Lab 6
APPLIED LINEAR ALGEBRA FOR IT - 501032

1 Exercises

Exercise 1: Write command to find 1-norm of the following matrices:

(a) $$A_1 = \begin{pmatrix} 1 & -7 \\ -2 & -3 \end{pmatrix}$$

(b) $$A_2 = \begin{pmatrix} -2 & 8 \\ 3 & 1 \end{pmatrix}$$

(c) $$A_3 = \begin{pmatrix} 2 & -8 \\ 3 & 1 \end{pmatrix}$$

(d) $$A_4 = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$$

(e) $$A_5 = \begin{pmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{pmatrix}$$

Note: Do not use the built-in function (i.e. norm function) to find norm.

Exercise 2: Write command to find infinity-norm of the following matrices:

(a) $$B_1 = \begin{pmatrix} 1 & -7 \\ -2 & -3 \end{pmatrix}$$

(b) $$B_2 = \begin{pmatrix} 3 & 6 \\ 1 & 0 \end{pmatrix}$$

(c) $$B_3 = \begin{pmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{pmatrix}$$

(d) $$B_4 = \begin{pmatrix} 3 & 6 & -1 \\ 3 & 1 & 0 \\ 2 & 4 & -7 \end{pmatrix}$$

(e) $$B_5 = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Note: Do not use the built-in function (i.e. norm function) to find norm.

Exercise 3: Write command to find calculate the Frobenius-norm

(a) $$C_1 = \begin{pmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{pmatrix}$$

(b) $$C_2 = \begin{pmatrix} 1 & 7 & 3 \\ 4 & -2 & -2 \\ -2 & -1 & 1 \end{pmatrix}$$

(c) $$C_3 = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$$

Note: Do not use the built-in function (i.e. norm function) to find norm.

Exercise 4: Let u and v be vectors in R^2. For the following u and v determine the angle between the vectors.

(a)
$$u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(b)
$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(c)
$$u = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \qquad v = \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$$

Exercise 5: Determine the unit vector $\hat{u}$ for each of the following vectors.

(a) $u = (2, \quad 3)^T$

(b) $u = (1, \quad 2, \quad 3)^T$

(c) $u = (1/2, \quad -1/2, \quad 1/4)^T$

(d) $u = (\sqrt{2}, \quad 2, \quad -\sqrt{2} \quad \sqrt{2})^T$

Exercise 6: Let $v_1 = (1, 2, 3)^T, \quad s_2 = (7, 4, 3)^T$, and $s_3 = (2, 1, 9)^T$ be the position of three satellites. Find the Euclidean distances between satellites.

Exercise 7: Write a function to decode the encoded following message $E = \begin{pmatrix} 80 & 98 & 99 & 85 & 106 & 94 \\ 71 & 92 & 76 & 95 & 100 & 92 \\ 124 & 163 & 140 & 160 & 176 & 161 \end{pmatrix}$. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 4 \end{pmatrix}$ and lookup table

Alphabet	A	B	C	D	...	W	X	Y	Z	
Position	1	2	3	4	...	23	24	25	26	27
Position +3	4	5	6	7	...	26	27	28	29	30

Hint: In order to decode the message D, you need to know the inverse matrix A^{-1} and then you calculate $A^{-1}E$ to get the matrix D. Remember that the matrix D contains the message. Finally, using the above lookup table to obtain the message which can read.

Exercise 8: Using lookup table in the Exercise 8, write a function to encode the following message

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using the matrix $A = \begin{pmatrix} 3 & 4 & 5 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

Hint: In order to encode the message, you need to create the matrix E with lookup table and then calculate AE.

Exercise 9: Write a function to calculate the similarities among documents. Consider the following document-term matrix

	T1	T2	T3	T4	T5	T6	T7	T8
Doc1	0	4	0	0	0	2	1	3
Doc2	3	1	4	3	1	2	0	1
Doc3	3	0	0	0	3	0	3	0
Doc4	0	1	0	3	0	0	2	0
Doc5	2	2	2	3	1	4	0	2

Using the Cosine Similarity $= \frac{Doc_i \cdot Doc_j}{\|Doc_i\|_2 \|Doc_j\|_2}$. Cosine Similarity is used to measure the angle between two unit length.

Exercise 10: Write a function reuse the Cosine Similarity measure to retrieve the documents which is the nearest with vector $\mathbf{q} = \begin{pmatrix} 0 & 0 & 0.7 & 0.5 & 0 & 0.3 \end{pmatrix}$. Given the documents are represented as vectors.

	nova	galaxy	heat	actor	film	role
D1	1.0	0.5	0.3	0	0	0
D2	0.5	1.0	0	0	0	0
D3	0	1.0	0.8	0.7	0	0
D4	0	0.9	1.0	0.5	0	0
D5	0	0	0	1.0	0	1.0
D6	0	0	0	0	0.7	0
D7	0.5	0	0.7	0	0	0.9
D8	0	0.6	0	1.0	0.3	0.2