Lab 10
APPLIED LINEAR ALGEBRA FOR IT - 501032

1 Exercises

Exercise 1: Find the eigenvalues of

(a)
$$A = \begin{pmatrix} -1 & 3.5 & 14 \\ 0 & 5 & -26 \\ 0 & 0 & 2 \end{pmatrix}$$

(b)
$$B = \begin{pmatrix} -2 & 0 & 0 \\ 99 & 0 & 0 \\ 10 & -4.5 & 10 \end{pmatrix}$$

(c)
$$C = \begin{pmatrix} 5 & 5 & 0 & 2 \\ 0 & 2 & -3 & 6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

(d)
$$D = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 6 & 2 & 0 & 0 \\ 0 & 3 & 6 & 0 \\ 2 & 3 & 3 & -5 \end{pmatrix}$$

(e)
$$E = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 & 0 \\ 3 & 8 & 0 & 0 & 0 \\ 0 & -7 & 2 & 1 & 0 \\ -4 & 1 & 9 & -2 & 3 \end{pmatrix}$$

Then, compute the determinant of A, B, C, D, E matrix based on the eigenvalues.

Exercise 2: Let
$$A = \begin{pmatrix} -6 & 28 & 21 \\ 4 & -15 & -12 \\ 8 & a & 25 \end{pmatrix}$$

For each a in the set { 32, 31.9, 31.8, 32.1, 32.2}, compute the characteristic of A and the eigenvalues. In each case, create a graph of the characteristic polynomial $p(t) = det(A - \lambda I)$ for $0 \le t \le 3$. If possible, construct all graphs on one coordinate system.

Exercise 3: Let $M \begin{bmatrix} -3 & -5 & -7 \\ -2 & 1 & 0 \\ 1 & 5 & 5 \end{bmatrix}$

(a) Use any appropriate software to find the eigenvalues of M

(b) For each eigenvalue λ found above, find the corresponding eigenvector **v** of **M** by using row reduction to solve $(M - \lambda I)v = 0$

(c) Construct a matrix **P** whose columns are the eigenvectors of **M**. Compute the product **D**=$P^{-1}MP$ and confirm that **D** is diagonal. Compute the determinants of **D** and **M**, then confirm that they are equal.

(d) Construct another matrix **Q** whose columns are also the eigenvectors of **M** but this time place them in a different order than in **P** (for example, the first of columns of the eigenvectors of M is the second column of Q) and then compute $Q^{-1}MQ$ again. What has changed?

Exercise 4: Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Use any appropriate software to find the eigenvalues and corresponding eigenvectors of A, A^T and A^{-1}. What do you observe ?

Exercise 5: Let $A_1 = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} A_4 = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

$A_5 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

Use any appropriate software to verify the matrices above are diagonalizable or not.

Exercise 6: Find the eigenvectors of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ Using it, construct a matrix **P** that diagonalizes **A**. Compute $P^{-1}AP$

Exercise 7: Find the singular values of the matrices

$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} -5 & 0 \\ 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} \sqrt{6} & 1 \\ 0 & \sqrt{6} \end{bmatrix}, A_4 = \begin{bmatrix} \sqrt{3} & 2 \\ 0 & \sqrt{3} \end{bmatrix}$

Exercise 8: Compute SVD of each matrix below.

$B_1 = \begin{bmatrix} -18 & 13 & -4 & 4 \\ 2 & 19 & -4 & 12 \\ -14 & 11 & -12 & 8 \\ -2 & 21 & 4 & 8 \end{bmatrix}, B_2 = \begin{bmatrix} 6 & -8 & -4 & 5 & -4 \\ 2 & 7 & -5 & -6 & 4 \\ 0 & -1 & -8 & 2 & 2 \\ -1 & -2 & 4 & 4 & -8 \end{bmatrix}$

Exercise 9: Write a program to compress image which an input image is given by user.

Exercise 10: Write a program to implement Recommender System. It will be able to recommend an item x to an user y. Let $M_{n\times m}$ matrix present the relation of users and items

$$M_{n\times m} = \left(\begin{array}{c|ccc} & item1 & item2 & item3 \\ \text{user 1} & \infty & \infty & 3 \\ \text{user 2} & 2 & \infty & 5 \\ \text{user 3} & 5 & 6 & 7 \\ \text{user 4} & \infty & \infty & \infty \end{array}\right)$$

where ∞ means that user y has never rated item x before.