



# Chapter Two

## **Budgetary and Other Constraints on Choice**



# Consumption Choice Sets

A **consumption choice set** is the collection of all consumption choices available to the consumer.

What constrains consumption choice?

- Budgetary, time and other resource limitations.

# Budget Constraints

A **consumption bundle** containing  $x_1$  units of commodity 1,  $x_2$  units of commodity 2 and so on up to  $x_n$  units of commodity  $n$  is denoted by the vector  $(x_1, x_2, \dots, x_n)$ .

Commodity prices are  $p_1, p_2, \dots, p_n$ .

# Budget Constraints

**Q: When is a consumption bundle  $(x_1, \dots, x_n)$  affordable at given prices  $p_1, \dots, p_n$ ?**

# Budget Constraints

**Q: When is a bundle  $(x_1, \dots, x_n)$  affordable at prices  $p_1, \dots, p_n$ ?**

**A: When**

$$p_1x_1 + \dots + p_nx_n \leq m$$

**where  $m$  is the consumer's (disposable) income.**

# Budget Constraints

The bundles that are only just affordable form the consumer's **budget constraint**. This is the set

$$\{ (x_1, \dots, x_n) \mid x_1 \geq 0, \dots, x_n \geq 0 \text{ and } p_1 x_1 + \dots + p_n x_n = m \}.$$

# Budget Constraints

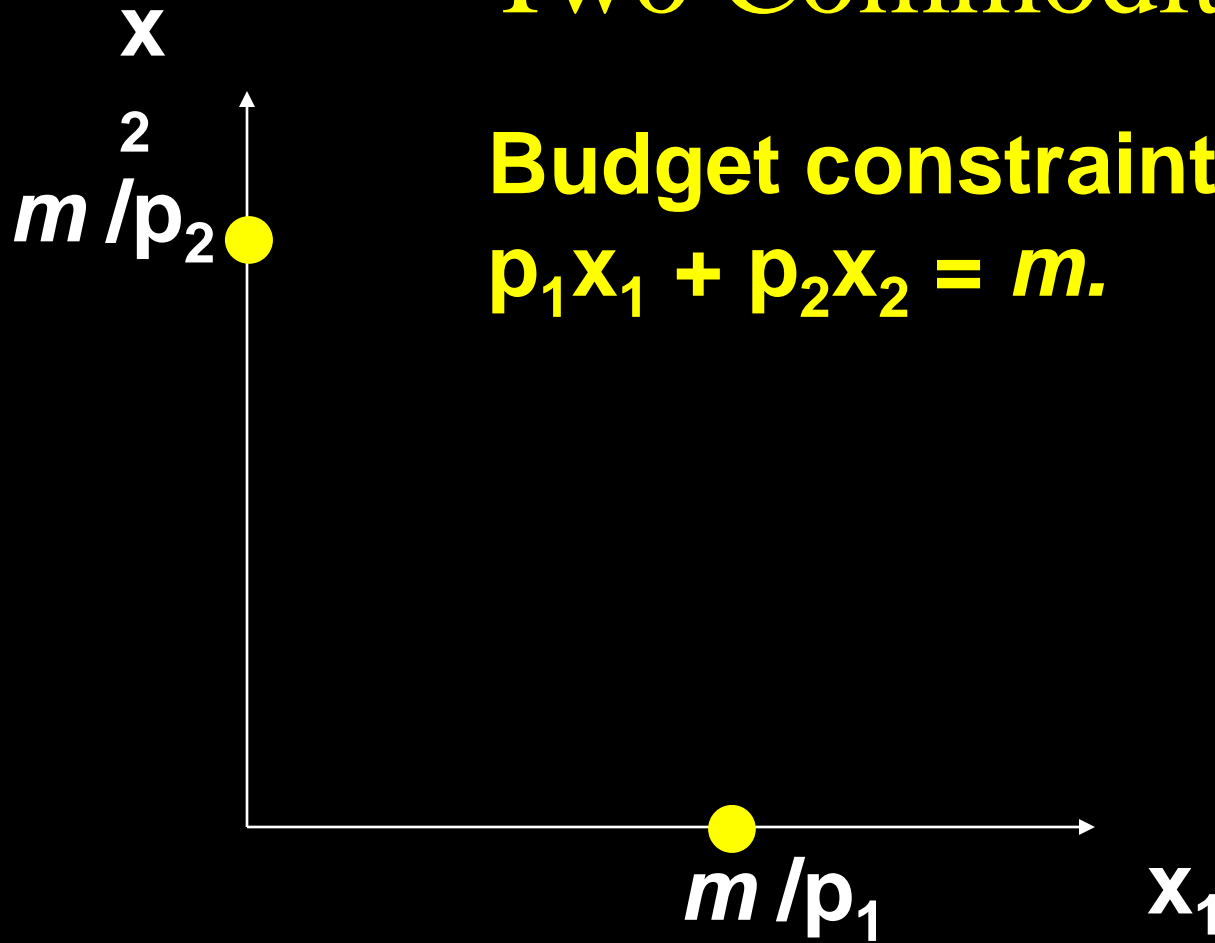
The consumer's **budget set** is the set of all affordable bundles;

$$B(p_1, \dots, p_n, m) = \\ \{ (x_1, \dots, x_n) \mid x_1 \geq 0, \dots, x_n \geq 0 \text{ and} \\ p_1 x_1 + \dots + p_n x_n \leq m \}$$

The budget constraint is the upper boundary of the budget set.

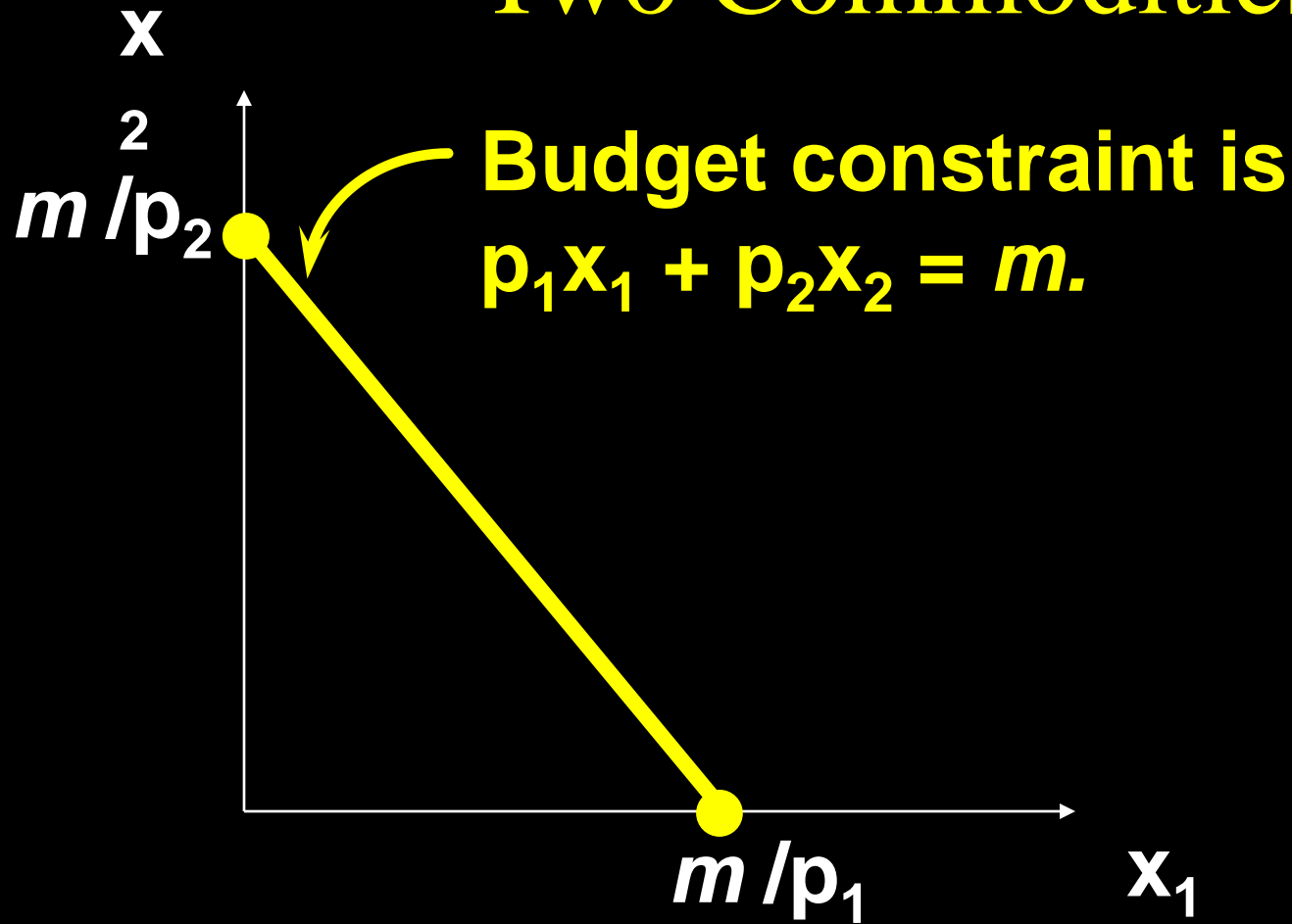
# Budget Set and Constraint for Two Commodities

**Budget constraint is**  
 **$p_1x_1 + p_2x_2 = m$ .**

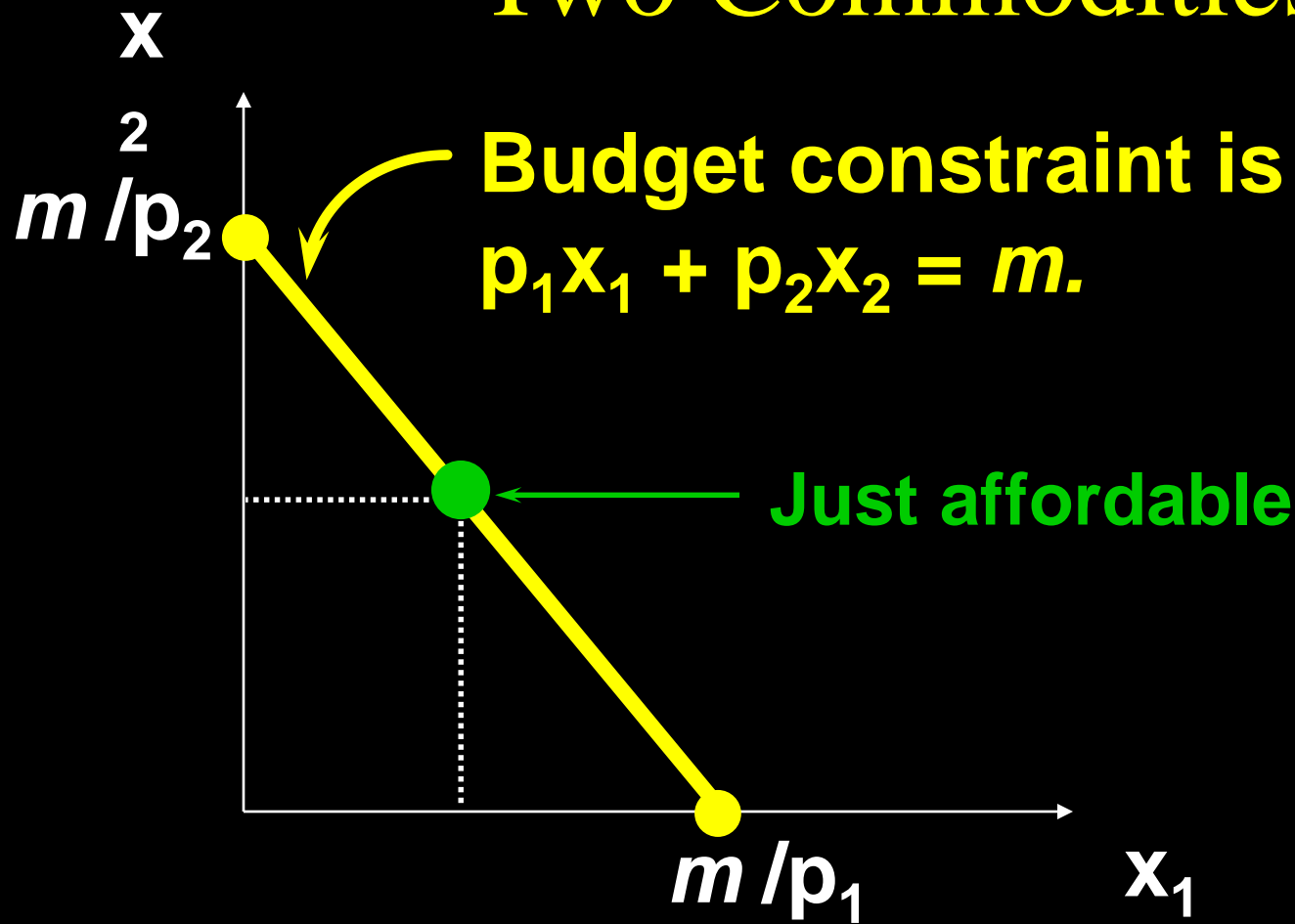




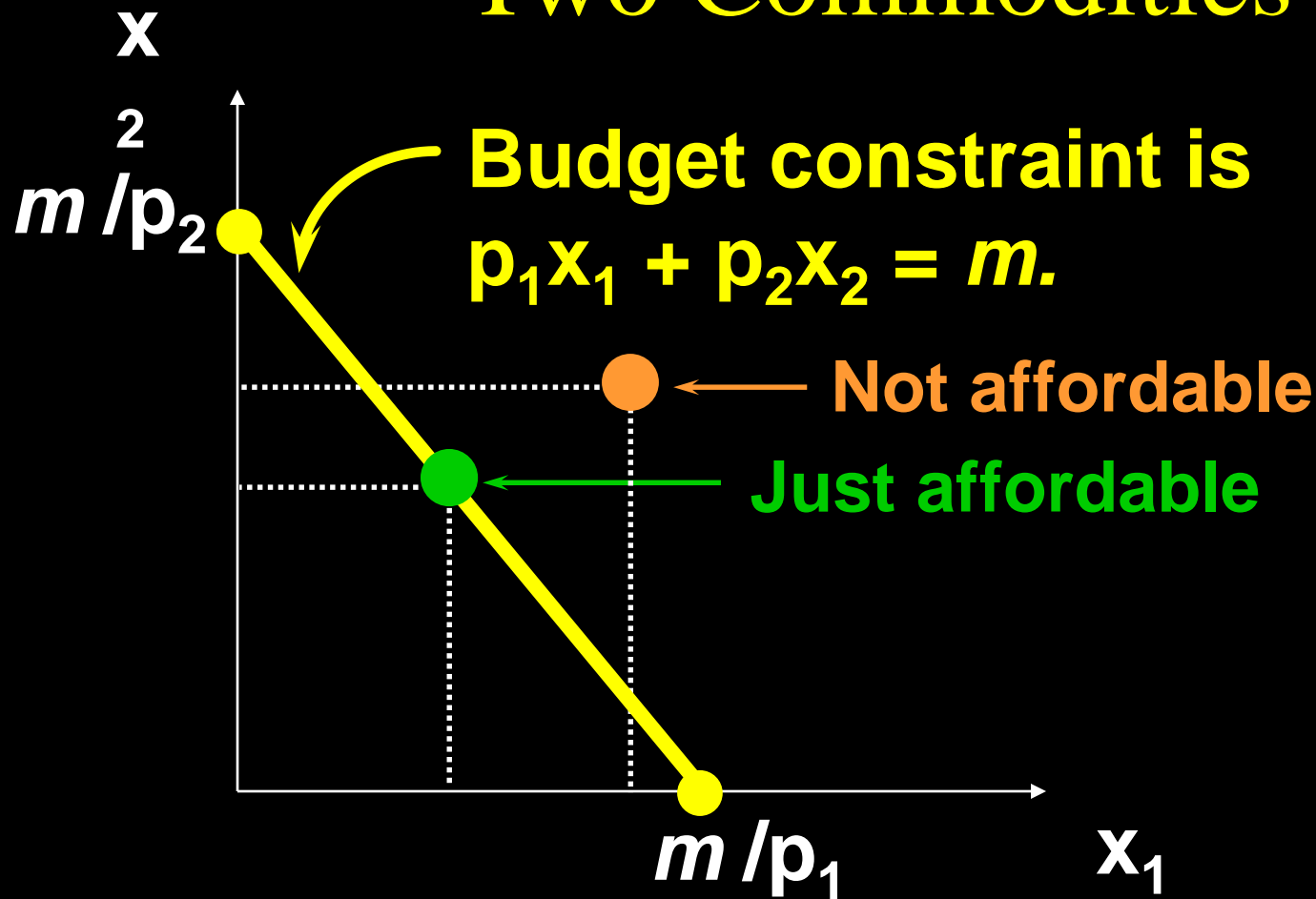
# Budget Set and Constraint for Two Commodities



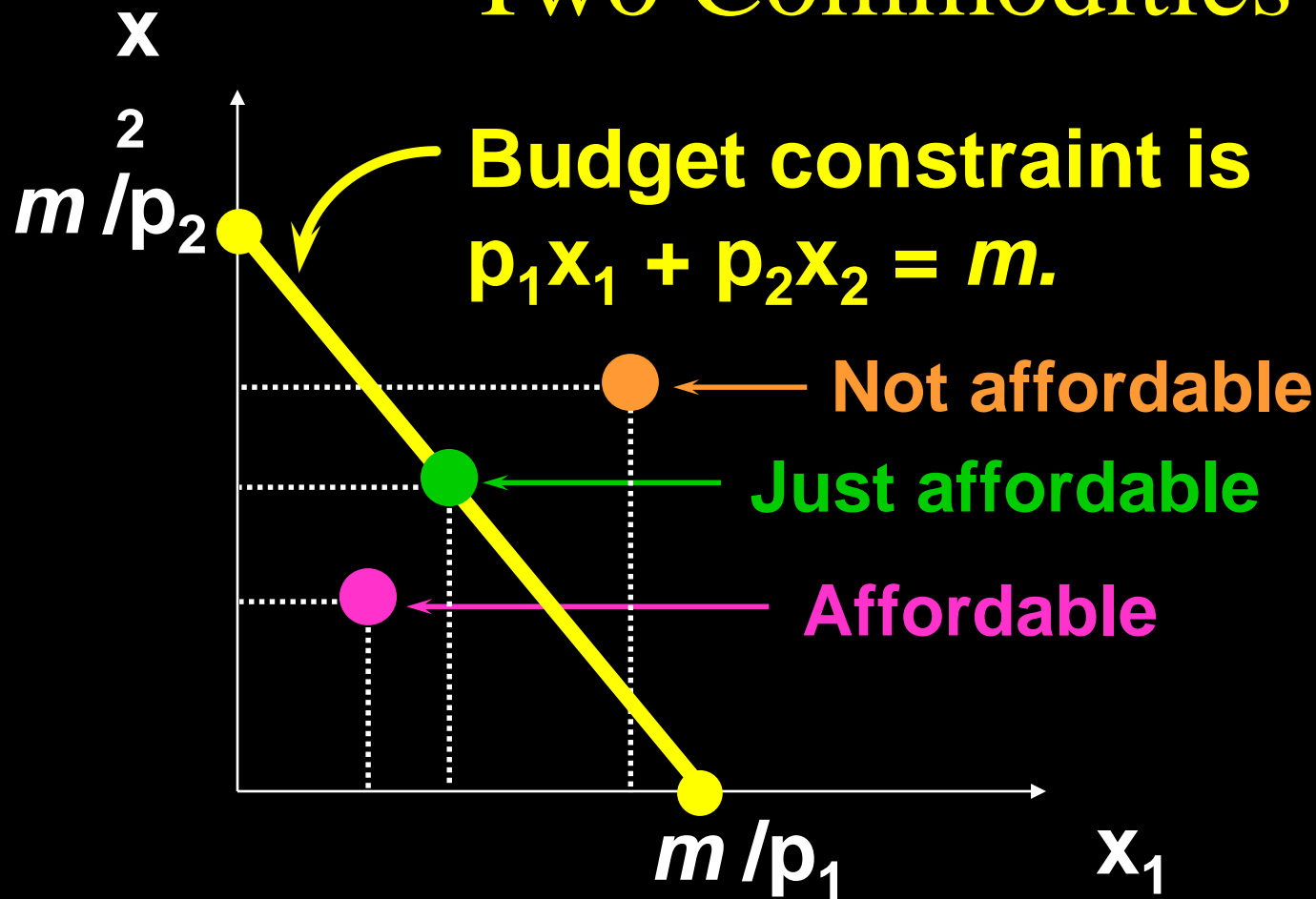
# Budget Set and Constraint for Two Commodities



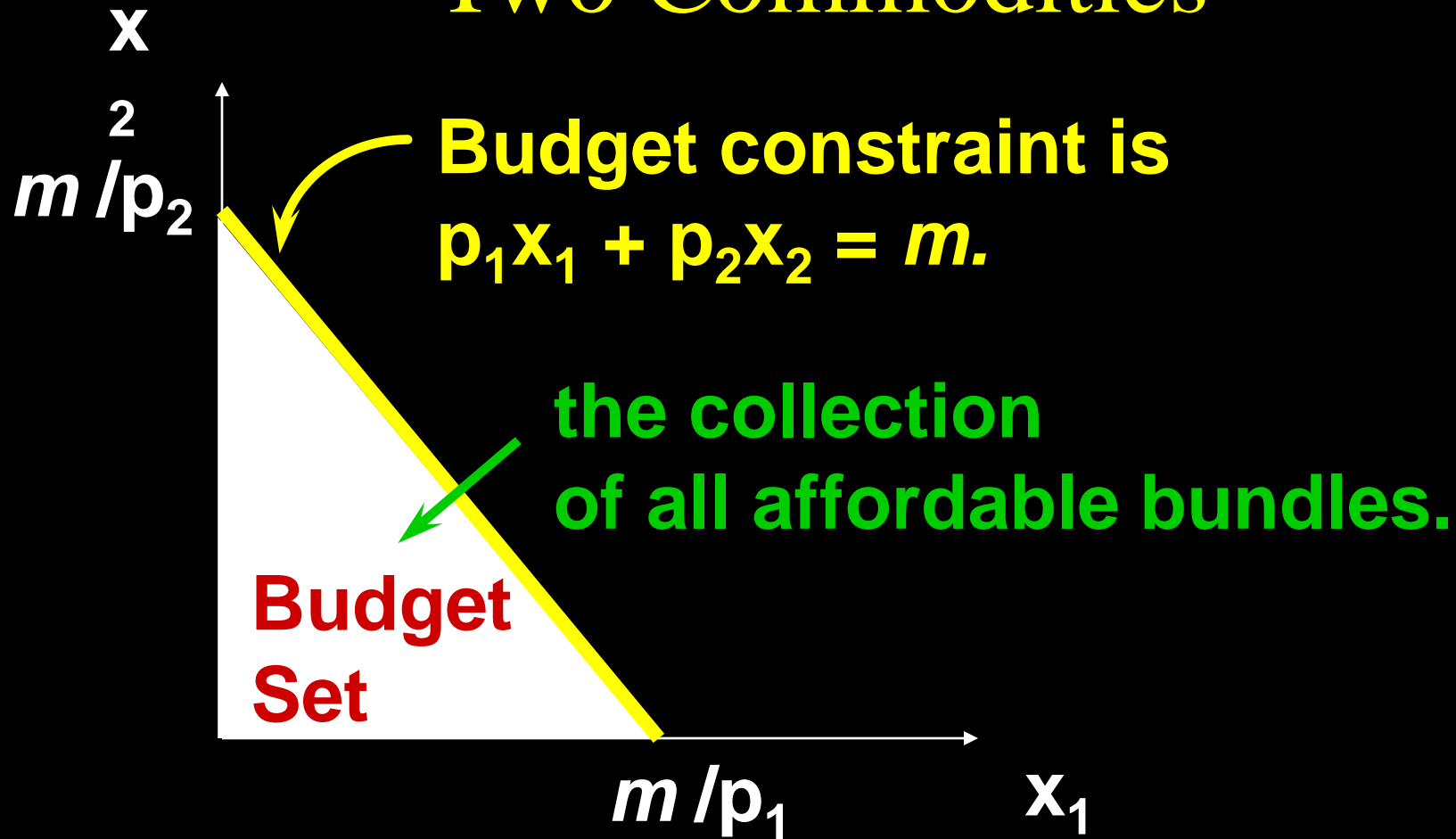
# Budget Set and Constraint for Two Commodities



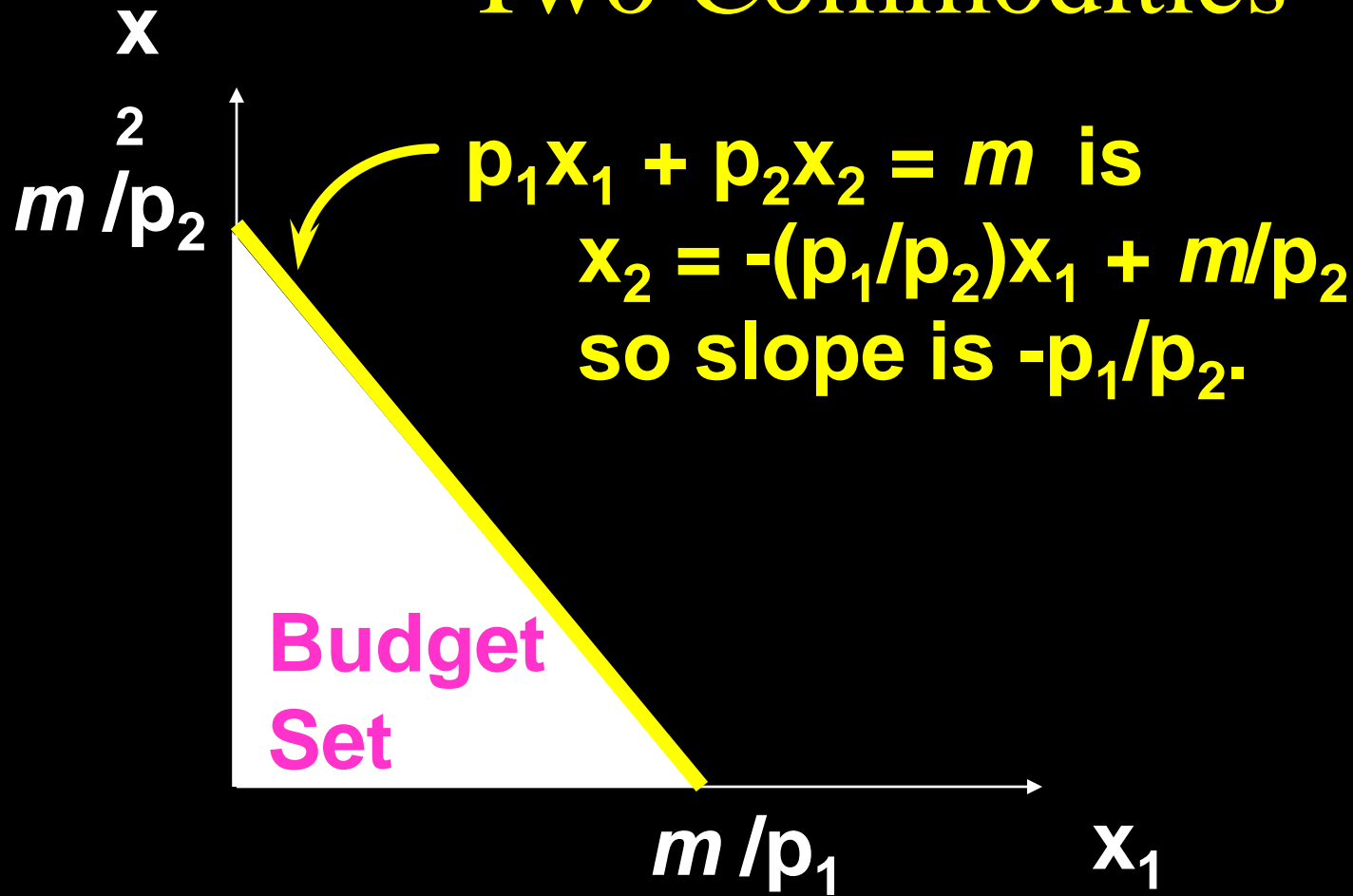
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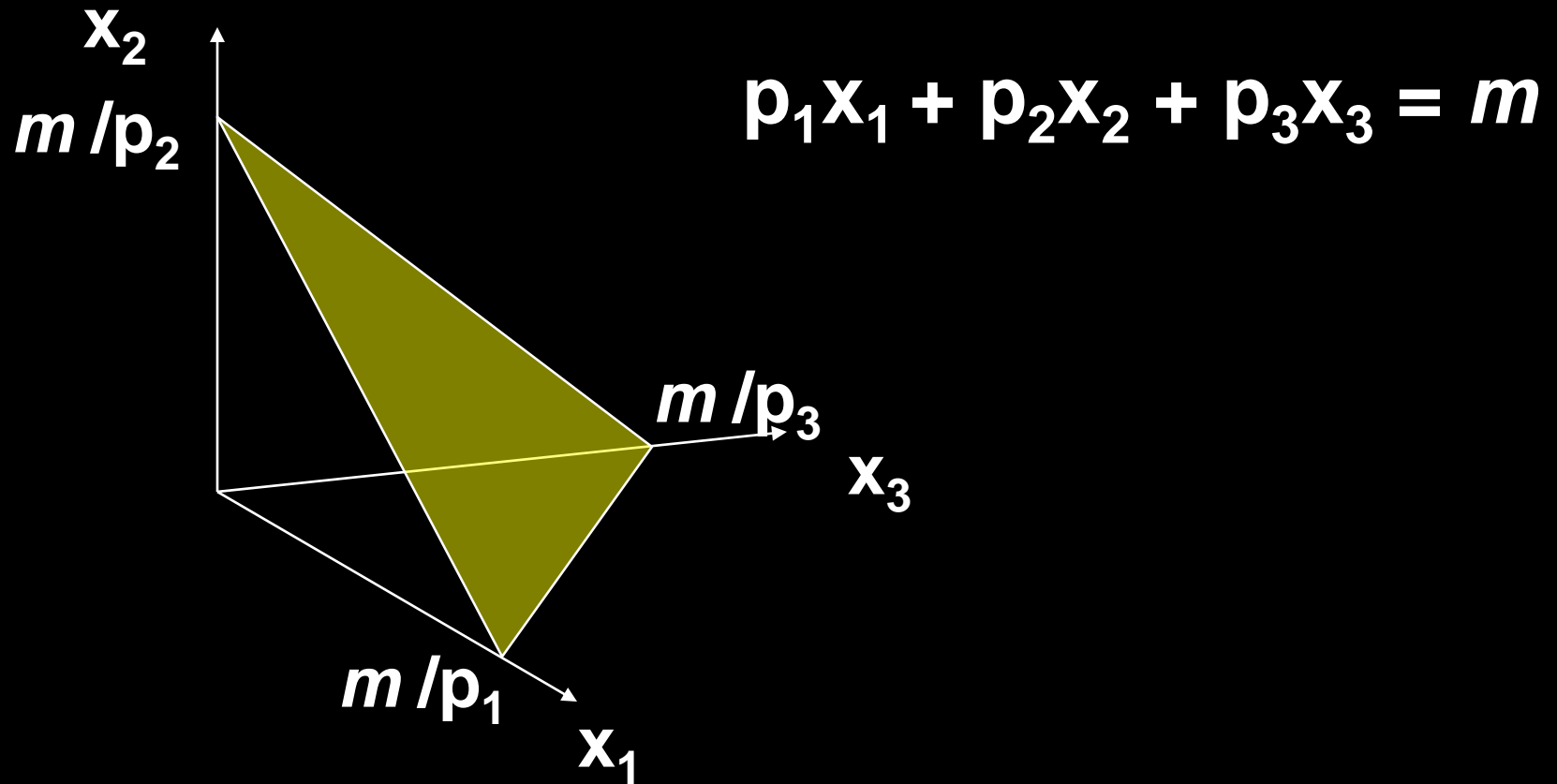
# Budget Set and Constraint for Two Commodities



# Budget Constraints

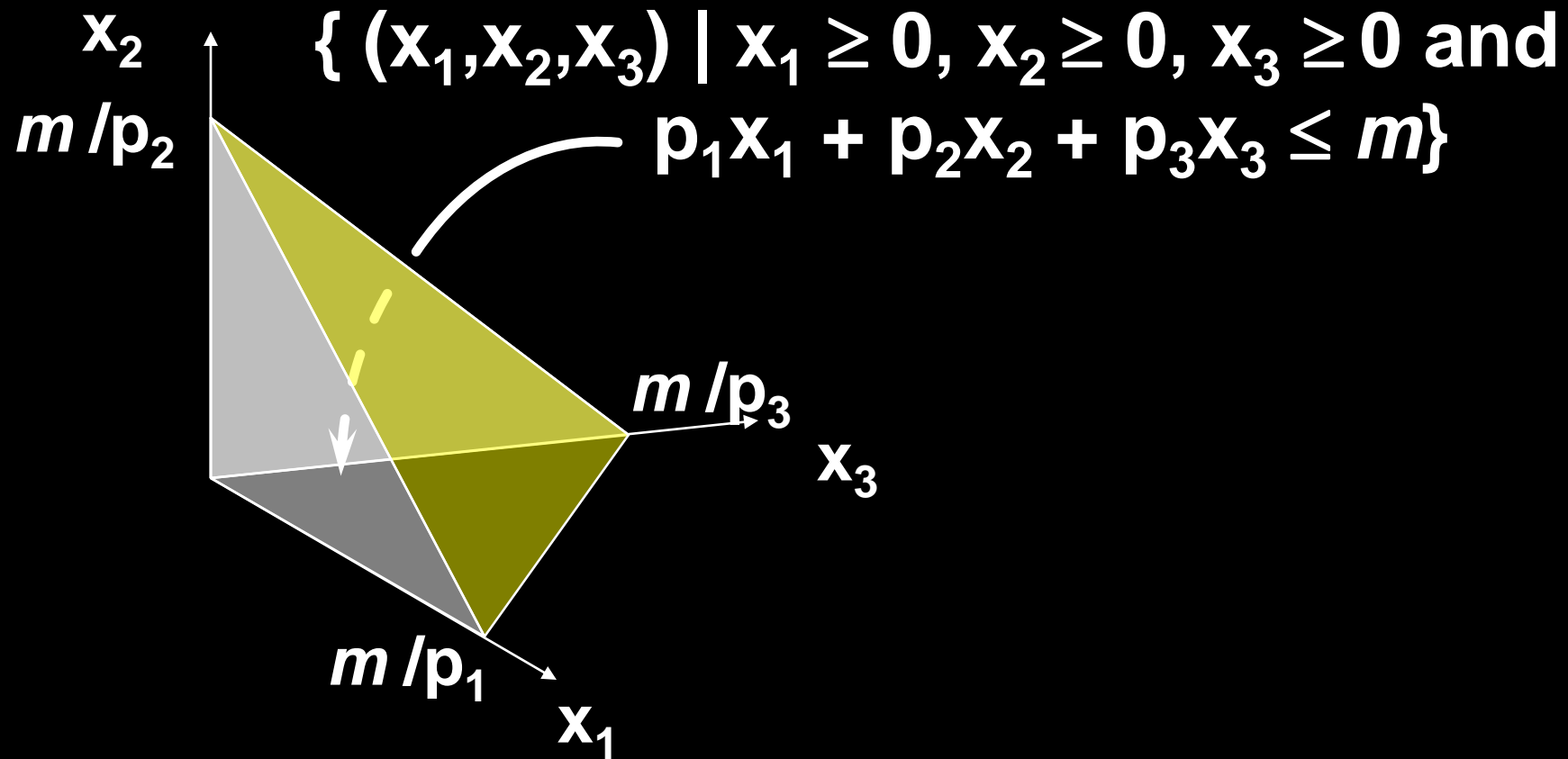
**If  $n = 3$  what do the budget constraint and the budget set look like?**

# Budget Constraint for Three Commodities





# Budget Set for Three Commodities



# Budget Constraints

For  $n = 2$  and  $x_1$  on the horizontal axis, the constraint's slope is  $-p_1/p_2$ .  
What does it mean?

$$x_2 = -\frac{p_1}{p_2}x_1 + \frac{m}{p_2}$$

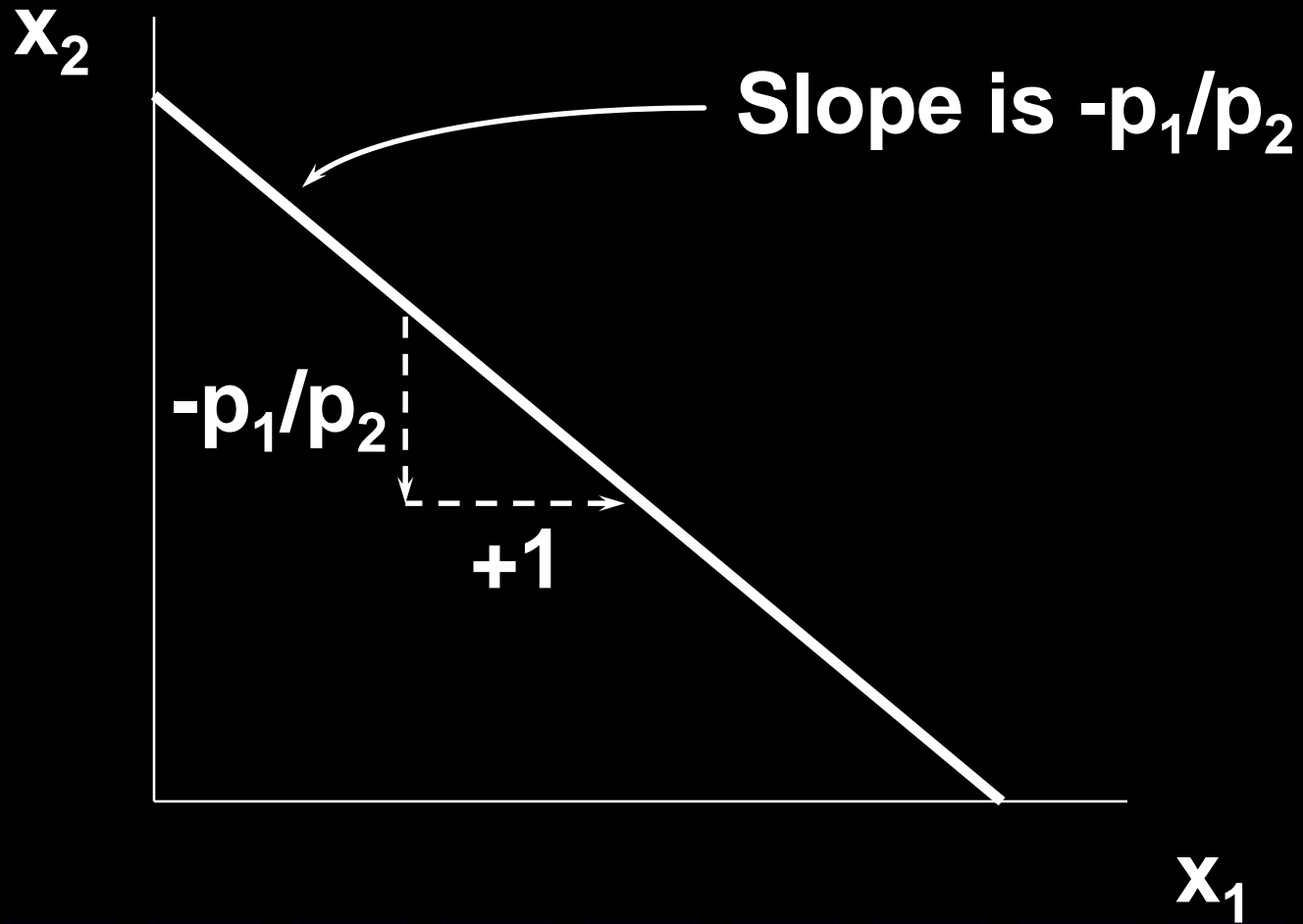
# Budget Constraints

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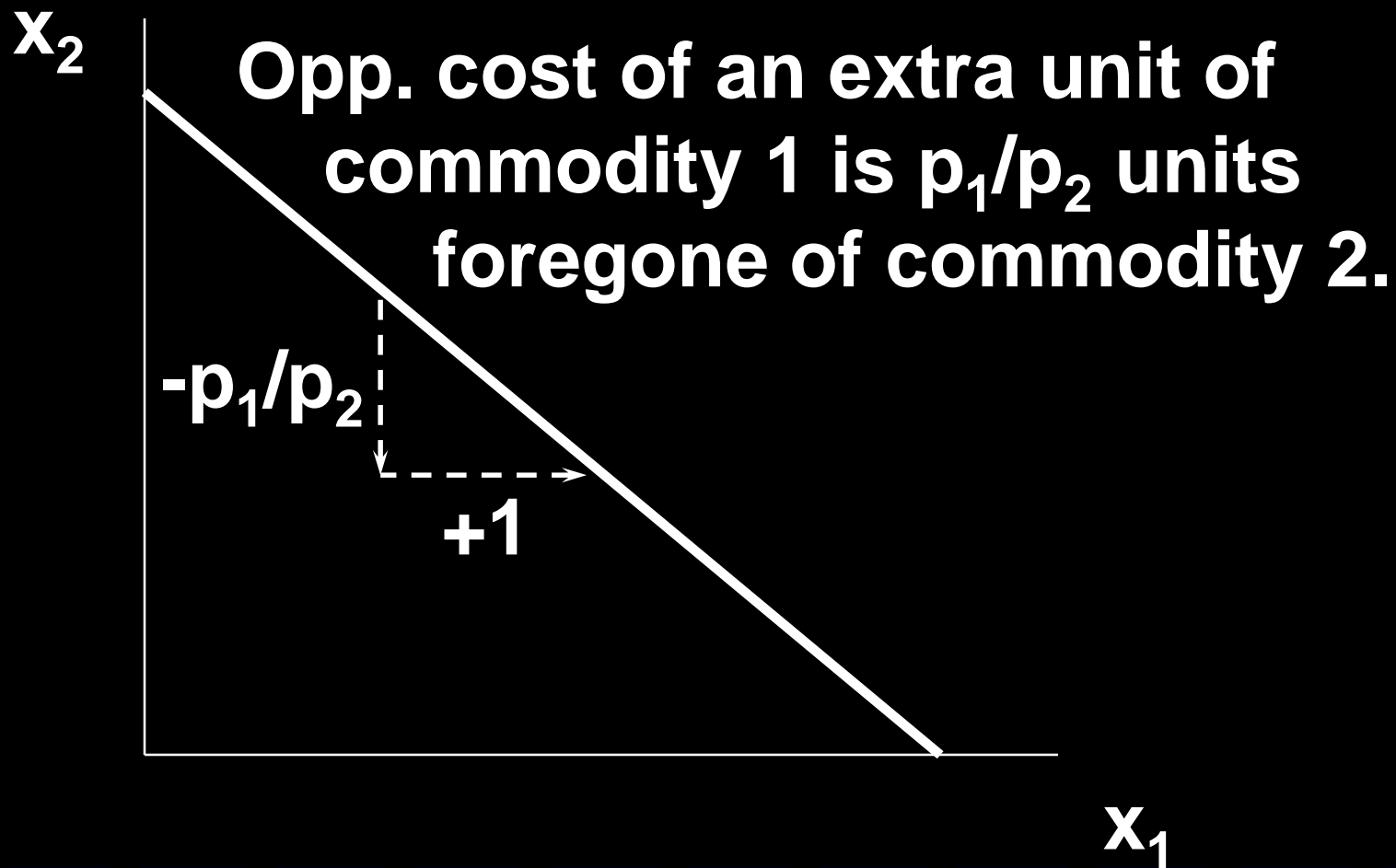
$$x_2 = -\frac{p_1}{p_2}x_1 + \frac{m}{p_2}$$

Increasing  $x_1$  by 1 must reduce  $x_2$  by  $p_1/p_2$ .

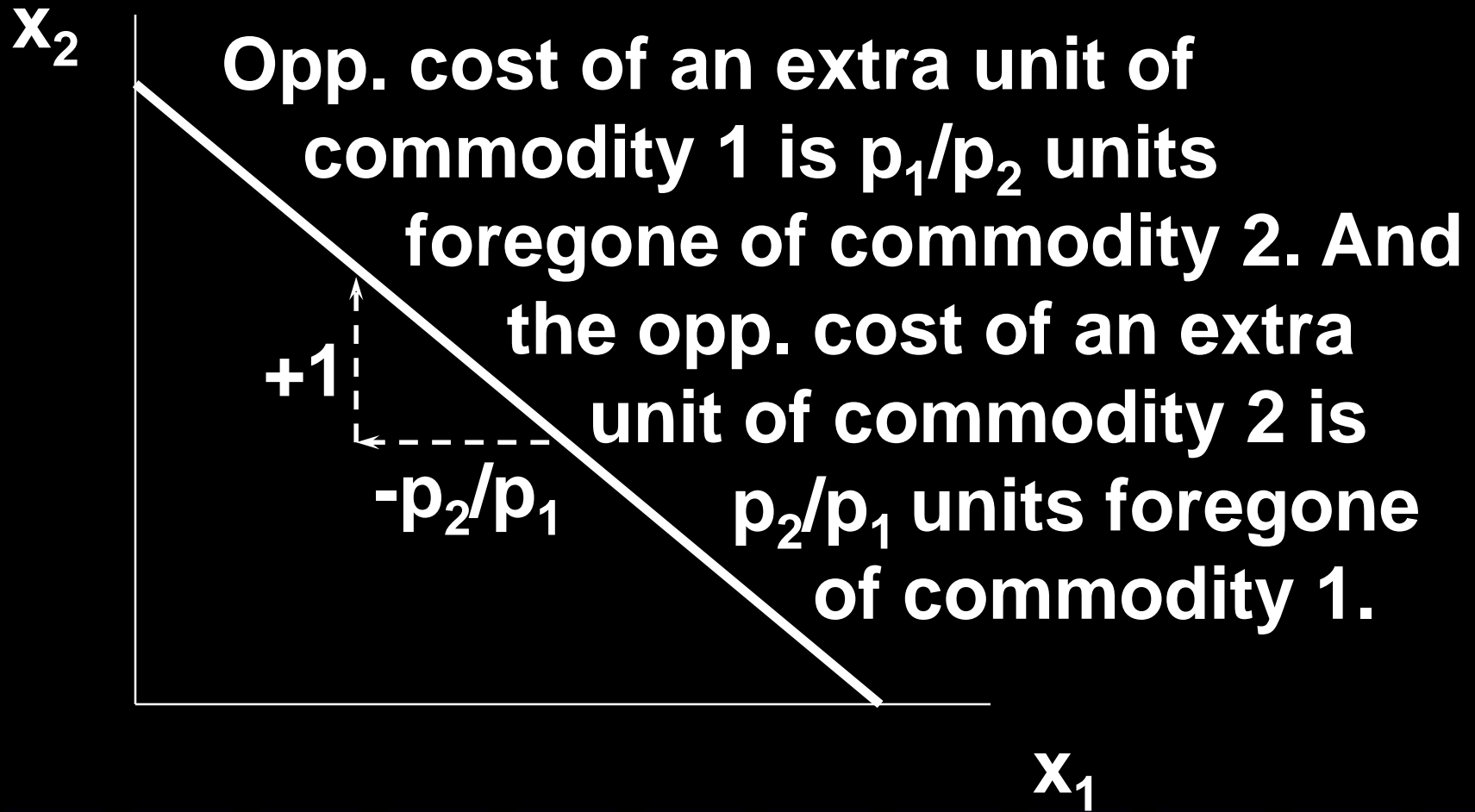
# Budget Constraints



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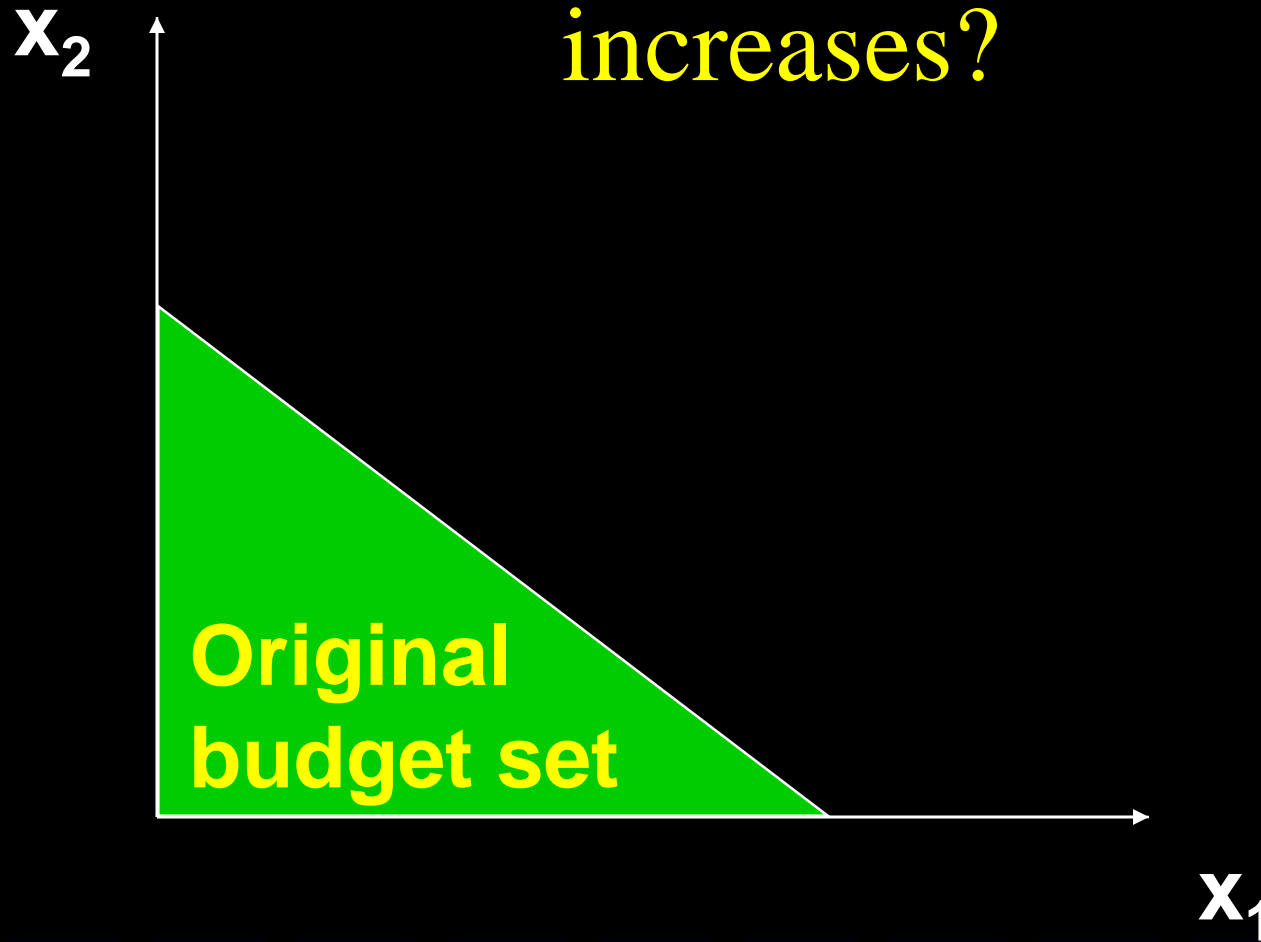
# Budget Constraints



# Budget Sets & Constraints; Income and Price Changes

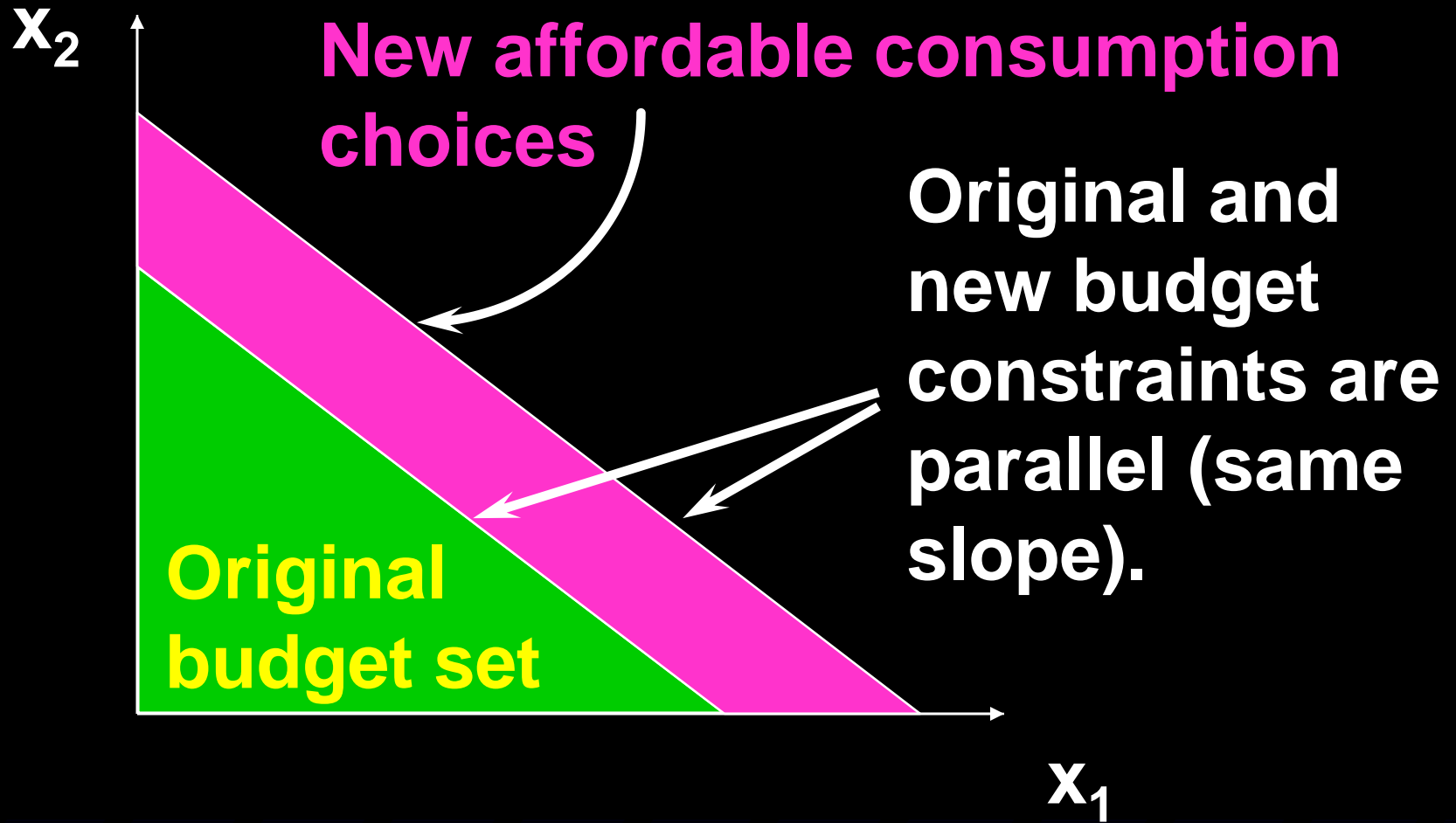
**The budget constraint and budget set depend upon prices and income. What happens as prices or income change?**

How do the budget set and budget constraint change as income  $m$  increases?

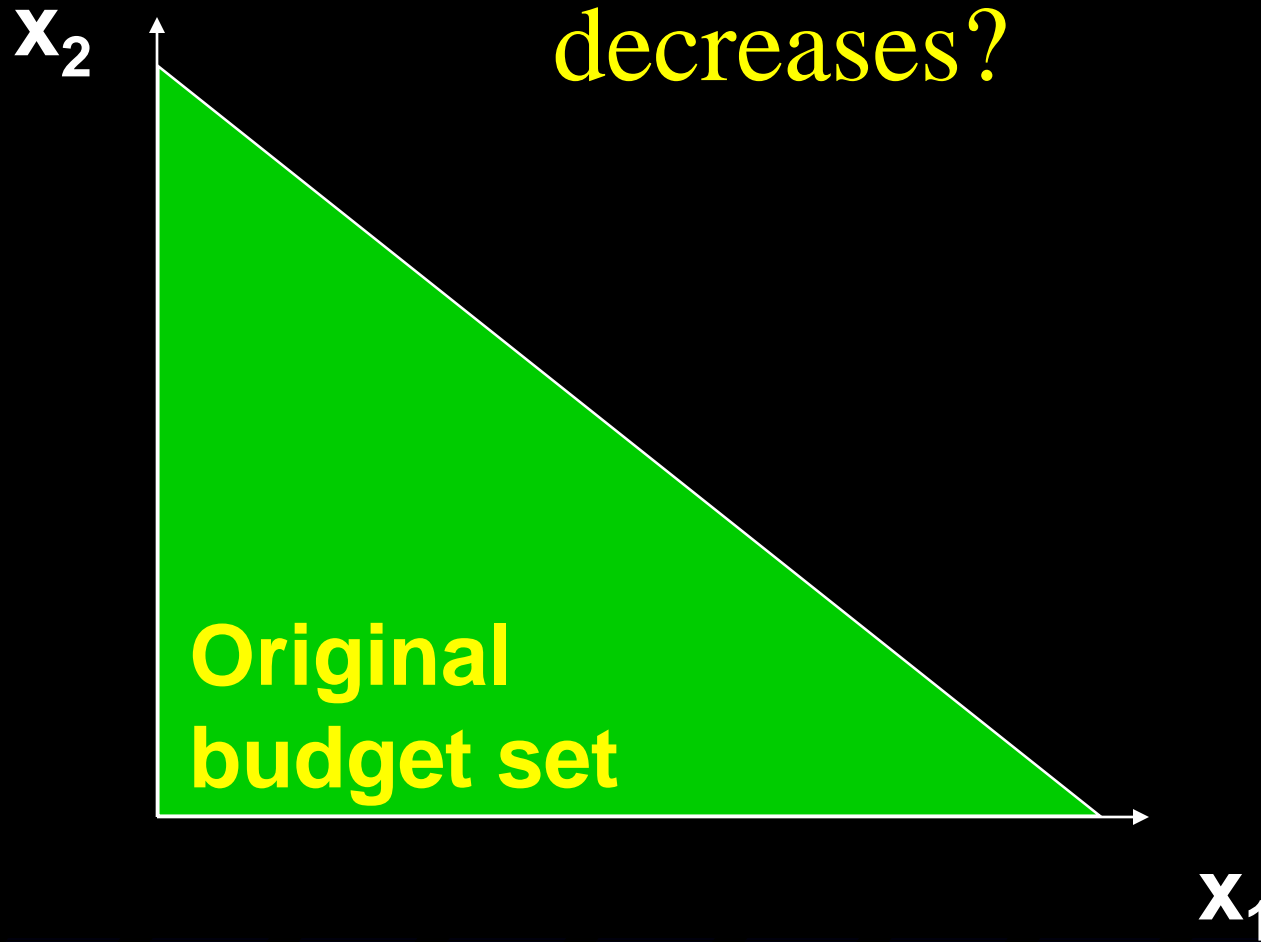




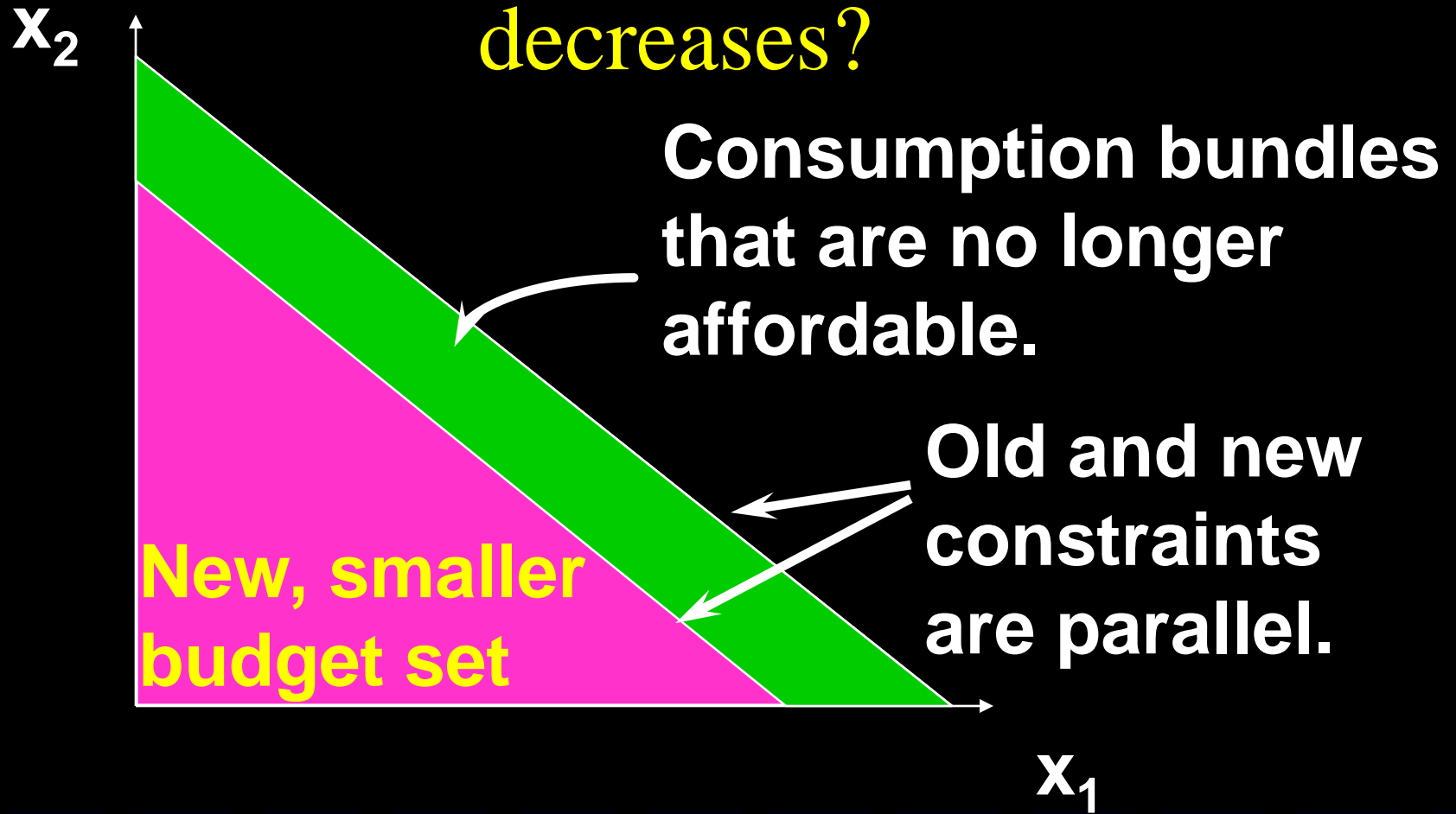
# Higher income gives more choice



How do the budget set and budget constraint change as income  $m$  decreases?



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# Budget Constraints - Income Changes

**Increases in income  $m$  shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.**

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**Increases in income  $m$  shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.**

**Decreases in income  $m$  shift the constraint inward in a parallel manner, thereby shrinking the budget set and reducing choice.**



# Budget Constraints - Income Changes

**No original choice is lost and new choices are added when income increases, so higher income cannot make a consumer worse off.**

**An income decrease may (typically will) make the consumer worse off.**

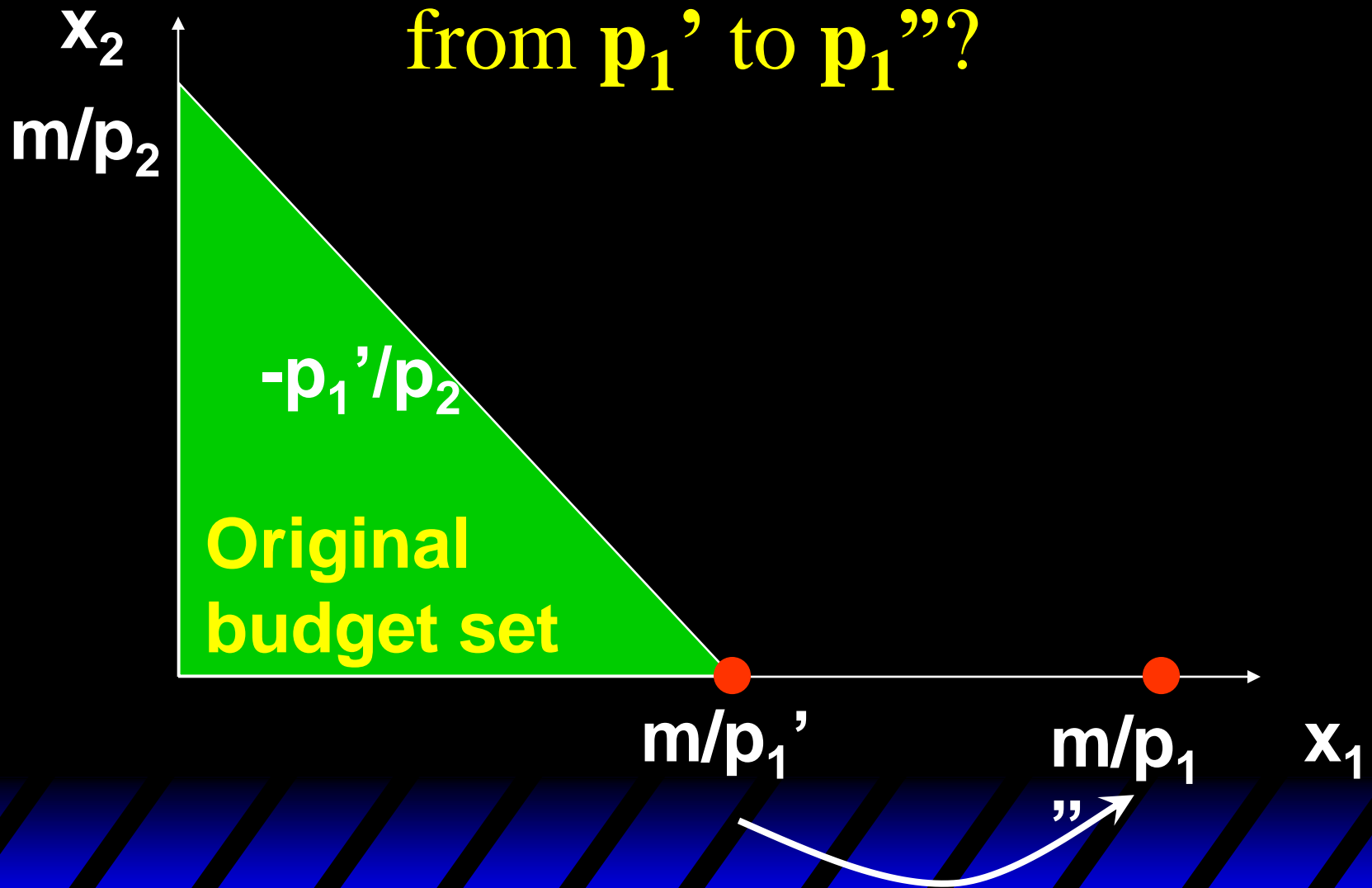


# Budget Constraints - Price Changes

**What happens if just one price decreases?**

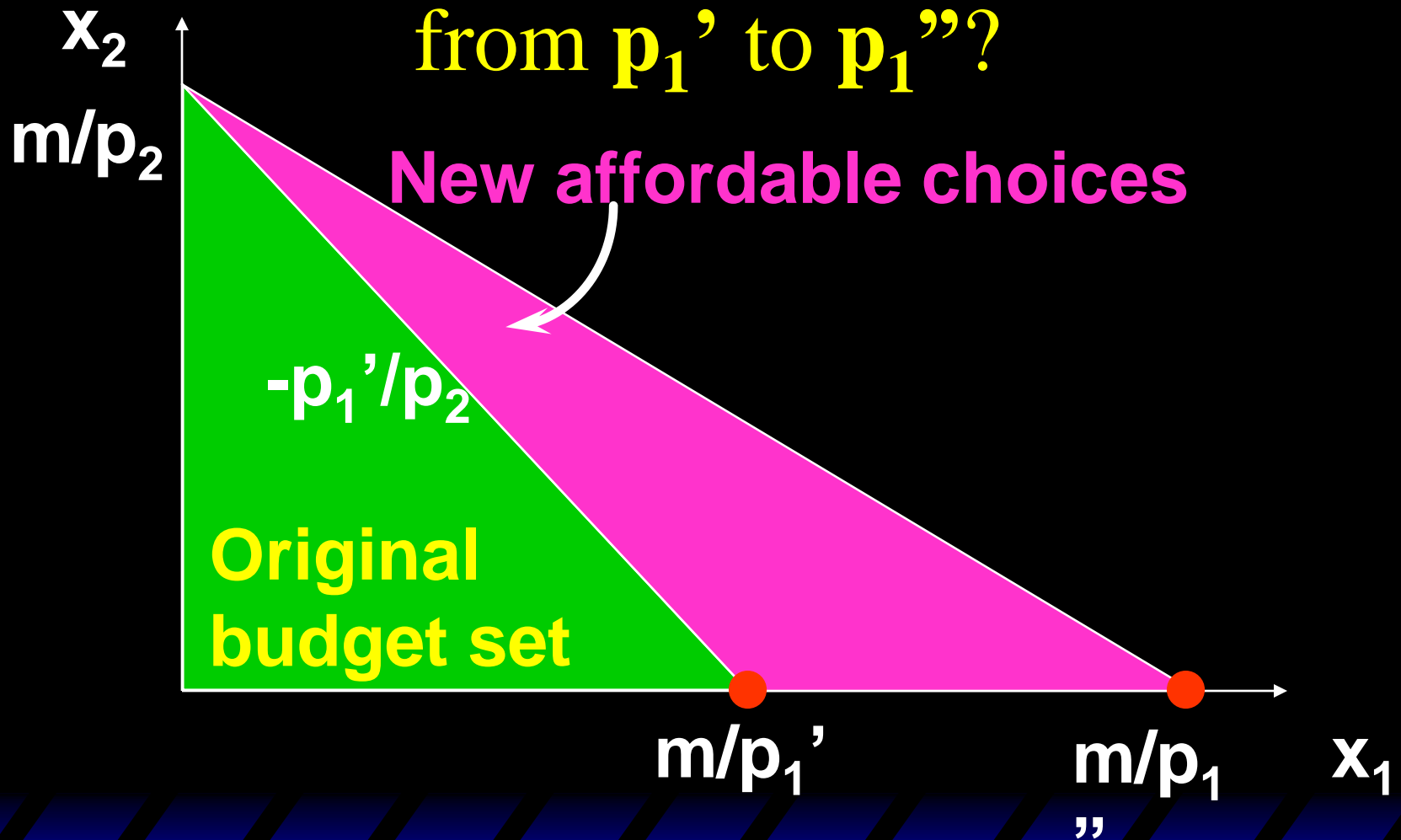
**Suppose  $p_1$  decreases.**

How do the budget set and budget constraint change as  $p_1$  decreases from  $p_1'$  to  $p_1''$ ?

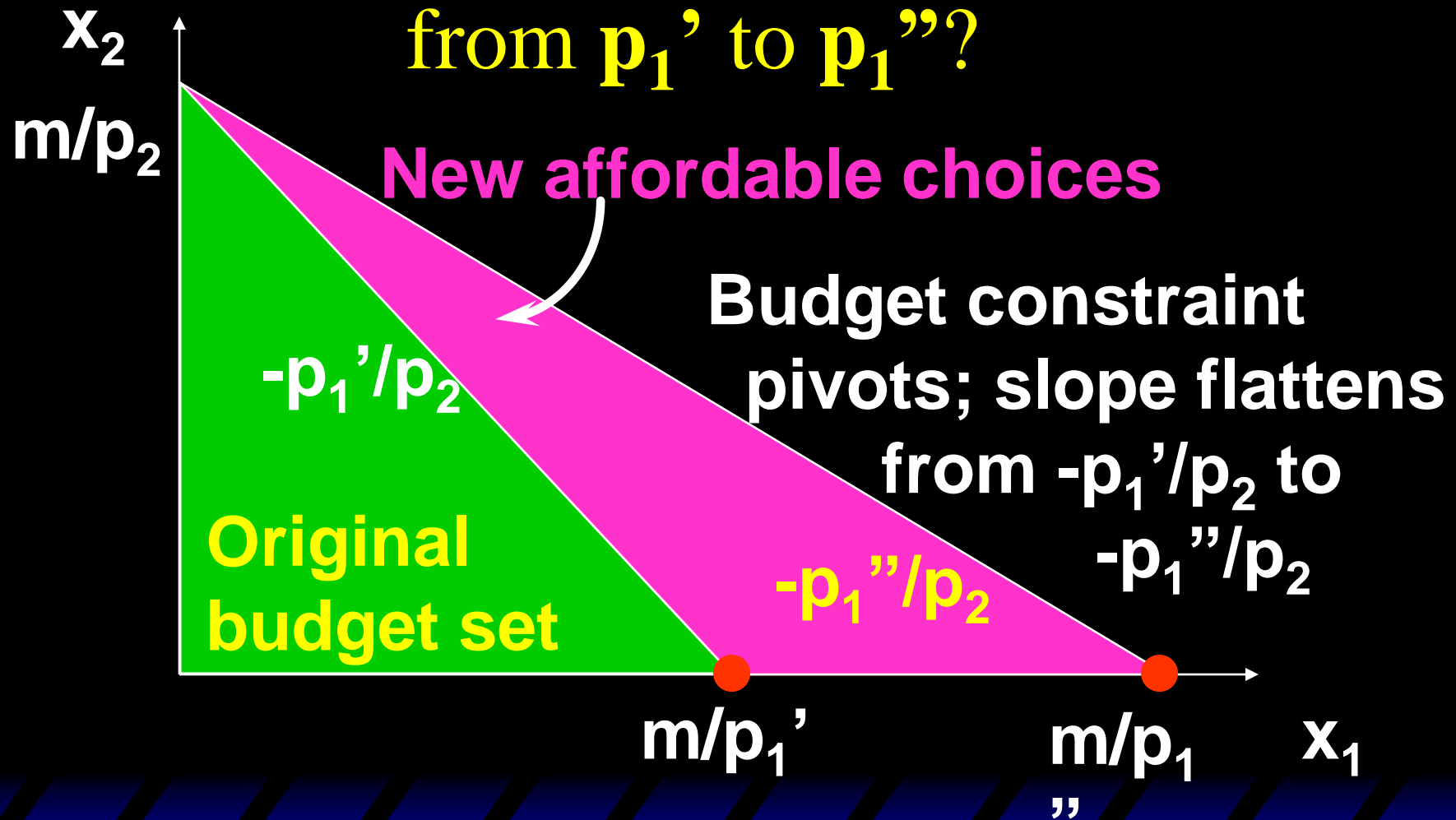




How do the budget set and budget constraint change as  $p_1$  decreases from  $p_1'$  to  $p_1''$ ?



How do the budget set and budget constraint change as  $p_1$  decreases from  $p_1'$  to  $p_1''$ ?



# Budget Constraints - Price Changes

**Reducing the price of one commodity pivots the constraint outward.** No old choice is lost and new choices are added, so reducing one price cannot make the consumer worse off.

# Budget Constraints - Price Changes

**Similarly, increasing one price pivots the constraint inwards, reduces choice and may (typically will) make the consumer worse off.**

# Uniform *Ad Valorem* Sales Taxes

**An *ad valorem* sales tax levied at a rate of 5% increases all prices by 5%, from  $p$  to  $(1+0.05)p = 1.05p$ .**

**An *ad valorem* sales tax levied at a rate of  $t$  increases all prices by  $tp$  from  $p$  to  $(1+t)p$ .**

**A uniform sales tax is applied uniformly to all commodities.**

# Uniform *Ad Valorem* Sales Taxes

A uniform sales tax levied at rate  $t$  changes the constraint from

$$p_1x_1 + p_2x_2 = m$$

to

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$

# Uniform *Ad Valorem* Sales Taxes

A uniform sales tax levied at rate  $t$  changes the constraint from

$$p_1x_1 + p_2x_2 = m$$

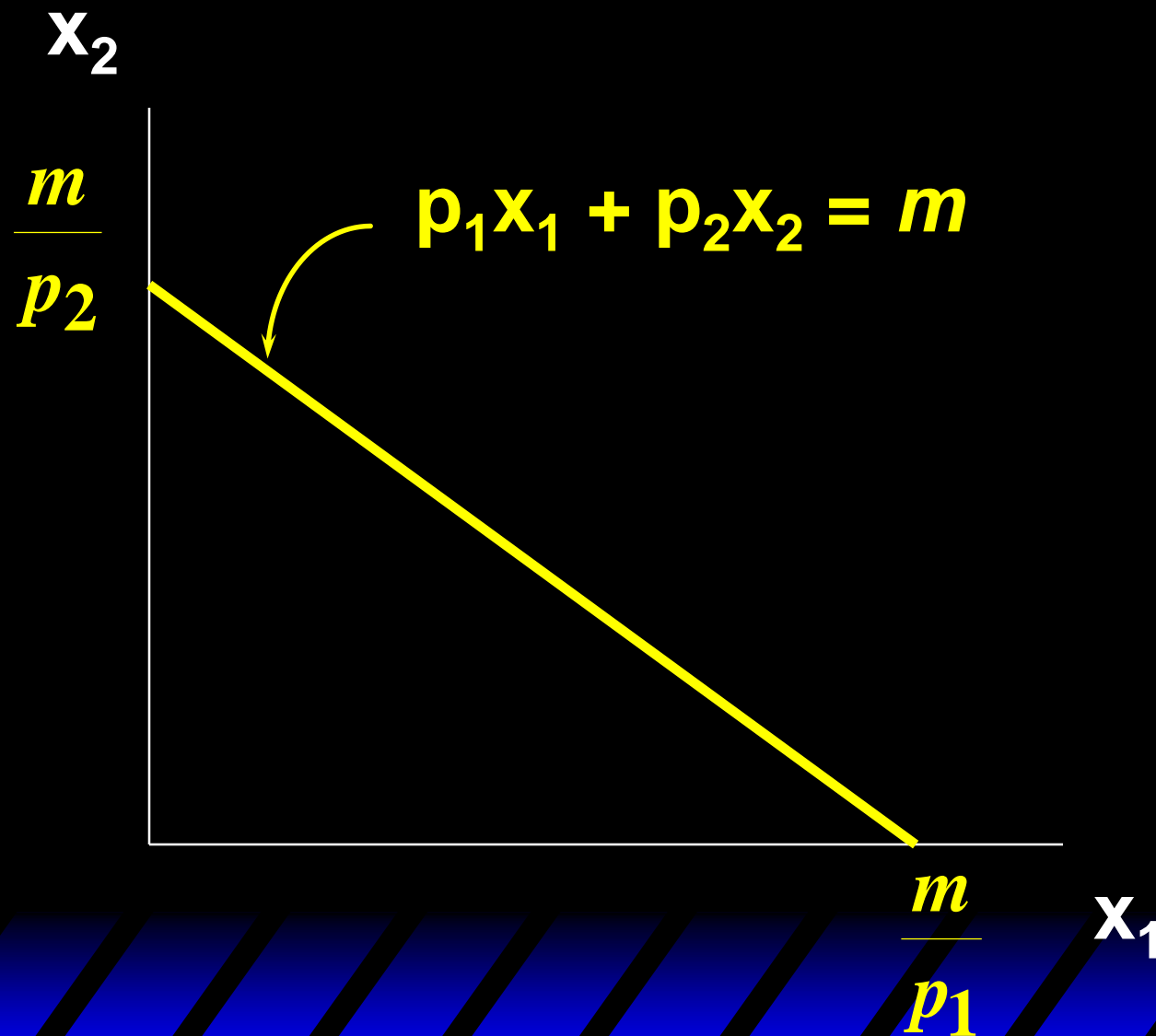
to

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$

i.e.

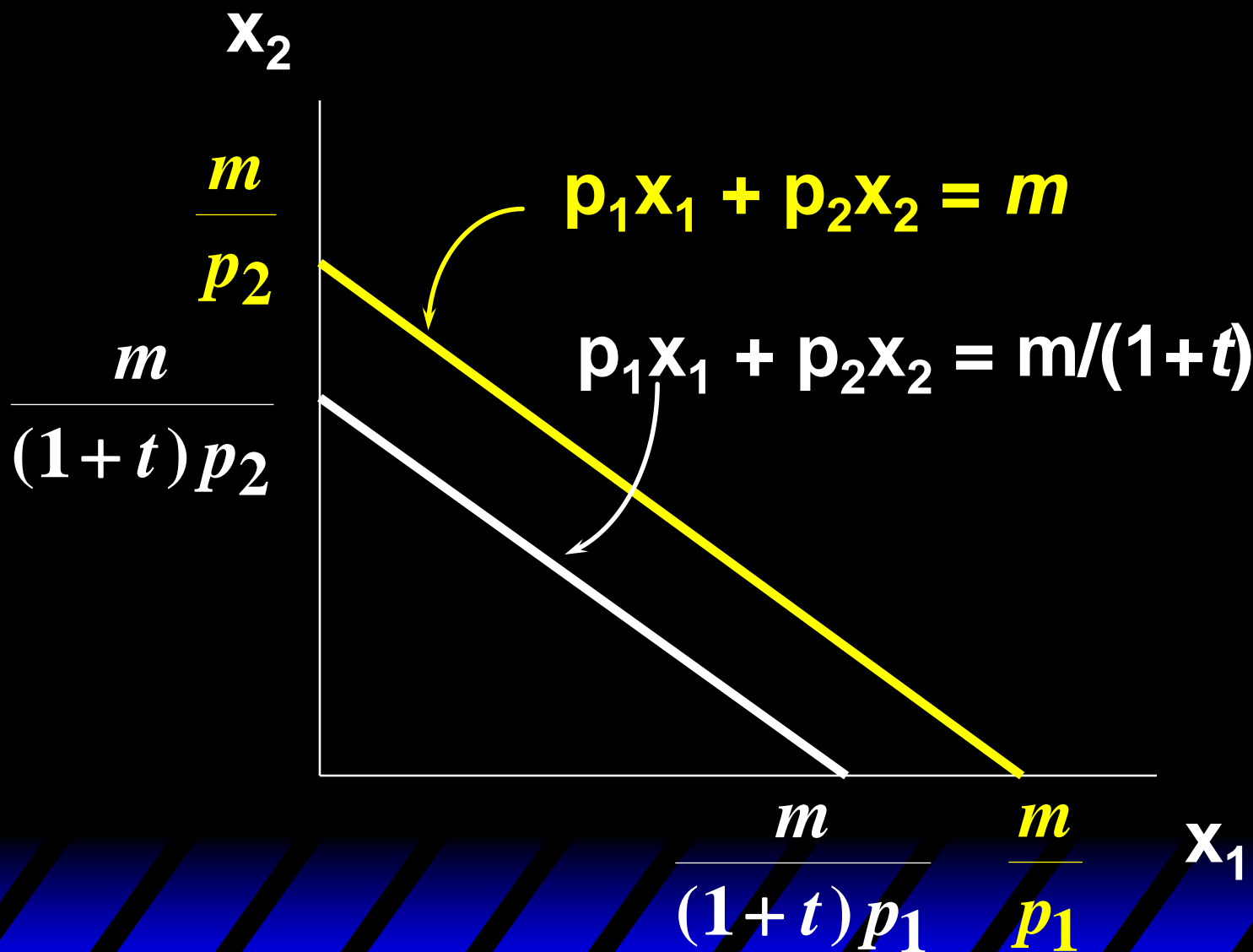
$$p_1x_1 + p_2x_2 = m/(1+t).$$

# Uniform *Ad Valorem* Sales Taxes

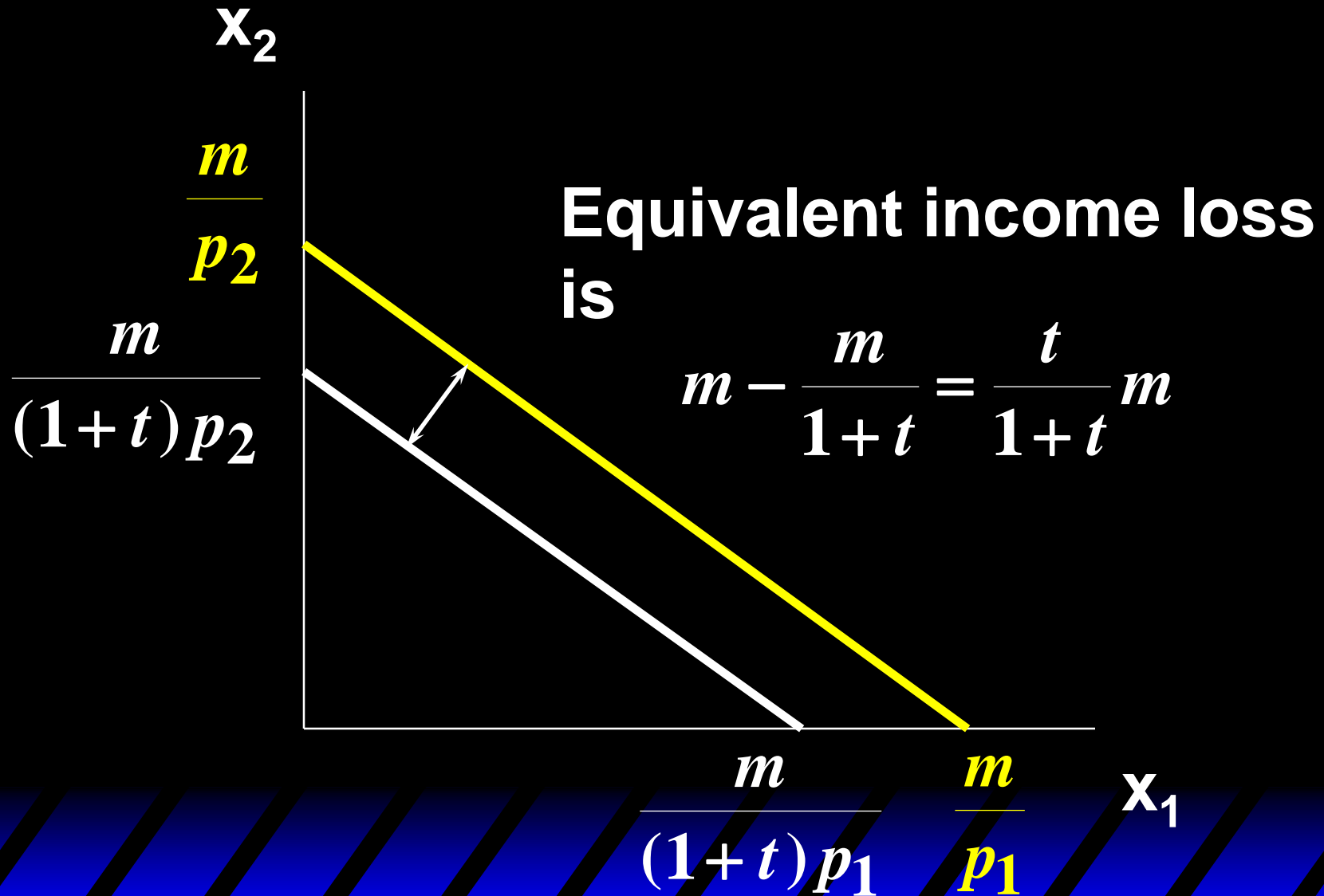




# Uniform *Ad Valorem* Sales Taxes

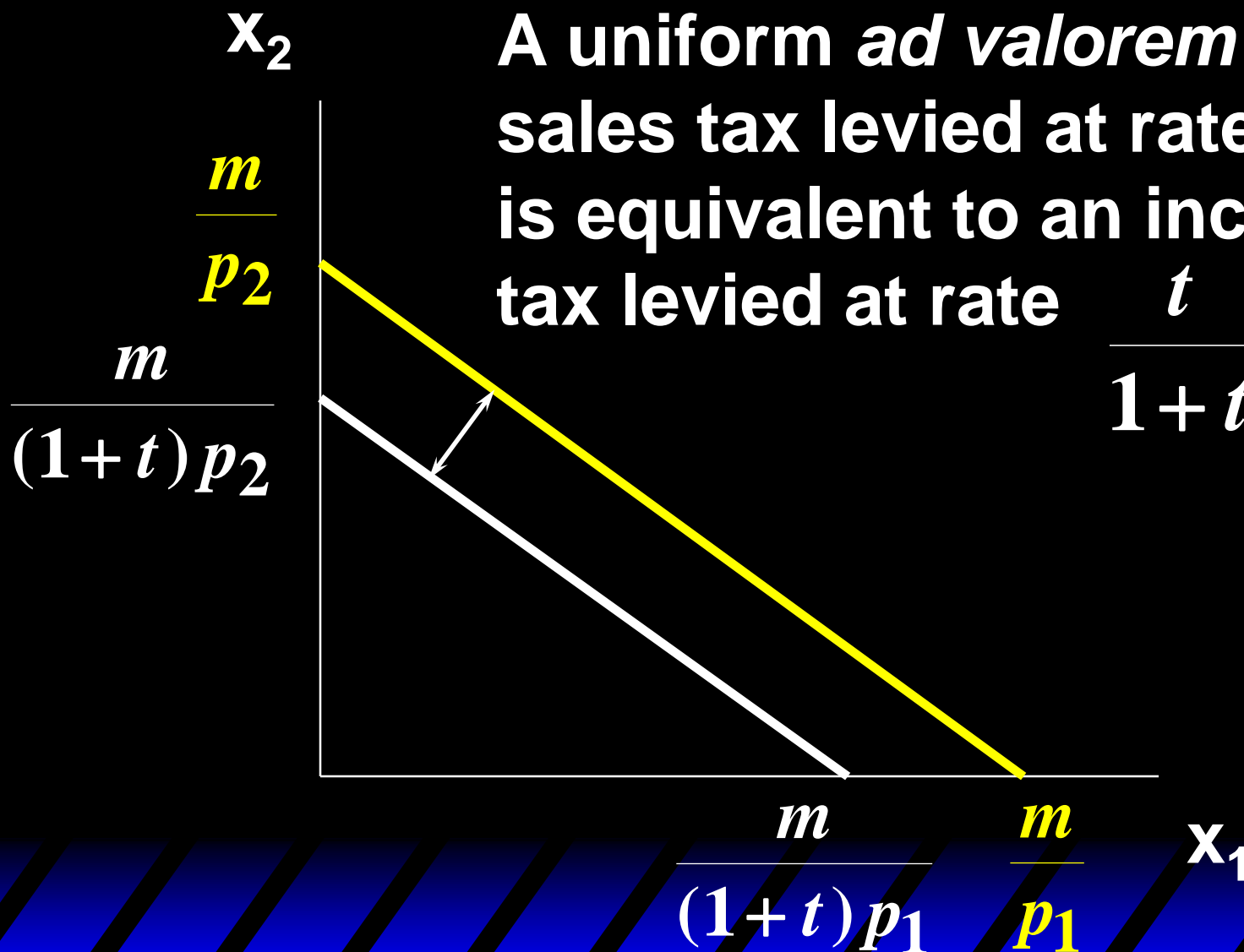


# Uniform *Ad Valorem* Sales Taxes



# Uniform *Ad Valorem* Sales Taxes

A uniform *ad valorem* sales tax levied at rate  $t$  is equivalent to an income tax levied at rate  $\frac{t}{1+t}$ .



# The Food Stamp Program

**Food stamps are coupons that can be legally exchanged only for food.**

**How does a commodity-specific gift such as a food stamp alter a family's budget constraint?**

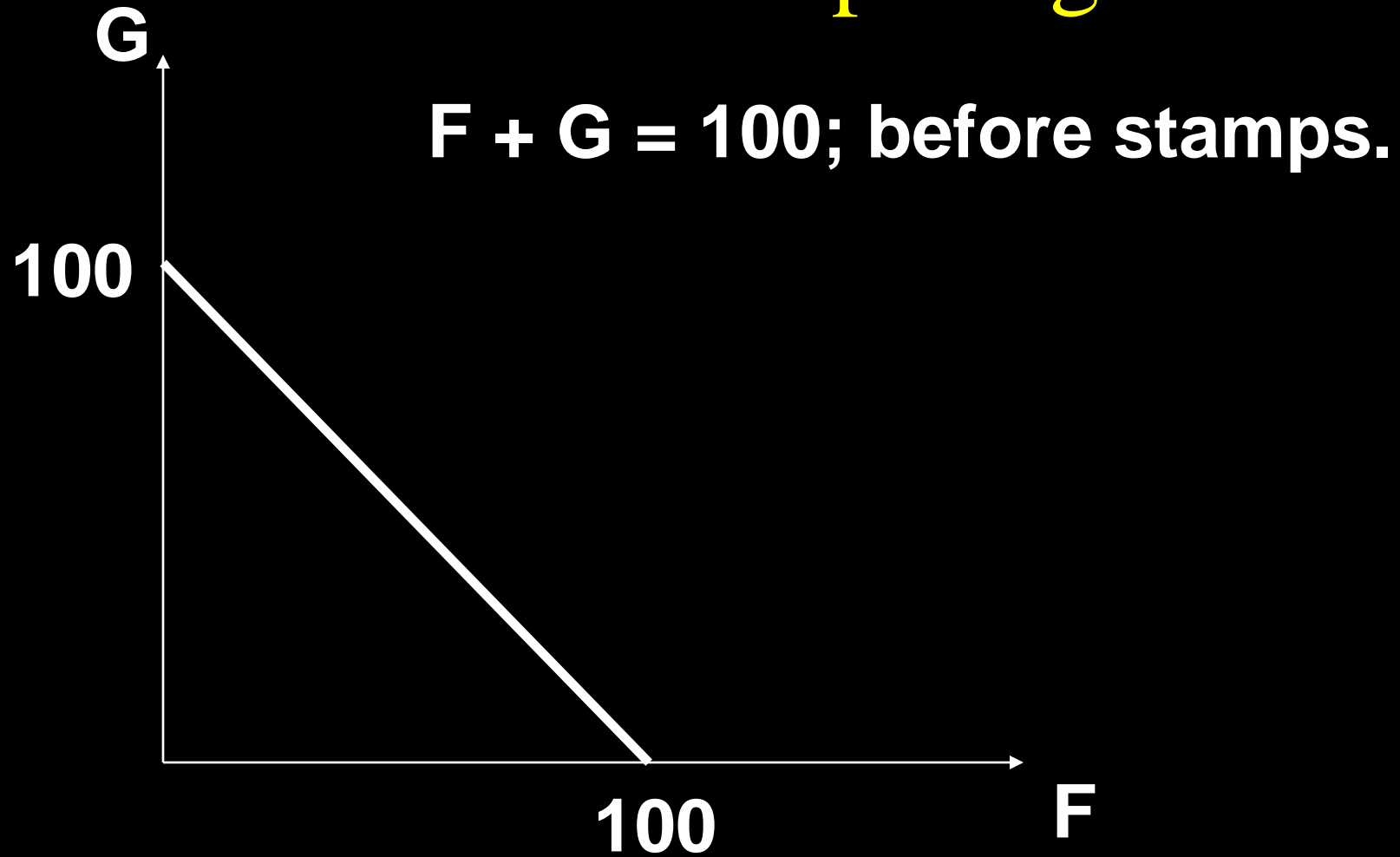
# The Food Stamp Program

**Suppose  $m = \$100$ ,  $p_F = \$1$  and the price of “other goods” is  $p_G = \$1$ .**

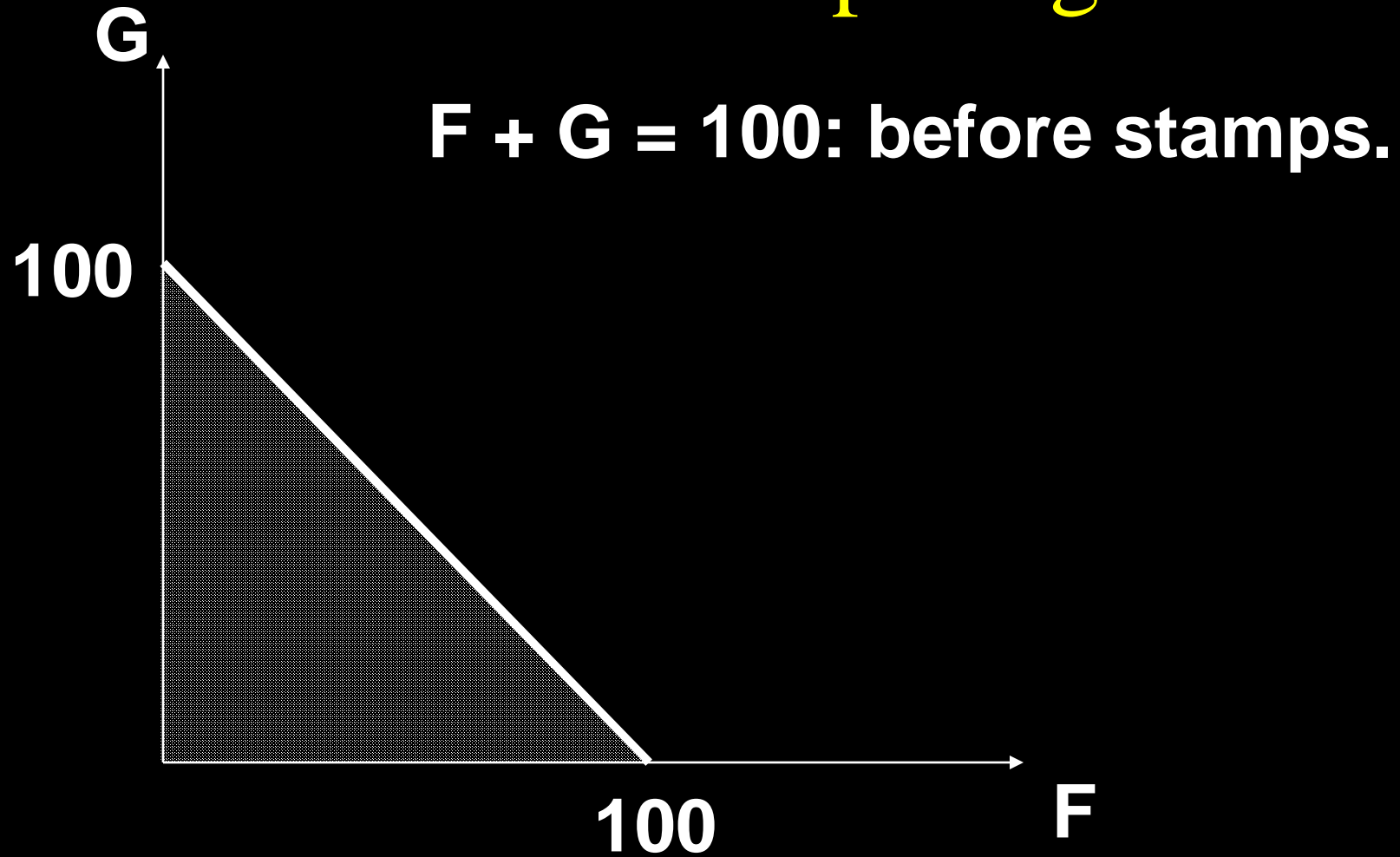
**The budget constraint is then**

$$F + G = 100.$$

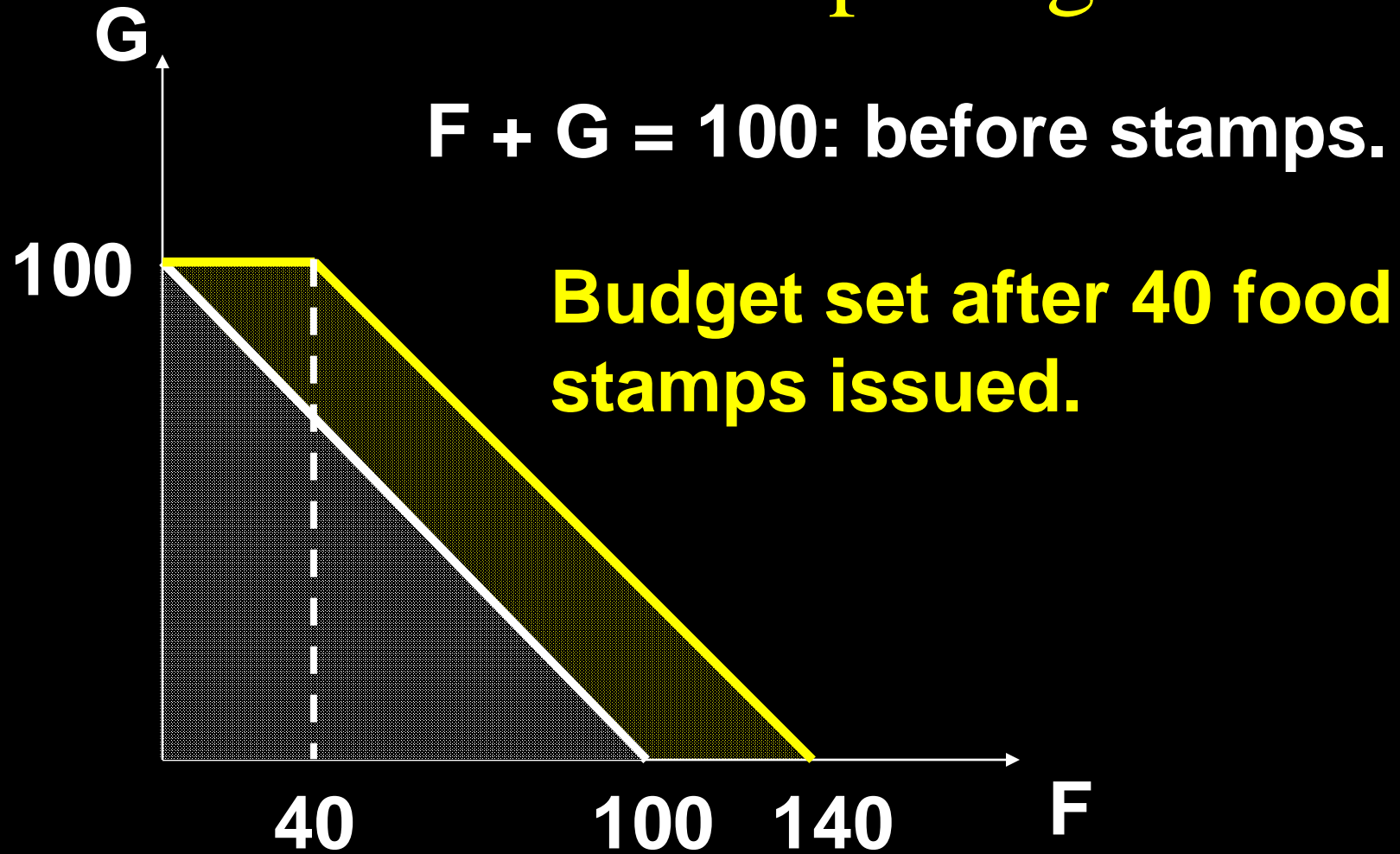
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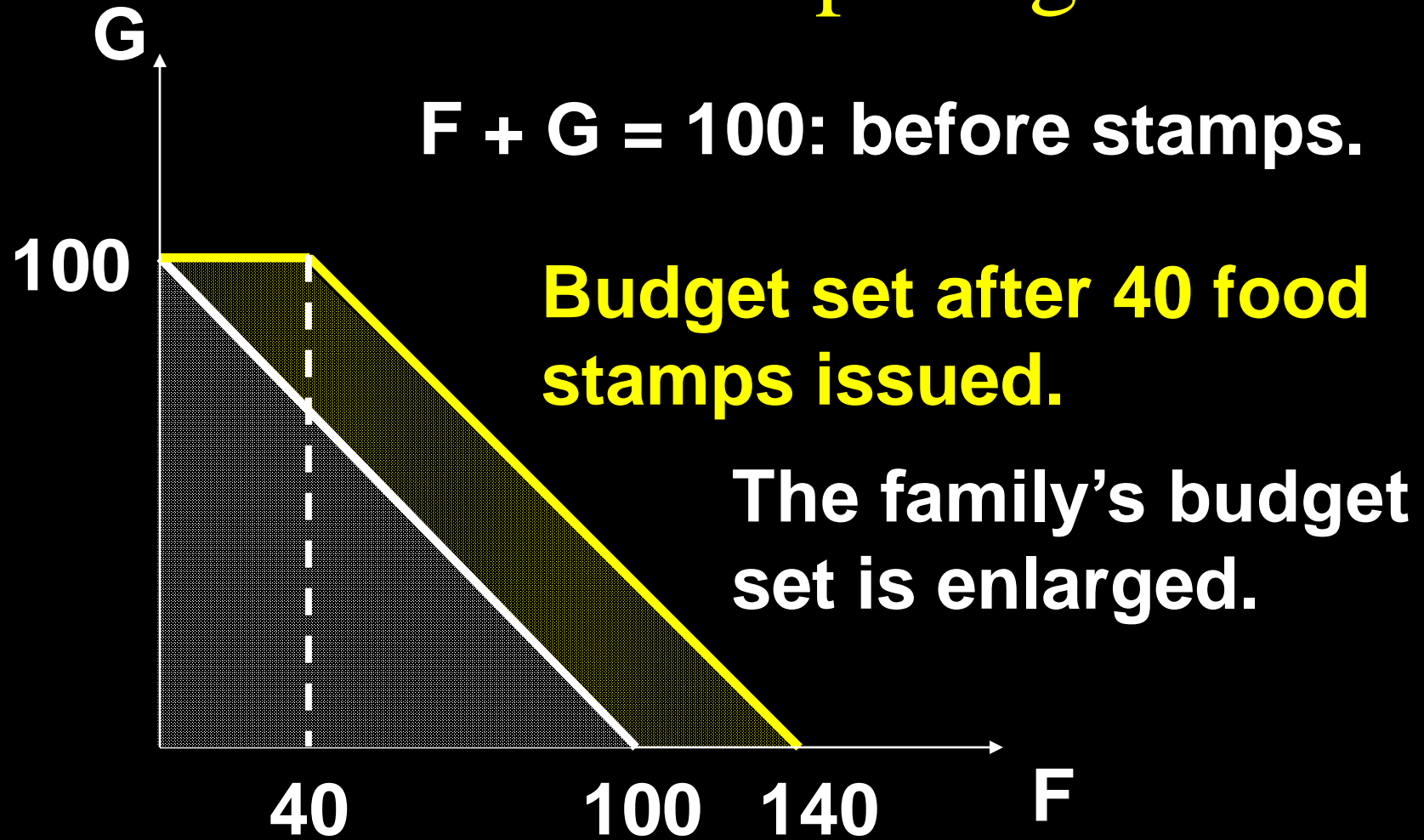


# The Food Stamp Program





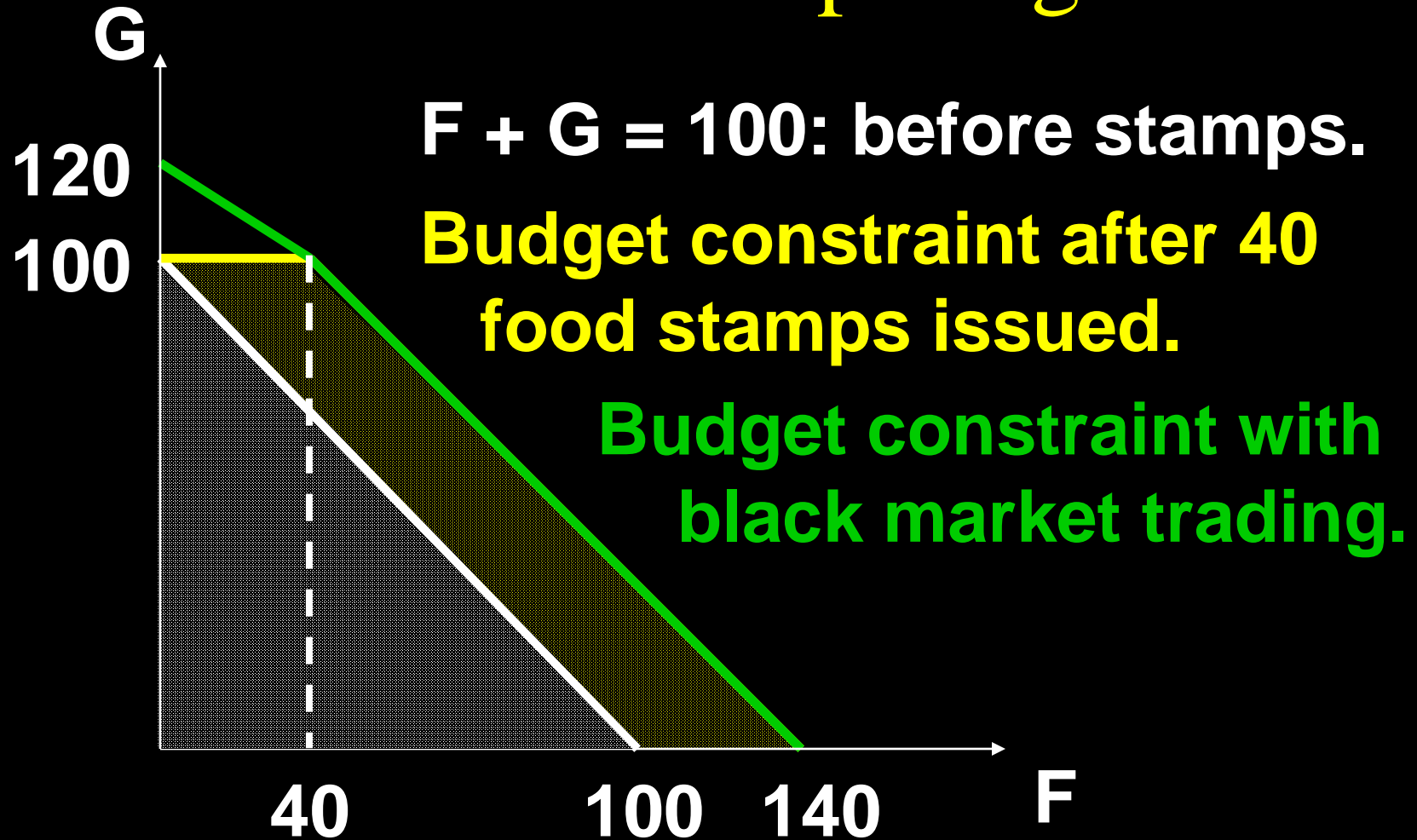
# The Food Stamp Program



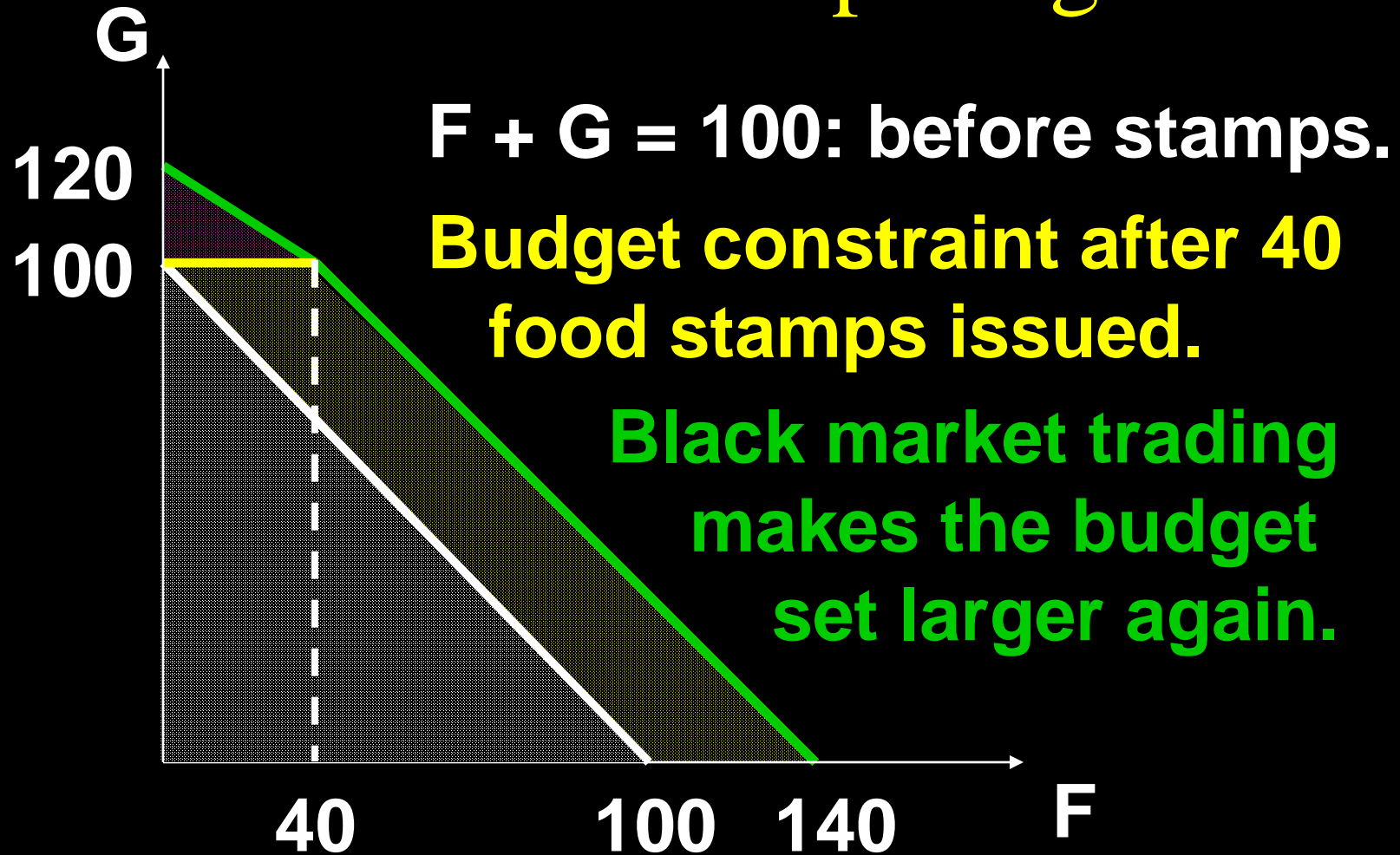
# The Food Stamp Program

**What if food stamps can be traded on a black market for \$0.50 each?**

# The Food Stamp Program



# The Food Stamp Program



# Budget Constraints - Relative Prices

**“Numeraire” means “unit of account”.**

**Suppose prices and income are measured in dollars. Say  $p_1 = \$2$ ,  $p_2 = \$3$ ,  $m = \$12$ . Then the constraint is**

$$2x_1 + 3x_2 = 12.$$

# Budget Constraints - Relative Prices

If prices and income are measured in cents, then  $p_1=200$ ,  $p_2=300$ ,  $m=1200$  and the constraint is

$$200x_1 + 300x_2 = 1200,$$

the same as

$$2x_1 + 3x_2 = 12.$$

**Changing the numeraire changes neither the budget constraint nor the budget set.**

# Budget Constraints - Relative Prices

The constraint for  $p_1=2$ ,  $p_2=3$ ,  $m=12$

$$2x_1 + 3x_2 = 12$$

is also  $1 \cdot x_1 + (3/2)x_2 = 6$ ,

the constraint for  $p_1=1$ ,  $p_2=3/2$ ,  $m=6$ .

Setting  $p_1=1$  makes commodity 1 the **numeraire** and defines all prices **relative to**  $p_1$ ; e.g.  $3/2$  is the price of commodity 2 relative to the price of commodity 1.

# Budget Constraints - Relative Prices

**Any** commodity can be chosen as the numeraire without changing the budget set or the budget constraint.



# Budget Constraints - Relative Prices

$p_1=2$ ,  $p_2=3$  and  $p_3=6 \Rightarrow$

price of commodity 2 relative to commodity 1 is  $3/2$ ,

price of commodity 3 relative to commodity 1 is 3.

Relative prices are the **rates of exchange** of commodities 2 and 3 for units of commodity 1.

# Shapes of Budget Constraints

**Q: What makes a budget constraint a straight line?**

**A: A straight line has a constant slope and the constraint is**

$$p_1x_1 + \dots + p_nx_n = m$$

**so if prices are constants then a constraint is a straight line.**

# Shapes of Budget Constraints

**But what if prices are not constants?**

***E.g.* bulk buying discounts, or price penalties for buying “too much”.**

**Then constraints will be curved.**



# Shapes of Budget Constraints - Quantity Discounts

**Suppose  $p_2$  is constant at \$1 but that  $p_1 = \$2$  for  $0 \leq x_1 \leq 20$  and  $p_1 = \$1$  for  $x_1 > 20$ .**

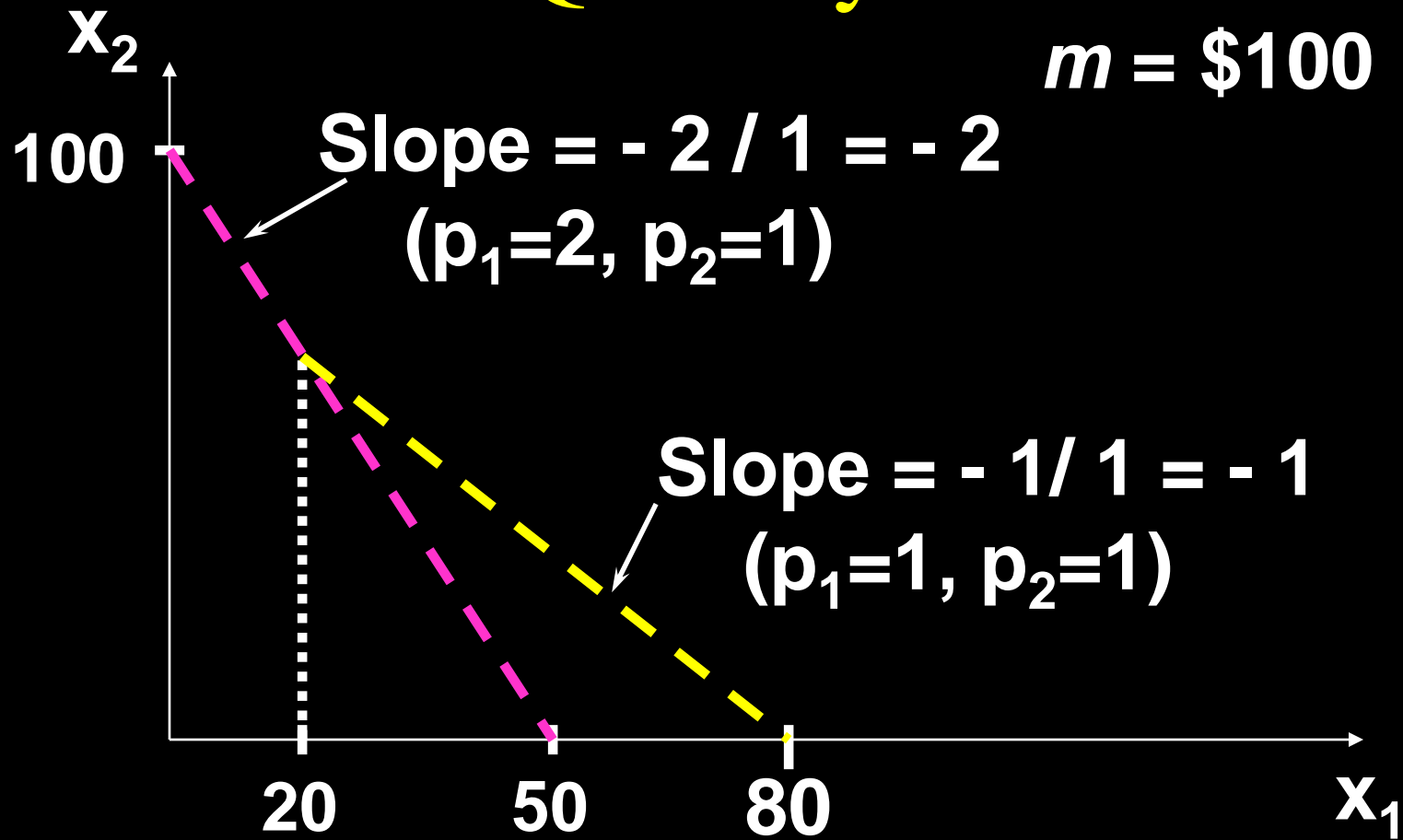
# Shapes of Budget Constraints - Quantity Discounts

Suppose  $p_2$  is constant at \$1 but that  $p_1 = \$2$  for  $0 \leq x_1 \leq 20$  and  $p_1 = \$1$  for  $x_1 > 20$ . Then the constraint's slope is

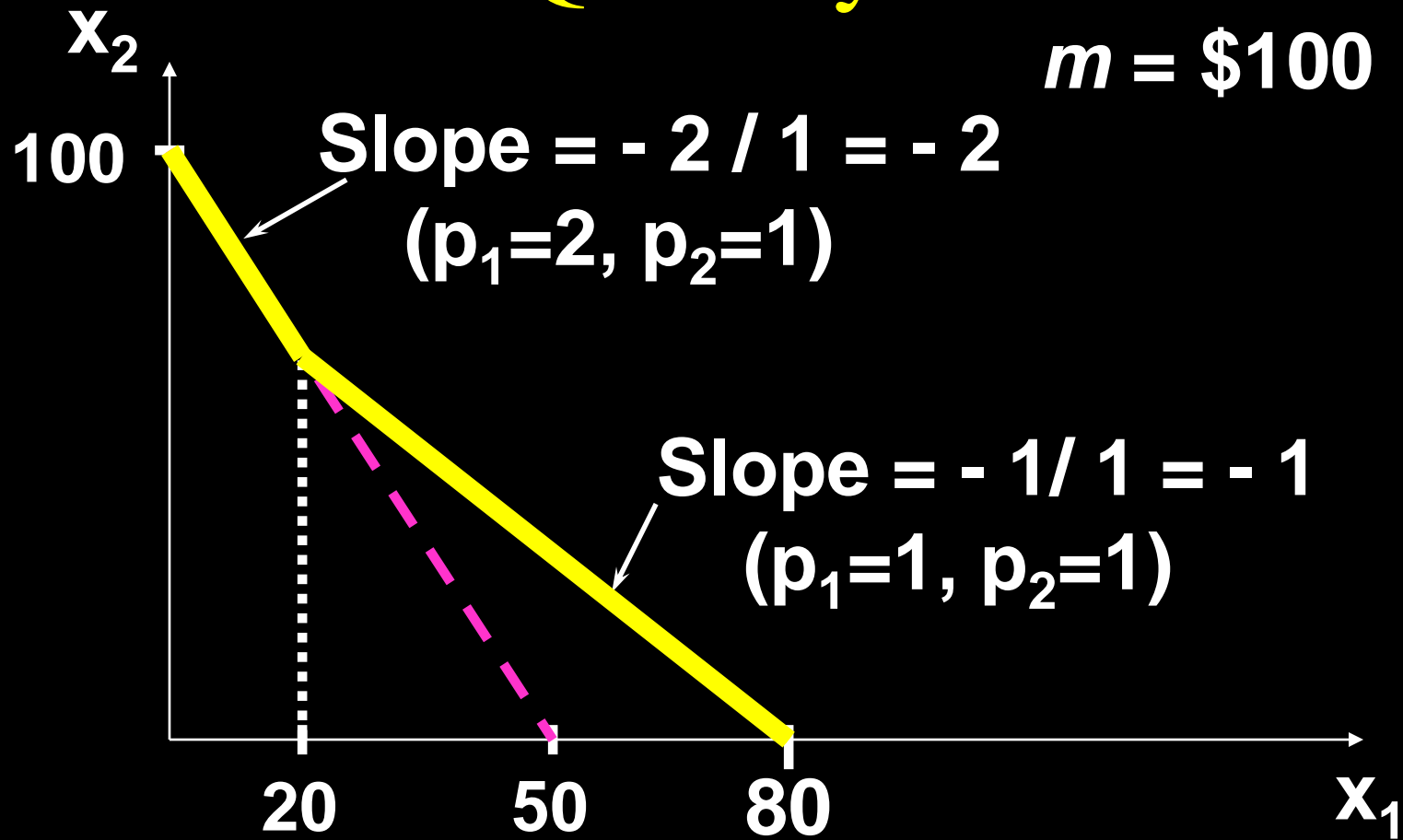
$$-p_1/p_2 = \begin{cases} -2, & \text{for } 0 \leq x_1 \leq 20 \\ -1, & \text{for } x_1 > 20 \end{cases}$$

and the constraint is

# Shapes of Budget Constraints with a Quantity Discount

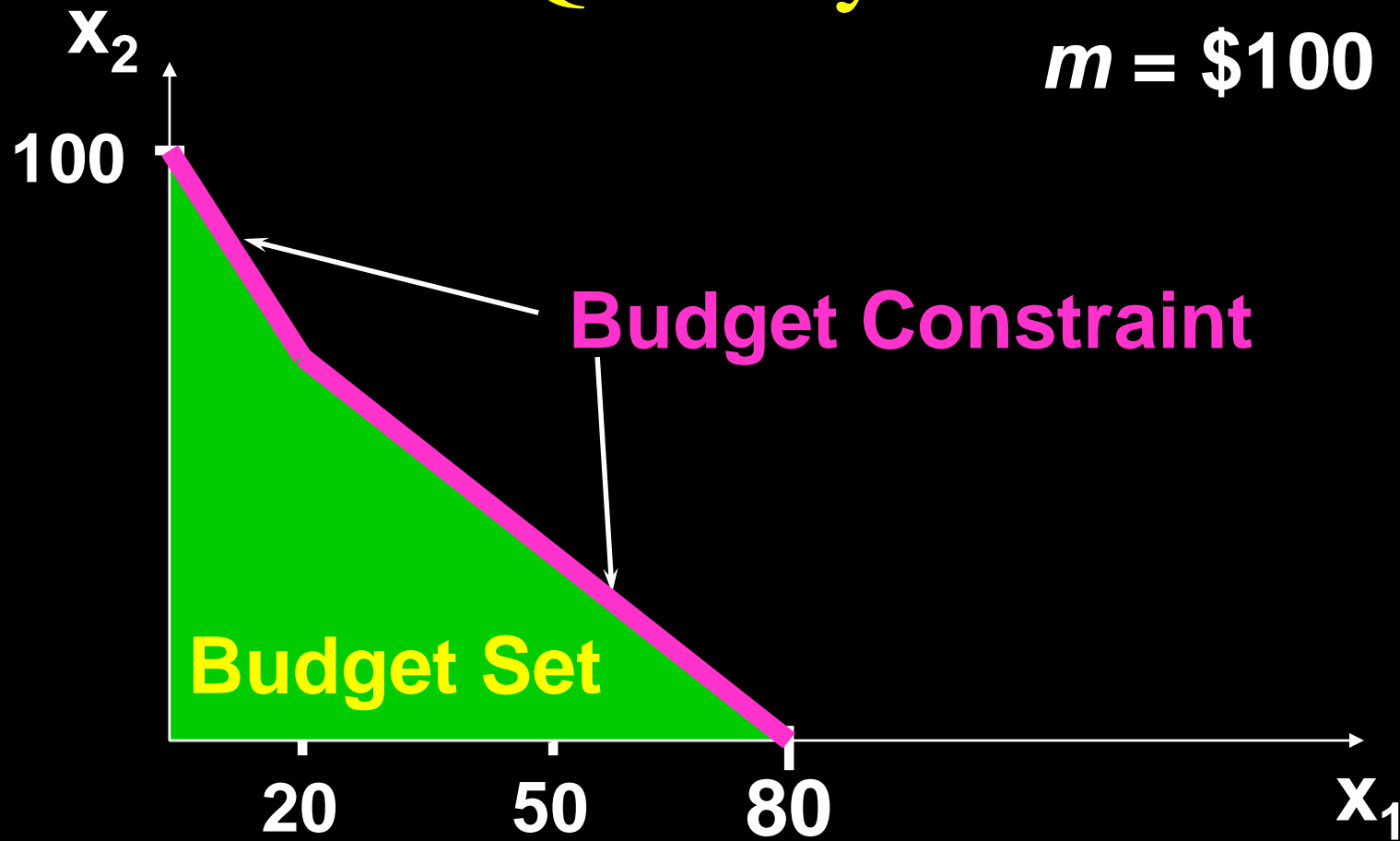


# Shapes of Budget Constraints with a Quantity Discount



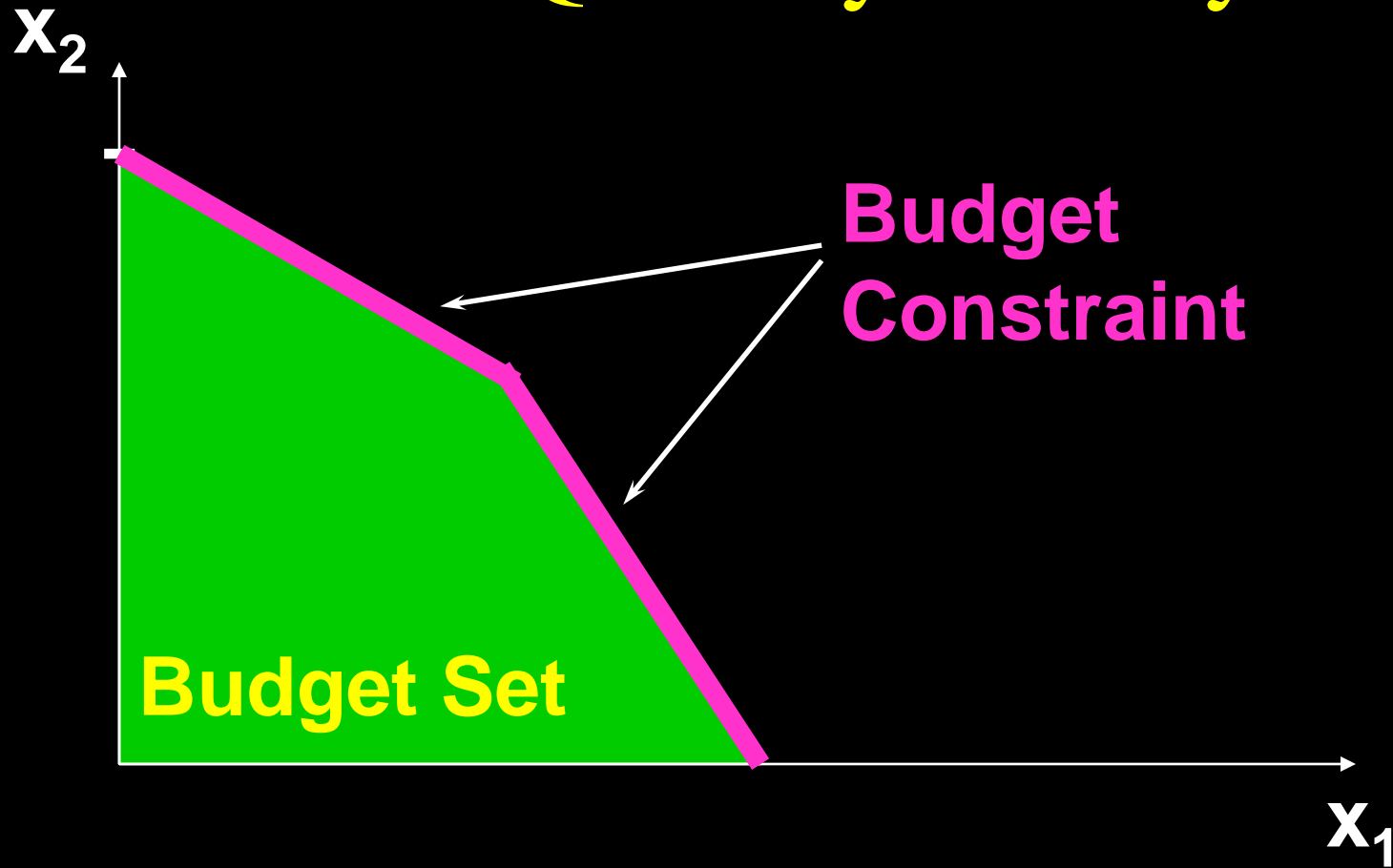
# Shapes of Budget Constraints with a Quantity Discount

$m = \$100$





# Shapes of Budget Constraints with a Quantity Penalty



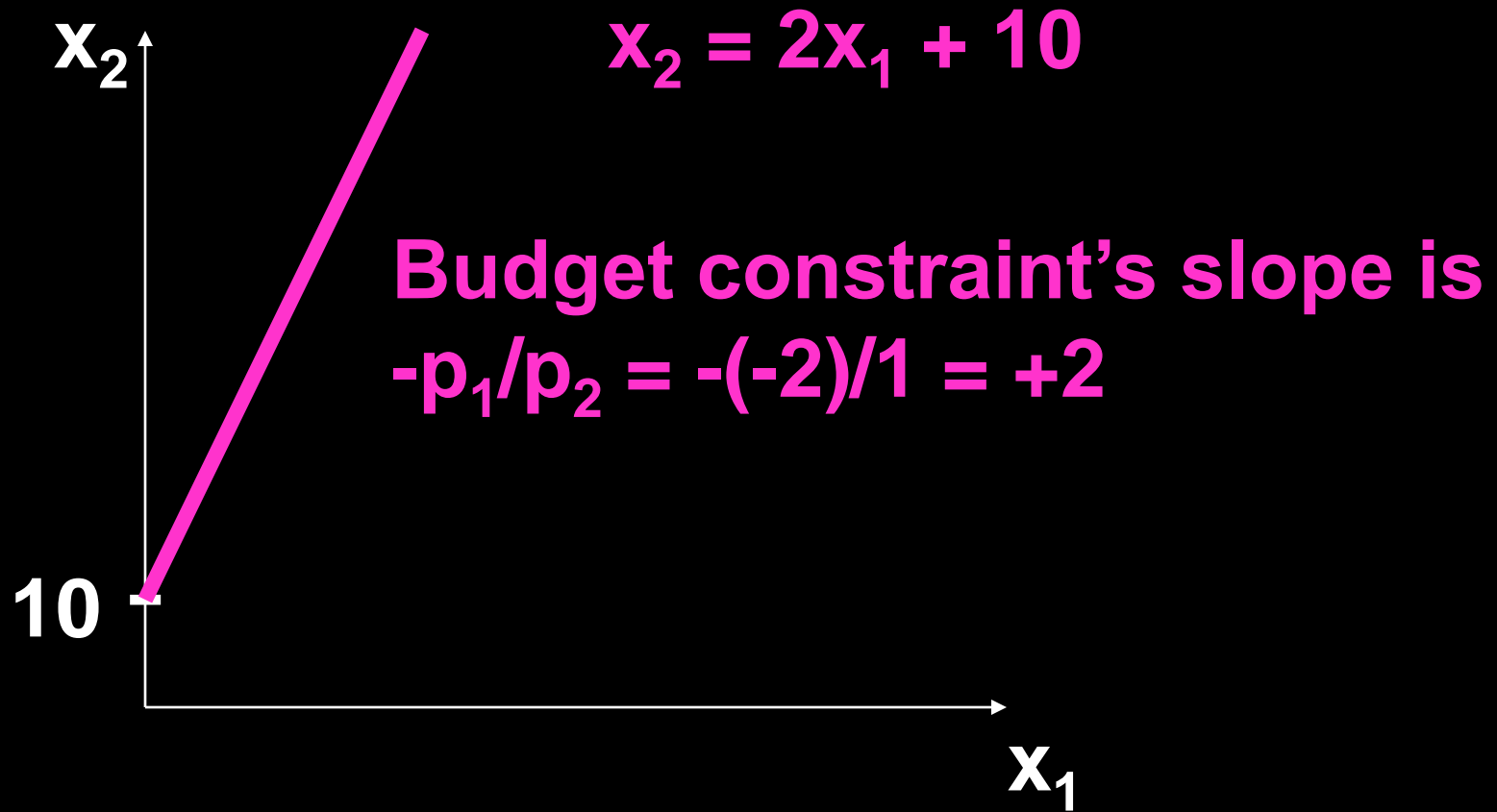
# Shapes of Budget Constraints - One Price Negative

**Commodity 1 is stinky garbage. You are paid \$2 per unit to accept it; *i.e.*  $p_1 = -\$2$ .  $p_2 = \$1$ . Income, other than from accepting commodity 1, is  $m = \$10$ .**

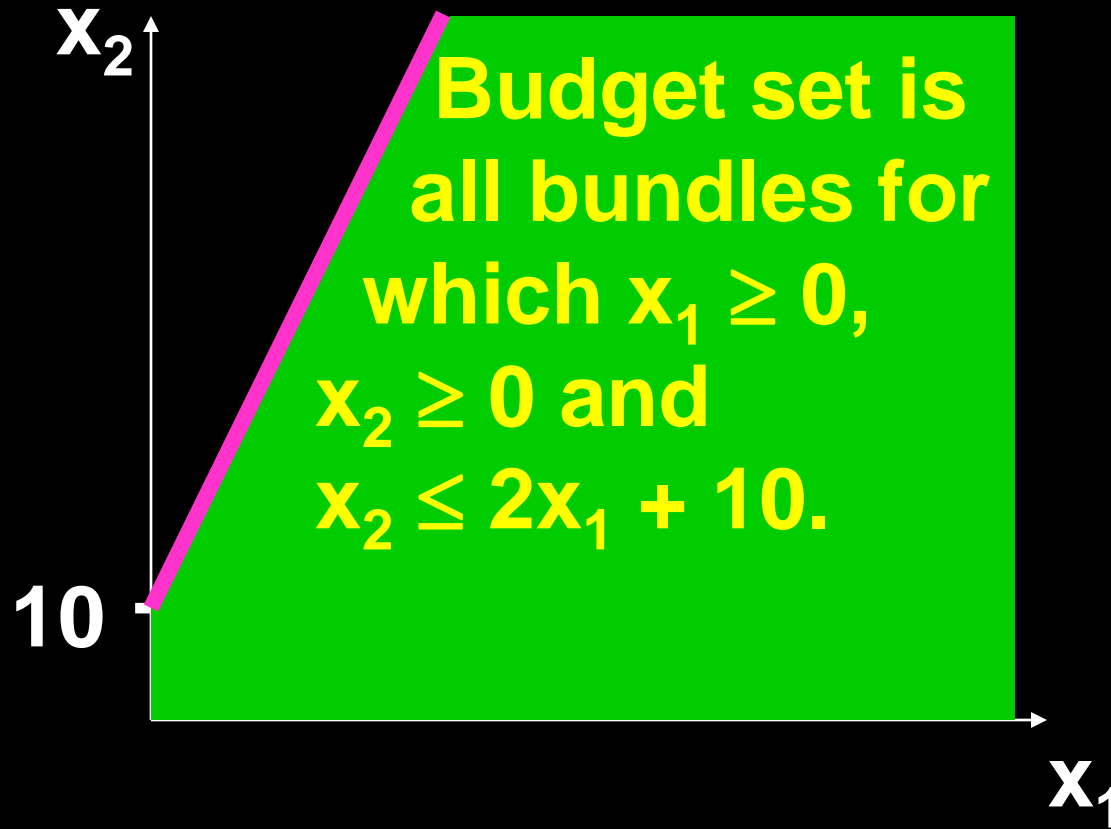
**Then the constraint is**

$$\text{- } 2x_1 + x_2 = 10 \quad \text{or} \quad x_2 = 2x_1 + 10.$$

# Shapes of Budget Constraints - One Price Negative



# Shapes of Budget Constraints - One Price Negative



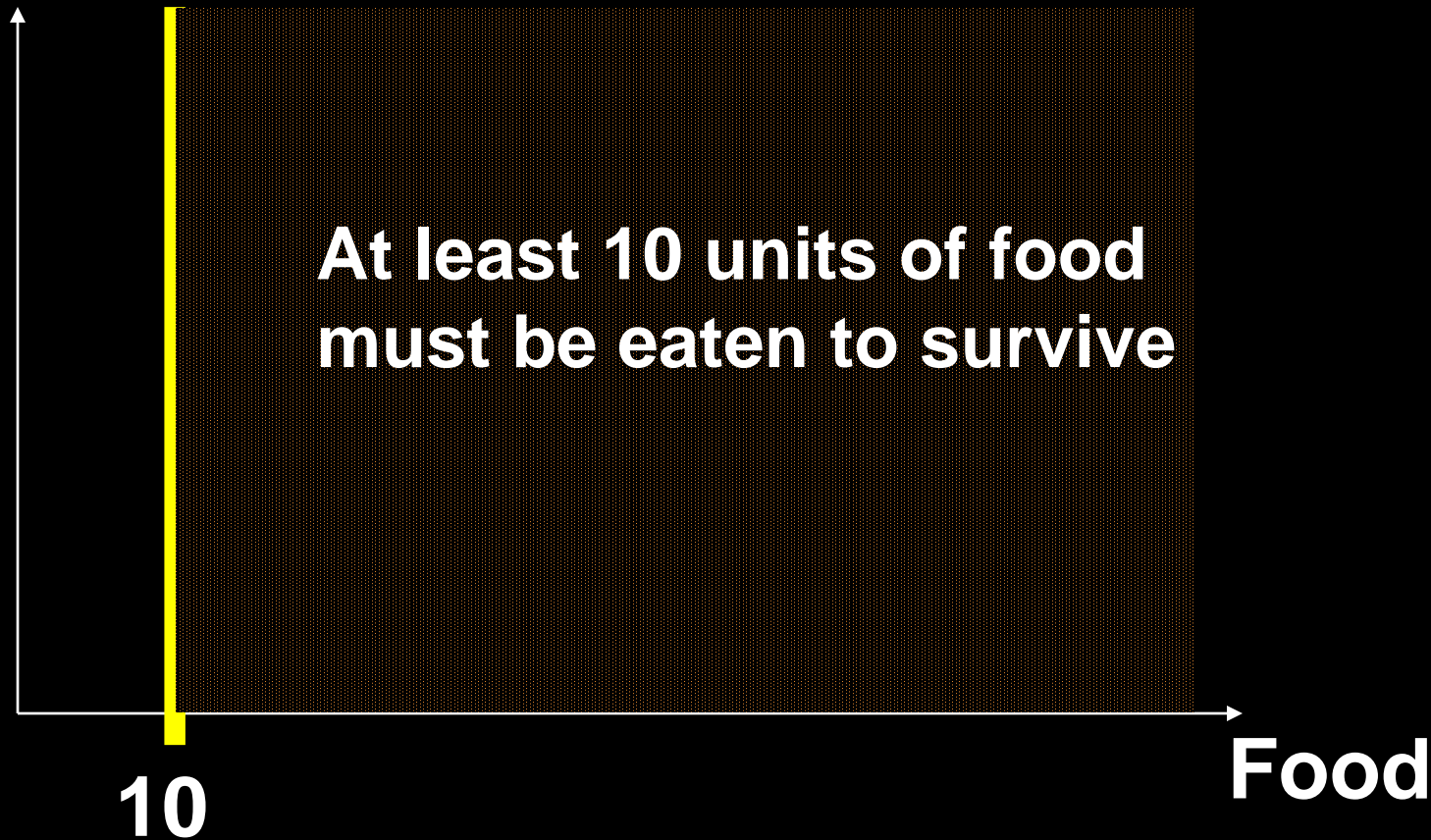
# More General Choice Sets

**Choices are usually constrained by more than a budget; e.g. time constraints and other resources constraints.**

**A bundle is available only if it meets **every** constraint.**

# More General Choice Sets

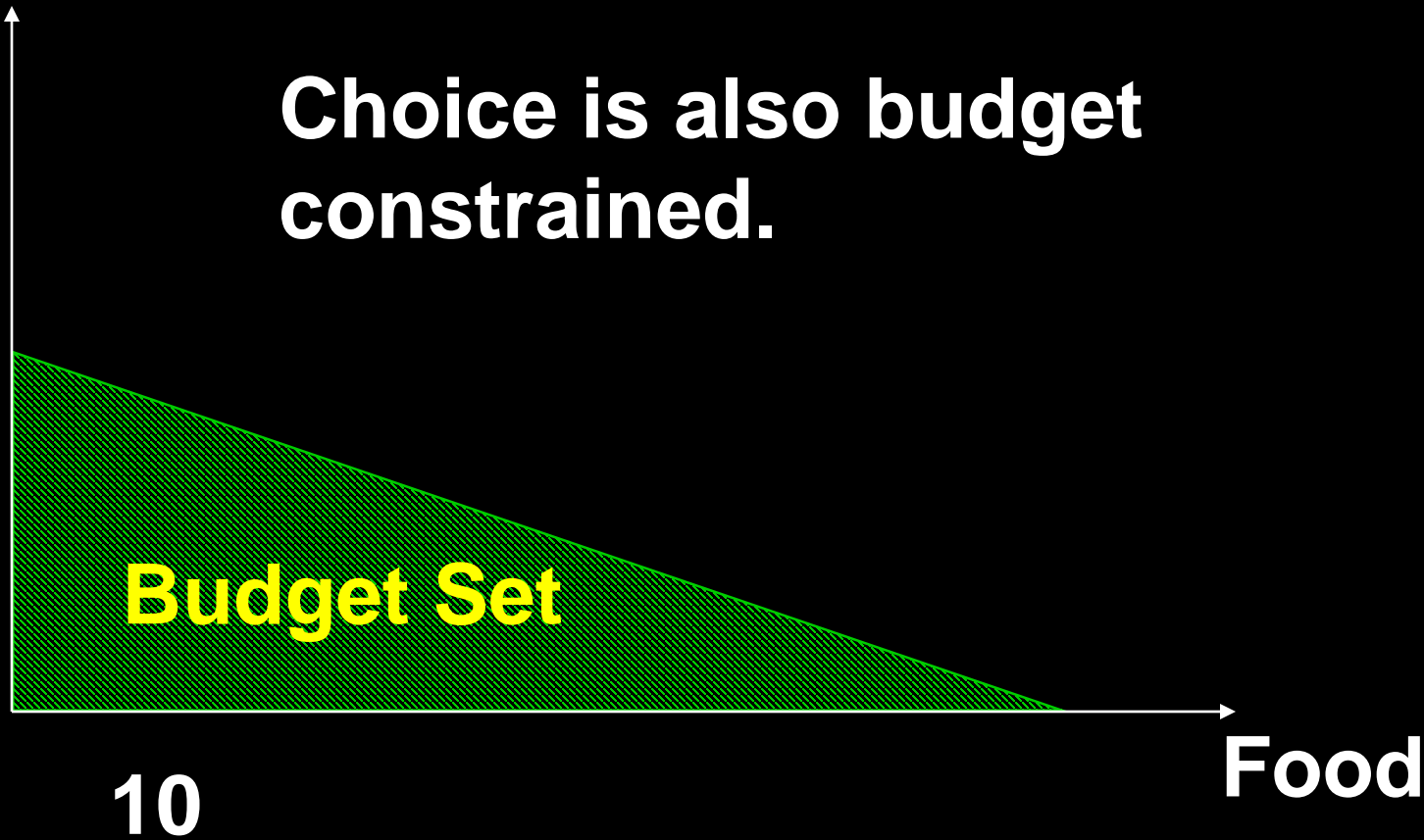
Other Stuff



# More General Choice Sets

Other Stuff

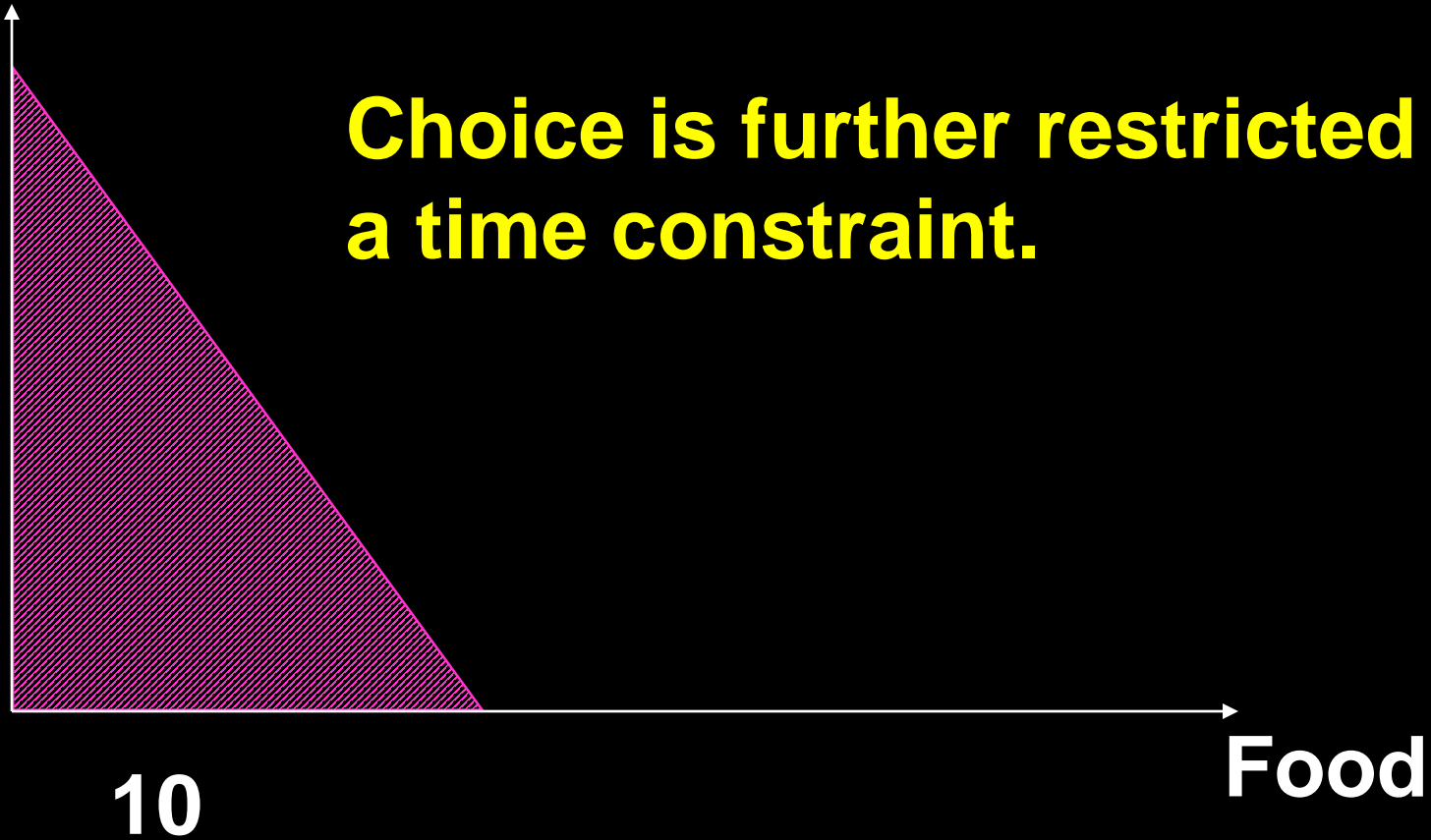
Choice is also budget constrained.



# More General Choice Sets

Other Stuff

**Choice is further restricted by a time constraint.**



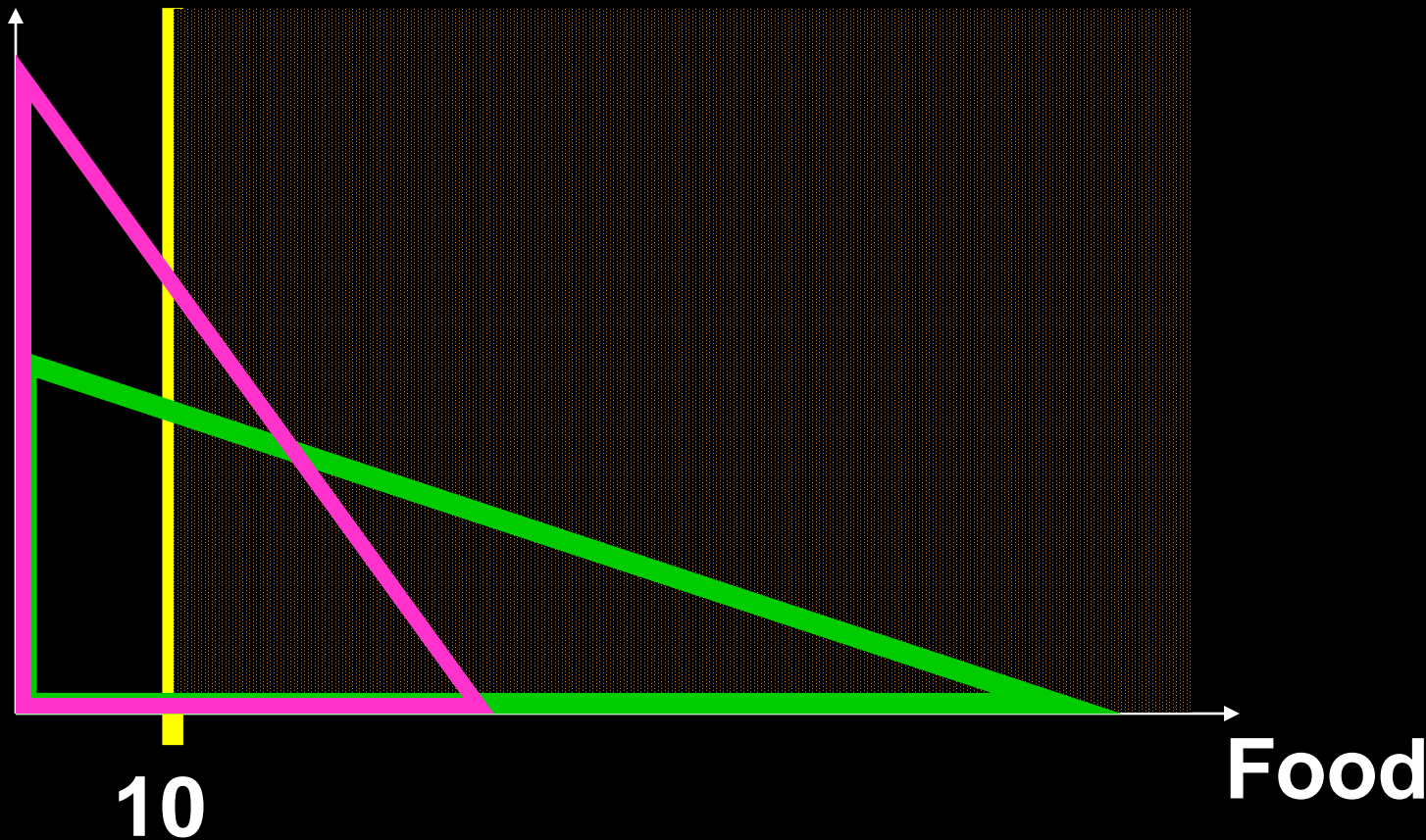


# More General Choice Sets

**So what is the choice set?**

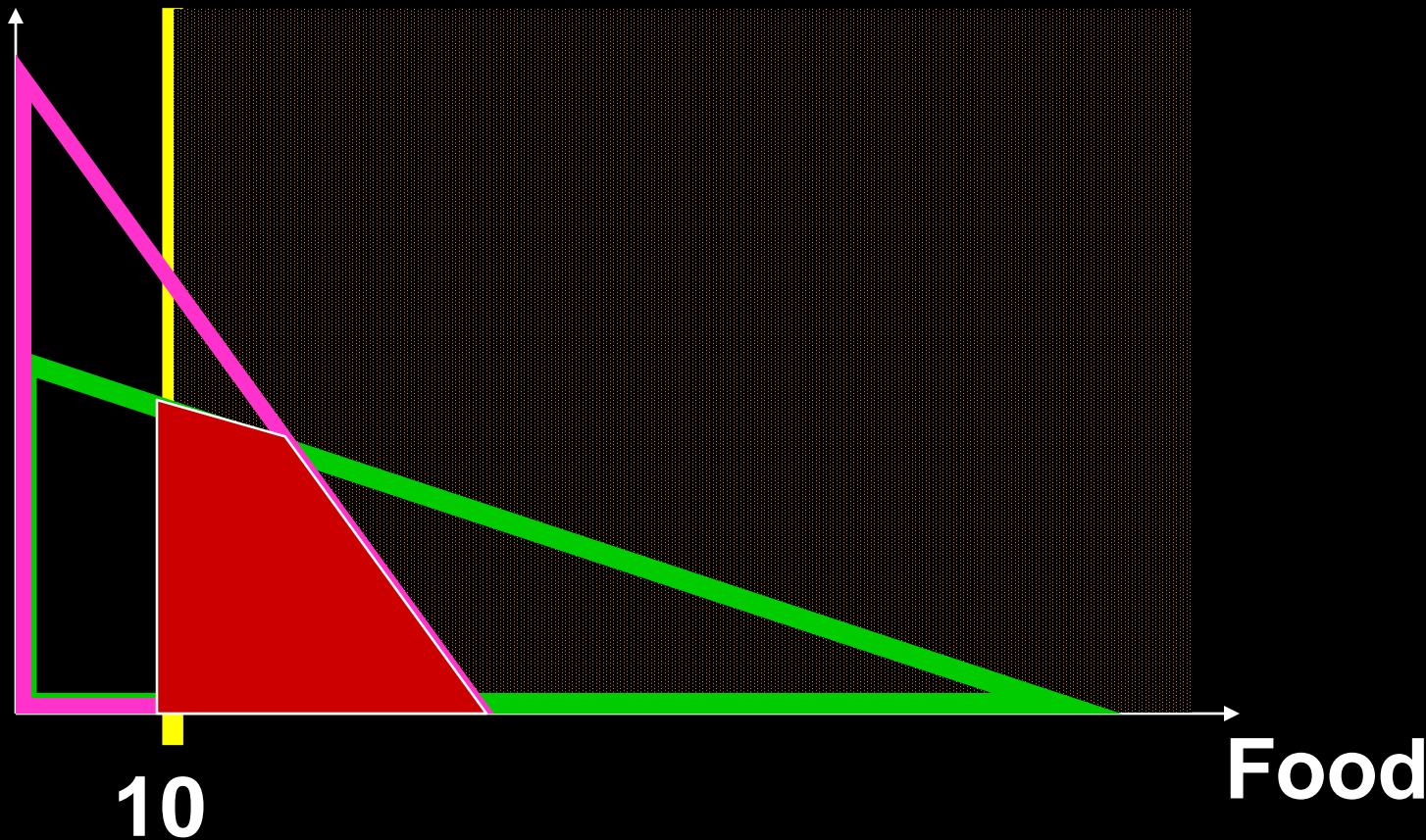
# More General Choice Sets

Other Stuff



# More General Choice Sets

Other Stuff



# More General Choice Sets

Other Stuff

The choice set is the intersection of all of the constraint sets.

