§1-8 基尔霍夫定律

- 基尔霍夫电流定律(KCL):

在集总电路中，在任何时刻，对任一结点，所有**流出结点**的支路电流的代数和恒等于零。即:

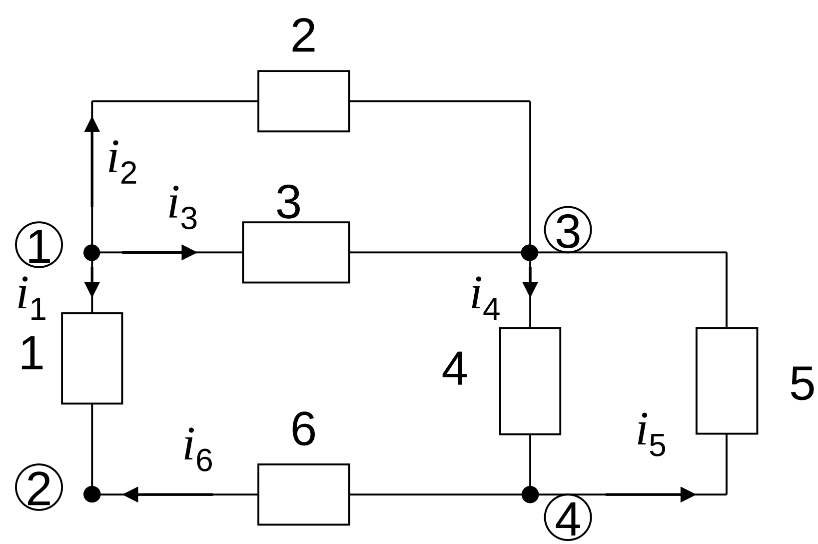
$$\Sigma i = 0$$

计算方法:

1、流出结点的电流前面取“+”，

流入结点的电流前面取“-”。

2、电流是流出结点还是流入结点，由参考方向判断。



§1-8 基尔霍夫定律

- 基尔霍夫电压定律(KVL):

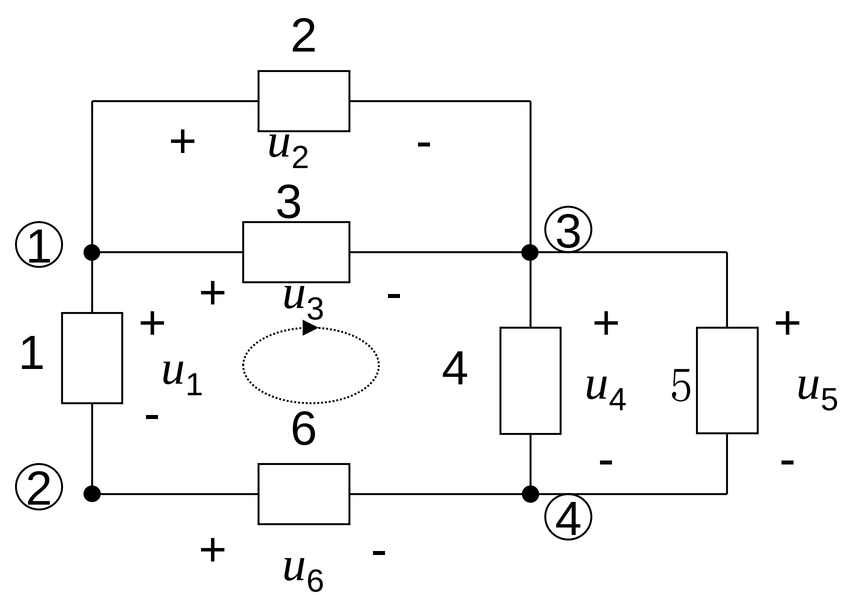
在集总电路中，在任何时刻，沿任一回路，所有支路电压的代数和恒等于零。

即：$\Sigma u = 0$

计算方法：

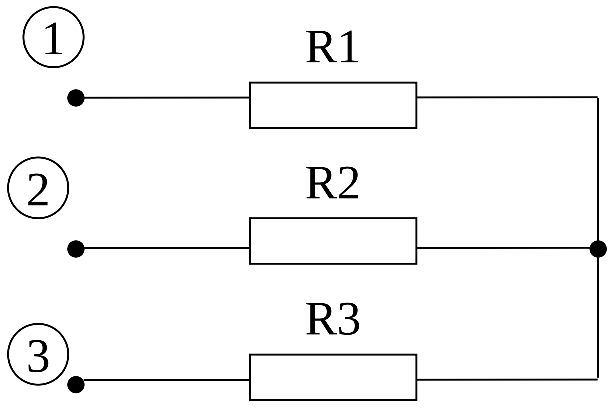
1、任意指定一个回路的绕行方向

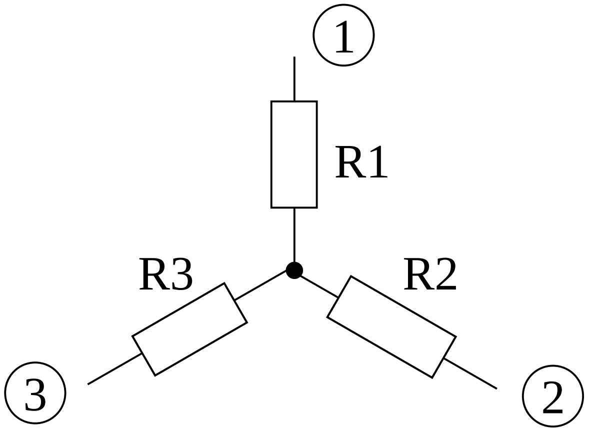
2、若支路电压参考方向与回路绕行方向一致，该电压前面取“+”；若相反，则取“-”。



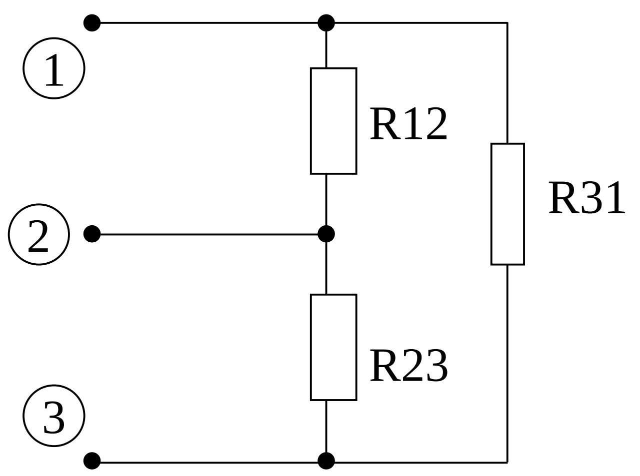
§2-4电阻的Y形和△形联结

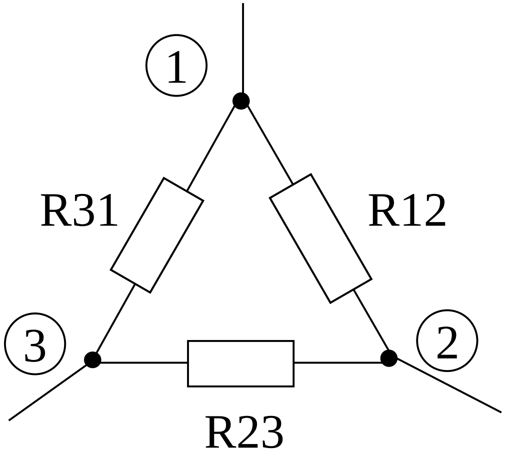
- Y形联结





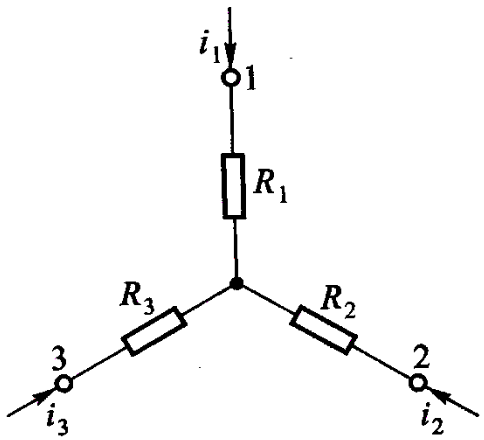
- △形联结

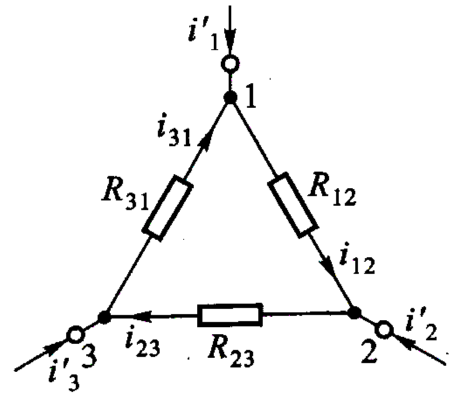




§ 2-4电阻的Y形和△形联结

电阻的Y形和△形联结之间的转换：





$$i_1 + i_2 + i_3 = 0$$

$$R_1 i_1 - R_2 i_2 = u_{12}$$

$$R_2 i_2 - R_3 i_3 = u_{23}$$

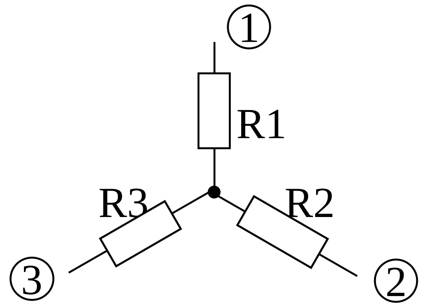
$$\left.\begin{aligned}
i_1 &= \frac{R_3 u_{12}}{R_1R_2 + R_2R_3 + R_3R_1} - \frac{R_2 u_{31}}{R_1R_2 + R_2R_3 + R_3R_1} \\
i_2 &= \frac{R_1 u_{23}}{R_1R_2 + R_2R_3 + R_3R_1} - \frac{R_3 u_{12}}{R_1R_2 + R_2R_3 + R_3R_1} \\
i_3 &= \frac{R_2 u_{31}}{R_1R_2 + R_2R_3 + R_3R_1} - \frac{R_1 u_{23}}{R_1R_2 + R_2R_3 + R_3R_1}
\end{aligned}\right\}$$

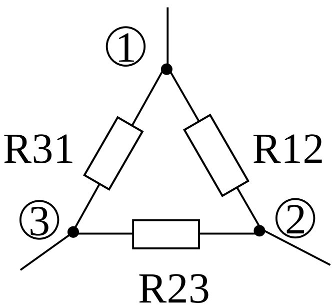
$$i_{12} = \frac{u_{12}}{R_{12}}, \quad i_{23} = \frac{u_{23}}{R_{23}}, \quad i_{31} = \frac{u_{31}}{R_{31}}$$

$$\left.\begin{aligned}
i'_1 &= \frac{u_{12}}{R_{12}} - \frac{u_{31}}{R_{31}} \\
i'_2 &= \frac{u_{23}}{R_{23}} - \frac{u_{12}}{R_{12}} \\
i'_3 &= \frac{u_{31}}{R_{31}} - \frac{u_{23}}{R_{23}}
\end{aligned}\right\}$$

§2-4电阻的Y形和△形联结

电阻的Y形和△形联结之间的转换:





$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

如果$R_{12}= R_{23}= R_{31}= R_{\triangle}$

得 $R_1 = R_2 = R_3 = \frac{R_\Delta}{3}$

$$R_{12} = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$$

$$R_{23} = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$$

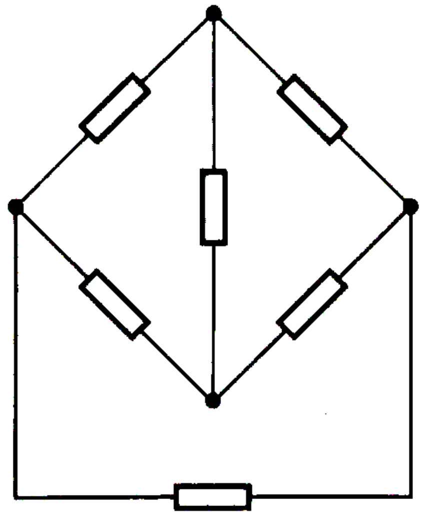
$$R_{31} = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$$

如果$R_1= R_2= R_3= R_Y$

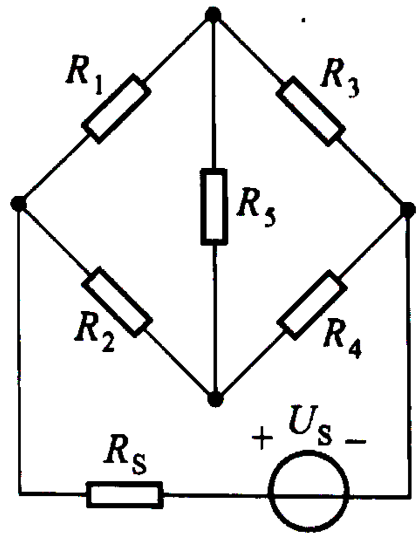
得$R_{12}= R_{23}= R_{31}= 3R_Y$

§2-4电阻的Y形和△形联结

- 桥形连接中的电阻既不是串联也不是并联



(a)



(b)

⇨ 使用电阻的Y-△等效变换

§3-4 网孔电流法

结点1： $i_2 - i_3 - i_1 = 0$

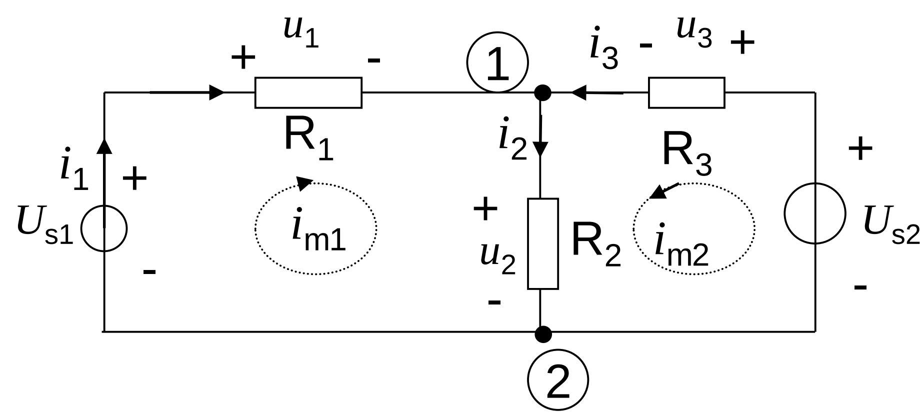
$$i_2 = i_3 + i_1$$

假设回路电流 i_{m1}, i_{m2}

$$i_1 = i_{m1}$$

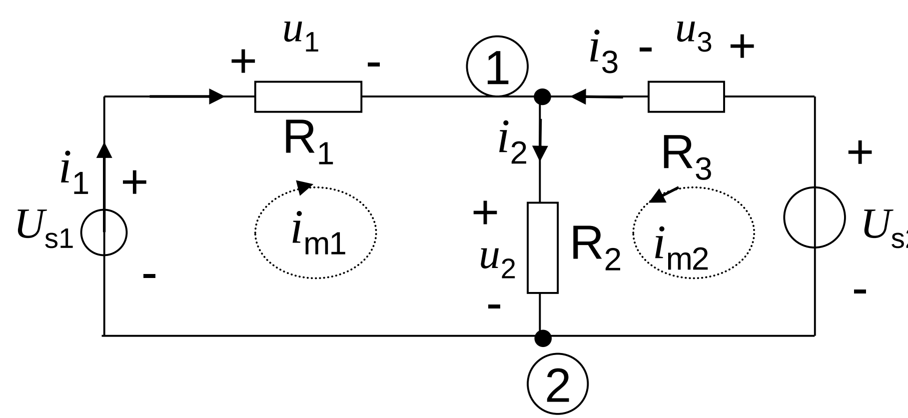
$$i_3 = i_{m2}$$

$$i_2 = i_{m1} + i_{m2}$$



§3-4 网孔电流法

以网孔电流为变量，列出每一个网孔回路的电压方程



回路1： $R_1 i_{m1} + R_2(i_{m1}+ i_{m2}) - u_{s1} = 0$

回路2： $R_3 i_{m2} + R_2(i_{m1}+ i_{m2}) - u_{s2} = 0$

整理两方程得：

回路1： $(R_1+R_2)\, i_{m1} + R_2\, i_{m2} = u_{s1}$

回路 2： $R_2 i_{m1} + (R_2+R_3)\, i_{m2} = u_{s2}$

自阻：回路中所有电阻之和，自阻总为正，例如R_1+R_2， R_2+R_3

互阻：两个回路的共有电阻，例如R_2, 互阻可能为正，也可能为负。

如果两个回路电流流过互阻的方向相同，则互阻为正；

如果两个回路电流流过互阻的方向相反，则互阻为负。

§3-4 网孔电流法

网孔电流方程的一般形式：

$$R_{11}i_{\mathrm{m1}}+R_{12}i_{\mathrm{m2}}+R_{13}i_{\mathrm{m3}}+\cdots+R_{1m}i_{\mathrm{m}m}=u_{\mathrm{S11}}$$

$$R_{21}i_{\mathrm{m1}}+R_{22}i_{\mathrm{m2}}+R_{23}i_{\mathrm{m3}}+\cdots+R_{2m}i_{\mathrm{m}m}=u_{\mathrm{S22}}$$

$$R_{31}i_{\mathrm{m1}}+R_{32}i_{\mathrm{m2}}+R_{33}i_{\mathrm{m3}}+\cdots+R_{3m}i_{\mathrm{m}m}=u_{\mathrm{S33}}$$

$$\cdots\cdots\cdots\cdots$$

$$R_{m1}i_{\mathrm{m1}}+R_{m2}i_{\mathrm{m2}}+R_{m3}i_{\mathrm{m3}}+\cdots+R_{mm}i_{\mathrm{m}m}=u_{\mathrm{S}mm}$$

下标m表示网孔（mesh）

下标*m*（斜体的）表示第*m*个网孔

双下标的电阻R_{11}、R_{22}、R_{33}、R_{mm}是各网孔的自阻

不同下标的电阻R_{12}、R_{13}、R_{21}是网孔间的互阻

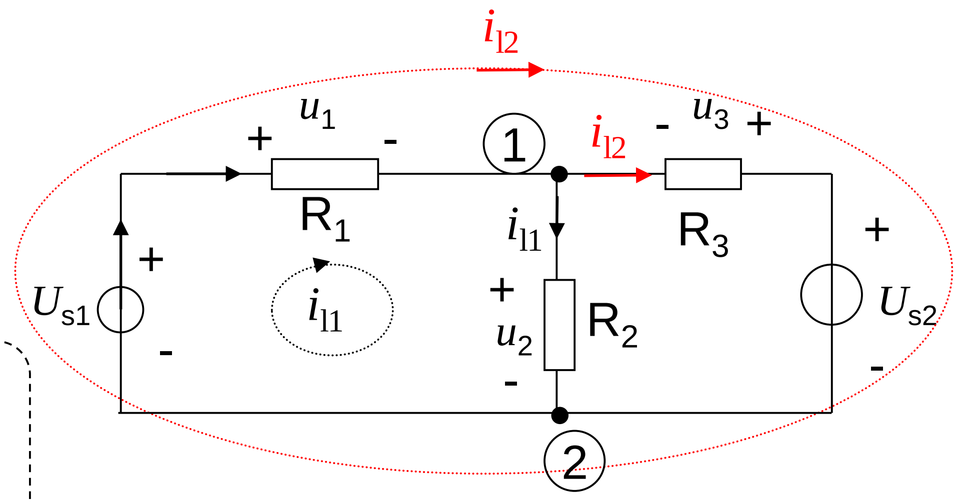
i_{m1}、i_{m2}、$i_{\mathrm{m}m}$是各网孔的电流

§ 3-5 回路电流法

以回路电流为变量，列出回路电压方程的分析方法。

回路1： $R_1(i_{l1}+i_{l2})+R_2 i_{l1}-u_{s1}=0$

回路2： $R_1(i_{l1}+i_{l2})+R_3 i_{l2}+u_{s2}-u_{s1}=0$



整理两方程得：

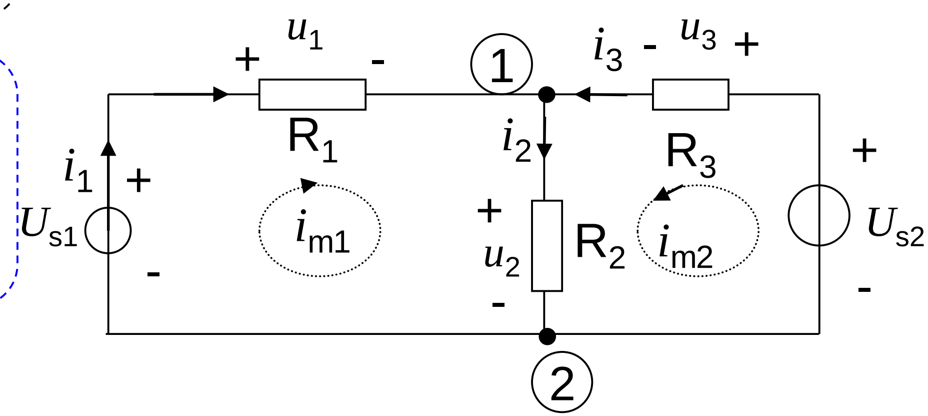
回路1： $(R_1+R_2)i_{l1}+R_1 i_{l2}=u_{s1}$

回路2： $R_1 i_{l1}+(R_1+R_3)i_{l2}=u_{s1}-u_{s2}$

与网孔电流法对比：

网孔1： $(R_1+R_2)i_{m1}+R_2 i_{m2}=u_{s1}$

网孔 2： $R_2 i_{m1}+(R_2+R_3)i_{m2}=u_{s2}$



$i_{m1}=i_{l1}+i_{l2}$　　$i_{m2}=-i_{l2}$

回路电流法与网孔电流法结果相同

回路电流法中的自阻、互阻的定义与网孔电流法相同

§3-5 回路电流法

回路电流方程的一般形式：

$$R_{11}i_{l1}+R_{12}i_{l2}+R_{13}i_{l3}+\cdots+R_{1l}i_{ll}=u_{S11}$$

$$R_{21}i_{l1}+R_{22}i_{l2}+R_{23}i_{l3}+\cdots+R_{2l}i_{ll}=u_{S22}$$

$$R_{31}i_{l1}+R_{32}i_{l2}+R_{33}i_{l3}+\cdots+R_{3l}i_{ll}=u_{S33}$$

$$\cdots\ \cdots\ \cdots\ \cdots$$

$$R_{l1}i_{l1}+R_{l2}i_{l2}+R_{l3}i_{l3}+\cdots+R_{ll}i_{ll}=u_{Sll}$$

下标l表示回路（loop）

下标*l*（斜体的）表示第*l*条回路

双下标的电阻R_{11}、R_{22}、R_{33}、R_{ll}是各回路的自阻

不同下标的电阻R_{12}、R_{13}、R_{21}是回路间的互阻

i_{l1}、i_{l2}、i_{ll}是各回路的电流

§ 3-6 结点电压法

以结点电压为变量的电路分析方法。

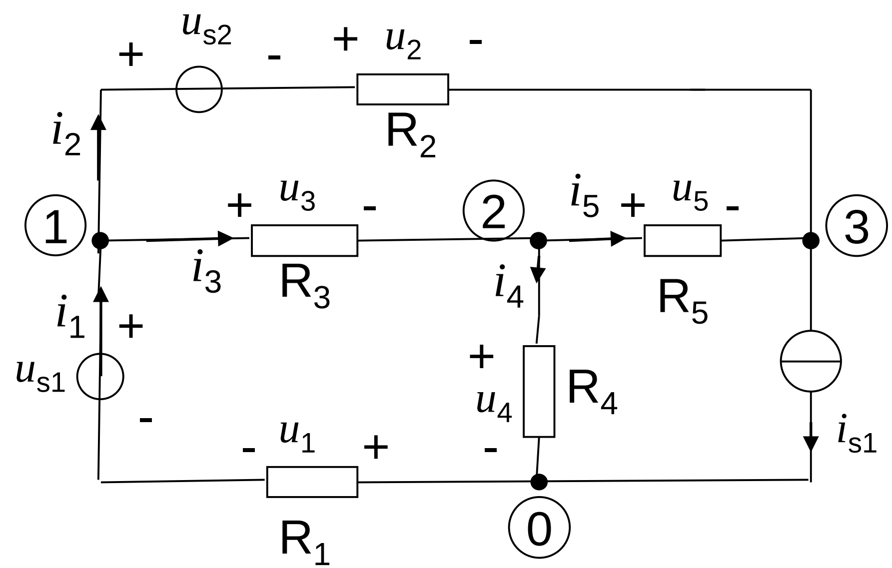
结点1： $i_2 + i_3 - i_1 = 0$

结点2： $i_4 + i_5 - i_3 = 0$

结点3： $-i_2 - i_5 + i_{s1} = 0$

取结点0作为参考，结点1、2、3的结点电压为u_{n1}、u_{n2}、u_{n3}，

代入上面的结点KCL方程：



$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)u_{n1}-\frac{1}{R_3}u_{n2}-\frac{1}{R_2}u_{n3}=\frac{u_{s1}}{R_1}+\frac{u_{s2}}{R_2}$$

$$-\frac{1}{R_3}u_{n1}+\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)u_{n2}-\frac{1}{R_5}u_{n3}=0$$

$$-\frac{1}{R_2}u_{n1}-\frac{1}{R_5}u_{n2}+\left(\frac{1}{R_2}+\frac{1}{R_5}\right)u_{n3}=-\frac{u_{s2}}{R_2}-i_{s1}$$

自导：结点所连接所有电导之和，总为正；

互导：两个独立结点之间的电导，总为负。

§ 3-6 结点电压法

结点电压方程的一般形式：

$$G_{11}u_{\mathrm{n}1} + G_{12}u_{\mathrm{n}2} + G_{13}u_{\mathrm{n}3} + \cdots + G_{1(n-1)}u_{\mathrm{n}(n-1)} = i_{\mathrm{S}11}$$

$$G_{21}u_{\mathrm{n}1} + G_{22}u_{\mathrm{n}2} + G_{23}u_{\mathrm{n}3} + \cdots + G_{2(n-1)}u_{\mathrm{n}(n-1)} = i_{\mathrm{S}22}$$

$$\cdots\cdots\ \cdots\ \cdots$$

$$G_{(n-1)1}u_{\mathrm{n}1} + G_{(n-1)2}u_{\mathrm{n}2} + G_{(n-1)3}u_{\mathrm{n}3} + \cdots + G_{(n-1)(n-1)}u_{\mathrm{n}(n-1)} = i_{\mathrm{S}(n-1)(n-1)}$$

下标n表示结点（node）

下标n（斜体的）表示第n个结点

双下标的电导G_{11}、G_{22}、G_{33}、$G_{(n\text{-}1)\,(n\text{-}1)}$是各结点的自导，总为正

不同下标的电导G_{12}、G_{13}、G_{21}是结点间的互导，总为负

等式右边i_{Sl1}、i_{S22}、$i_{\mathrm{S}(n\text{-}1)(n\text{-}1)}$是流向结点的电流源的代数和，流入取+，流出取-

§3-6 结点电压法

- 备注：

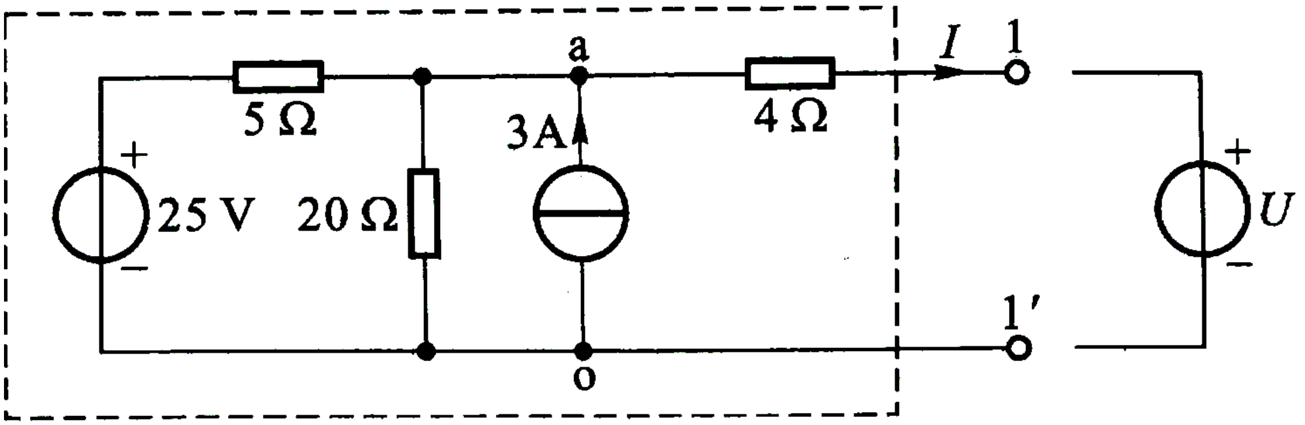
1、含无伴电压源支路的电路，应用结点法进行分析时，如果电压源的低电位端就是参考结点时，那么电压源另一端的结点电压就是电压源的值。

2、含无伴电压源支路的电路，应用结点法进行分析时，如果电压源的两端都不是参考结点时，增加电压源支路的电流作为附加变量，列入KCL方程，同时增加电压源支路两端结点电压与电压源电压的关系方程。

3、当电路中含有受控电源时，把它们按独立电源处理。

§ 4.3 戴维宁及诺顿定理

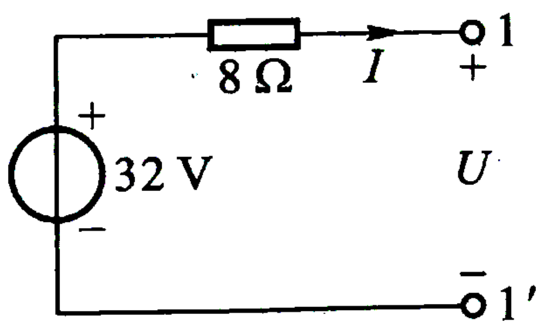
含电阻、电源的一端口如何简化？



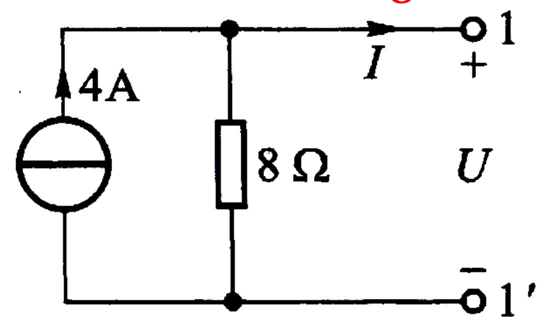
(a)

$U = 32 - 8I$

$I = 4 - \frac{U}{8}$



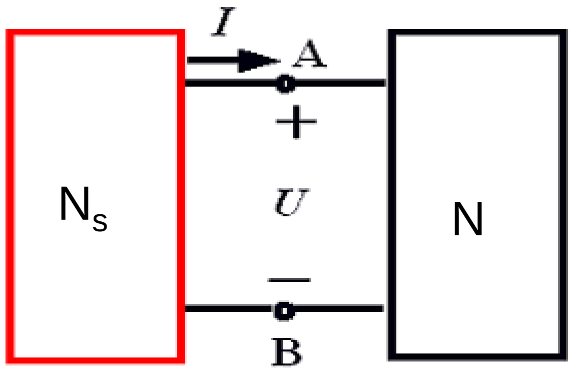
(b)

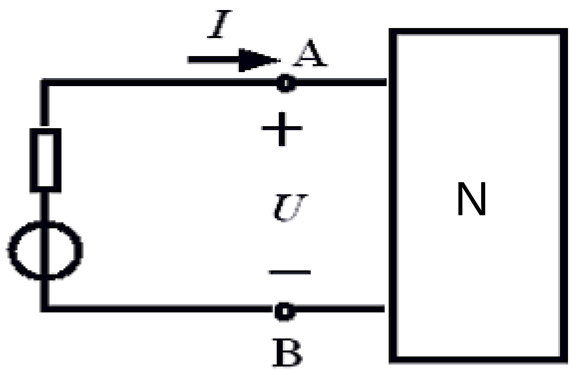


(c)

§4-3 戴维宁及诺顿定理

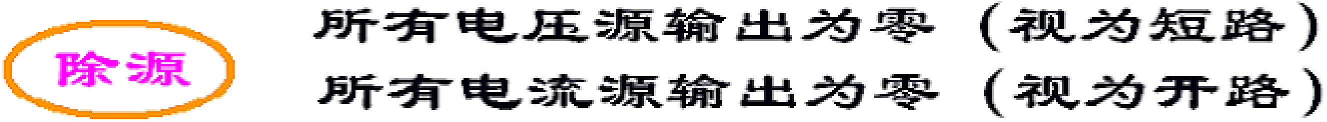
戴维宁等效定理：任一有源二端线性网络Ns，可用一电压源与一电阻串联的组合模型等效代替。





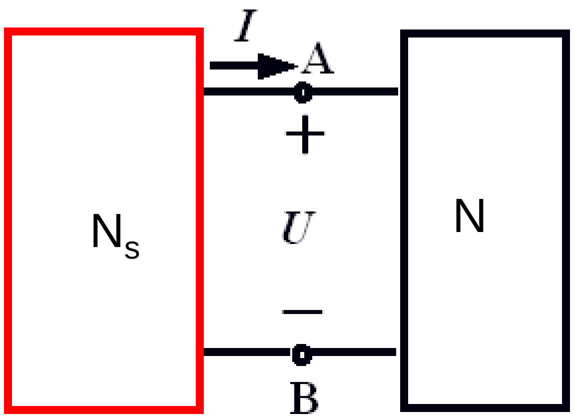
等效电阻：一端口内全部独立电源置零后的输入电阻。

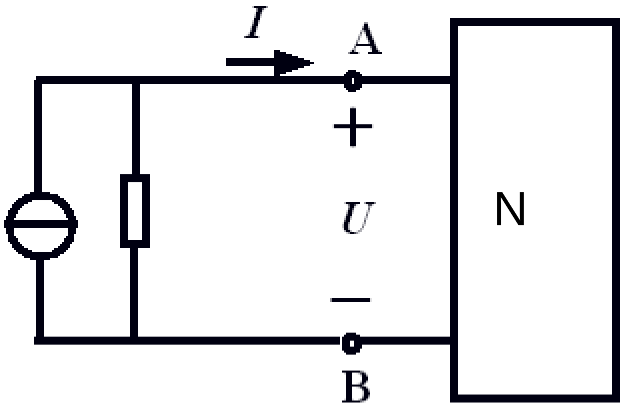
等效电压：一端口的开路电压。



§4-3 戴维宁及诺顿定理

诺顿等效定理：任一有源二端线性网络Ns，可用一电流源与一电阻并联的组合模型等效代替。





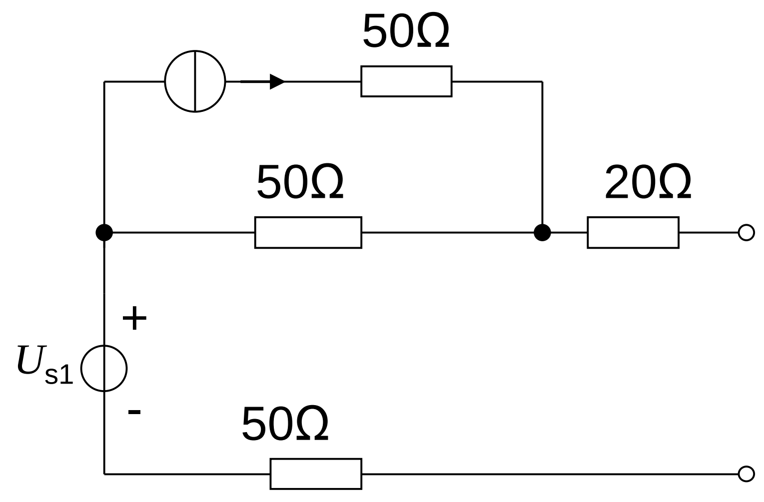
等效电阻：一端口内全部独立电源置零后的输入电阻。

等效电流：一端口的短路电流。

§4-3 戴维宁及诺顿定理

- 求输入端电阻Rin的方法：

1. 电阻等效变换：

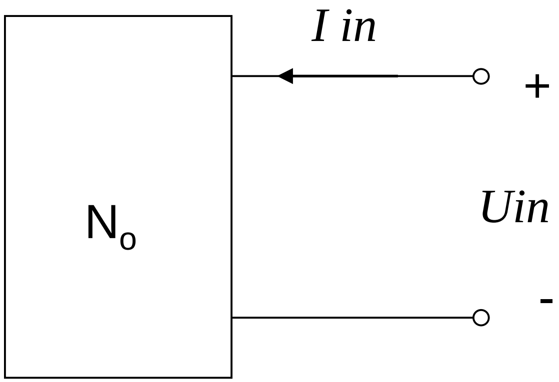


§4-3 戴维宁及诺顿定理

- 求输入端电阻Rin的方法：

2、比例法：

$$R_{in} = \frac{U_{in}}{I_{in}}$$



独立源置零

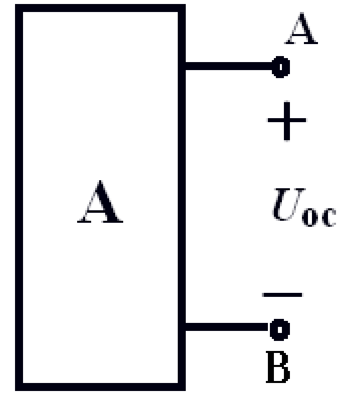
受控源保留

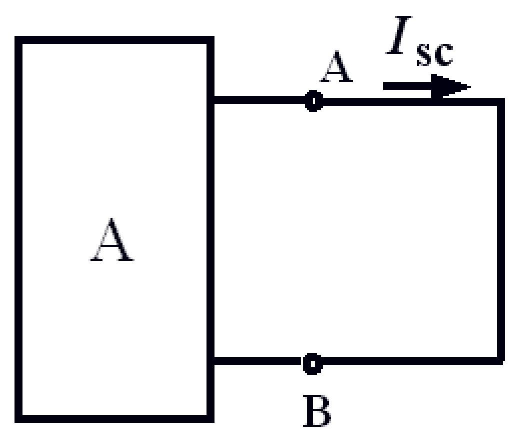
§4-3 戴维宁及诺顿定理

- 求输入端电阻Rin的方法：

3、开路短路法：

$$R_{in} = \frac{U_{oc}}{I_{sc}}$$





独立源和受控源都保留

§4-3 戴维宁及诺顿定理

戴维宁和诺顿定理应用说明：

1、此定理主要用于化简电路，或与其它定理相结合，分析复杂电路；

2、输入端电阻Rin的值可能为正，也可能为负，也可能为无穷大或零；

$R_{eq}=0, u_{oc}$有限值，则存在无伴电压源，不存在诺顿等效

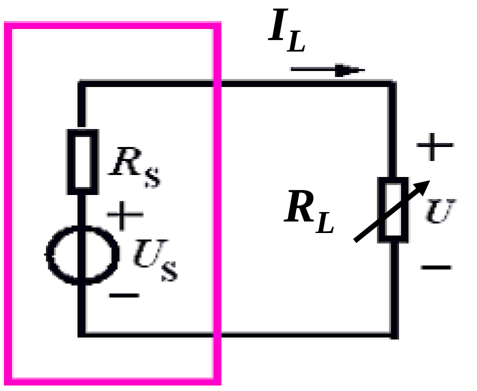
$G_{eq}=0, i_{sc}$有限值，则存在无伴电流源，不存在戴维宁等效

§4-4 最大功率传输定理

- 负载R_L所获得的功率：

$$P_L = I_L^2 R_L = \left(\frac{U_S}{R_S + R_L}\right)^2 R_L$$

$$= \frac{U_S^2}{R_S + R_L} \cdot \frac{R_L}{R_S + R_L} = P_S \cdot \eta$$



$\boldsymbol{P_S}$为电源发出的功率，η 为传输效率。

$$\frac{\mathrm{d}P_L}{\mathrm{d}R_L} = U_S^2 \left[\frac{(R_S + R_L)^2 - R_L \times 2(R_S + R_L)}{(R_S + R_L)^4}\right] = 0$$

当$R_L = R_S$时，$P_{L\max} = \dfrac{U_S^2 R_S}{(2R_S)^2} = \dfrac{U_S^2}{4R_S}$

§4-4 最大功率传输定理

- **最大功率传输定理说明：**

1、传输功率最大时，传输效率不一定最大。

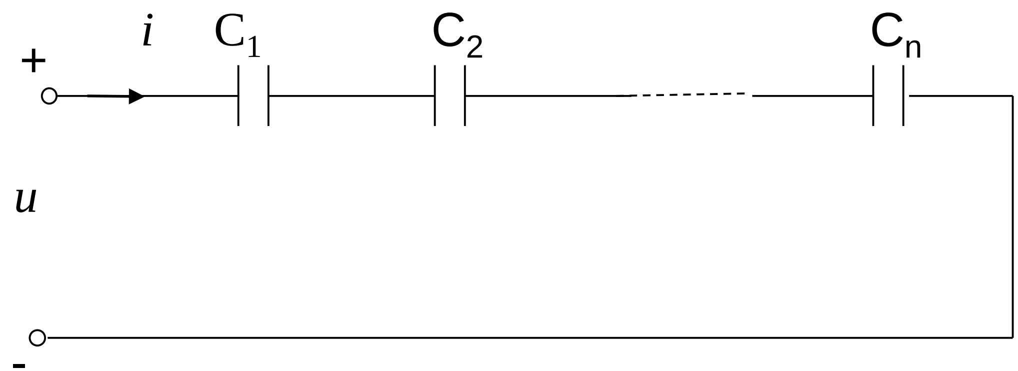
一般在电力传输时，要求传输效率尽量大，

信号传输时，要求传输功率尽量大。

2、当R_S=R_L时，传输功率最大，此时又称负载匹配。在高频电路设计中负载匹配是很重要的考虑问题。

§6-3 电容、电感元件的串并联

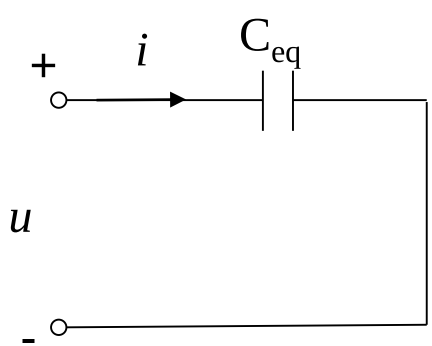
电容的串联



$$u = u_1 + u_2 + \cdots + u_n = u_1(t_0) + \frac{1}{C_1}\int_{t_0}^{t} i\,\mathrm{d}\xi + \cdots + u_n(t_0) + \frac{1}{C_n}\int_{t_0}^{t} i\,\mathrm{d}\xi$$

$$= u_1(t_0) + u_2(t_0) + \cdots + u_n(t_0) + \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right)\int_{t_0}^{t} i\,\mathrm{d}\xi$$

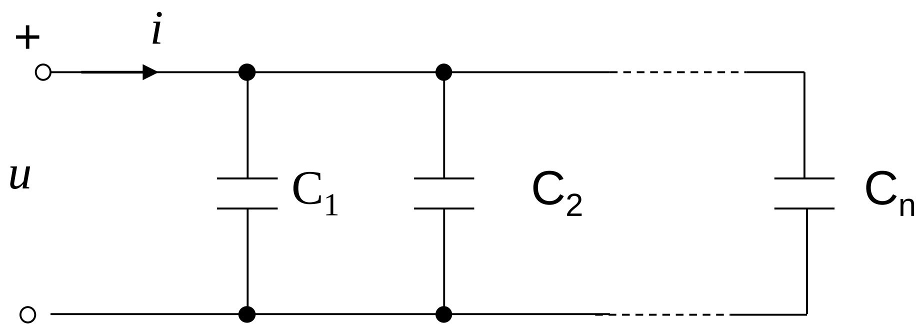
$$= u(t_0) + \frac{1}{C_{eq}}\int_{t_0}^{t} i\,\mathrm{d}\xi$$



$$\frac{1}{C_{eq}} = \sum_{k=1}^{n} \frac{1}{C_k}$$

§6-3 电容、电感元件的串并联

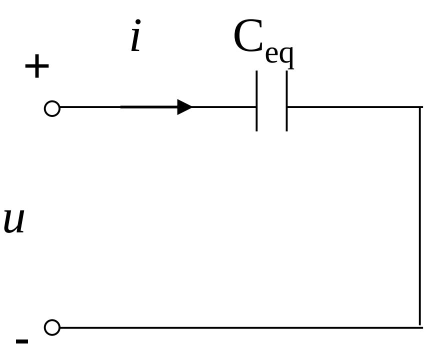
- 电容的并联



$$i = i_1 + i_2 + \cdots + i_n = C_1\frac{\mathrm{d}u}{\mathrm{d}t} + C_2\frac{\mathrm{d}u}{\mathrm{d}t} + \cdots + C_n\frac{\mathrm{d}u}{\mathrm{d}t}$$

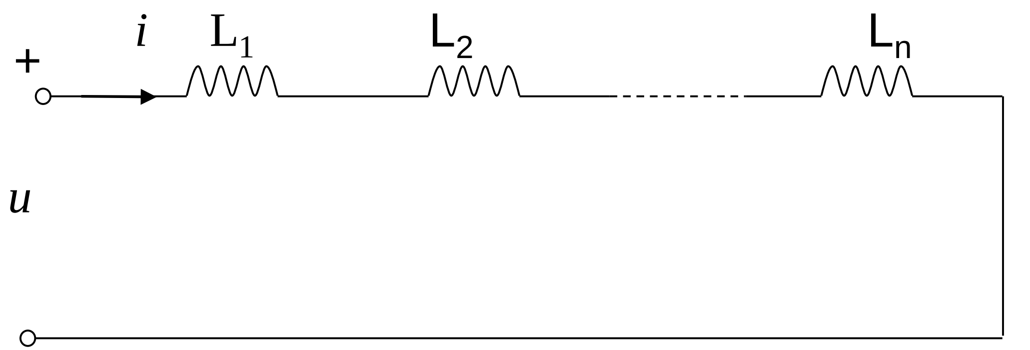
$$= C_{eq}\frac{\mathrm{d}u}{\mathrm{d}t}$$

$$C_{eq} = \sum_{k=1}^{n} C_k$$



§6-3 电容、电感元件的串并联

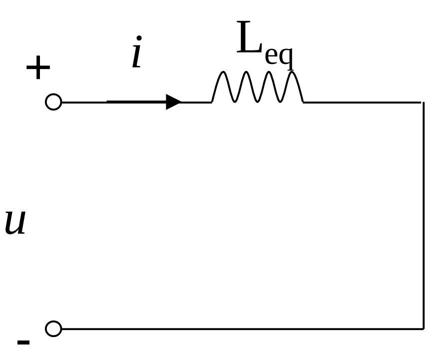
- 电感的串联



$$u=u_1+u_2+\cdots+u_n=L_1\frac{\mathrm{d}i}{\mathrm{d}t}+L_2\frac{\mathrm{d}i}{\mathrm{d}t}+\cdots+L_n\frac{\mathrm{d}i}{\mathrm{d}t}$$

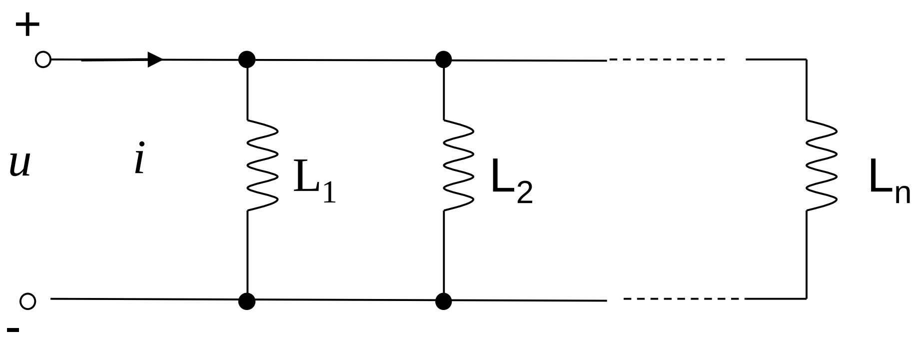
$$=(L_1+L_2+\cdots+L_n)\frac{\mathrm{d}i}{\mathrm{d}t}=L_{eq}\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$L_{eq}=\sum_{k=1}^{n}L_k$$

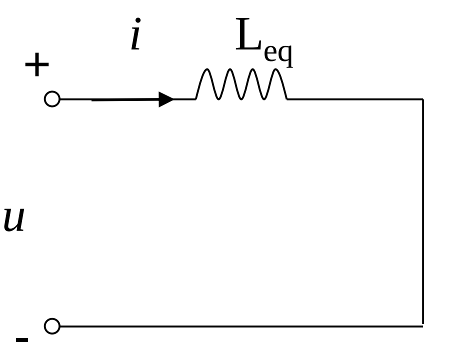


§6-3 电容、电感元件的串并联

- 电感的并联



$$\frac{1}{L_{eq}}=\sum_{k=1}^{n}\frac{1}{L_k}$$

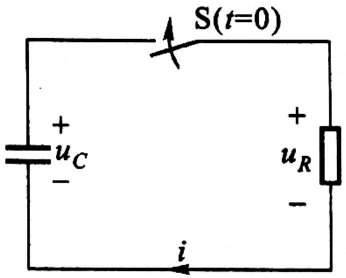


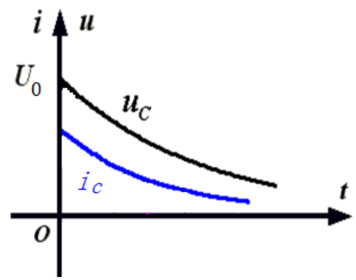
小结：一阶电路的零输入响应和零状态响应

零输入响应　　　　零状态响应

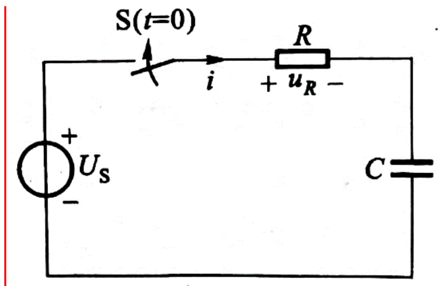
RC

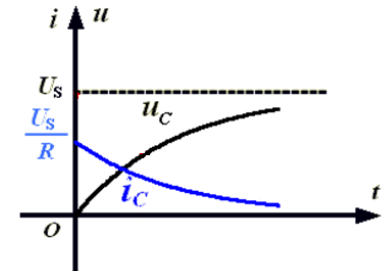
$\tau = RC$





$$u_C = U_0 e^{-\frac{t}{\tau}} \qquad i_C = -\frac{U_0}{R} e^{-\frac{t}{\tau}}$$

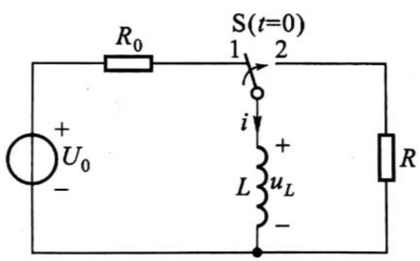


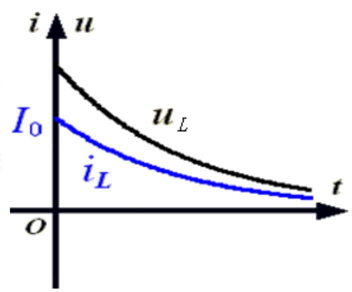


$$u_C = U_S\left(1 - e^{-\frac{t}{\tau}}\right) \qquad i = \frac{U_S}{R} e^{-\frac{t}{\tau}}$$

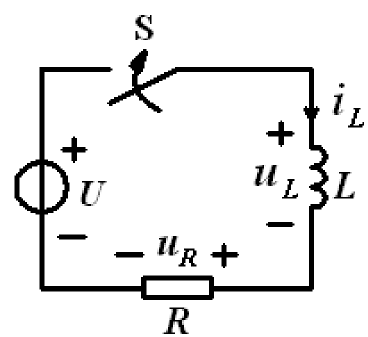
RL

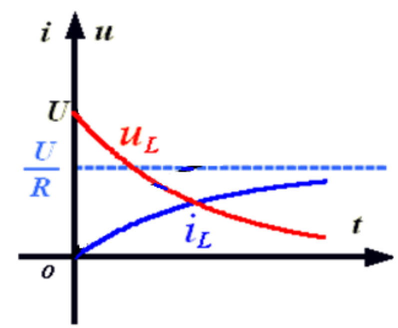
$$\tau = \frac{L}{R}$$





$$i = I_0 e^{-\frac{t}{\tau}} = \frac{U_0}{R_0} e^{-\frac{t}{\tau}} \quad u_L = -RI_0 e^{-\frac{t}{\tau}}$$





$$i_L = \frac{U}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad u_L = U e^{-\frac{t}{\tau}}$$

§ 7.4 一阶电路的全响应

三要素法： $f(0_+)$、$f(\infty)$、τ

$$f(t) = \text{特解} \; + Ae^{-\frac{t}{\tau}} = \text{稳态解} \; + Ae^{-\frac{t}{\tau}} = f(\infty) + Ae^{-\frac{t}{\tau}}$$

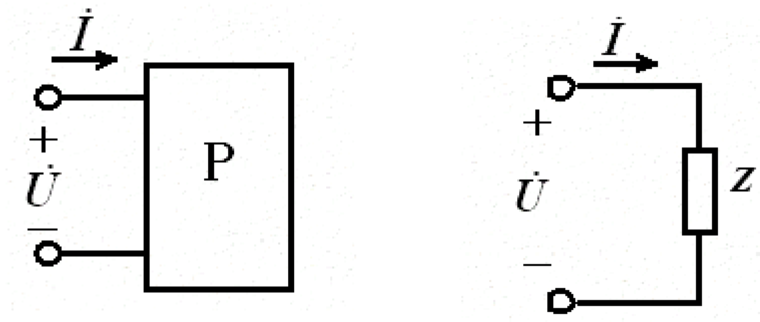
$$f(0_+) = f(\infty) + Ae^{-\frac{0}{\tau}} = f(\infty) \; + A \qquad A = f(0_+) - f(\infty)$$

$$\boxed{f(t) = f(\infty) + [f(0_+) - f(\infty)]e^{-\frac{t}{\tau}}}$$

§9-1 阻抗和导纳

1.复阻抗

$$Z=\frac{\dot{U}}{\dot{I}}=\frac{U\angle\phi_u}{I\angle\phi_i}=|Z|\angle\phi_Z$$



Z不是正弦量，而是一个复数。

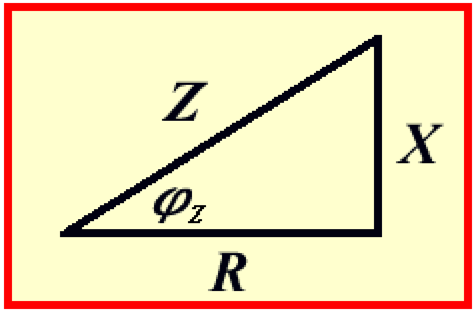
阻抗的模： $|Z|$ 阻抗角： $\phi_Z=\phi_u-\phi_i$

$$Z=R+\mathrm{j}X$$

电阻 电抗

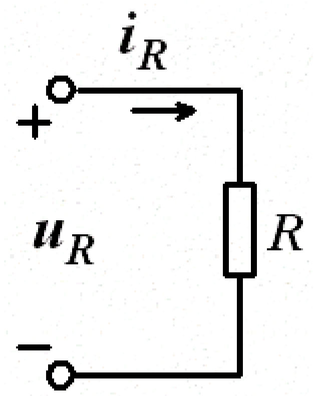
$$|Z|=\sqrt{R^2+X^2}$$

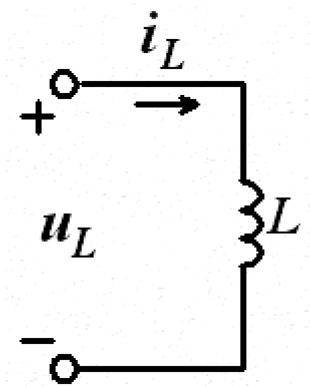
$$\phi_Z=\arctan\frac{X}{R}$$

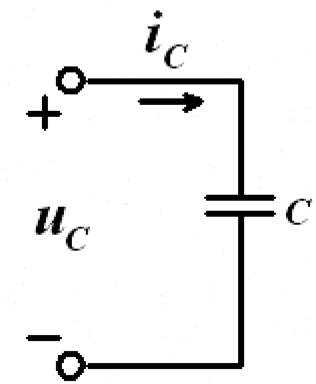


§9-1 阻抗和导纳

特例：







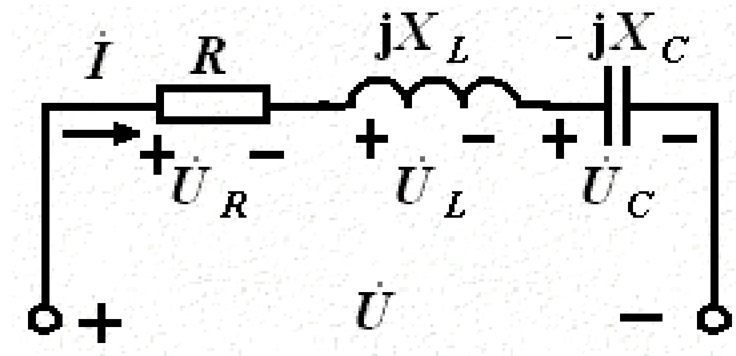
$$Z = \frac{\dot{U}}{\dot{I}} = R$$

$$Z = \frac{\dot{U}}{\dot{I}} = \mathrm{j}X_L = \mathrm{j}\omega L$$

$$Z = \frac{\dot{U}}{\dot{I}} = -\mathrm{j}X_C = \frac{1}{\mathrm{j}\omega C}$$

§9-1 阻抗和导纳

2.*RLC* 串联复阻抗



根据KVL：

$$\dot{U} = \dot{U}_R + \dot{U}_L + \dot{U}_C \quad = \dot{I}(R + \mathrm{j}X_L - \mathrm{j}X_C)$$

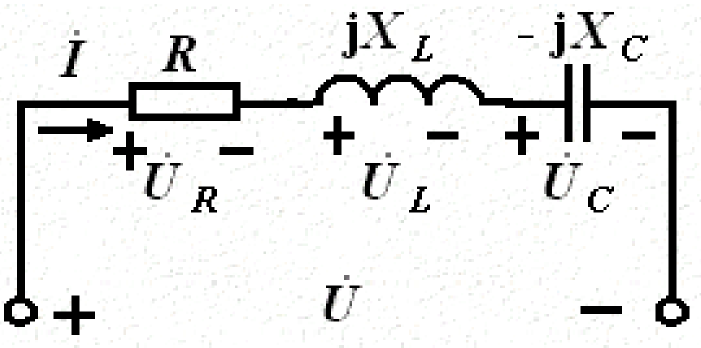
$$Z = \frac{\dot{U}}{\dot{I}} = R + \mathrm{j}(X_L - X_C) = R + \mathrm{j}X = |Z| \angle \phi_Z$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + X^2}$$

$$\phi_Z = \arctan\frac{X_L - X_C}{R}$$

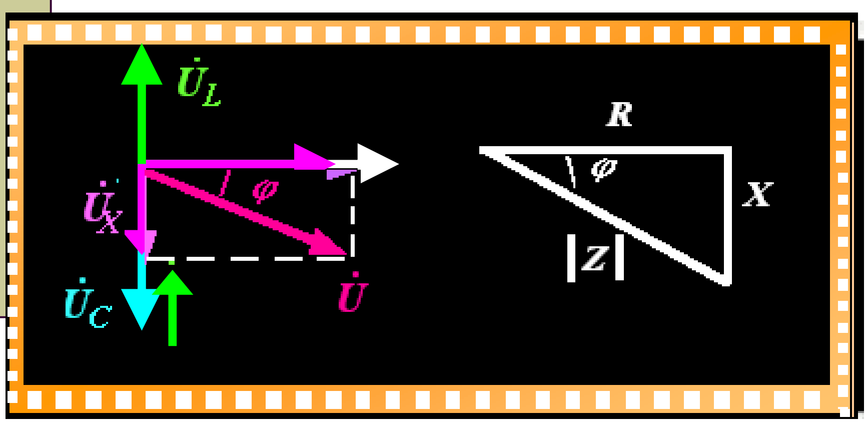
§9-1 复阻抗和复导纳

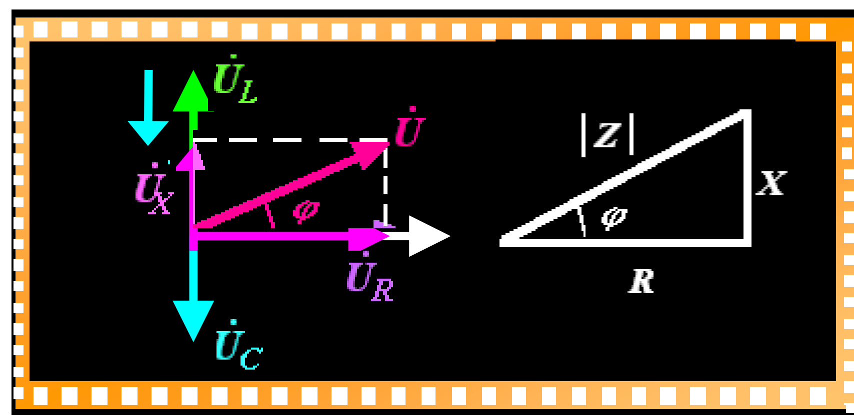
2.*RLC*串联复阻抗



$X_L - X_C = 0 \quad \varphi = 0$

阻性电路





$X_L - X_C < 0 \quad \varphi < 0$　　容性电路

电压滞后电流

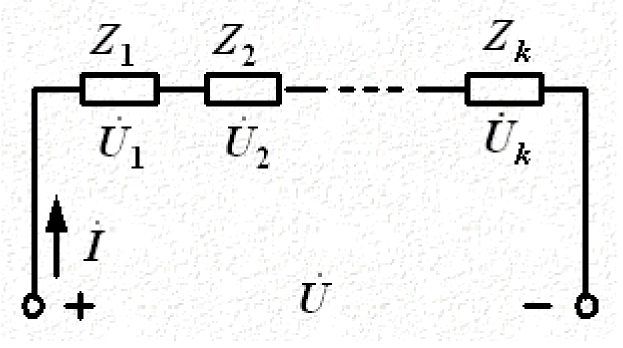
$$\frac{1}{\omega C_{eq}} = |X|$$

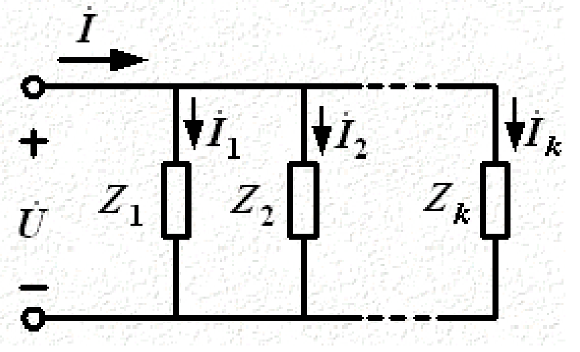
$X_L - X_C > 0 \quad \varphi > 0$　　感性电路

电压超前电流

$$\omega L_{eq} = |X|$$

3.复阻抗串并联





由KVL可证明：

$$\dot{U} = \dot{U}_1 + \dot{U}_2 + \cdots + \dot{U}_k$$

$$\dot{I}Z_S = \dot{I}Z_1 + \dot{I}Z_2 + \cdots + \dot{I}Z_k$$

$$Z_S = \sum_{k=1}^{n} Z_k = \sum_{k=1}^{n} R_k + j\sum_{k=1}^{n} X_k$$

$$= R + jX$$

由KCL可证明：

$$\dot{I} = \dot{I}_1 + \dot{I}_2 + \cdots + \dot{I}_k$$

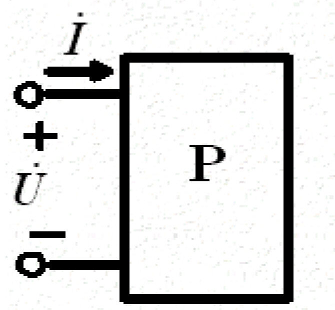
$$\frac{\dot{U}}{Z_p} = \frac{\dot{U}}{Z_1} + \frac{\dot{U}}{Z_2} + \cdots + \frac{\dot{U}}{Z_k}$$

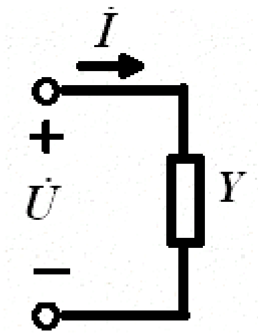
$$Z_P = \frac{1}{\sum_{k=1}^{n} \frac{1}{Z_k}}$$

§9-1 复阻抗和复导纳

二、复导纳

$$Y=\frac{\dot{I}}{\dot{U}}=\frac{I\angle\varphi_i}{U\angle\varphi_u}=|Y|\angle\varphi_Y$$





导纳的模：$|\mathbf{Y}|$　　导纳角：　$\phi_Y=\phi_i-\phi_u$

$$Y=\frac{1}{Z}=\frac{1}{R+jX}=G+jB$$

电导　电纳

容性：$\omega C_{eq}=B\ (B>0)$

感性：$\frac{1}{\omega L_{eq}}=|B|\ (B<0)$

$$|Y|=\sqrt{G^2+B^2}\qquad \phi_Y=arctan\frac{B}{G}$$

§9-1 复阻抗和复导纳

二、阻抗导纳的等效变换

$$YZ = 1$$

如果 $Z = R + \mathrm{j}X$

$$Y = \frac{1}{Z} = \frac{1}{R + \mathrm{j}X} = \frac{\mathrm{R}}{\mathrm{R}^2 + \mathrm{X}^2} - \mathrm{j}\frac{\mathrm{X}}{\mathrm{R}^2 + \mathrm{X}^2}$$

如果 $Y = G + jB$

$$Z = \frac{1}{Y} = \frac{1}{G - \mathrm{j}B} = \frac{\mathrm{G}}{\mathrm{G}^2 + \mathrm{B}^2} + \mathrm{j}\frac{\mathrm{B}}{\mathrm{G}^2 + \mathrm{B}^2}$$

§9-2 正弦交流电路的分析

当电路中电量都是同频率的电量时：

- KCL相量形式

$$\dot{I}_1+\dot{I}_2+\ldots\ldots+\dot{I}_n+\ldots\ldots=0$$

- KVL相量形式

$$\dot{U}_1+\dot{U}_2+\ldots\ldots+\dot{U}_n+\ldots\ldots=0$$

§9-2 正弦交流电路的分析

1.解析法：

利用电路分析方法求解。

注意：已知或未知的物理量要以相量形式表示，阻抗、感抗、容抗要以复数形式表示。

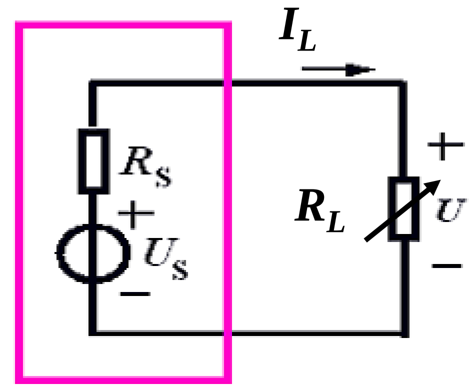
2.相量图法：

选定参考相量，分别画出相关物理量的相量图，通过平行四边形法则，确定所要求的解。

§9-4 最大功率传输

- 负载R_L所获得的功率

直流电路中：



$$P_L = I_L^2 R_L = \left(\frac{Us}{Rs + R_L}\right)^2 R_L$$

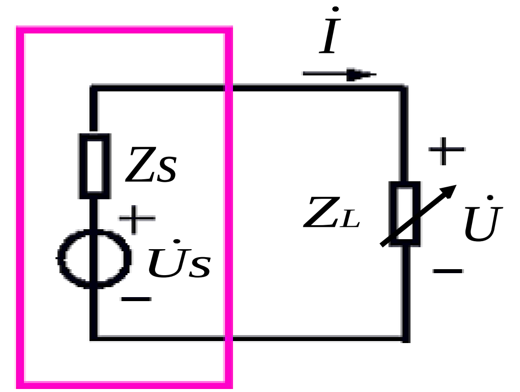
$$\frac{\mathrm{d}P_L}{\mathrm{d}R_L} = Us^2 \left[\frac{(Rs + R_L)^2 - R_L \times 2(Rs + R_L)}{(Rs + R_L)^4}\right] = 0$$

当$R_L = Rs$时，$P_{L\,\max} = \dfrac{Us^2 Rs}{(2Rs)^2} = \dfrac{Us^2}{4Rs^2}$

§9-4 最大功率传输

正弦交流电路中：

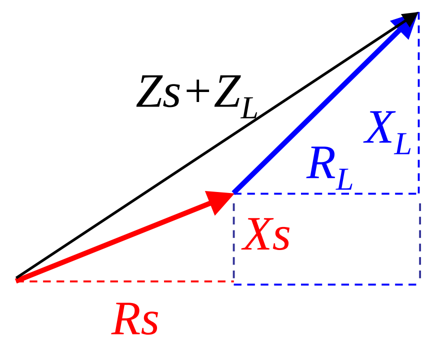
$Z_S = R_s + \mathrm{j}X_s \qquad Z_L = R_L + \mathrm{j}X_L$



- 负载Z_L所获得的有功功率：

$$P_L = I^2 R_L = \left[\frac{Us^2}{(Rs + R_L)^2 + (Xs + X_L)^2} \right] R_L$$

$$X_L = -Xs \qquad \frac{\mathrm{d}}{\mathrm{d}R_L} \left[\frac{R_L}{(Rs + R_L)^2} \right] = 0$$



当$Z_L = Zs^* = Rs - \mathrm{j}Xs$时，$P_{L\max} = \dfrac{Us^2 Rs}{(2Rs)^2} = \dfrac{Us^2}{4Rs}$

§ 10-2　含有耦合电感电路的计算

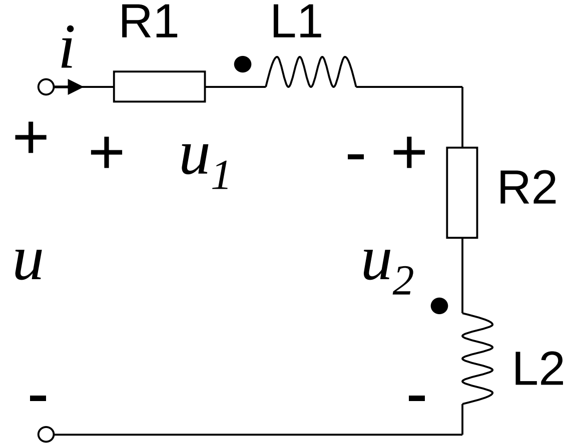
同相串联电路：

$$u_1 = R_1 i + L_1 \frac{di}{dt} + M \frac{di}{dt}$$

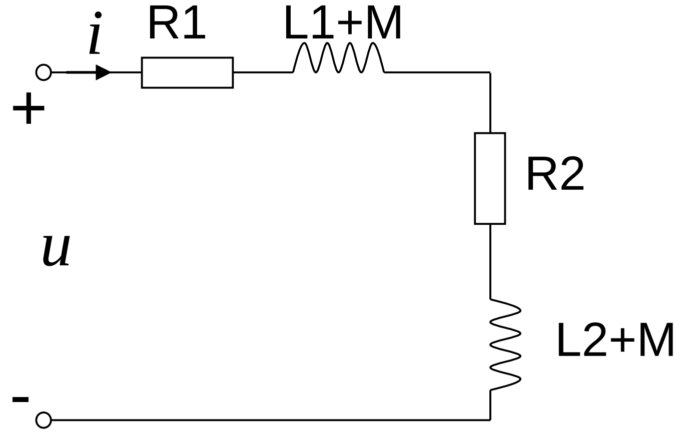
$$u_2 = R_2 i + L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$u = u_1 + u_2$$

$$= (R_1 + R_2) i + (L_2 + L_1 + 2M) \frac{di}{dt}$$



无耦合等效电路：



相量形式：

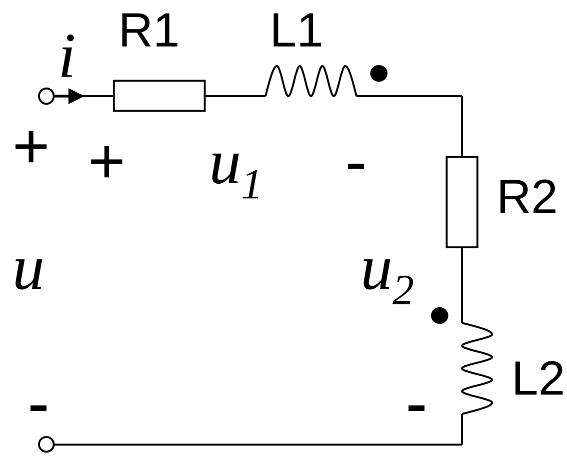
$$\dot{U} = \dot{U}_1 + \dot{U}_2$$

$$= (R_1 + R_2)\dot{I} + \mathrm{j}\omega(L_2 + L_1 + 2M)\dot{I}$$

$$Z = \frac{\dot{U}}{\dot{I}} = (R_1 + R_2) + \mathrm{j}\omega(L_2 + L_1 + 2M)$$

§ 10-2　含有耦合电感电路的计算

反相串联电路：



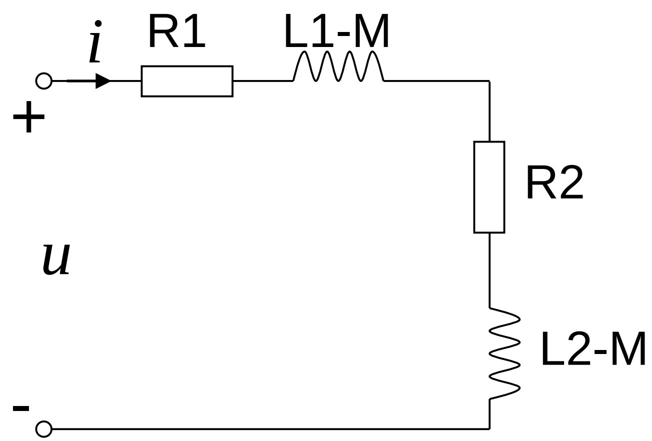
$$u_1 = R_1 i + L_1 \frac{di}{dt} - M\frac{di}{dt}$$

$$u_2 = R_2 i + L_2 \frac{di}{dt} - M\frac{di}{dt}$$

$$u = u_1 + u_2$$

$$= (R_1 + R_2)i + (L_2 + L_1 - 2M)\frac{di}{dt}$$

无耦合等效电路：



相量形式：

$$\dot{U} = \dot{U}_1 + \dot{U}_2$$

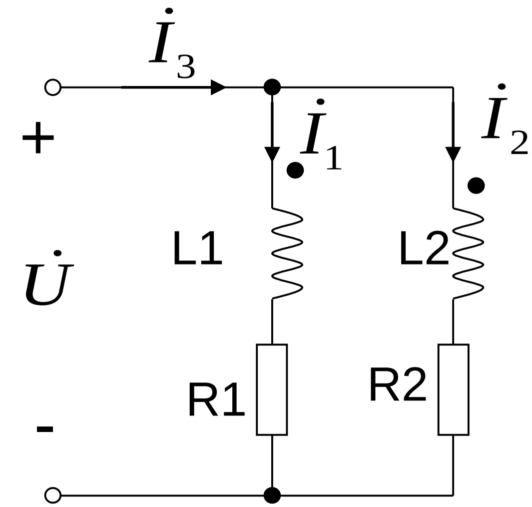
$$= (R_1 + R_2)\dot{I} + \mathrm{j}\omega(L_2 + L_1 - 2M)\dot{I}$$

$$+ \quad Z = \frac{\dot{U}}{\dot{I}} = (R_1 + R_2) + \mathrm{j}\omega(L_2 + L_1 - 2M)$$

§ 10-2　含有耦合电感电路的计算

同侧并联电路：

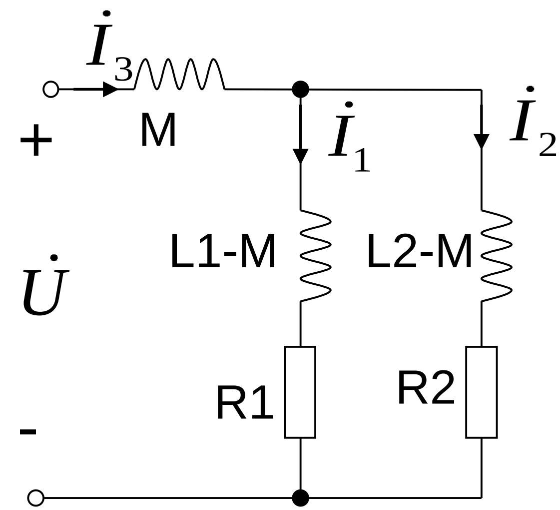
$$\begin{cases} \dot{U} = (R_1 + j\omega L_1)\dot{I}_1 + j\omega M\dot{I}_2 \\ \dot{U} = j\omega M\dot{I}_1 + (R_2 + j\omega L_2)\dot{I}_2 \\ \dot{I}_3 = \dot{I}_1 + \dot{I}_2 \end{cases}$$



$$\begin{aligned} \dot{U} &= (R_1 + j\omega L_1)\dot{I}_1 + j\omega M\dot{I}_2 \\ &= (R_1 + j\omega L_1 - j\omega M)\dot{I}_1 + j\omega M\dot{I}_1 + j\omega M\dot{I}_2 \\ &= (R_1 + j\omega(L_1 - M))\dot{I}_1 + j\omega M\dot{I}_3 \end{aligned}$$

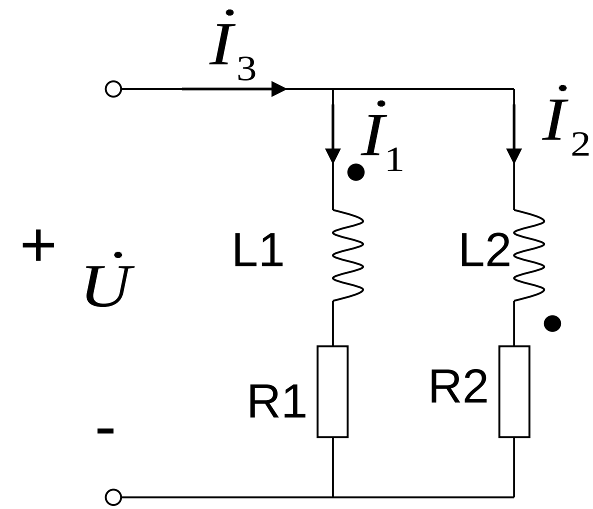
$$\dot{U} = (R_2 + j\omega(L_2 - M))\dot{I}_2 + j\omega M\dot{I}_3$$

无耦合等效电路：



§ 10-2　含有耦合电感电路的计算

异侧并联电路：

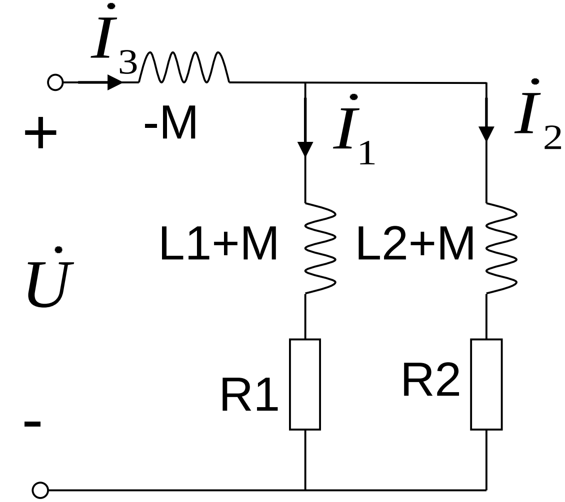


$$
\begin{cases}
\dot{U} = (R_1 + \mathrm{j}\omega L_1)\dot{I}_1 - \mathrm{j}\omega M\dot{I}_2 \\
\dot{U} = -\mathrm{j}\omega M\dot{I}_1 + (R_2 + \mathrm{j}\omega L_2)\dot{I}_2 \\
\dot{I}_3 = \dot{I}_1 + \dot{I}_2
\end{cases}
$$

$$
\begin{aligned}
\dot{U} &= (R_1 + \mathrm{j}\omega L_1)\dot{I}_1 - \mathrm{j}\omega M\dot{I}_2 \\
&= (R_1 + \mathrm{j}\omega L_1 + \mathrm{j}\omega M)\dot{I}_1 - \mathrm{j}\omega M\dot{I}_1 - \mathrm{j}\omega M\dot{I}_2 \\
&= (R_1 + \mathrm{j}\omega(L_1 + M))\dot{I}_1 - \mathrm{j}\omega M\dot{I}_3
\end{aligned}
$$

$$
\dot{U} = (R_2 + \mathrm{j}\omega(L_2 + M))\dot{I}_2 - \mathrm{j}\omega M\dot{I}_3
$$

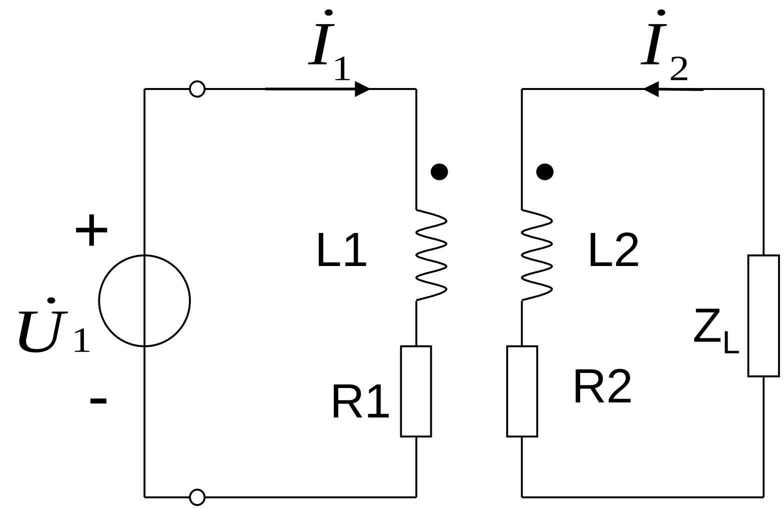
无耦合等效电路：



§10-4　变压器原理

一次回路（原边回路、初级回路）：

$$\dot{U}_1=(R_1+j\omega L_1)\dot{I}_1+\mathrm{j}\omega M\dot{I}_2$$



二次回路（副边回路、次级回路）：

$$\mathrm{j}\omega M\dot{I}_1+(R_2+\mathrm{j}\omega L_2+Z_L)\dot{I}_2=0$$

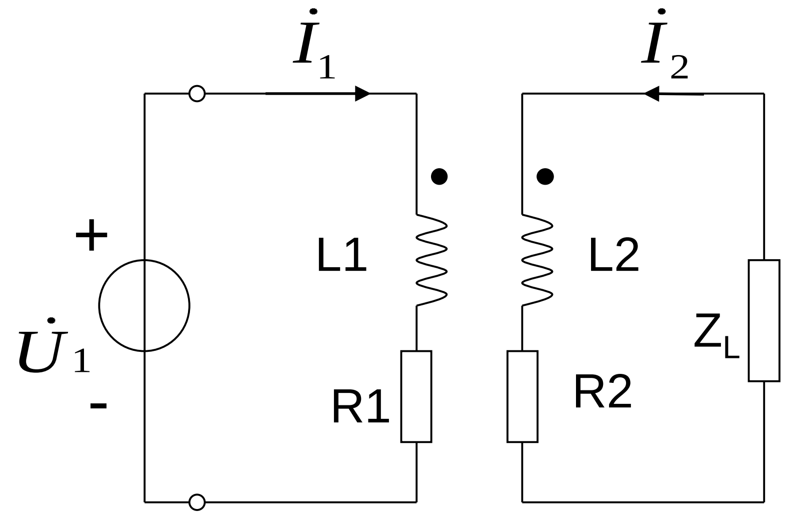
令：　$Z_M=\mathrm{j}\omega M$，　$Z_{11}=R_1+\mathrm{j}\omega L_1$，　$Z_{22}=R_2+\mathrm{j}\omega L_2+Z_L$

$$Z_{11}\dot{I}_1+Z_M\dot{I}_2=\dot{U}_1$$

$$Z_M\dot{I}_1+Z_{22}\dot{I}_2=0$$

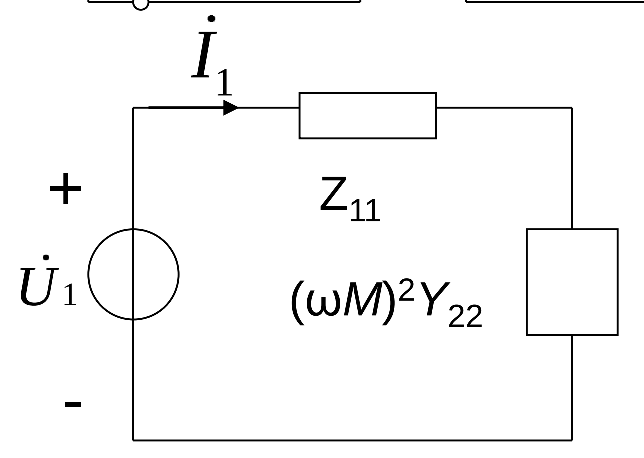
§ 10-4　变压器原理

$$\begin{cases} Z_{11}\dot{I}_1 + Z_M\dot{I}_2 = \dot{U}_1 \\ Z_M\dot{I}_1 + Z_{22}\dot{I}_2 = 0 \end{cases} \Rightarrow \dot{I}_2 = -\frac{Z_M}{Z_{22}}\dot{I}_1 = -Z_M Y_{22}\dot{I}_1$$



$$(Z_{11} - Z_M{}^2 Y_{22})\dot{I}_1 = \dot{U}_1$$

$$\dot{I}_1 = \frac{\dot{U}_1}{Z_{11} - Z_M{}^2 Y_{22}} = \frac{\dot{U}_1}{Z_{11} + (\omega M)^2 Y_{22}} = \frac{\dot{U}_1}{Z_1}$$

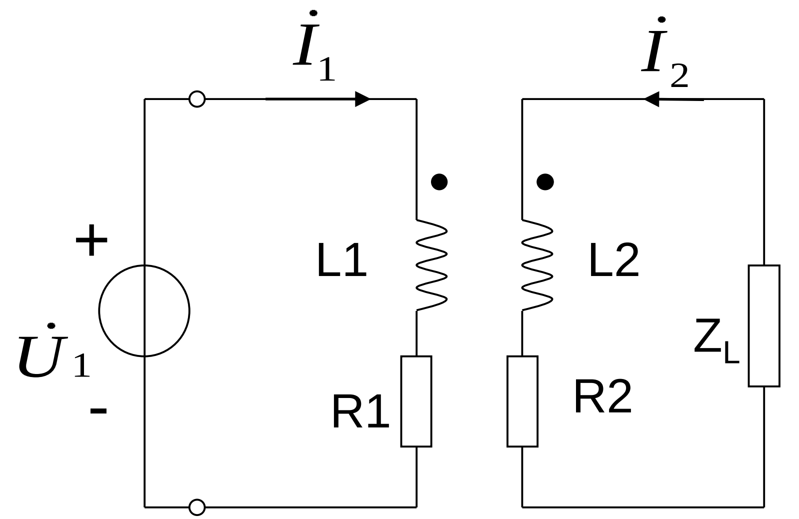


一次等效电路

注：$Z_M = \mathrm{j}\omega M$，　$Z_{11} = R_1 + \mathrm{j}\omega L_1$，　$Z_{22} = R_2 + \mathrm{j}\omega L_2 + Z_L$

§ 10−4　变压器原理

$$\begin{cases} Z_{11}\dot{I}_1 + Z_M\dot{I}_2 = \dot{U}_1 \\ Z_M\dot{I}_1 + Z_{22}\dot{I}_2 = 0 \end{cases} \Rightarrow \dot{I}_2 = -\frac{Z_M}{Z_{22}}\dot{I}_1$$

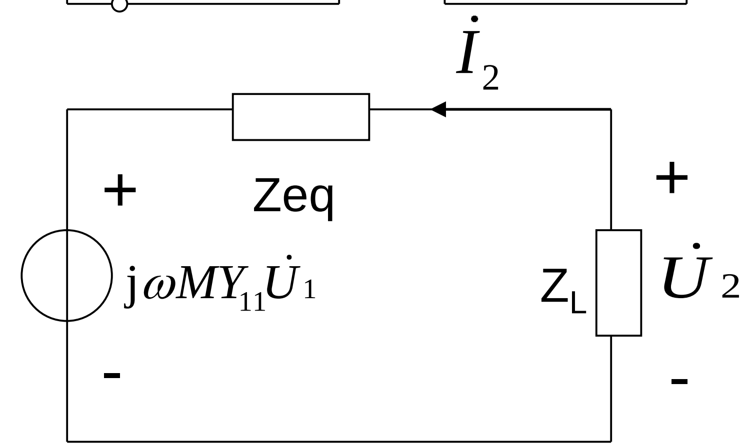


$$\dot{U}_2 = -Z_L\dot{I}_2 = Z_L\frac{Z_M}{Z_{22}}\dot{I}_1$$

$$\dot{I}_2 = -\frac{Z_M}{Z_{22}} * \dot{I}_1 = -\frac{Z_M}{Z_{22}} * \frac{\dot{U}_1}{Z_{11} + (\omega M)^2 Y_{22}}$$

$$= -\frac{Z_M\dot{U}_1}{Z_{22}Z_{11} + (\omega M)^2} = -\frac{Z_M\dot{U}_1 / Z_{11}}{Z_{22} + (\omega M)^2 Y_{11}}$$

$$= -\frac{Z_M\dot{U}_1 / Z_{11}}{R_2 + \mathrm{j}\omega L_2 + Z_L + (\omega M)^2 Y_{11}} = -\frac{\dot{U}_{OC}}{Z_{\mathrm{eq}} + Z_L}$$



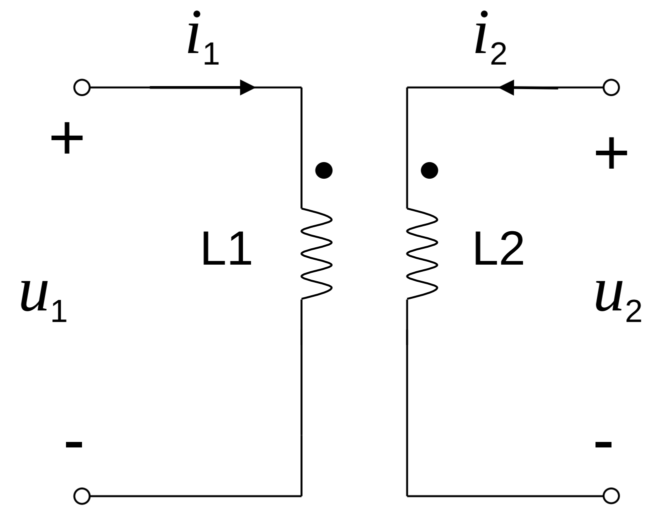
式中，$Z_{\mathrm{eq}} = R_2 + \mathrm{j}\omega L_2 + (\omega M)^2 Y_{11}$

二次等效电路

注：$Z_M = \mathrm{j}\omega M$，　$Z_{11} = R_1 + \mathrm{j}\omega L_1$，　$Z_{22} = R_2 + \mathrm{j}\omega L_2 + Z_L$

§10-5　理想变压器

设线圈中的磁通为Φ，线圈1、线圈2的匝数分别为N_1，N_2，有：



$$\psi_1 = N_1\phi$$

$$\psi_2 = N_2\phi$$

$$u_1 = \frac{d\psi_1}{dt} = N_1\frac{d\phi}{dt}$$

$$u_2 = \frac{d\psi_2}{dt} = N_2\frac{d\phi}{dt}$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{\psi_1}{\psi_2} = \frac{N_1}{N_2}$$

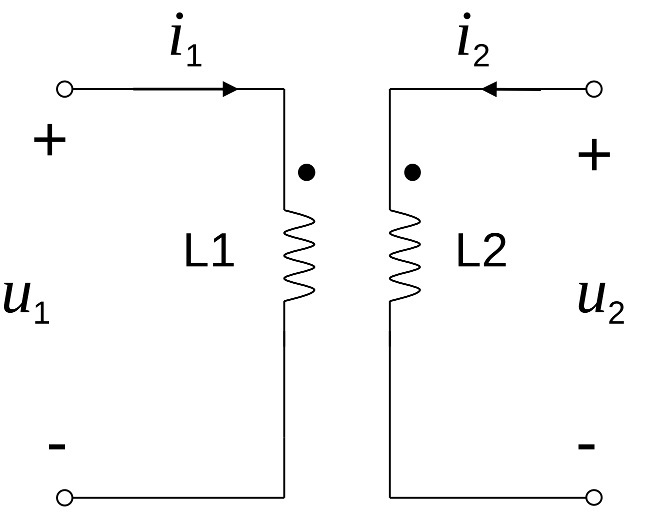
§10-5 理想变压器

$$u_1 = \frac{\mathrm{d}\psi_1}{\mathrm{d}t} = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$\frac{di_1}{dt} = \frac{1}{L_1}u_1 - \frac{M}{L_1}\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$\int \mathrm{d}i_1 = \frac{1}{L_1}\int u_1\mathrm{d}t - \frac{M}{L_1}\int\frac{\mathrm{d}i_2}{\mathrm{d}t}\mathrm{d}t$$

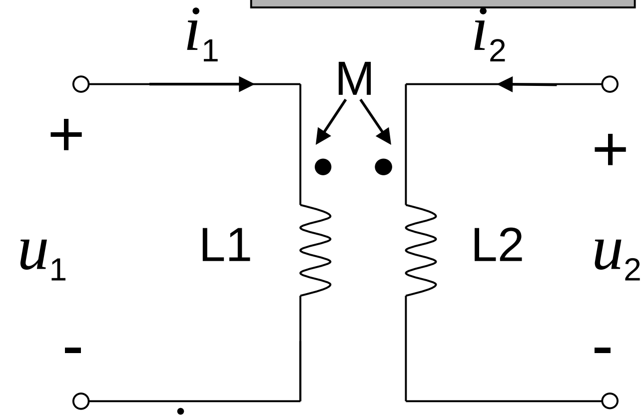
$$i_1 = -\frac{M}{L_1}i_2 = -\frac{\sqrt{L_2}}{\sqrt{L_1}}i_2$$



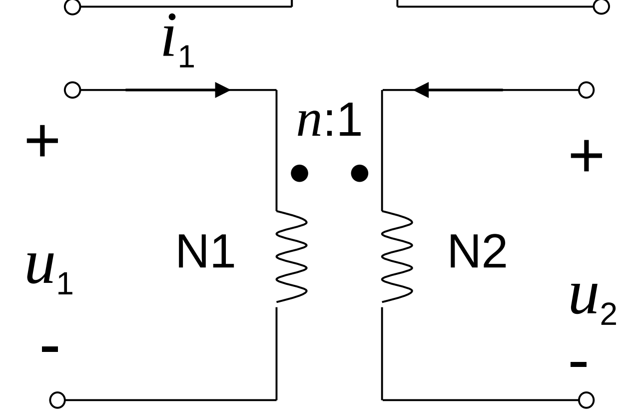
$$\Rightarrow \frac{i_1}{i_2} = -\frac{\sqrt{L_2}}{\sqrt{L_1}} = -\frac{N_2}{N_1}$$

§10-5　理想变压器

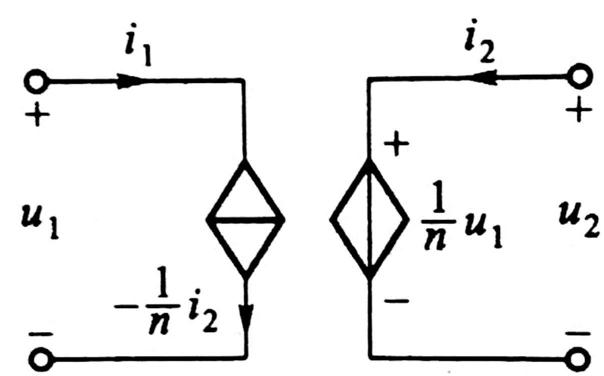
$$\frac{u_1}{u_2}=\frac{N_1}{N_2} \qquad \frac{i_1}{i_2}=-\frac{N_2}{N_1}$$



设 $n=\frac{N_1}{N_2}$ 为理想变压器的匝数比，又称为变比



$$\begin{cases} u_1 = nu_2 \\ i_1 = -\dfrac{1}{n} i_2 \end{cases} \Rightarrow u_1 i_1 + u_2 i_2 = 0$$

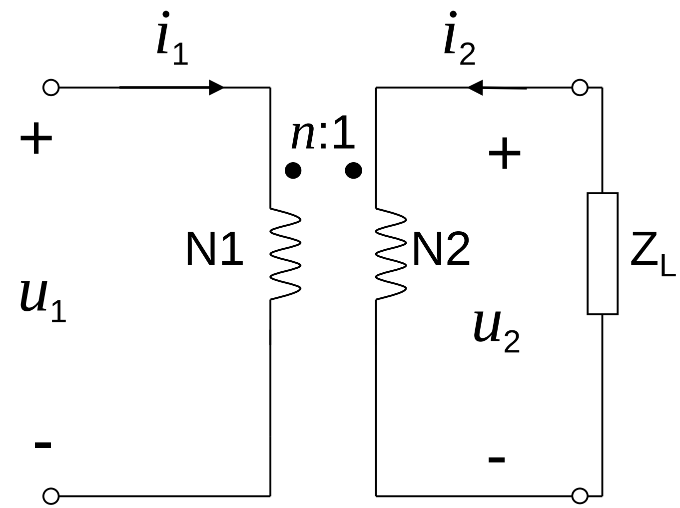


理想变压器将一侧吸收的能量全部传输到另一侧输出。

在传输过程中，仅仅将电压、电流按变比作数值的变换，既不耗能也不储能，是一个非动态无损耗的磁耦合元件。

§ 10-5　理想变压器

理想变压器的阻抗转换



二次侧接阻抗Z_L，折合到一次侧的等效阻抗：

$$Z_{11'} = \frac{\dot{U}_1}{\dot{I}_1} = \frac{n\dot{U}_2}{-\frac{1}{n}\dot{I}_2} = n^2 Z_L$$

§13.3 平均值、有效值、平均功率

一、有效值

$$I=\sqrt{\frac{1}{T}\int_0^T i^2\mathrm{d}t}$$

$$I=\sqrt{\frac{1}{T}\int_0^T [I_0+\sum_{k=1}^{\infty} I_{km}\cos(k\omega_1 t+\varphi_k)]^2\mathrm{d}t}$$

$$I=\sqrt{I_0{}^2+I_1^2+I_2^2+\cdots}=\sqrt{I_0{}^2+\sum_{k=1}^{\infty} I_k^2}$$

$$U=\sqrt{U_0{}^2+U_1^2+U_2^2+\cdots}=\sqrt{U_0{}^2+\sum_{k=1}^{\infty} U_k^2}$$

§13.3 平均值、有效值、平均功率

三、平均功率

$$u(t) = U_0 + \sum_{k=1}^{\infty}[U_{km}\sin(k\omega_1 t + \varphi_{ku})]$$

$$i(t) = I_0 + \sum_{k=1}^{\infty}[I_{km}\sin(k\omega_1 t + \varphi_{ki})]$$

$$P = \frac{1}{T}\int_0^T p\mathrm{d}t = \frac{1}{T}\int_0^T ui\ \mathrm{d}t$$

不同频率的正弦电压与电流乘积的积分为0

$$P = P_0 + \sum_{k=1}^{\infty} P_k = U_0 I_0 + \sum_{k=1}^{\infty} U_k I_k \cos(\varphi_{ku} - \varphi_{ki})$$

$$U_k = \frac{U_{km}}{\sqrt{2}}, I_k = \frac{I_{km}}{\sqrt{2}}$$

四、视在功率

$$S = UI = \sqrt{U_0^{\ 2} + \sum_{k=1}^{\infty} U_k^2}\sqrt{I_0^{\ 2} + \sum_{k=1}^{\infty} I_k^2}$$

§13.3 平均值、有效值、平均功率

- 注意：

1、不同频率的电压和电流不构成平均功率。

2、对于非正弦周期信号电路：

$$I \neq I_0 + I_1 + I_2 + \ldots\ldots$$

$$I = \sqrt{I_0{}^2 + I_1^2 + I_2^2 + \cdots} = \sqrt{I_0{}^2 + \sum_{k=1}^{\infty} I_k^2}$$

$$U \neq U_0 + U_1 + U_2 + \ldots\ldots$$

$$U = \sqrt{U_0{}^2 + U_1^2 + U_2^2 + \cdots} = \sqrt{U_0{}^2 + \sum_{k=1}^{\infty} U_k^2}$$

$$S \neq U_0 I_0 + U_1 I_1 + U_2 I_2 + \ldots\ldots$$

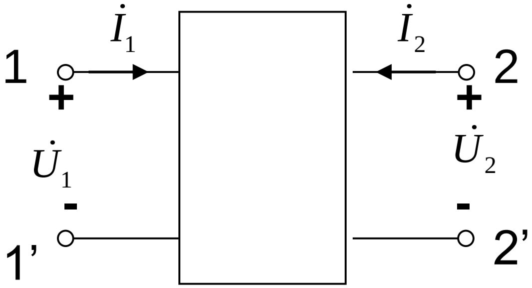
$$S = UI = \sqrt{U_0{}^2 + \sum_{k=1}^{\infty} U_k^2}\sqrt{I_0{}^2 + \sum_{k=1}^{\infty} I_k^2}$$

§16-2　二端口方程和参数

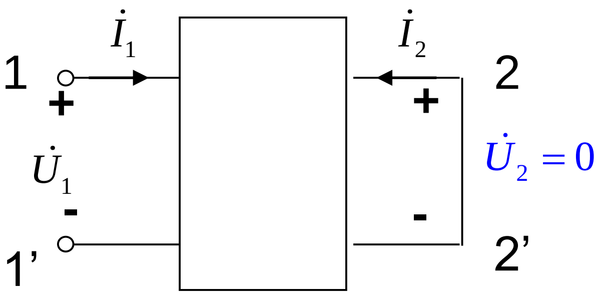
$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$

$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

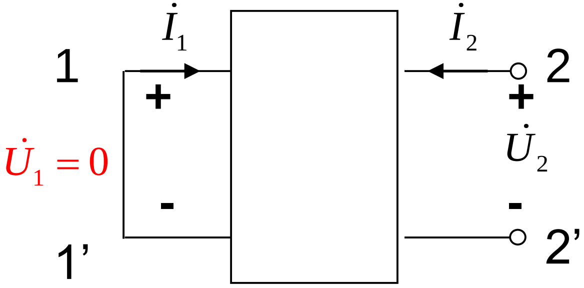
$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$



$$Y_{11} = \left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0} \qquad Y_{21} = \left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0}$$



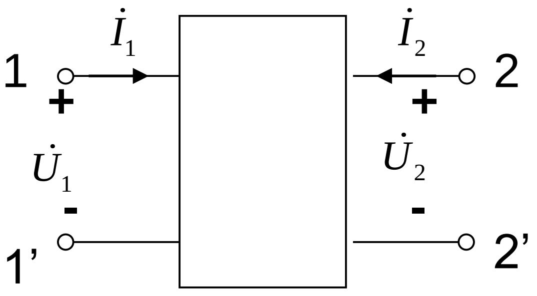
$$Y_{12} = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{U}_1=0} \qquad Y_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{U}_1=0}$$



Y：短路导纳参数

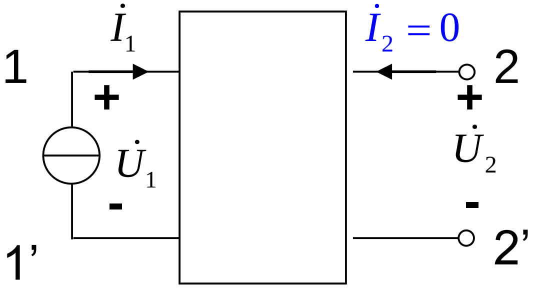
§ 16-2　二端口方程和参数

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

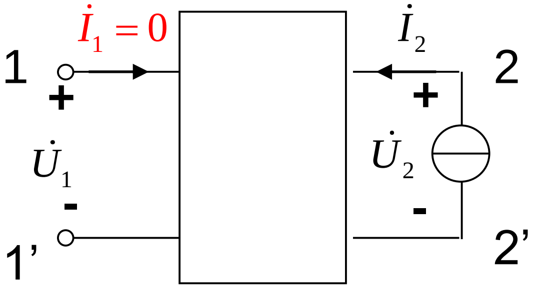


$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0} \qquad Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0}$$



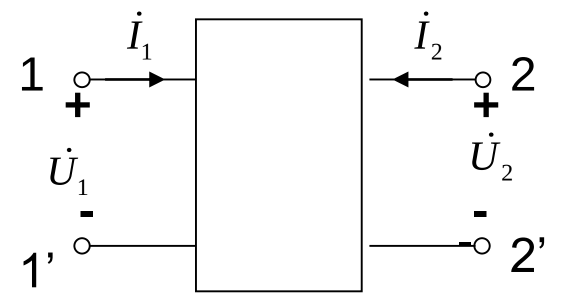
$$Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0} \qquad Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$



Z：开路阻抗矩阵

§ 16-2　二端口方程和参数

Y参数方程：
$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$
$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$



Y：短路导纳矩阵 $\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

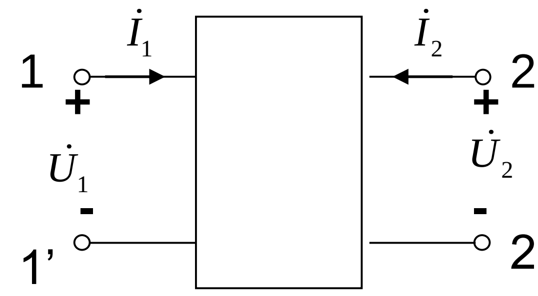
Z参数方程：
$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

Z：开路阻抗矩阵 $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$　　$Z = Y^{-1}$　　$Y = Z^{-1}$

§ 16-2　二端口方程和参数

A参数方程： $\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$

$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$



$$A_{11} = \left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0} \qquad A_{21} = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}$$

$$A_{12} = \left.\frac{\dot{U}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0} \qquad A_{22} = \left.\frac{\dot{I}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}$$

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

H参数方程： $\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2$

$$\dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2$$

$$H_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0} \qquad H_{21} = \left.\frac{\dot{I}_2}{\dot{I}_1}\right|_{\dot{U}_2=0}$$

$$H_{12} = \left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_1=0} \qquad H_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{I}_1=0}$$

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix}$$

§ 16-2　二端口方程和参数

	互易条件	对称条件
Y参数：	$Y_{12}=Y_{21}$	$Y_{12}=Y_{21}$　$Y_{11}=Y_{22}$
Z参数：	$Z_{12}=Z_{21}$	$Z_{12}=Z_{21}$　$Z_{11}=Z_{22}$
A参数：	$A_{11}A_{22}-A_{12}A_{21}=1$	$A_{11}A_{22}-A_{12}A_{21}=1$ $A_{11}=A_{22}$
H参数：	$H_{12}=-H_{21}$	$H_{11}H_{22}-H_{12}H_{21}=1$ $H_{12}=-H_{21}$