# Chapter Fourteen

## Consumer’s Surplus

# Monetary Measures of Gains-to-Trade

- You can buy as much gasoline as you wish at $1 per gallon once you enter the gasoline market.
- Q: What is the most you would pay to enter the market?

# Monetary Measures of Gains-to-Trade

- **A: You would pay up to the dollar value of the gains-to-trade you would enjoy once in the market.**
- **How can such gains-to-trade be measured?**

# Monetary Measures of Gains-to-Trade

- **Three such measures are:**
  - **Consumer's Surplus**
  - **Equivalent Variation, and**
  - **Compensating Variation.**
- **Only in one special circumstance do these three measures coincide.**

# $ Equivalent Utility Gains

- **Suppose gasoline can be bought only in lumps of one gallon.**
- **Use $r_1$ to denote the most a single consumer would pay for a 1st gallon -- call this her reservation price for the 1st gallon.**
- **$r_1$ is the dollar equivalent of the marginal utility of the 1st gallon.**

# $ Equivalent Utility Gains

- Now that she has one gallon, use $r_2$ to denote the most she would pay for a 2nd gallon -- this is her reservation price for the 2nd gallon.
- $r_2$ is the dollar equivalent of the marginal utility of the 2nd gallon.

# $ Equivalent Utility Gains

- **Generally, if she already has n-1 gallons of gasoline then $r_n$ denotes the most she will pay for an nth gallon.**
- **$r_n$ is the dollar equivalent of the marginal utility of the nth gallon.**

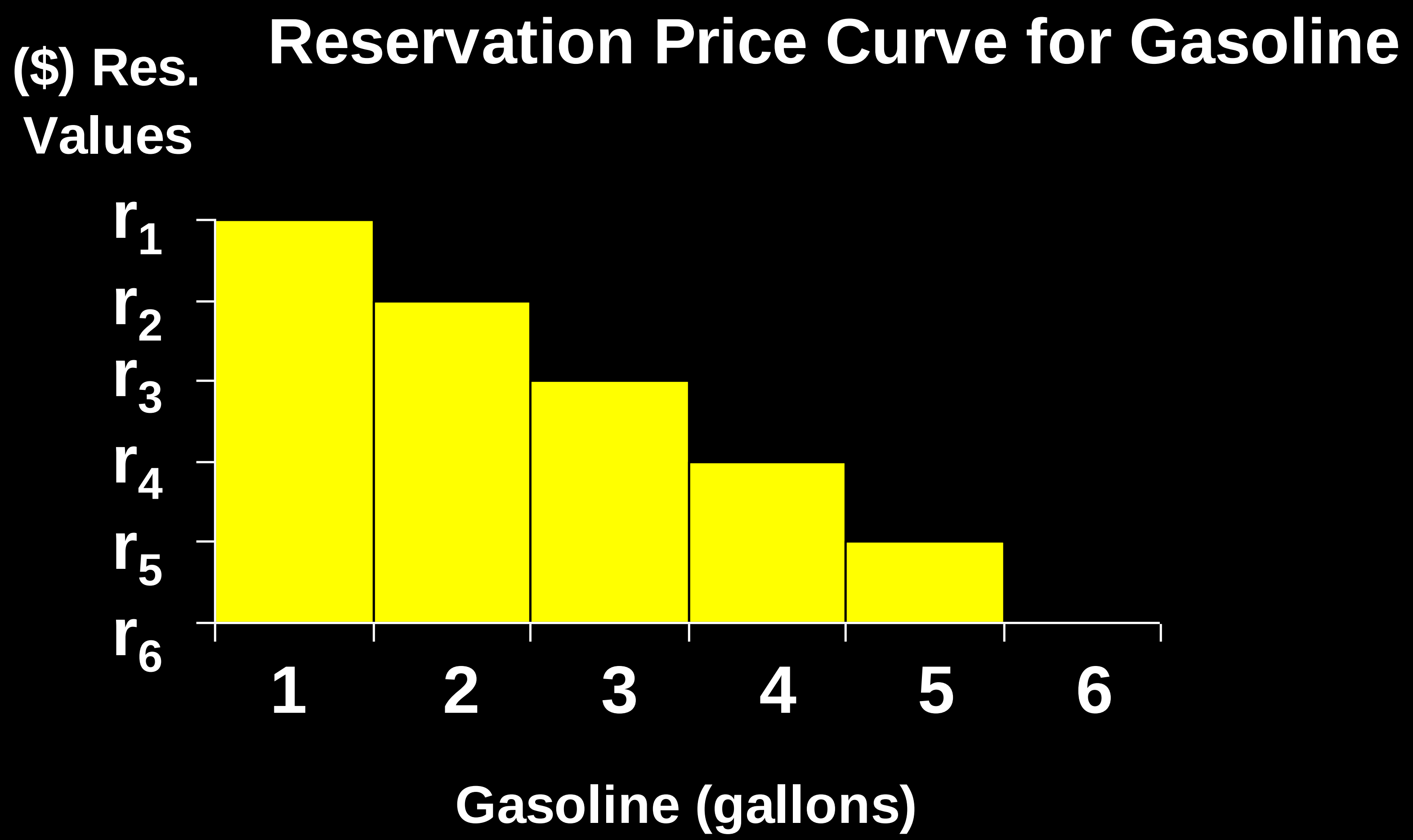
# \$ Equivalent Utility Gains

- $r_1 + \ldots + r_n$ will therefore be the dollar equivalent of the total change to utility from acquiring n gallons of gasoline at a price of \$0.
- So $r_1 + \ldots + r_n - p_G n$ will be the dollar equivalent of the total change to utility from acquiring n gallons of gasoline at a price of $\$p_G$ each.

# $ Equivalent Utility Gains

- **A plot of $r_1$, $r_2$, … , $r_n$, … against n is a reservation-price curve. This is not quite the same as the consumer's demand curve for gasoline.**

# $ Equivalent Utility Gains



# $ Equivalent Utility Gains

- What is the monetary value of our consumer's gain-to-trading in the gasoline market at a price of $\$p_G$?

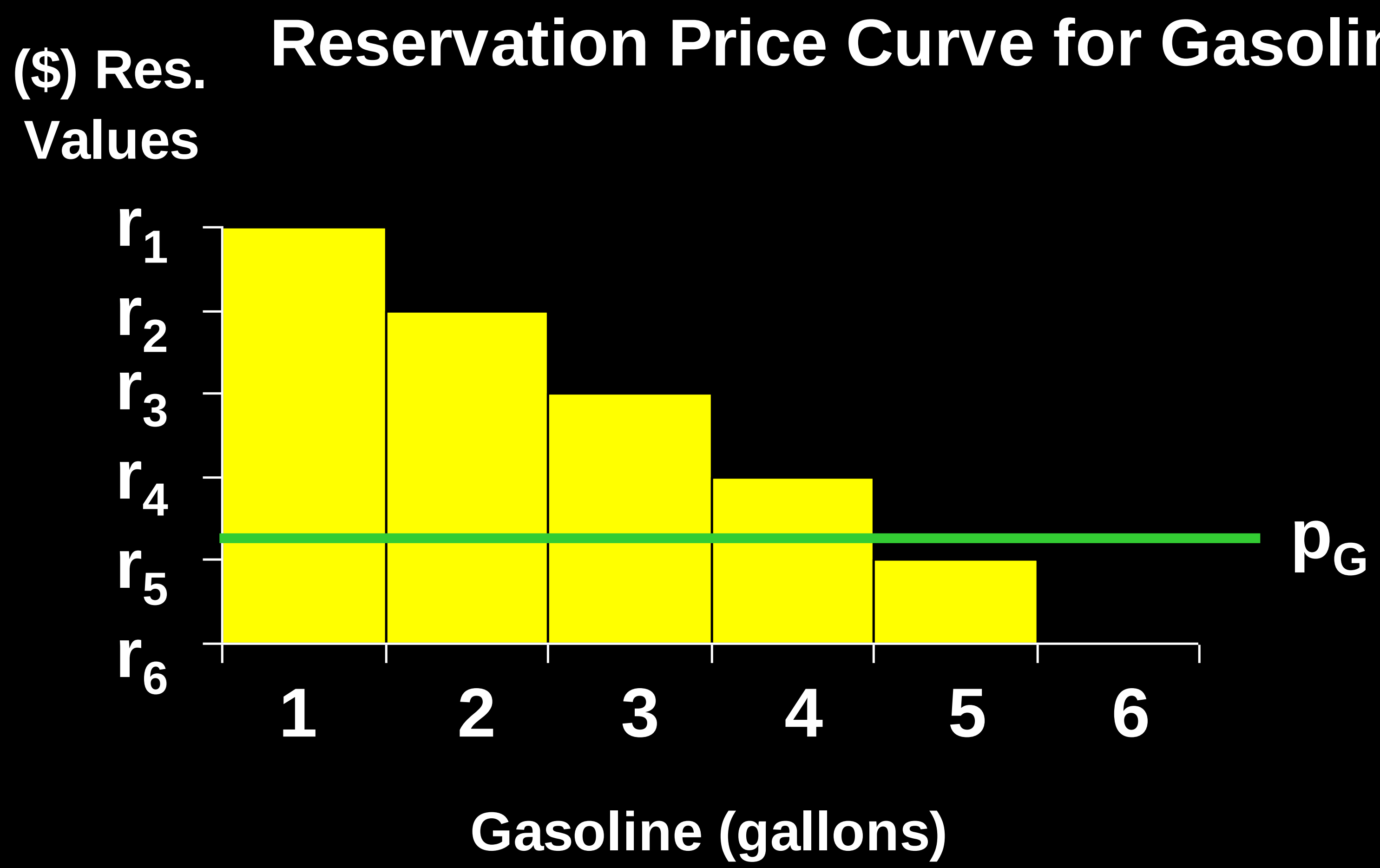
# $ Equivalent Utility Gains

- The dollar equivalent net utility gain for the 1st gallon is $\$(r_1 - p_G)$
- and is $\$(r_2 - p_G)$ for the 2nd gallon,
- and so on, so the dollar value of the gain-to-trade is

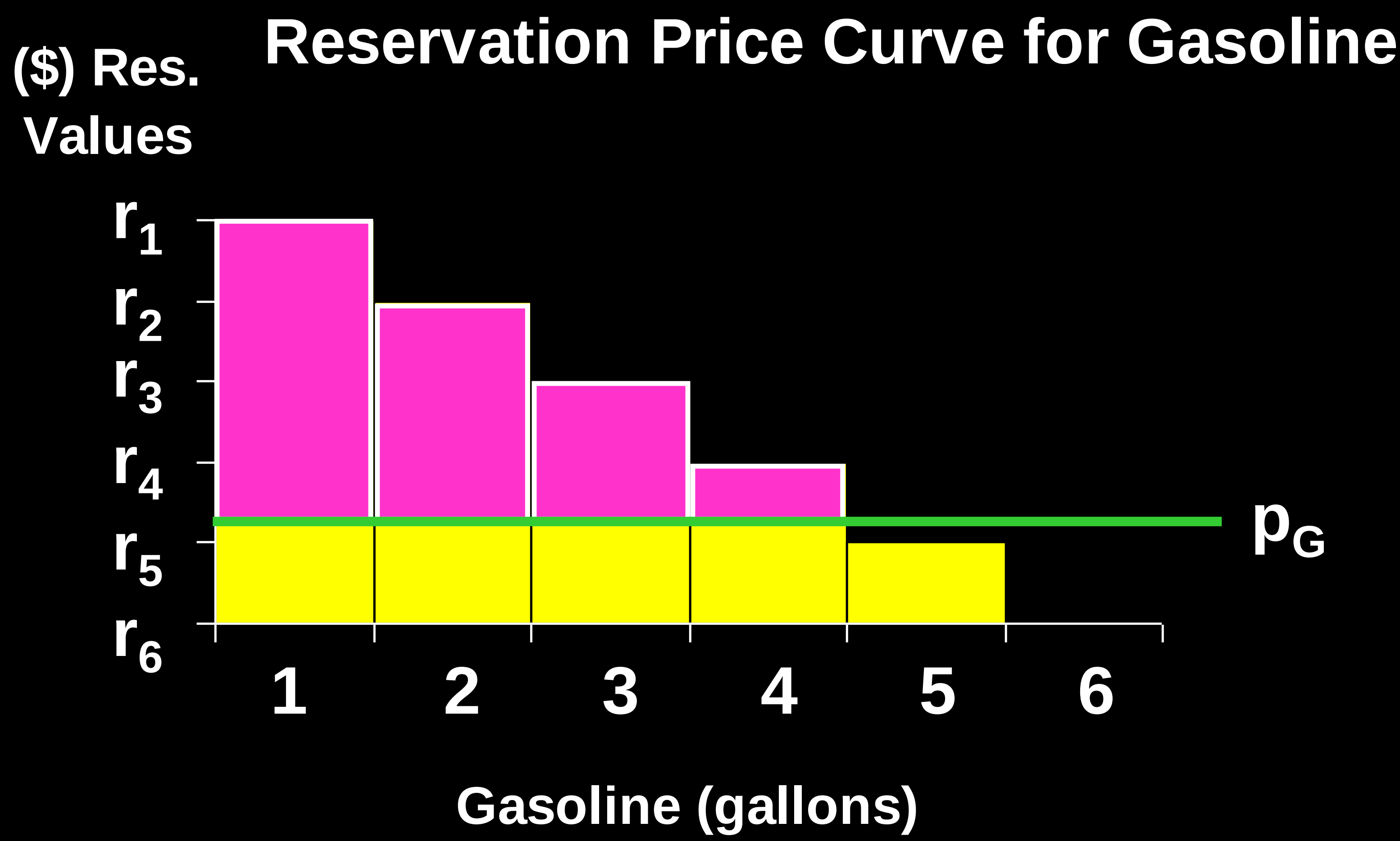
$$\$(r_1 - p_G) + \$(r_2 - p_G) + \ldots$$

for as long as $r_n - p_G > 0$.

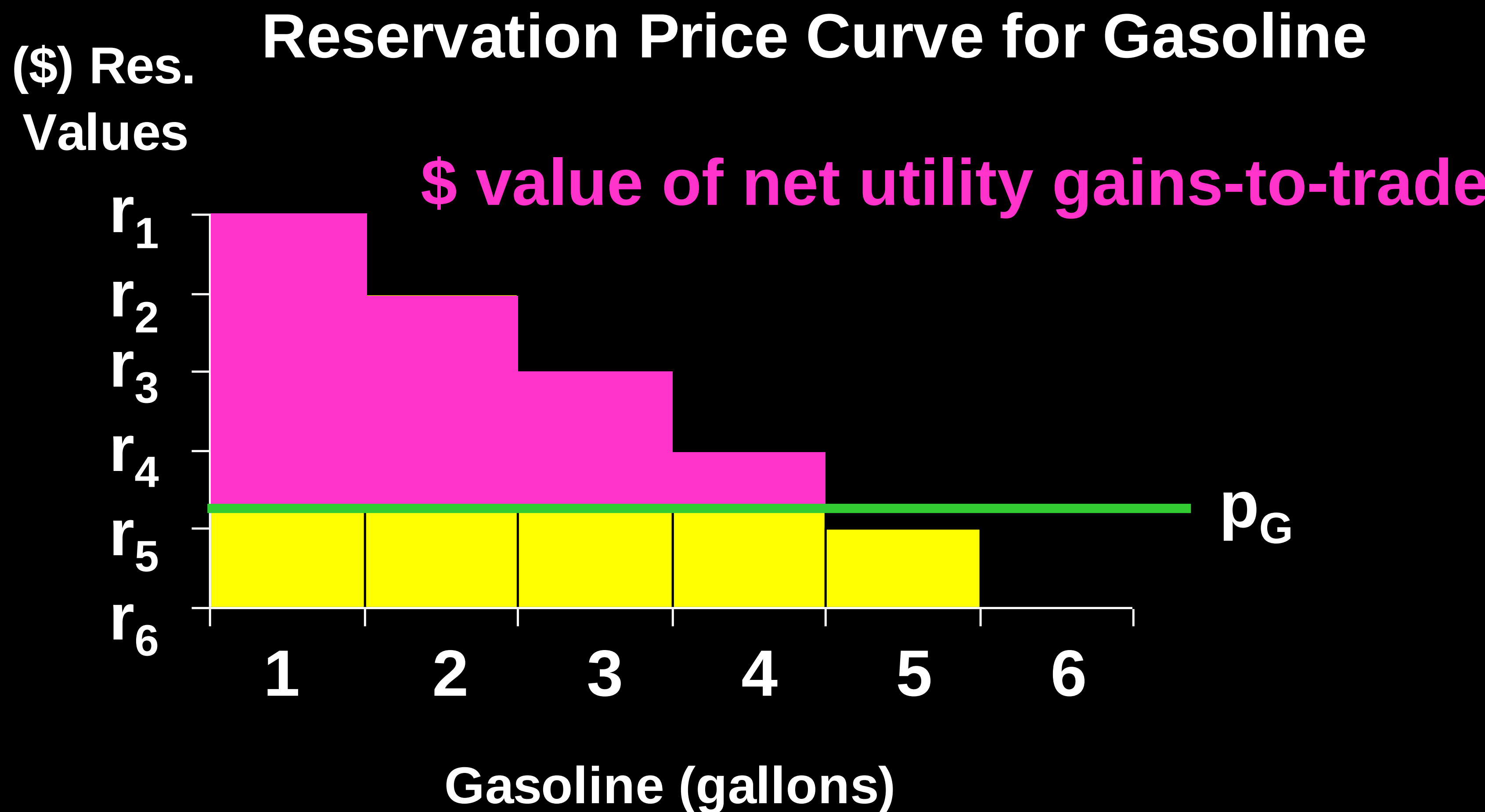
# $ Equivalent Utility Gains



# $ Equivalent Utility Gains



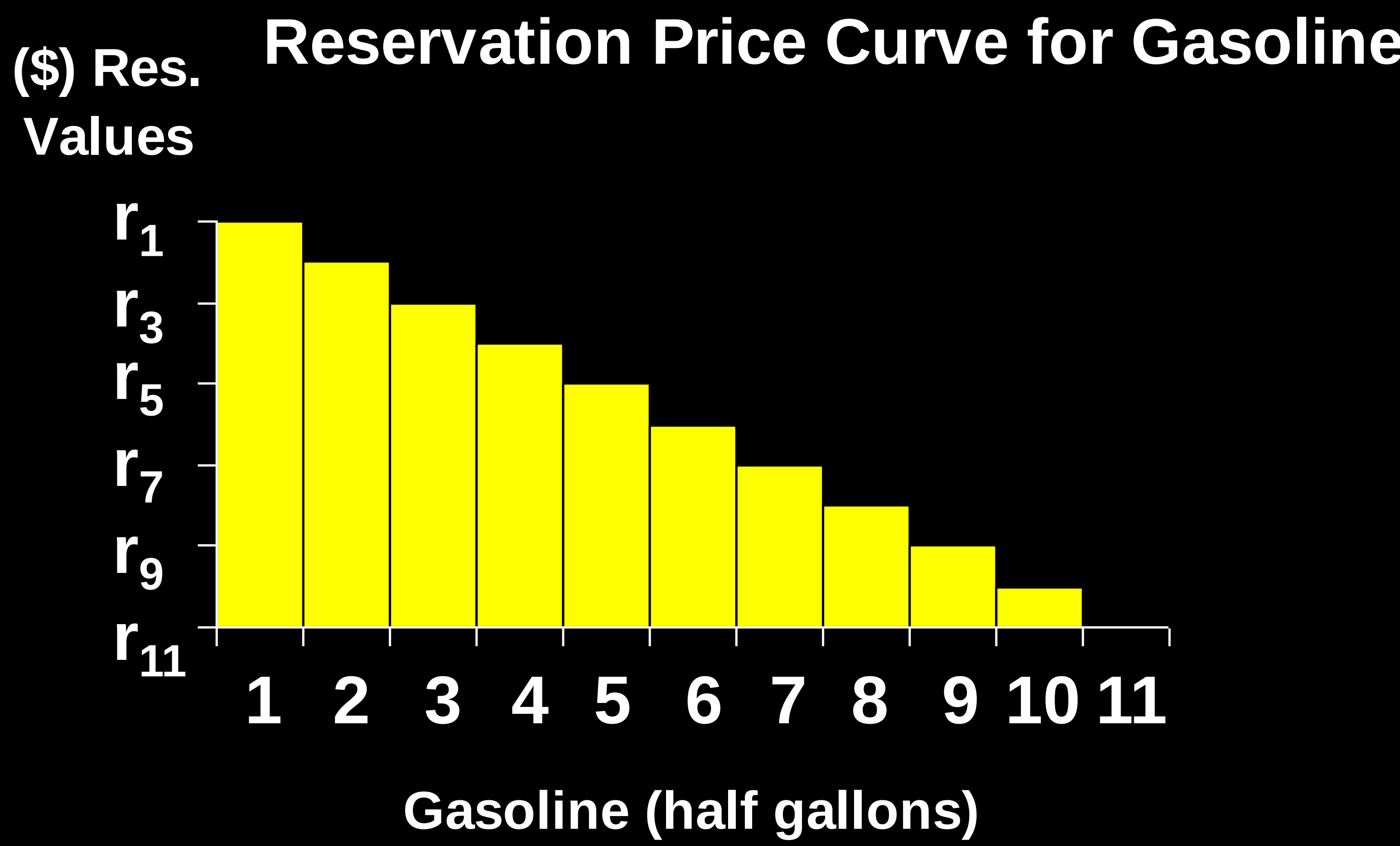
# $ Equivalent Utility Gains



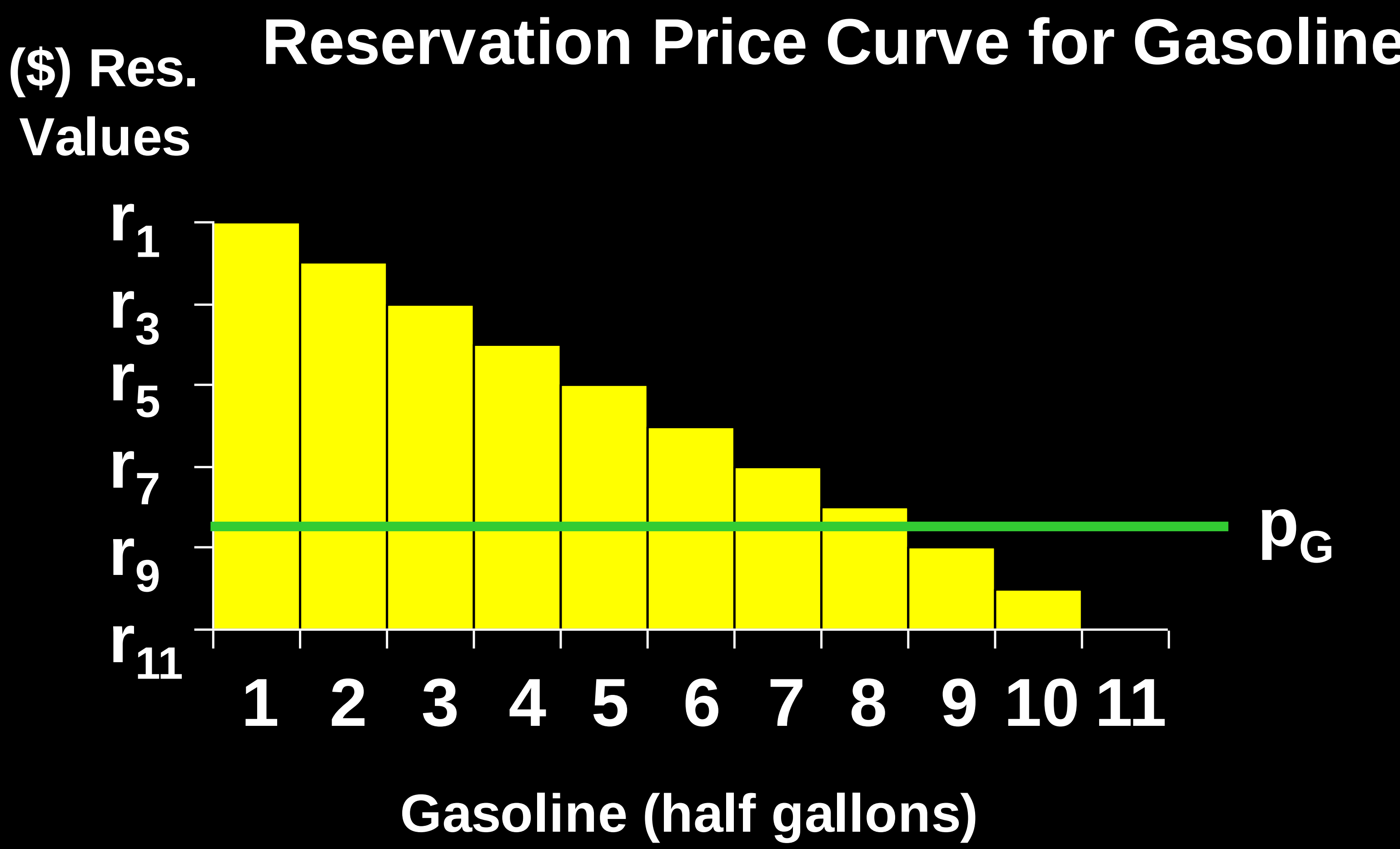
# $ Equivalent Utility Gains

- Now suppose that gasoline is sold in half-gallon units.
- $r_1$, $r_2$, … , $r_n$, … denote the consumer's reservation prices for successive half-gallons of gasoline.
- Our consumer's new reservation price curve is

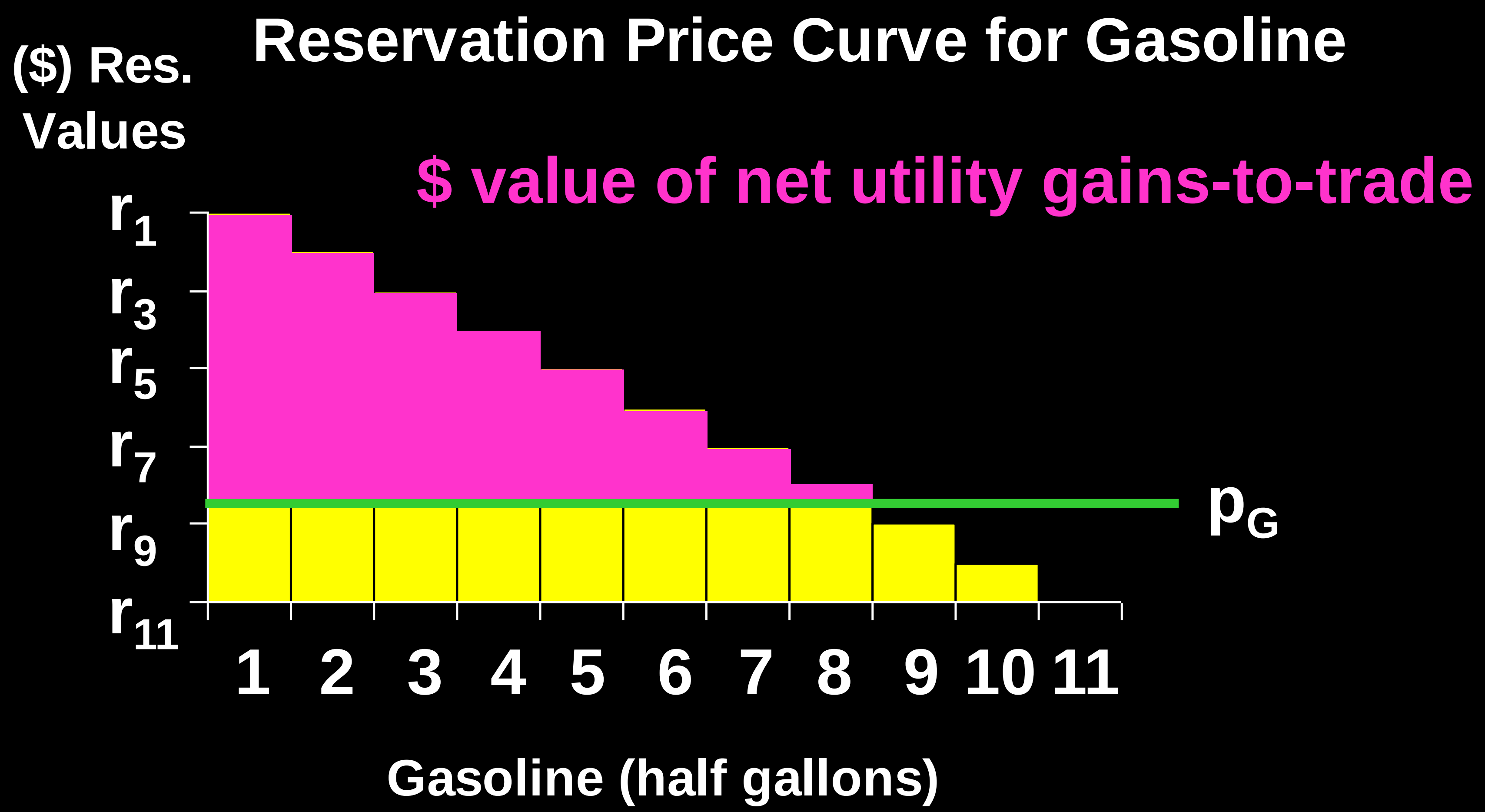
# $ Equivalent Utility Gains



# $ Equivalent Utility Gains



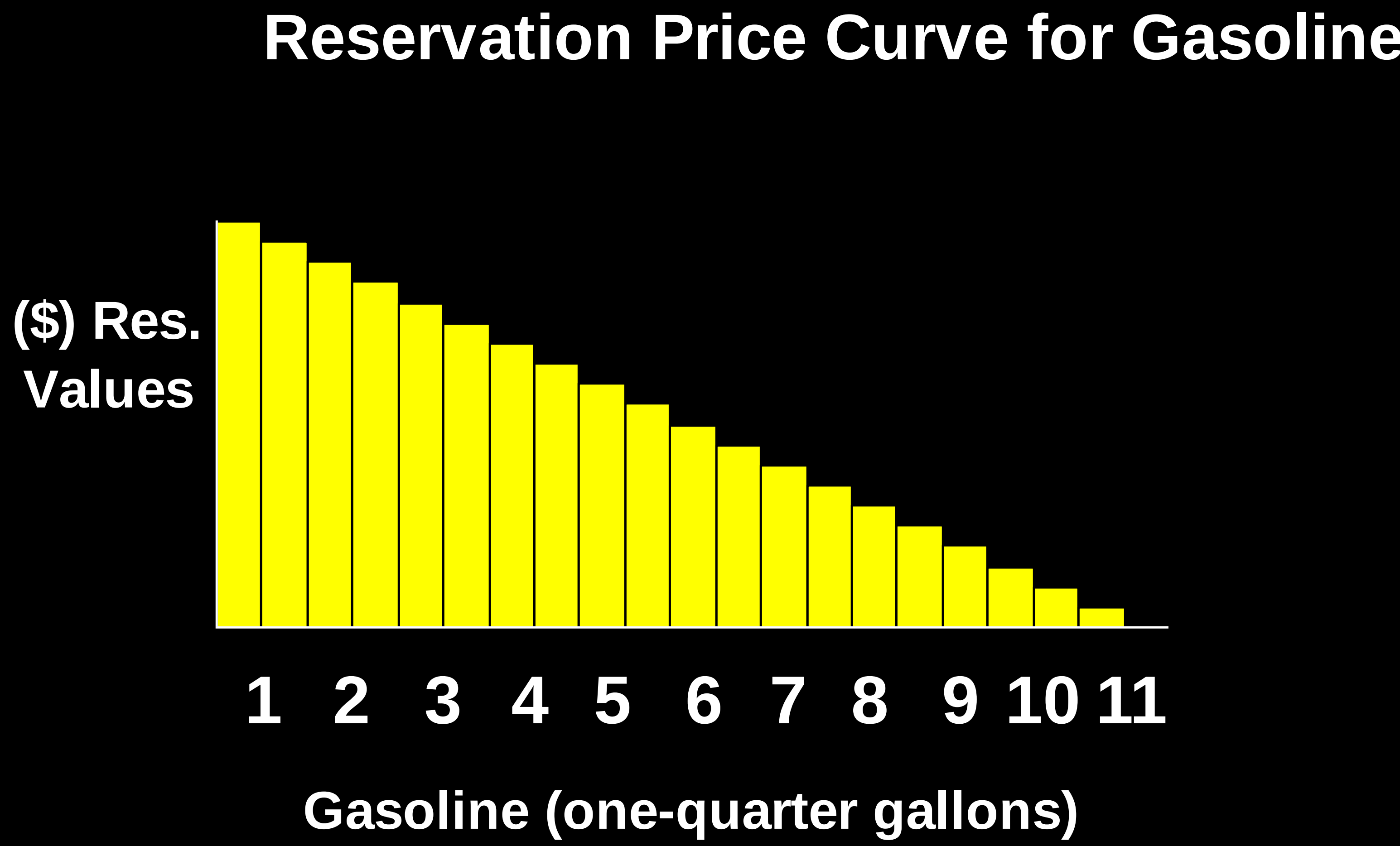
# $ Equivalent Utility Gains



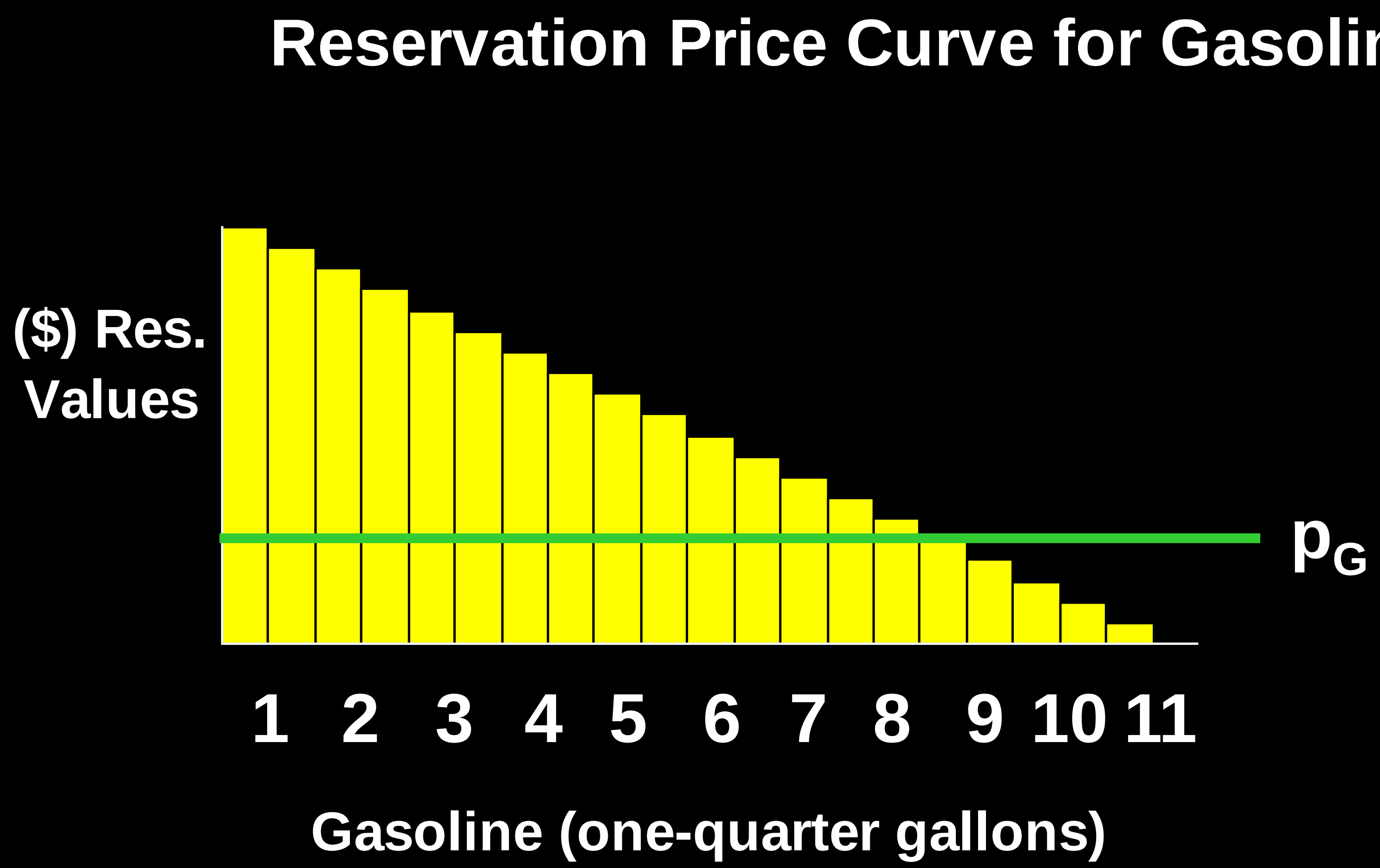
# $ Equivalent Utility Gains

- **And if gasoline is available in one-quarter gallon units ...**

# $ Equivalent Utility Gains

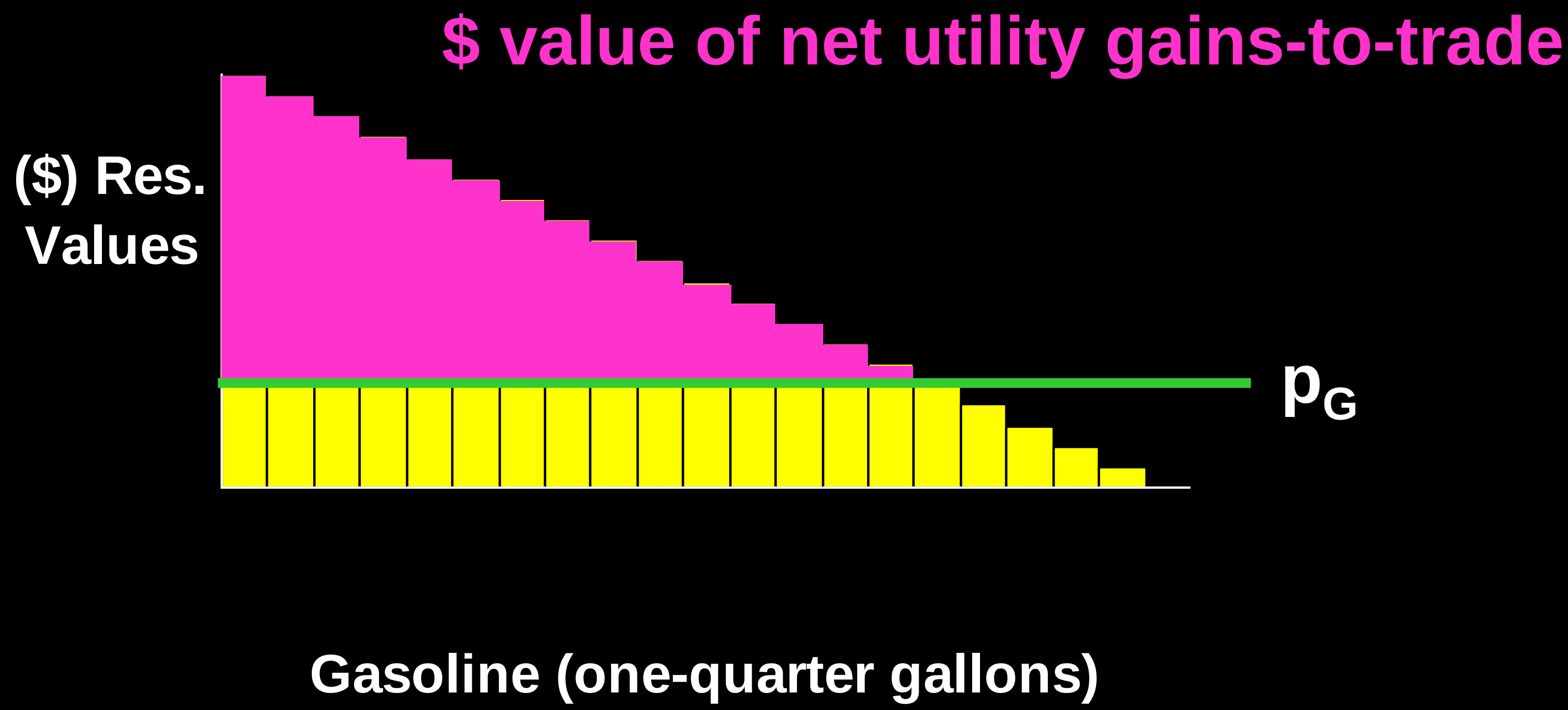


# $ Equivalent Utility Gains



# $ Equivalent Utility Gains

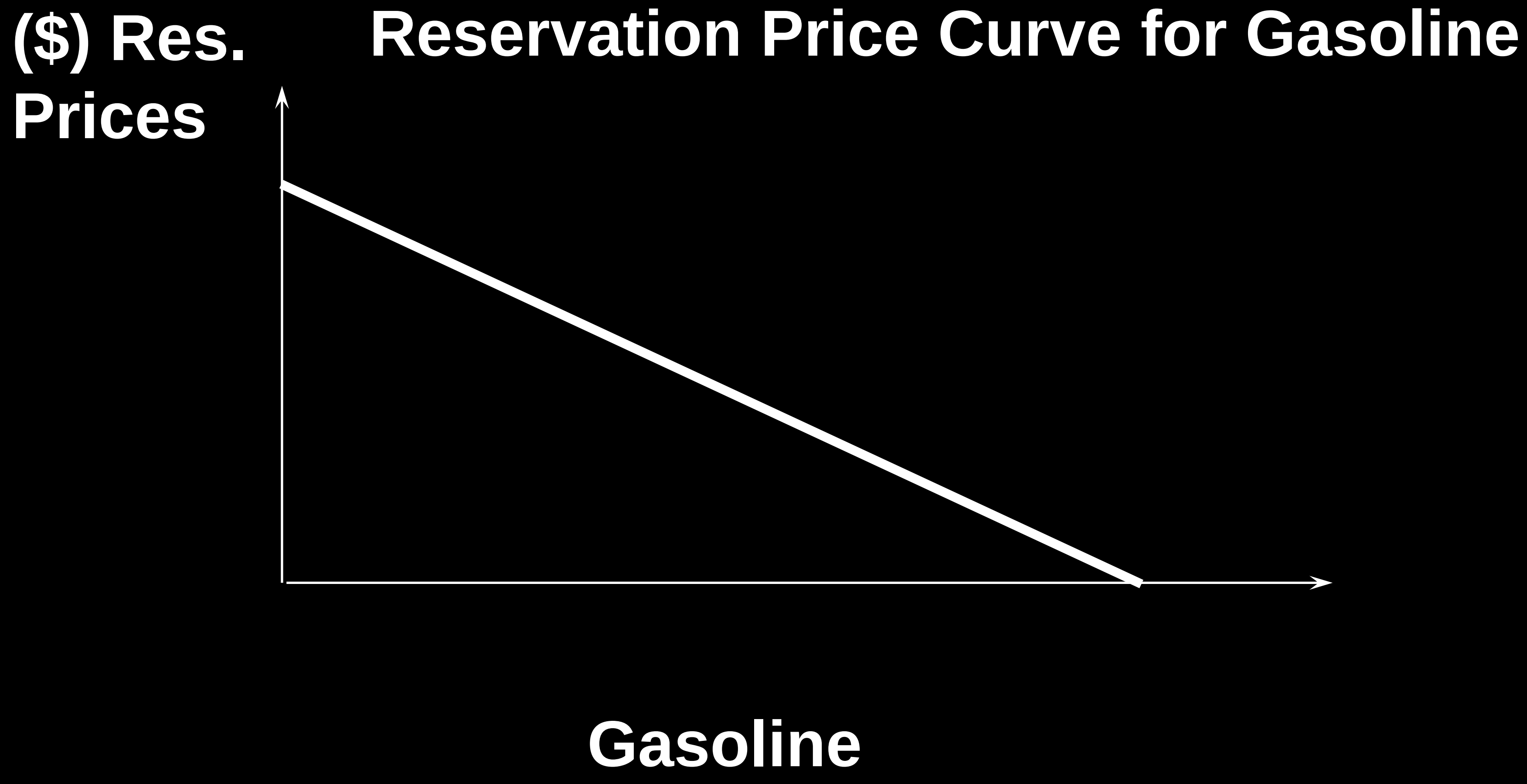
**Reservation Price Curve for Gasoline**



# $ Equivalent Utility Gains

- **Finally, if gasoline can be purchased in any quantity then ...**

# $ Equivalent Utility Gains



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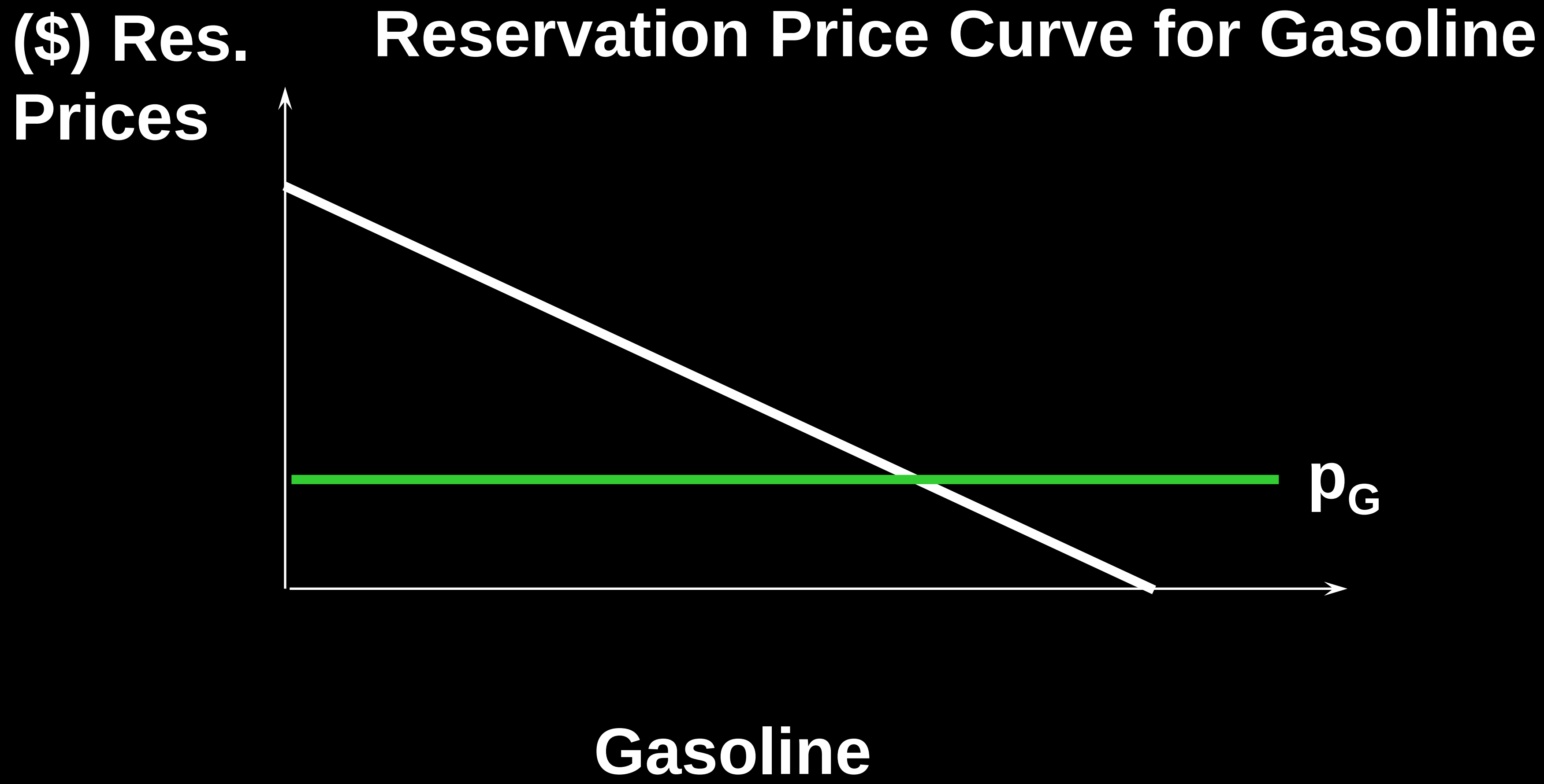

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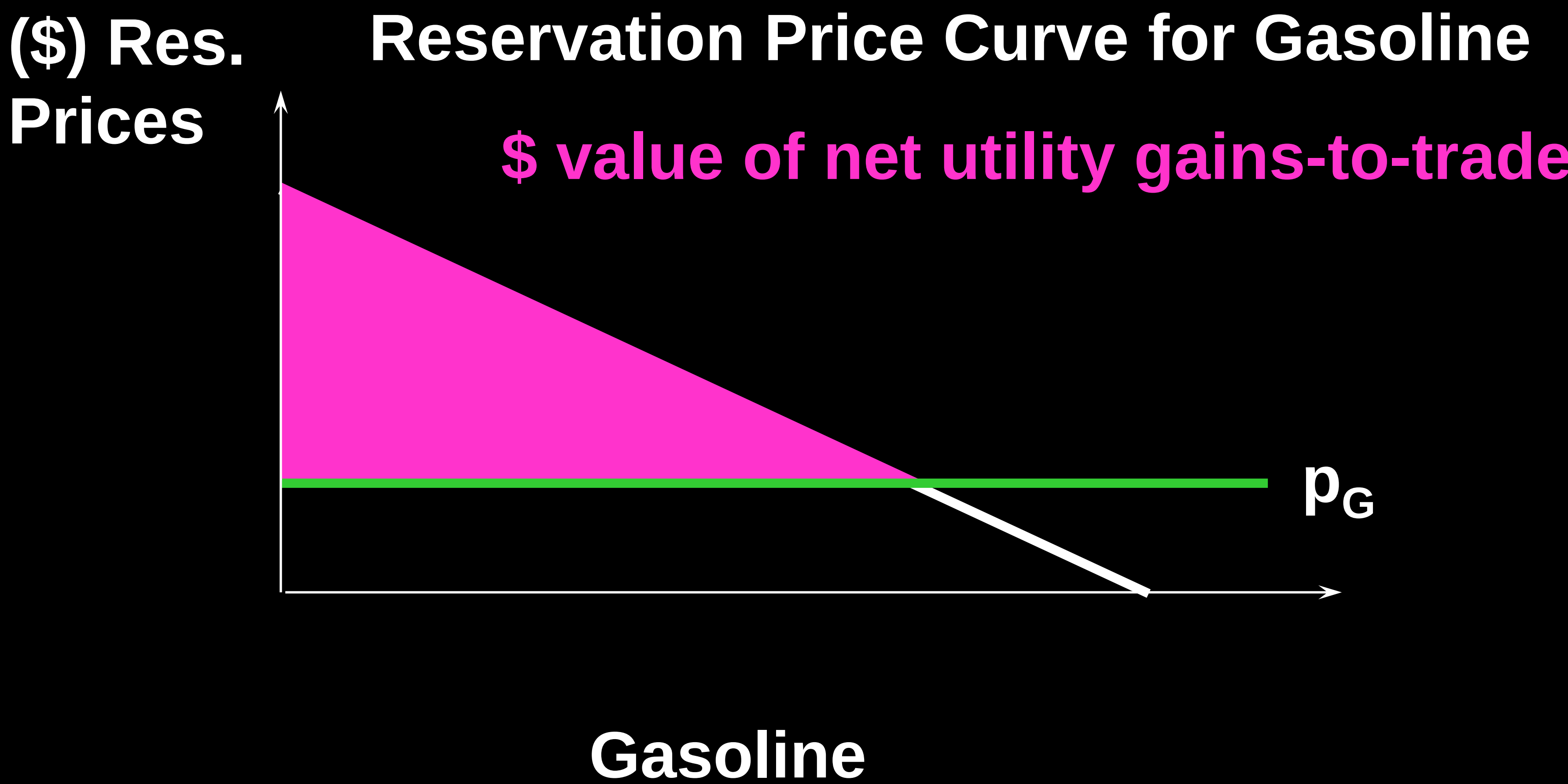


# $ Equivalent Utility Gains

- Unfortunately, estimating a consumer's reservation-price curve is difficult,
- so, as an approximation, the reservation-price curve is replaced with the consumer's ordinary demand curve.

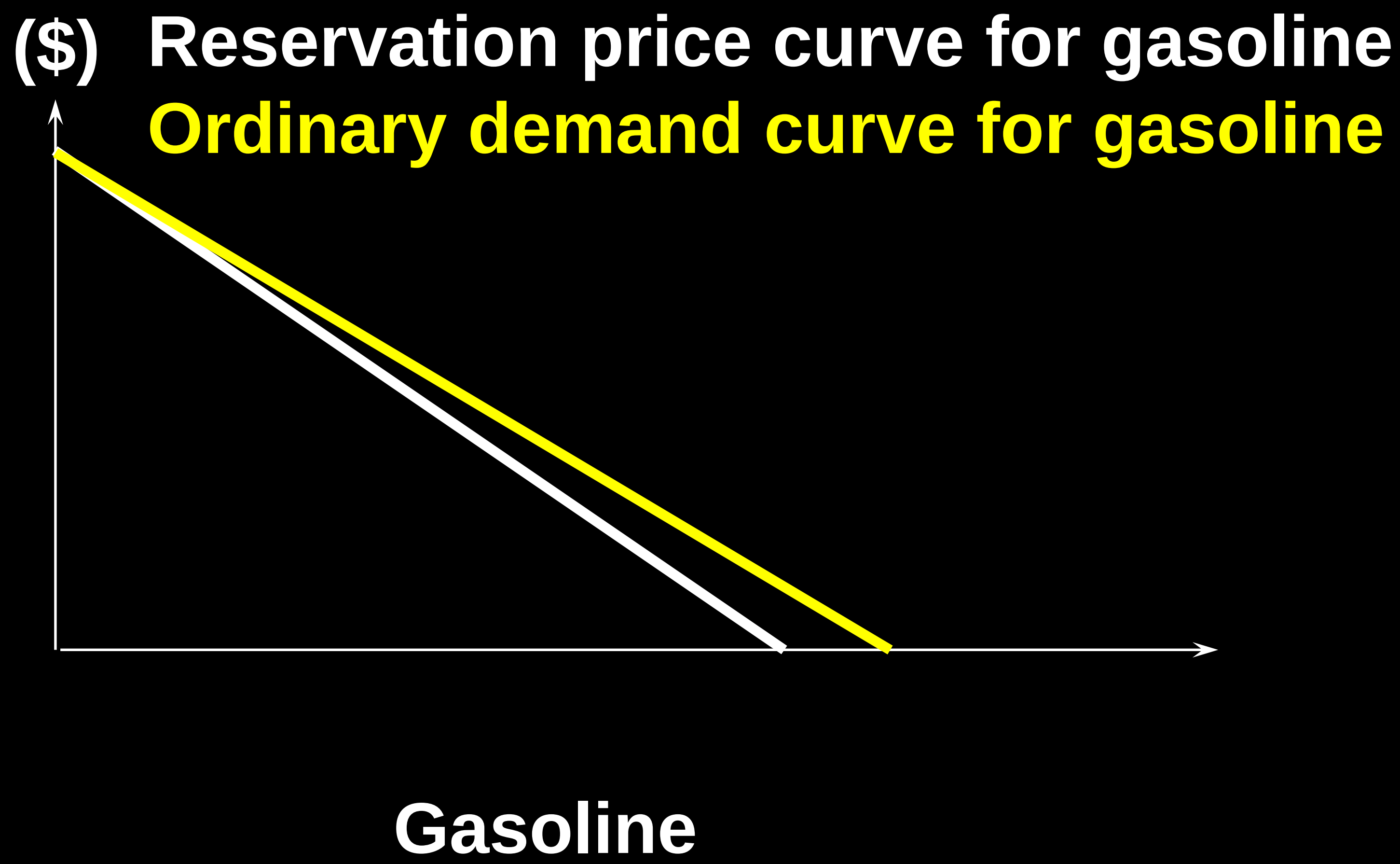
# Consumer's Surplus

- A consumer's reservation-price curve is not quite the same as her ordinary demand curve.  Why not?
- A reservation-price curve describes **sequentially** the values of successive single units of a commodity.
- An ordinary demand curve describes the most that would be paid for q units of a commodity purchased **simultaneously**.

# Consumer's Surplus

- **Approximating the net utility gain area under the reservation-price curve by the corresponding area under the ordinary demand curve gives the Consumer's Surplus measure of net utility gain.**

# Consumer's Surplus



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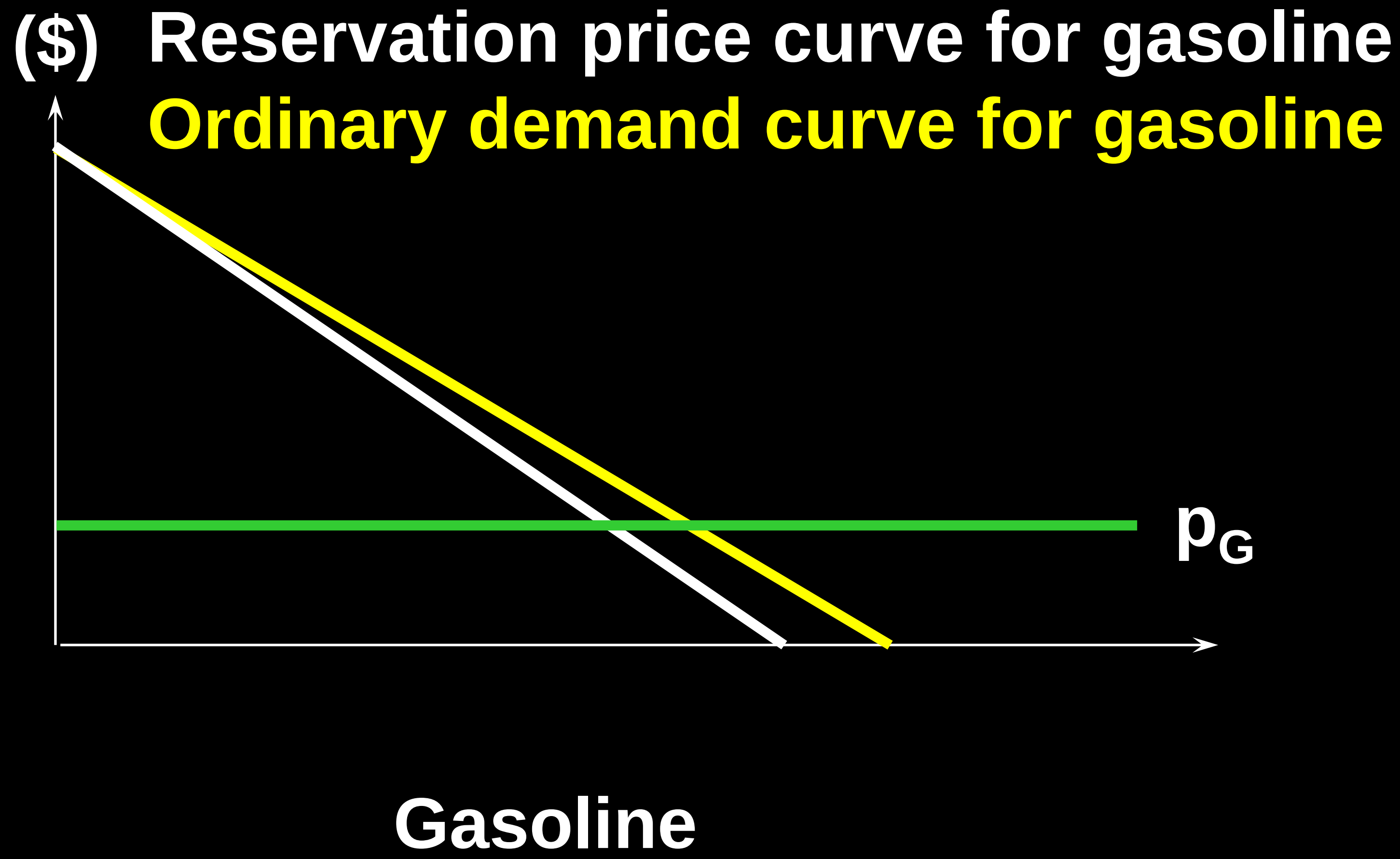

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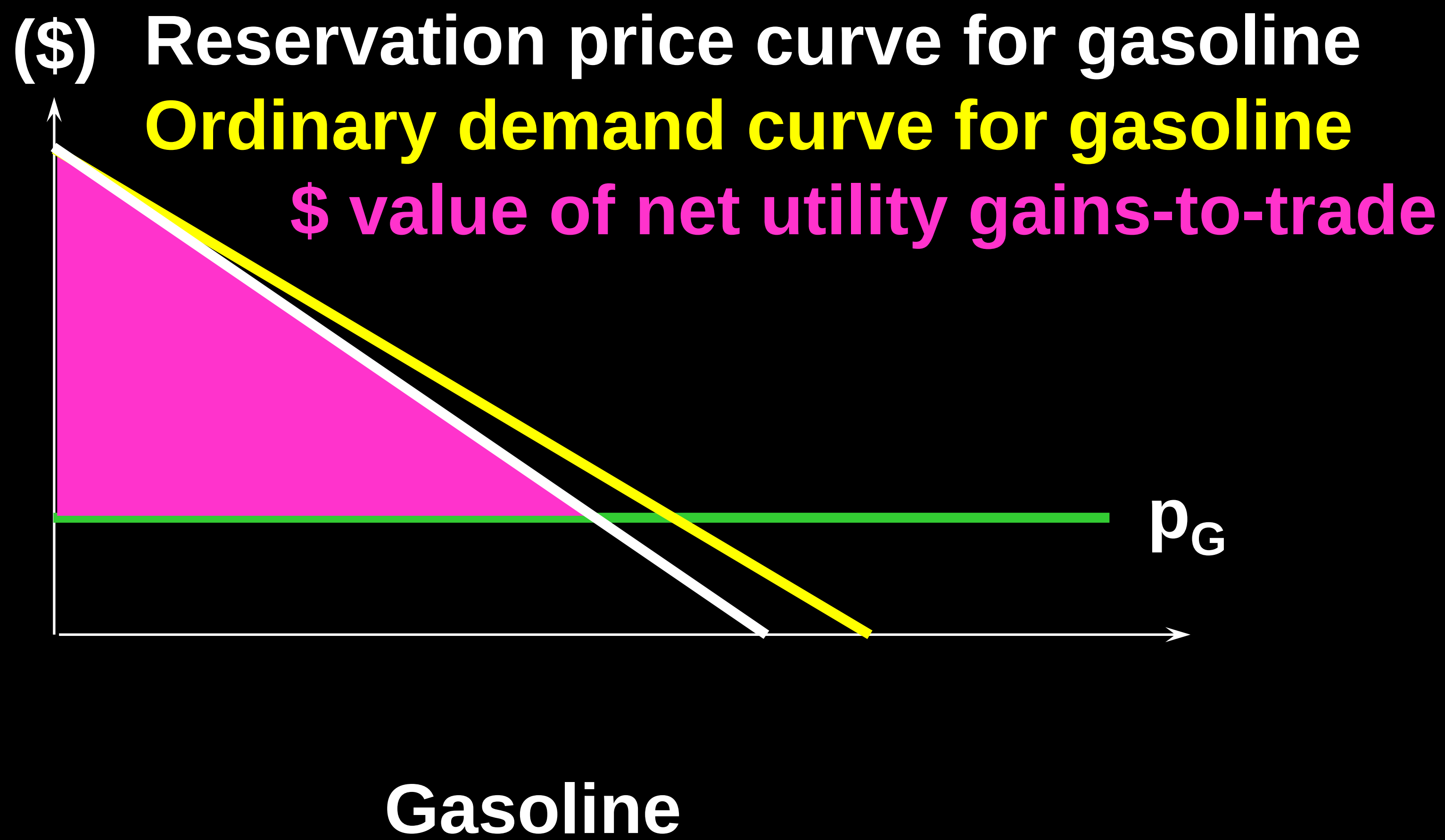

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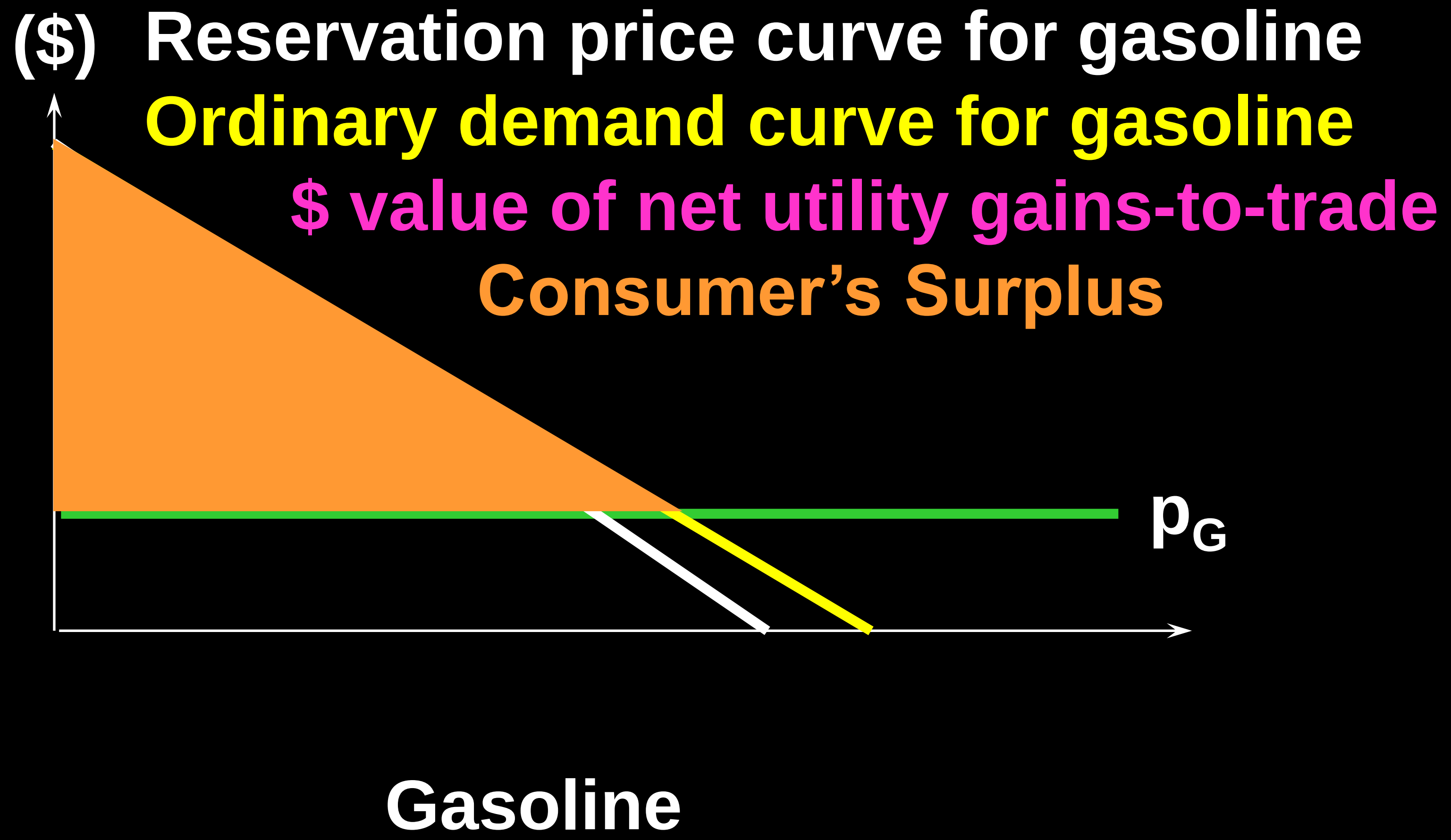
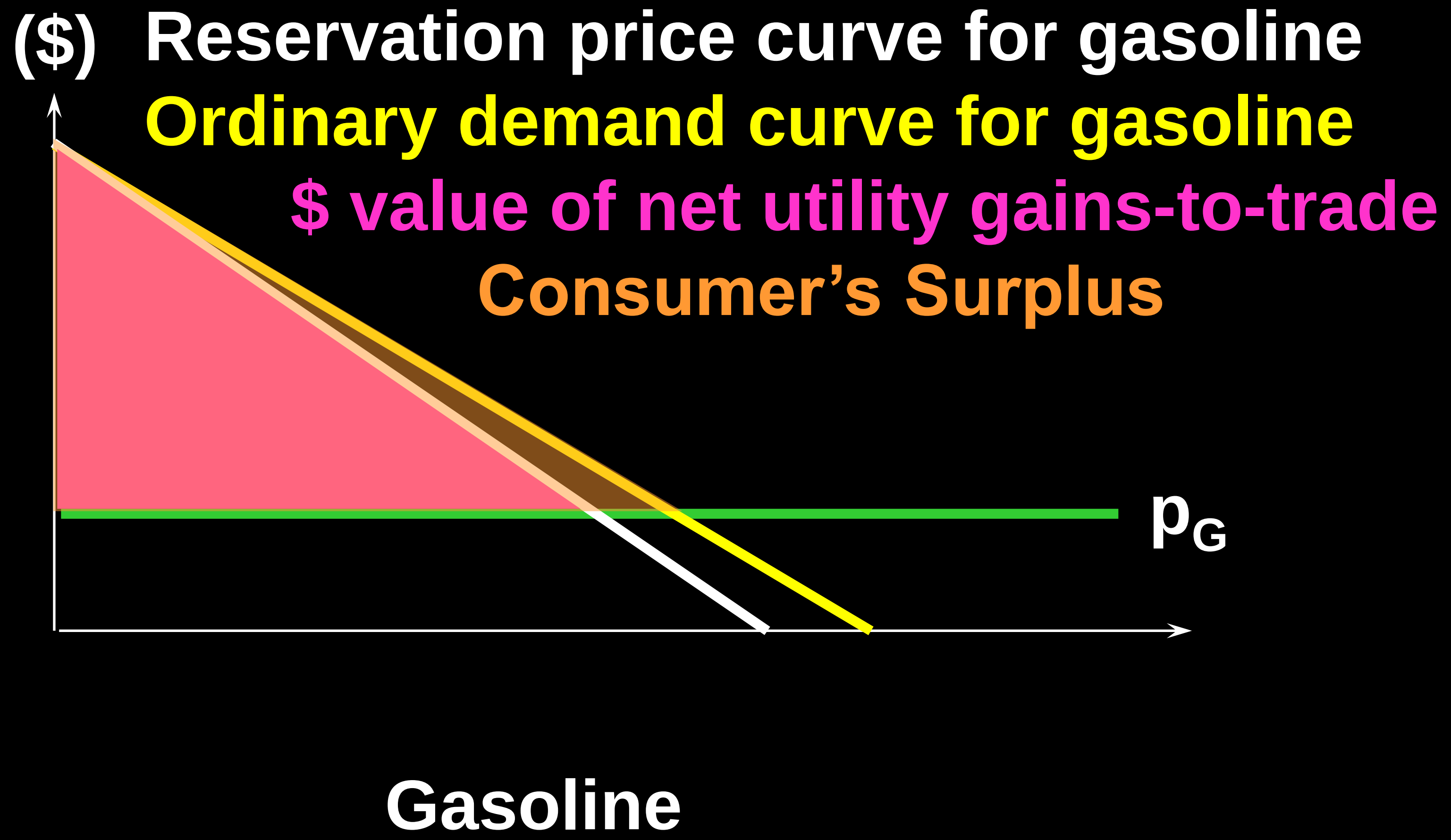


# Consumer's Surplus



# Consumer's Surplus

- The difference between the consumer's reservation-price and ordinary demand curves is due to income effects.
- But, if the consumer's utility function is quasilinear in income then there are no income effects and Consumer's Surplus is an exact $ measure of gains-to-trade.

# Consumer's Surplus

The consumer's utility function is quasilinear in $x_2$.

$$U(x_1, x_2) = v(x_1) + x_2$$

Take $p_2 = 1$. Then the consumer's choice problem is to maximize

$$U(x_1, x_2) = v(x_1) + x_2$$

subject to

$$p_1 x_1 + x_2 = m.$$

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$$p_1 x_1 + x_2 = m.$$

# Consumer's Surplus

That is, choose $x_1$ to maximize

$$v(x_1) + m - p_1 x_1.$$

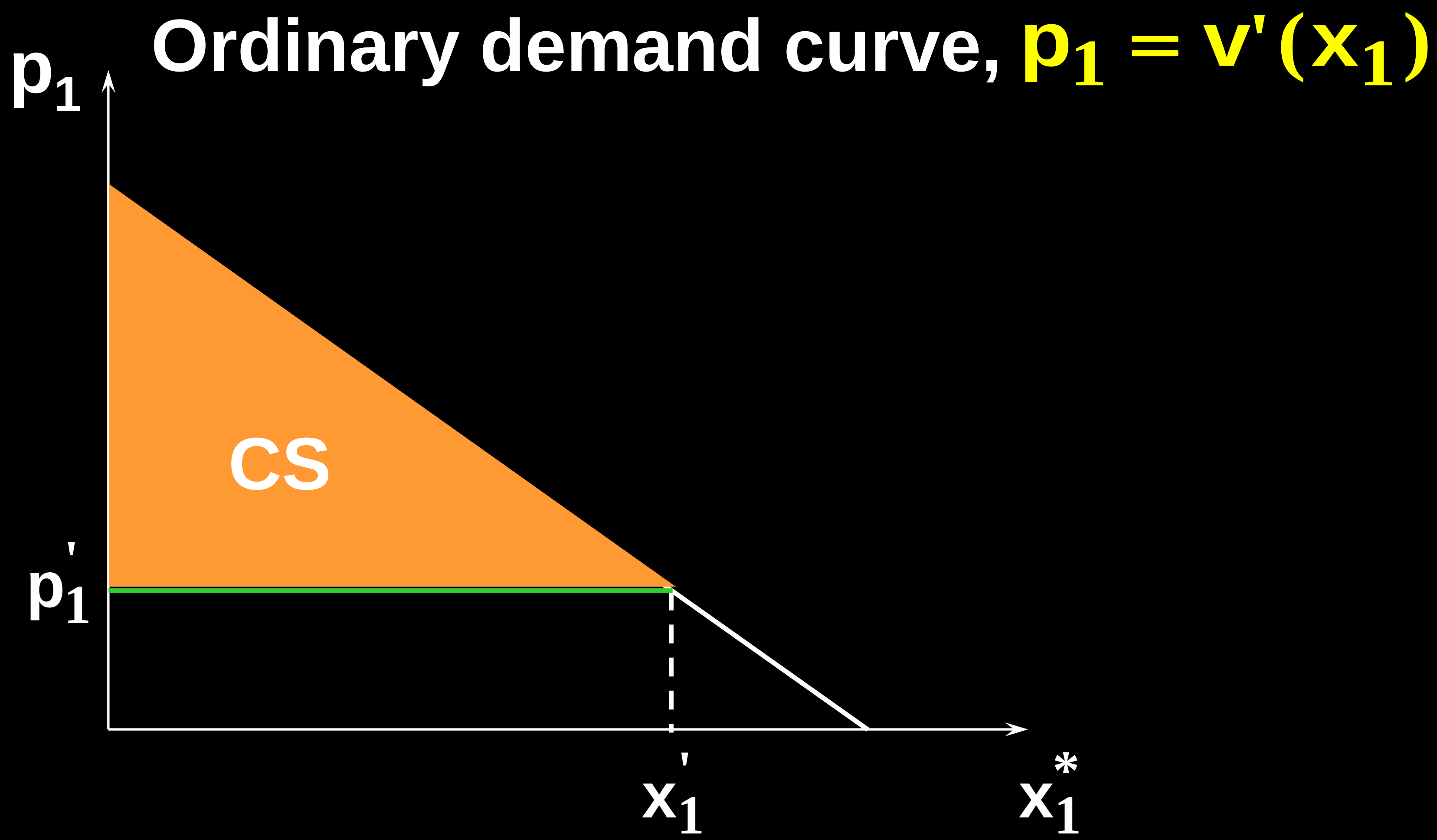
The first-order condition is

$$v'(x_1) - p_1 = 0$$

That is, $p_1 = v'(x_1).$

This is the equation of the consumer's ordinary demand for commodity 1.

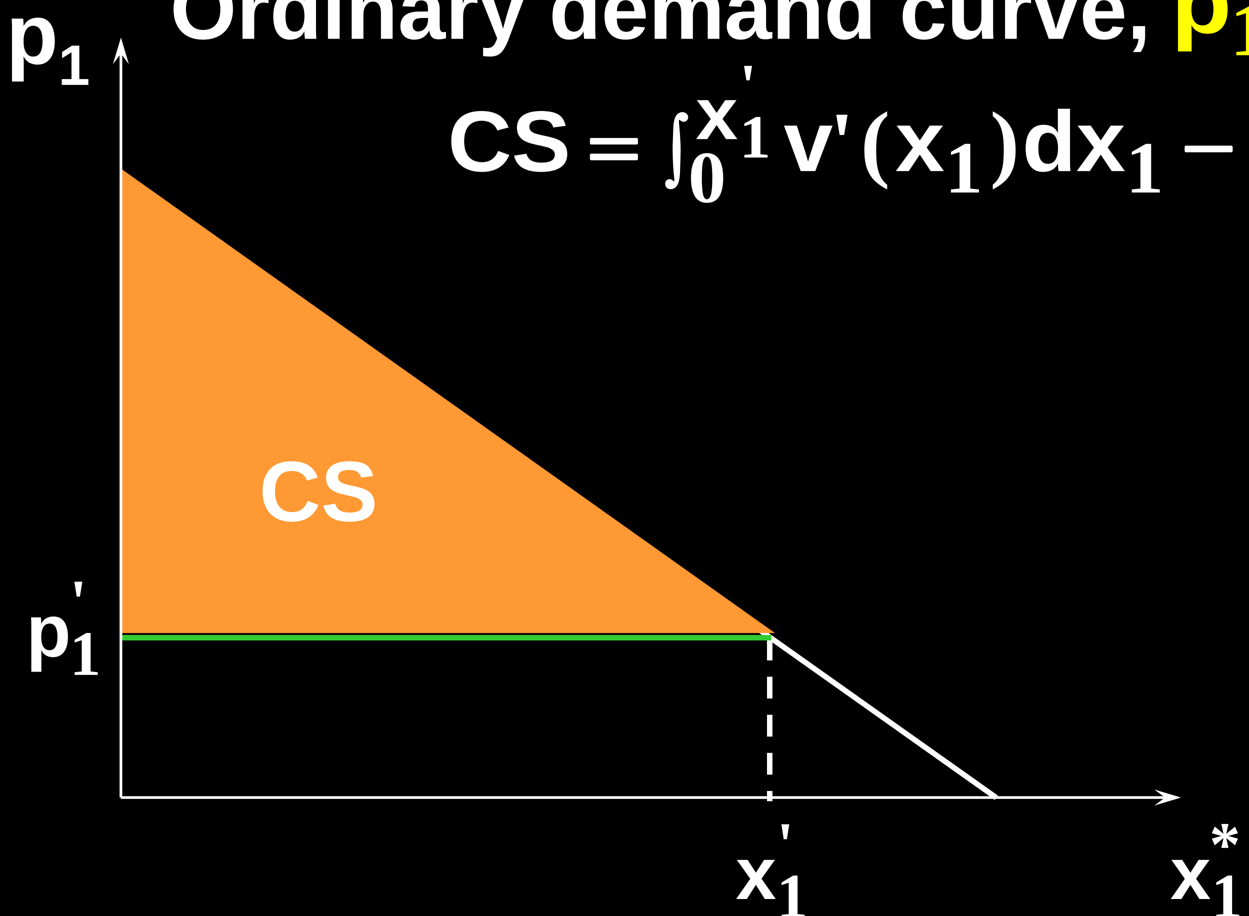
# Consumer's Surplus



# Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1' x_1'$$

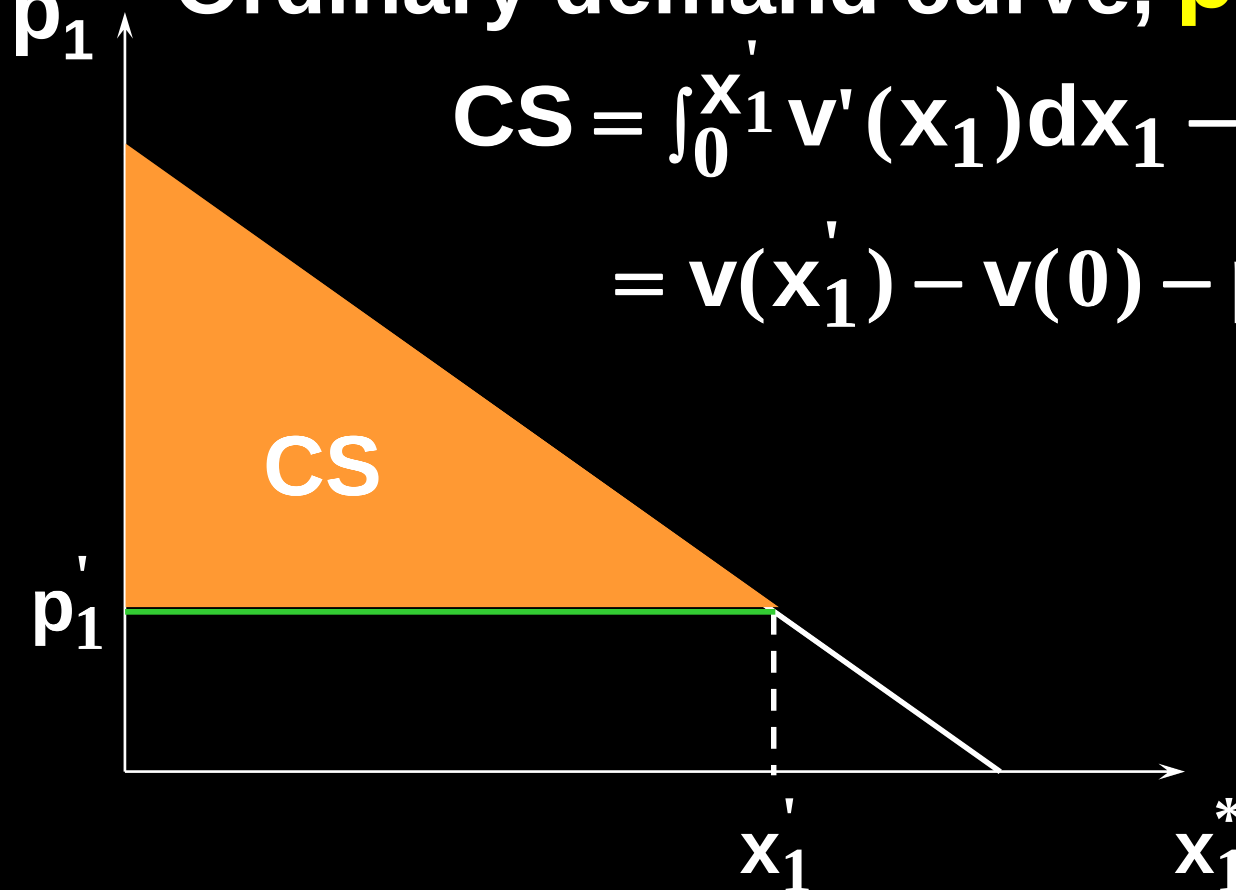


# Consumer’s Surplus

**Ordinary demand curve,** $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1' x_1'$$

$$= v(x_1') - v(0) - p_1' x_1'$$



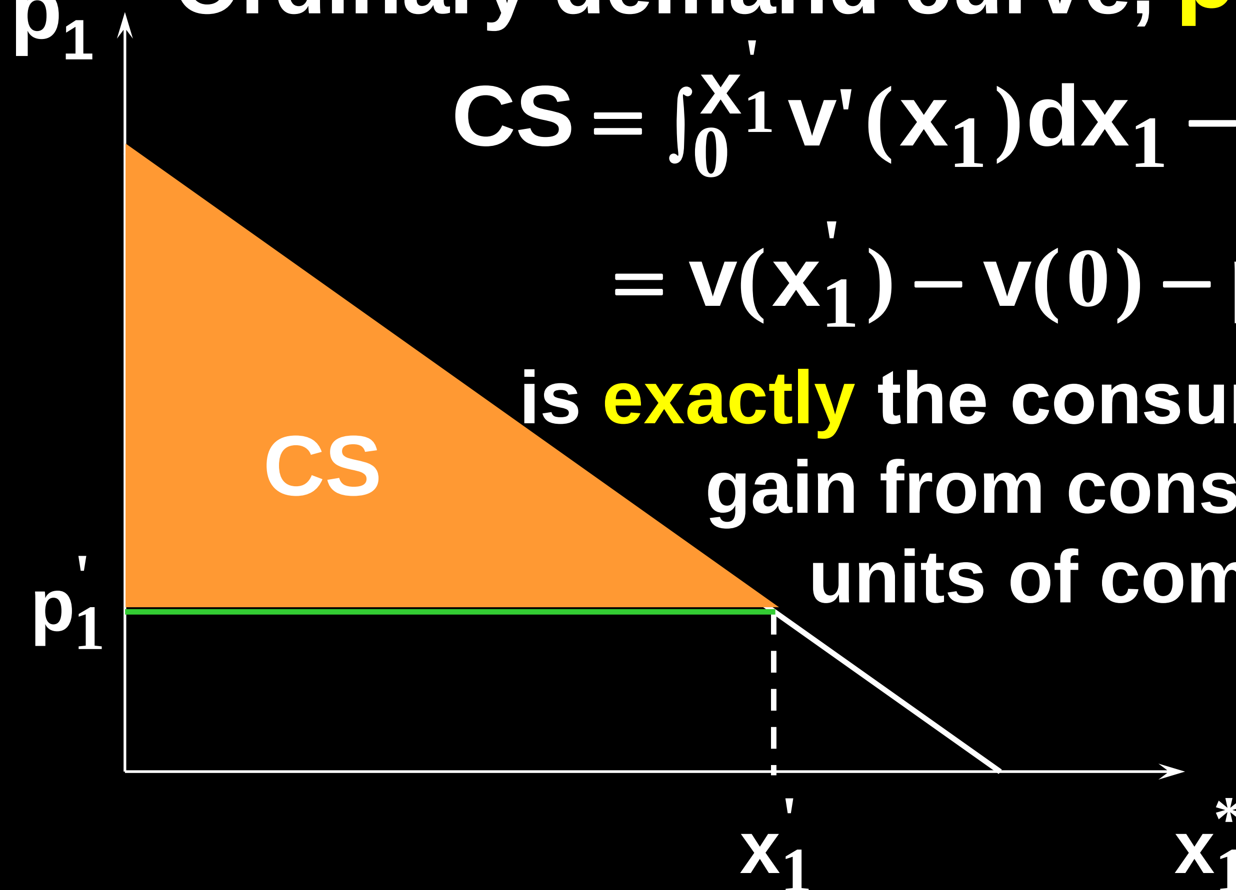
# Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1' x_1'$$

$$= v(x_1') - v(0) - p_1' x_1'$$

is **exactly** the consumer's utility gain from consuming $x_1$' units of commodity 1.



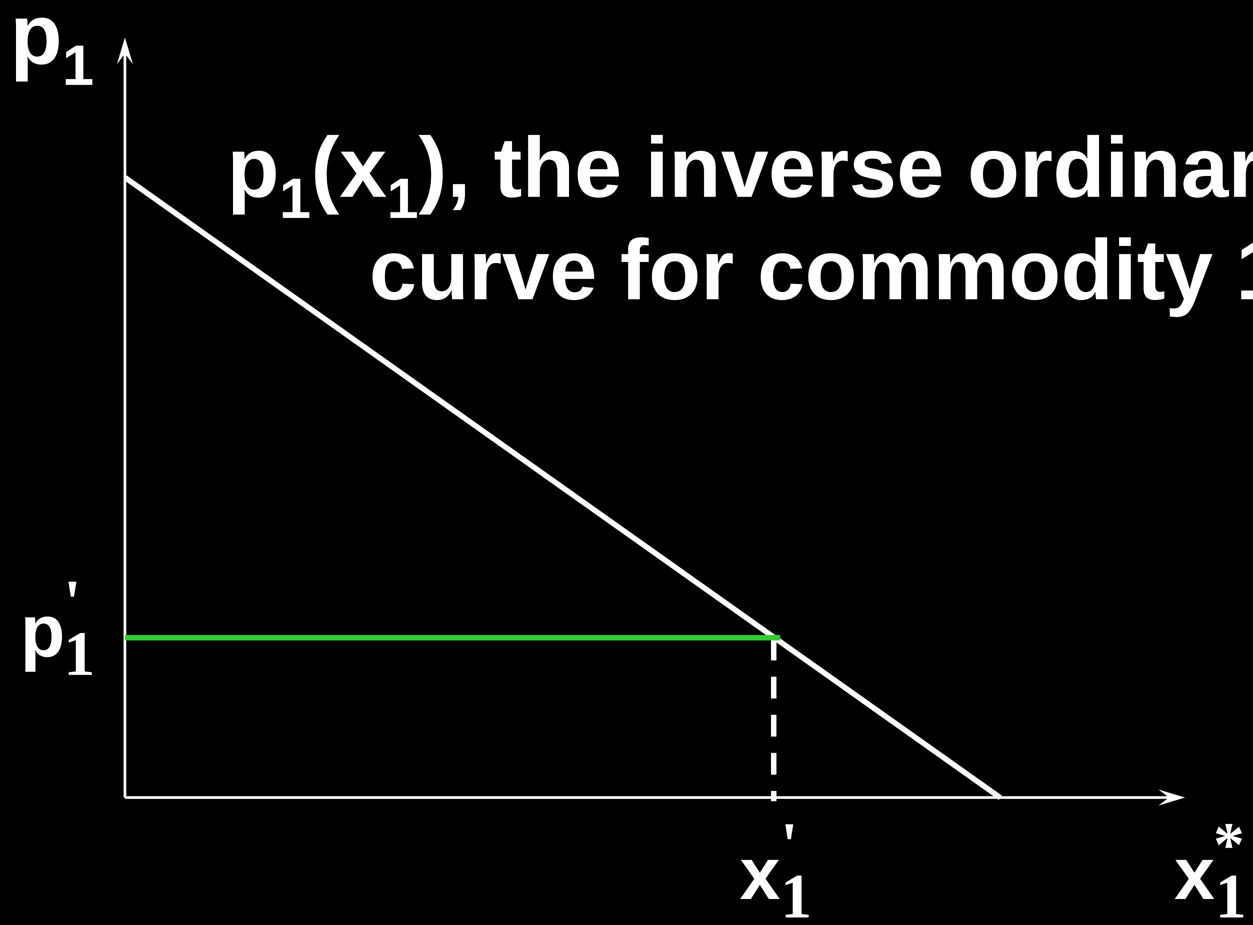
# Consumer’s Surplus

- Consumer’s Surplus is an exact dollar measure of utility gained from consuming commodity 1 when the consumer’s utility function is quasilinear in commodity 2.
- Otherwise Consumer’s Surplus is an approximation.

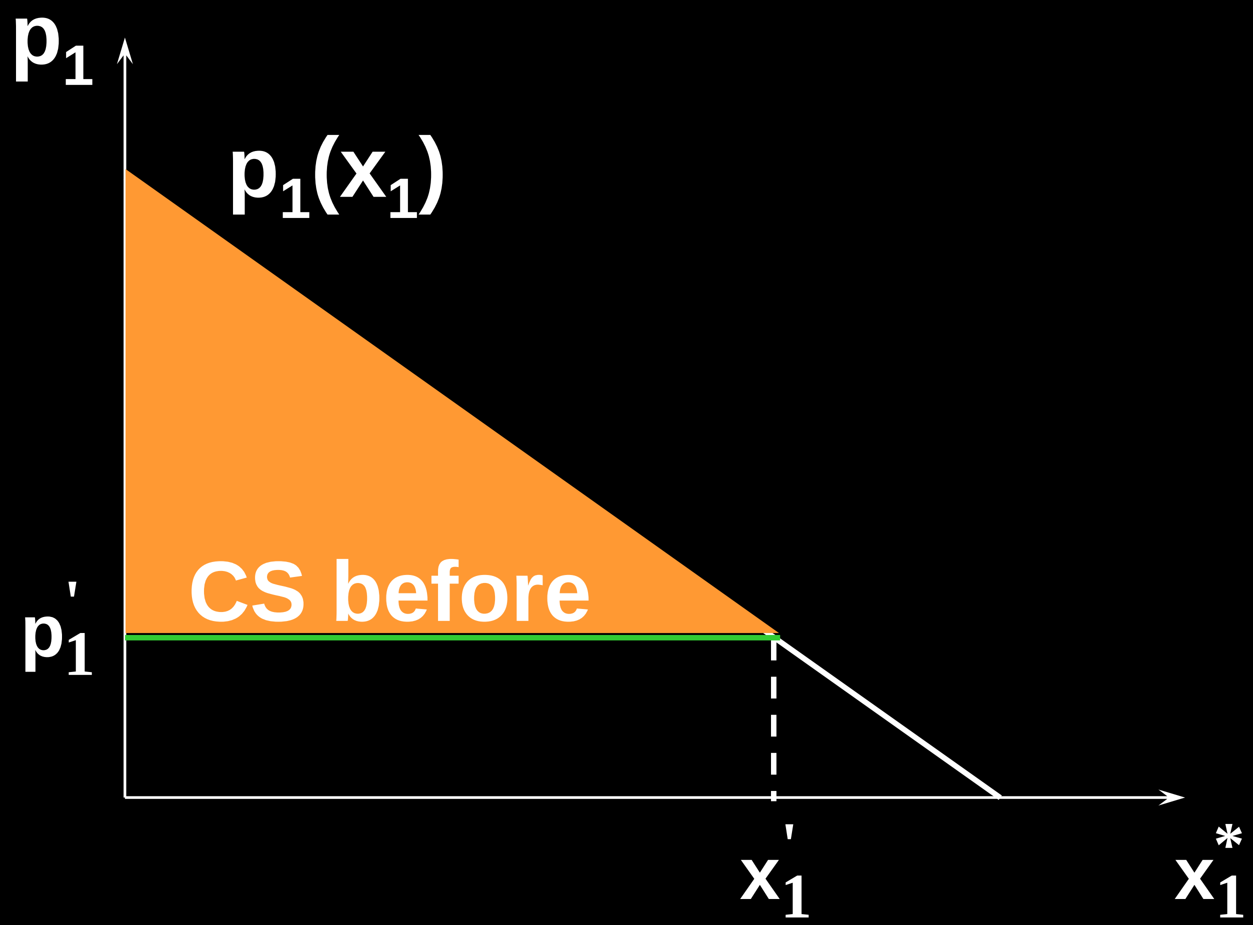
# Consumer's Surplus

- **The change to a consumer's total utility due to a change to $p_1$ is approximately the change in her Consumer's Surplus.**

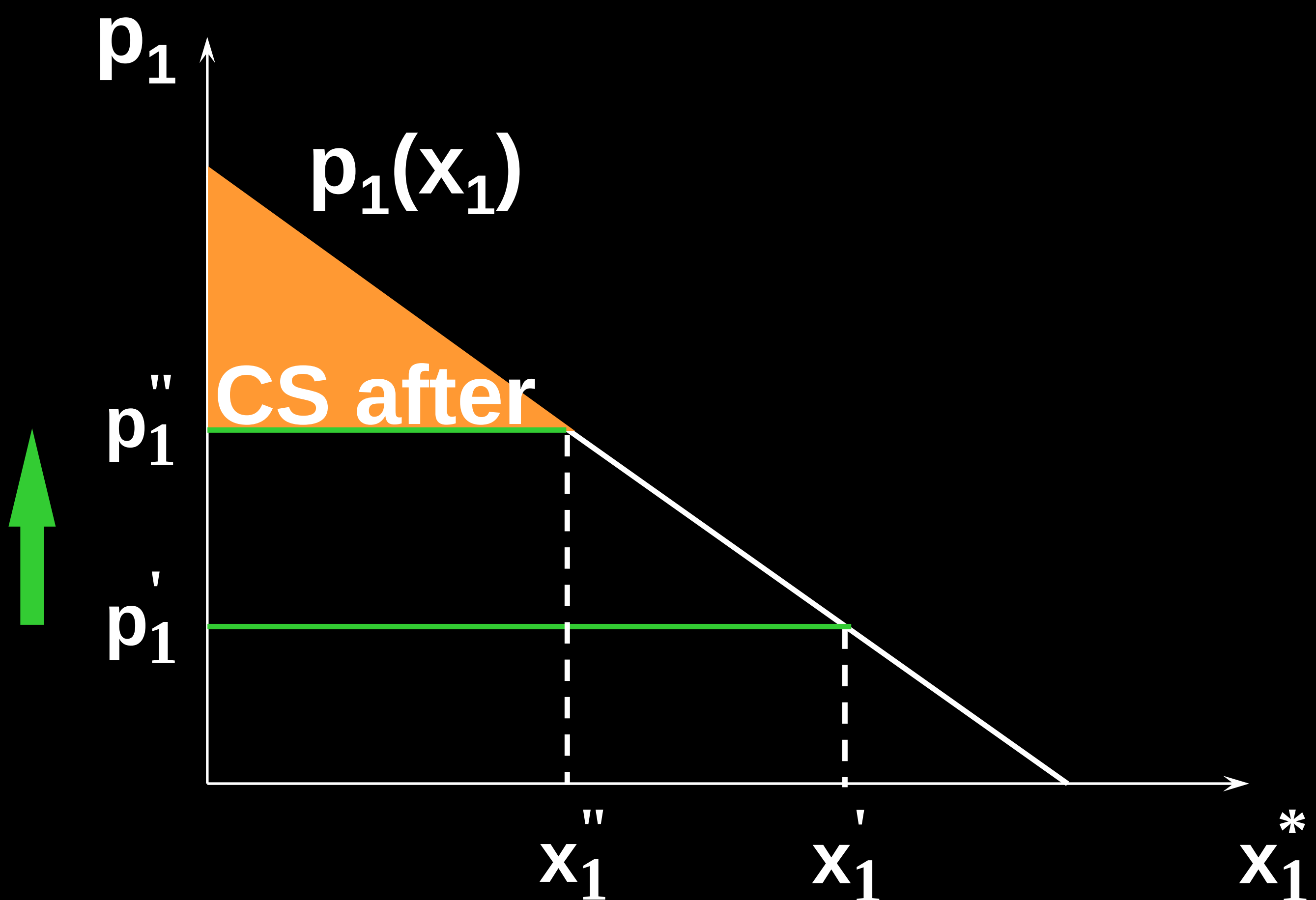
# Consumer's Surplus



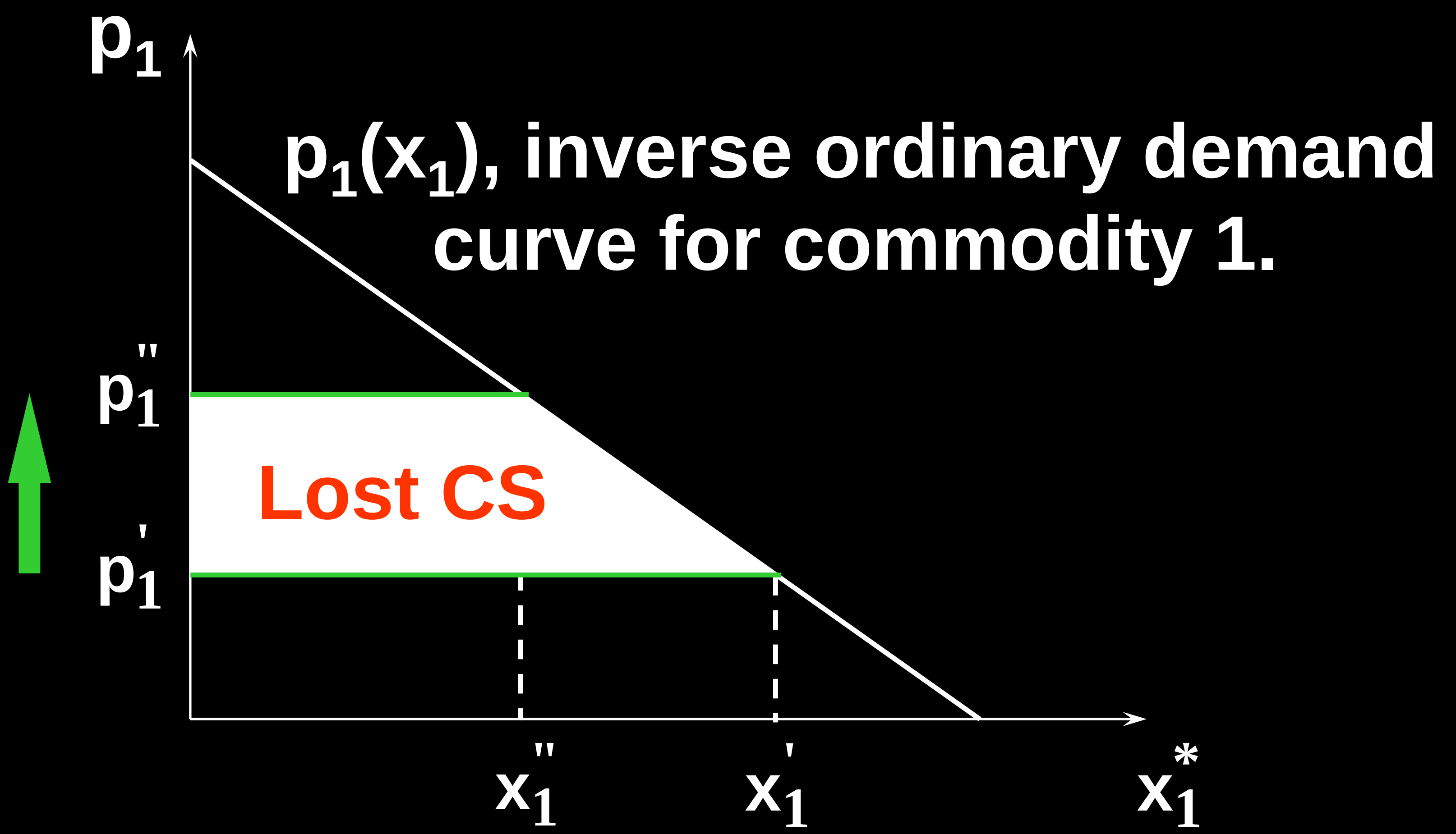
# Consumer's Surplus



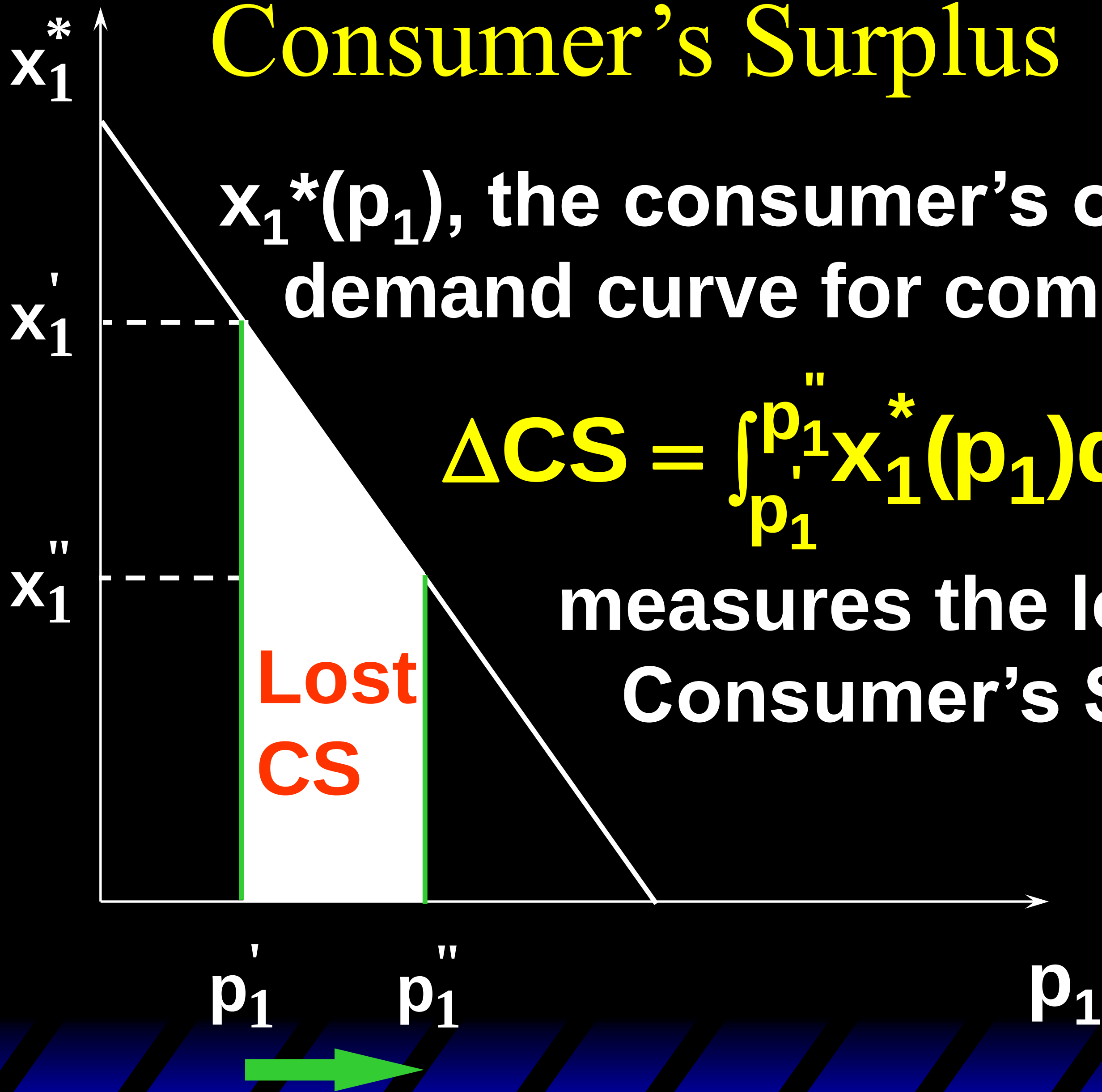
# Consumer's Surplus



# Consumer's Surplus



# Consumer's Surplus




$x_1^*(p_1)$, the consumer's ordinary demand curve for commodity 1.

$$\Delta CS = \int_{p_1'}^{p_1''} x_1^*(p_1)\,dp_1$$

measures the loss in Consumer's Surplus.

# Compensating Variation and Equivalent Variation

- **Two additional dollar measures of the total utility change caused by a price change are Compensating Variation and Equivalent Variation.**

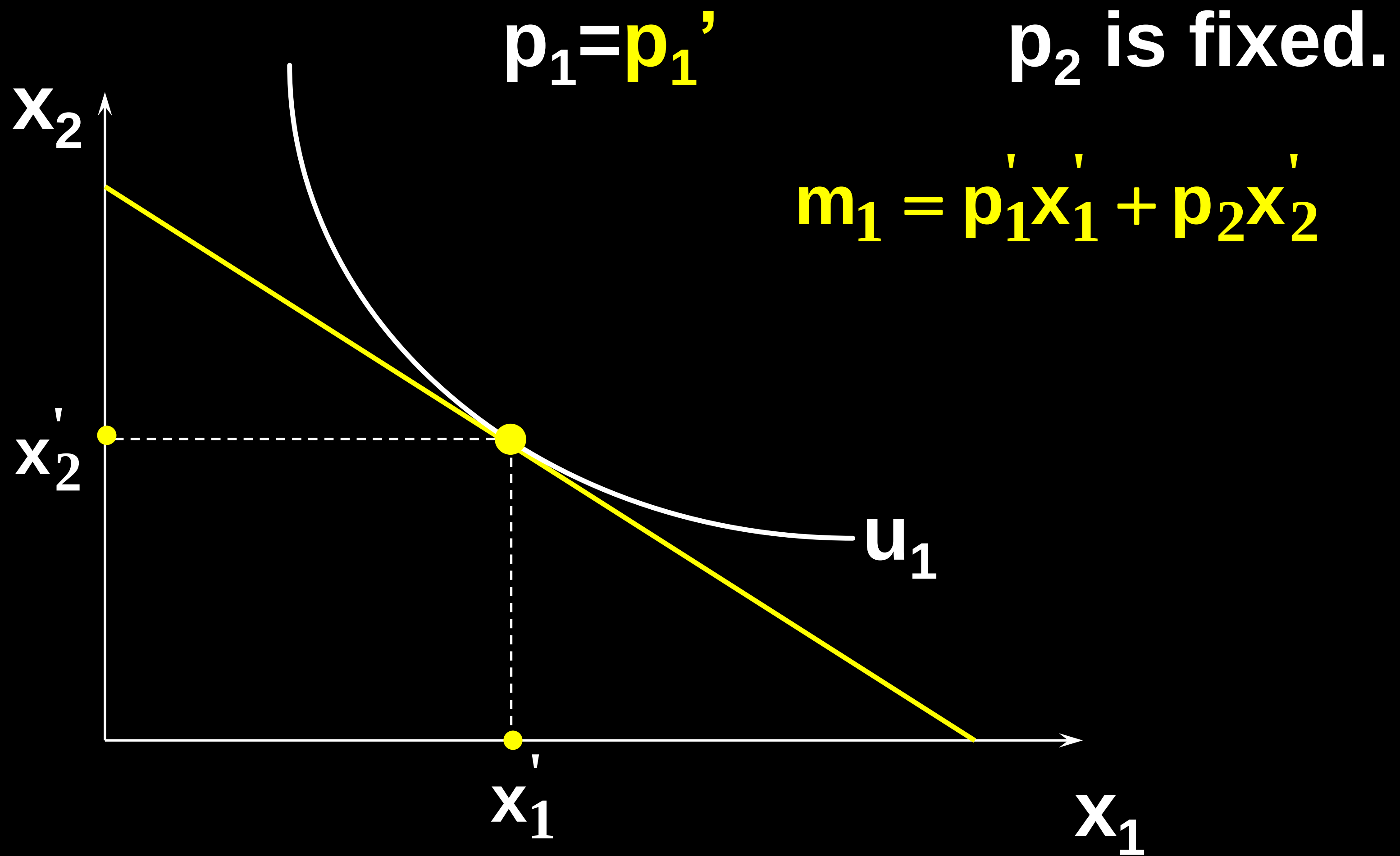
# Compensating Variation

- $p_1$ rises.
- Q: What is the least extra income that, at the **new prices**, just restores the consumer's original utility level?

# Compensating Variation

- $p_1$ rises.
- Q: What is the least extra income that, at the new prices, just restores the consumer's original utility level?
- A: The Compensating Variation.

# Compensating Variation



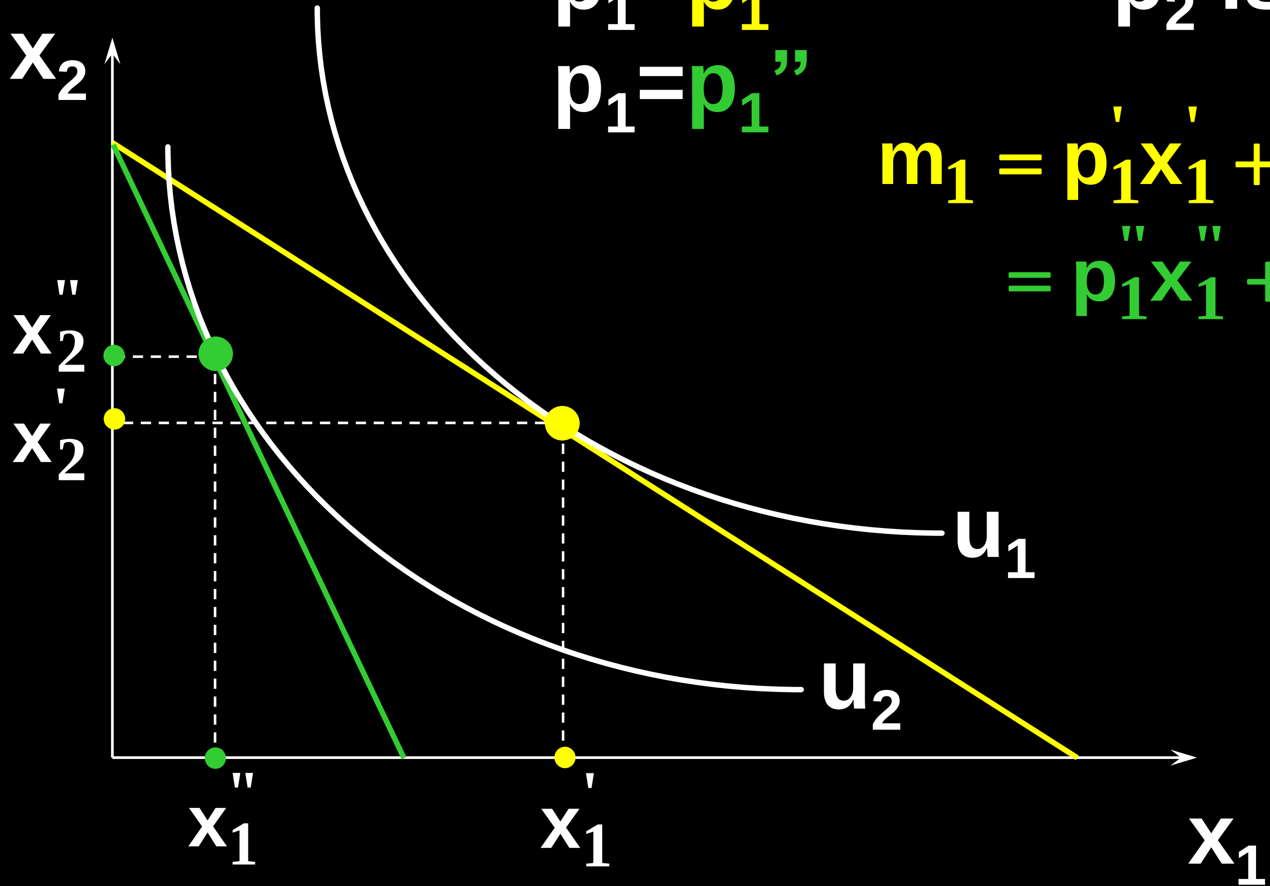
# Compensating Variation

$p_1 = p_1'$

$p_1 = p_1''$

$p_2$ is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$



# Compensating Variation

$p_1 = p_1'$

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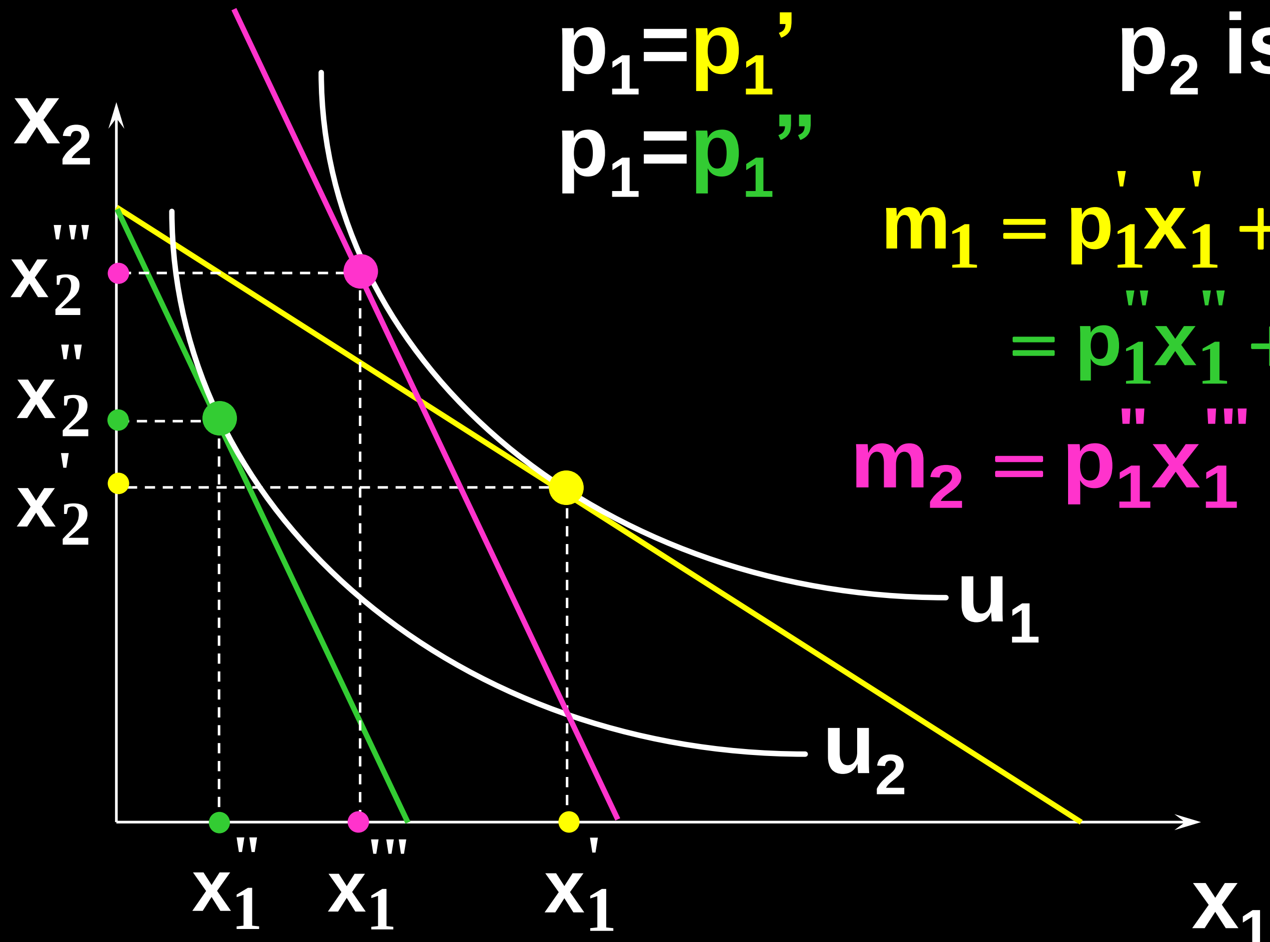
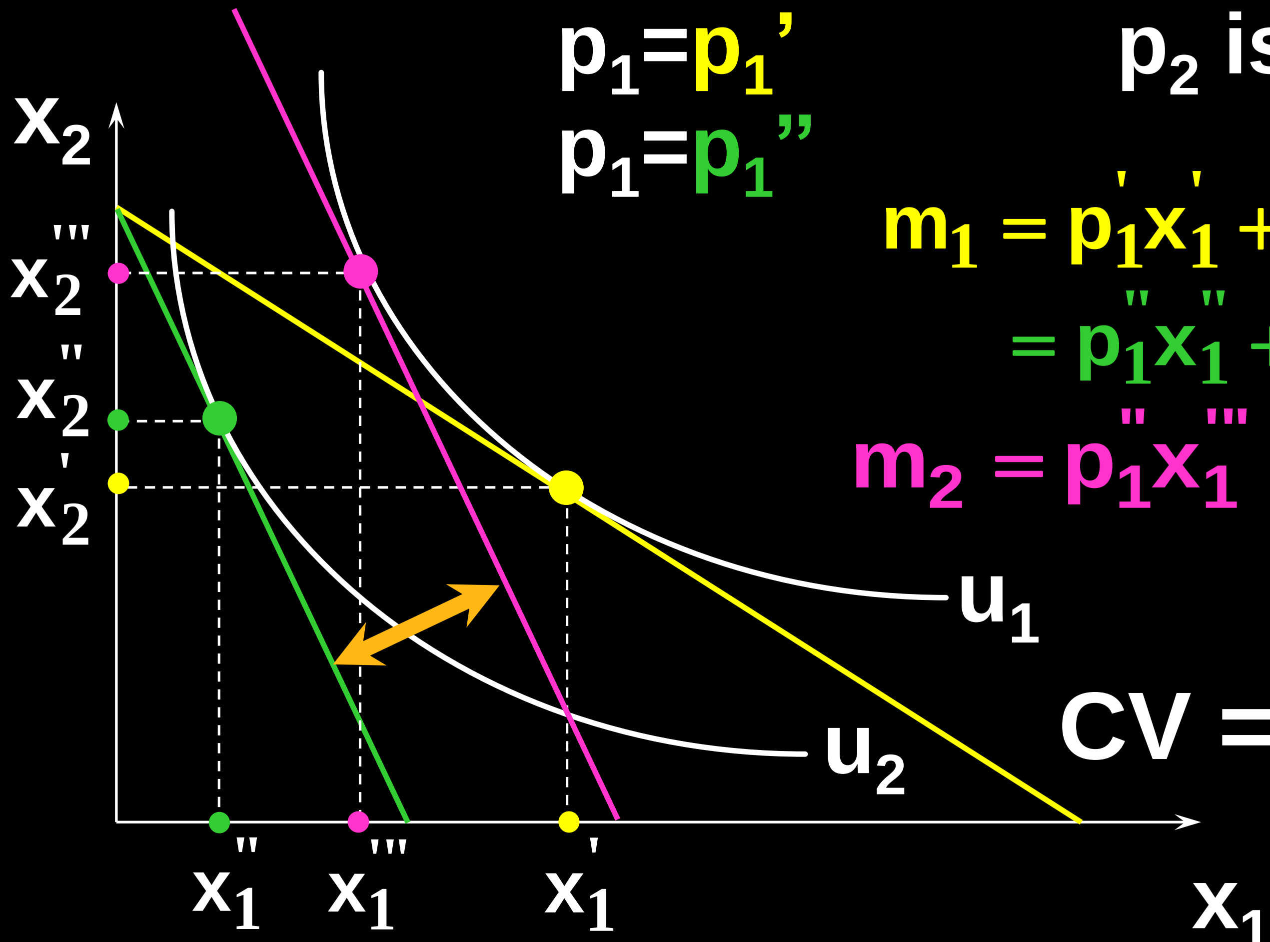
$p_2$ is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
$$m_2 = p_1'' x_1''' + p_2 x_2'''$$



# Compensating Variation




$p_2$ is fixed.

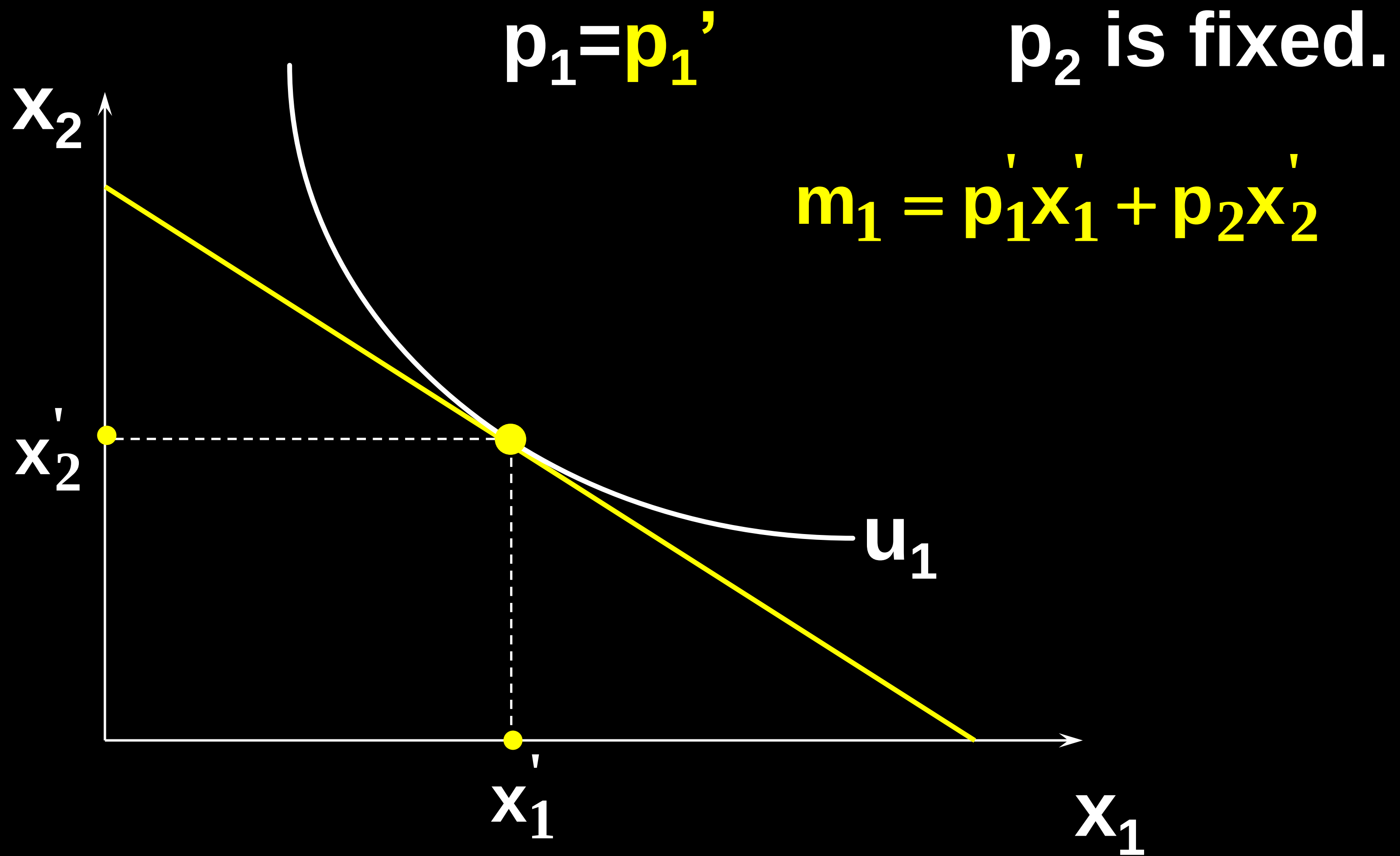
$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
$$m_2 = p_1'' x_1''' + p_2 x_2'''$$

$CV = m_2 - m_1$.

# Equivalent Variation

- $p_1$ rises.
- Q: What is the least income taken away that, at the original prices, just achieve the consumer's New utility level?
- A:  The Equivalent Variation.

# Equivalent Variation

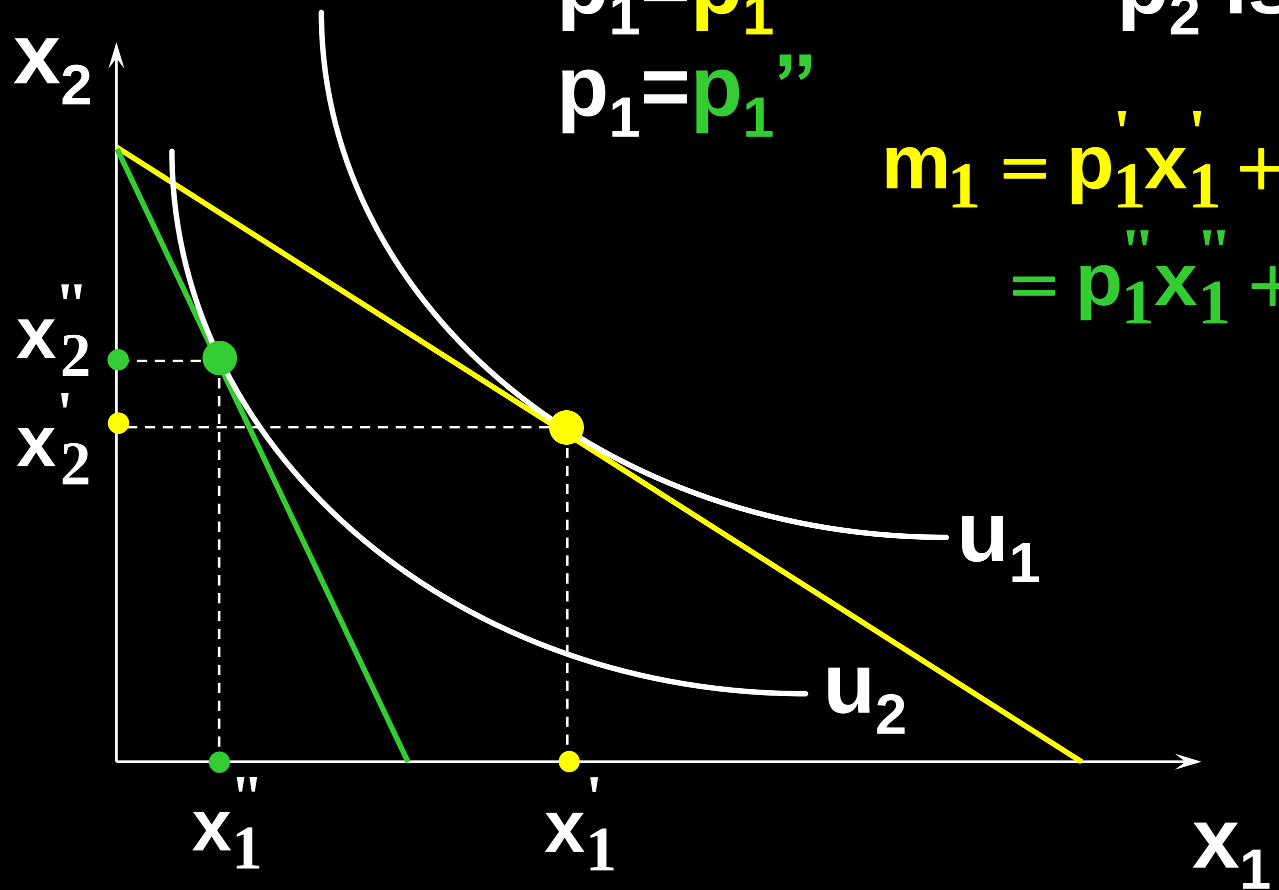


# Equivalent Variation

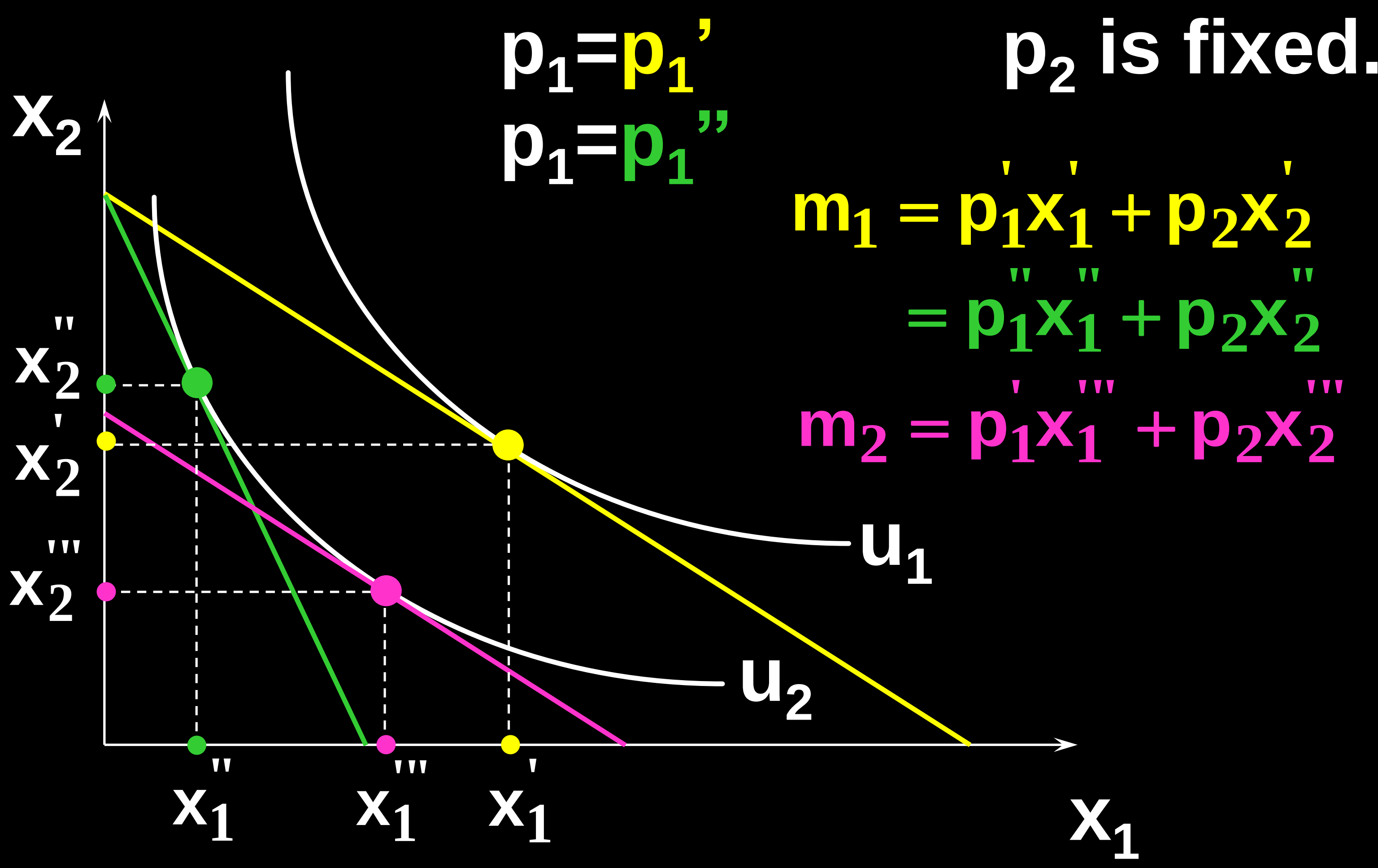
$p_1 = p_1'$
$p_1 = p_1''$

$p_2$ is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$



# Equivalent Variation



# Equivalent Variation

$p_1 = p_1'$

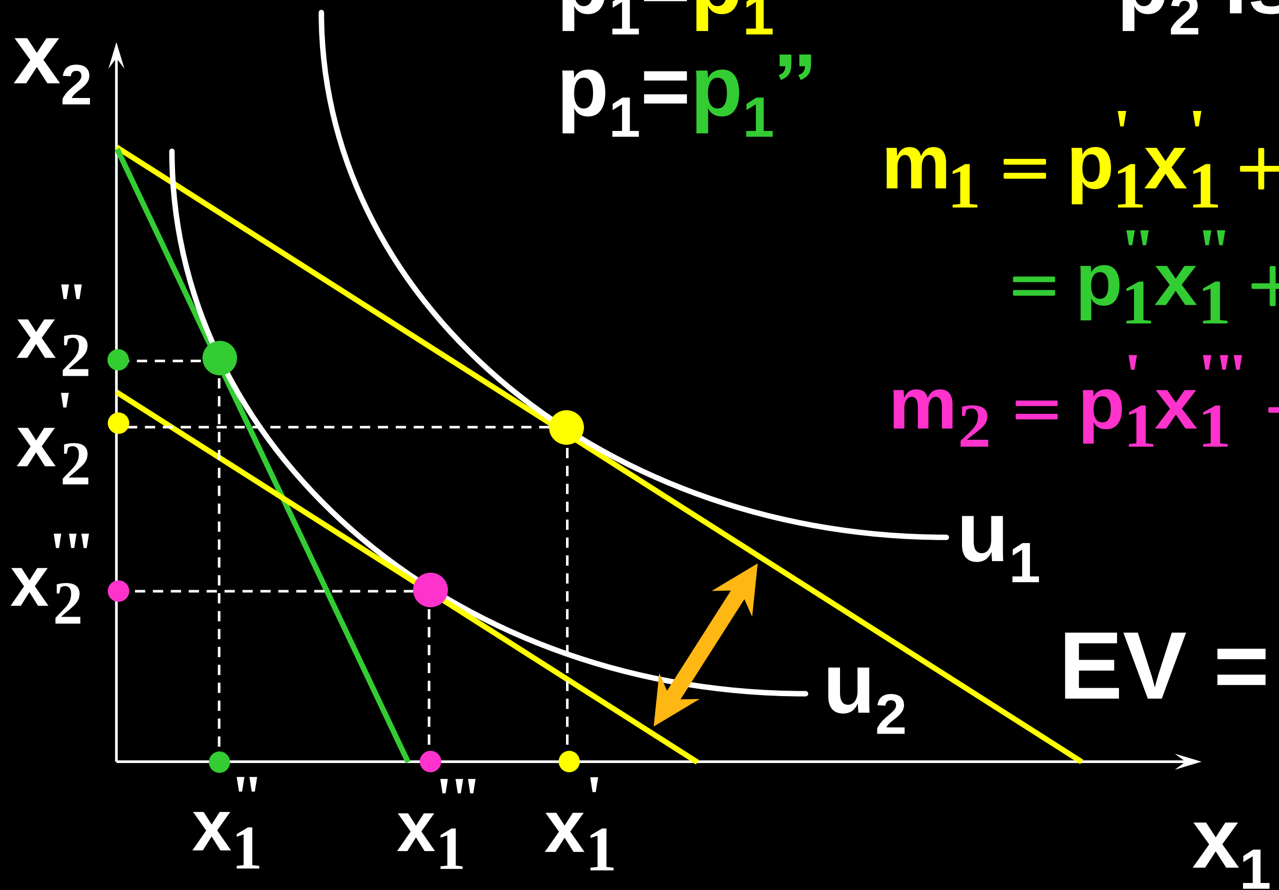
$p_1 = p_1''$

$p_2$ is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$

$$= p_1'' x_1'' + p_2 x_2''$$

$$m_2 = p_1' x_1''' + p_2 x_2'''$$




EV = $m_1$ - $m_2$.

# Consumer's Surplus, Compensating Variation and Equivalent Variation

- **Relationship 1: When the consumer's preferences are quasilinear, all three measures are the same.**

# Consumer's Surplus, Compensating Variation and Equivalent Variation

- **Consider first the change in Consumer's Surplus when $p_1$ rises from $p_1$' to $p_1$".**

# Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when $p_1$ rises from $p_1$' to $p_1$" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1' x_1'$$

and so the change in CS when $p_1$ rises from $p_1$' to $p_1$" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

$$= v(x_1') - v(0) - p_1' x_1' - \left[ v(x_1'') - v(0) - p_1'' x_1'' \right]$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

If $$U(x_1, x_2) = v(x_1) + x_2$$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when $p_1$ rises from $p_1$' to $p_1$" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

$$= v(x_1') - v(0) - p_1'x_1' - \left[v(x_1'') - v(0) - p_1''x_1''\right]$$

$$= v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'').$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

- Now consider the change in CV when $p_1$ rises from $p_1'$ to $p_1''$.
- The consumer's utility for given $p_1$ is

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and CV is the extra income which, at the new prices, makes the consumer's utility the same as at the old prices. That is, ...

# Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + CV - p_1'' x_1''.$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + CV - p_1'' x_1''.$$

So

$$CV = v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'')$$
$$= \Delta CS.$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

- Now consider the change in EV when $p_1$ rises from $p_1'$ to $p_1''$.
- The consumer's utility for given $p_1$ is

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and EV is the extra income which, at the old prices, makes the consumer's utility the same as at the new prices. That is, ...

# Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + EV - p_1'' x_1''.$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + EV - p_1'' x_1''.$$

**That is,**

$$EV = v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'')$$
$$= \Delta CS.$$

# Consumer's Surplus, Compensating Variation and Equivalent Variation

**So when the consumer has quasilinear utility,**

**CV = EV = $\Delta$CS.**

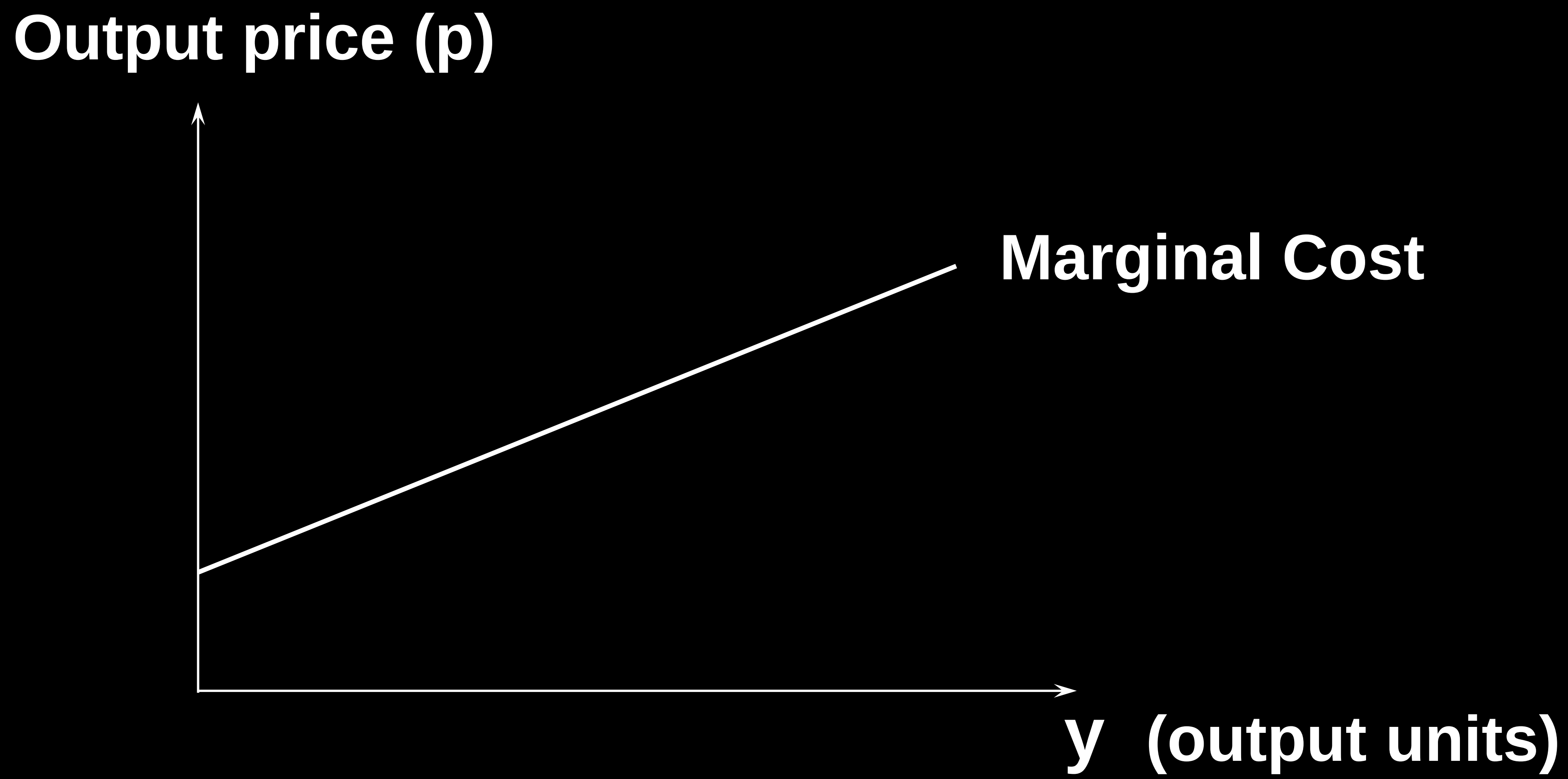
**But, otherwise, we have:**

**Relationship 2: In size, EV < $\Delta$CS < CV.**

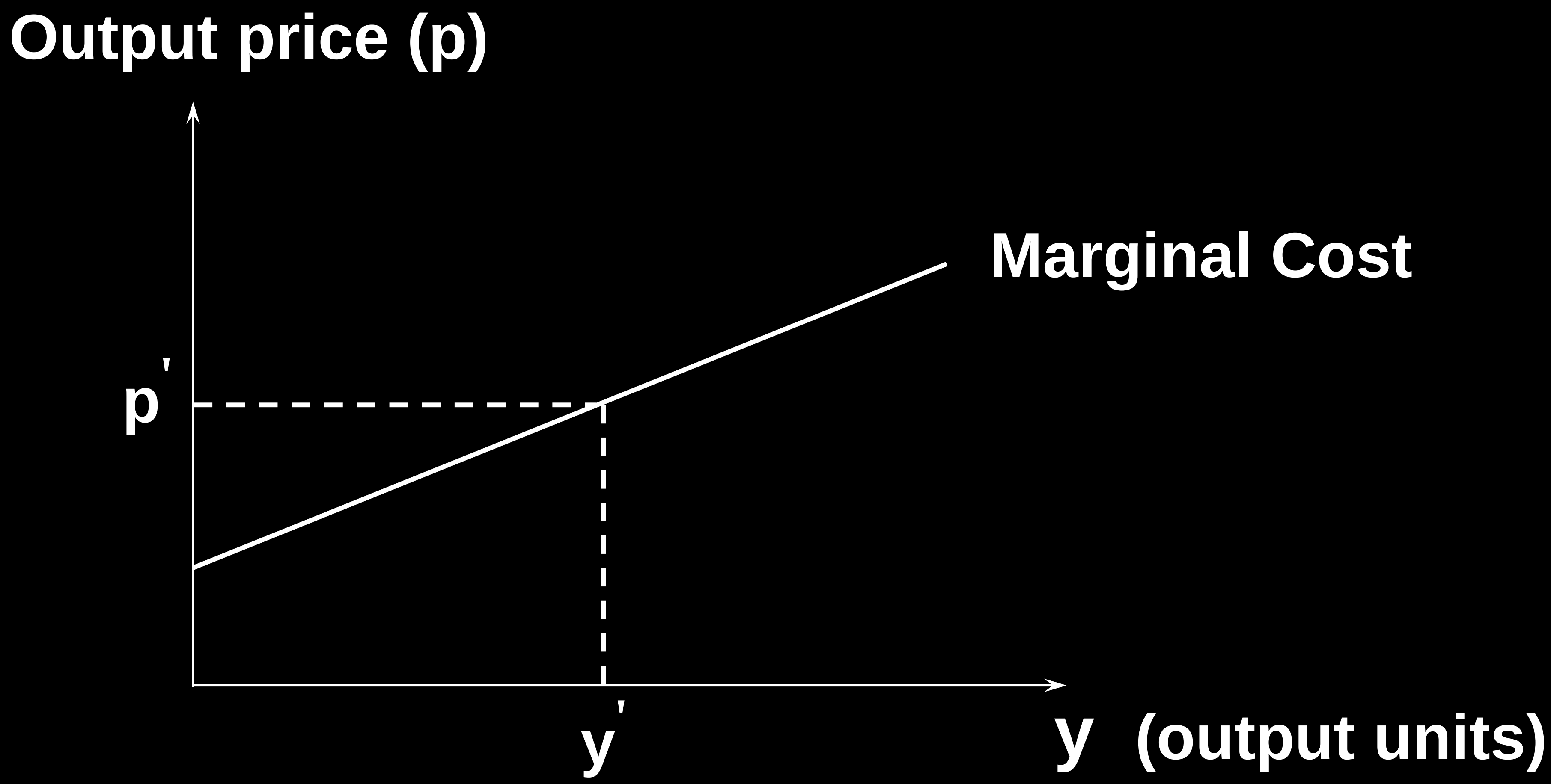
# Producer's Surplus

- Changes in a firm's welfare can be measured in dollars much as for a consumer.

# Producer's Surplus



# Producer's Surplus



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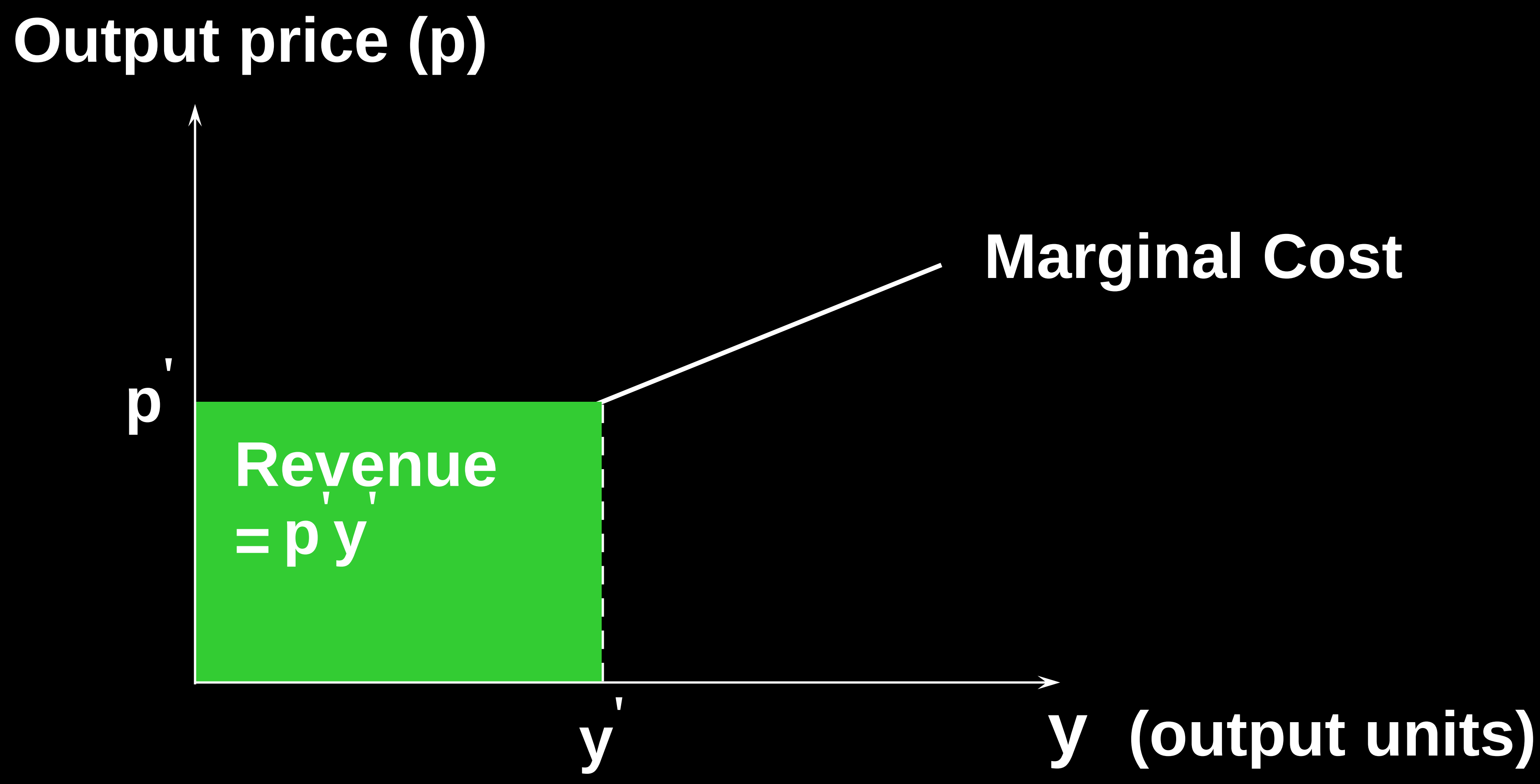
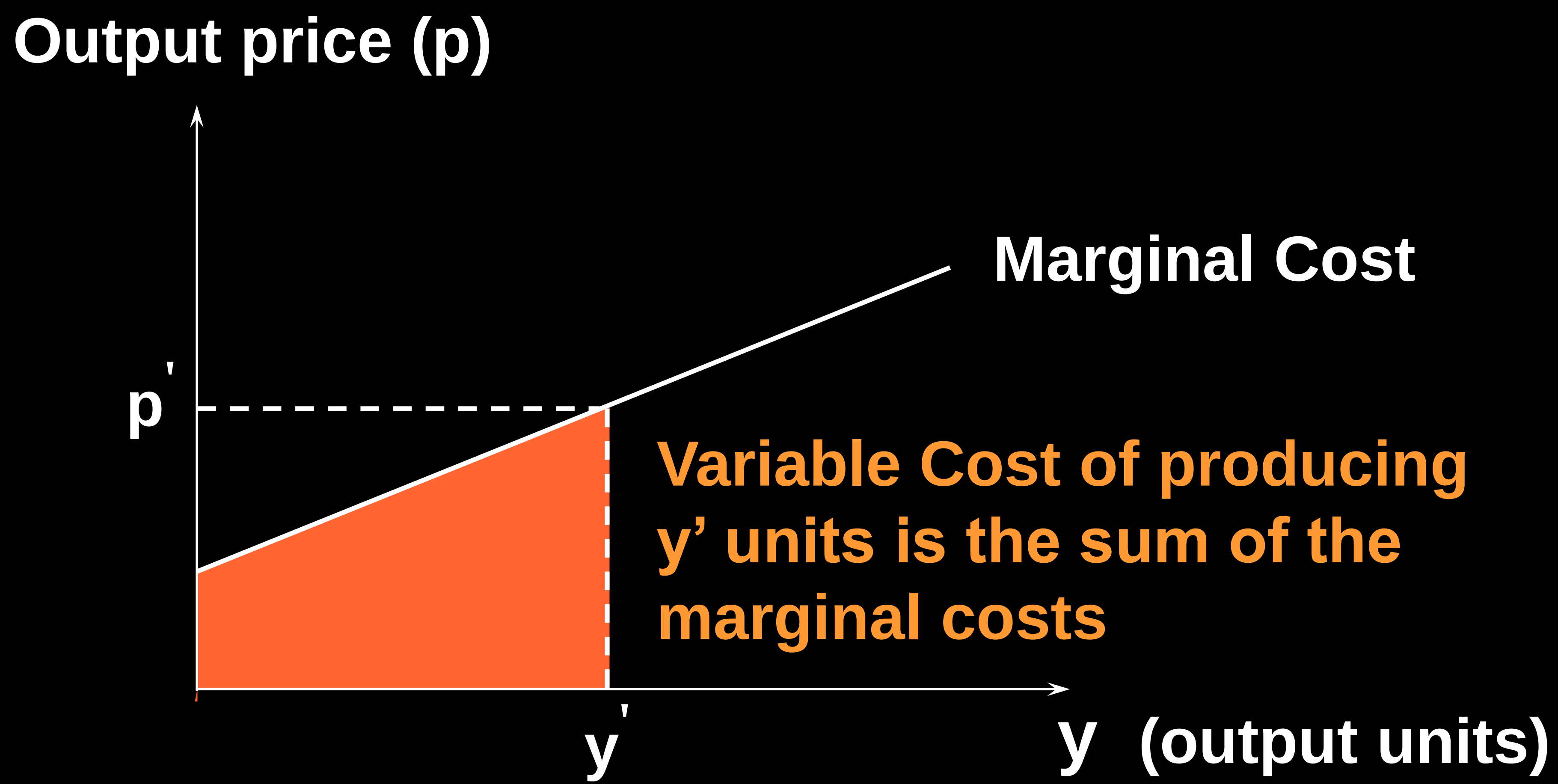


# Producer's Surplus



# Producer's Surplus