

2017-2018 学年第一学期期中模拟练习

一、判断题(每小题 3 分, 共 15 分)

1. () In a binary search tree, the keys on the same level from left to right must be in sorted (non-decreasing) order. (3 分)
2. () For a sequentially stored linear list of length N , the time complexities for query and insertion are $O(1)$ and $O(N)$, respectively. (3 分)
3. () If keys are pushed onto a stack in the order $\{1, 2, 3, 4, 5\}$, then it is impossible to obtain the output sequence $\{3, 4, 1, 2, 5\}$. (3 分)
4. () $N\log N^2$ and $N\log N$ have the same speed of growth. (3 分)
5. () In a directed graph, the sum of the in-degrees and out-degrees of all the vertices is twice the total number of edges. (3 分)

二、选择题(共 65 分)

- 1、Given a quadtree(四叉树) with 3 nodes of degree 2, 2 nodes of degree 3, 4 nodes of degree 4. The number of leaf nodes in this tree is ___. (5 分)
 - I. 12
 - J. 21
 - K. 20
 - L. 10
- 2、In a complete binary tree with 1102 nodes, there must be ___ leaf nodes. (6 分)
 - A. 79
 - B. 551
 - C. 1063
 - D. cannot be determined
- 3、If an undirected graph $G = (V, E)$ contains 10 vertices. Then to guarantee that G is connected in any cases, there has to be at least ___ edges. (6 分)
 - I. 45
 - J. 37
 - K. 36
 - L. 9
- 4、Insert $\{5, 11, 13, 1, 3, 6\}$ one by one into an initially empty binary search tree. The post-order traversal sequence of the resulting tree is: (6 分)
 - I. 3, 1, 5, 6, 13, 11
 - J. 1, 3, 11, 6, 13, 5
 - K. 1, 3, 11, 6, 13, 5

L. 3, 1, 6, 13, 11, 5

5、 In-order traversal of a binary tree can be done iteratively. Given the stack operation sequence as the following:

push(1), push(2), push(3), pop(), push(4), pop(), pop(), push(5), pop(), pop(), push(6), pop()

Which one of the following statements is TRUE? (6 分)

I. 2 is the parent of 4

J. 2 and 6 are siblings

K. 6 is the root

L. None of the above

6、 The array representation of a disjoint set is given by $\{4, 6, 5, 2, -3, -4, 3\}$. If the elements are numbered from 1 to 7, the resulting array after invoking Union(Find(7), Find(1)) with union-by-size and path-compression is: (5 分)

A. $\{4, 6, 5, 2, 6, -7, 3\}$

B. $\{6, 6, 5, 6, -7, 5, 5\}$

C. $\{4, 6, 5, 2, -7, 5, 3\}$

D. $\{6, 6, 5, 6, 6, -7, 5\}$

7、 The recurrent equations for the time complexities of programs P1 and P2 are:

- P1: $T(1)=1, T(N)=T(N/2)+1$;
- P2: $T(1)=1, T(N)=2T(N/2)+1$;

Then the correct conclusion about their time complexities is: (5 分)

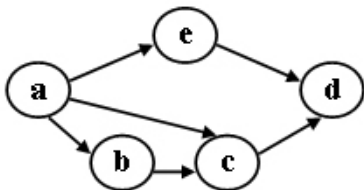
I. they are both $O(N)$

J. $O(\log N)$ for P1, and $O(N)$ for P2

K. they are both $O(\log N)$

L. $O(\log N)$ for P1, and $O(N \log N)$ for P2

8、 From the given graph shown by the figure, how many different topological orders can we obtain? (5 分)



A. 1

B. 3

C. 2

D. 4

9、 Suppose that an array of size m is used to store a circular queue. If the head pointer front and the current size variable size are used to represent the range of the queue instead of front and rear, then the maximum capacity of this queue can be: (5 分)

I. cannot be determined

J. $m+1$

K. $m-1$

L. m

10 、 The result of performing three DeleteMin operations in the min-heap $\{1,3,2,6,7,5,4,15,14,12,9,10,11,13,8\}$ is: (5 分)

A. 4,6,5,12,7,10,8,15,14,9,13,11

B. 4,5,6,12,7,10,8,15,14,13,9,11

C. 4,6,5,13,7,10,8,15,14,12,9,11

D. 4,5,6,7,8,9,10,11,12,13,14,15

11、 Which of the following statements is TRUE about topological sorting? (5 分)

A. In a DAG, if for any pair of distinct vertices V_i and V_j , there is a path either from V_i to V_j or from V_j to V_i , then the DAG must have a unique topological sequence.

B. If a graph has a topological sequence, then its adjacency matrix must be triangular.

C. If the adjacency matrix is triangular, then the corresponding directed graph must have a unique topological sequence.

D. If V_i precedes V_j in a topological sequence, then there must be a path from V_i to V_j .

12、 To insert s after p in a doubly linked circular list, we must do: (6 分)

I. $p \rightarrow next = s$; $s \rightarrow prior = p$; $p \rightarrow next \rightarrow prior = s$; $s \rightarrow next = p \rightarrow next$;

J. $p \rightarrow next \rightarrow prior = s$; $p \rightarrow next = s$; $s \rightarrow prior = p$; $s \rightarrow next = p \rightarrow next$;

K. $s \rightarrow prior = p$; $s \rightarrow next = p \rightarrow next$; $p \rightarrow next = s$; $p \rightarrow next \rightarrow prior = s$;

L. $s \rightarrow prior = p$; $s \rightarrow next = p \rightarrow next$; $p \rightarrow next \rightarrow prior = s$; $p \rightarrow next = s$;

三、程序填空题（满分 20 分）

1、 (10 分) Find.

Please fill in the blanks in the program which performs Find as a Union/Find operation with path compression.

```
SetType Find ( ElementType X, DisjSet S )
```

```
{
```

```
    ElementType root, trail, lead;
```

```
    for ( root = X; S[root] > 0; _____ (5 分) );
```

```
    for ( trail = X; trail != root; trail = lead ) {
```

```

        lead = S[trail] ;
        _____(5 分);
    }
    return root;
}

```

2、（10 分）FindKthLarges.

The function is to find the K-th largest element in a list A of N elements. The function BuildMinHeap(H, K) is to arrange elements H[1] ... H[K] into a min-heap. Please complete the following program.

```

ElementType FindKthLargest ( int A[], int N, int K )
{
    /* it is assumed that K<=N */

    ElementType *H;
    int i, next, child;

    H = (ElementType *)malloc((K+1)*sizeof(ElementType));
    for ( i=1; i<=K; i++) H[i] = A[i-1];
    BuildMinHeap(H, K);
    for ( next=K; next<N; next++) {
        H[0] = A[next];
        if ( H[0] > H[1] ) {
            for ( i=1; i*2<=K; i=child ) {
                child = i*2;
                if ( child!=K && _____(5 分) ) child++;
                if ( _____(5 分) )
                    H[i] = H[child];
                else break;
            }
            H[i] = H[0];
        }
    }
    return H[1];
}

```

2017-2018 学年第一学期期中模拟练习参考答案

一、判断题(每小题 3 分, 共 15 分)

1、【正解】T

【解析】二叉搜索树中任意一个结点的值一定大于其左孩子的值而小于其右孩子的值(如果存在的话)。其中序遍历序列是非递减的, 因此层序遍历也是非递减的。

【考点延伸】二叉搜索树

2、【正解】T

【解析】线性表支持随机访问, 因此访问的时间复杂度为 $O(1)$ 。但是在第 i 个位置插入时, 需将其后的 $n-i$ 个元素分别向后移 1 位, 因此时间复杂度为 $O(n)$ 。

【考点延伸】线性表

3、【正解】T

【解析】模拟出入栈, 当 3, 4 出栈后, 栈中的元素序列为 1 2, 此时 1 不可能在 2 之前出栈。

【考点延伸】栈

4、【正解】T

【解析】前者可化为 $2N\log N$, 可以将系数忽略, 因此二者增长的速度相等

【考点延伸】时间复杂度

5、【正解】T

【解析】有向图中, 每条边连接两个顶点, 而顶点的入度对应的边也是其他顶点出度对应的边, 将入度和出度相加会得到边数的二倍。

【考点延伸】有向图

二、单选题(每小题 3 分, 共 60 分)

1 【正解】C

【解析】四叉树中每一个结点最多有 4 个孩子。因此边数+1 = 度为 4 的结点数+度为 3 的结点数+度为 2 的结点数+度为 1 的结点数+叶子结点数; 而度为 1 的结点数+2*度为 2 的结点数+3*度为 3 的结点数+4*度为 4 的结点数 = 边数。叶子结点数为 20

【考点延伸】树

2 【正解】B

【解析】 $512 - [(1102 - 1023) + 1] / 2 = 472$, $472 + 79 = 551$

【考点延伸】完全二叉树

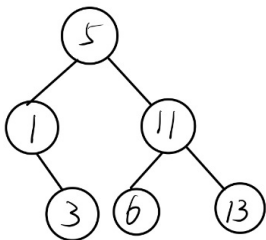
3 【正解】A

【解析】要保证 G 在任何情况都是连通的, 所有结点都相连, 因此需要 $9 * 10 / 2 = 45$ 条边

【考点延伸】最小生成树

4 【正解】D

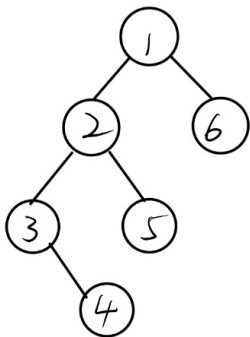
【解析】二叉搜索树如下图所示



【考点延伸】二叉树搜索树

5 【正解】B

【解析】我们可以发现入栈的元素序列是此二叉树的前序遍历，出栈的元素序列构成中序遍历，构建的二叉树如下



【考点延伸】二叉树的非递归遍历

6 【正解】D

【解析】执行 Find(7)后，7 会直接连接在 5 上；执行 Find(1)后，1，4 会直接连在 6 上，合并时，5 会连在 6 上，5，6 的值都会改变。因此最后的序列为 6 6 5 6 6 -7 5

【考点延伸】并查集，路径压缩

7 【正解】B

【解析】通过迭代可以计算出 P1: $T(n) = \log_3 n + 1$, P2: $T(n) = \frac{3}{2}n - \frac{1}{2}$ 因此 $O(\log N)$ for

P1, $O(N)$ for P2

【考点延伸】时间复杂度

8 【正解】B

【解析】a b c d; a e b c d; a b e c d 共三种

【考点延伸】拓扑排序

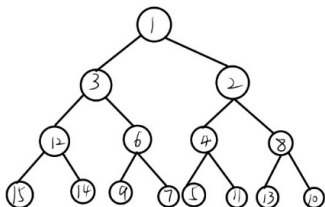
9 【正解】D

【解析】已知循环队列的大小，在数组中能存储的最大元素数量就是数组的大小 m

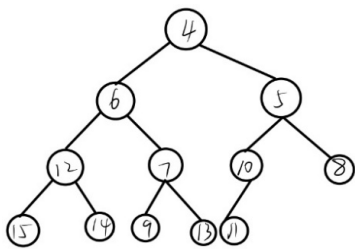
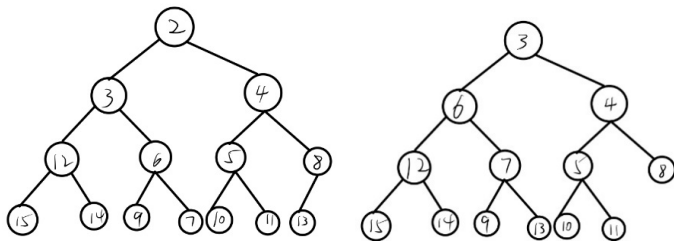
【考点延伸】循环队列

10 【正解】A

【解析】最小堆如下图所示



经过三次删除操作:



【考点延伸】最小堆

11 【正解】A

【解析】 $2 \rightarrow 3 \rightarrow 1$ 这样顺序的图直接就排除对称和三角；对角线以下元素均为零，表明只有顶点 i 到顶点 j ($i < j$) 可能有边，而顶点 j 到顶点 i 一定没有边，即有向图是一个无环图，因此一定存在拓扑序列，但是该拓扑序列不一定唯一，可以举反例证明。另外，若题目说对角线以上均为 1，以下均为 0，则拓扑序列唯一。如果 V_i 在拓扑序列中位于 V_j 之前，则不一定存在从 V_i 到 V_j 的路径。

【考点延伸】拓扑排序

12 【正解】B

【解析】对于链表的操作，画图是最容易理解的。在 p 后插入 s ，先把 p 后面的结点 q 的前驱结点指向 s ，再把 p 的下一个结点指向 s ，然后把 s 的前驱结点指向 p ， s 的后继结点指向 q 。

【考点延伸】双向循环链表

三、程序填空题（满分 20 分）

1、【解析】(1) $root = S[root]$ (5 分)

(2) $S[trail] = root$ (5 分)

【考点延伸】并查集和路径压缩

2、【解析】(1) $H[child] < H[child + 1]$ (5 分)

(2) $H[child] > H[0]$ (5 分)

【考点延伸】堆排序