Supplemental Note S2 for:

Complex genetic admixture histories reconstructed with Approximate Bayesian Computation

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We used the rectangular hyperbola class of functions to obtain increasing/decreasing patterns using only one shape parameter. We give here the derivation of the equations used, giving the example of a decreasing pattern.

A decreasing hyperbola is given by the function:

$$f(x) = \frac{a(1-x)}{a+x}$$
 (S2.1)

with $x \in [0; 1]$, $f(x) \in [0; 1]$ and $a \in [0; +\infty[$. Parameter a controls the shape ("steepness") of the curve obtained (figure S2.1).

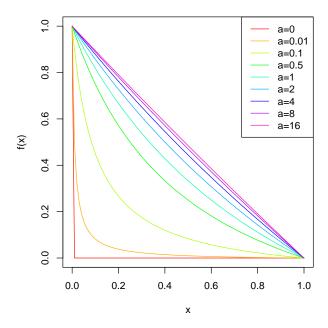


Figure S2.1: Influence of a on equation S2.1

The intersection between the hyperbola and y = x is given by

$$x = y = -a + \sqrt{a + a^2}$$

thus, we can sample an uniform deviate $u \in [0; \frac{1}{2}]$ and set parameter a:

$$a = \frac{u^2}{1 - 2u}$$

to obtain all hyperbola shapes.

We then transformed equation S2.1 to rescale the ranges of x and f(x):

$$f(x) = \frac{a(y_{max} - y_{min})(1 - \frac{x - x_{min}}{x_{max} - x_{min}})}{a + \frac{x - x_{min}}{x_{max} - x_{min}}} + y_{min}$$
(S2.2)

with $x \in [x_{min}; x_{max}]$ and $f(x) \in [y_{min}; y_{max}]$ (figure S2.2).

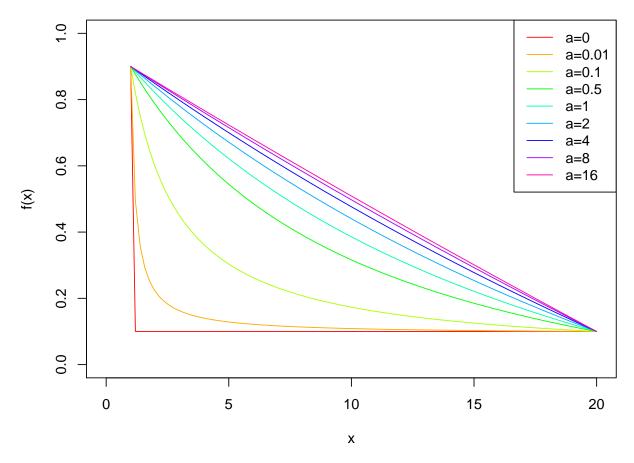


Figure S2.2: Influence of a on equation S2.2 with $x_{min}=1,\,x_{max}=20,\,y_{min}=0.1$ and $y_{max}=0.9$

With the notation used in the main text for contributions, and considering 20 generations of admixture, we obtain:

$$s_{S,g} = \frac{a(s_{S,1} - s_{S,20})(1 - \frac{g-1}{20-1})}{a + \frac{g-1}{20-1}} + s_{S,20}$$
(S2.3)

and an example of the patterns obtained for different u values is given in figure S2.3.

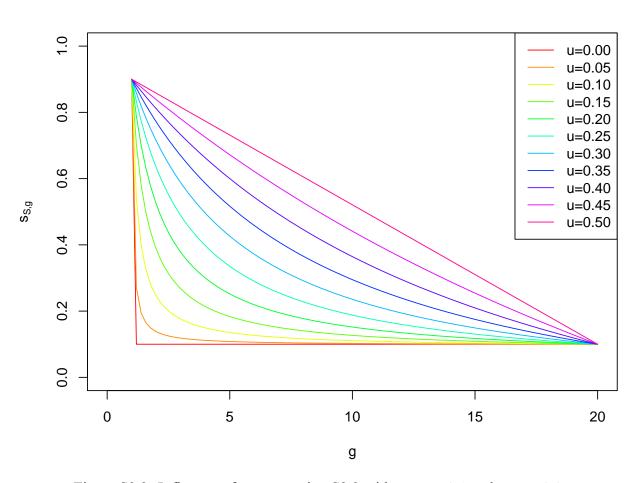


Figure S2.3: Influence of u on equation S2.3 with $s_{S,20}=0.1$ and $s_{S,1}=0.9$