Lab 08 - 09
APPLIED LINEAR ALGEBRA FOR IT - 501032

1 Exercises

Exercise 1: To find the least square solution to $\mathbf{Ax} = \mathbf{b}$. For this case, the equation $A^TAx = A^Tb$ with

$$\mathbf{A} = \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

Exercise 2: Solve the linear system below and find the least squares solutions.

$$\begin{cases} e = 0.5 \\ d + e = 1.6 \\ c + 2d + e = 2.8 \\ c + e = 0.8 \\ 4c + d + e = 5.1 \\ 4c + 2d + e = 5.9 \end{cases} \Leftrightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \\ 4 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} c \\ d \\ e \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1.6 \\ 2.8 \\ 0.8 \\ 5.1 \\ 5.9 \end{pmatrix} \Leftrightarrow \mathbf{Ax} = \mathbf{b}$$

Exercise 3: Find the equation $y = a_0 + a_1x$ of the least squares line that best fits the given data points.

(a) $(0, 1), (1, 1), (2, 2), (3, 2)$

(b) $(1, 0), (2, 1), (4, 2), (5, 3)$

(c) $(-1, 0), (0, 1), (1, 2), (2, 4)$

(d) $(2, 3), (3, 2), (5, 1), (6, 0)$

Exercise 4: An engineer is tracking the friction index over mileage of a breaking system of a vehicle. She expects that the mileage-friction relationship is approximately linear and she collects five data points that are show in the table below.

Mileage	2000	6000	20000	30000	40000
Friction Index	20	18	10	6	2

Write a function to describe these points above and the approximately points by graph.

Hint: Consider $\mathbf{b}$ is the vector of friction index data values and $\mathbf{y}$ values when we plug in the mileage data for $\mathbf{x}$ and find $\mathbf{y}$ by the equation of the line $ax + b = y$. We want minimizes the distance between $\mathbf{b}$ and $\mathbf{y}$.

Exercise 5: A cetain experiment produces the data $(1, 7.9), (2, 5.4)$ and $(3, -9)$. Describe the model that produces a least squares fit of these points by a function of the f orm

$$y = Acosx + Bsinx$$

Exercise 6: A bioengineer is studying the growth of a genetically engineered bacteria culture and suspects that is it approximately follows a cubic model. He collects six data points that are show table below.

Time in Days	1	2	3	4	5	6
Grams	2.1	3.5	4.2	3.1	4.4	6.8

He assumes the equation has the cubic form $ax^3 + bx^2 + cx + d = y$. Write a function to describe these points above by graph.

Exercise 7: We define the 2×2 matrix $S_{\lambda,\mu}$ as

$$S_{\lambda,\mu} = \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$$

When we multiply $S_{\lambda,\mu}$ to any vector $v \in R^2$, we get

$$S_{\lambda,\mu} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda x \\ \mu y \end{bmatrix}$$

Execute commands trans 2×2.

(a) Describe geometrically the action of $S_{2,2}$ with $\lambda = 2$ and $\mu = 2$.

(b) Describe geometrically the action of $S_{0.5,0.5}$ with $\lambda = 0.5$ and $\mu = 0.5$.

(c) Describe geometrically the action of $S_{1,-1}$ with $\lambda = 1$ and $\mu = -1$.

(d) Describe geometrically the action of $S_{-1,1}$ with $\lambda = -1$ and $\mu = 1$.

Exercise 8: We define the 2×2 matrix R_θ as

$$R_\theta = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

Execute commands trans 2×2.

(a) Describe geometrically the action of R_θ with $\theta = \pi$.

(b) Describe geometrically the action of R_θ with $\theta = \dfrac{\pi}{3}$.

Exercise 9: Draw the house and its image when you perform transformations:

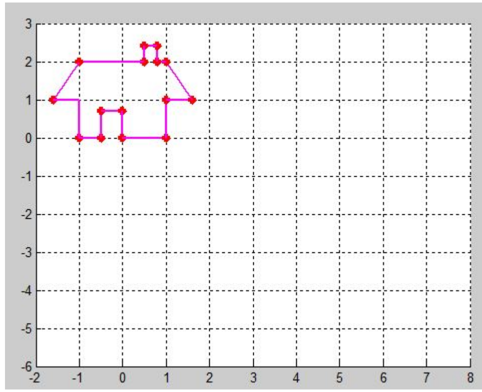
(a) **Translation** with $t_x = 2$ and $t_y = 4$

(b) **Rotation** with $\alpha = \pi/3$

(c) **Scaling** with $S_x = 2$ and $S_y = 3$

(d) **Shear**

- along x with $Shear_x$ = 0.5
- along y with $Shear_y$ = -1.5

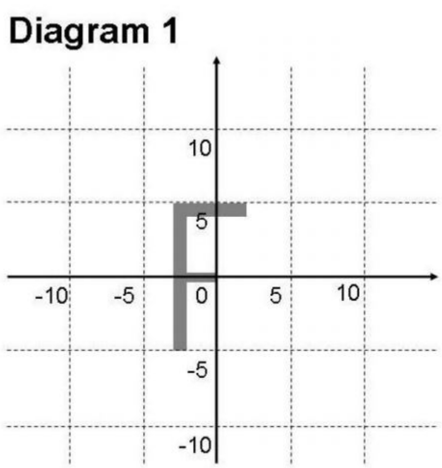


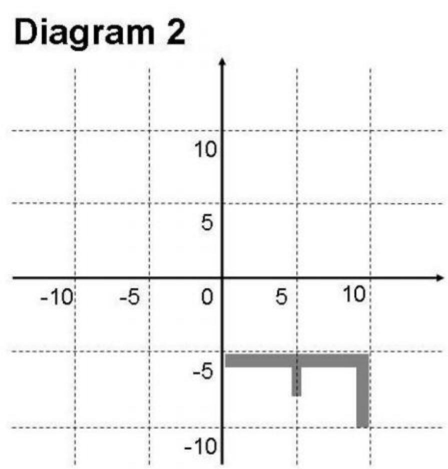
Exercise 10: Let $P(1,1), Q(3,1), R(1,3)$ are the vertices of a triangle, and matrix A represent the vertices of this triangle. Determine the image of the triangle PQR under the transformation $(-I)A$.

Exercise 11: Define the following map from R^2 to R^3:

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Hence representing each 2D point by a 3D coordinate ($w = 1$).





(a) Perform transformations on the original image of Figure F using each of the following transformation matrices, and observe the corresponding geometrical changes,

$$T_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, T_2 = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}, T_3 = \begin{bmatrix} 1 & 0 & \frac{5}{4} \\ 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 1 \end{bmatrix}, T_4 = \begin{bmatrix} \frac{1}{4} & 0 & \frac{5}{4} \\ 0 & \frac{1}{4} & -\frac{5}{4} \\ 0 & 0 & 1 \end{bmatrix},$$

$$T_5 = \begin{bmatrix} -\frac{1}{4} & 0 & 0 \\ 0 & -\frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}, T_6 = \begin{bmatrix} -\frac{1}{4} & 0 & -\frac{5}{4} \\ 0 & -\frac{1}{4} & \frac{5}{4} \\ 0 & 0 & 1 \end{bmatrix}, T_7 = \begin{bmatrix} 1 & -1 & \frac{5}{4} \\ 0 & 0 & \frac{5}{4} \\ 0 & 0 & 1 \end{bmatrix},$$

(b) Draw on Diagram 2, the image of the transformation of the Figure F in Diagram 1, represented by the matrix

$$T = \begin{bmatrix} 1 & 0 & -2 \\ 0 & -2 & 1 \\ 9 & 0 & 1 \end{bmatrix}$$