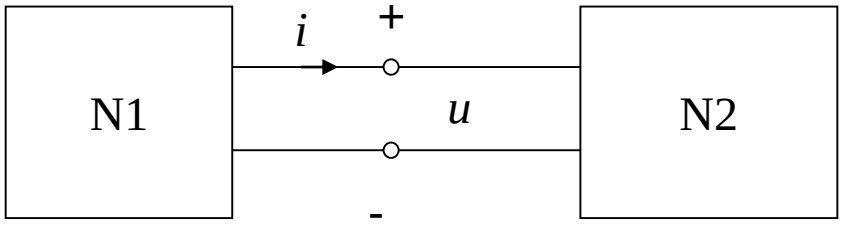
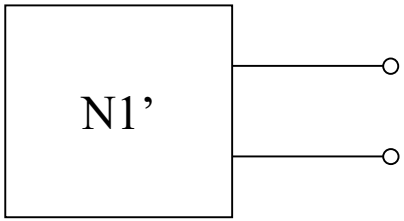
第二章 电阻电路的等效变换

- 等效电路的概念
- 电阻的串联和并联
- 电阻的Y形和△形联结
- 电压源和电流源的串联和并联
- 实际电源的两种模型及其等效变换
- 输入电阻

§2-2等效电路的概念

- 等效电路的概念



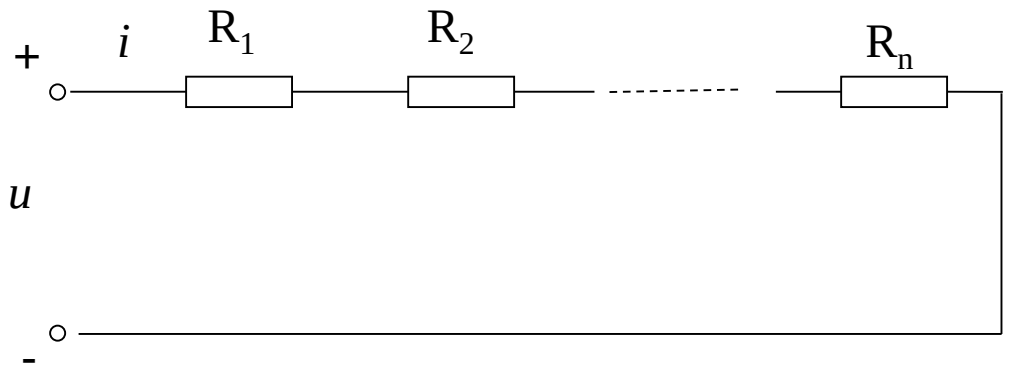


端口外特性：二端口外部端子的u和i之间的关系

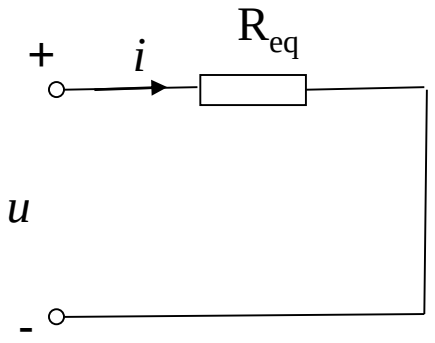
如果 N1 和 N1’ 的外特性相同，则称两者等效。

§2-3电阻的串联和并联

- 电阻的串联

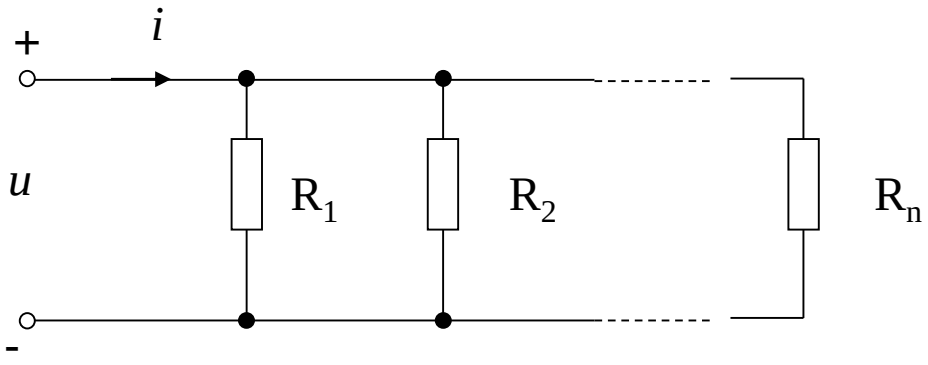


$$R_{eq}=\sum_{k=1}^{n}R_k$$

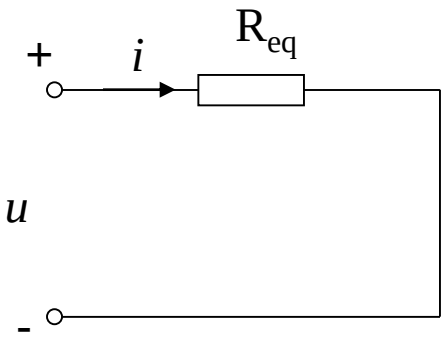


§2-3电阻的串联和并联

- 电阻的并联

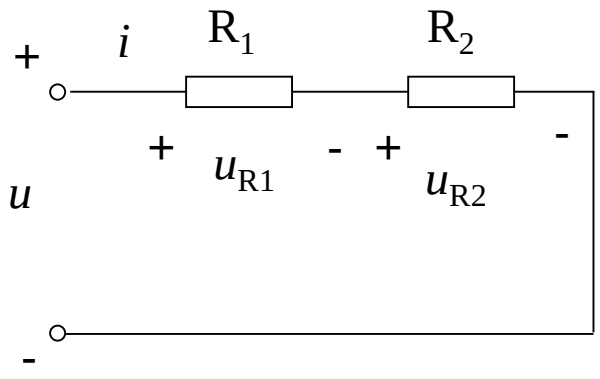


$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{\sum_{k=1}^{n} \frac{1}{R_k}}$$



§2-3电阻的串联和并联

- 两个电阻的串联



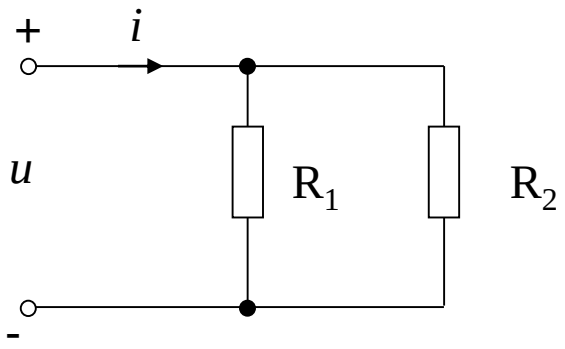
$$R_{eq}=R_1+R_2$$

$$u_{R1}=\frac{R_1}{R_1+R_2}u$$

$$u_{R2}=\frac{R_2}{R_1+R_2}u$$

§2-3电阻的串联和并联

- 两个电阻的并联



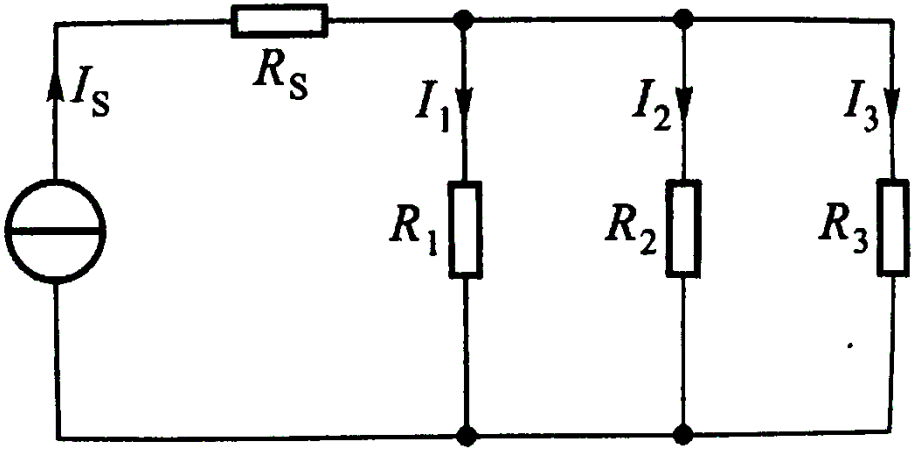
$$R_{eq}=\frac{R_1R_2}{R_1+R_2}$$

$$i_{R1}=\frac{R_2}{R_1+R_2}i$$

$$i_{R2}=\frac{R_1}{R_1+R_2}i$$

§2-3电阻的串联和并联

例2-1



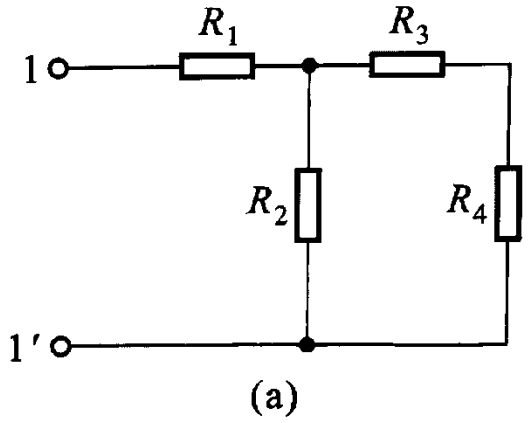
$$I_1 = \frac{G_1}{G_1 + G_2 + G_3} I_S = \frac{0.025}{0.025 + 0.1 + 0.04} \times 16.5\ \text{mA} = 2.5\ \text{mA}$$

$$I_2 = \frac{G_2}{G_1 + G_2 + G_3} I_S = \frac{0.1}{0.025 + 0.1 + 0.04} \times 16.5\ \text{mA} = 10\ \text{mA}$$

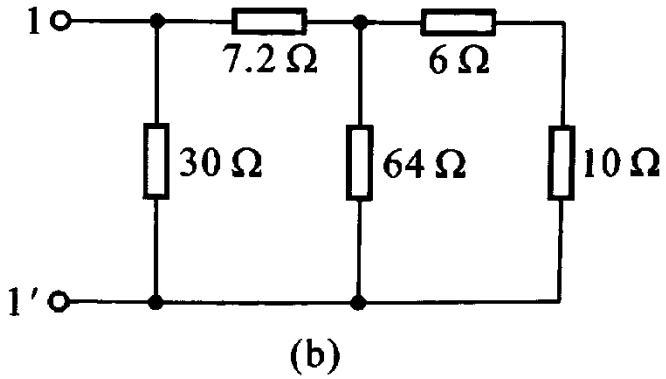
$$I_3 = \frac{G_3}{G_1 + G_2 + G_3} I_S = \frac{0.04}{0.025 + 0.1 + 0.04} \times 16.5\ \text{mA} = 4\ \text{mA}$$

§2-3电阻的串联和并联

- 既有串联又有并联时，称为电阻的串、并联，或简称**混联**



(a)

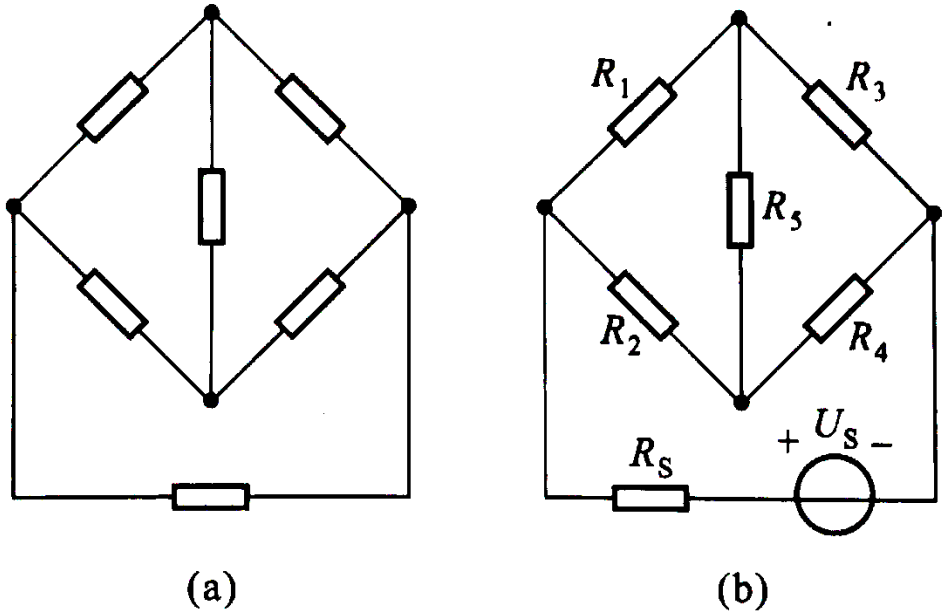


(b)

$$R_{eq} = R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

§2-4电阻的Y形和△形联结

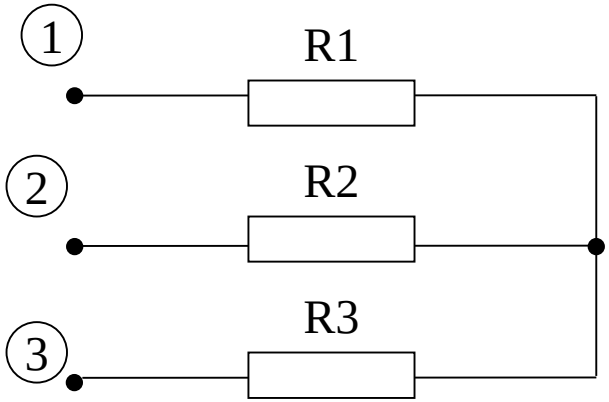
- 桥形连接中的电阻既不是串联也不是并联

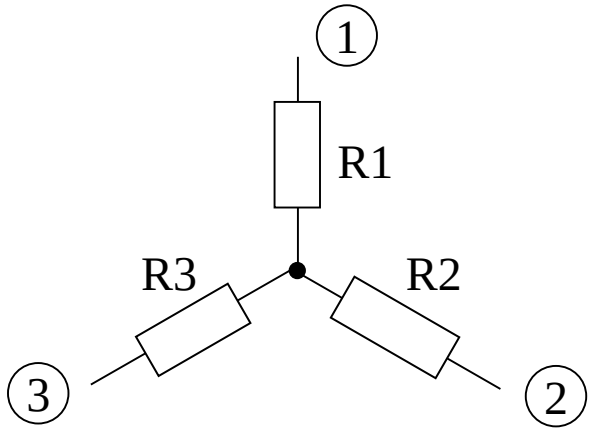


⇨ 使用电阻的Y-△等效变换

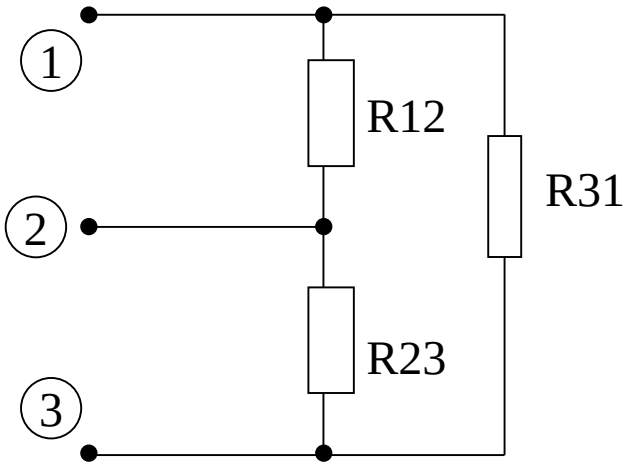
§2-4电阻的Y形和△形联结

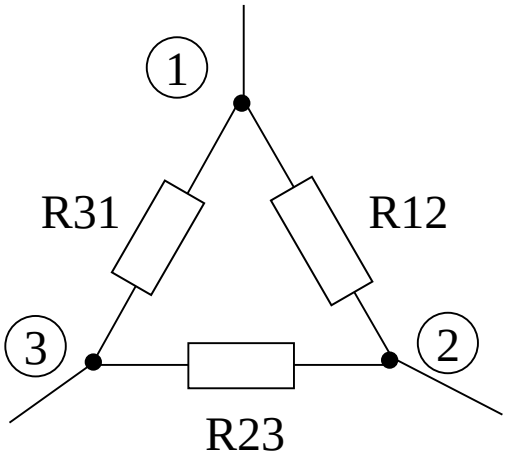
- Y形联结





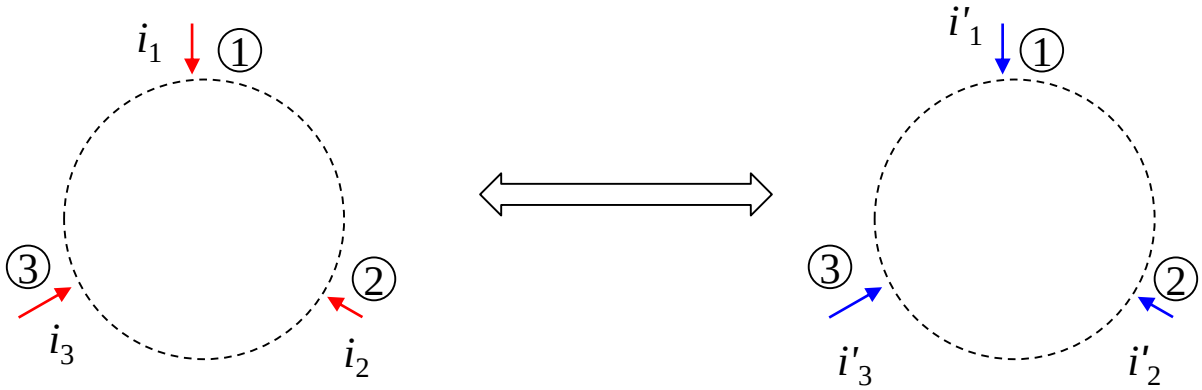
- △形联结





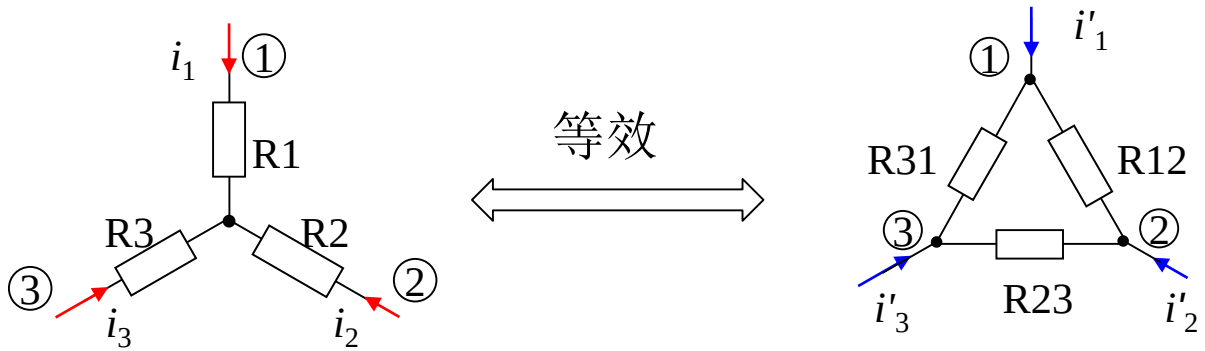
§2-4电阻的Y形和△形联结

- 电阻的Y形和△形联结之间的转换：



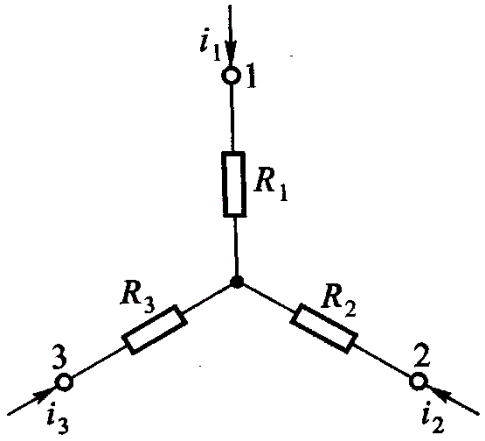
等效：

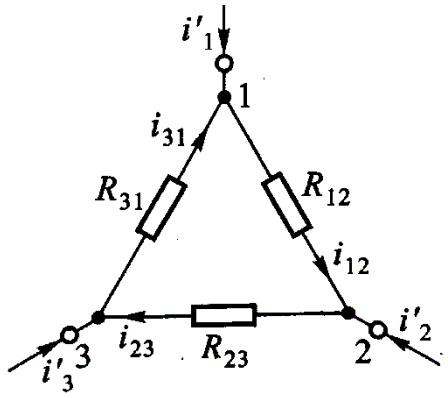
- 对应端子间有相同的电压u_{12}、u_{23}、u_{31}
- 流入端子的电流相等：$i_1=i'_1$、$i_2=i'_2$、$i_3=i'_3$



§2-4电阻的Y形和△形联结

电阻的Y形和△形联结之间的转换：





$$i_1 + i_2 + i_3 = 0$$

$$R_1 i_1 - R_2 i_2 = u_{12}$$

$$R_2 i_2 - R_3 i_3 = u_{23}$$

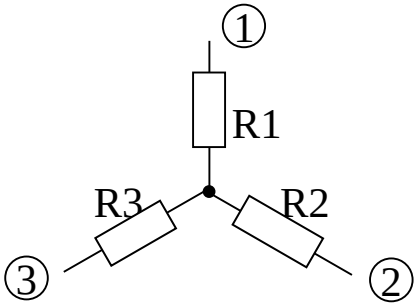
$$\left.\begin{aligned} i_1 &= \frac{R_3 u_{12}}{R_1R_2 + R_2R_3 + R_3R_1} - \frac{R_2 u_{31}}{R_1R_2 + R_2R_3 + R_3R_1} \\ i_2 &= \frac{R_1 u_{23}}{R_1R_2 + R_2R_3 + R_3R_1} - \frac{R_3 u_{12}}{R_1R_2 + R_2R_3 + R_3R_1} \\ i_3 &= \frac{R_2 u_{31}}{R_1R_2 + R_2R_3 + R_3R_1} - \frac{R_1 u_{23}}{R_1R_2 + R_2R_3 + R_3R_1} \end{aligned}\right\}$$

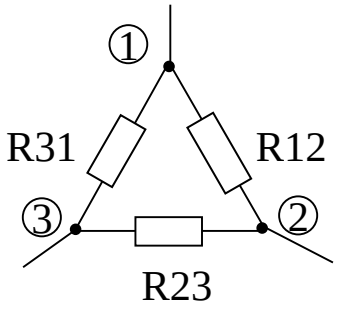
$$i_{12} = \frac{u_{12}}{R_{12}},\ i_{23} = \frac{u_{23}}{R_{23}},\ i_{31} = \frac{u_{31}}{R_{31}}$$

$$\left.\begin{aligned} i'_1 &= \frac{u_{12}}{R_{12}} - \frac{u_{31}}{R_{31}} \\ i'_2 &= \frac{u_{23}}{R_{23}} - \frac{u_{12}}{R_{12}} \\ i'_3 &= \frac{u_{31}}{R_{31}} - \frac{u_{23}}{R_{23}} \end{aligned}\right\}$$

§2-4电阻的Y形和△形联结

电阻的Y形和△形联结之间的转换：





$$R_1=\frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$R_2=\frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$$

$$R_3=\frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$R_{12}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$$

$$R_{23}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$$

$$R_{31}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$$

如果$R_{12}=R_{23}=R_{31}=R_\triangle$

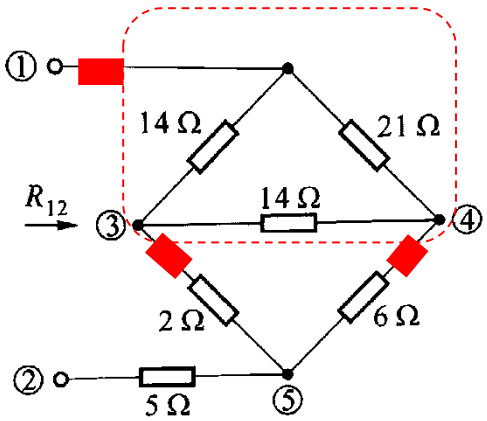
得 $R_1=R_2=R_3=\frac{R_\Delta}{3}$

如果$R_1=R_2=R_3=R_Y$

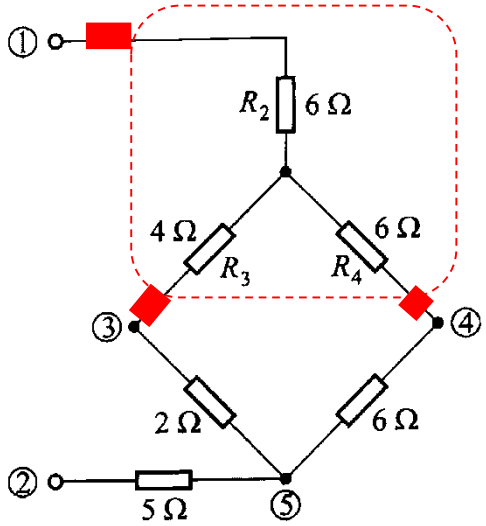
得$R_{12}=R_{23}=R_{31}=3R_Y$

§2-4电阻的Y形和△形联结

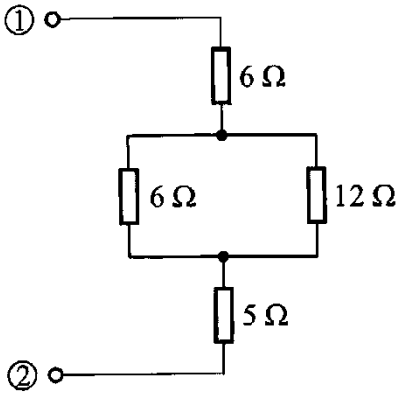
例2-2 解法一： △ -> Y



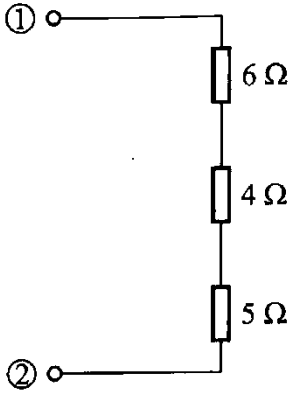
(a)



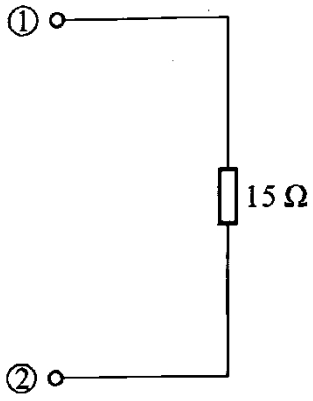
(b)



(c)



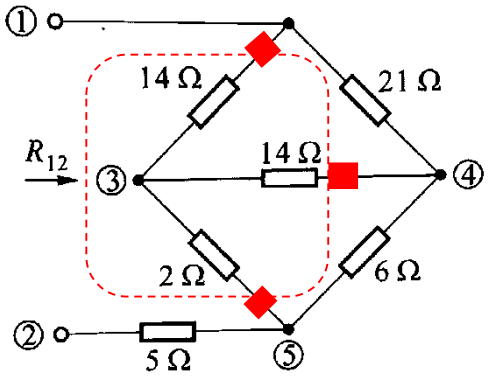
(d)

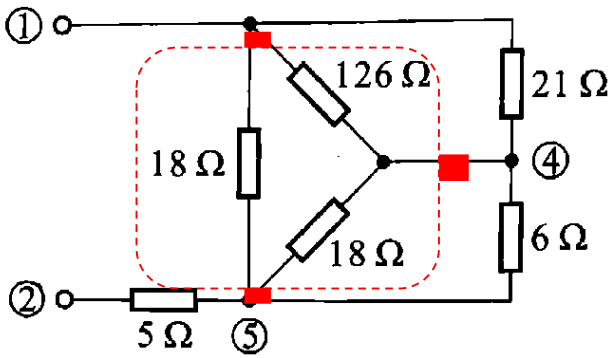


(e)

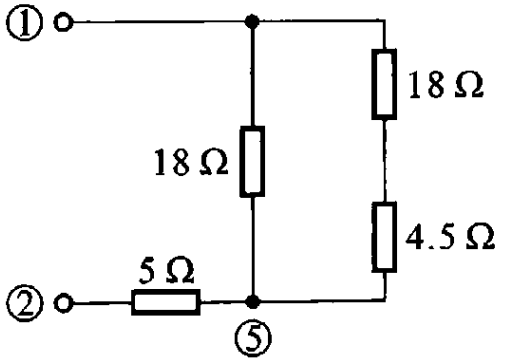
§2-4电阻的Y形和△形联结

例2-2 解法二： Y -> △





(a)



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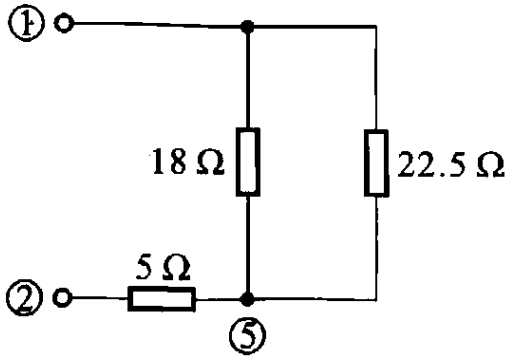
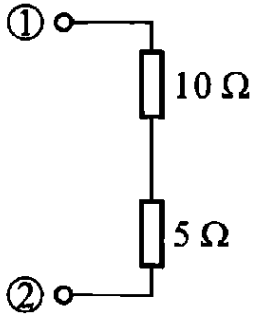
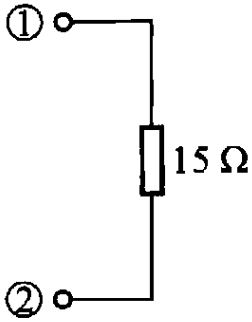


(c)



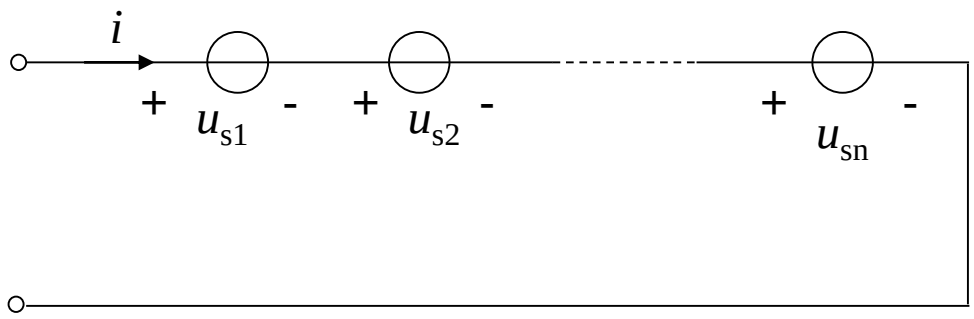
(d)



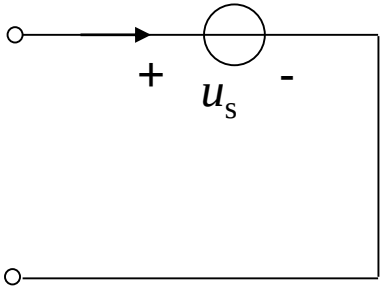
(e)

§2-5电压源和电流源的串联和并联

- 电压源的串联



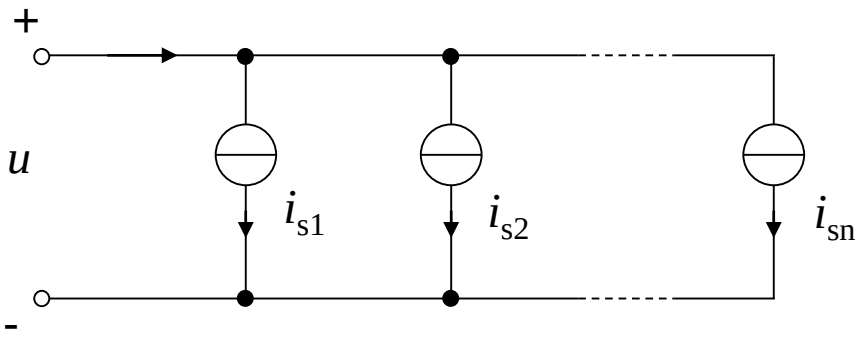
$$u_{eq}=\sum_{k=1}^{n}u_{sk}$$



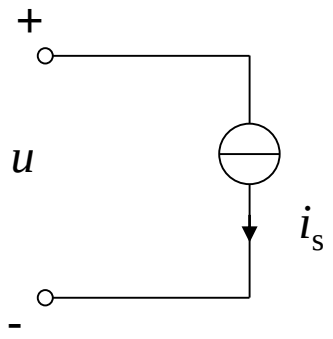
注意：如果u_{sk}与u_s的参考方向一致，u_{sk}前面取“+”，不一致时取“-”。

§2-5电压源和电流源的串联和并联

- 电流源的并联



$$i_{eq}=\sum_{k=1}^{n} i_{sk}$$



注意：如果i_{sk}与i_s的参考方向一致，i_{sk}前面取“+”，不一致时取“-”。

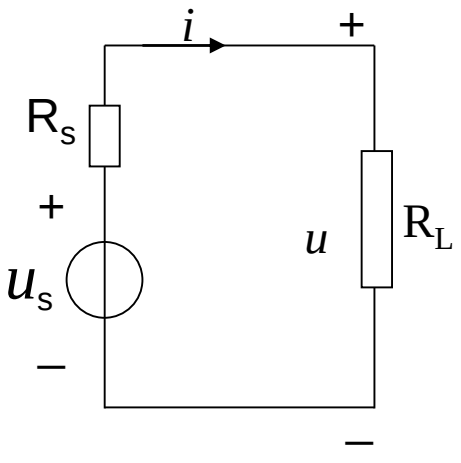
§2-5电压源和电流源的串联和并联

- 注意：

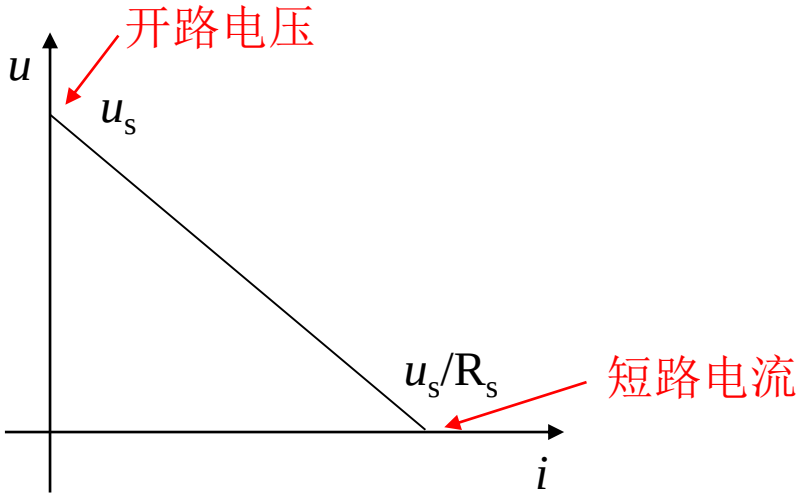
1. 只有电压相等、极性一致的电压源才允许并联；
2. 只有电流相等、方向一致的电流源才允许串联；
3. 一个电压源与其他元件并联的支路，可等效为一个电压源；
4. 一个电流源与其他元件串联的支路，可等效为一个电流源；

§2-6 实际电源的两种模型及其等效变换

- 实际电压源



$u = u_s - R_s i$



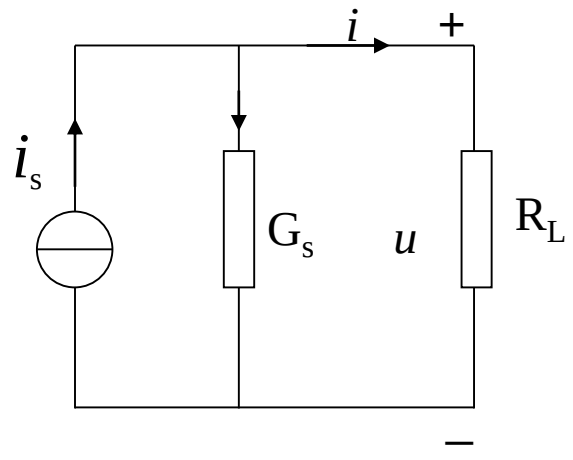
端口的伏安特性曲线

当$R_L=\infty$，即开路时，$u = u_s$，称为开路电压；

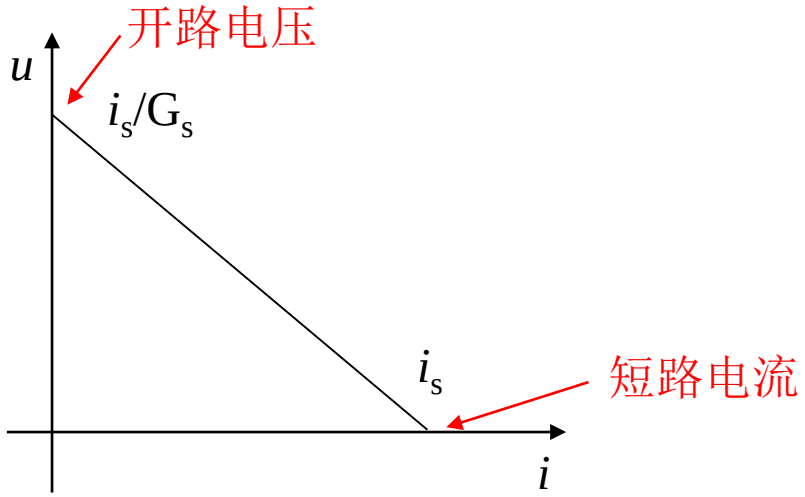
当$R_L=0$，即短路时，$i = u_s/R_s$，称为短路电流；

§2-6 实际电源的两种模型及其等效变换

- 实际电流源



$$i = i_s - G_s u$$



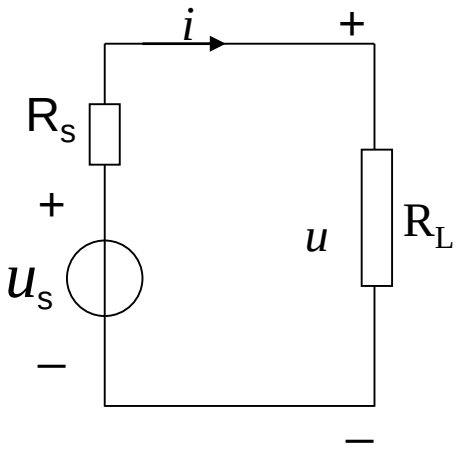
端口的伏安特性曲线

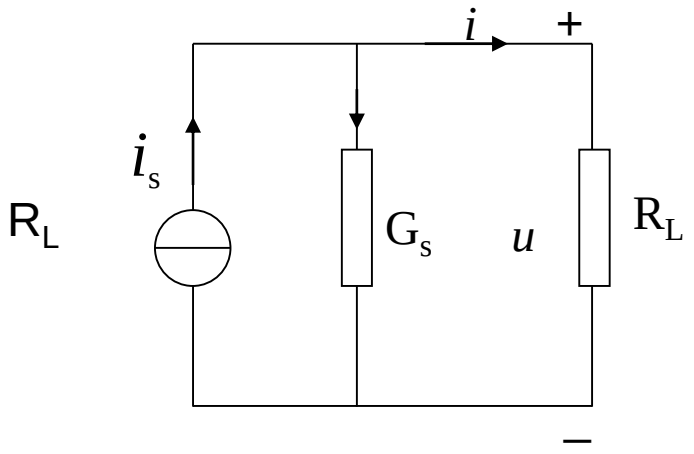
当$R_L=\infty$，即开路时，$u = i_s / G_s = i_s R_s$，称为开路电压；

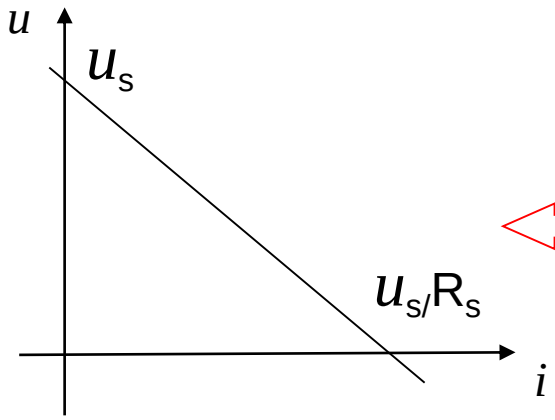
当$R_L=0$，即短路时，$i = i_s$，称为短路电流；

§2-6 实际电源的两种模型及其等效变换

- 两种模型的等效变换



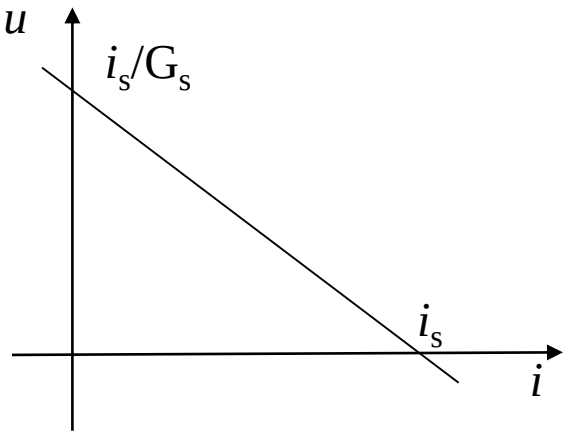




$u_s=i_s/G_s$

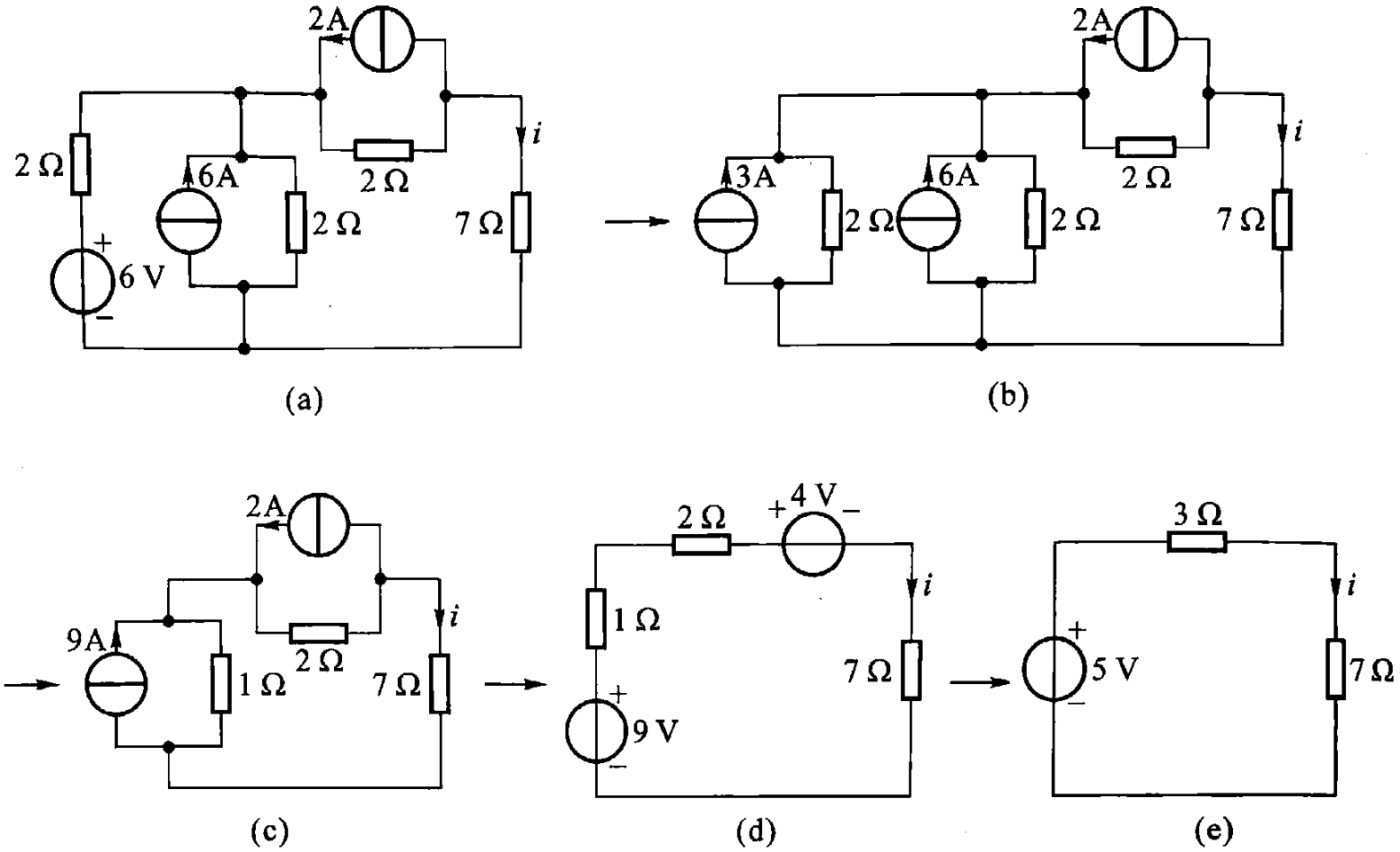
$R_s=1/G_s$

等效



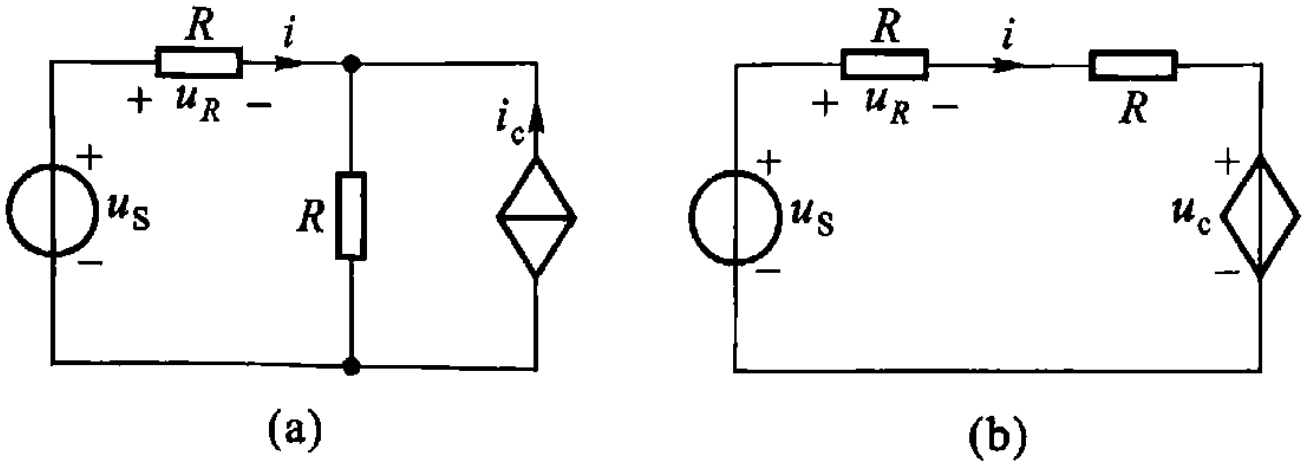
§2-6 实际电源的两种模型及其等效变换

例2-3



§2-6 实际电源的两种模型及其等效变换

例2-4



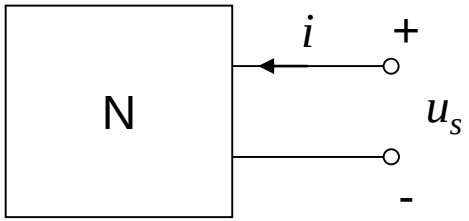
$$Ri + Ri + u_c = u_S$$

$$2u_R + 4u_R = u_S$$

$$u_R = \frac{u_S}{6} = 2\ \text{V}$$

§2-7 输入电阻

- 等效电阻与输入电阻



- 如果一端口网络内部仅含有电阻，可以利用电阻电路的等效变换，求得它的等效电阻。
- 如果一端口网络内部除了含有电阻外，还含有控制电源，但不含独立电源，此一端口仍可以等效为一个电阻。一般假设在端口加一电压源u_s，求端口电流i，等效电阻

$$R_{eq} = \frac{u_s}{i}$$

R_{eq}又称为输入电阻。

§2-7 输入电阻

例2-5

例 2－5　求图 2－18(a)所示一端口的输入电阻。

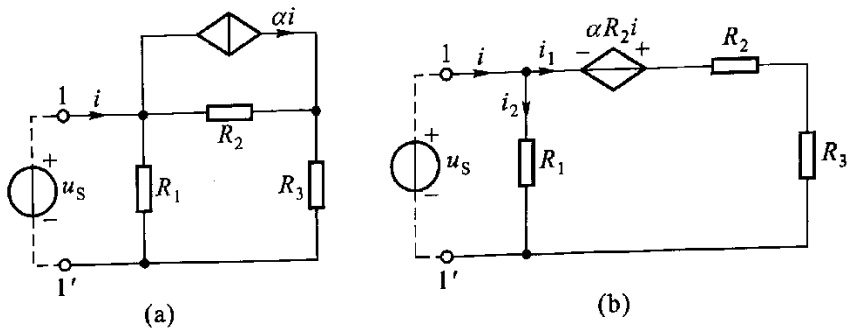


图 2－18　例 2－5 图

解　在端口 1－1′处加电压 u_S，求出 i，再由式(2－12)求输入电阻 R_i。

将 CCCS 和电阻 R_2 的并联组合等效变换为 CCVS 和电阻的串联组合，如图 2－18(b)所示。根据 KVL，有

$$u_S = -R_2\alpha i + (R_2 + R_3)i_1 \tag{1}$$

$$u_S = R_1 i_2 \tag{2}$$

再由 KCL，$i = i_1 + i_2$，可得 $i_1 = i - i_2 = i - \dfrac{u_S}{R_1}$，代入(1)式，整理后，有

$$R_i = \frac{u_S}{i} = \frac{R_1R_3 + (1-\alpha)R_1R_2}{R_1 + R_2 + R_3}$$

上式分子中有负号出现，因此，当存在受控源时，在一定的参数条件下，R_i 有可能是零，也有可能是负值。例如，当 $R_1 = R_2 = 1\ \Omega$，$R_3 = 2\ \Omega$，$\alpha = 5$ 时，$R_i = -0.5\ \Omega$。§1—5 中曾指出负电阻元件实际是一个发出功率的元件。本例中一端口向外发出功率是由于受控源发出功率。

课后作业

- P30 1-18　1-19　1-20

- P46

  2-5

  2-10

  2-11

  2-13

  2-14