Lab 4-5
APPLIED LINEAR ALGEBRA FOR IT - 501032

1 Exercises

Exercise 1: Find the solution of the linear systems

(a)
$$\begin{aligned} x + 2y + z &= 0 \\ 2x - y + z &= 0 \\ 2x + y \quad &= 0 \end{aligned}$$

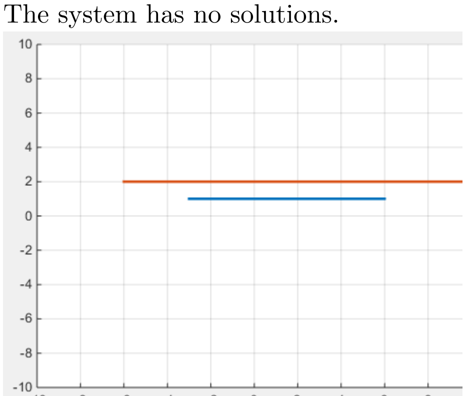
(b)
$$\begin{aligned} 2x + y + z + t &= 1 \\ x + 2y + z + t &= 1 \\ x + y + 2z + 2t &= 1 \\ x + y + z + 2t &= 1 \end{aligned}$$

Exercise 2: Consider the following system of linear equations:

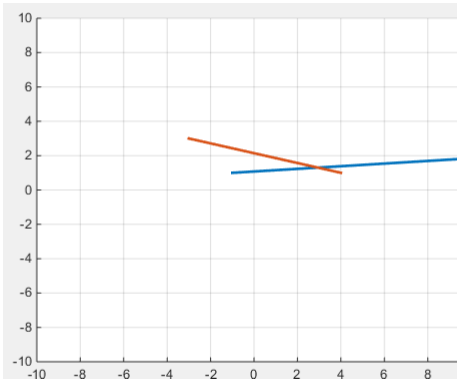
$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \\ a_3x + b_3y &= c_3 \end{aligned}$$

For real values of a_i,b_i and c_i, the graphs of these equations are lines in a two dimensional (x, y) coordinate system. Write a function to investigate the number of solutions for system in two variables such that

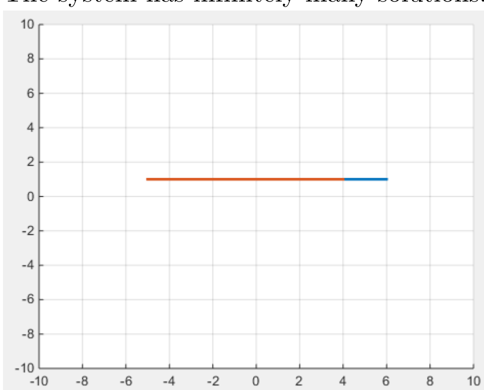
(a) The system has no solutions.



(b) The system has a unique solution.



(c) The system has infinitely many solutions.



Exercise 3: Consider the following system of linear equations:

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

For real values of a_i,b_i, c_i and d_i, the graphs of these equations are the planes in a three dimensional (x, y, z) coordinate system. You should write a function to investigate the number of solutions for system in two variables such that

(a) The system has no solutions.

(b) The system has a unique solution.

(c) The system has infinitely many solutions.

Exercise 4: Let

$$\begin{aligned} x + y + 2z &= 1 \\ 3x + 6y - 5z &= -1 \\ 2x + 4y - 3z &= 0 \end{aligned}$$

(a) Find the determinant of A. Is A invertible?

(b) Use the inverse matrix to find the solution for this linear system.

(c) (*) Use Gaussian elimination method to find the solution of this linear system.

(d) (*) Use Gaussian elimination with Partial Pivot method to find the solution of this linear system.

(e) (*) Use LU method to find the solution of this linear system.

Exercise 5: Try to use the three different methods to solve the following system:

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + 2y + 2z &= 1 \\ 2x + 4y + z &= 2 \end{aligned}$$

Exercise 6: Find the interpolating polynomial $p(t) = a_0 + a_1 t + a_2 t^2$ for the data (1,6), (2,15), (3,38). That is, find a_0, a_1 and a_2

Exercise 7: A group took a trip on a bus, at \$3 per child and \$3.2 per adult for a total of \$118.4. They took the train back at \$3.5 per child and \$3.6 per adult for a total of \$135.2. How many children and how many adults?

Exercise 8: Global positioning system (GPS) is used to find our location. Satellites send signals to a receiver on earth with information about the location of the satellite in xyz-coordinate system and the time t when the signal was transmitted. After some calculations we obtain the following three linear equations. Solve these equations.

$$\begin{aligned} 2x - 4y + 4z + 0.077t &= 3.86 \\ -2y + 2z - 0.056t &= -3.47 \\ 2x - 2y &= 0 \end{aligned}$$

Exercise 9: The actual color a viewer sees on a screen is influenced by the specific type and amount of phosphors on the the screen. So each computer screen manufacturer must convert between the (R,G,B) data and an international CIE standard for color, which uses three primary colors, called X, Y, and Z. A typical conversion for short-persistence phosphors is

$$\begin{pmatrix} 0.61 & 0.29 & 0.15 \\ 0.35 & 0.59 & 0.063 \\ 0.04 & 0.12 & 0.787 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Find the equation that converts the standard CIE data to (R,G,B)

Exercise 10: The Leontief model represents the economy as a linear system. Consider a particular economy which depends on oil (O), energy (E) and services (S). the input-output matrix A of such an economy is given by

$$\begin{array}{c c} & \begin{array}{ccc} \mathrm{O} & \mathrm{E} & \mathrm{S} \end{array} \\ \begin{array}{c} \mathrm{O} \\ \mathrm{E} \\ \mathrm{S} \end{array} & \begin{pmatrix} 0.25 & 0.15 & 0.1 \\ 0.4 & 0.15 & 0.2 \\ 0.15 & 0.2 & 0.2 \end{pmatrix} = \mathbf{A} \end{array}$$

The numbers in the first row are produced as follows: To produce one unit of oil industry uses 0.25 units of oil, 0.15 units of energy and 0.1 units of services. Similarly the numbers in the other rows are established. The production vector $\mathbf{p}$ and the demand vector $\mathbf{d}$ satisfies $\mathbf{p} = \mathbf{A}\mathbf{p} + \mathbf{d}$. Determine the production vector $\mathbf{p}$ if $\mathbf{d} = \begin{pmatrix} 100 & 100 & 100 \end{pmatrix}^T$

Exercise 11: Chemical equations describe the quantities of substances consumed and produced by chemical reactions. For instance

$$(x_1)C_3H_8 + (x_2)O_2 \longrightarrow (x_3)CO_2 + (x_4)H_2O$$

To *balance* this equation, a chemist must find whole numbers $x_1, ..., x_4$ such that the total numbers of carbon (C), hydrogen (H), and oxygen (O) atoms on the left match the corresponding numbers of atoms on the right.

Hint: Three types of atoms (C, H, O) construct a vector in R^3.