中级微观经济学

成本最小化与成本曲线

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一、判断题

1. ×

$f(x,y)=x+2y$，$x>0, y>0$　$P_x \times 2$，$P_y \times 3$　生产成本

使用当前的方式 说明 $2P_x=P_y$．设 $P_x=t$，$P_y=2t$ $\Rightarrow P_x'=2t$，$P_y'=6t$

修改后，大家都会购买 x，成本无法上升一倍，错误

2. ×

可以是平的

3. √

4. √

$C=10+3y$　边际成本为3，平均成本为 $3+\frac{10}{y}>3$

5. ×

应该是总可变成本

二、选择题

1. C

$$\begin{cases} c_1=2y_1^2+90 \\ c_2=6y_2^2+40 \end{cases}$$

$C=2y_1^2+90+6y_2^2+40$，s.t. $y_1+y_2=32$

$C=2y_1^2+90+6(32-y_1)^2+40$

即成本多出32单位

$$\frac{\partial c}{\partial y_1}=4y_1+6(2y_1-64)=0 \qquad 16y_1=64\times 6 \qquad y_1=24$$

$\therefore y_2=32-y_1=8$，选C．

2. B

$f(S,L)=S^{\frac{1}{2}}L^{\frac{1}{2}}$

A厂: S与L价格为7

B厂: S为8, L为6

使单位成本 min.

单位成本1 $= \dfrac{7S+7L}{S^{\frac{1}{2}}L^{\frac{1}{2}}} = 7\left(\dfrac{S}{L}\right)^{\frac{1}{2}} + 7\left(\dfrac{L}{S}\right)^{\frac{1}{2}} \geq 7$

单位成本2 $= \dfrac{6S+8L}{S^{\frac{1}{2}}L^{\frac{1}{2}}} = 6\left(\dfrac{S}{L}\right)^{\frac{1}{2}} + 8\left(\dfrac{L}{S}\right)^{\frac{1}{2}} \geq 2\sqrt{48}$

∴ 选2厂.

3. C

$ATC\uparrow$, $MC > ATC$.

4. A

$Q = X_1^{0.5}X_2$, $\overline{X_2} = 15$. $w_1 = 75$, $w_2 = 2$. 短期总成本?

$\min\limits_{X_1, X_2} (w_1X_1 + w_2\overline{X_2})$, s.t. $X_1^{0.5}X_2 = Q$.

$\mathcal{L}(X_1, \overline{X_2}, \lambda) = w_1X_1 + w_2X_2 + \lambda(Q - X_1^{0.5}\overline{X_2})$

求导得 $w_1 - \frac{1}{2}\lambda x_1^{-\frac{1}{2}}\overline{x_2} = 0 \Rightarrow \frac{1}{2}\lambda x_1^{-\frac{1}{2}} 15 = 75$

$\Rightarrow x_1 = \left(\frac{10}{\lambda}\right)^{-2} = \frac{\lambda^2}{100}$, $x_2 = 15$ $Q = \frac{\lambda}{10} \times 15 = \frac{3\lambda}{2}$.

$\therefore TC = 75x_1 + 2\times 15 = 75 \times \frac{\lambda^2}{100} + 30 = \frac{3\lambda^2}{4} + 30 = \frac{Q^2}{3} + 30$

$MC = (TC)' = \frac{2Q}{3}$, 选A

三、计算题

$Q = A^{0.25} B^{0.25} C^{0.25}$，价格 $P_A、P_B、P_C$

(1) 总成本函数

成本最小化： $\min\limits_{A,B,C} (P_A A + P_B B + P_C C)$， s.t. $A^{\frac{1}{4}} B^{\frac{1}{4}} C^{\frac{1}{4}} = Q$

$$Z(A,B,C,\lambda) = P_A A + P_B B + P_C C + \lambda(Q - A^{\frac{1}{4}} B^{\frac{1}{4}} C^{\frac{1}{4}})$$

$$\begin{cases} \frac{\partial Z}{\partial A} = 0 \\ \frac{\partial Z}{\partial B} = 0 \\ \frac{\partial Z}{\partial C} = 0 \\ \frac{\partial Z}{\partial \lambda} = 0 \end{cases} \Rightarrow \quad B = \frac{P_A}{P_B} A, \quad C = \frac{P_A}{P_C} A$$

代入得 $Q = A^{\frac{1}{4}} (\frac{P_A}{P_B} A)^{\frac{1}{4}} (\frac{P_A}{P_C} A)^{\frac{1}{4}} = A^{\frac{3}{4}} \frac{P_A^{\frac{1}{2}}}{P_B^{\frac{1}{4}} P_C^{\frac{1}{4}}}$

$$\therefore A = \frac{P_B^{\frac{1}{3}} P_C^{\frac{1}{3}} Q^{\frac{4}{3}}}{P_A^{\frac{2}{3}}}$$

$\therefore TC = 3P_A A = 3P_A^{\frac{1}{3}} P_B^{\frac{1}{3}} P_C^{\frac{1}{3}} Q^{\frac{4}{3}}$

$AC = \frac{TC}{Q} = 3P_A^{\frac{1}{3}} P_B^{\frac{1}{3}} P_C^{\frac{1}{3}} Q^{\frac{1}{3}}$

$MC = (TC)' = 4P_A^{\frac{1}{3}} P_B^{\frac{1}{3}} P_C^{\frac{1}{3}} Q^{\frac{1}{3}}$.

(2) $W(A,B,\bar{C},\lambda) = P_A A + P_B B + P_C C + \lambda(Q - A^{\frac{1}{4}} B^{\frac{1}{4}} C^{\frac{1}{4}})$

求偏导得 $A = \frac{P_B^{\frac{1}{2}} Q^2}{P_A^{\frac{1}{2}} C^{\frac{1}{2}}}$ 代入得 $TC = P_A A + P_B \frac{P_A}{P_B} A + P_C C = 2P_A \frac{P_B^{\frac{1}{2}} Q^2}{P_A^{\frac{1}{2}} C^{\frac{1}{2}}} + P_C C$

$= 2P_A^{\frac{1}{2}} P_B^{\frac{1}{2}} C^{-\frac{1}{2}} Q + P_C C$

$\therefore AC = \frac{TC}{Q} = 2P_A^{\frac{1}{2}} P_B^{\frac{1}{2}} C^{-\frac{1}{2}} + \frac{P_C C}{Q}$

$MC = (TC)' = 2P_A^{\frac{1}{2}} P_B^{\frac{1}{2}} C^{-\frac{1}{2}}$.