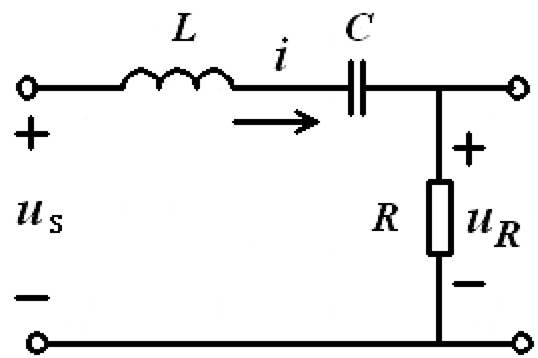
第十一章 电路的频率响应

- 谐振的补充内容

1. RLC串联

$$N(\mathrm{j}\omega)=\frac{\dot U_R}{\dot U_S}=\frac{R}{R+\mathrm{j}\left(\omega L-\dfrac{1}{\omega C}\right)}=\frac{R^2}{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}-\mathrm{j}\frac{R\left(\omega L-\dfrac{1}{\omega C}\right)}{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}$$



幅频特性

$$A(\omega)=\frac{R}{\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}}$$

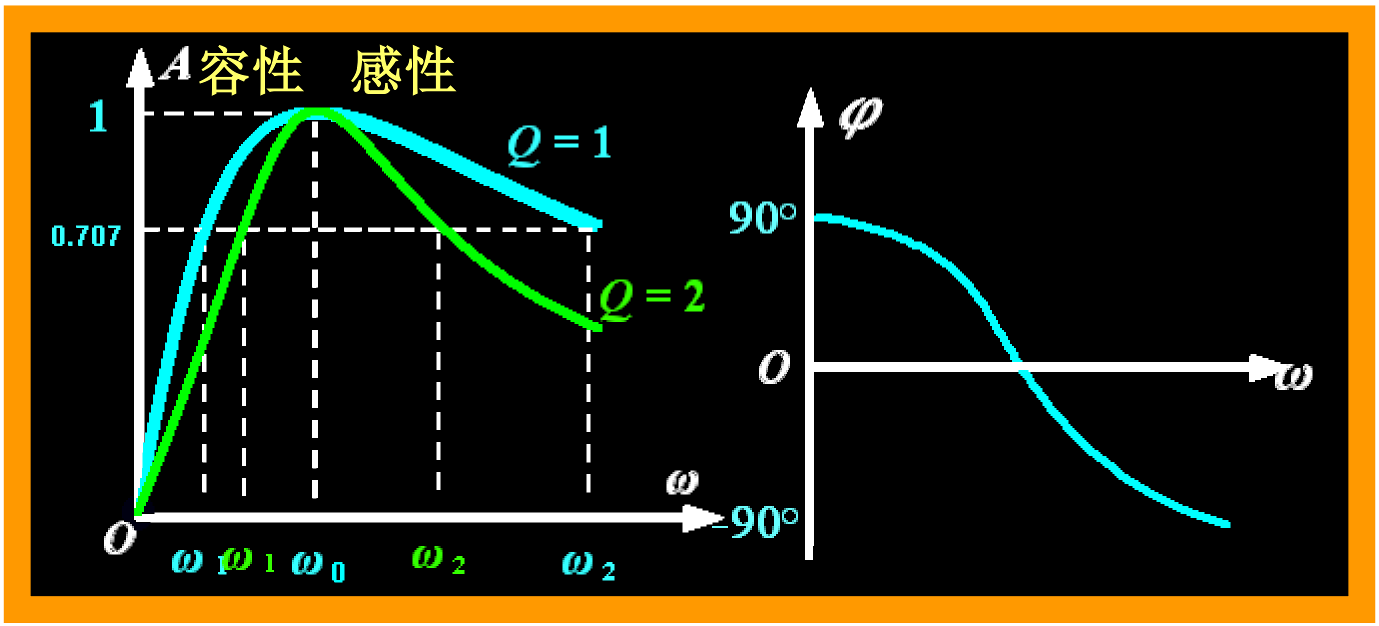
相频特性

$$\varphi(\omega)=-\arctan\frac{\omega L-\dfrac{1}{\omega C}}{R}$$

谐振

$$\omega_0 L=\frac{1}{\omega_0 C}$$

$$\omega_0=\frac{1}{\sqrt{LC}}$$



1. RLC串联

幅频特性为 $$A(\omega)=\frac{R}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

由 $A(\omega)=\frac{1}{\sqrt{2}}$ 可得

$$\left(\omega L-\frac{1}{\omega C}\right)^2=R^2$$

解上式方程，由 $\omega L-\frac{1}{\omega C}=R$ 解得

$$\begin{cases}\omega=\dfrac{CR+\sqrt{C^2R^2+4LC}}{2LC} & (1)\\[2ex] \omega=\dfrac{CR-\sqrt{C^2R^2+4LC}}{2LC} & (2)\end{cases}$$

由 $\omega L-\frac{1}{\omega C}=-R$ 解得

$$\begin{cases}\omega=\dfrac{-CR+\sqrt{C^2R^2+4LC}}{2LC} & (3)\\[2ex] \omega=\dfrac{-CR-\sqrt{C^2R^2+4LC}}{2LC} & (4)\end{cases}$$

式(2)、(4)为$\omega<0$，无意义。因此解为(1)---- ω_2、(3)---- ω_1

得 $BW=\omega_2-\omega_1=\frac{R}{L}$

品质因数定义为 $Q=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 CR}$，上式可写为 $BW=\frac{\omega_0}{Q}$

1. *RLC*串联

相频特性为

$$\varphi(\omega) = -\arctan\frac{\omega L - \dfrac{1}{\omega C}}{R}$$

由 $A(\omega) = \dfrac{1}{\sqrt{2}}$ 可得

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

(1)---- ω_2

当 $\omega L - \dfrac{1}{\omega C} = R$ 时，解得 $\phi(\omega) = -\arctan\dfrac{\omega L - \dfrac{1}{\omega C}}{R} = -\arctan 1 = -45^\circ$

当 $\omega L - \dfrac{1}{\omega C} = -R$ 时，解得 $\phi(\omega) = -\arctan\dfrac{\omega L - \dfrac{1}{\omega C}}{R} = -\arctan(-1) = 45^\circ$

(3)---- ω_1

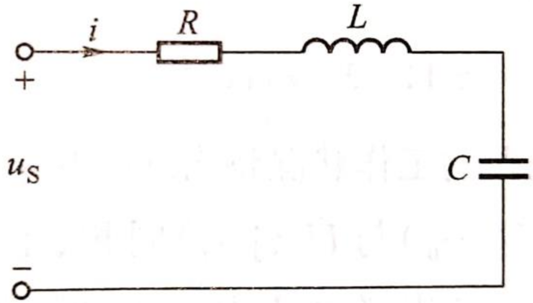


图 11－2　RLC 串联电路

例 11－2　图 11－2 所示电路中，$U_s=0.1\ \text{V}$，$R=1\Omega$，$L=2\ \mu\text{H}$，$C=200\ \text{pF}$ 时，电流 $I=0.1\ \text{A}$。求正弦电压源 u_S 的频率 ω 和电压 U_C、U_L 以及电路的 Q 值。

解　令 $\dot{U}_s=0.1\angle 0°\ \text{V}$，设电流为 $\dot{I}=0.1\angle\phi_i\ \text{A}$，则有

$$0.1\angle\phi_i=\frac{0.1\angle 0°}{1+\text{j}X(\text{j}\omega)}$$

显然有 $X(\text{j}\omega)=0$，$\phi_i=0°$，所以电流 $\dot{I}$ 与电压 $\dot{U}_s$ 同相，电路处于谐振状态。谐振频率 ω_0 为

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2\times10^{-6}\times200\times10^{-12}}}\text{rad/s}=50\times10^6\ \text{rad/s}$$

ω_0 即为电压源 u_S 的频率。电路的 Q 值为

$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}=100$$

谐振时电压 U_L 和 U_C 为

$$U_L=U_C=QU_s=10\ \text{V}$$

例 11－3 RLC 串联电路中 $U_s = 200\ \text{V}$，$C = 6.34\ \mu\text{F}$，电路的固有频率 $\omega_0 = 314\ \text{rad/s}$，带通函数的带宽 $BW = 6.28\ \text{rad/s}$。求 L、R 和 U_L、U_C。

解 电路的 Q 值为

$$Q = \frac{\omega_0}{BW} = \frac{314}{6.28} = 50$$

谐振时容抗等于感抗，则 L 为

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(314)^2 \times 6.34 \times 10^{-6}}\text{H} = 1.60\text{H}$$

R 的值为

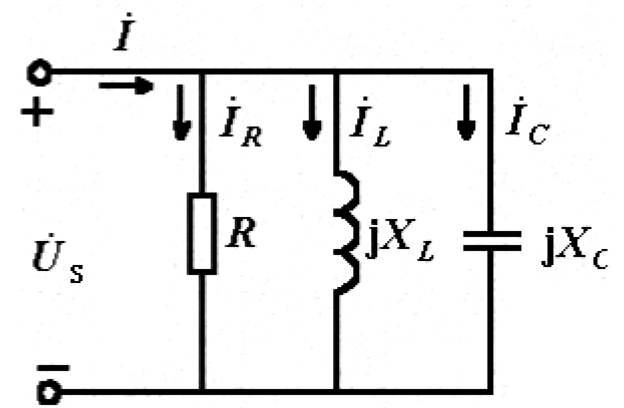
$$R = \frac{\omega_0 L}{Q} = \frac{314 \times 1.6}{50}\Omega = 10\Omega$$

$U_L(\text{j}\omega_0)$、$U_C(\text{j}\omega_0)$为

$$U_L(\text{j}\omega_0) = U_C(\text{j}\omega_0) = QU_s(\text{j}\omega_0) = 50 \times 200\ \text{V} = 10\ 000\ \text{V}$$

2. *RLC*并联

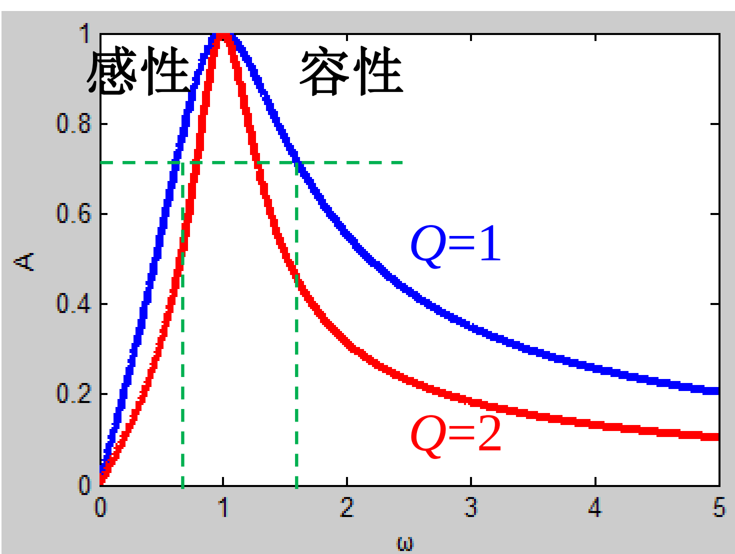
$$\frac{\dot{I}}{\dot{U}_S}=\frac{1}{R}+\mathrm{j}\left(\omega C-\frac{1}{\omega L}\right)\qquad \frac{\dot{I}_R}{\dot{U}_S}=\frac{1}{R}$$

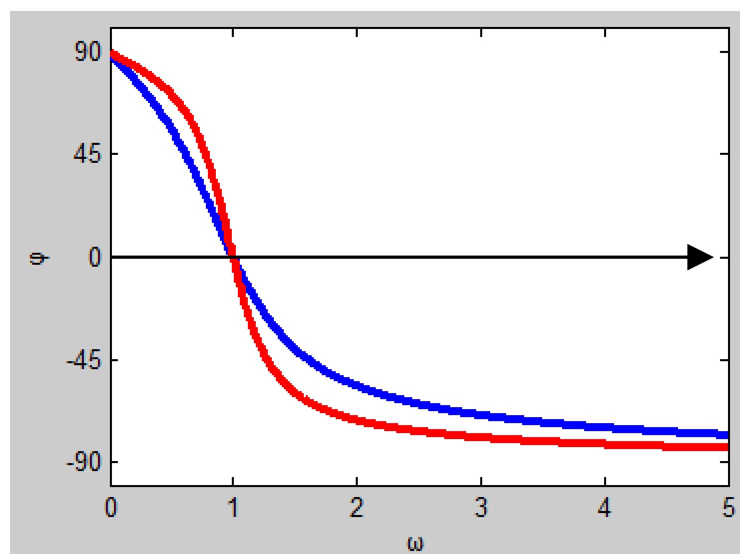


$$N(\mathrm{j}\omega)=\frac{\dot{I}_R}{\dot{I}}=\frac{(1/R)^2}{\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2}-\mathrm{j}\frac{(1/R)\left(\omega C-\frac{1}{\omega L}\right)}{\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2}$$

幅频特性 $$A(\omega)=\frac{1/R}{\sqrt{\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2}}$$

相频特性 $$\phi(\omega)=-\arctan\frac{\omega C-\frac{1}{\omega L}}{1/R}$$





谐振

$$\omega_0 C=\frac{1}{\omega_0 L}$$

$$\omega_0=\frac{1}{\sqrt{LC}}$$

2. RLC并联

幅频特性为 $A(\omega)=\dfrac{1/R}{\sqrt{\dfrac{1}{R^2}+\left(\omega C-\dfrac{1}{\omega L}\right)^2}}$

由 $A(\omega)=\dfrac{1}{\sqrt{2}}$ 可得

$$\left(\omega C-\frac{1}{\omega L}\right)^2=\left(\frac{1}{R}\right)^2$$

解上式方程，由 $\omega C-\dfrac{1}{\omega L}=\dfrac{1}{R}$ 解得
$$\begin{cases}\omega=\dfrac{L/R+\sqrt{(L/R)^2+4LC}}{2LC} & (1)\\[2ex] \omega=\dfrac{L/R-\sqrt{(L/R)^2+4LC}}{2LC} & (2)\end{cases}$$

由 $\omega C-\dfrac{1}{\omega L}=-\dfrac{1}{R}$ 解得
$$\begin{cases}\omega=\dfrac{-L/R+\sqrt{(L/R)^2+4LC}}{2LC} & (3)\\[2ex] \omega=\dfrac{-L/R-\sqrt{(L/R)^2+4LC}}{2LC} & (4)\end{cases}$$

式(2)、(4)为$\omega<0$，无意义。因此解为(1)---- ω_2、(3)---- ω_1

得 $BW=\omega_2-\omega_1=\dfrac{1}{RC}$

品质因数的定义，注意串联和并联中互为倒数

品质因数定义为 $Q=\dfrac{R}{\omega_0 L}=\omega_0 CR$，上式可写为 $BW=\dfrac{\omega_0}{Q}$，与串联相同

2. *RLC*并联

相频特性为

$$\phi(\omega) = \arctan \frac{\omega C - \dfrac{1}{\omega L}}{1/R}$$

由 $A(\omega) = \dfrac{1}{\sqrt{2}}$ 可得

$$A(\omega) = \frac{1/R}{\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}}$$

(1)---- ω_2

当 $\omega C - \dfrac{1}{\omega L} = \dfrac{1}{R}$时，解得 $\phi(\omega) = -\arctan \dfrac{\omega C - \dfrac{1}{\omega L}}{1/R} = -\arctan 1 = -45^\circ$

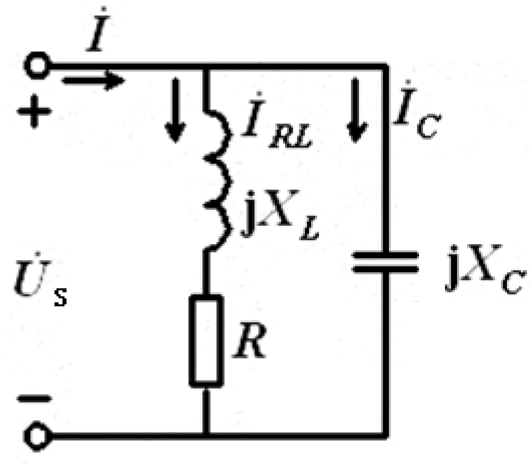
当 $\omega C - \dfrac{1}{\omega L} = -\dfrac{1}{R}$ 时，解得 $\phi(\omega) = -\arctan \dfrac{\omega C - \dfrac{1}{\omega L}}{1/R} = -\arctan(-1) = 45^\circ$

(3)---- ω_1

3. 非理想情况下的*RLC*并联

$$N(\mathrm{j}\omega)=\frac{\dot{I}}{\dot{U}_S}=Y=\frac{1}{(R+\mathrm{j}\omega L)}+\mathrm{j}\omega C$$

$$=\frac{R}{R^2+(\omega L)^2}+\mathrm{j}\left(\omega C-\frac{\omega L}{R^2+(\omega L)^2}\right)$$



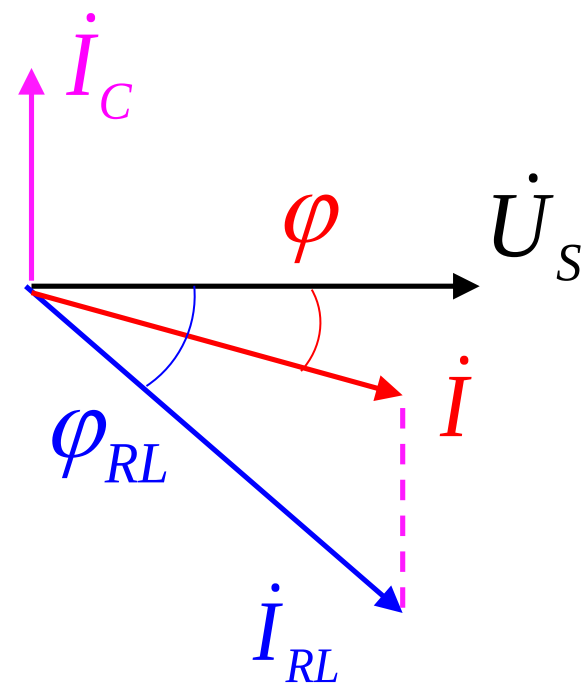
LR支路 $I_{RL}=\dfrac{U_S}{\sqrt{R^2+(\omega L)^2}}$

C支路 $I_C=\dfrac{U_S}{1/\omega C}=\omega CU_S$

总电流

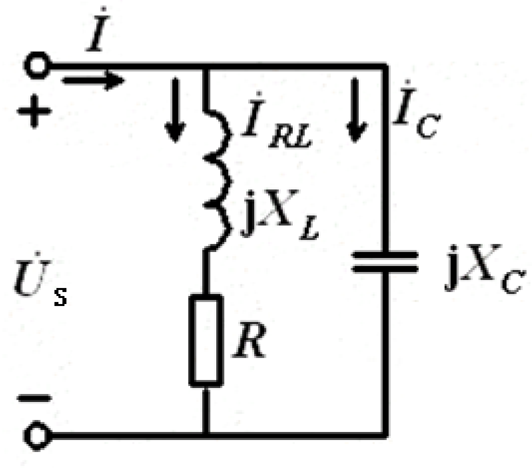
$$I=\sqrt{(I_{RL}\cos\varphi_{RL})^2+(I_{RL}\sin\varphi_{RL}-I_C)^2}$$

$$\varphi=\arctan\frac{I_C-I_{RL}\sin\varphi_{RL}}{I_{RL}\cos\varphi_{RL}}$$



3. 非理想情况下的*RLC*并联

$$N(\mathrm{j}\omega)=\frac{\dot I}{\dot U_S}=Y=\frac{R}{R^2+(\omega L)^2}+\mathrm{j}\left(\omega C-\frac{\omega L}{R^2+(\omega L)^2}\right)$$

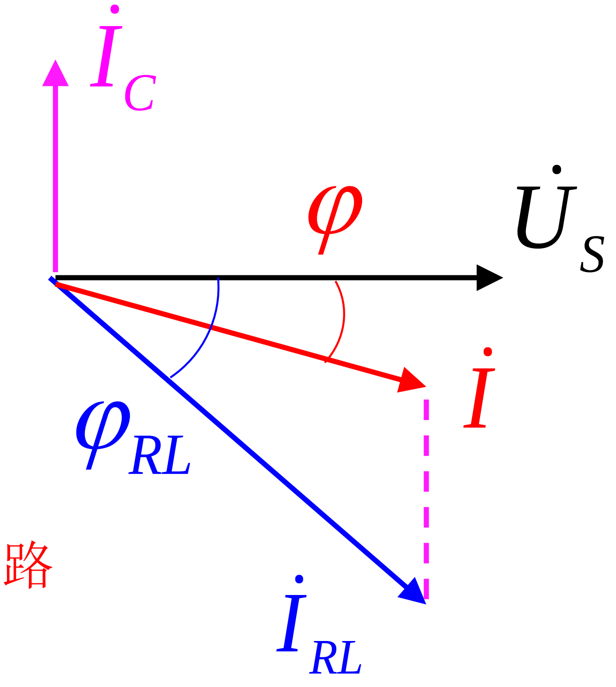


$$I_{RL}\sin\varphi_{RL}=\frac{U_S}{Z_{RL}}\frac{X_L}{Z_{RL}}\qquad I_C=\frac{U_S}{X_C}$$

当 $I_{RL}\sin\varphi_{RL}-I_C=0$ 时，谐振，即

$$\frac{1}{X_C}-\frac{X_L}{Z_{RL}^{\ 2}}=0$$

谐振频率： $\omega_0=\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{L^2}}$



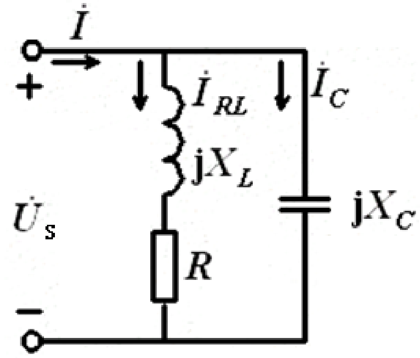
当 $\dfrac{1}{LC}\gg\dfrac{R^2}{L^2}$ 时，$\omega_0\approx\dfrac{1}{\sqrt{LC}}$ 与RLC串联电路谐振频率相同

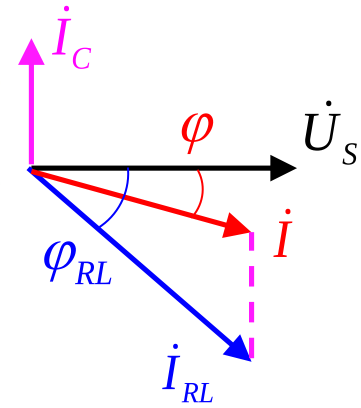
3. 非理想情况下的*RLC*并联

$$I_{RL}\sin\varphi_{RL}=\frac{U_S}{Z_{RL}}\frac{X_L}{Z_{RL}}$$

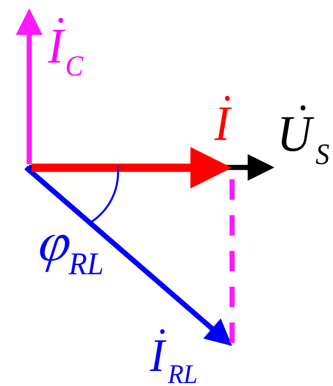
$$=\frac{U_S}{\sqrt{R^2+(\omega L)^2}}\frac{\omega L}{\sqrt{R^2+(\omega L)^2}}$$

$$=U_S\frac{\omega L}{R^2+(\omega L)^2}$$





$$I_{RL}\cos\varphi_{RL}=\frac{U_S}{\sqrt{R^2+X_L{}^2}}\frac{R}{\sqrt{R^2+X_L{}^2}}=U_S\frac{R}{R^2+X_L{}^2}$$



谐振时， $I_0=I_{RL}\cos\varphi_{RL}=U_S\dfrac{R}{R^2+X_L{}^2}$

$$Z_0=\frac{U_S}{I_0}=\frac{R^2+X_L{}^2}{R}=\frac{L}{CR}$$

$$I_C=I_{RL}\sin\varphi_{RL}=\frac{U_S}{Z_0}\frac{X_L}{R}=I_0\frac{X_L}{R}$$

品质因数 $Q=\dfrac{X_L}{R}=\dfrac{\omega L}{R}$ $I_C=I_0Q$

与RLC串联电路的品质因数定义相同