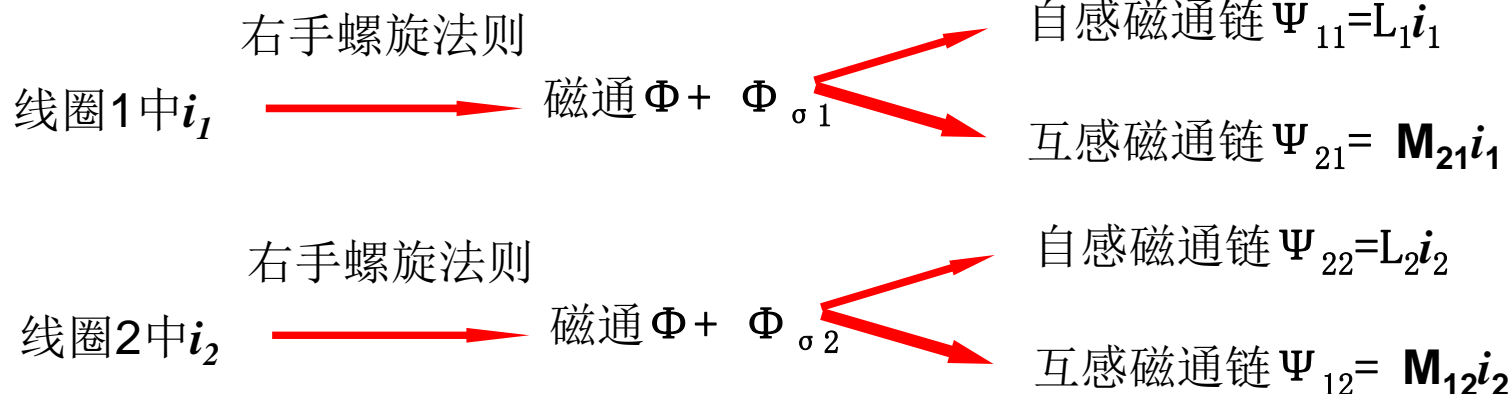


第十章 含有耦合电感的电路

- 互感
- 含有耦合电感电路的计算
- 耦合电感的功率
- 变压器原理
- 理想变压器

§ 10-1 互感

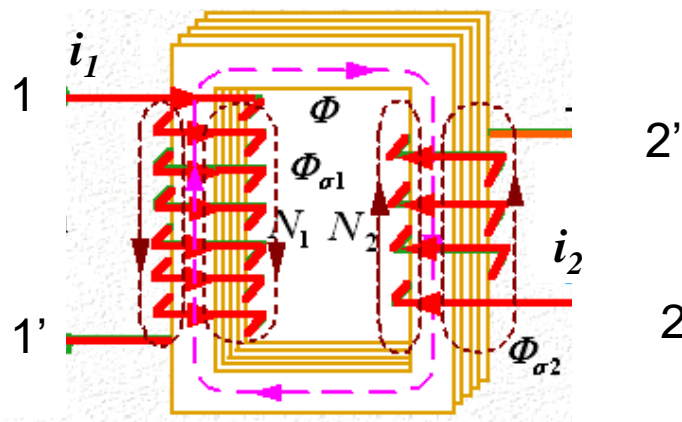
一、互感磁通：



互感系数 $M_{12} = M_{21} = M$

$$\Psi_1 = \Psi_{11} + \Psi_{12} = L_1 i_1 + M i_2$$

$$\Psi_2 = \Psi_{22} + \Psi_{21} = L_2 i_2 + M i_1$$



双下标的含义：第1个表示磁通链所在的线圈

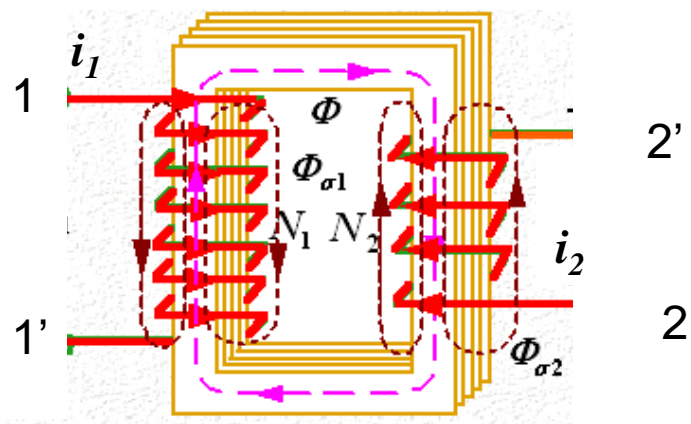
第2个表示产生该磁通链的施感电流所在的线圈

§ 10-1 互感

一、同名端定义：

如果一对施感电流流过耦合线圈时，自感磁通链与互感磁通链的方向一致，则这对施感电流的入端（或出端）定义为耦合电感的同名端。

注意：同名端是由耦合线圈的绕向决定的，与电流的参考方向无关。



§ 10-1 互感

若电流从同名端输入,

$$\Psi_1 = \Psi_{11} + \Psi_{12} = L_1 i_1 + M i_2$$

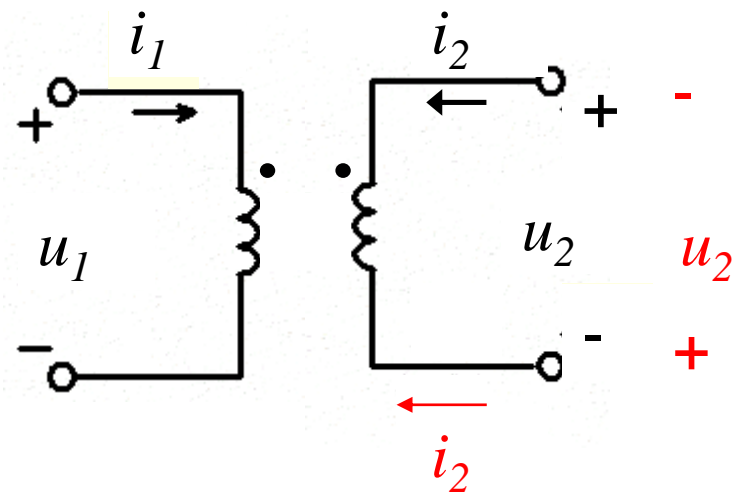
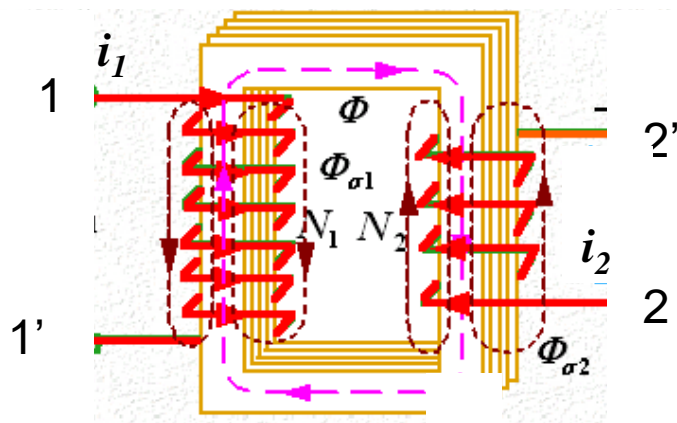
$$\Psi_2 = \Psi_{22} + \Psi_{21} = L_2 i_2 + M i_1$$

若电流从异名端输入,

$$\Psi_1 = \Psi_{11} - \Psi_{12} = L_1 i_1 - M i_2$$

$$\Psi_2 = \Psi_{22} - \Psi_{21} = L_2 i_2 - M i_1$$

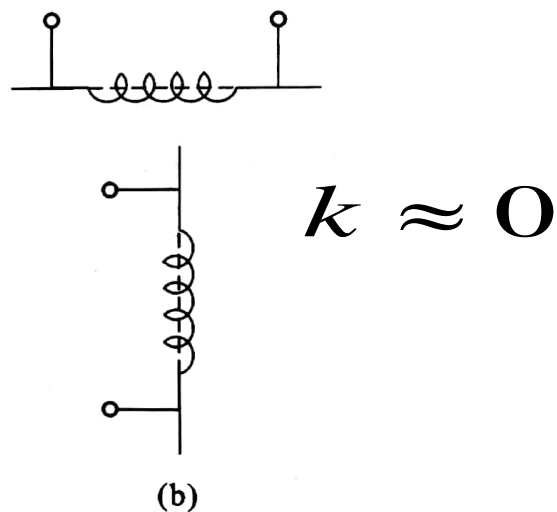
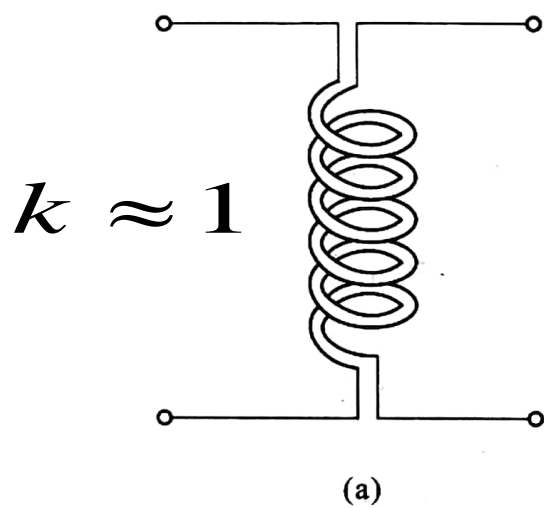
耦合系数 $k = \frac{M}{\sqrt{L_1 L_2}}$



§ 10-1 互感

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

耦合系数与线圈之间的位置有关：



§ 10-1 互感

二、互感电压：

若电流从同名端输入，

$$\Psi_1 = \Psi_{11} + \Psi_{12} = L_1 i_1 + M i_2$$

$$\Psi_2 = \Psi_{22} + \Psi_{21} = L_2 i_2 + M i_1$$



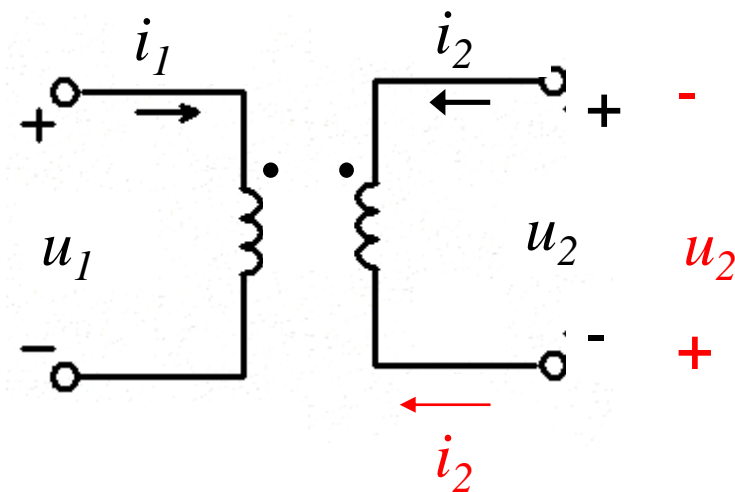
$$u_1 = \frac{d\psi_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = \frac{d\psi_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

若电流从异名端输入，

$$u_1 = \frac{d\psi_1}{dt} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$u_2 = \frac{d\psi_2}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



§ 10-2 含有耦合电感电路的计算

同相串联电路:

$$u_1 = R_1 i + L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$u_2 = R_2 i + L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$u = u_1 + u_2$$

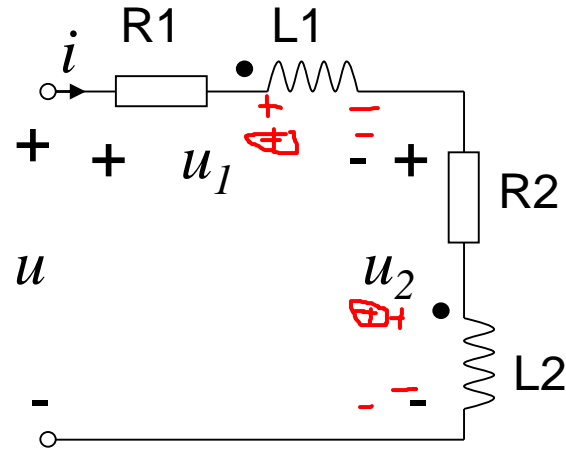
$$= (R_1 + R_2)i + (L_2 + L_1 + 2M) \frac{di}{dt}$$

相量形式:

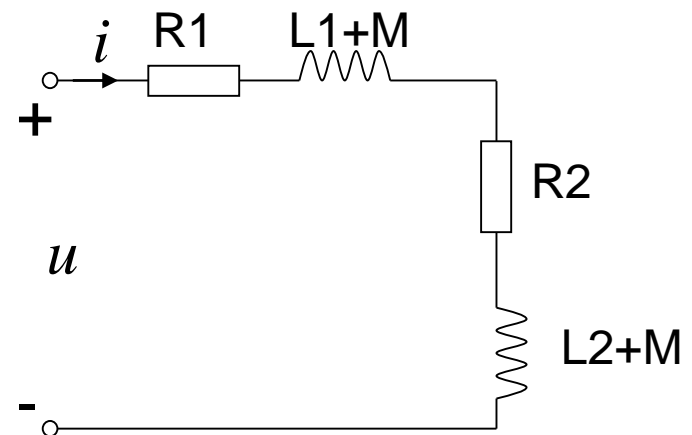
$$\dot{U} = \dot{U}_1 + \dot{U}_2$$

$$= (R_1 + R_2)\dot{I} + j\omega(L_2 + L_1 + 2M)\dot{I}$$

$$Z = \frac{\dot{U}}{\dot{I}} = (R_1 + R_2) + j\omega(L_2 + L_1 + 2M)$$



无耦合等效电路:



§ 10-2 含有耦合电感电路的计算

反相串联电路:

$$u_1 = R_1 i + L_1 \frac{di}{dt} - M \frac{di}{dt}$$

$$u_2 = R_2 i + L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$u = u_1 + u_2$$

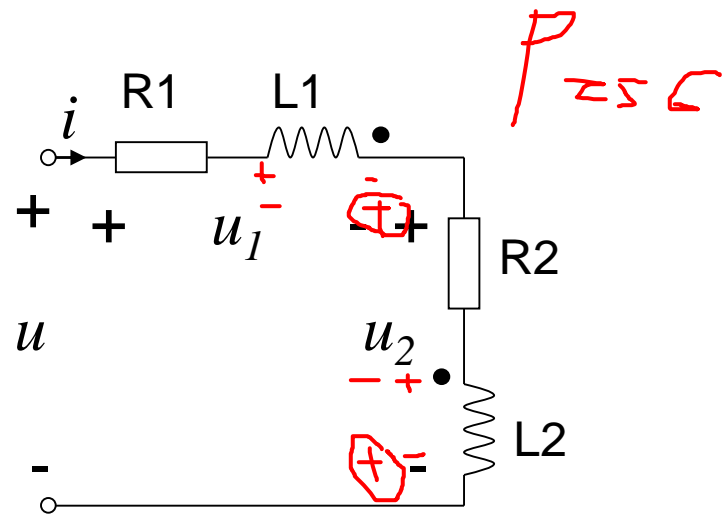
$$= (R_1 + R_2)i + (L_2 + L_1 - 2M) \frac{di}{dt}$$

相量形式:

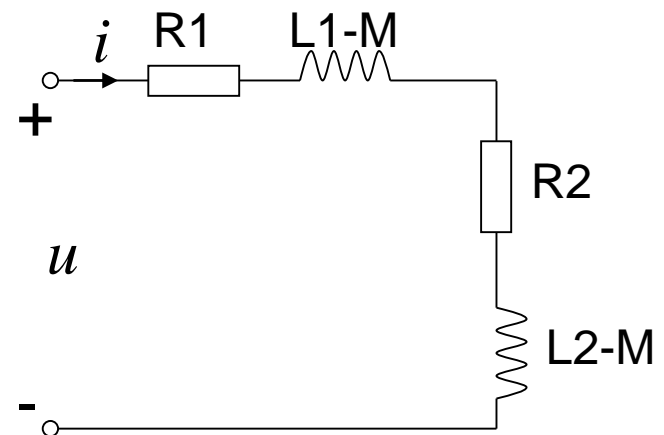
$$\dot{U} = \dot{U}_1 + \dot{U}_2$$

$$= (R_1 + R_2)\dot{I} + j\omega(L_2 + L_1 - 2M)\dot{I}$$

$$Z = \frac{\dot{U}}{\dot{I}} = (R_1 + R_2) + j\omega(L_2 + L_1 - 2M)$$



无耦合等效电路:

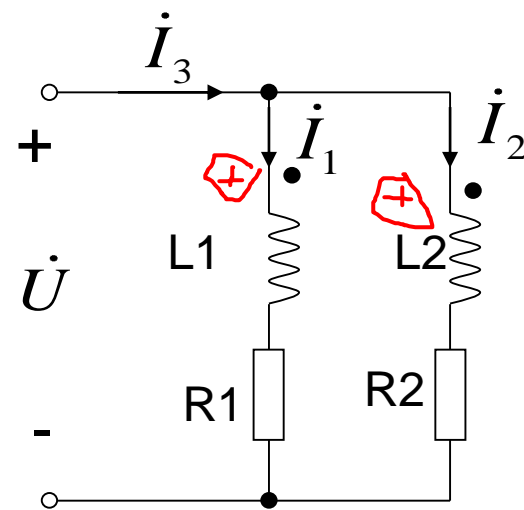


§ 10-2 含有耦合电感电路的计算

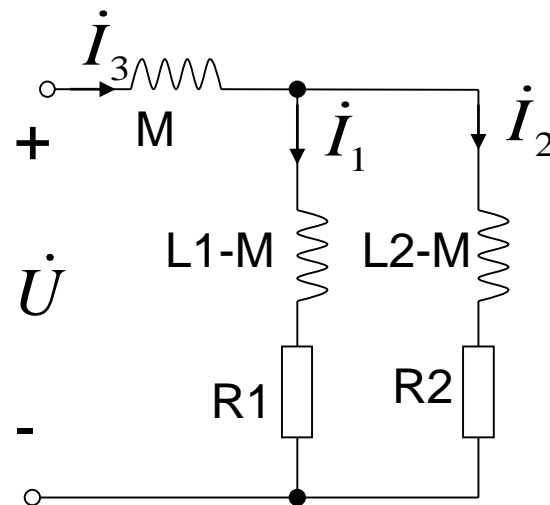
同侧并联电路:

$$\begin{cases} \dot{U} = (R_1 + j\omega L_1)\dot{I}_1 + j\omega M\dot{I}_2 \\ \dot{U} = j\omega M\dot{I}_1 + (R_2 + j\omega L_2)\dot{I}_2 \\ \dot{I}_3 = \dot{I}_1 + \dot{I}_2 \end{cases}$$

$$\begin{aligned} \dot{U} &= (R_1 + j\omega L_1)\dot{I}_1 + j\omega M\dot{I}_2 \\ &= (R_1 + j\omega L_1 - j\omega M)\dot{I}_1 + j\omega M\dot{I}_1 + j\omega M\dot{I}_2 \\ &= (R_1 + j\omega(L_1 - M))\dot{I}_1 + j\omega M\dot{I}_3 \\ \dot{U} &= (R_2 + j\omega(L_2 - M))\dot{I}_2 + j\omega M\dot{I}_3 \end{aligned}$$



无耦合等效电路:

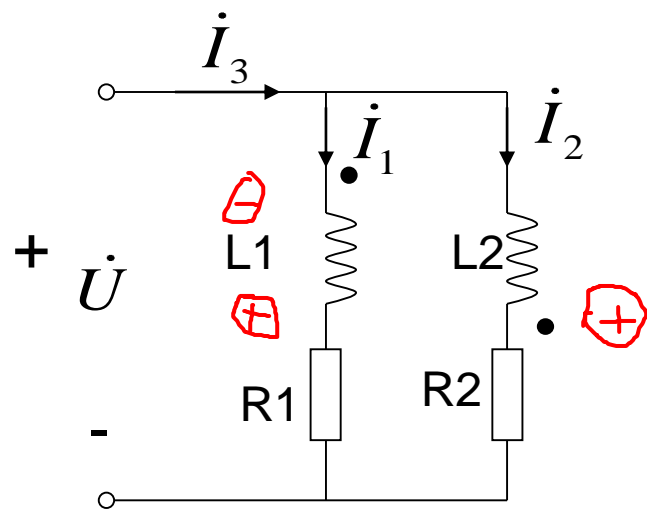


§ 10-2 含有耦合电感电路的计算

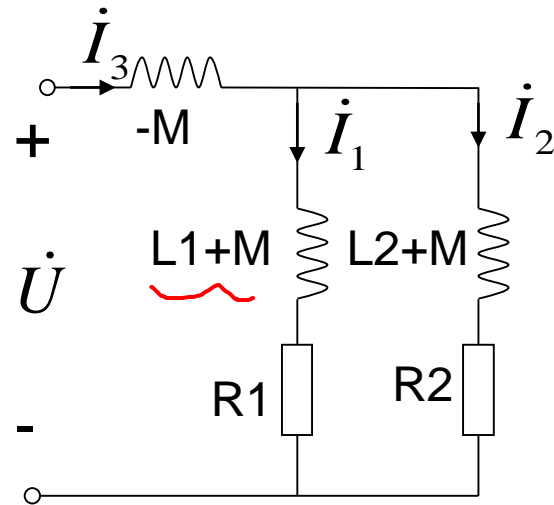
异侧并联电路：

$$\begin{cases} \dot{U} = (R_1 + j\omega L_1)\dot{I}_1 - j\omega M\dot{I}_2 \\ \dot{U} = -j\omega M\dot{I}_1 + (R_2 + j\omega L_2)\dot{I}_2 \\ \dot{I}_3 = \dot{I}_1 + \dot{I}_2 \end{cases}$$

$$\begin{aligned} \dot{U} &= (R_1 + j\omega L_1)\dot{I}_1 - j\omega M\dot{I}_2 \\ &= (R_1 + j\omega L_1 + j\omega M)\dot{I}_1 - j\omega M\dot{I}_1 - j\omega M\dot{I}_2 \\ &= (R_1 + j\omega(L_1 + M))\dot{I}_1 - j\omega M\dot{I}_3 \\ \dot{U} &= (R_2 + j\omega(L_2 + M))\dot{I}_2 - j\omega M\dot{I}_3 \end{aligned}$$



无耦合等效电路：



§ 10-2 含有耦合电感电路的计算

去耦法:

■ 如果耦合电感的两条支路各有一端与第3支路形成一个仅含3条支路的共同节点，则可用3条无耦合的电感支路等效替代

■ 3条支路的等效电感分别为:

支路3: $L_3 = \pm M$ 同侧取+, 异侧取-

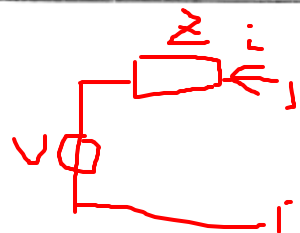
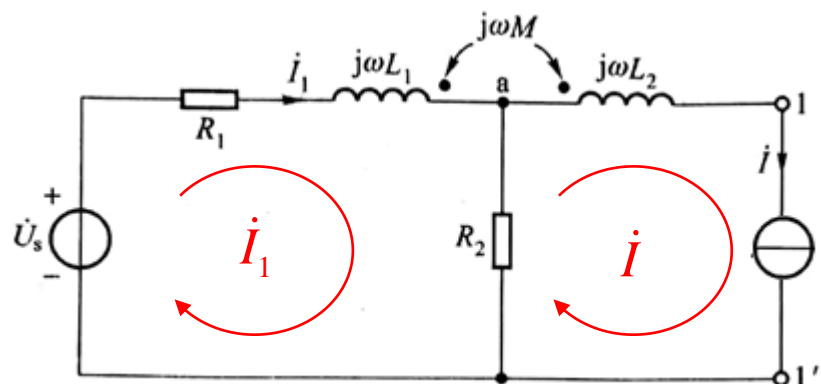
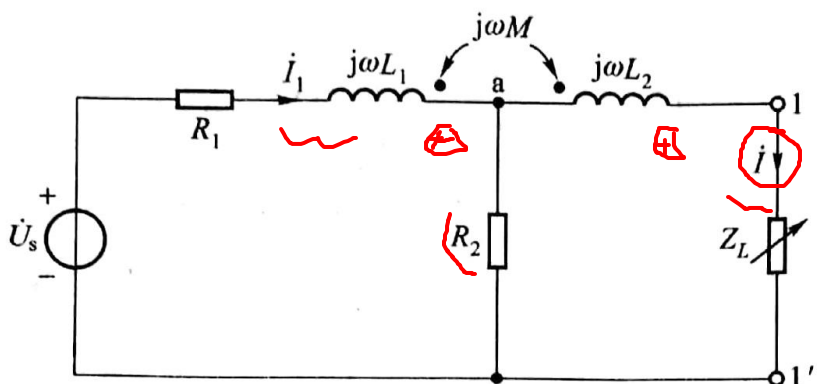
支路1: $L'_1 = L_1 \mp M$

支路2: $L'_2 = L_2 \mp M$

} M 前的符号与 L_3 中的相反

例10-5

例 10-5 图 10-6(a)所示电路中 $\omega L_1 = \omega L_2 = 10\Omega$, $\omega M = 5\Omega$, $R_1 = R_2 = 6\Omega$, $U_s = 12\text{ V}$ 。求 Z_L 最佳匹配时获得的功率 P 。



方法一：戴维宁（诺顿）等效电路，含耦合电感时，方法与含受控源的一端电路相同

在端口 1-1' 置电流源 \dot{I} 替代 Z_L ，求外特性 $\dot{U}_{11'} = f(\dot{I})$

$$(R_1 + R_2 + j\omega L_1)\dot{I}_1 - (R_2 + j\omega M)\dot{I} = \dot{U}_s \quad (\text{左网孔})$$

$$\dot{U}_{11'} = -(R_2 + j\omega L_2)\dot{I} + (R_2 + j\omega M)\dot{I}_1 \quad (\text{右网孔})$$

得
$$\dot{U}_{11'} = \frac{1}{2}\dot{U}_s - (3 + j7.5)\dot{I}$$

从上式可得戴维宁等效电路参数：

$$\dot{U}_{\infty} = \frac{1}{2}\dot{U}_s = 6 \angle 0^\circ \text{ V}$$

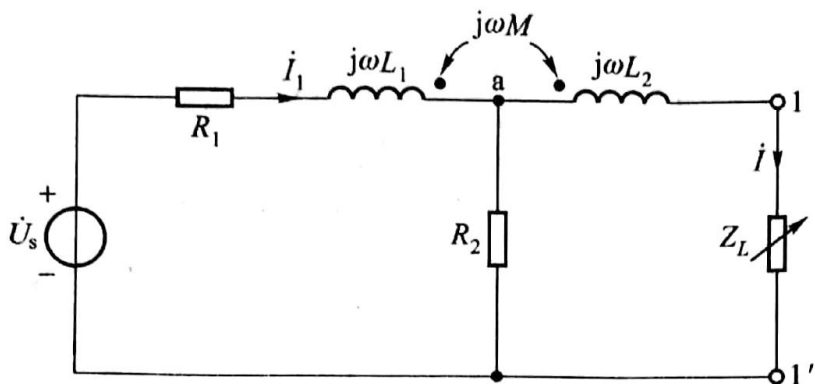
$$Z_{eq} = (3 + j7.5) \Omega$$

最佳匹配时， $Z_L = Z_{eq}^* = (3 - j7.5) \Omega$

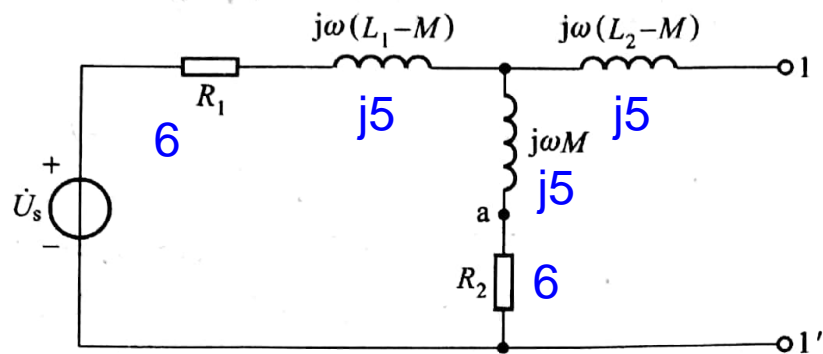
功率为
$$P = \frac{U_{\infty}^2}{4R_{eq}} = \frac{36}{12} \text{ W} = 3 \text{ W}$$

例10-5

例 10-5 图 10-6(a)所示电路中 $\omega L_1 = \omega L_2 = 10\Omega$, $\omega M = 5\Omega$, $R_1 = R_2 = 6\Omega$, $U_s = 12\text{ V}$ 。求 Z_L 最佳匹配时获得的功率 P 。



方法二：去耦法，去耦后的等效电路为：



从电路图中可得戴维宁等效电路参数：

$$\dot{U}_{11'\text{oc}} = \frac{U_s}{2(6 + j5)}(6 + j5) = \frac{1}{2} \dot{U}_s$$

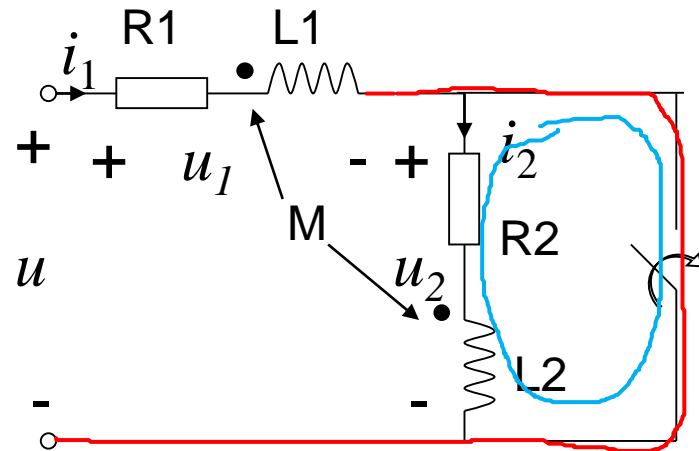
$$Z_{\text{eq}} = \left[\frac{1}{2}(6 + j5) + j5 \right] \Omega = (3 + j7.5) \Omega$$

§ 10-3 耦合电感的功率

S闭合后，环路电压方程：

$$R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = u$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = 0$$



瞬时功率方程：

$$R_1 i_1^2 + i_1 L_1 \frac{di_1}{dt} + i_1 M \frac{di_2}{dt} = u i_1$$

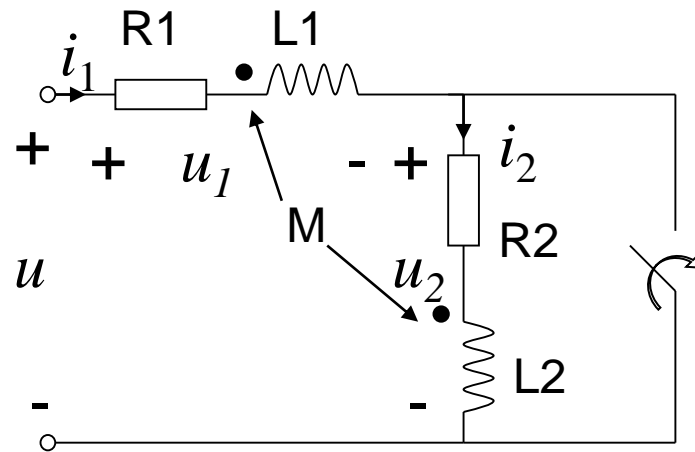
$$R_2 i_2^2 + i_2 L_2 \frac{di_2}{dt} + i_2 M \frac{di_1}{dt} = 0$$

它们实现电磁能的转换和传输

§ 10-3 耦合电感的功率

线圈1吸收的复功率:

$$\begin{aligned}\bar{S}_1 &= \dot{U} \dot{I}_1^* \\ &= (R_1 \dot{I}_1 + j\omega L_1 \dot{I}_1 + j\omega M \dot{I}_2) \dot{I}_1^* \\ &= (R_1 + j\omega L_1) I_1^2 + j\omega M \dot{I}_2 \dot{I}_1^*\end{aligned}$$



线圈2吸收的复功率:

$$\begin{aligned}\bar{S}_2 &= 0 \\ &= (R_2 \dot{I}_2 + j\omega L_2 \dot{I}_2 + j\omega M \dot{I}_1) \dot{I}_2^* \\ &= (R_2 + j\omega L_2) I_2^2 + j\omega M \dot{I}_1 \dot{I}_2^*\end{aligned}$$

§ 10-3 耦合电感的功率

线圈1互感电压耦合功率: $j\omega M \dot{I}_2 \dot{I}_1^*$

线圈2互感电压耦合功率: $j\omega M \dot{I}_1 \dot{I}_2^*$

假设: $\dot{I}_2 \dot{I}_1^* = X + jY$, $\dot{I}_1 \dot{I}_2^* = X - jY$,

$$j\omega M \dot{I}_2 \dot{I}_1^* = j\omega M (X + jY) = j\omega MX - \omega MY$$

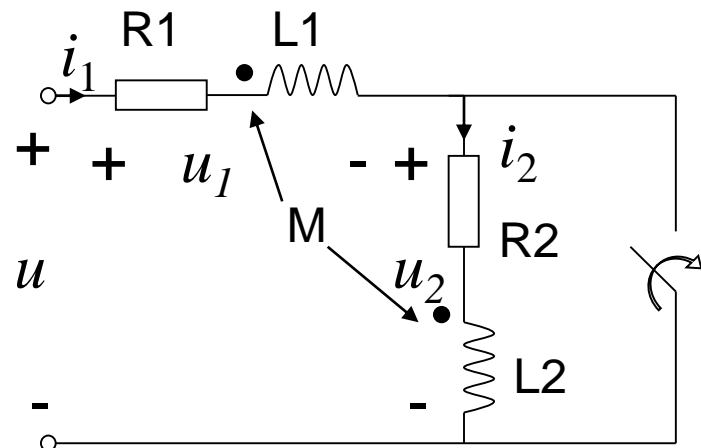
$$j\omega M \dot{I}_1 \dot{I}_2^* = j\omega M (X - jY) = j\omega MX + \omega MY$$

$\dot{I}_2 \dot{I}_1^*$ 与 $\dot{I}_1 \dot{I}_2^*$ 互为共轭复数, 实部同号, 虚部异号。

它们乘以j以后, 虚部同号, 实部异号。

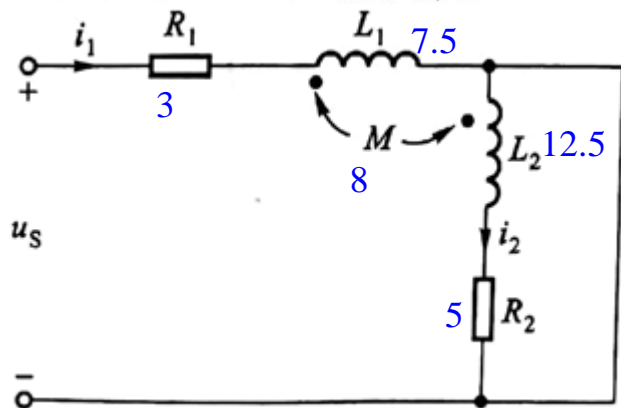
互感 M 的非能耗特性: 耦合功率中的有功功率相互异号, 表明有功功率从一个端口进入(正号, 吸收), 必须从另一个端口输出(负号, 发出)。

有功功率通过耦合电感的电磁场传播。



例10-6

例 10-6 求图 10-7 所示电路的复功率,并说明互感在功率转换和传输中的作用。图中 $U_s = 50 \text{ V}$, $R_1 = 3\Omega$, $\omega L_1 = 7.5\Omega$, $R_2 = 5\Omega$, $\omega L_2 = 12.5\Omega$, $\omega M = 8\Omega$ (与例 10-3 相同)。



电路的方程为(令 $\dot{U}_s = 50 \angle 0^\circ \text{ V}$)

$$(3 + j7.5) \dot{I}_1 + j8 \dot{I}_2 = \dot{U}_s$$

$$j8 \dot{I}_1 + (5 + j12.5) \dot{I}_2 = 0$$

$$\dot{I}_1 = \frac{5 + j12.5}{(3 + j7.5)(5 + j12.5) + (j8)^2} \dot{U}_s = 8.81 \angle -32.93^\circ \text{ A}$$

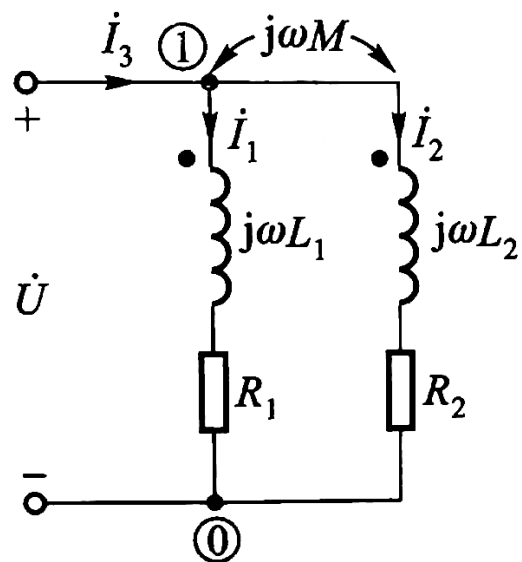
$$\dot{I}_2 = \frac{-j8 \dot{I}_1}{5 + j12.5} = 5.24 \angle 168.87^\circ \text{ A}$$

电源的复功率 $\bar{S}_s = \dot{U}_s \dot{I}_1^* = (3 + j7.5) \dot{I}_1^2 + j8 \dot{I}_2 \dot{I}_1^*$
 $= [(232.85 + j582.12) + (137.15 - j342.91)] \text{ V} \cdot \text{A}$

右边网络复功率 $\bar{S}_2 = j8 \dot{I}_1 \dot{I}_2^* + (5 + j12.5) \dot{I}_2^2$
 $= [(-137.15 - j342.91) + (137.15 + j342.91)] \text{ V} \cdot \text{A}$
 $= 0$

例10-7

例 10-7 对例 10-4 中的复功率的转换和传输作进一步分析。



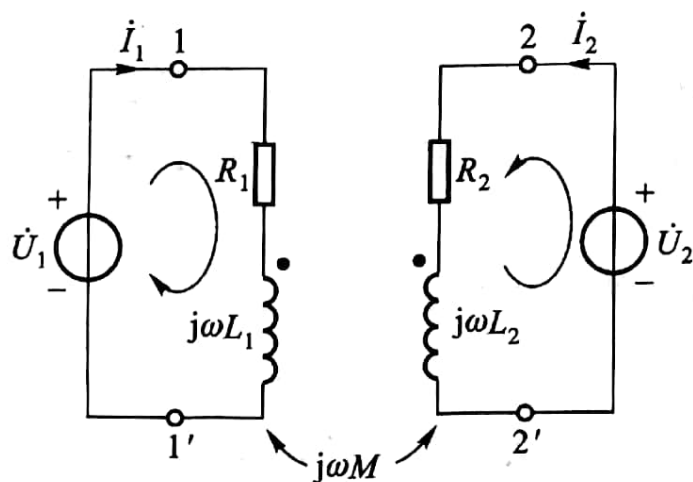
复功率 \bar{S}_1 和 \bar{S}_2 分别为

$$\bar{S}_1 = \dot{U} \dot{I}_1^* = (3 + j7.5) I_1^2 + j8(1.99 \angle -110.59^\circ \times 4.40 \angle 59.14^\circ)$$

$$\bar{S}_2 = \dot{U} \dot{I}_2^* = j8(1.99 \angle 110.59^\circ \times 4.40 \angle -59.14^\circ) + (5 + j12.5) I_2^2$$

$$\bar{S}_1 = [(58.08 + j145.2) + (54.78 + j43.65)] \text{ V} \cdot \text{A}$$

$$\bar{S}_2 = [(-54.78 + j43.65) + (19.80 + j49.50)] \text{ V} \cdot \text{A}$$



从结果可以看出,互感 M 起同向耦合作用,耦合电感中的无功功率都增加了同一个值,而有功功率的传输情况是:线圈 1 多吸收的 54.78W 传输给线圈 2,并由线圈 2 发出,扣除线圈 2 中电阻 R_2 的消耗后,尚有 34.98W 多余功率,这部分有功功率又返回电源,表明系统对有功功率有过量吸收的情况,出现“过冲”现象。如果将图 10-4(a)所示电路改接成图 10-8 的形式,而其中的参数值与例 10-4 所述相同,而两边的电压源为 $U_1 = U_2 = U$,这样计算结果完全相同,但可以看得更清楚。有功功率从左边的电压源发出,供给耦合电感中的电阻消耗后,又将多余部分传输给右边的电压源吸收。

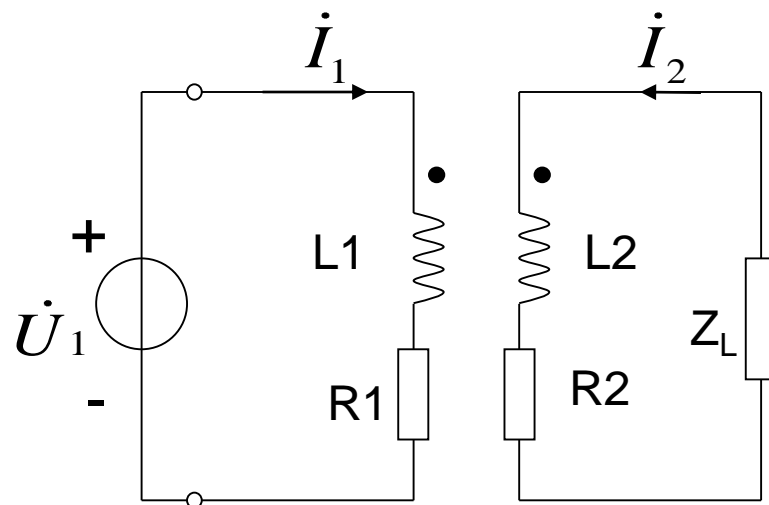
§ 10-4 变压器原理

一次回路（原边回路、初级回路）：

$$\dot{U}_1 = (R_1 + j\omega L_1)\dot{I}_1 + j\omega M\dot{I}_2$$

二次回路（副边回路、次级回路）：

$$j\omega M\dot{I}_1 + (R_2 + j\omega L_2 + Z_L)\dot{I}_2 = 0$$



$$\text{令： } Z_M = j\omega M, \quad Z_{11} = R_1 + j\omega L_1, \quad Z_{22} = R_2 + j\omega L_2 + Z_L$$

$$Z_{11}\dot{I}_1 + Z_M\dot{I}_2 = \dot{U}_1$$

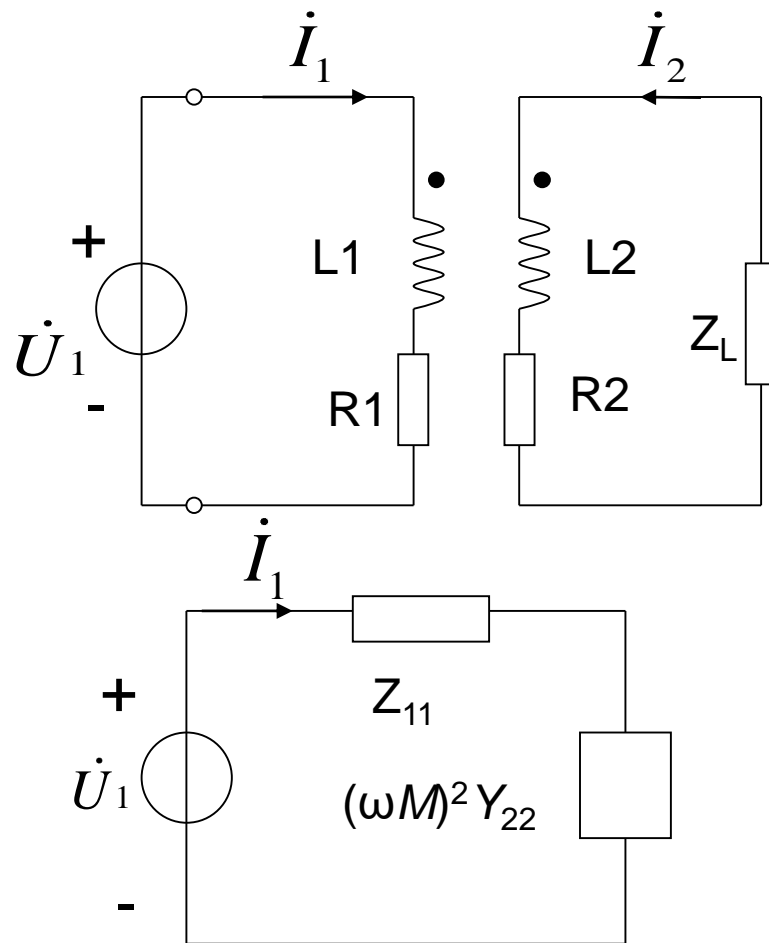
$$Z_M\dot{I}_1 + Z_{22}\dot{I}_2 = 0$$

§ 10-4 变压器原理

$$\begin{cases} Z_{11}\dot{I}_1 + Z_M\dot{I}_2 = \dot{U}_1 \\ Z_M\dot{I}_1 + Z_{22}\dot{I}_2 = 0 \end{cases} \Rightarrow \dot{I}_2 = -\frac{Z_M}{Z_{22}}\dot{I}_1 = -Z_M Y_{22}\dot{I}_1$$

$$(Z_{11} - Z_M^2 Y_{22})\dot{I}_1 = \dot{U}_1$$

$$\begin{aligned} \dot{I}_1 &= \frac{\dot{U}_1}{Z_{11} - Z_M^2 Y_{22}} = \frac{\dot{U}_1}{Z_{11} + (\omega M)^2 Y_{22}} \\ &= \frac{\dot{U}_1}{Z_1} \end{aligned}$$



一次等效电路

注: $Z_M = j\omega M$, $Z_{11} = R_1 + j\omega L_1$, $Z_{22} = R_2 + j\omega L_2 + Z_L$

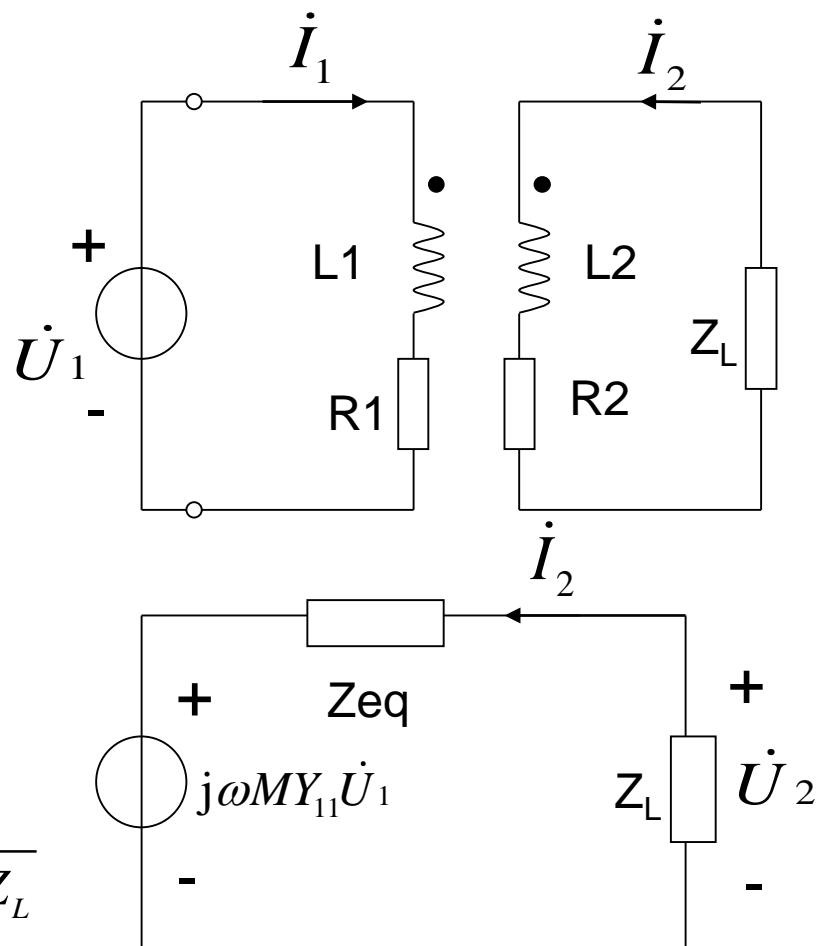
§ 10-4 变压器原理

$$\begin{cases} Z_{11}\dot{I}_1 + Z_M\dot{I}_2 = \dot{U}_1 \\ Z_M\dot{I}_1 + Z_{22}\dot{I}_2 = 0 \end{cases} \Rightarrow \dot{I}_2 = -\frac{Z_M}{Z_{22}}\dot{I}_1$$

$$\dot{U}_2 = -Z_L\dot{I}_2 = Z_L \frac{Z_M}{Z_{22}}\dot{I}_1$$

$$\begin{aligned} \dot{I}_2 &= -\frac{Z_M}{Z_{22}} * \dot{I}_1 = -\frac{Z_M}{Z_{22}} * \frac{\dot{U}_1}{Z_{11} + (\omega M)^2 Y_{22}} \\ &= -\frac{Z_M \dot{U}_1}{Z_{22} Z_{11} + (\omega M)^2} = -\frac{Z_M \dot{U}_1 / Z_{11}}{Z_{22} + (\omega M)^2 Y_{11}} \\ &= -\frac{Z_M \dot{U}_1 / Z_{11}}{R_2 + j\omega L_2 + Z_L + (\omega M)^2 Y_{11}} = -\frac{\dot{U}_{oc}}{Z_{eq} + Z_L} \end{aligned}$$

式中, $Z_{eq} = R_2 + j\omega L_2 + (\omega M)^2 Y_{11}$



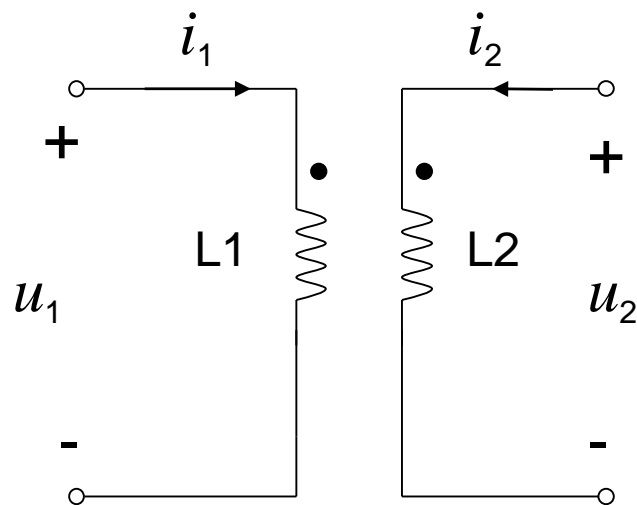
二次等效电路

注: $Z_M = j\omega M$, $Z_{11} = R_1 + j\omega L_1$, $Z_{22} = R_2 + j\omega L_2 + Z_L$

§ 10-5 理想变压器

理想变压器条件

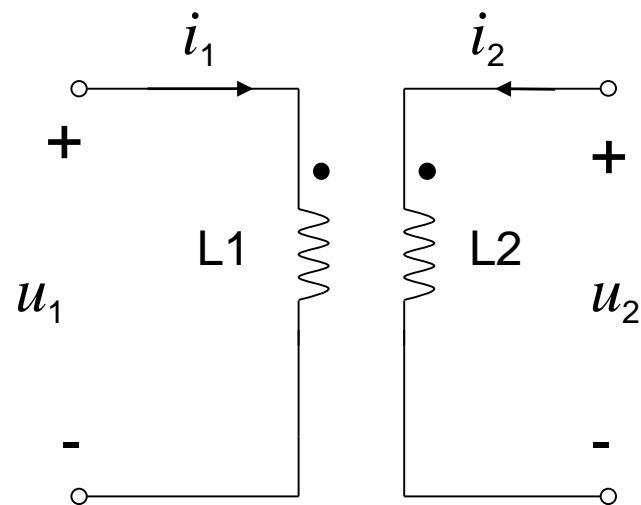
- 1、无损耗 ($R=0$) ,
- 2、耦合系数 $k=1$,
- 3、 L_1 、 L_2 、 M 趋于无穷大。



§ 10-5 理想变压器

$$\begin{aligned}\psi_1 &= L_1 i_1 + M i_2 = L_1 i_1 + \sqrt{L_1 L_2} i_2 \\ &= \sqrt{L_1} (\sqrt{L_1} i_1 + \sqrt{L_2} i_2)\end{aligned}$$

$$\begin{aligned}\psi_2 &= M i_1 + L_2 i_2 = \sqrt{L_1 L_2} i_1 + L_2 i_2 \\ &= \sqrt{L_2} (\sqrt{L_1} i_1 + \sqrt{L_2} i_2)\end{aligned}$$



$$\begin{aligned}u_1 &= \frac{d\psi_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = \sqrt{L_1} \left(\sqrt{L_1} \frac{di_1}{dt} + \sqrt{L_2} \frac{di_2}{dt} \right) \\ u_2 &= \frac{d\psi_2}{dt} = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} = \sqrt{L_2} \left(\sqrt{L_1} \frac{di_1}{dt} + \sqrt{L_2} \frac{di_2}{dt} \right)\end{aligned} \Rightarrow \frac{u_1}{u_2} = \frac{\psi_1}{\psi_2} = \frac{\sqrt{L_1}}{\sqrt{L_2}}$$

§ 10-5 理想变压器

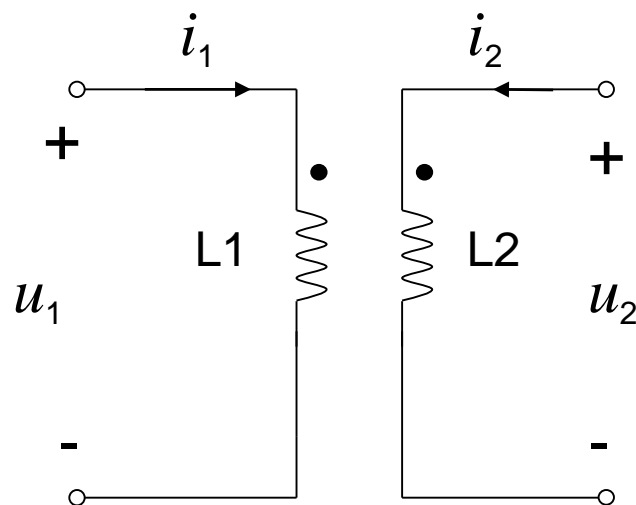
设线圈中的磁通为 Φ ，线圈1、线圈2的匝数分别为 N_1 ， N_2 ，有：

$$\psi_1 = N_1 \phi$$

$$\psi_2 = N_2 \phi$$

$$u_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\phi}{dt}$$

$$u_2 = \frac{d\psi_2}{dt} = N_2 \frac{d\phi}{dt}$$



$$\Rightarrow \frac{u_1}{u_2} = \frac{\psi_1}{\psi_2} = \frac{N_1}{N_2}$$

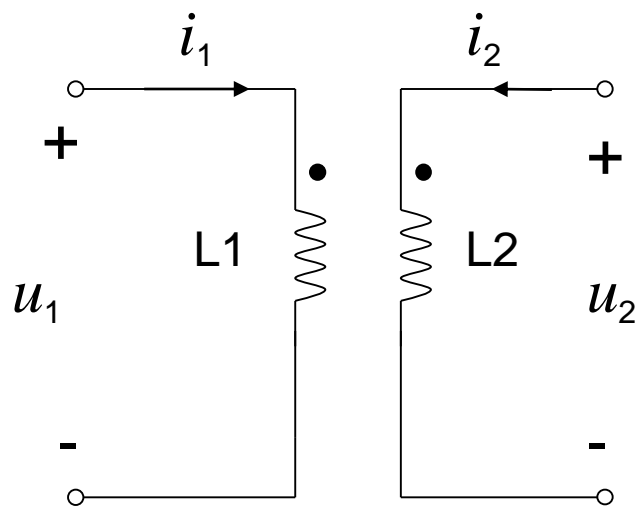
§ 10-5 理想变压器

$$u_1 = \frac{d\psi_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \frac{1}{L_1} u_1 - \frac{M}{L_1} \frac{di_2}{dt}$$

$$\int di_1 = \frac{1}{L_1} \int u_1 dt - \frac{M}{L_1} \int \frac{di_2}{dt} dt$$

$$i_1 = -\frac{M}{L_1} i_2 = -\frac{\sqrt{L_2}}{\sqrt{L_1}} i_2$$



$$\Rightarrow \frac{i_1}{i_2} = -\frac{\sqrt{L_2}}{\sqrt{L_1}} = -\frac{N_2}{N_1}$$

§ 10-5 理想变压器

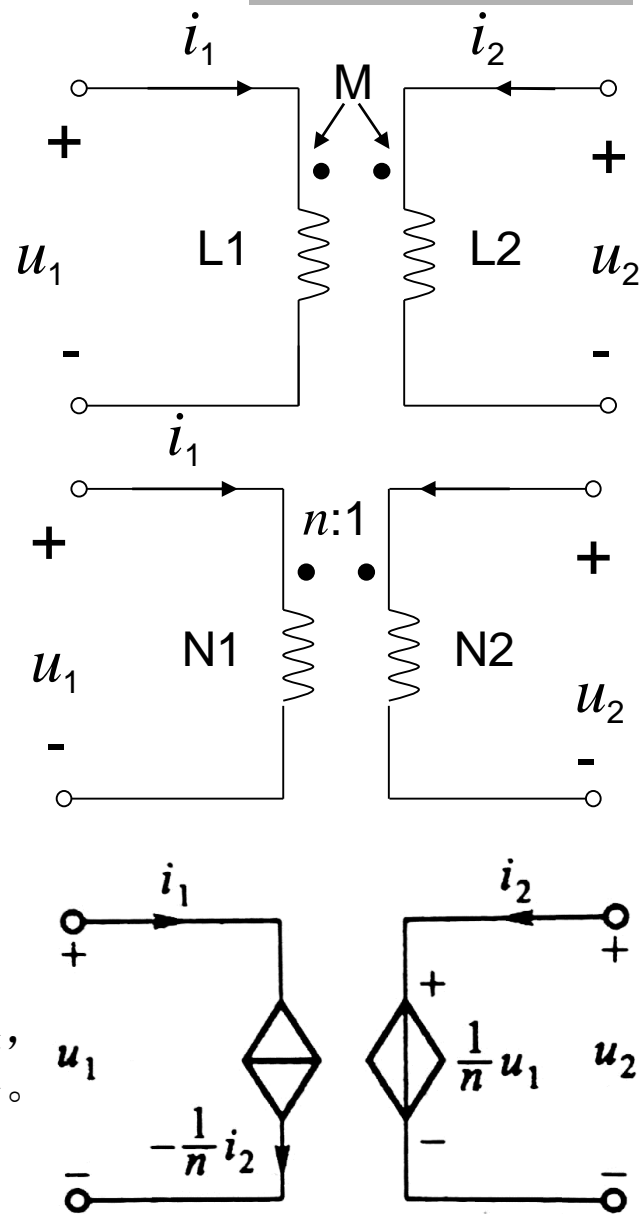
$$\frac{u_1}{u_2} = \frac{N_1}{N_2} \quad \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

设 $n = \frac{N_1}{N_2}$ 为理想变压器的匝数比，又称为变比

$$\begin{cases} u_1 = nu_2 \\ i_1 = -\frac{1}{n}i_2 \end{cases} \Rightarrow u_1 i_1 + u_2 i_2 = 0$$

理想变压器将一侧吸收的能量全部传输到另一侧输出。

在传输过程中，仅仅将电压、电流按变比作数值的变换，既不耗能也不储能，是一个非动态无损耗的磁耦合元件。

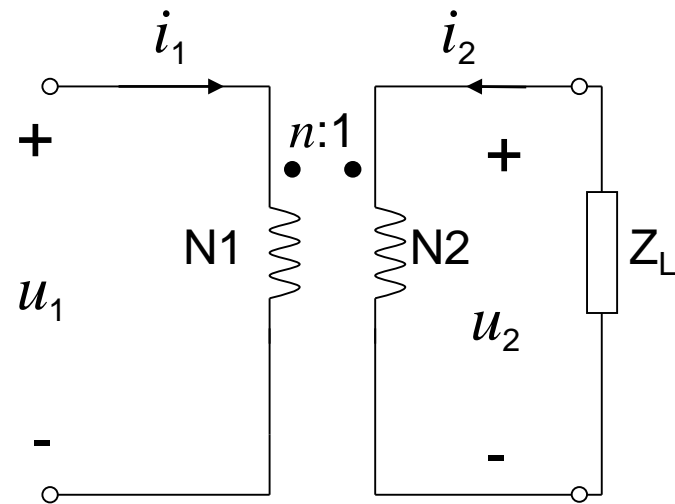


§ 10-5 理想变压器

理想变压器的阻抗转换

二次侧接阻抗 Z_L ，折合到一次侧的等效阻抗：

$$Z_{11'} = \frac{\dot{U}_1}{\dot{I}_1} = \frac{n\dot{U}_2}{-\frac{1}{n}\dot{I}_2} = n^2 Z_L$$



作业

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