Chapter Twenty-Seven

Oligopoly

Oligopoly

A monopoly is an industry consisting a single firm.

A duopoly is an industry consisting of two firms.

An **oligopoly** is an industry consisting of a few firms. Particularly, each firm's own price or output decisions affect its competitors' profits.

Oligopoly

- How do we analyze markets in which the supplying industry is oligopolistic?
- Consider the duopolistic case of two firms supplying the same product.

Quantity Competition

- Assume that firms compete by choosing output levels.
- If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. The market price will be $p(y_1 + y_2)$.
- The firms' total cost functions are $c_1(y_1)$ and $c_2(y_2)$.

Quantity Competition

Suppose firm 1 takes firm 2's output level choice y_2 as given. Then firm 1 sees its profit function as

$$\Pi_1(y_1;y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

Given y_2, what output level y_1 maximizes firm 1's profit?

Quantity Competition; An Example

Suppose that the market inverse demand function is

$$p(y_T) = 60 - y_T$$

and that the firms' total cost functions are

$c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2.$

Quantity Competition; An Example

Then, for given y_2, firm 1's profit function is

$$\Pi(y_1; y_2) = (60 - y_1 - y_2)y_1 - y_1^2.$$

Quantity Competition; An Example

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So, given y_2, firm 1's profit-maximizing output level solves

$$\frac{\partial \Pi}{\partial y_1} = 60 - 2y_1 - y_2 - 2y_1 = 0.$$

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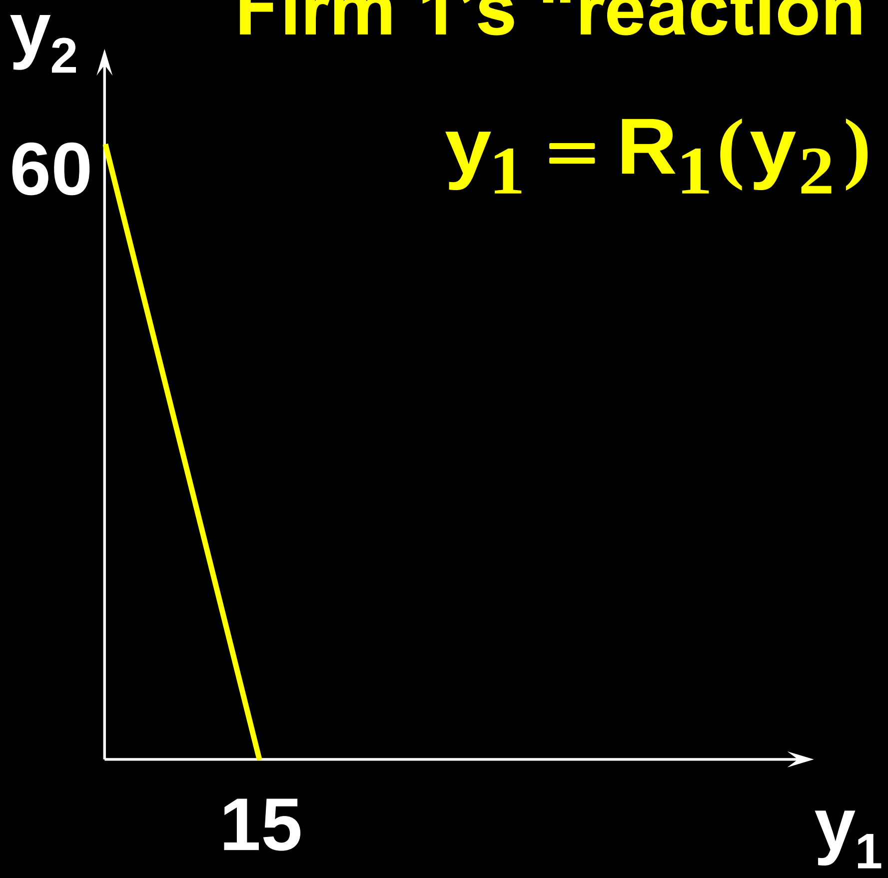
I.e. firm 1's best response to y_2 is

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Quantity Competition; An Example

Firm 1's "reaction curve"



$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Quantity Competition; An Example

- Similarly, given y_1, firm 2's profit function is

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Quantity Competition; An Example

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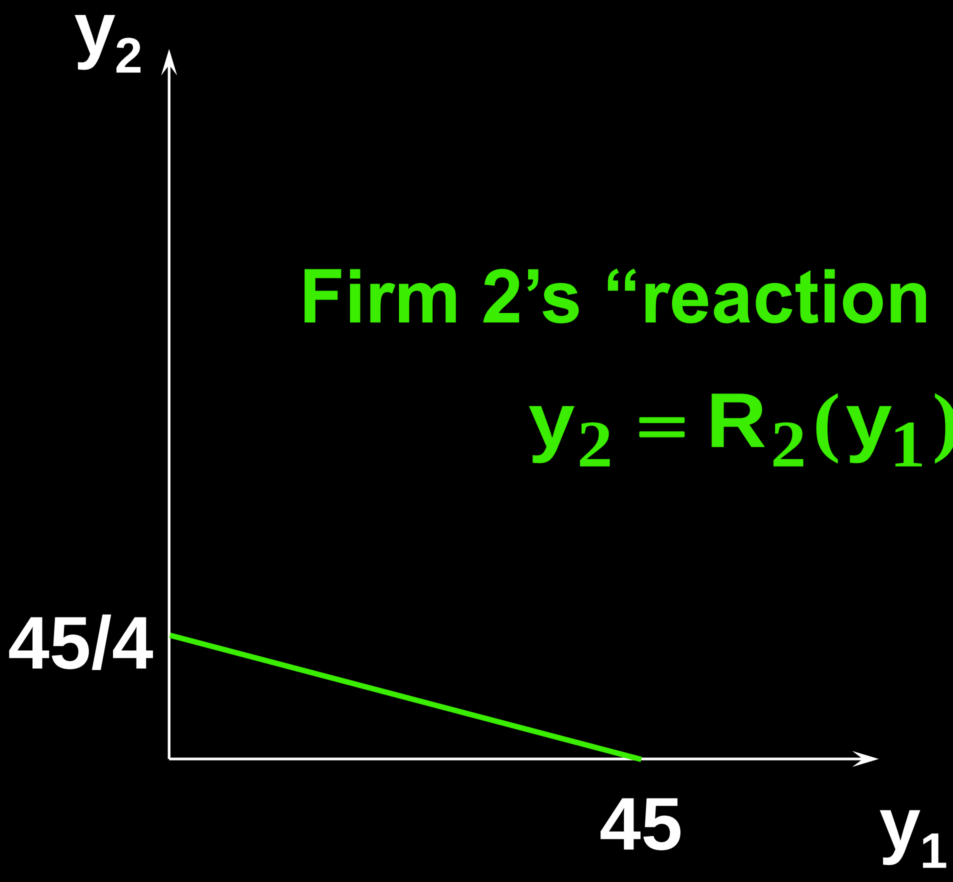
Quantity Competition; An Example



$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Quantity Competition; An Example

An equilibrium is when each firm's output level is a best response to the other firm's output level, for then neither wants to deviate from its output level.

A pair of output levels (y_1^*, y_2^*) is a Cournot-Nash equilibrium if

$$y_1^* = R_1(y_2^*) \quad \text{and} \quad y_2^* = R_2(y_1^*).$$

Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

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Substitute for y_2* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right)$$

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Hence

$$y_2^* = \frac{45 - 13}{4} = 8.$$

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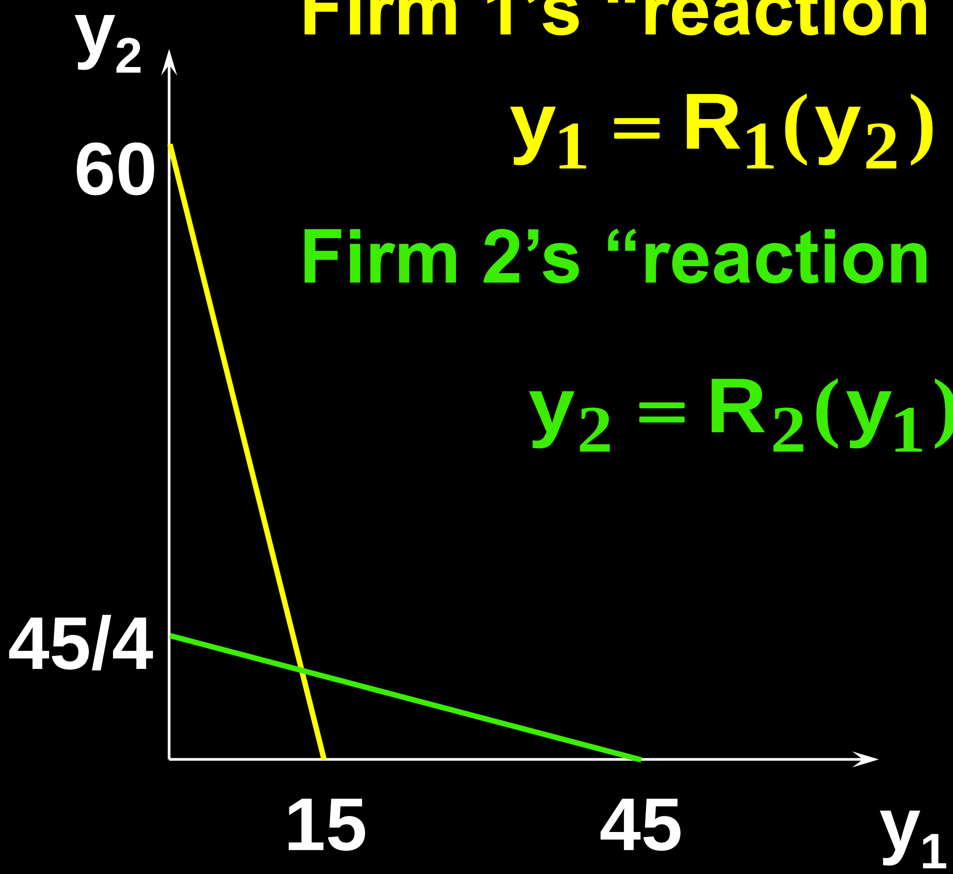
Hence $$y_2^* = \frac{45 - 13}{4} = 8.$$

So the Cournot-Nash equilibrium is

$$(y_1^*, y_2^*) = (13, 8).$$

Quantity Competition; An Example



Firm 1's "reaction curve"

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Firm 2's "reaction curve"

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Quantity Competition; An Example

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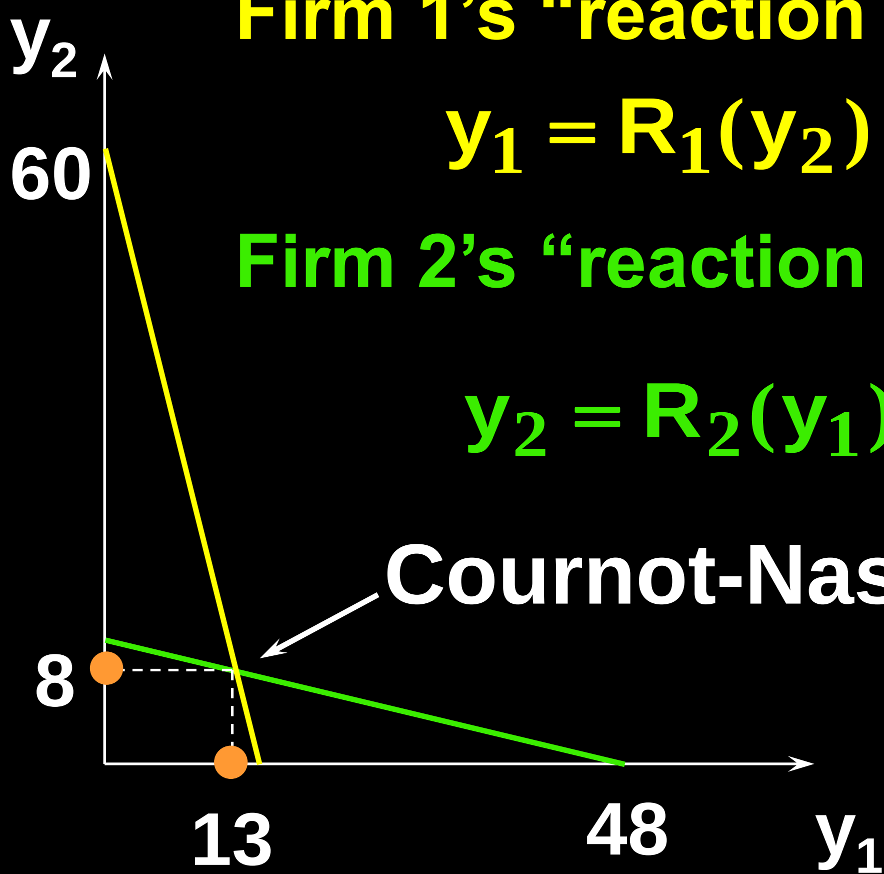


Cournot-Nash equilibrium

$$\left(y_1^*, y_2^*\right) = (13, 8).$$

Quantity Competition

Generally, given firm 2's chosen output level y_2, firm 1's profit function is

$$\Pi_1(y_1;y_2) = p(y_1 + y_2)y_1 - c_1(y_1)$$

and the profit-maximizing value of y_1 solves

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1 + y_2) + y_1 \frac{\partial p(y_1 + y_2)}{\partial y_1} - c_1{}'(y_1) = 0.$$

The solution, $y_1 = R_1(y_2)$, is firm 1's Cournot-Nash reaction to y_2.

Quantity Competition

- Similarly, given firm 1's chosen output level y_1, firm 2's profit function is

$$\Pi_2(y_2;y_1) = p(y_1 + y_2)y_2 - c_2(y_2)$$

and the profit-maximizing value of y_2 solves

$$\frac{\partial \Pi_2}{\partial y_2} = p(y_1 + y_2) + y_2 \frac{\partial p(y_1 + y_2)}{\partial y_2} - c_2{}'(y_2) = 0.$$

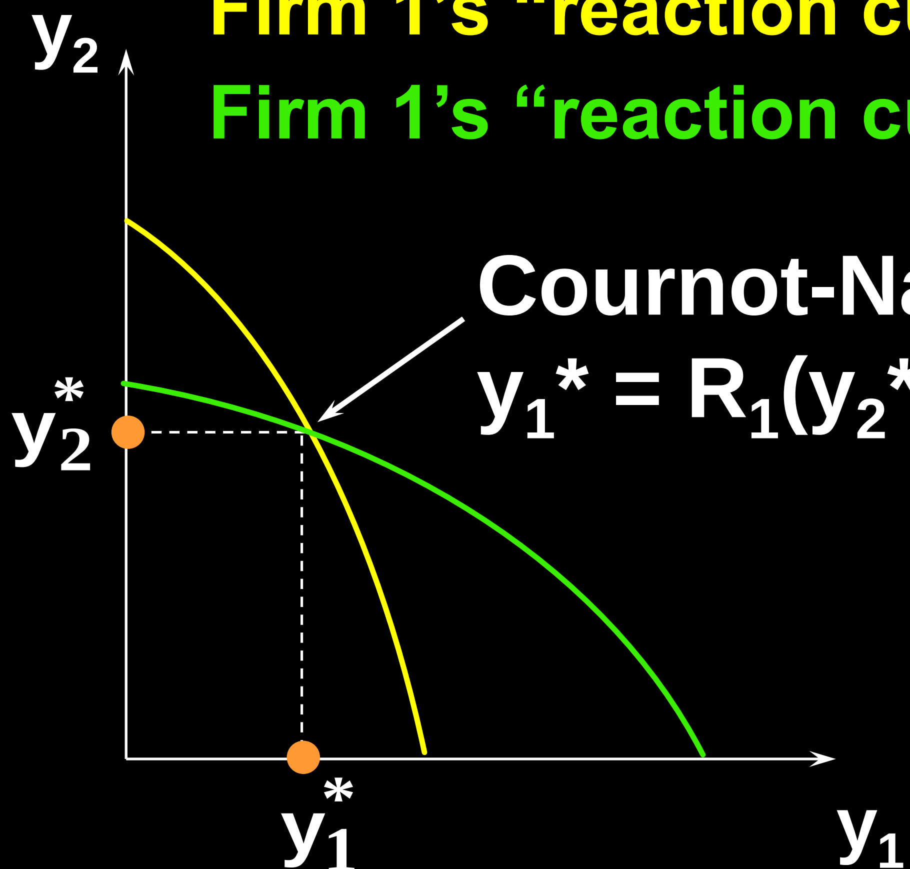
- The solution, $y_2 = R_2(y_1)$, is firm 2's Cournot-Nash reaction to y_1.

Quantity Competition

Firm 1's "reaction curve" $y_1 = R_1(y_2)$.

Firm 1's "reaction curve" $y_2 = R_2(y_1)$.

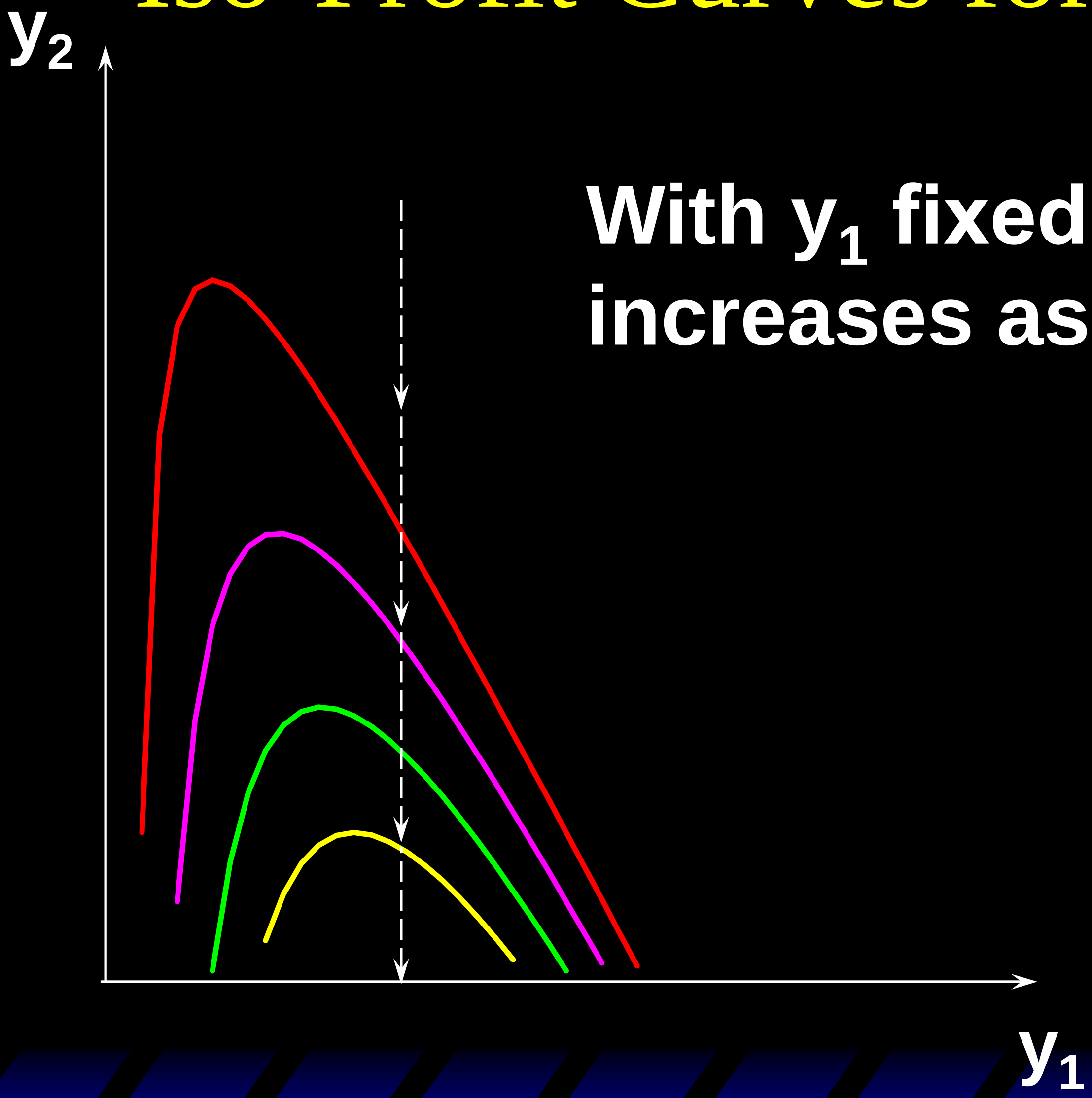
Cournot-Nash equilibrium
$y_1^* = R_1(y_2^*)$ and $y_2^* = R_2(y_1^*)$



Iso-Profit Curves

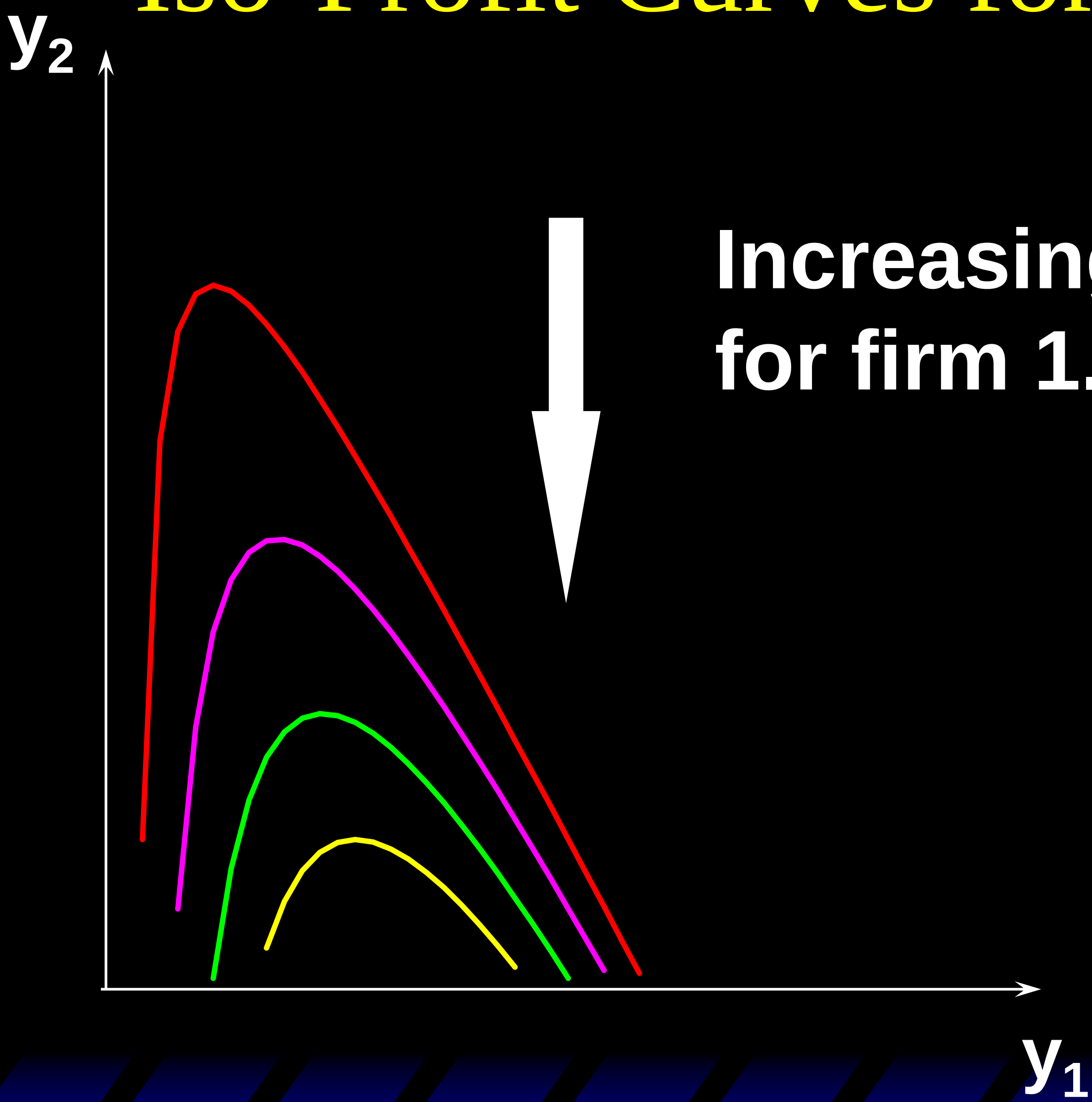
- For firm 1, an iso-profit curve contains all the output pairs (y_1,y_2) giving firm 1 the same profit level Π_1.
- What do iso-profit curves look like?

Iso-Profit Curves for Firm 1



With y_1 fixed, firm 1's profit increases as y_2 decreases.

Iso-Profit Curves for Firm 1



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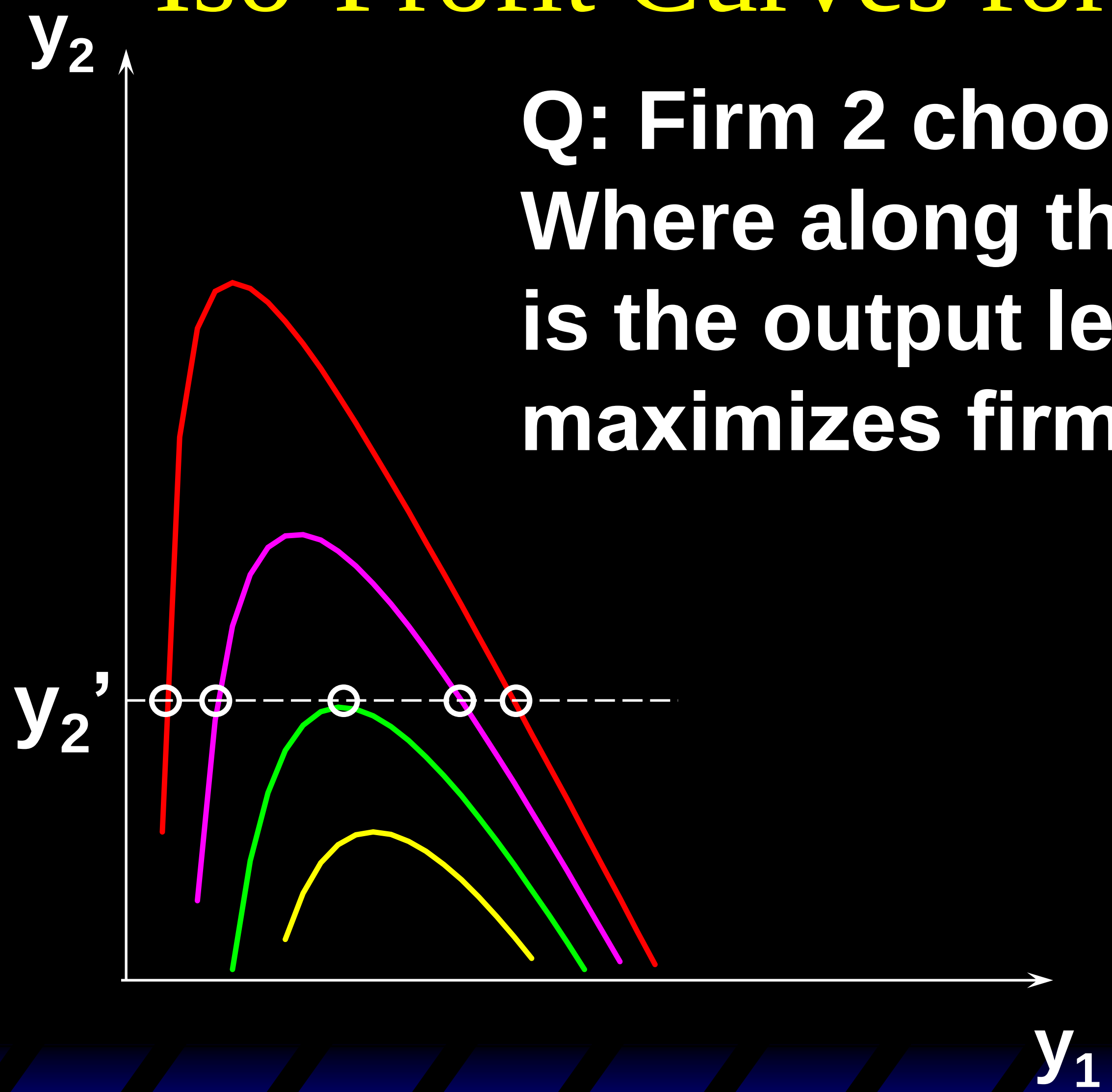

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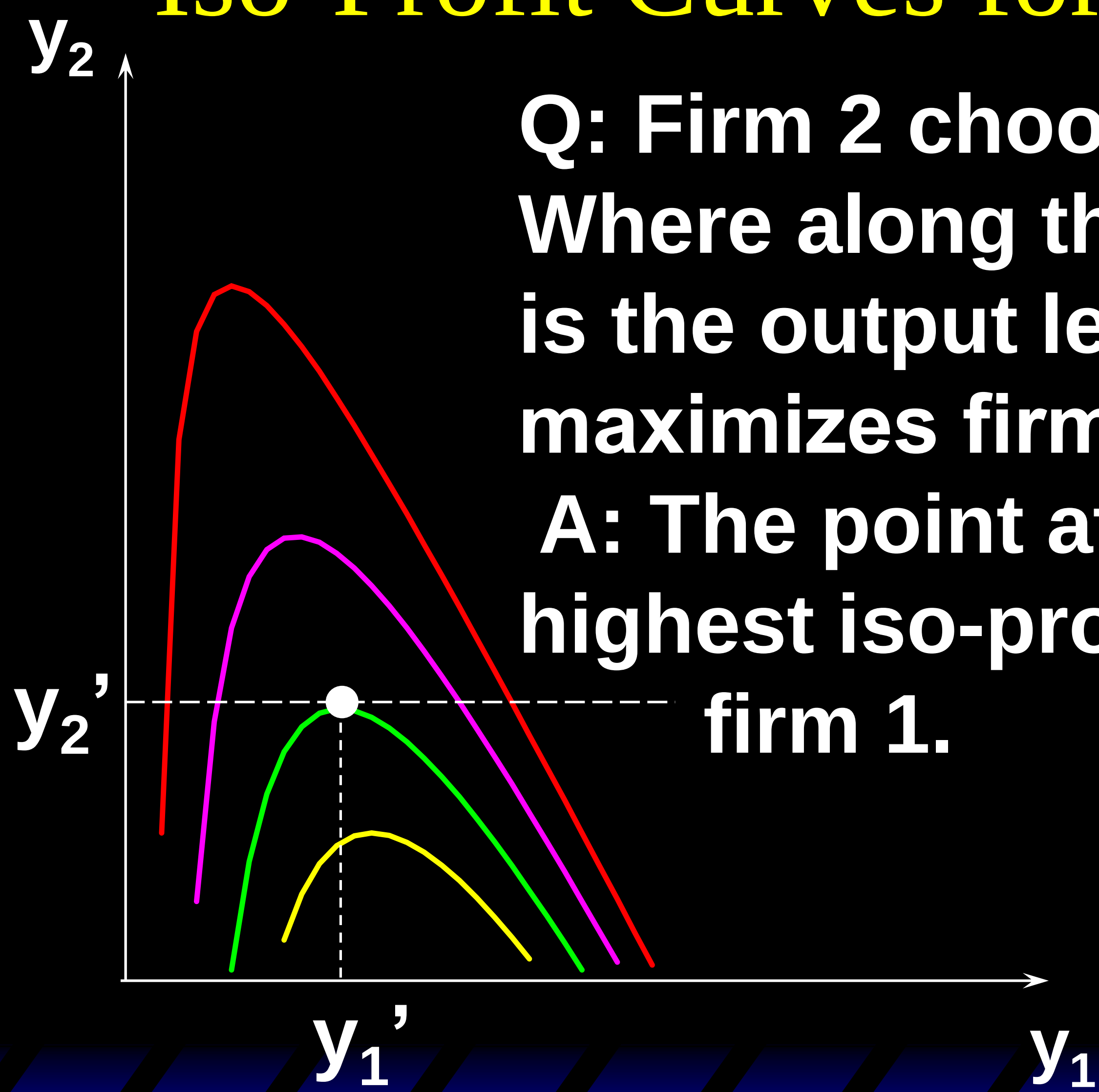
Iso-Profit Curves for Firm 1



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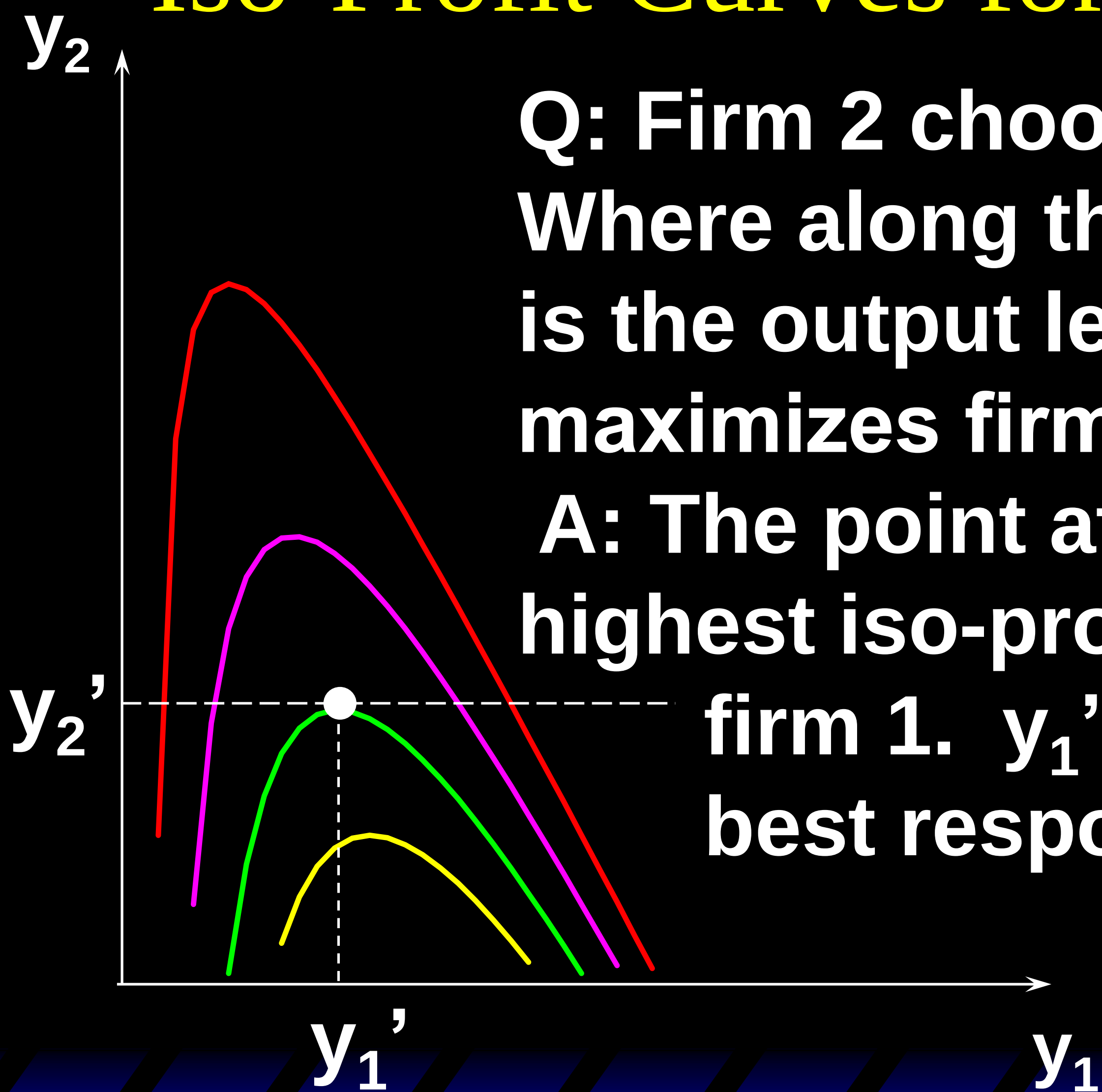

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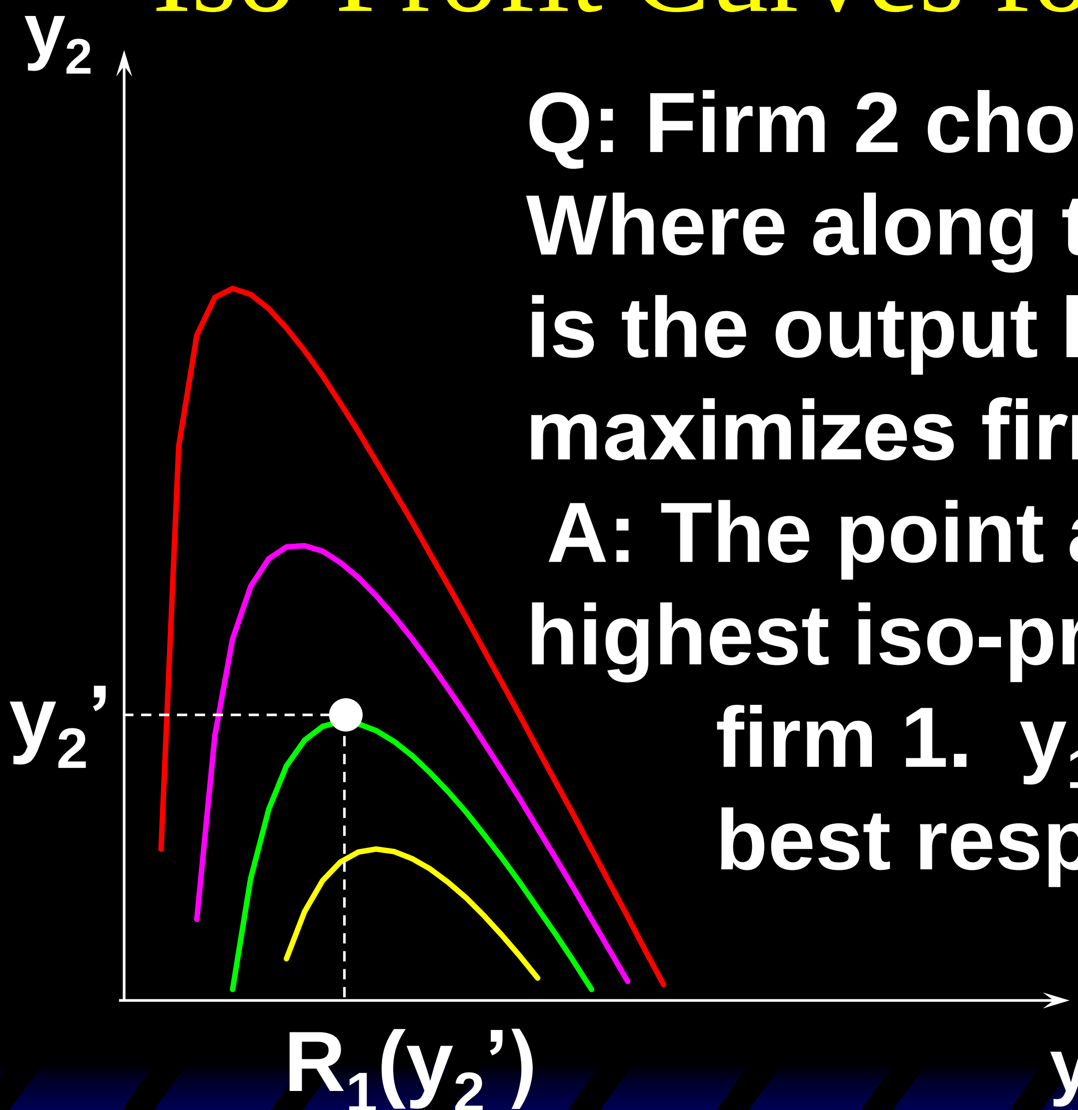

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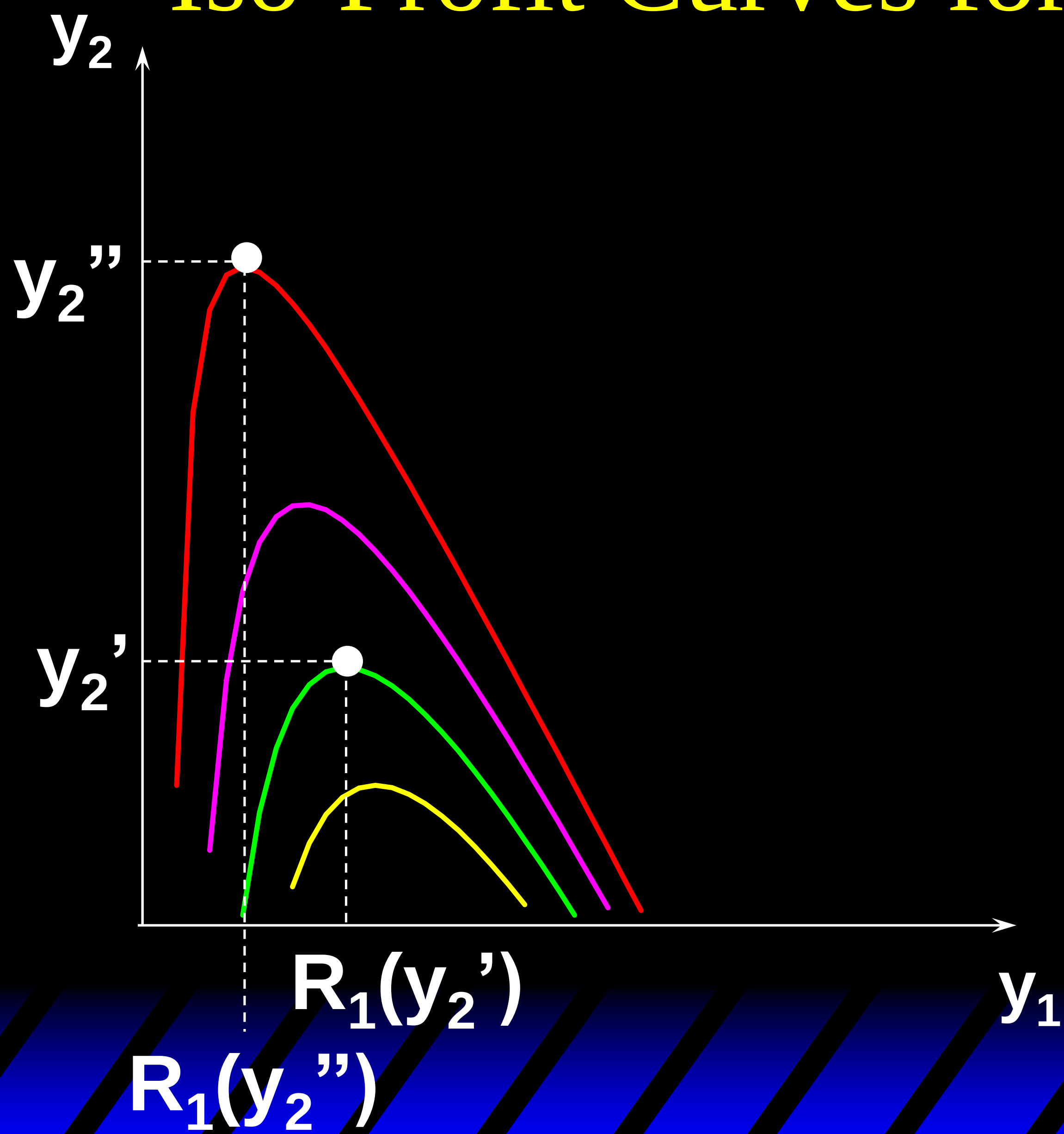

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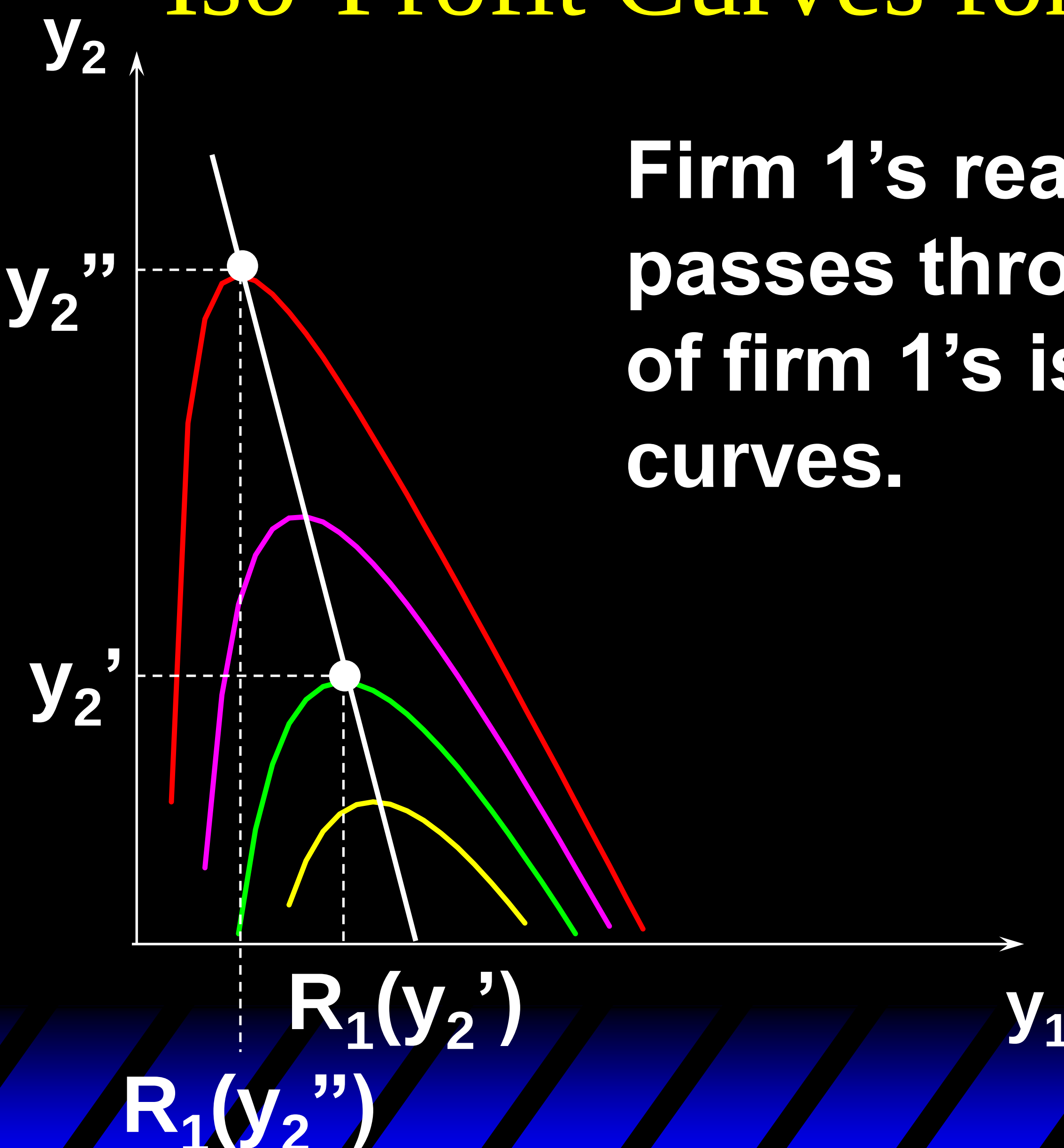
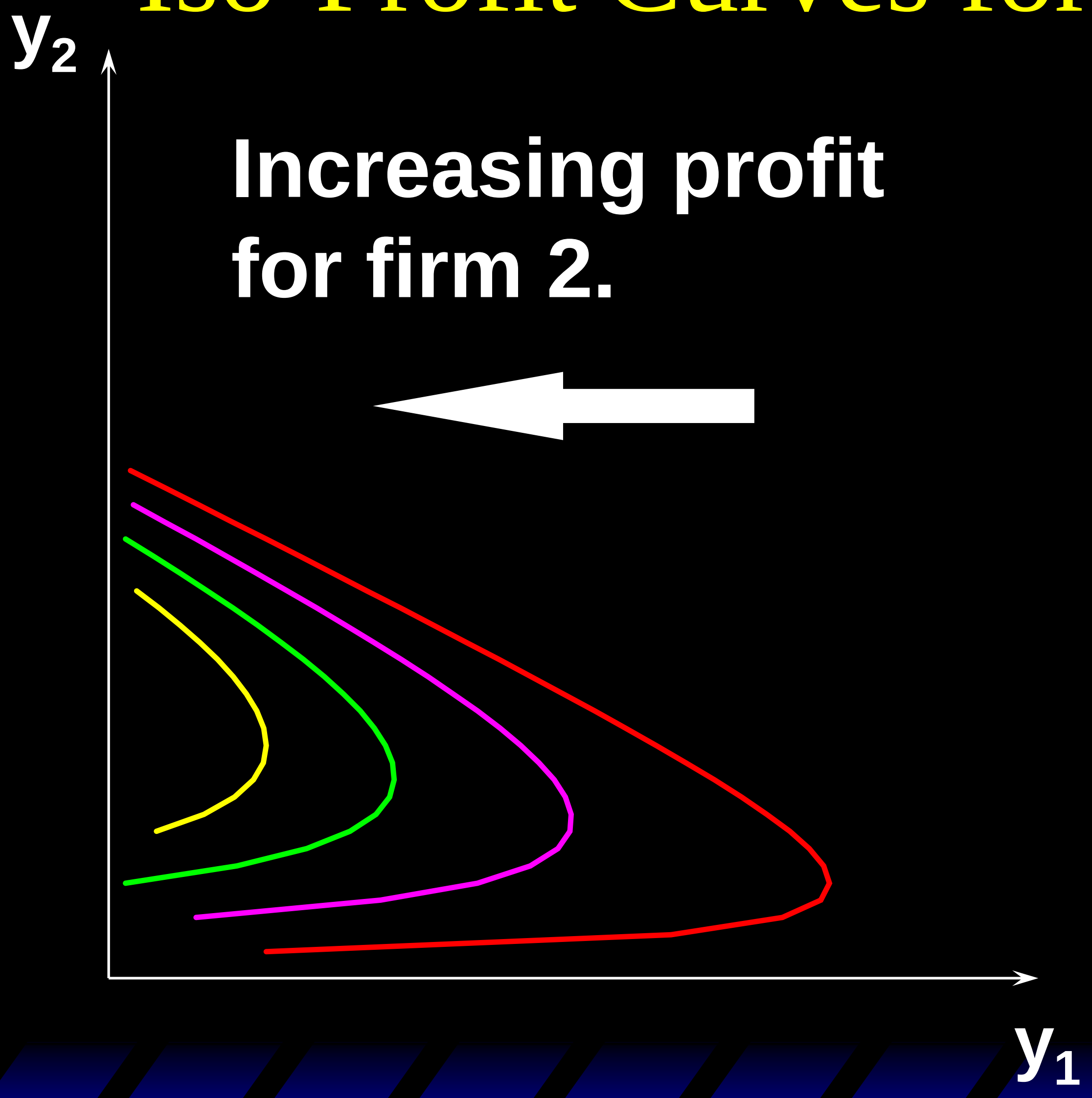
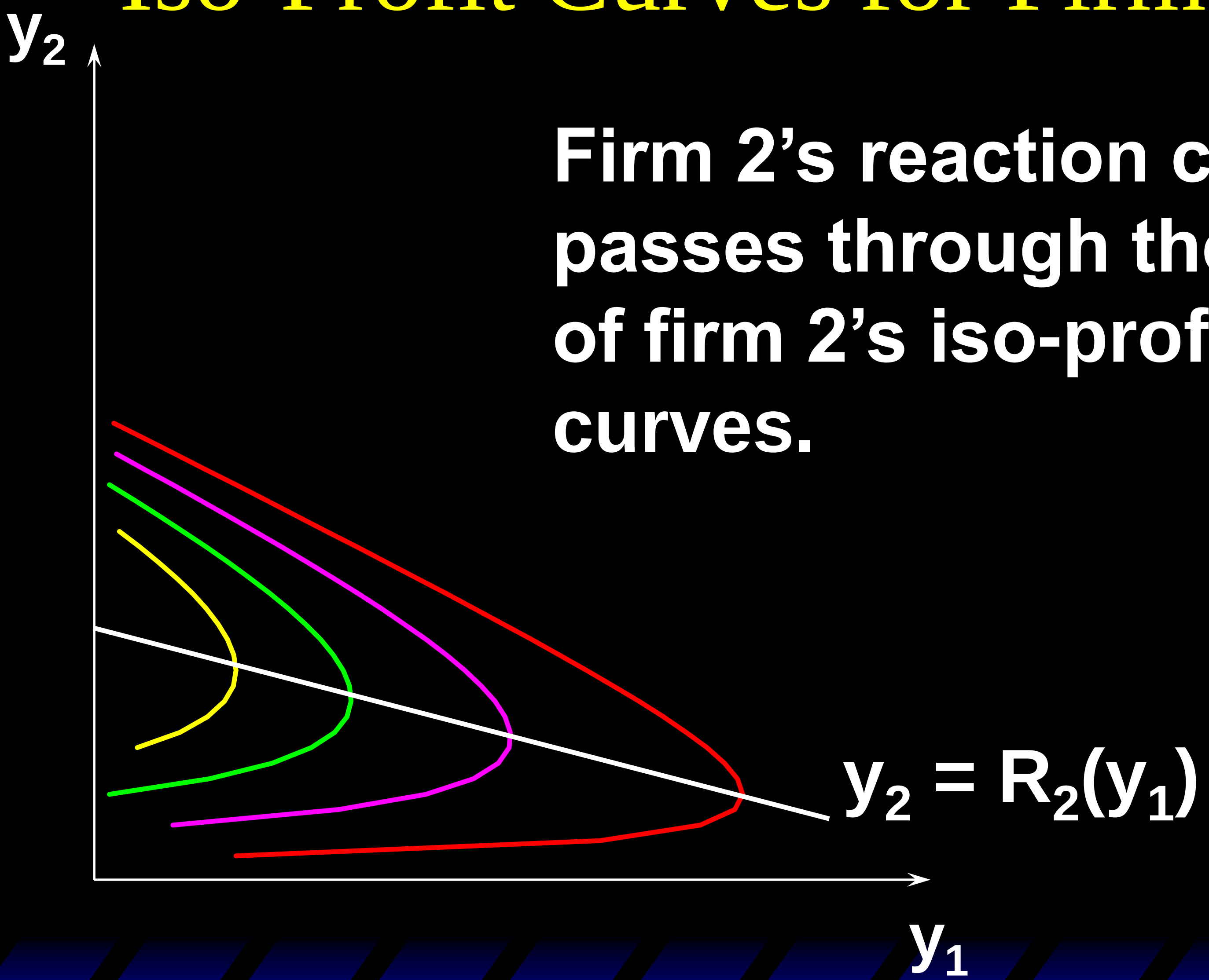


Firm 1’s reaction curve passes through the “tops” of firm 1’s iso-profit curves.

Iso-Profit Curves for Firm 2



Iso-Profit Curves for Firm 2

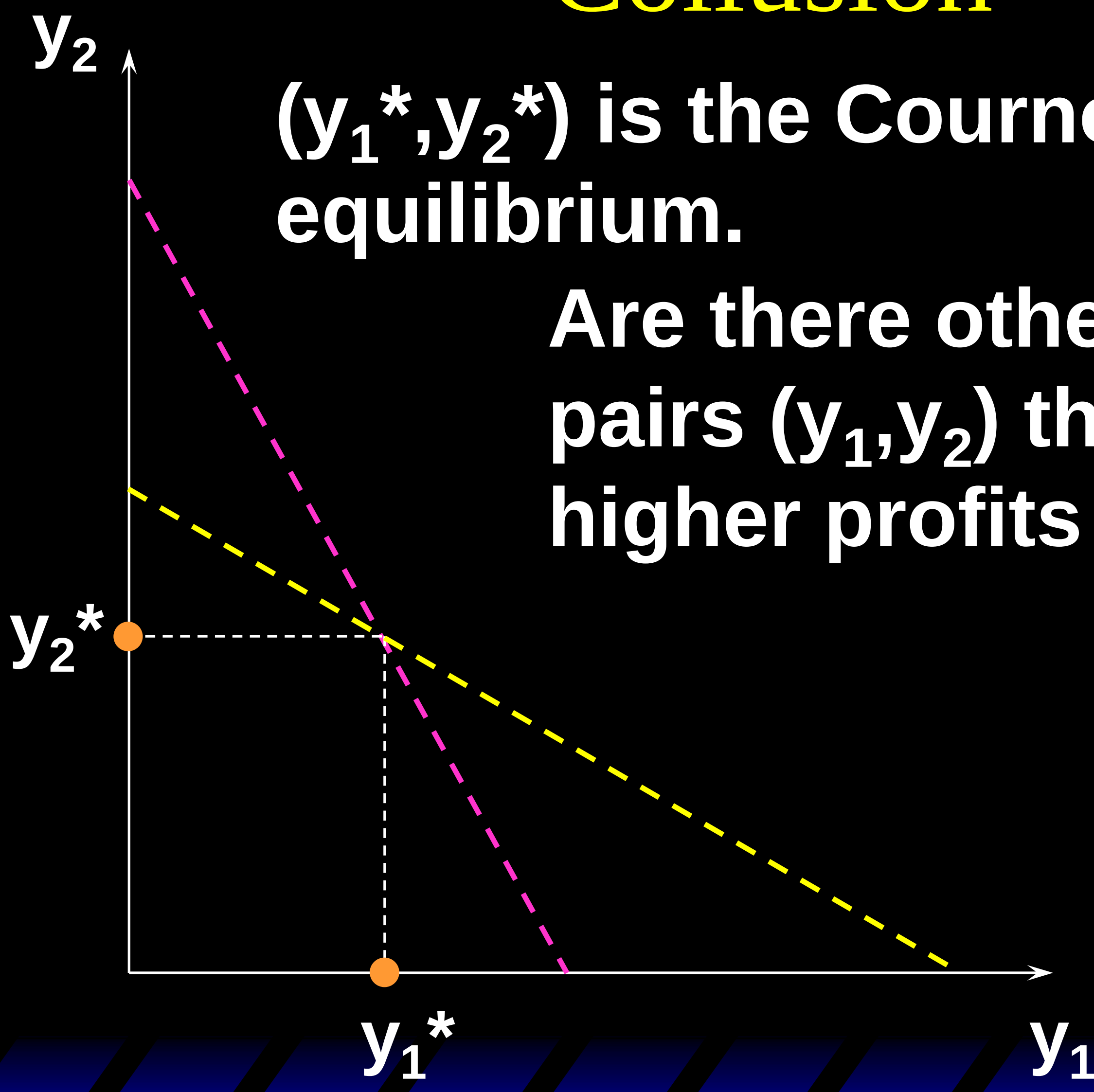


Firm 2's reaction curve passes through the "tops" of firm 2's iso-profit curves.

Collusion

Q: Are the Cournot-Nash equilibrium profits the largest that the firms can earn in total?

Collusion



(y_1^*,y_2^*) is the Cournot-Nash equilibrium.

Are there other output level pairs (y_1,y_2) that give higher profits to both firms?

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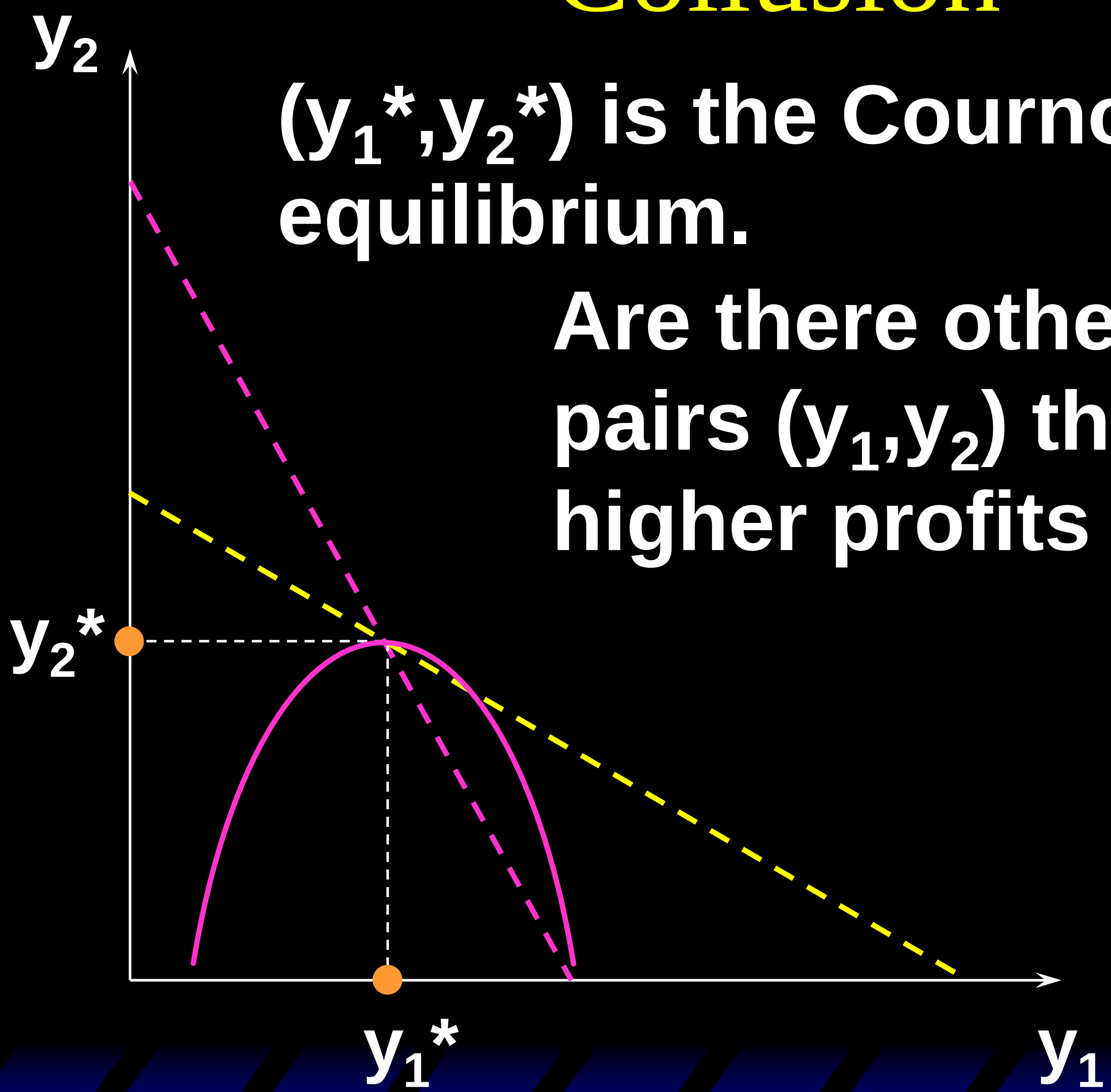

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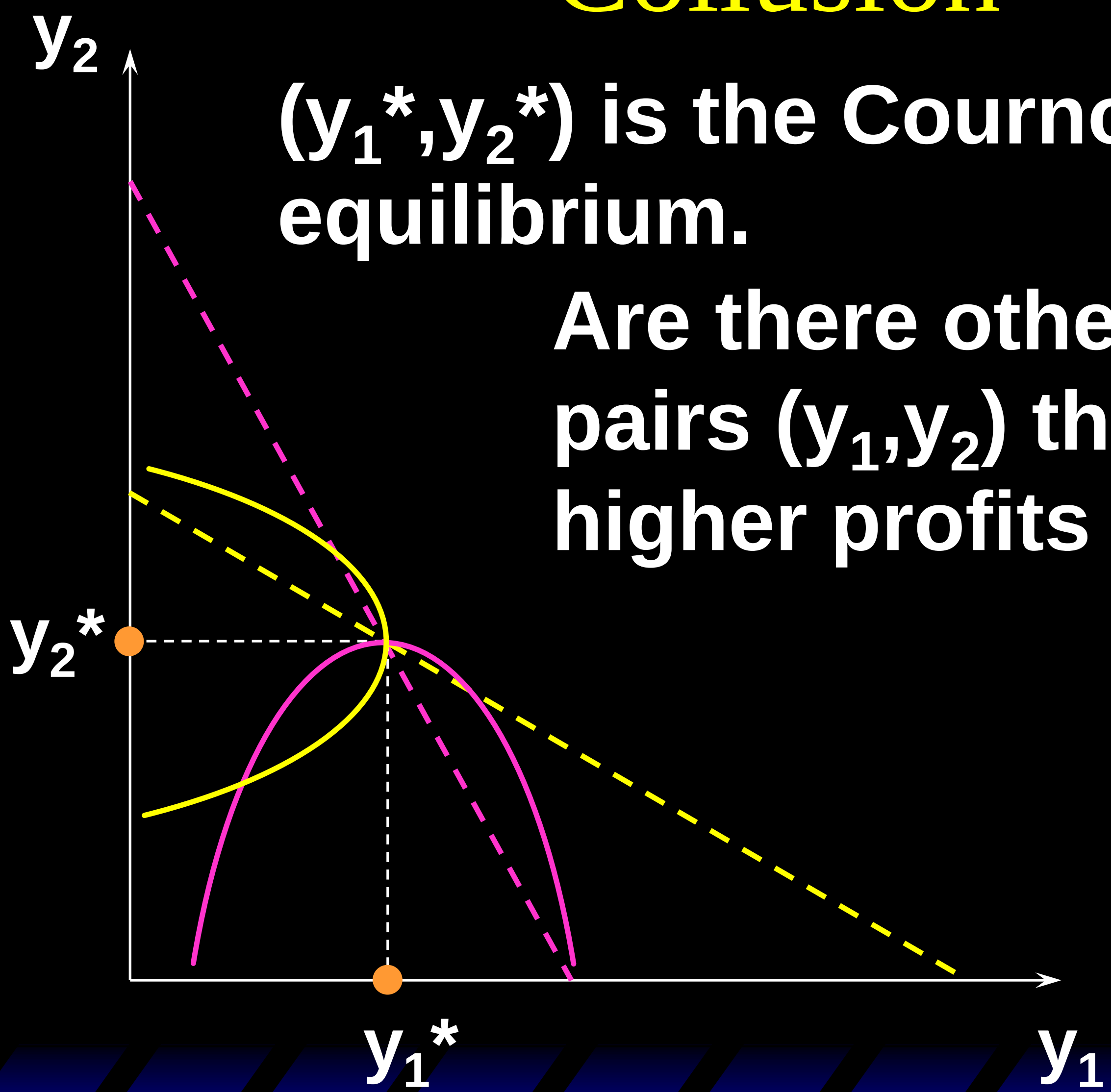

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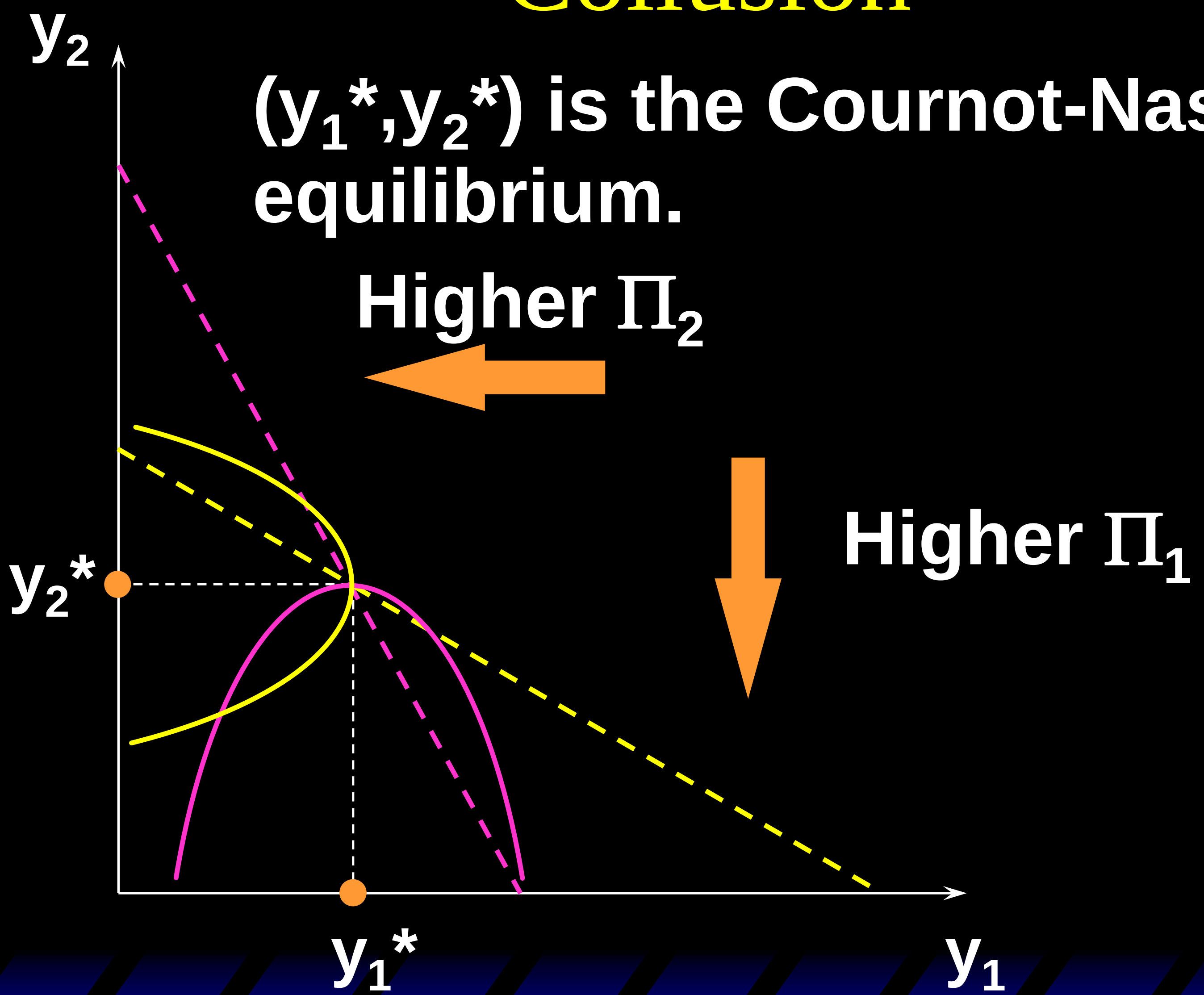

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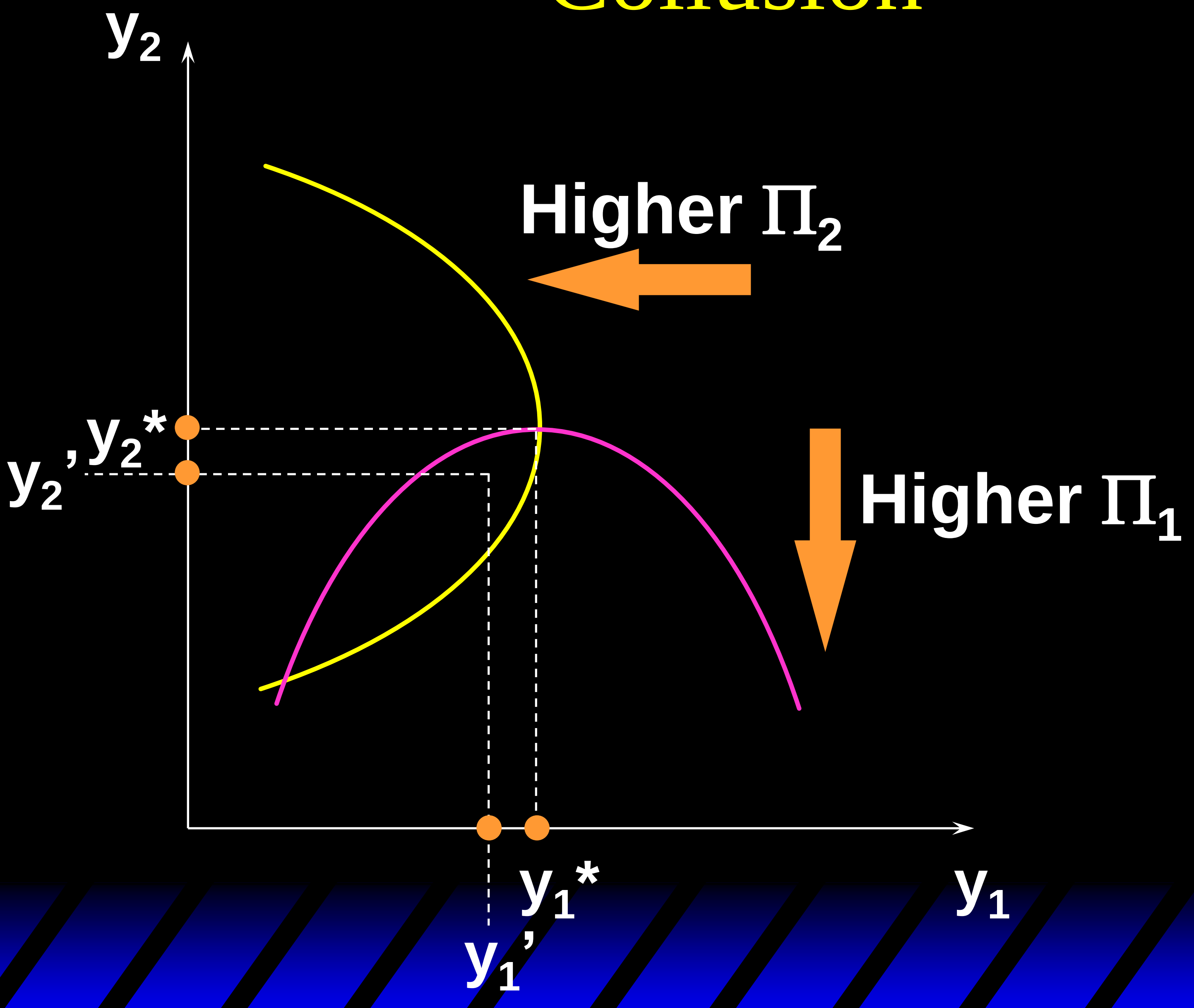

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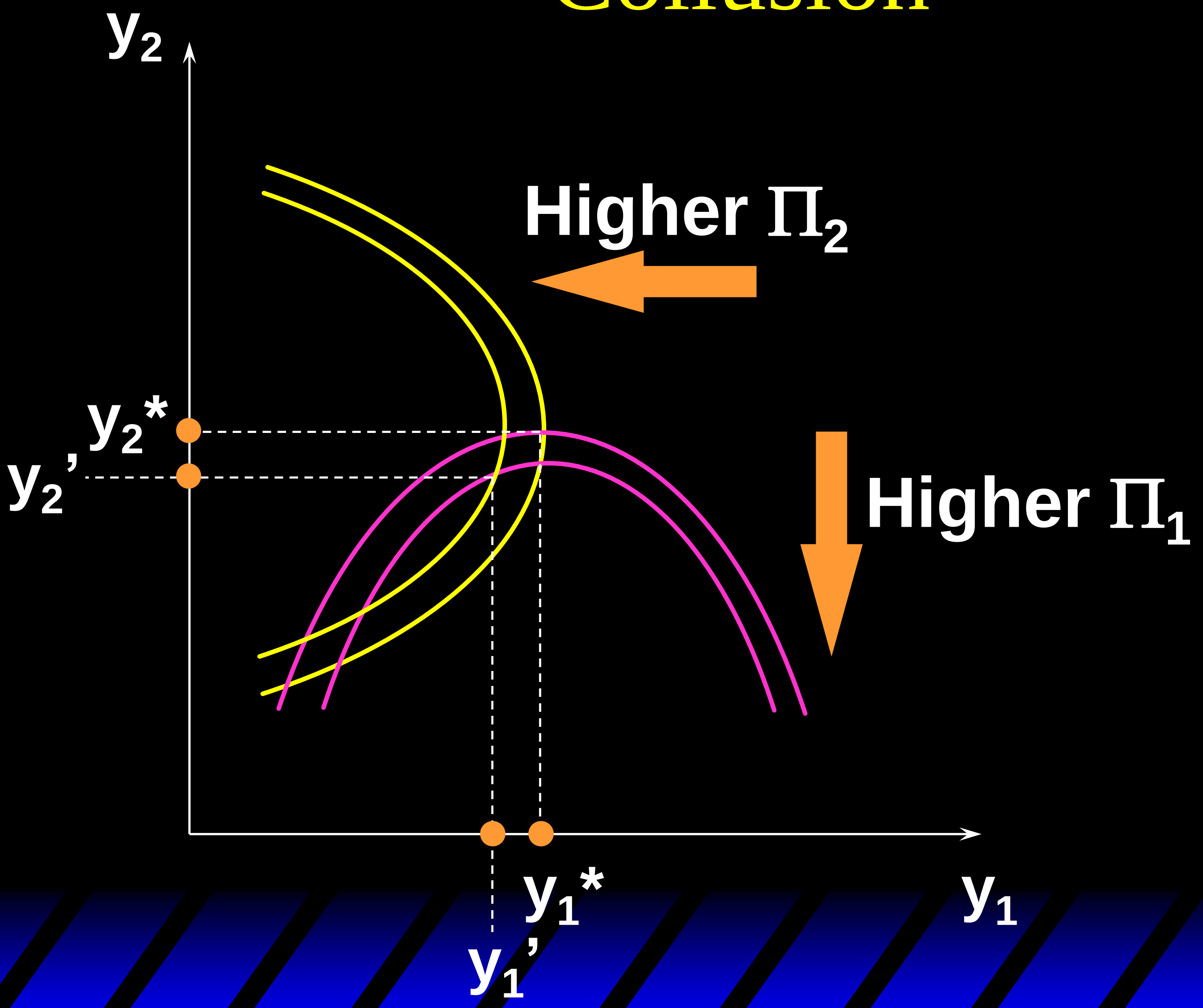

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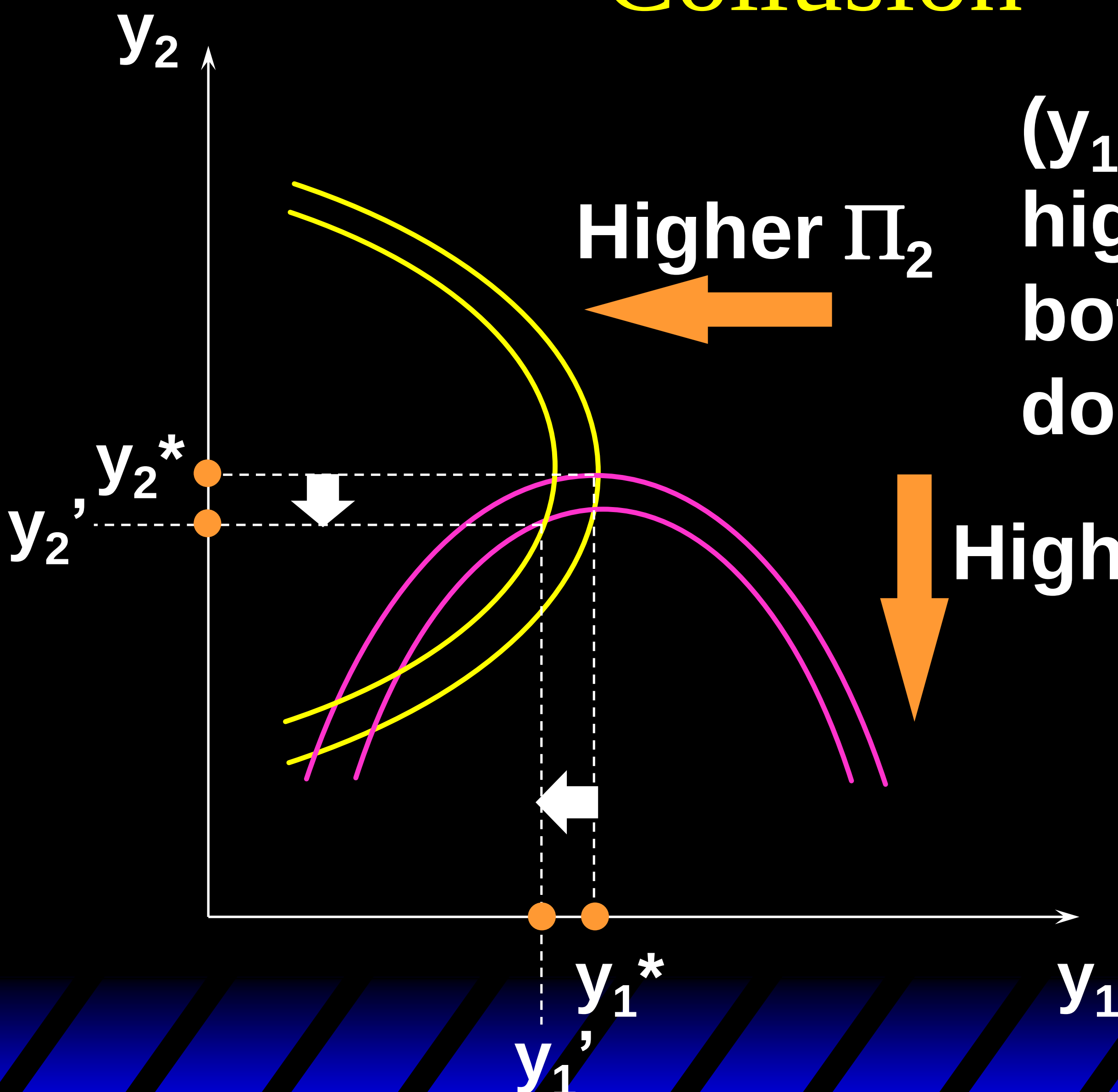


(y_1',y_2') earns higher profits for both firms than does (y_1*,y_2*).

Collusion

- So there are profit incentives for both firms to "cooperate" by lowering their output levels.
- This is **collusion**.
- Firms that collude are said to have formed a **cartel**.
- If firms form a cartel, how should they do it?

Collusion

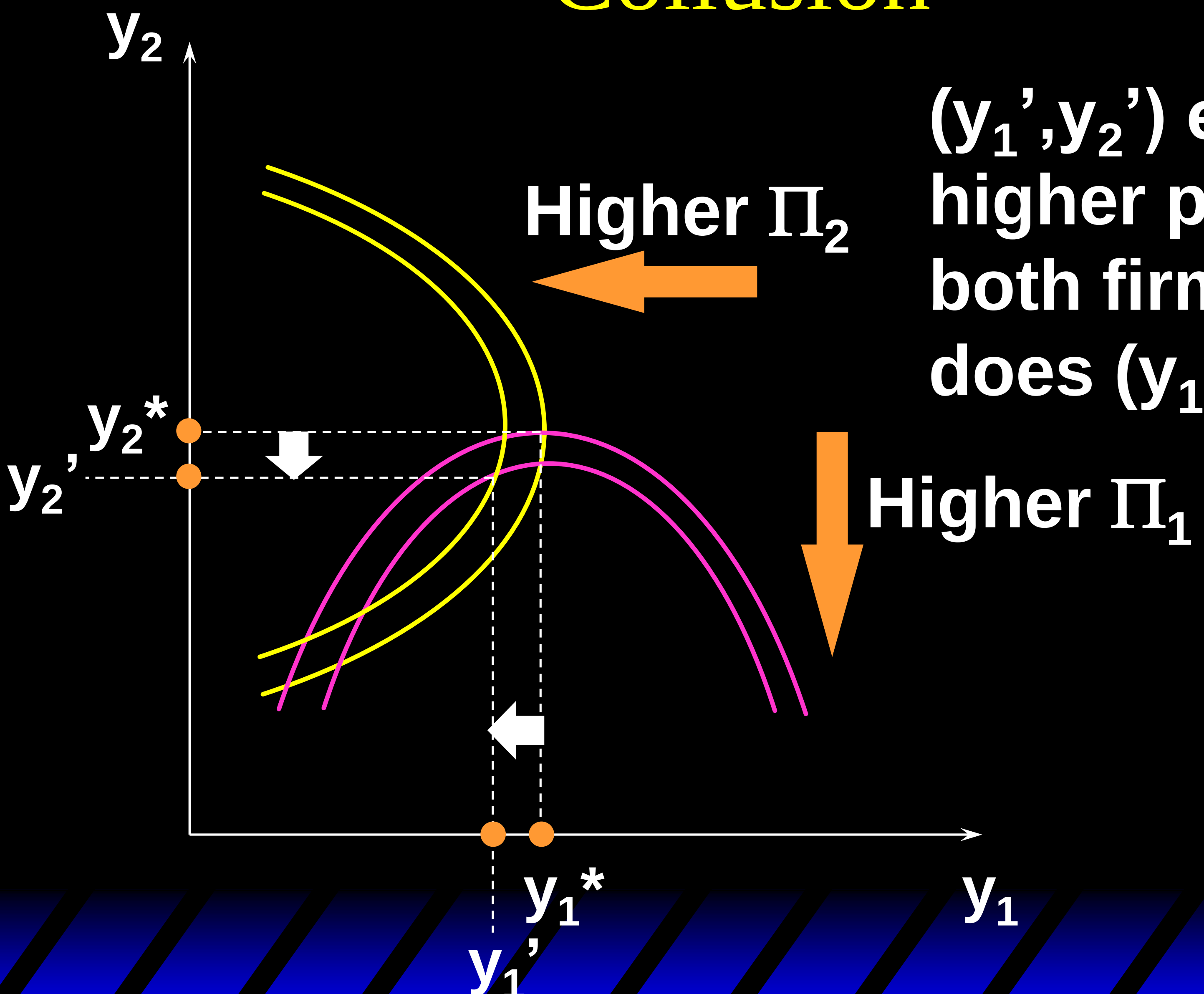
Suppose the two firms want to maximize their total profit and divide it between them. Their goal is to choose cooperatively output levels y_1 and y_2 that maximize

$$\Pi^m(y_1, y_2) = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2).$$

Collusion

The firms cannot do worse by colluding since they can cooperatively choose their Cournot-Nash equilibrium output levels and so earn their Cournot-Nash equilibrium profits. So collusion must provide profits at least as large as their Cournot-Nash equilibrium profits.

Collusion



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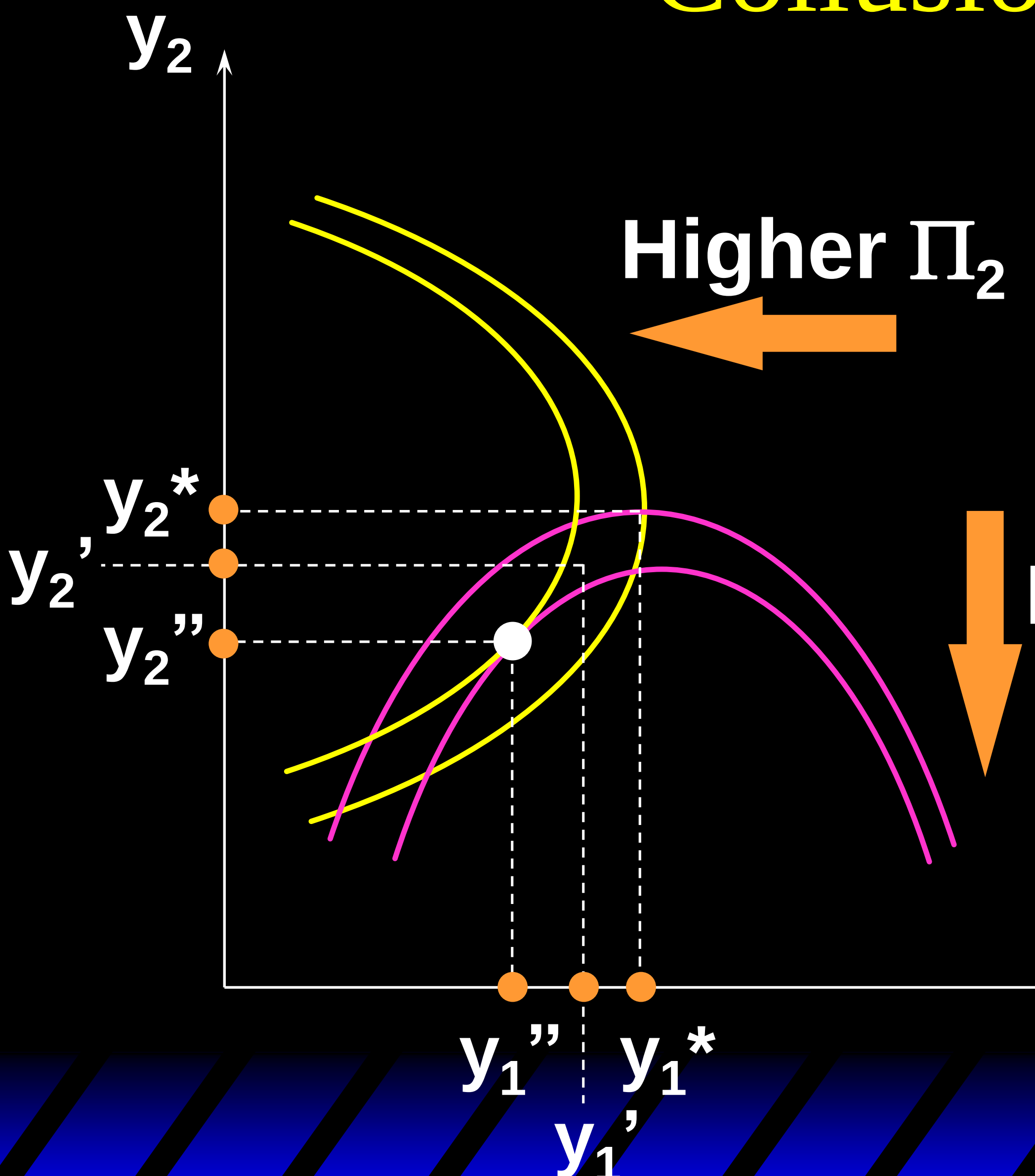
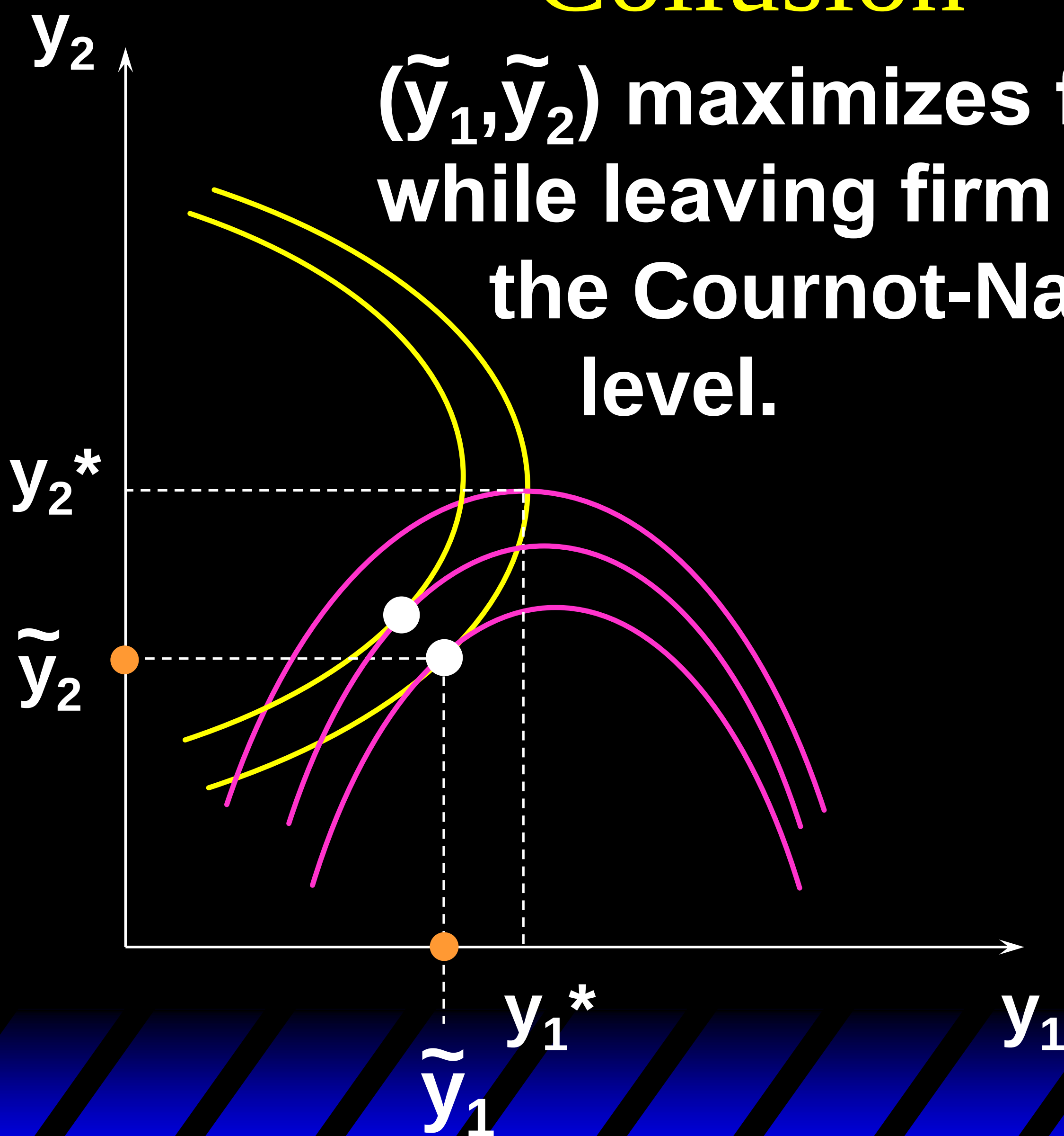


(y_1', y_2') earns higher profits for both firms than does (y_1^*, y_2^*).

(y_1'', y_2'') earns still higher profits for both firms.

Collusion

$(\tilde{y}_1, \tilde{y}_2)$ maximizes firm 1's profit while leaving firm 2's profit at the Cournot-Nash equilibrium level.



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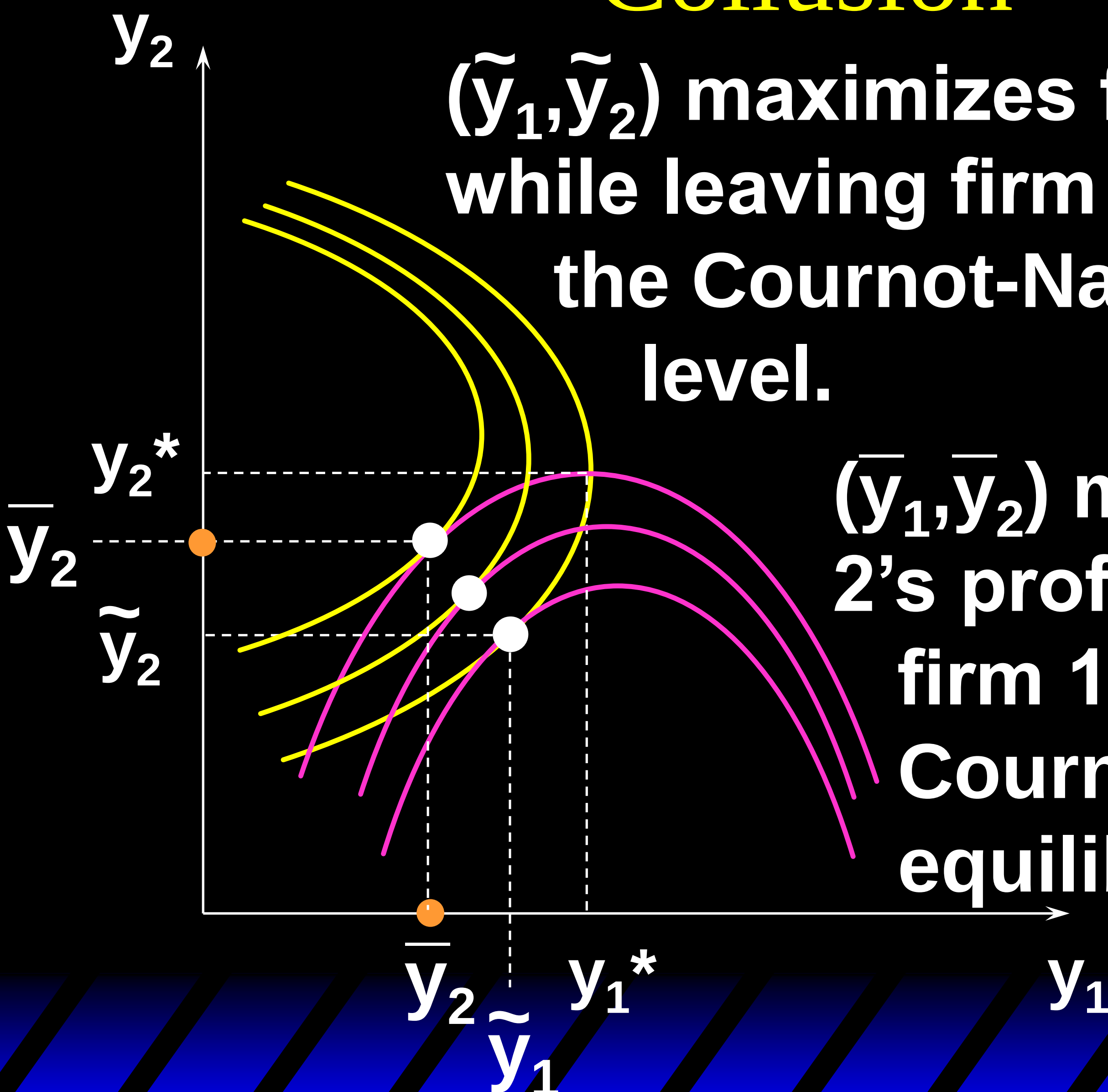
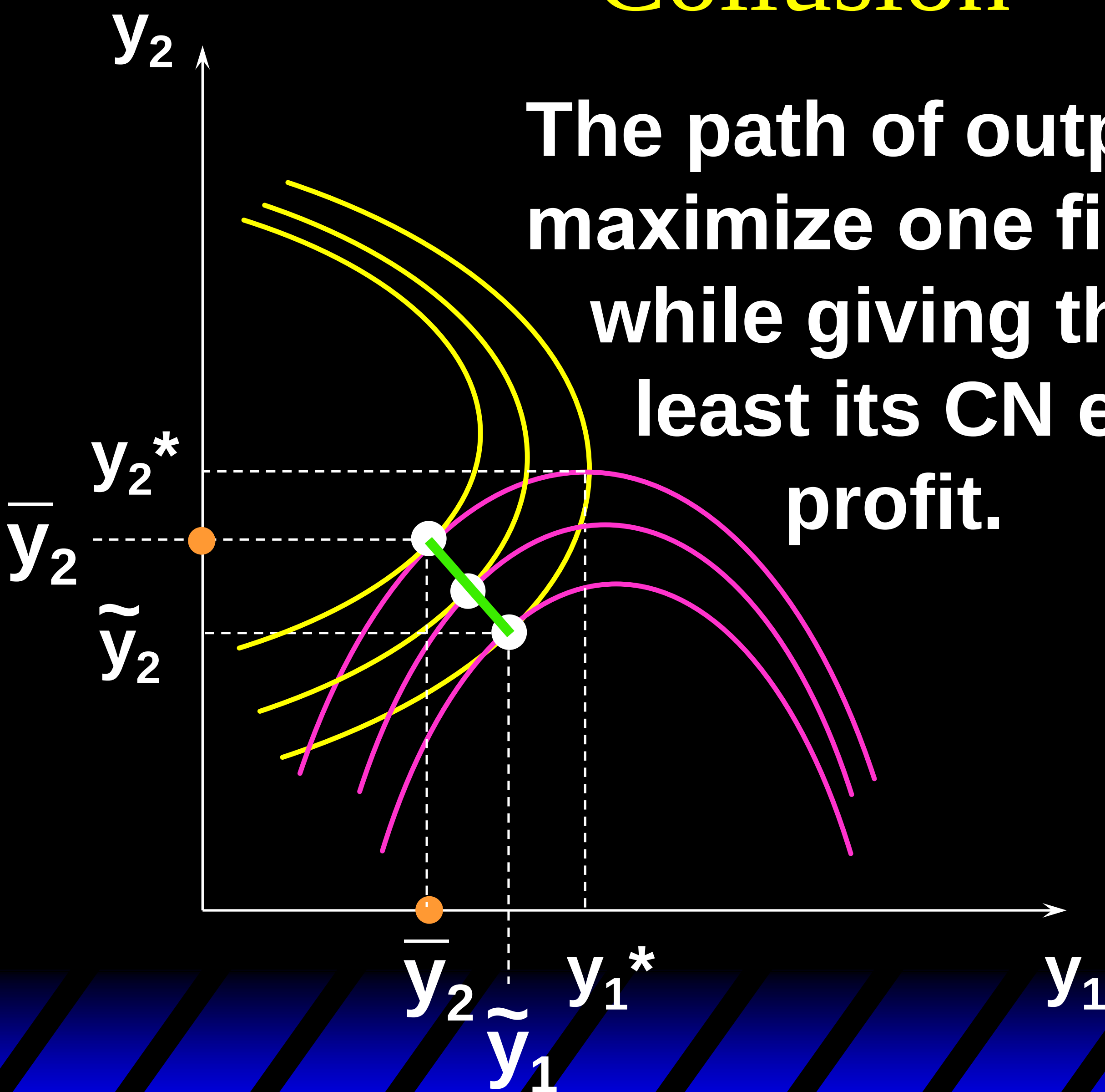


$(\tilde{y}_1,\tilde{y}_2)$ maximizes firm 1's profit while leaving firm 2's profit at the Cournot-Nash equilibrium level.

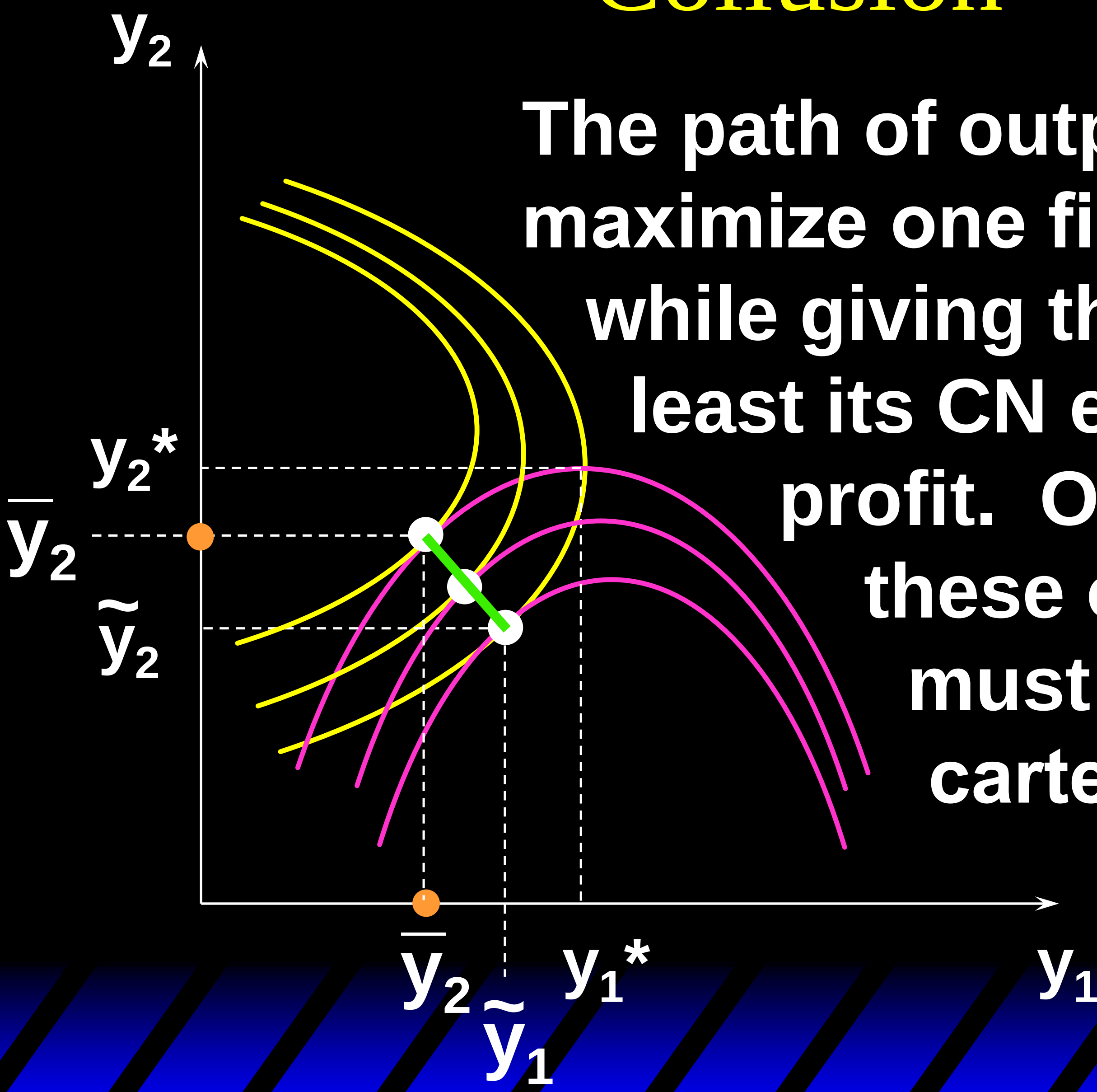
$(\bar{y}_1,\bar{y}_2)$ maximizes firm 2's profit while leaving firm 1's profit at the Cournot-Nash equilibrium level.

Collusion



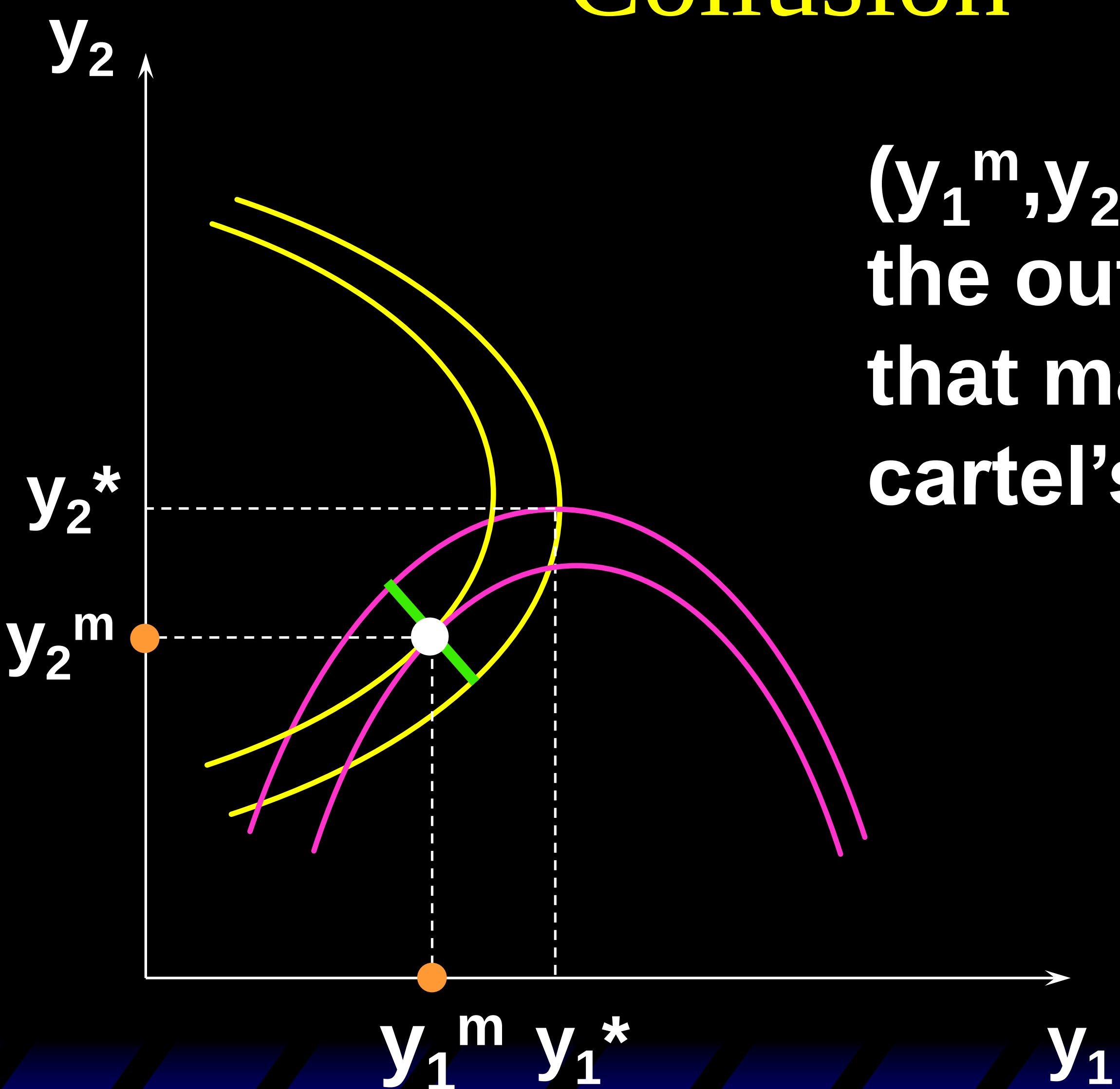
The path of output pairs that maximize one firm's profit while giving the other firm at least its CN equilibrium profit.

Collusion



The path of output pairs that maximize one firm's profit while giving the other firm at least its CN equilibrium profit. One of these output pairs must maximize the cartel's joint profit.

Collusion



(y_1^m, y_2^m) denotes the output levels that maximize the cartel's total profit.

Collusion

Is such a cartel stable?

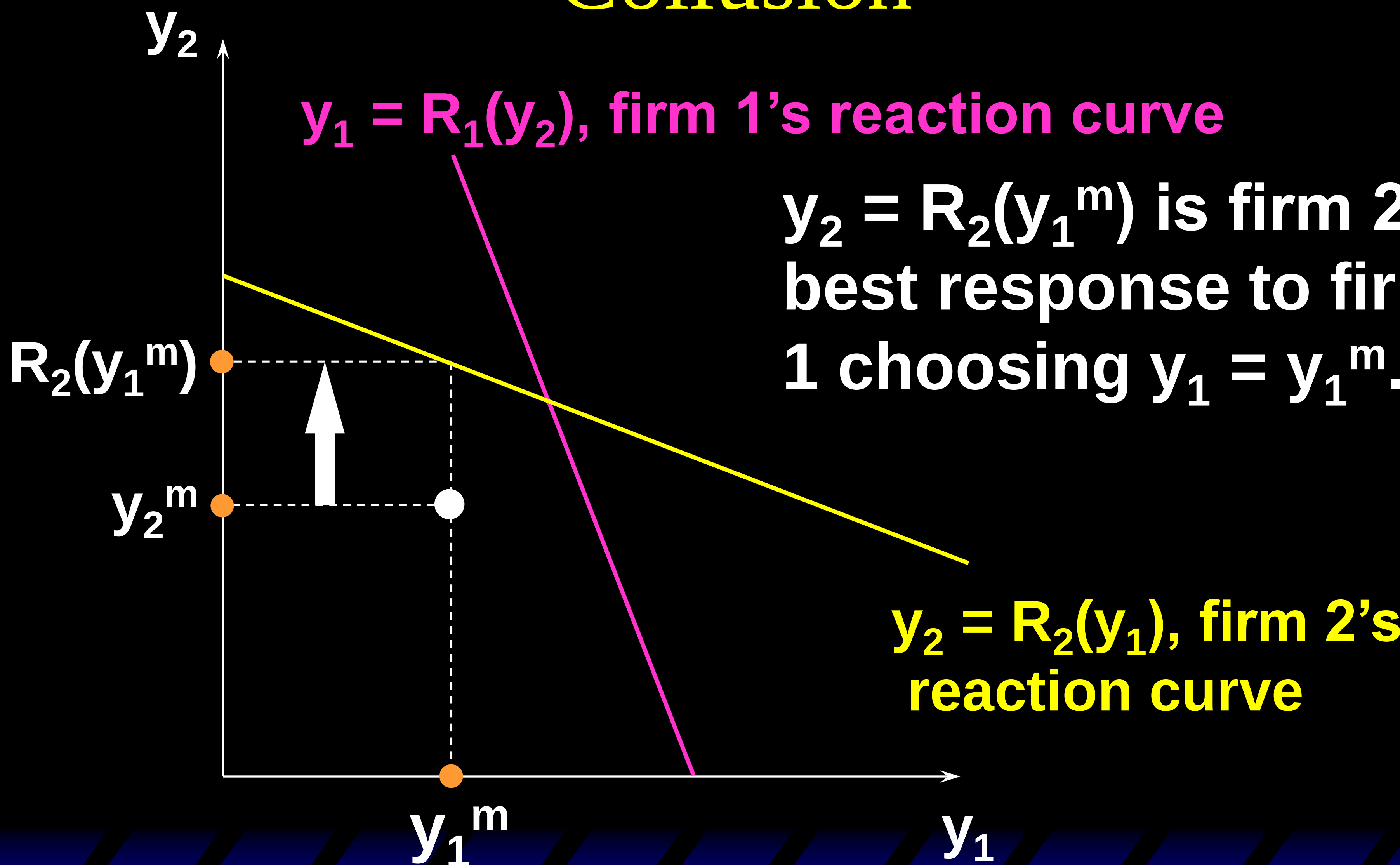
Does one firm have an incentive to cheat on the other?

I.e. if firm 1 continues to produce y_1^m units, is it profit-maximizing for firm 2 to continue to produce y_2^m units?

Collusion

- **Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m)$.**

Collusion



$y_2 = R_2(y_1^m)$ is firm 2's best response to firm 1 choosing $y_1 = y_1^m$.

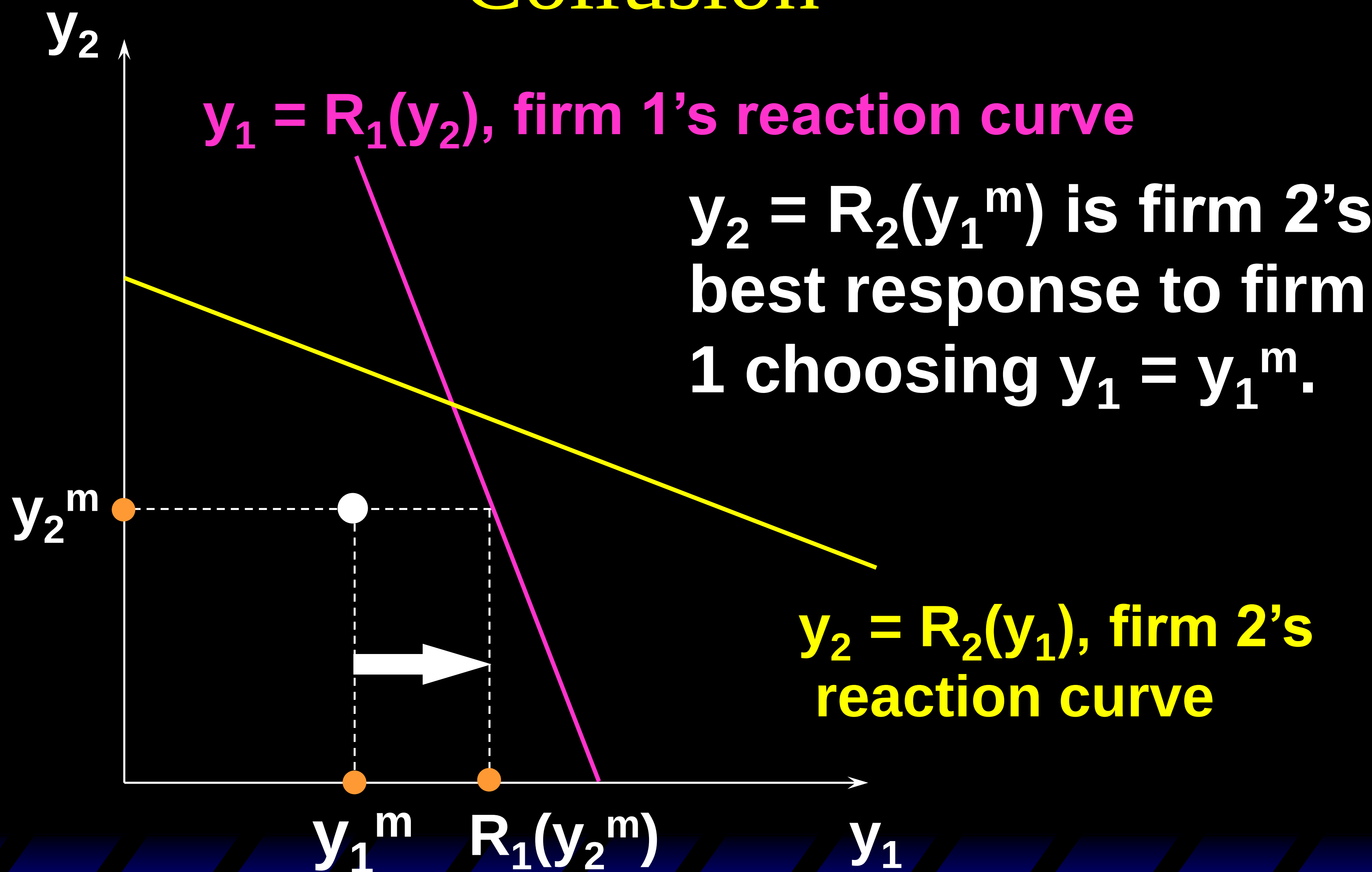
Collusion

- Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m) > y_2^m$.
- Firm 2's profit increases if it cheats on firm 1 by increasing its output level from y_2^m to $R_2(y_1^m)$.

Collusion

Similarly, firm 1's profit increases if it cheats on firm 2 by increasing its output level from y_1^m to $R_1(y_2^m)$.

Collusion



y_2
$y_1 = R_1(y_2)$, firm 1's reaction curve
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y_2^m
$y_2 = R_2(y_1)$, firm 2's reaction curve
y_1^m
$R_1(y_2^m)$
y_1

Collusion

So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.

Penalty:

- Firm 1 announce that if firm 2 cheats, firm 1 will produce Cournot equilibrium Forever in the future
- Need to be multi-period game
- Cheating provides profit in this period, but breaking the cartel leads to loss

The Order of Play

- So far it has been assumed that firms choose their output levels **simultaneously**.
- The competition between the firms is then a **simultaneous play game** in which the output levels are the strategic variables.

The Order of Play

- What if firm 1 chooses its output level first and then firm 2 responds to this choice?
- Firm 1 is then a **leader**. Firm 2 is a **follower**.
- The competition is a **sequential game** in which the output levels are the strategic variables.

The Order of Play

Such games are von Stackelberg games.

Is it better to be the leader?

Or is it better to be the follower?

Stackelberg Games

Q: What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?

Stackelberg Games

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A: Choose $y_2 = R_2(y_1)$.

Stackelberg Games

Q: What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?

A: Choose $y_2 = R_2(y_1)$.

Firm 1 knows this and so perfectly anticipates firm 2's reaction to any y_1 chosen by firm 1.

Stackelberg Games

- This makes the leader's profit function
  $$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

Stackelberg Games

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$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

- The leader then chooses y_1 to maximize its profit level.

Stackelberg Games

This makes the leader's profit function

$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

The leader then chooses y_1 to maximize its profit level.

Q: Will the leader make a profit at least as large as its Cournot-Nash equilibrium profit?

Stackelberg Games

A: Yes. The leader could choose its Cournot-Nash output level, knowing that the follower would then also choose its C-N output level. The leader’s profit would then be its C-N profit. But the leader does not have to do this, so its profit must be at least as large as its C-N profit.

Stackelberg Games; An Example

The market inverse demand function is $p = 60 - y_T$. The firms' cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.

Firm 2 is the follower. Its reaction function is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Stackelberg Games; An Example

The leader's profit function is therefore

$$\Pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$

$$= (60 - y_1 - \frac{45 - y_1}{4})y_1 - y_1^2$$

$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

Stackelberg Games; An Example

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$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

For a profit-maximum,

$$\frac{195}{4} = \frac{7}{2}y_1 \Rightarrow y_1^s = 13\cdot 9.$$

Stackelberg Games; An Example

Q: What is firm 2's response to the leader's choice $y_1^s = 13 \cdot 9$?

Stackelberg Games; An Example

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Stackelberg Games; An Example

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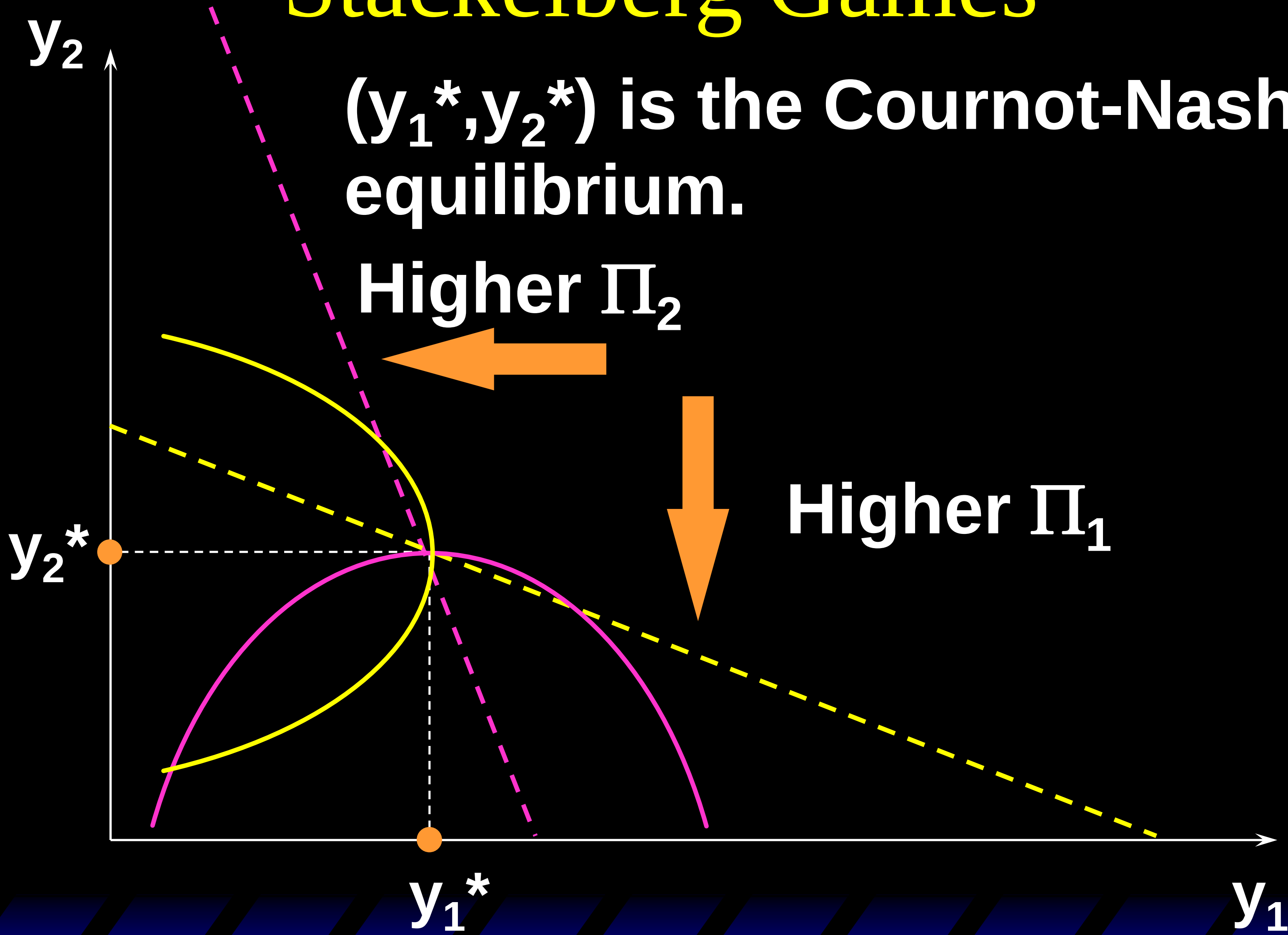
A: $y_2^s = R_2(y_1^s) = \dfrac{45 - 13 \cdot 9}{4} = 7 \cdot 8.$

The output levels are $(y_1^*, y_2^*) = (13.9, 7.8)$ so the leader produces more than its C-N output and the follower produces less than its C-N output. This is true generally.

Stackelberg Games

(y_1^*, y_2^*) is the Cournot-Nash equilibrium.



Stackelberg Games

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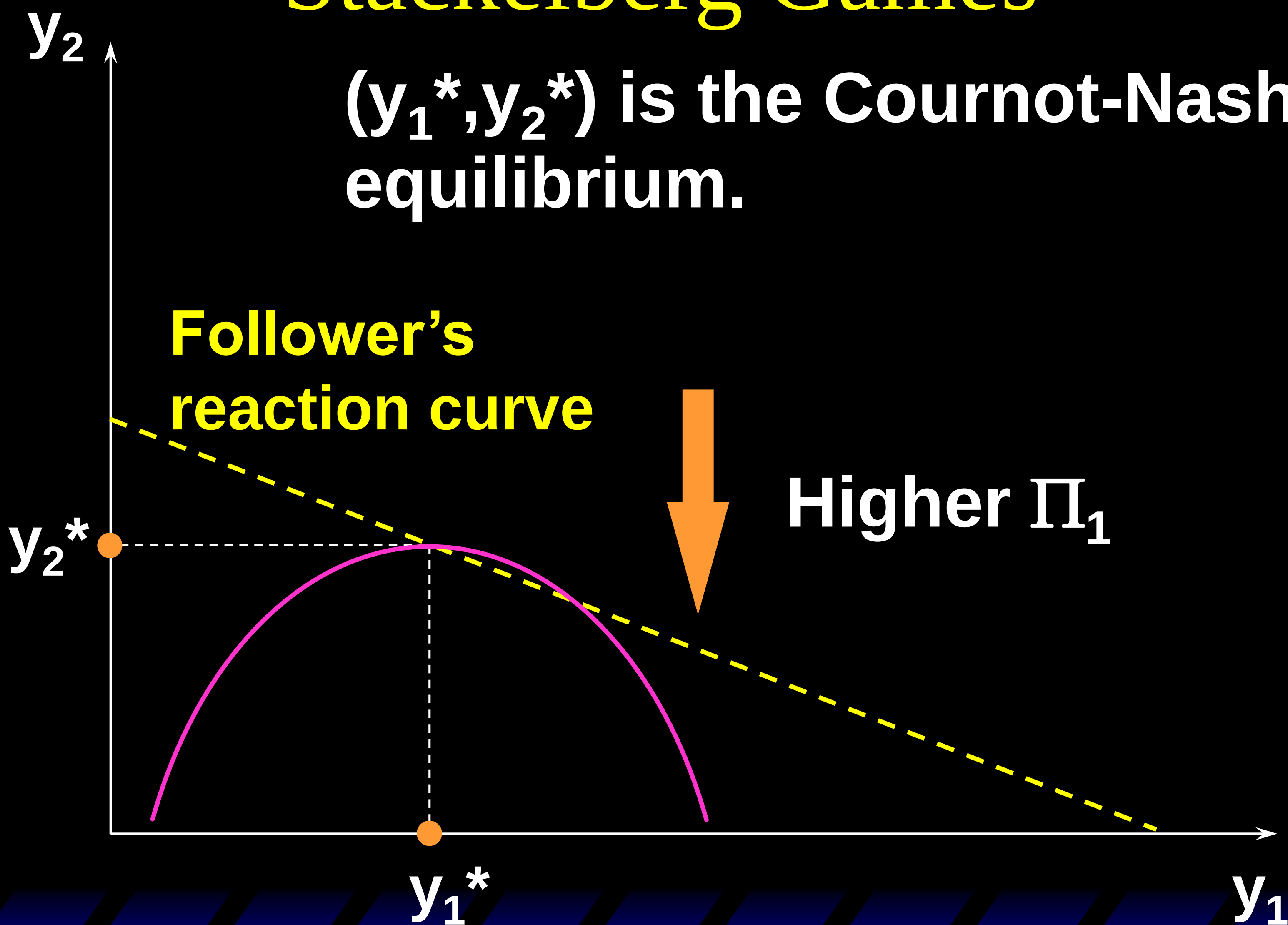
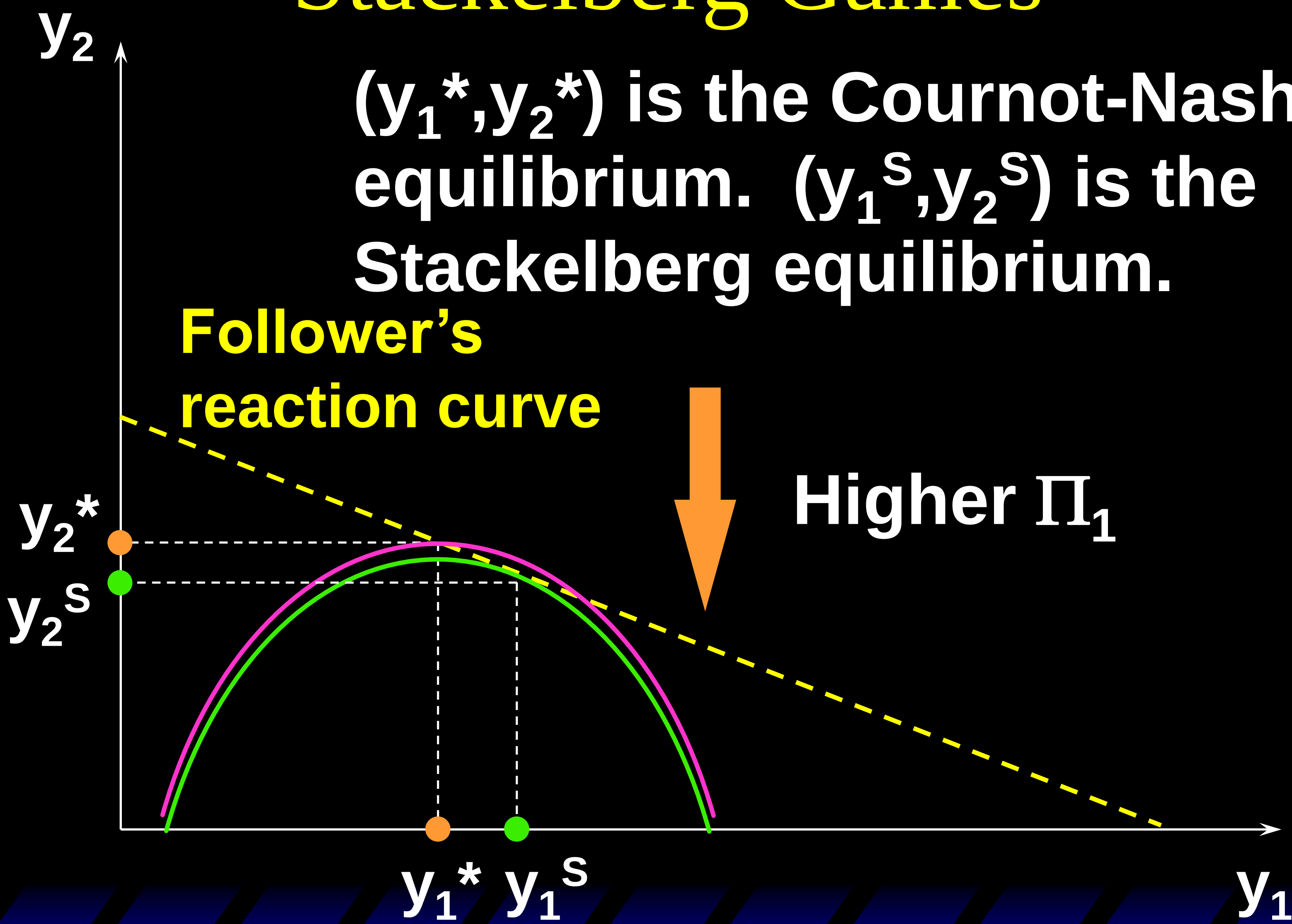


Stackelberg Games

(y_1^*, y_2^*) is the Cournot-Nash equilibrium. (y_1^S, y_2^S) is the Stackelberg equilibrium.



Stackelberg Games

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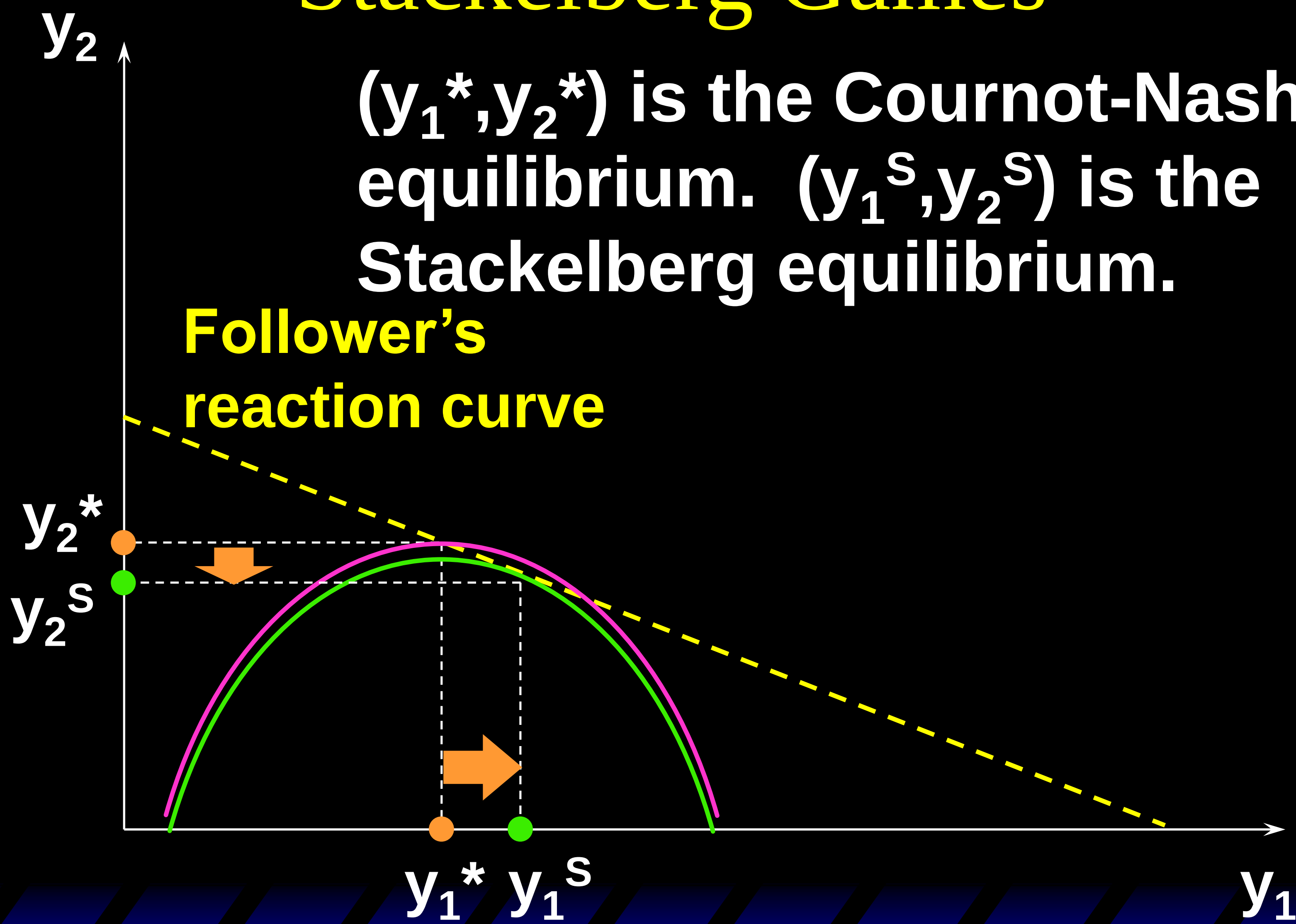

Price Competition

What if firms compete using only price-setting strategies, instead of using only quantity-setting strategies?

Games in which firms use only price strategies and play simultaneously are **Bertrand** games.

Bertrand Games

- Each firm's marginal production cost is constant at c.
- All firms set their prices simultaneously.
- Q: Is there a Nash equilibrium?

Bertrand Games

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A: Yes. Exactly one.

Bertrand Games

- Each firm's marginal production cost is constant at c.
- All firms set their prices simultaneously.
- Q: Is there a Nash equilibrium?
- A: Yes. Exactly one. All firms set their prices equal to the marginal cost c. Why?

Bertrand Games

- Suppose one firm sets its price higher than another firm's price.

Bertrand Games

- Suppose one firm sets its price higher than another firm’s price.
- Then the higher-priced firm would have no customers.

Bertrand Games

Suppose one firm sets its price higher than another firm’s price.

Then the higher-priced firm would have no customers.

Hence, at an equilibrium, all firms must set the same price.

Bertrand Games

Suppose the common price set by all firm is higher than marginal cost c.

Bertrand Games

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Then one firm can just slightly lower its price and sell to all the buyers, thereby increasing its profit.

Bertrand Games

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Then one firm can just slightly lower its price and sell to all the buyers, thereby increasing its profit.

The only common price which prevents undercutting is c. Hence this is the only Nash equilibrium.

Sequential Price Games

What if, instead of simultaneous play in pricing strategies, one firm decides its price ahead of the others.

This is a sequential game in pricing strategies called a **price-leadership game**.

The firm which sets its price ahead of the other firms is the price-leader.

Sequential Price Games

- Think of one large firm (the leader) and many competitive small firms (the followers).
- The small firms are price-takers and so their collective supply reaction to a market price p is their aggregate supply function $Y_f(p)$.

Sequential Price Games

The market demand function is D(p).

So the leader knows that if it sets a price p the quantity demanded from it will be the **residual demand**

$$L(p) = D(p) - Y_f(p).$$

Hence the leader's profit function is

$$\Pi_L(p) = p(D(p) - Y_f(p)) - c_L(D(p) - Y_f(p)).$$

Sequential Price Games

The leader's profit function is

$$\Pi_L(p) = p(D(p) - Y_f(p)) - c_L(D(p) - Y_F(p))$$

so the leader chooses the price level p^* for which profit is maximized.

The followers collectively supply $Y_f(p^*)$ units and the leader supplies the residual quantity $D(p^*) - Y_f(p^*)$.