Lab 7
APPLIED LINEAR ALGEBRA FOR IT - 501032

1 Exercises

Exercise 1: Determine if w is a linear combination of the vectors below

(a) $v_1 = (1, 2, 3, 4), v_2 = (-1, 0, 1, 3), v_3 = (0, 5, -6, 8)$ and $w = (3, -6, 17, 11)$

(b) $v_1 = (1, 1, 2, 2), v_2 = (2, 3, 5, 6), v_3 = (2, -1, 3, 6)$ and $w = (0, 5, 3, 0)$

(c) $v_1 = (1, 1, 2, 2), v_2 = (2, 3, 5, 6), v_3 = (2, -1, 3, 6)$ and $w = (-1, 6, 1, -4)$

(d) $v_1 = (1, 2, 3, 4), v_2 = (-1, 0, 1, 3), v_3 = (0, 5, -6, 8), v_4 = (1, 15, -12, 8)$ and $w = (0, -6, 17, 11)$

Exercise 2: Write program to verify

(a) Whether the vectors $\{(1, -2, 0)^T, (0, -4, 1)^T, (1, -1, 1)^T\}$ are linearly independent or not.

> **Hint:** let a linear combination which is zero and find solution.

(b) Whether the vectors $\{(1, 0, 2)^T, (0, 1, 4)^T, (2, -2, -4)^T\}$ are linearly independent or not. Find (a, b, c) that $a(1, 0, 2)^T + b(0, 1, 4)^T + c(2, -2, -4)^T = 0$. Store the values of (a, b, c) in a vector x.

(c) Whether the vectors $\{(1, -2, 3, 4), (2, 4, 5, 0), (-2, 0, 0, 4), (3, 2, 1, -1)\}$ are linearly independent or not.

(d) Whether the vectors $\{(0, 0, 1, 2, 3), (0, 0, 2, 3, 1), (1, 2, 3, 4, 5), (2, 1, 0, 0, 0), (-1, -3, -5, 0, 0)\}$ are linearly independent or not.

Exercise 3: Let the matrix

$$C = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 4 & -1 & 0 & 2 \\ 0 & -1 & -8 & -10 \end{bmatrix}$$

(a) Find a basis for the col(C)

(b) Find a basis for the row(C)

Exercise 4: Let the matrix

$$A_2 = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 4 & -1 & 0 & 2 \\ 0 & -1 & -8 & -10 \end{bmatrix}$$

(a) Find a basis for the null-space of A_2 and the stored answer as column vectors:
- v_1 = the first vector in basis
- v_2 = the second vector in basis

(b) Check to find if any linear combination of v_1 and v_2 is in null(A2). For example, $A_{2_{LC}} = A_2*(a*v_1 - b*v_2)$, find a, b

Exercise 5: Write the program to determine whether **w** is the column space of A, the null space of A, or both, where

$$\text{(a)}\quad w = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -3 \end{bmatrix}, A = \begin{bmatrix} 7 & 6 & -4 & 1 \\ -5 & -1 & 0 & -2 \\ 9 & -11 & 7 & -3 \\ 19 & -9 & 7 & 1 \end{bmatrix}$$

$$\text{(b)}\quad w = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} -8 & 5 & -2 & 0 \\ -5 & 2 & 1 & -2 \\ 10 & -8 & 6 & -3 \\ 3 & -2 & 1 & 0 \end{bmatrix}$$

Exercise 6: Let $a_1, a_2, ...a_5$ donate the columns of the matrix A, where

$$B = [a_1 a_2 a_4], A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix}$$

Explain why a_3 and a_5 are in the column space of B

Exercise 7: What is the dimention of span $\{(1,0,2)^T, (0,1,4)^T, (2,-2,-4)^T\}$. What is the basis for their span?

Exercise 8: Find a basis for the null space of the given matrix A.

(a) $A =$ Hilbert matrix with size 5

(b) $A =$ Pascal matrix with size 5

(c) $A =$ Magic matrix with size 5

Note:

- **Hilbert matrix**: A square with entries being the unit fractions

$$H(i,j) = \frac{1}{i+j-1}$$

- **Pascal matrix**: A is a symmetric positive definite matrix with integer entries taken from Pascal's triangle. There are three ways to achieve this: as either an upper-triangular matrix, a lower-triangular matrix, or a symmetric matrix.

$$P(i,j) = \frac{n!}{r!(n-r)!}$$

- **Magic matrix**: A square with numbers so that the total of each row, each column and each main diagonal are all the same.

Exercise 9: Write a function to show that $\{u_1, u_2, ..., u_n\}$ is an orthogonal set.

Hint: Consider each possible pairs of distinct vectors $< u_i u_j >= 0$ whenever $i \neq j$

For example $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $u_3 = \begin{pmatrix} \frac{-1}{2} \\ 2 \\ \frac{7}{2} \end{pmatrix}$

Exercise 10: Let **y** and **u** vector. Write a function to find the orthogonal projection of **y** on **u**.

Hint: $proj_u y = \dfrac{< y \cdot u >}{< u \cdot u >} u$. For example, $y = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$ and $u = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Exercise 11: Let a matrix $m \times n$, write a function to check that has orthonormal columns.

Hint: An $m \times n$ matrix U has orthonormal columns if and only if $U^T U = I$

Exercise 12: Use the Gram - Schmidt process to produce an orthogonal basis for column space of

$$\mathbf{A} = \begin{pmatrix} -10 & 13 & 7 & -11 \\ 2 & 1 & -5 & 3 \\ -6 & 3 & 13 & -3 \\ 16 & -16 & -2 & 5 \\ 2 & 1 & -5 & -7 \end{pmatrix}$$