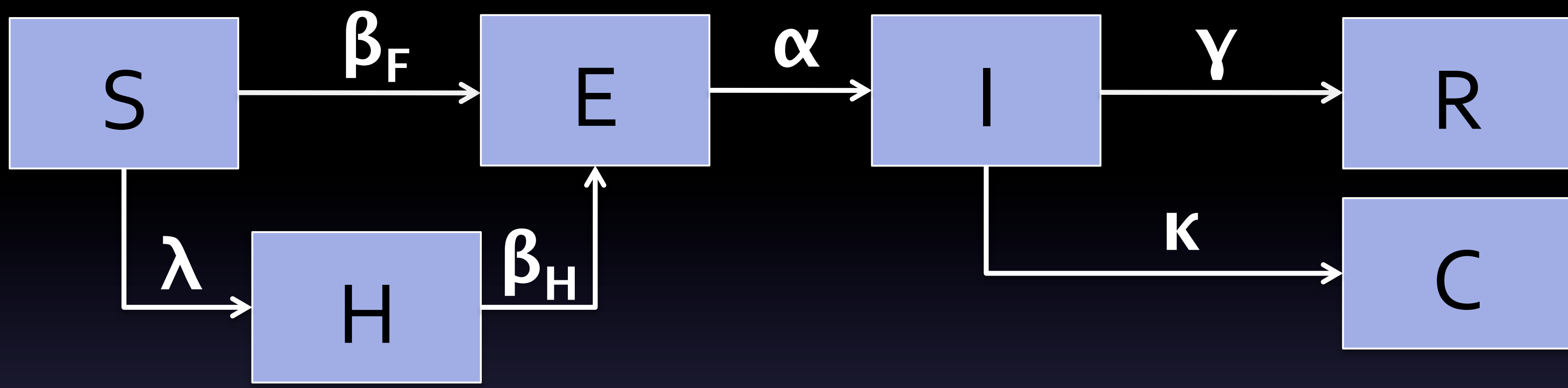
- Periodicity and seasonality
- Higher-order interactions between S and I
- Other vaccination strategies
- Quarantine
- Pulsed eradication/control
- Trade-offs: More difficult to program, results become more susceptible to odd interactions between variables, harder to come up with defensible parameters for all values

One More Elaborate Model

- Survivors find safe havens (H) at an average rate of 14 days
- *Much* lower interaction rate between shelter survivors and zombies ($\beta_F = 2x\beta_H$)
- Some number of zombies are killed by those in safe havens



Equation for the "Safe Haven" Model



$$S_{t+1} = S_t - \beta_F I_t S_t - \lambda S_t$$

$$H_{t+1} = H_t + \lambda S_t - \beta_H I_t H_t$$

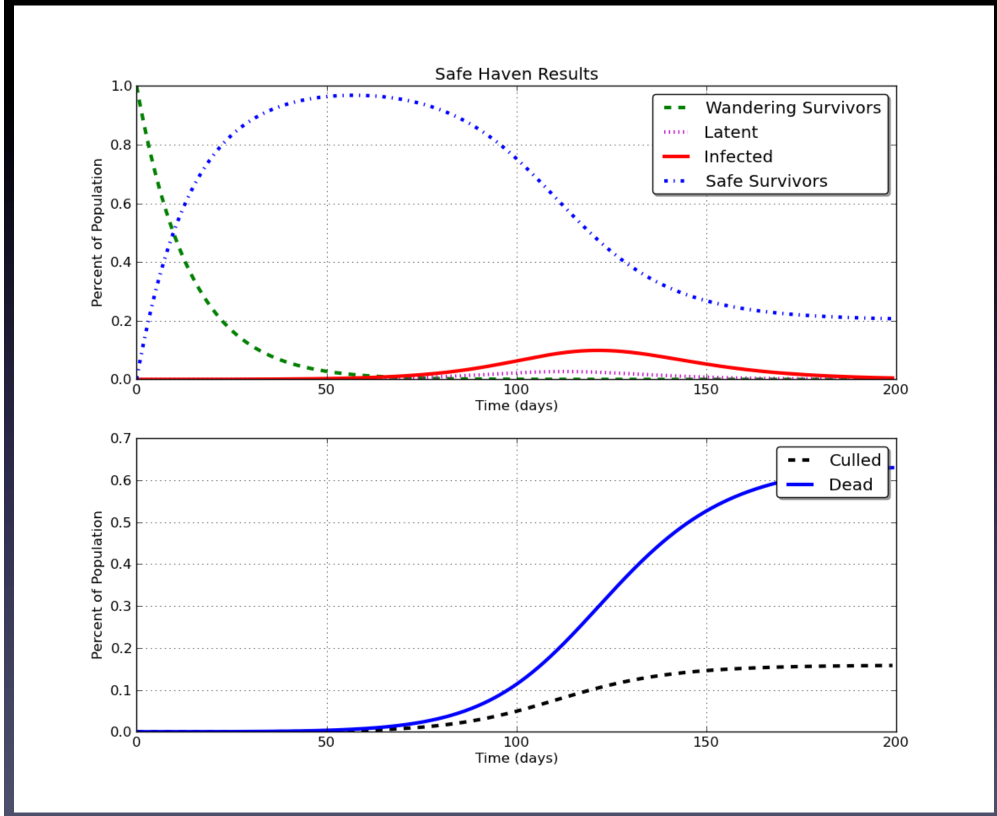
$$E_{t+1} = E_t + \beta I_t S_t + \beta_H I_t H_t - \alpha E_t$$

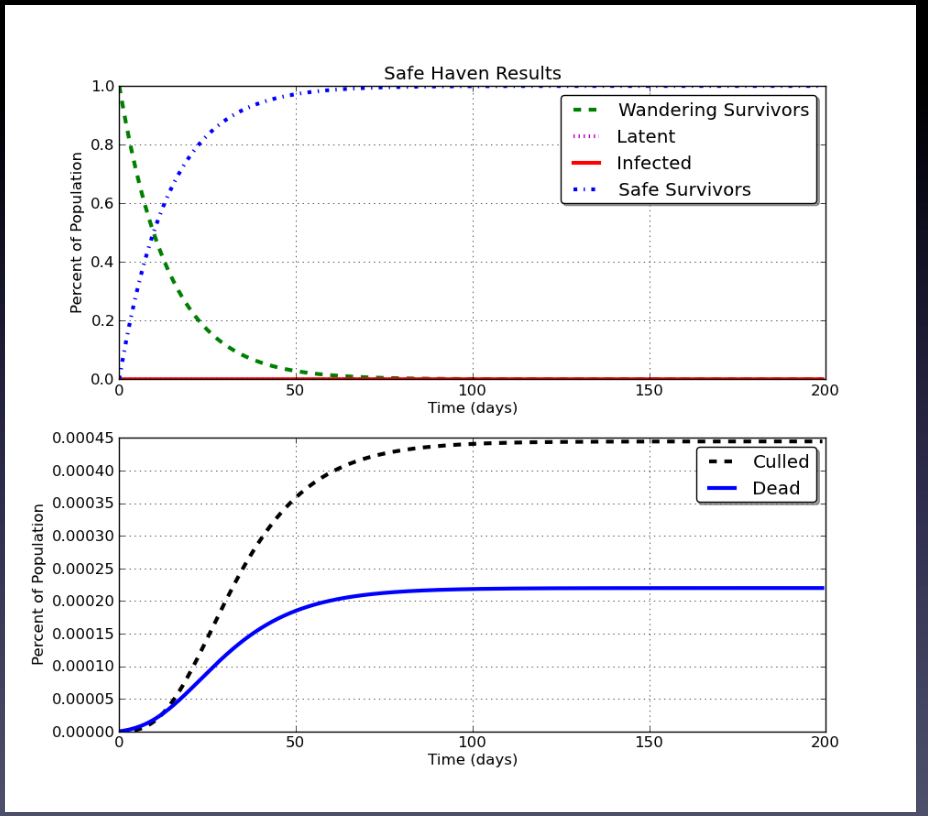
$$I_{t+1} = I_t + \alpha E_t - \gamma I_t - \kappa H_t I_t$$

$$C_{t+1} = C_t + \kappa H_t I_t$$

$$R_{t+1} = R_t + \gamma I_t$$

Results of "Safe Haven"





Kappa = 0.05

Kappa = 0.25

Where Do Parameters Come From?

- Can be obtained from data, leaving one or two parameters (usually β) unknown, and using the value that best fits the data
  - Problems with data quality
  - Right way is hard, easy way doesn't tell you what you think
- Estimates from the literature for some/all parameters
  - Good for established systems with good observational studies
  - Problem for novel diseases
- Guess
  - Expert opinion
  - Monte Carlo sensitivity analysis