Chapter Fifteen

Market Demand

From Individual to Market Demand Functions

- **Think of an economy containing n consumers, denoted by i = 1, … ,n.**
- **Consumer i's ordinary demand function for commodity j is**

$$x_j^{*i}(p_1, p_2, m^i)$$

From Individual to Market Demand Functions

- **When all consumers are price-takers, the market demand function for commodity j is**

$$X_j(p_1, p_2, m^1, \cdots, m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i).$$

- **If all consumers are identical then**

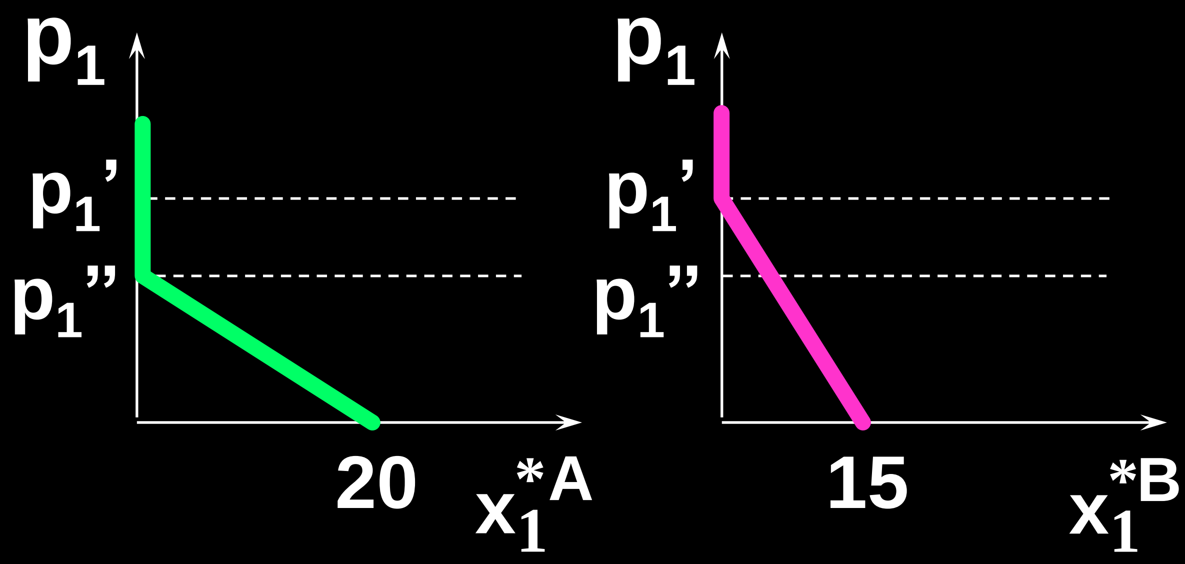
$$X_j(p_1, p_2, M) = n \times x_j^*(p_1, p_2, m)$$

where M = nm.

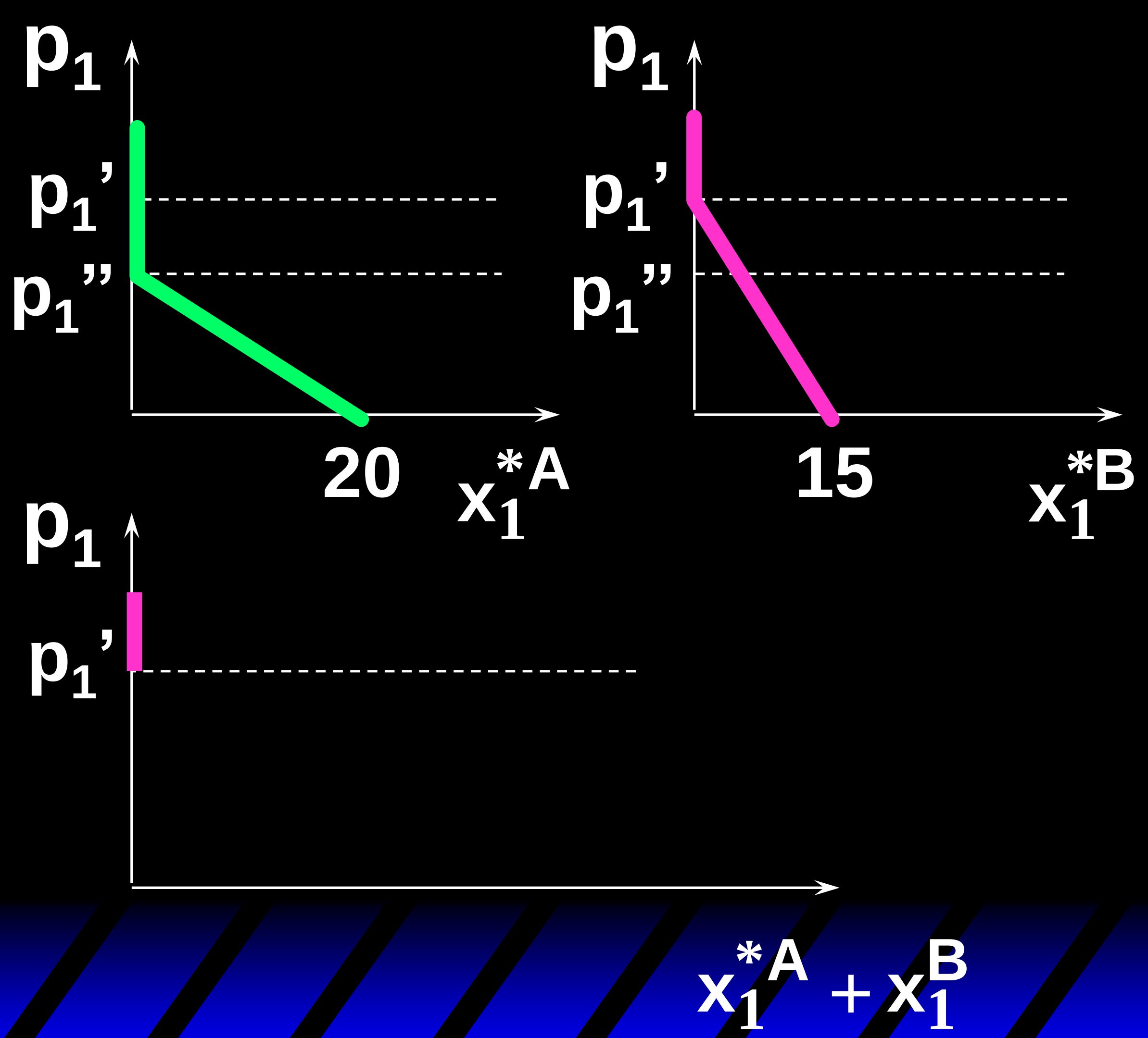
From Individual to Market Demand Functions

- **The market demand curve is the "horizontal sum" of the individual consumers' demand curves.**
- **E.g. suppose there are only two consumers; i = A,B.**

From Individual to Market Demand Functions



From Individual to Market Demand Functions



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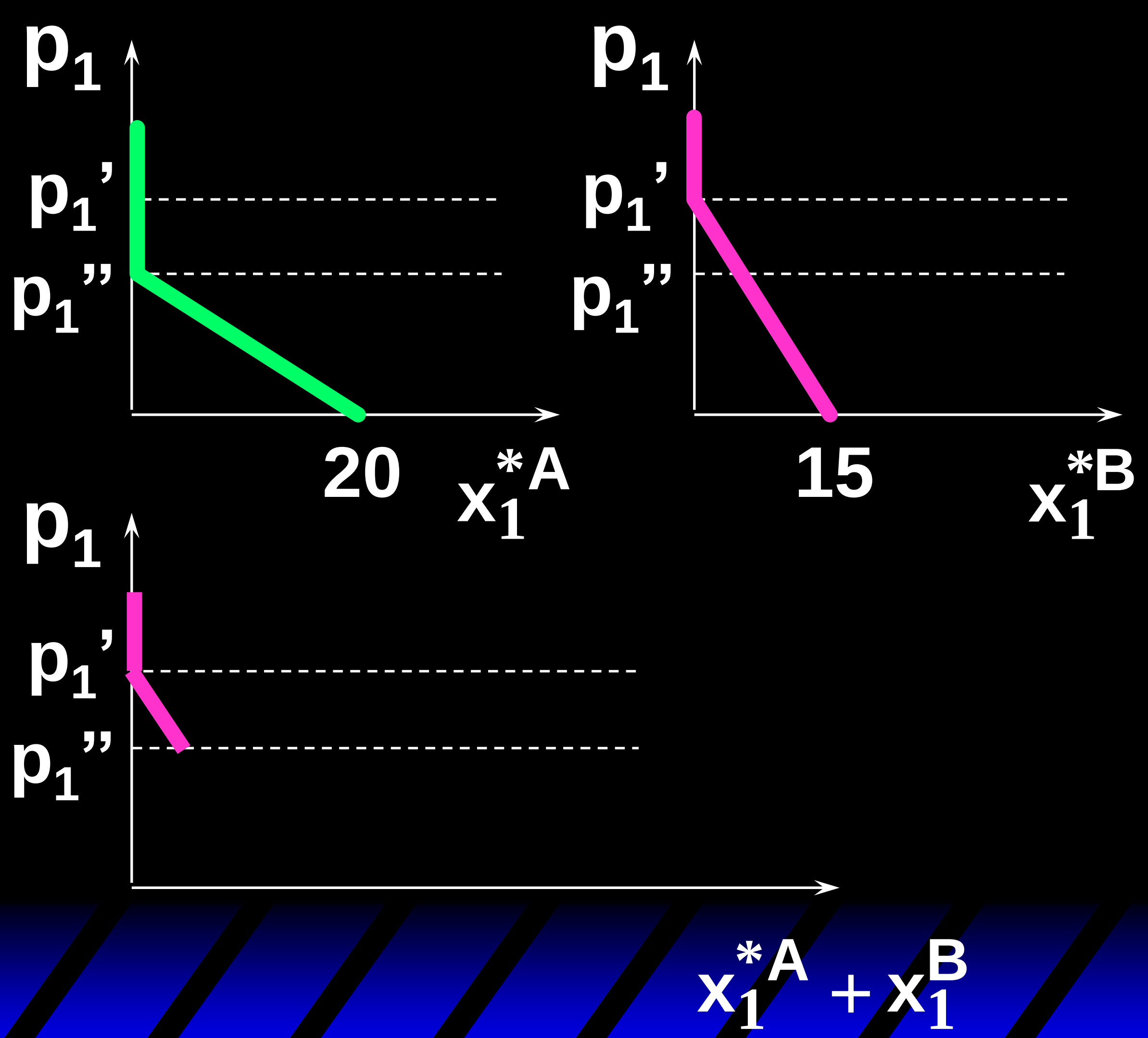
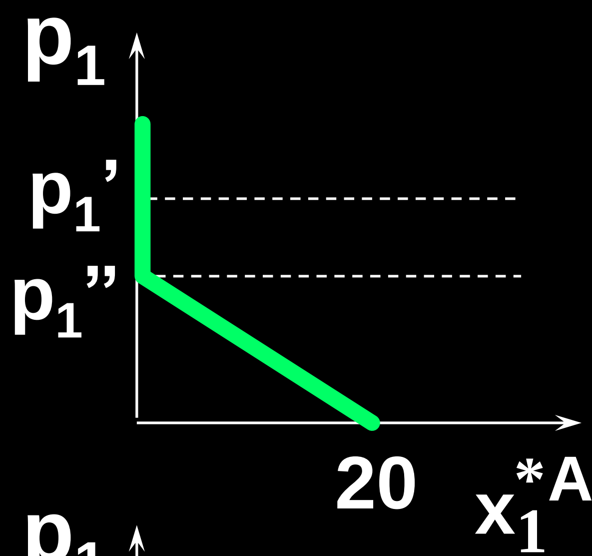
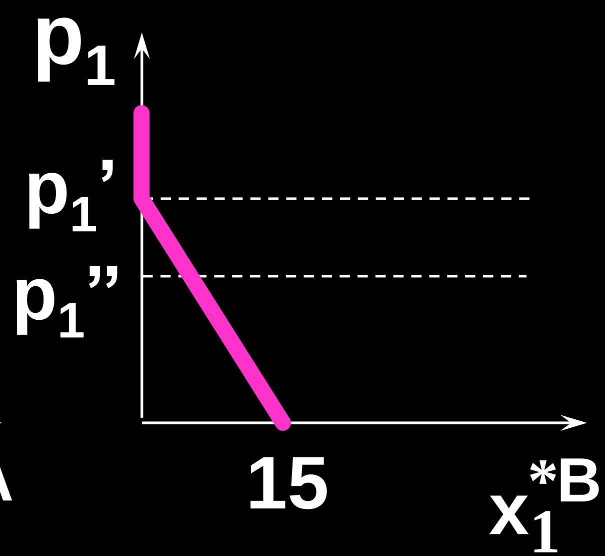
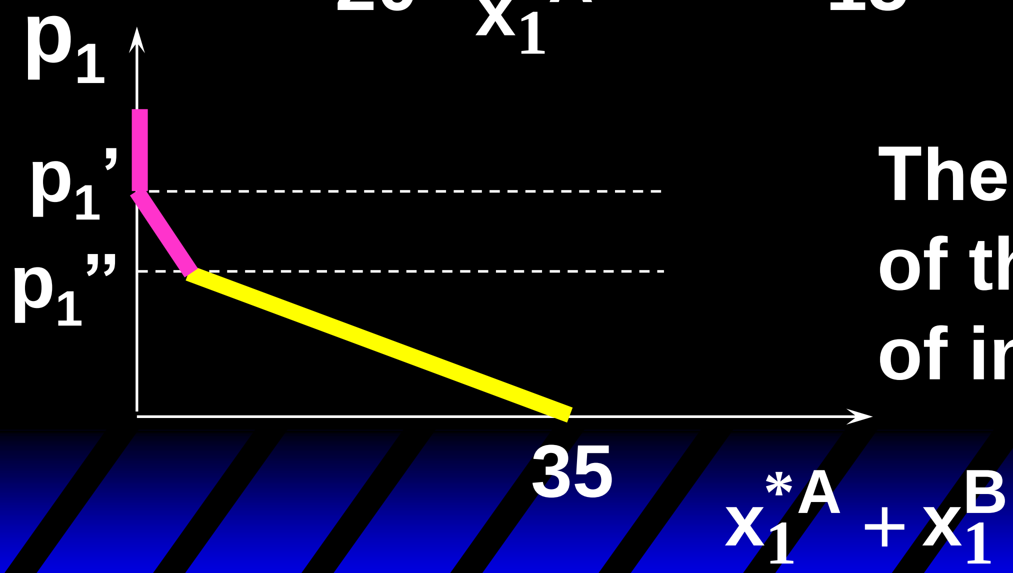


From Individual to Market Demand Functions







The "horizontal sum" of the demand curves of individuals A and B.

Elasticities

- Elasticity measures the "sensitivity" of one variable with respect to another.
- The elasticity of variable X with respect to variable Y is

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}.$$

Economic Applications of Elasticity

- Economists use elasticities to measure the sensitivity of
  - quantity demanded of commodity i with respect to the price of commodity i (own-price elasticity of demand)
  - demand for commodity i with respect to the price of commodity j (cross-price elasticity of demand).

Economic Applications of Elasticity

- demand for commodity i with respect to income (income elasticity of demand)
- quantity supplied of commodity i with respect to the price of commodity i (own-price elasticity of supply)

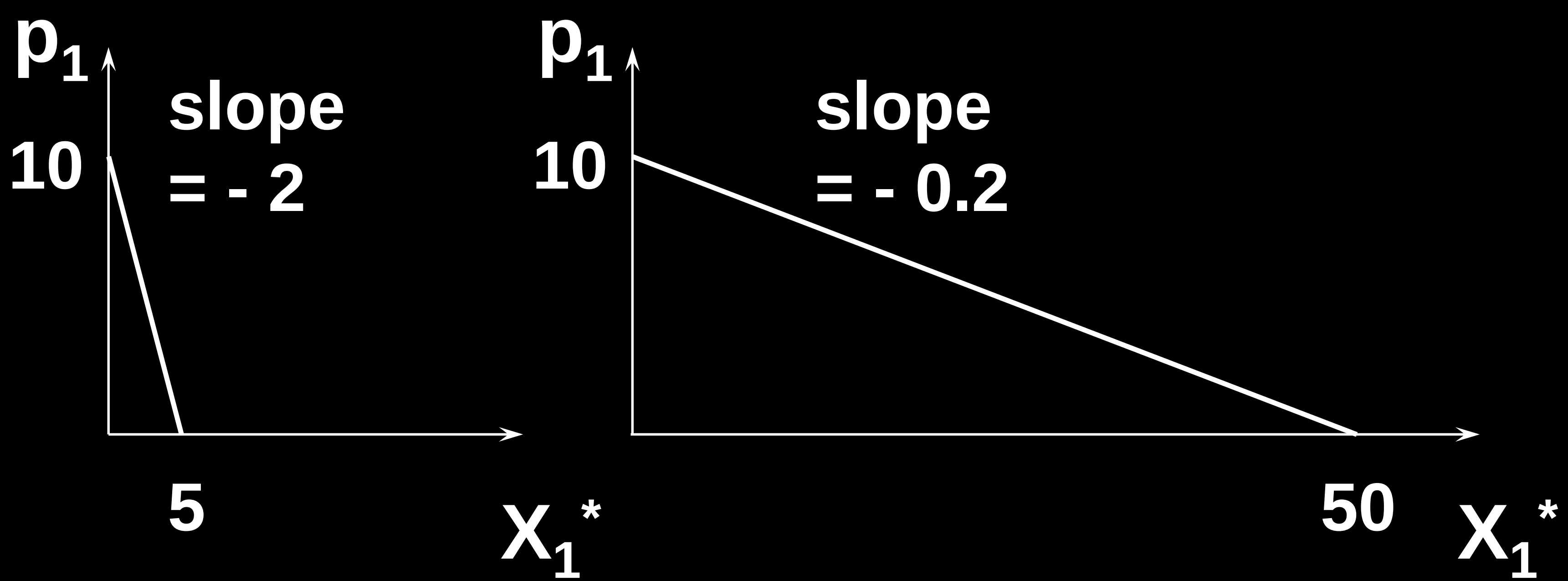
Economic Applications of Elasticity

- quantity supplied of commodity i with respect to the wage rate (elasticity of supply with respect to the price of labor)
- and many, many others.

Own-Price Elasticity of Demand

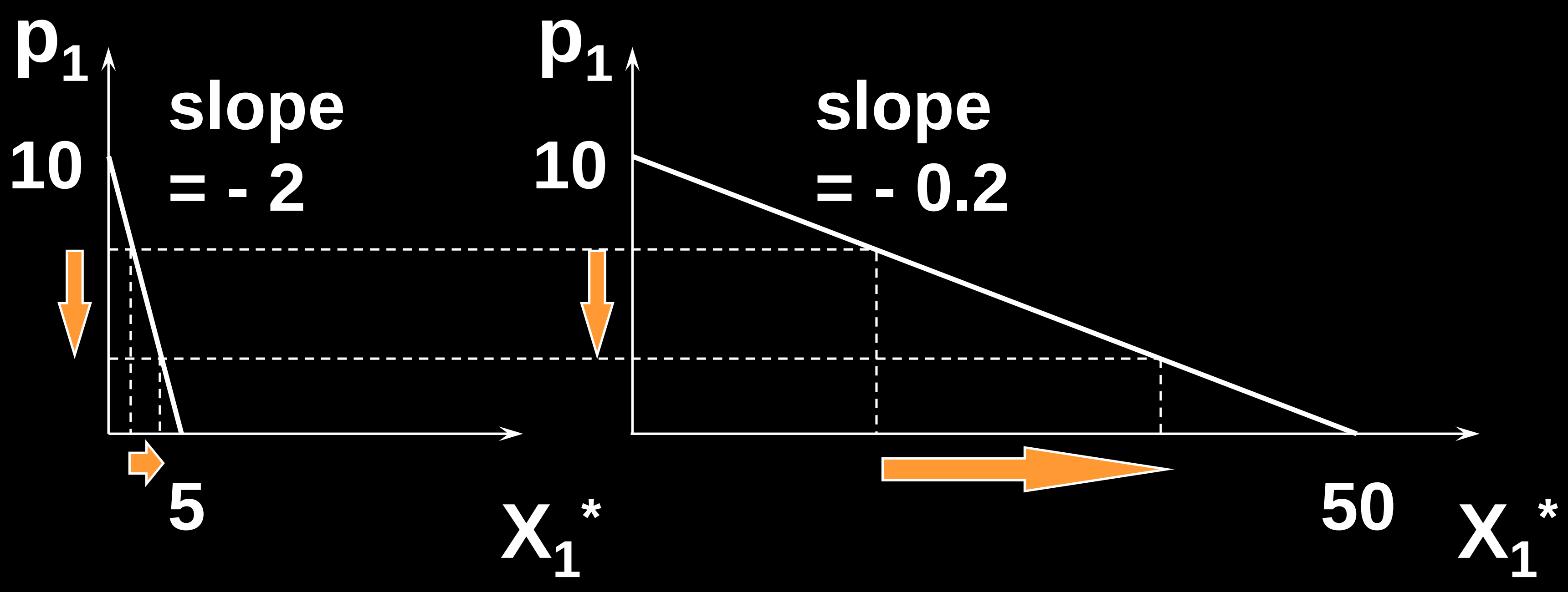
- **Q: Why not use a demand curve's slope to measure the sensitivity of quantity demanded to a change in a commodity's own price?**

Own-Price Elasticity of Demand



In which case is the quantity demanded X_1^* more sensitive to changes to p_1?

Own-Price Elasticity of Demand



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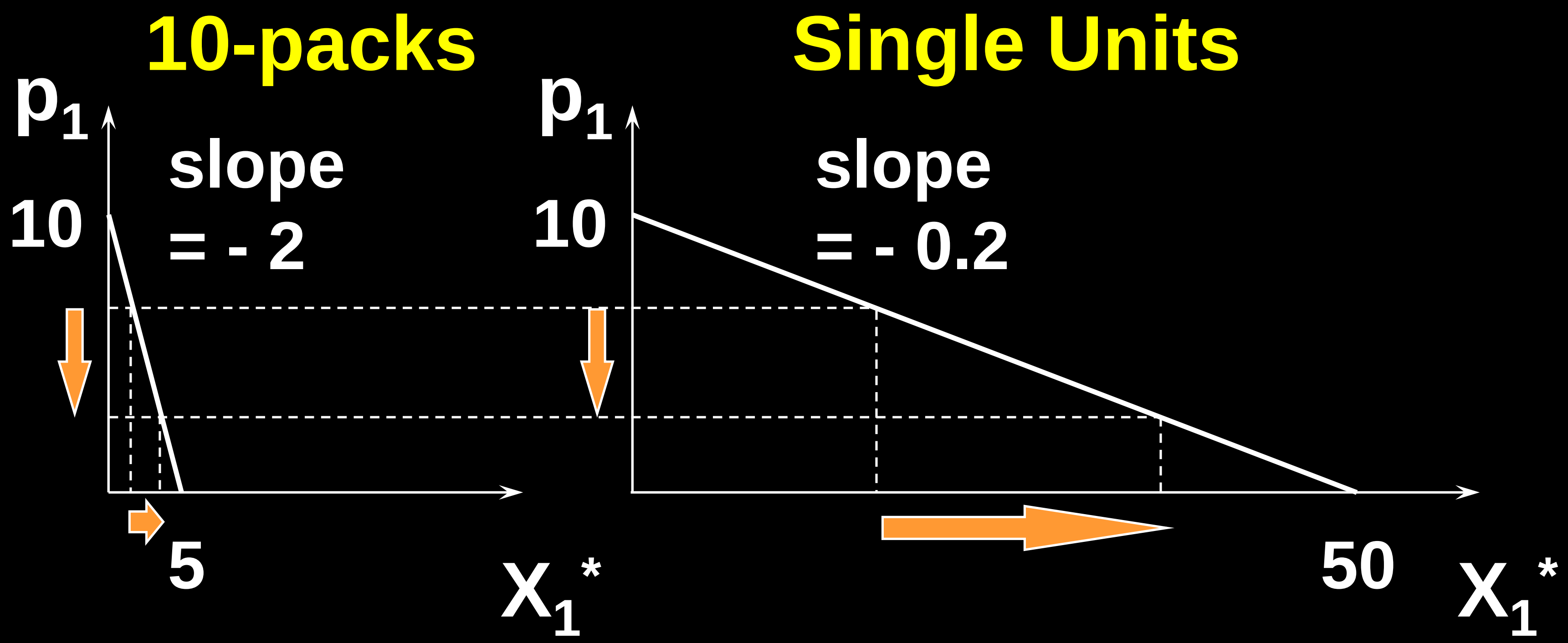

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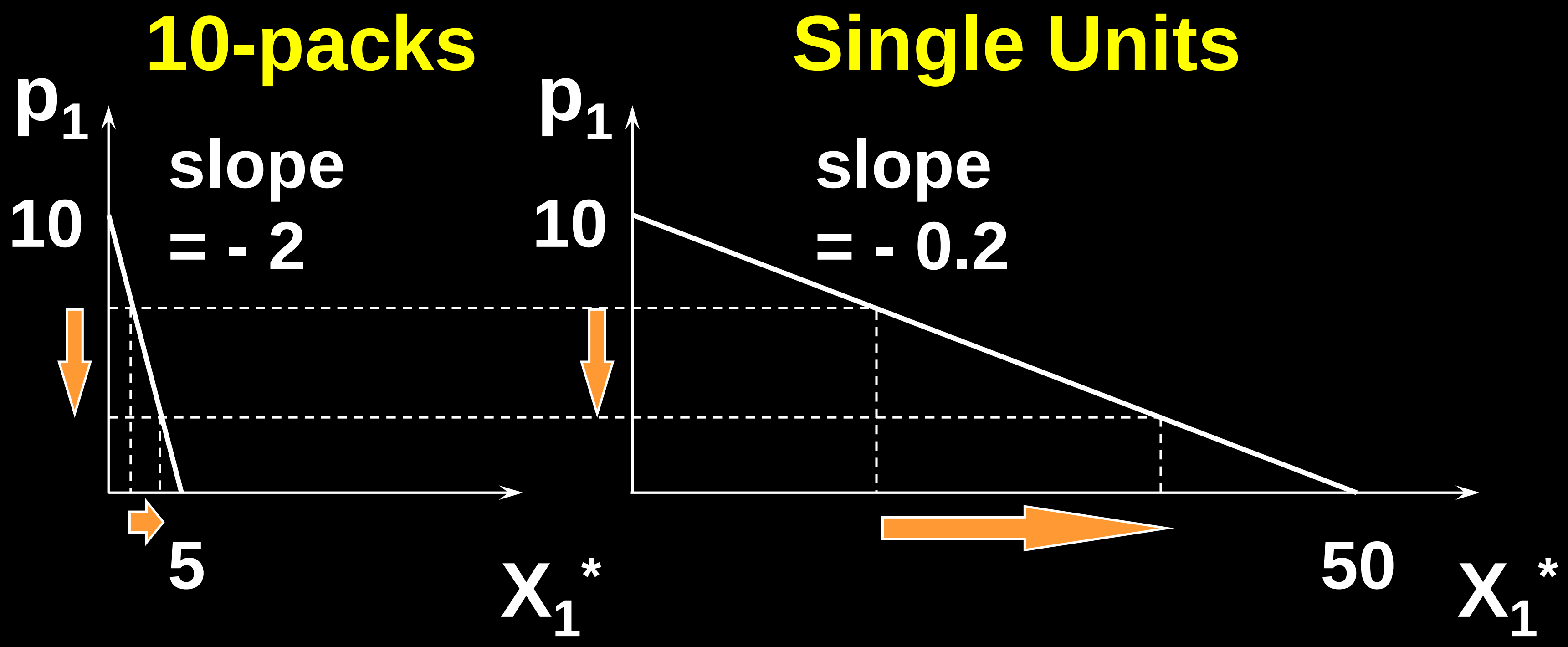


In which case is the quantity demanded X_1^* more sensitive to changes to p_1?
It is the same in both cases.

Own-Price Elasticity of Demand

- **Q: Why not just use the slope of a demand curve to measure the sensitivity of quantity demanded to a change in a commodity’s own price?**
- **A: Because the value of sensitivity then depends upon the (arbitrary) units of measurement used for quantity demanded.**

Own-Price Elasticity of Demand

$$\varepsilon_{x_1^*,p_1} = \frac{\%\Delta x_1^*}{\%\Delta p_1}$$

is a ratio of percentages and so has no units of measurement.

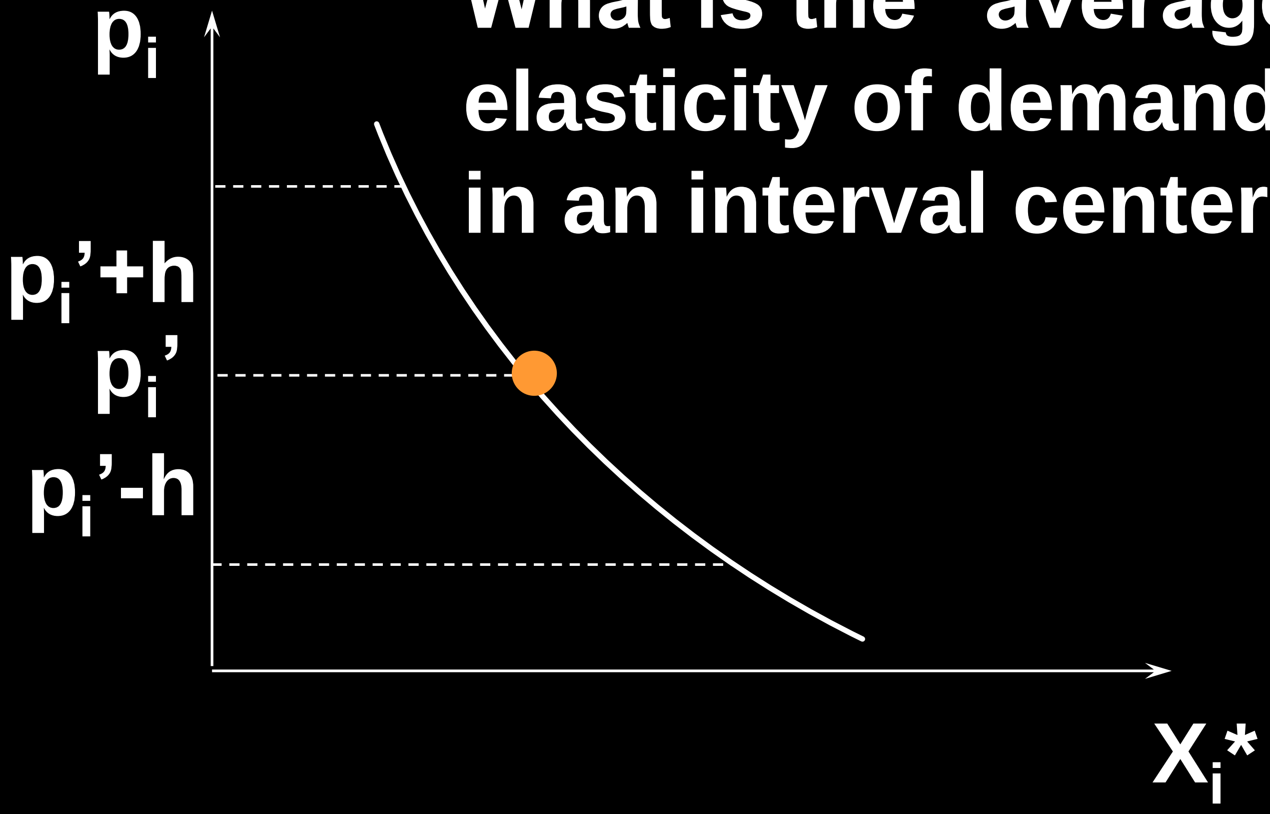
Hence own-price elasticity of demand is a sensitivity measure that is independent of units of measurement.

Arc and Point Elasticities

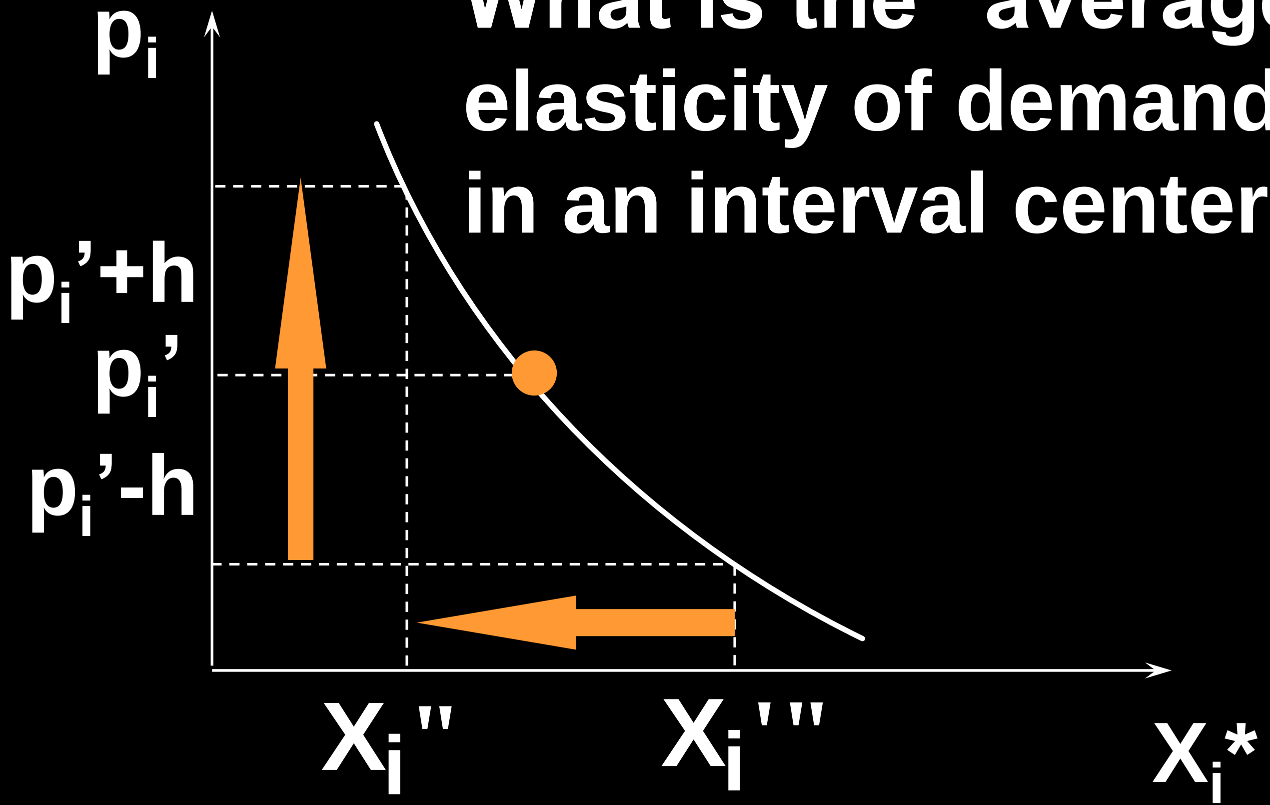
- An "average" own-price elasticity of demand for commodity i over an **interval of values for p_i** is an **arc-elasticity**, usually computed by a mid-point formula.
- Elasticity computed for a **single value of p_i** is a **point elasticity**.

Arc Own-Price Elasticity

What is the "average" own-price elasticity of demand for prices in an interval centered on p_i'?



Arc Own-Price Elasticity



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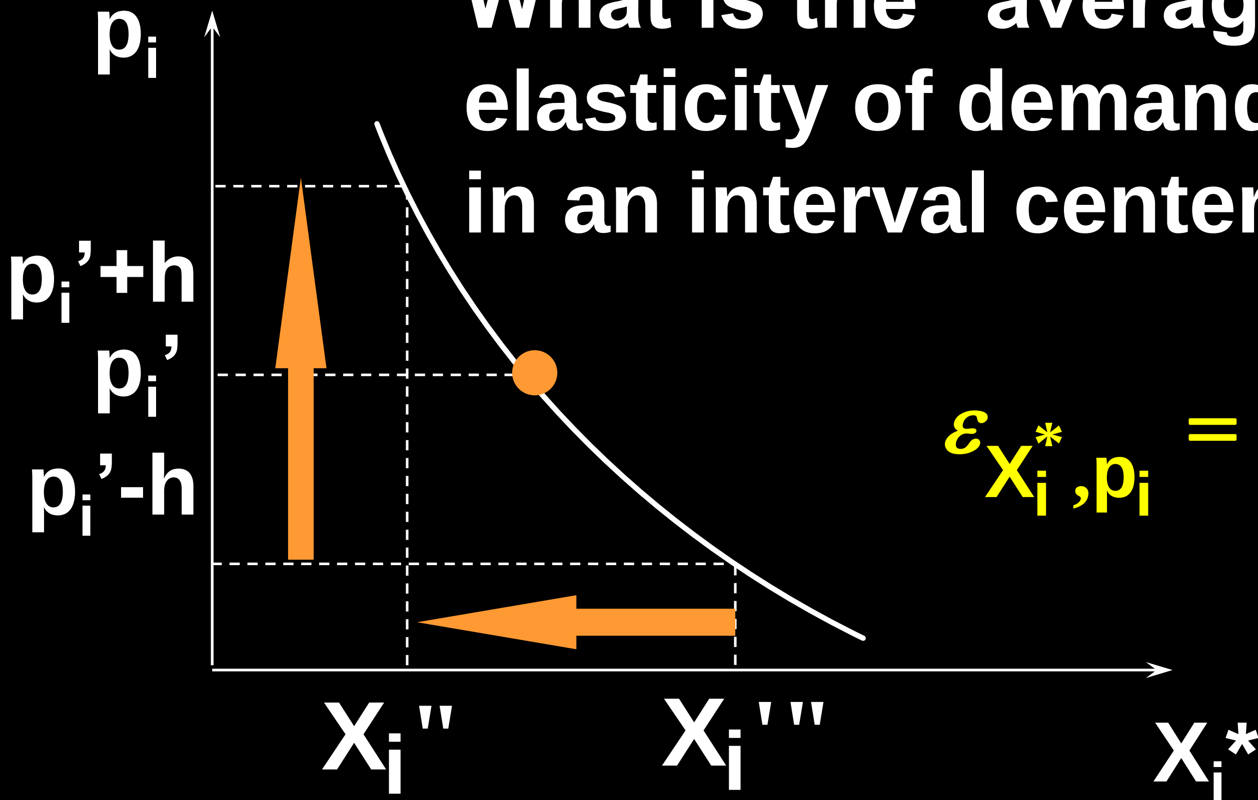
Arc Own-Price Elasticity

What is the “average” own-price elasticity of demand for prices in an interval centered on p_i’?



$$\varepsilon_{x_i^*, p_i} = \frac{\% \Delta x_i^*}{\% \Delta p_i}$$

Arc Own-Price Elasticity

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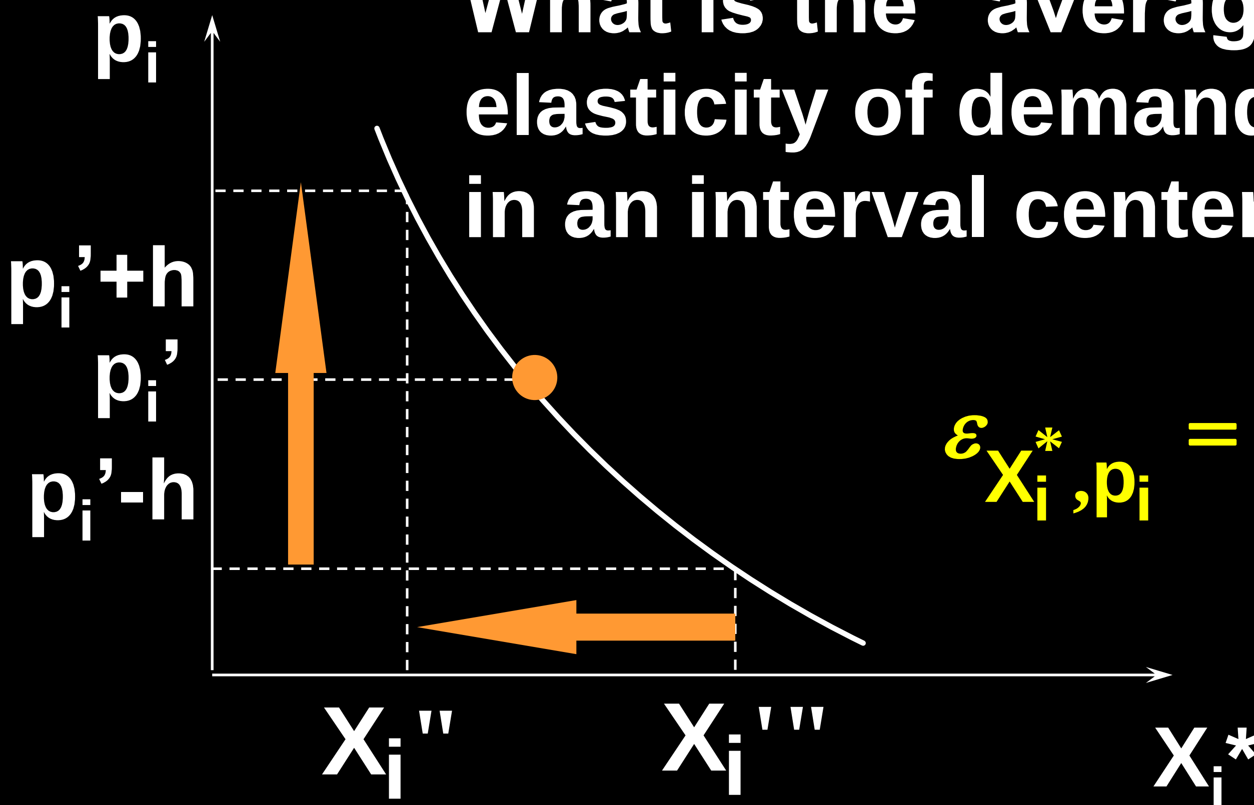
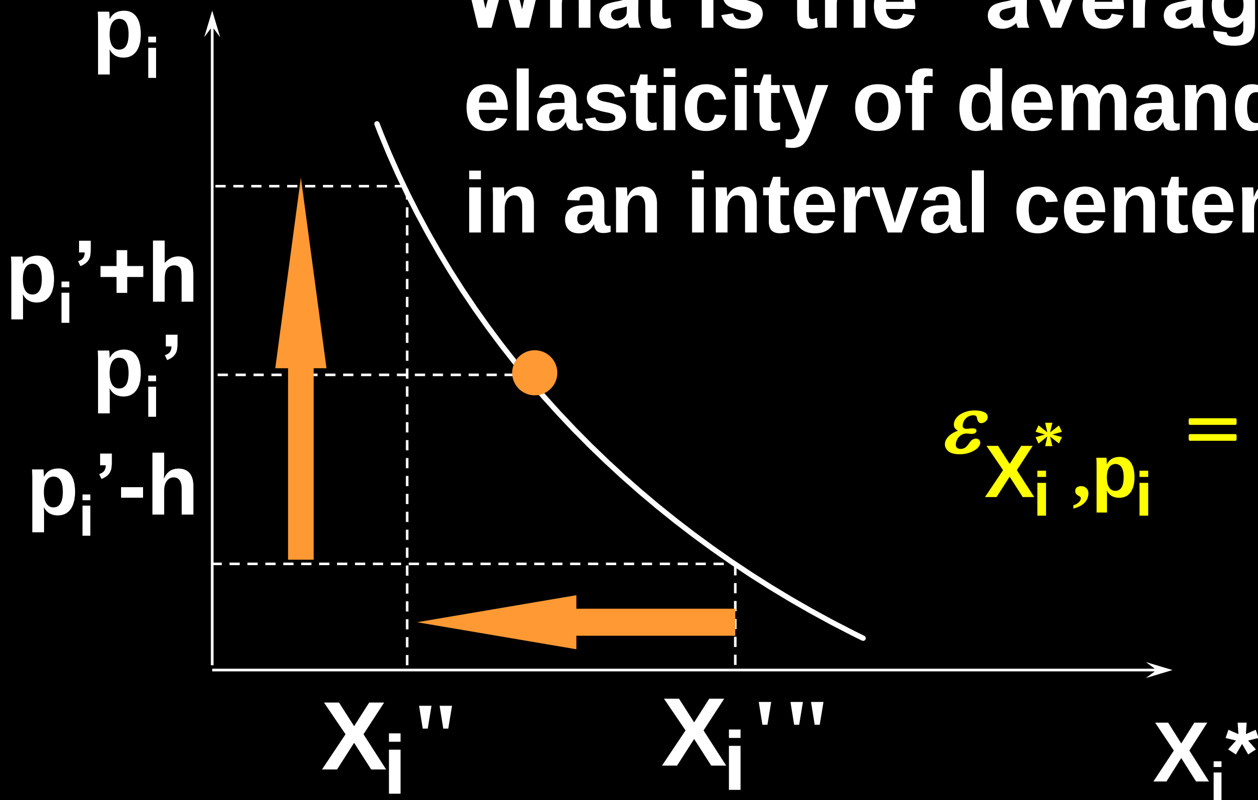


$$\varepsilon_{x_i^*,p_i} = \frac{\%\Delta x_i^*}{\%\Delta p_i}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

Arc Own-Price Elasticity



What is the "average" own-price elasticity of demand for prices in an interval centered on p_i'?

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'} \qquad \%\Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

Arc Own-Price Elasticity

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

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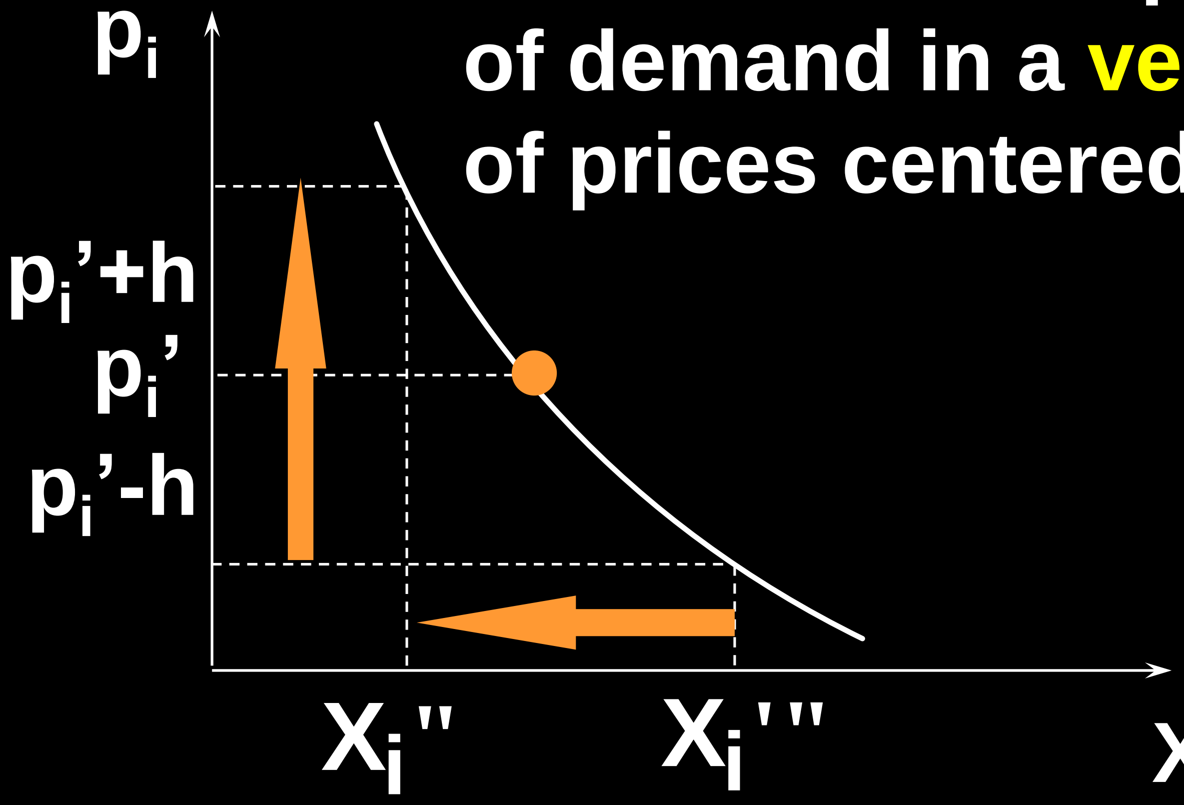
So

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

is the arc own-price elasticity of demand.

Point Own-Price Elasticity

What is the own-price elasticity of demand in a **very small interval** of prices centered on p_i'?



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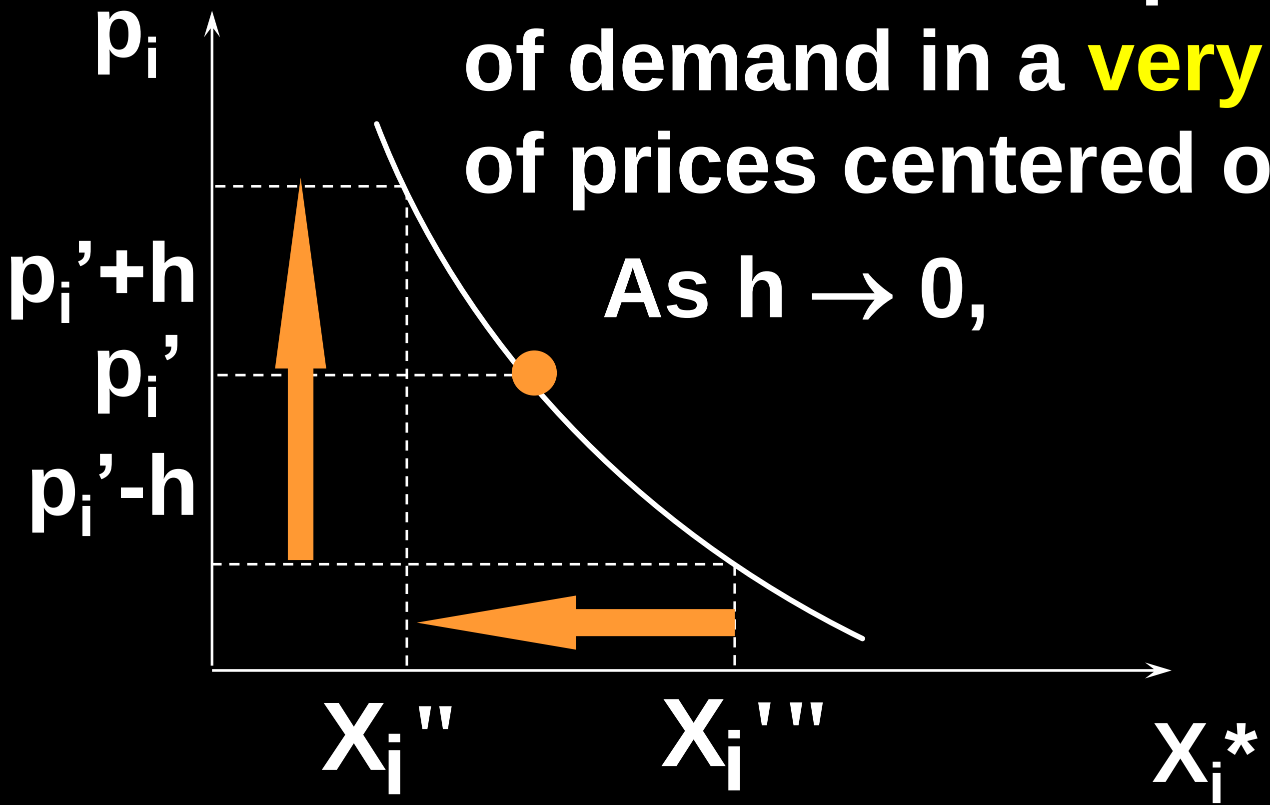

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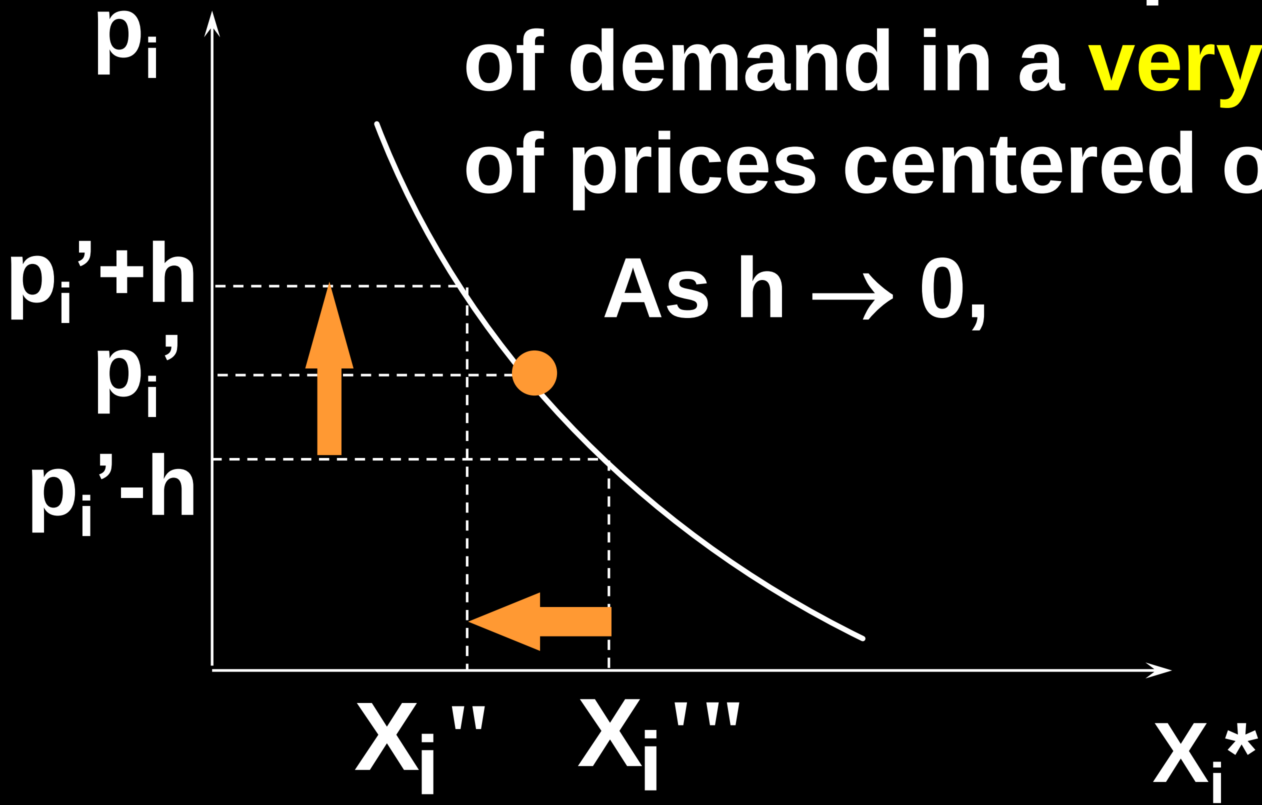

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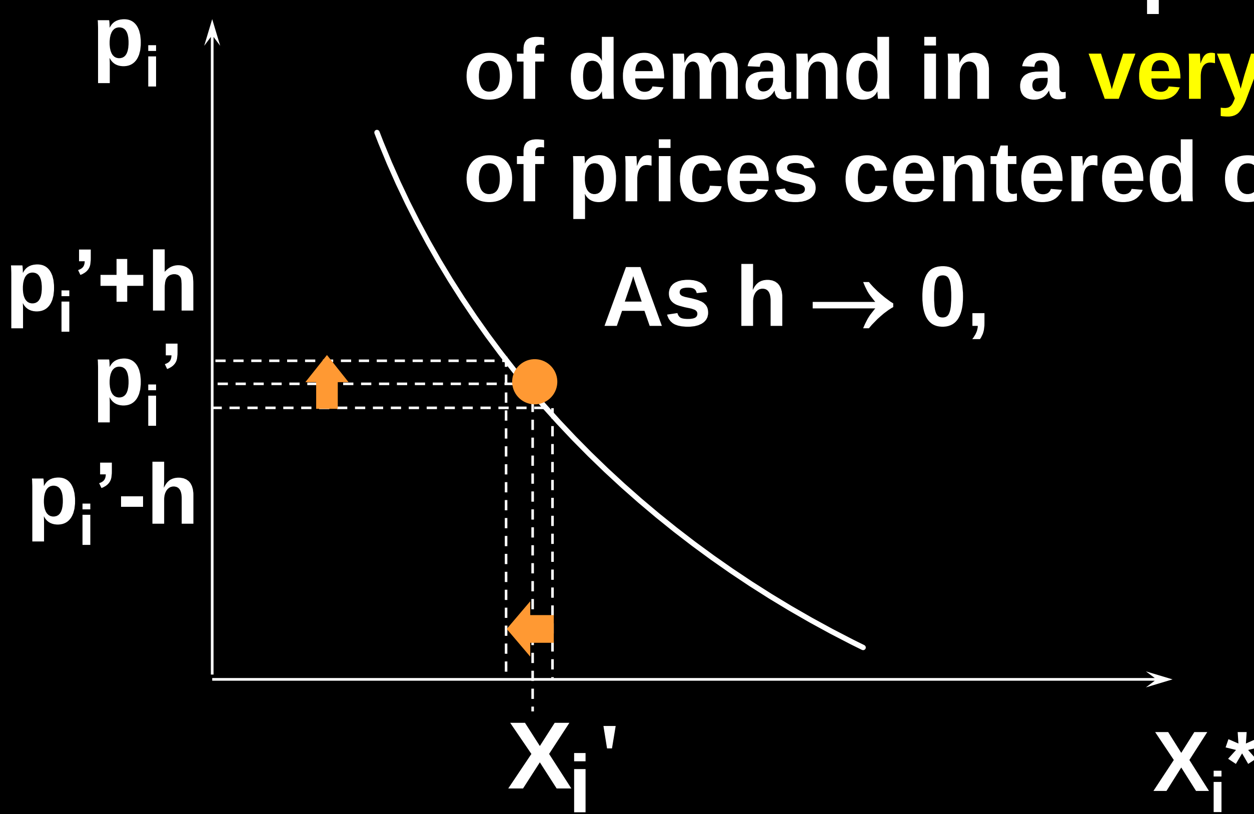

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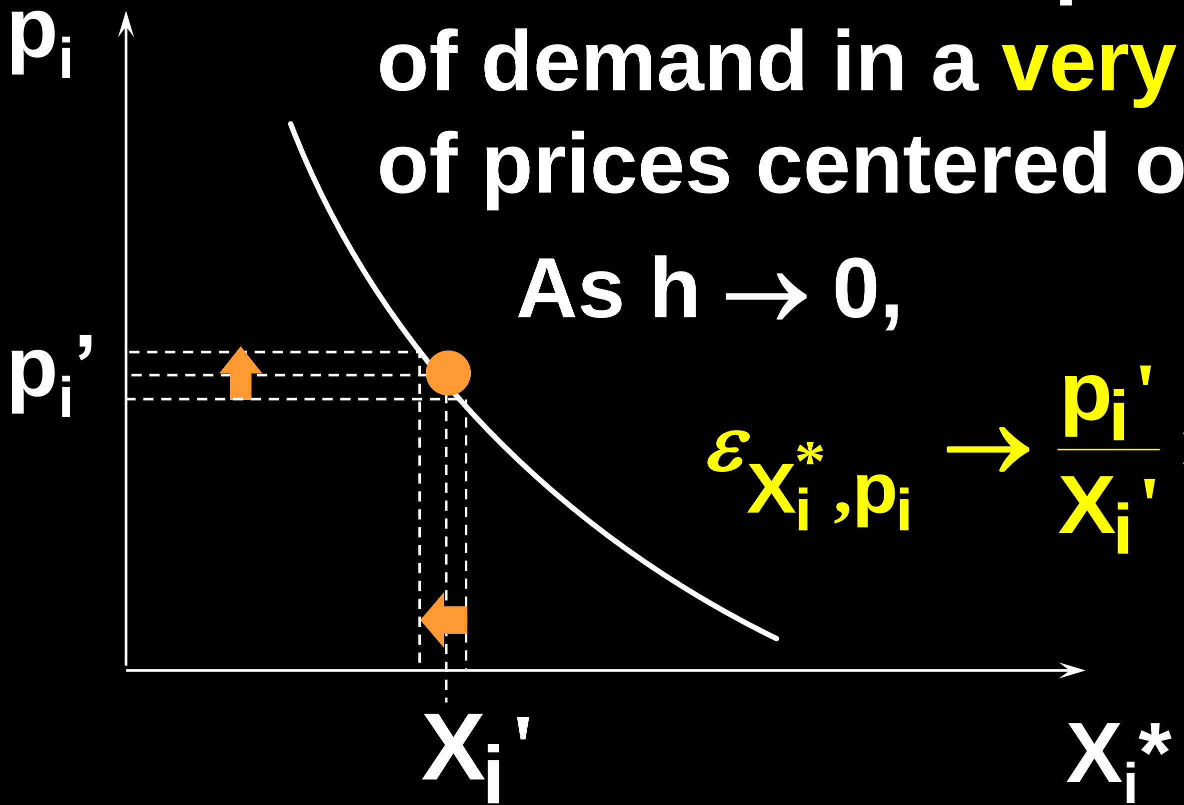

As $h \to 0$,

$$\varepsilon_{X_i^*,p_i} \to \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$

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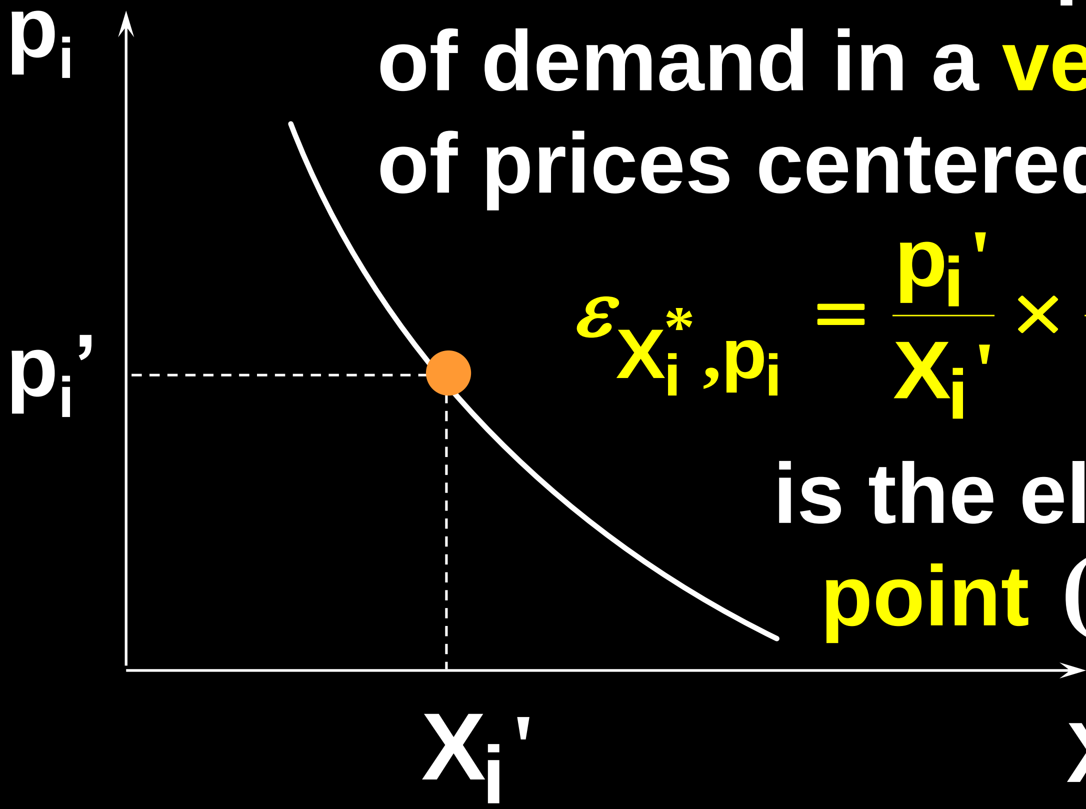


$$\varepsilon_{X_i^*,p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

is the elasticity **at the point** (X_i', p_i').

Point Own-Price Elasticity

$$\varepsilon_{X_i^*,p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

E.g. Suppose $p_i = a - bX_i$.

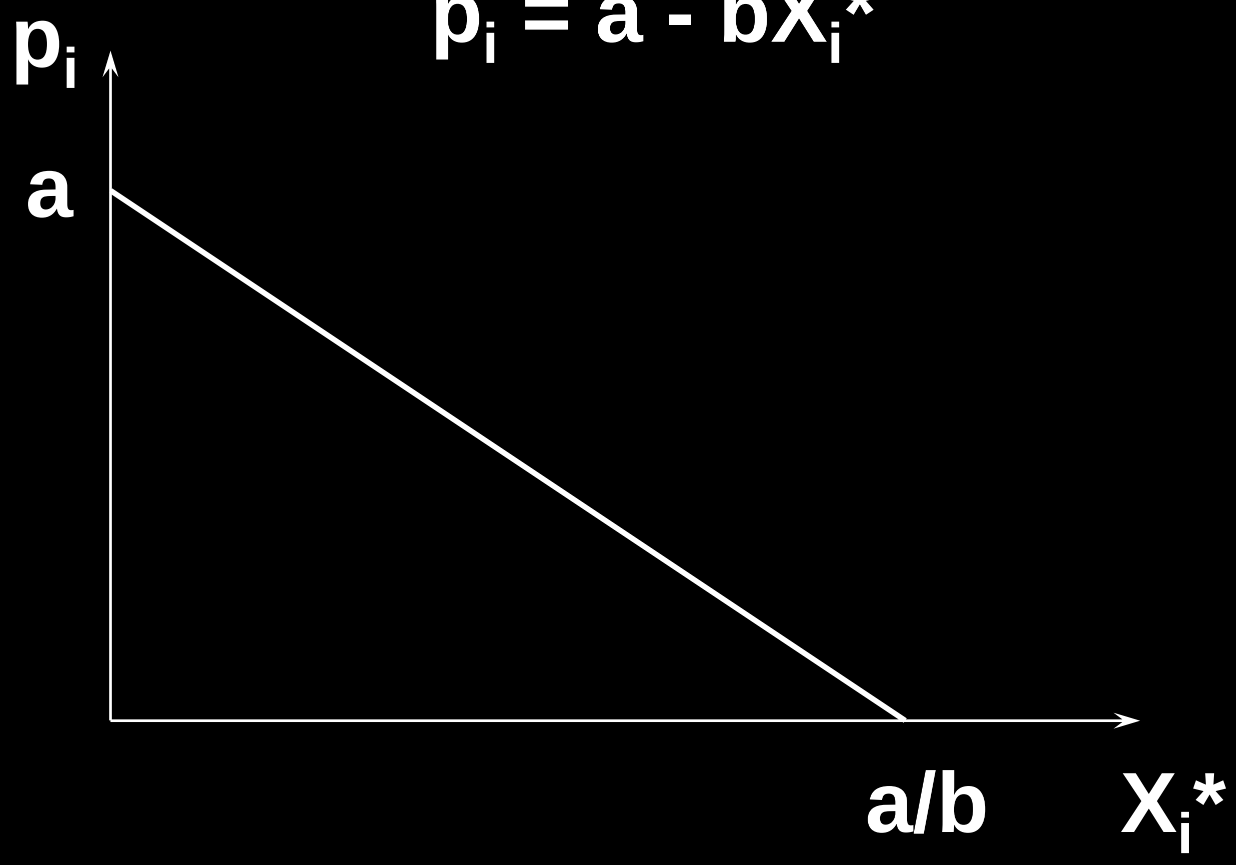
Then $X_i = (a-p_i)/b$ and

$$\frac{dX_i^*}{dp_i} = -\frac{1}{b}.$$ Therefore,

$$\varepsilon_{X_i^*,p_i} = \frac{p_i}{(a-p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a-p_i}.$$

Point Own-Price Elasticity

$$p_i = a - bX_i^*$$



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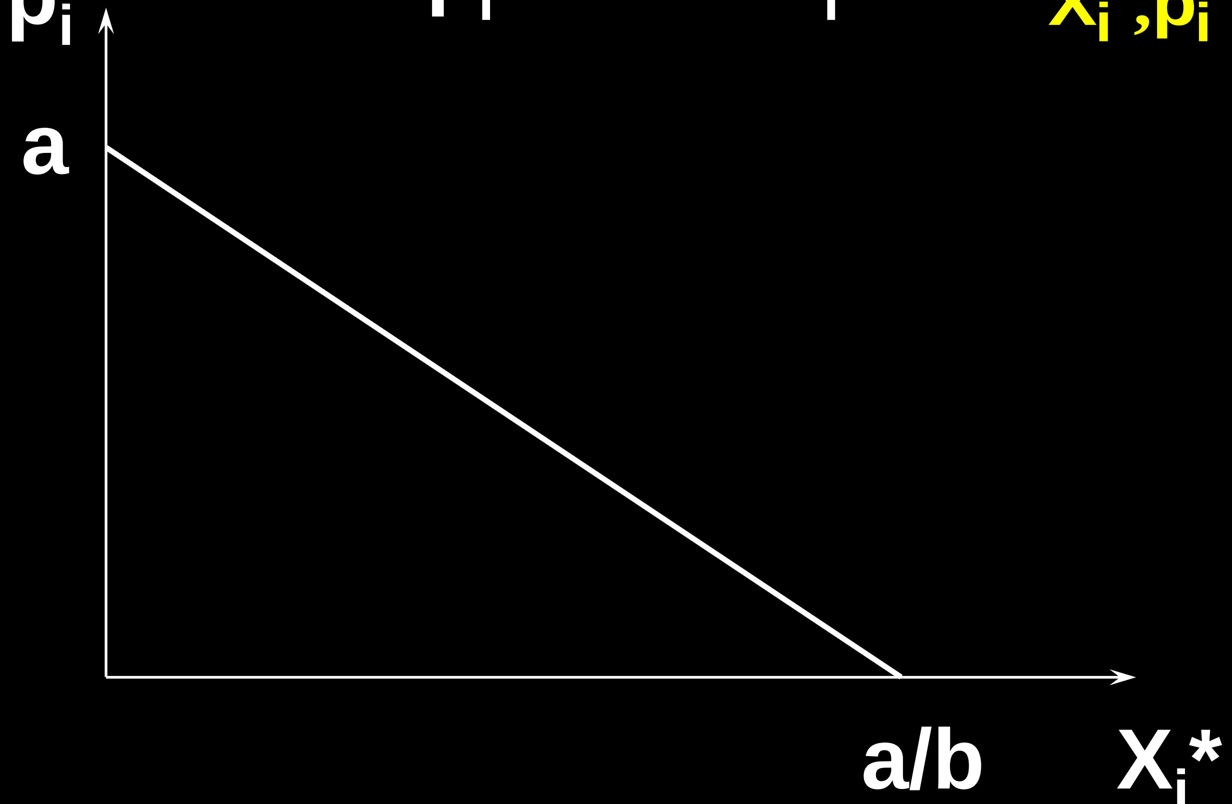

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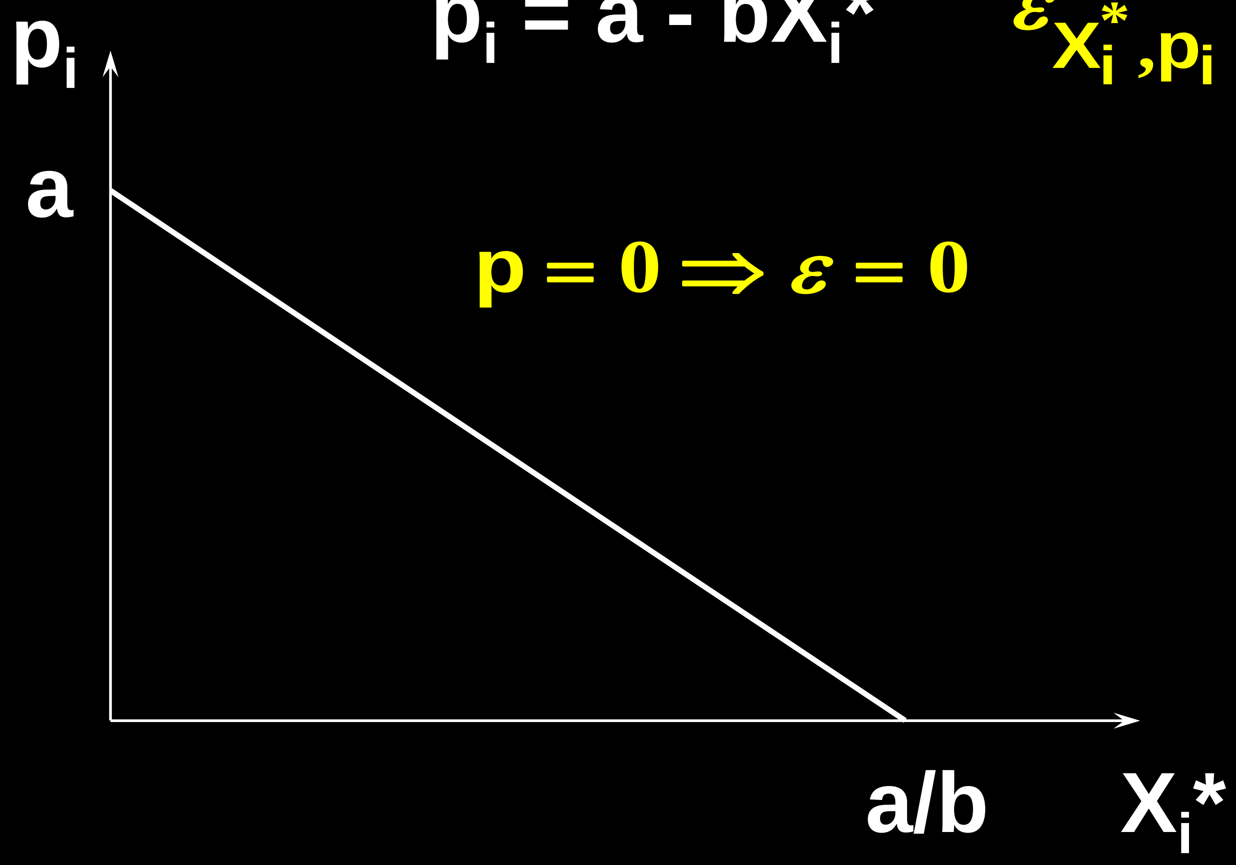
$$p = 0 \Rightarrow \varepsilon = 0$$



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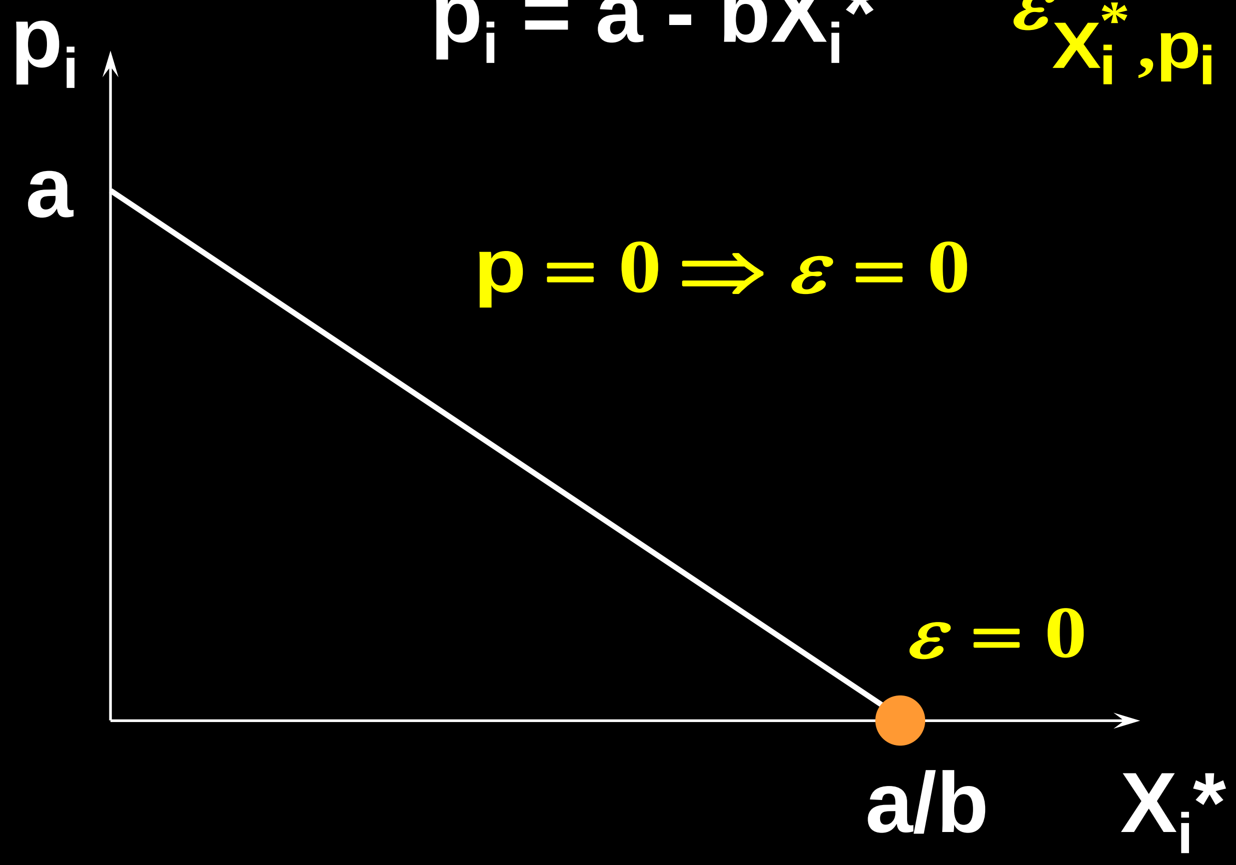
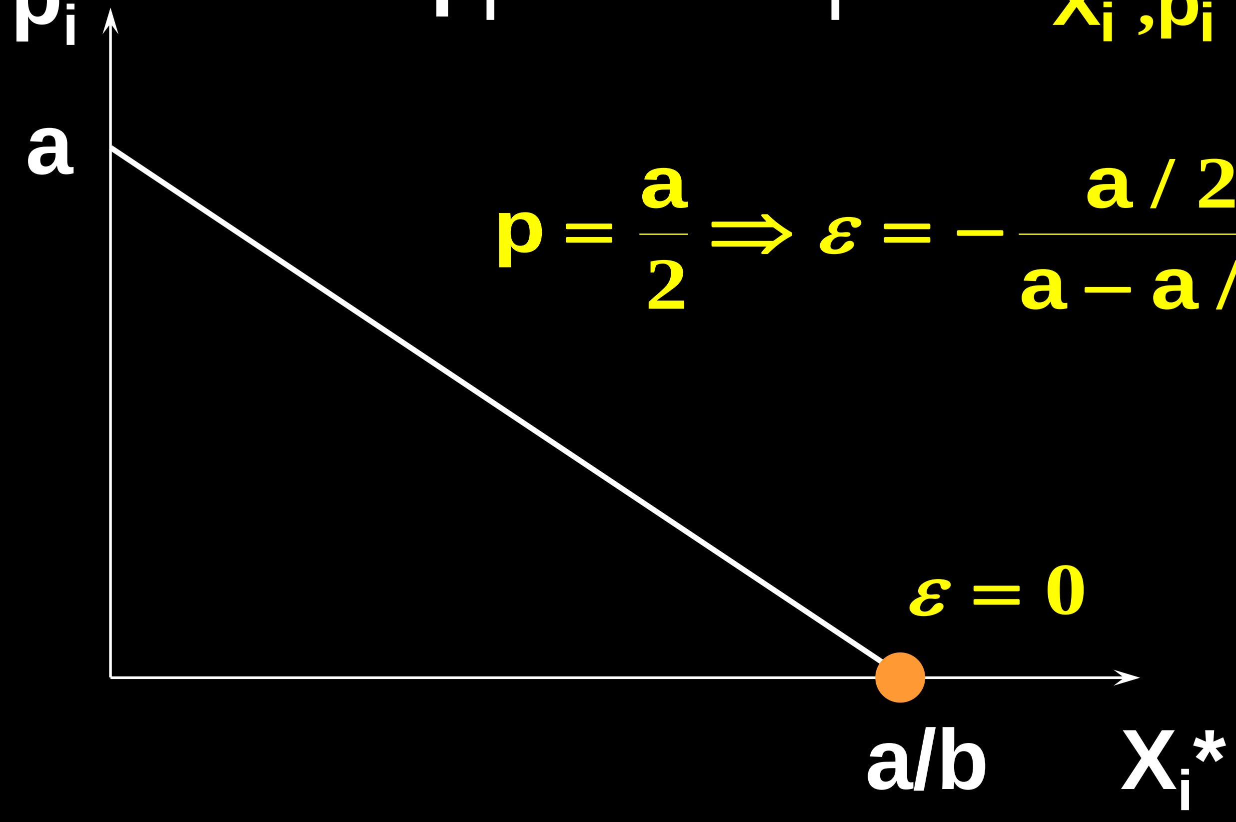


Point Own-Price Elasticity

$p_i = a - bX_i^*$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = \frac{a}{2} \Rightarrow \varepsilon = -\frac{a/2}{a - a/2} = -1$$



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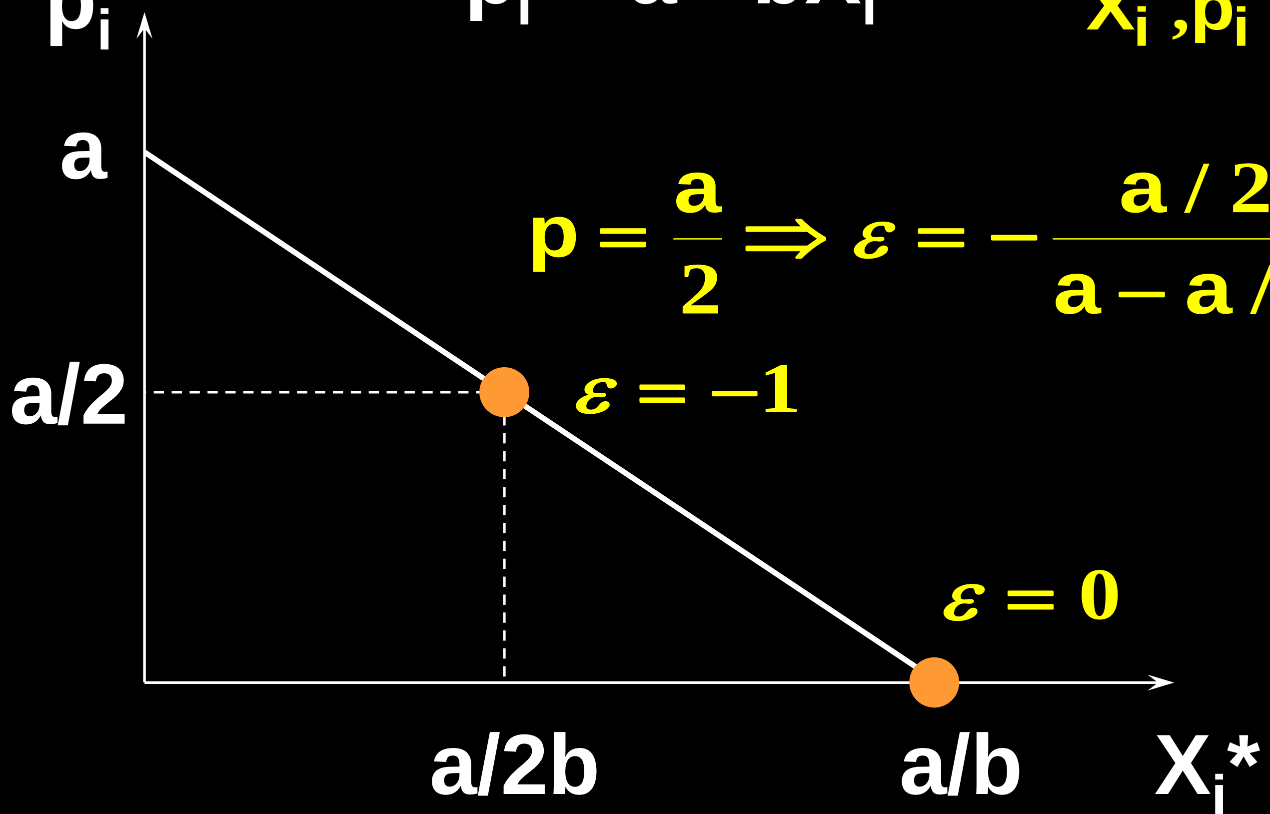
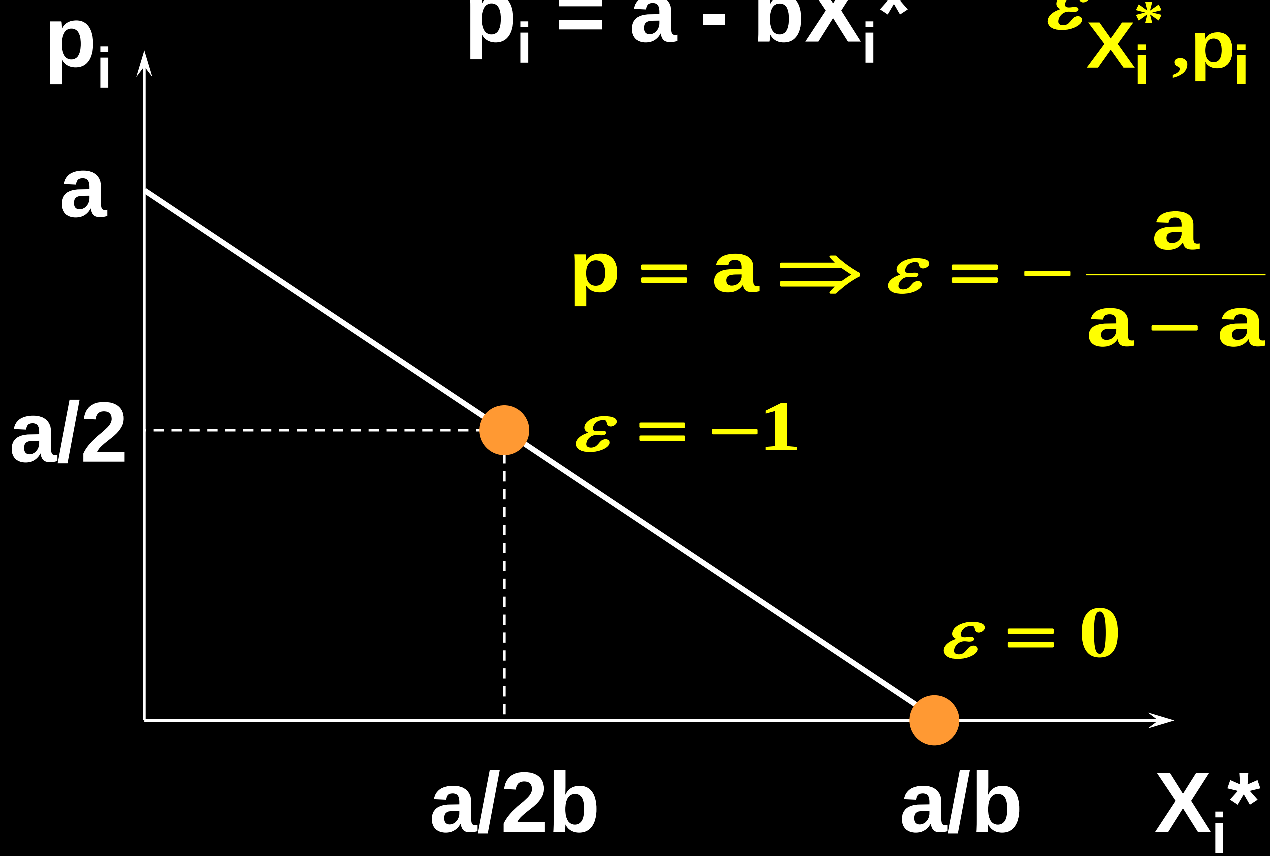


Point Own-Price Elasticity

$p_i = a - bX_i^*$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = a \Rightarrow \varepsilon = -\frac{a}{a-a} = -\infty$$



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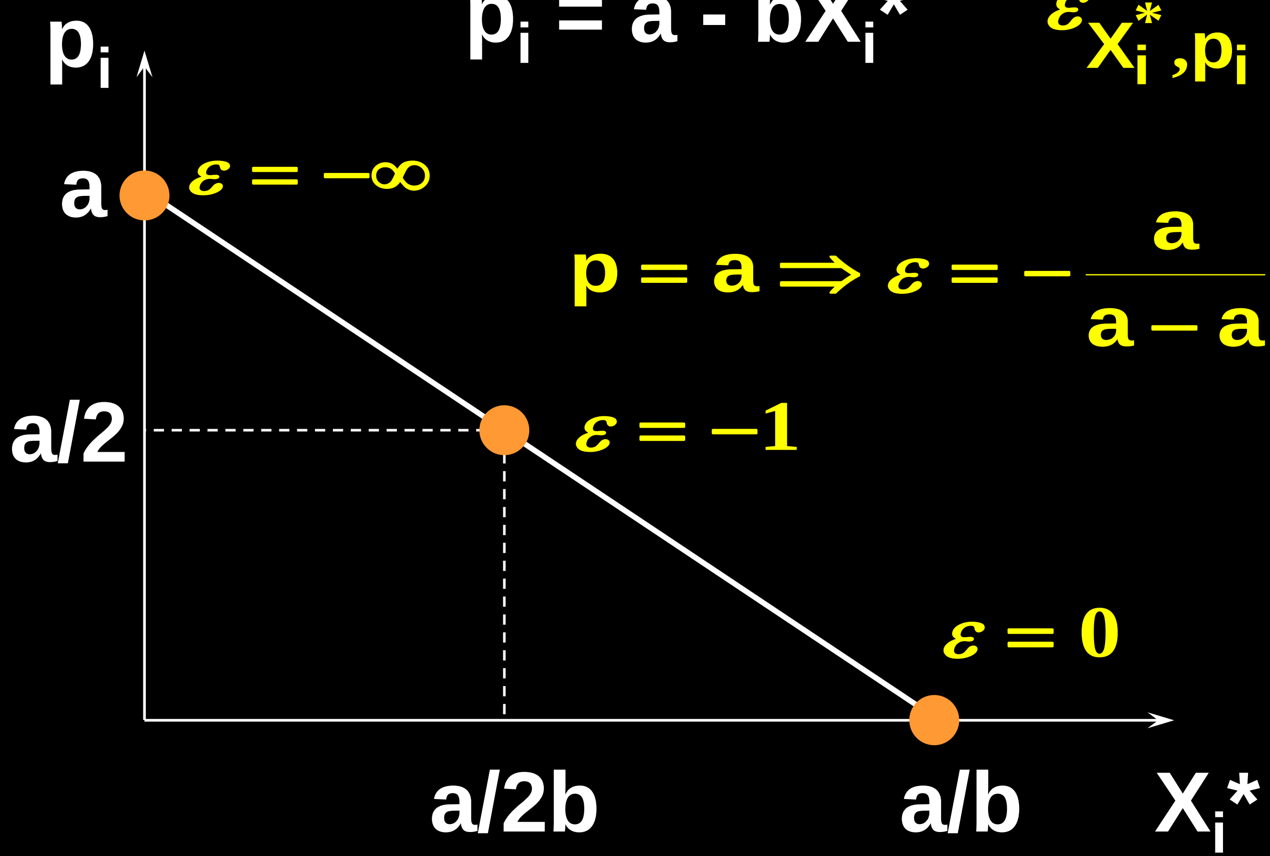

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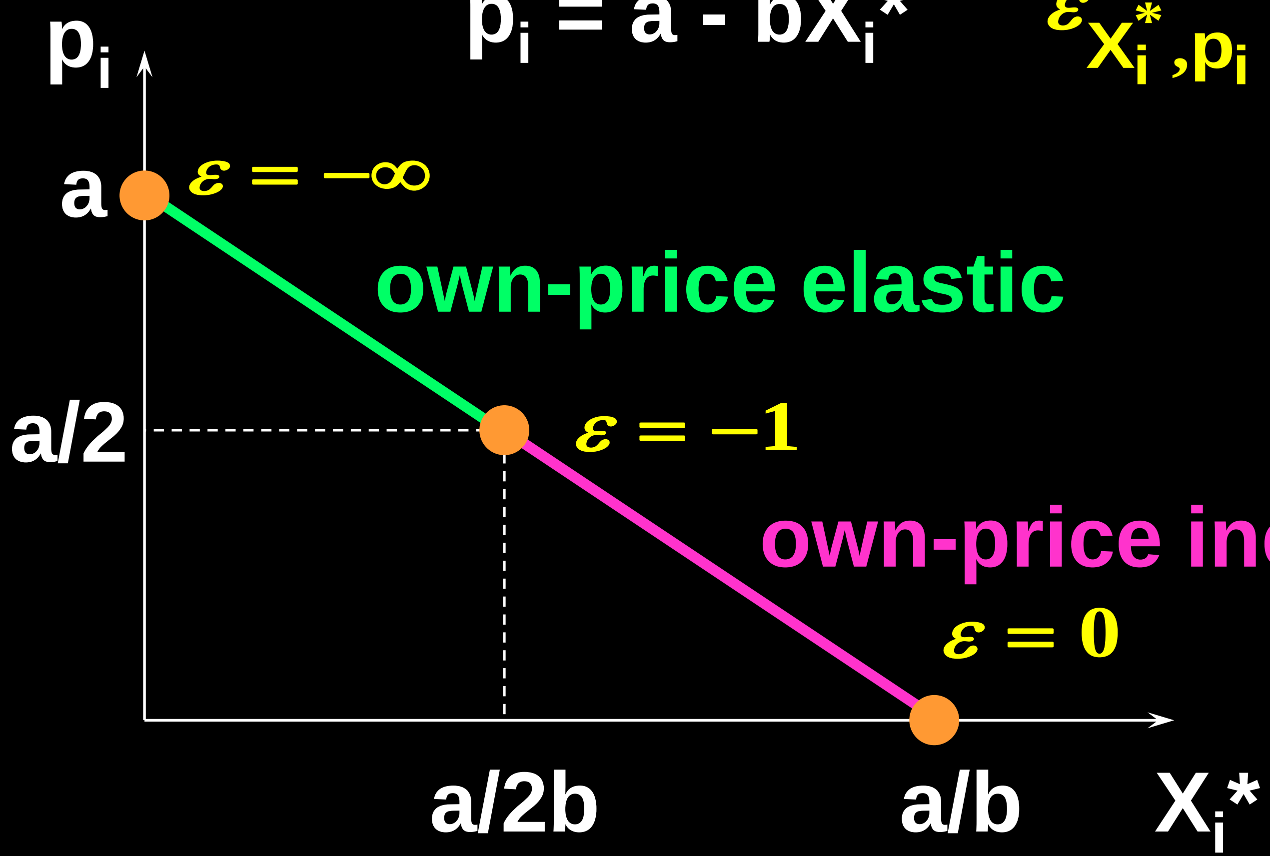

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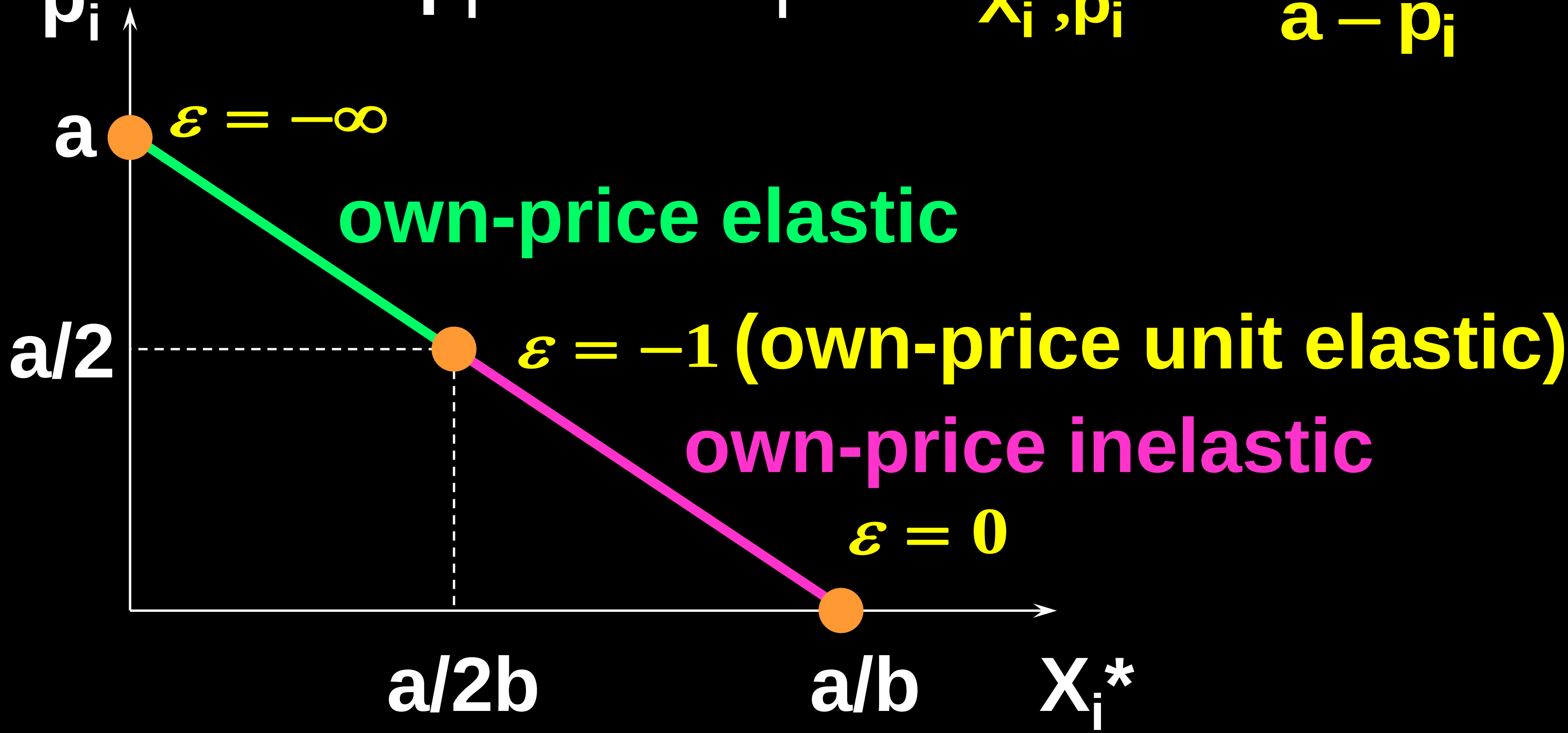


Point Own-Price Elasticity

$$\varepsilon_{x_i^*,p_i} = \frac{p_i}{x_i^*} \times \frac{dx_i^*}{dp_i}$$

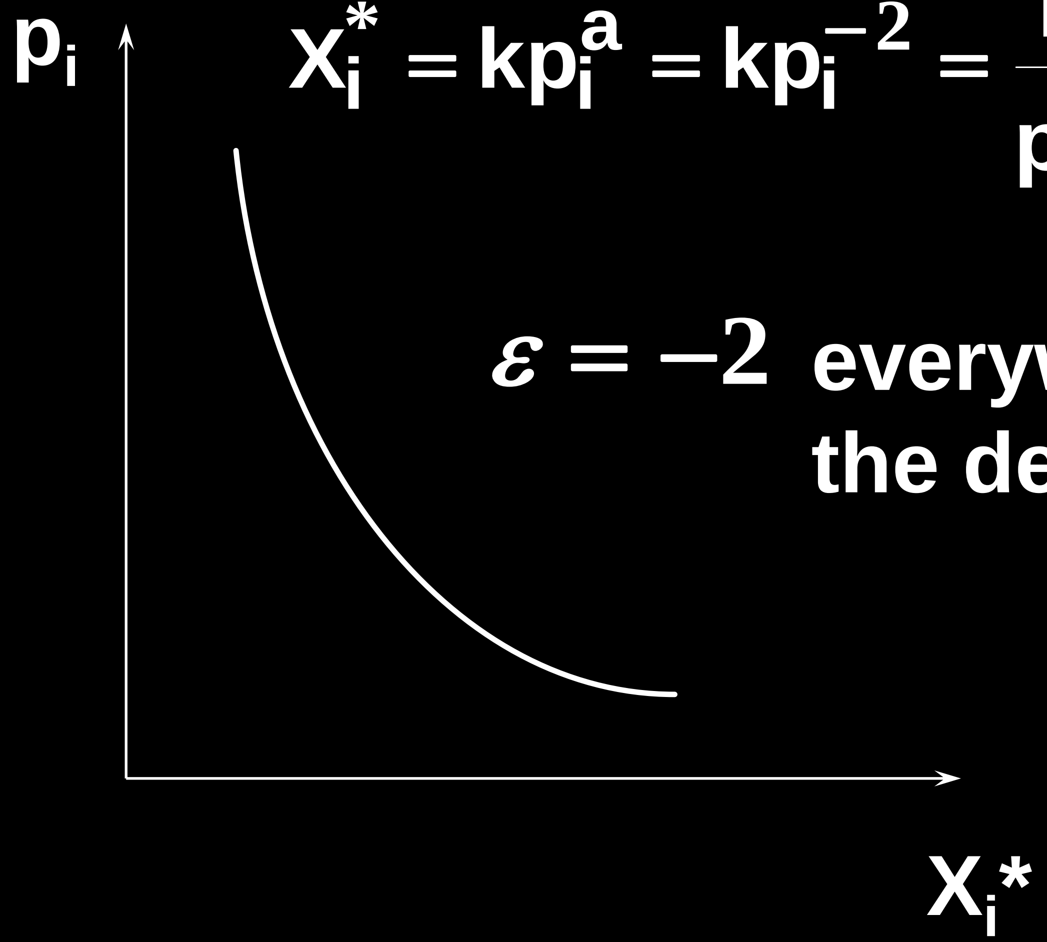
E.g. $x_i^* = kp_i^a$. Then $\dfrac{dx_i^*}{dp_i} = ap_i^{a-1}$

so

$$\varepsilon_{x_i^*,p_i} = \frac{p_i}{kp_i^a} \times kap_i^{a-1} = a\frac{p_i^a}{p_i^a} = a.$$

Point Own-Price Elasticity

$$X_i^* = kp_i^a = kp_i^{-2} = \frac{k}{p_i^2}$$



$\varepsilon = -2$ everywhere along the demand curve.

Revenue and Own-Price Elasticity of Demand

- **If raising a commodity's price causes little decrease in quantity demanded, then sellers' revenues rise.**
- **Hence own-price inelastic demand causes sellers' revenues to rise as price rises.**

Revenue and Own-Price Elasticity of Demand

- **If raising a commodity's price causes a large decrease in quantity demanded, then sellers' revenues fall.**
- **Hence own-price elastic demand causes sellers' revenues to fall as price rises.**

Revenue and Own-Price Elasticity of Demand

- Sellers' revenue is $R(p) = p \times X^*(p).$

Revenue and Own-Price Elasticity of Demand

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So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

Revenue and Own-Price Elasticity of Demand

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So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1 + \varepsilon].$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)\left[1 + \varepsilon\right]$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

so if $\varepsilon = -1$ then $\dfrac{dR}{dp} = 0$

and a change to price does not alter sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^{*}(p)[1+\varepsilon]$$

but if $-1 < \varepsilon \le 0$ **then** $\dfrac{dR}{dp} > 0$

and a price increase raises sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

And if $\varepsilon < -1$ then $\dfrac{dR}{dp} < 0$

and a price increase reduces sellers' revenue.

Revenue and Own-Price Elasticity of Demand

In summary:

Own-price inelastic demand; $-1 < \varepsilon \le 0$
price rise causes rise in sellers' revenue.

Own-price unit elastic demand; $\varepsilon = -1$
price rise causes no change in sellers' revenue.

Own-price elastic demand; $\varepsilon < -1$
price rise causes fall in sellers' revenue.

Marginal Revenue and Own-Price Elasticity of Demand

- A seller's **marginal revenue** is the rate at which revenue changes with the number of units sold by the seller.

$$MR(q) = \frac{dR(q)}{dq}.$$

Marginal Revenue and Own-Price Elasticity of Demand

p(q) denotes the seller's inverse demand function; i.e. the price at which the seller can sell q units. Then

$$R(q) = p(q) \times q$$

so

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} q + p(q)$$

$$= p(q)\left[1 + \frac{q}{p(q)} \frac{dp(q)}{dq}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right].$$

and $$\varepsilon = \frac{dq}{dp} \times \frac{p}{q}$$

so $$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$ says that the rate

at which a seller's revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price; *i.e.*, upon the of the own-price elasticity of demand.

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$

If $\varepsilon = -1$ then $MR(q) = 0.$

If $-1 < \varepsilon \le 0$ then $MR(q) < 0.$

If $\varepsilon < -1$ then $MR(q) > 0.$

Marginal Revenue and Own-Price Elasticity of Demand

If $\varepsilon = -1$ then $MR(q) = 0.$ Selling one more unit does not change the seller's revenue.

If $-1 < \varepsilon \le 0$ then $MR(q) < 0.$ Selling one more unit reduces the seller's revenue.

If $\varepsilon < -1$ then $MR(q) > 0.$ Selling one more unit raises the seller's revenue.

Marginal Revenue and Own-Price Elasticity of Demand

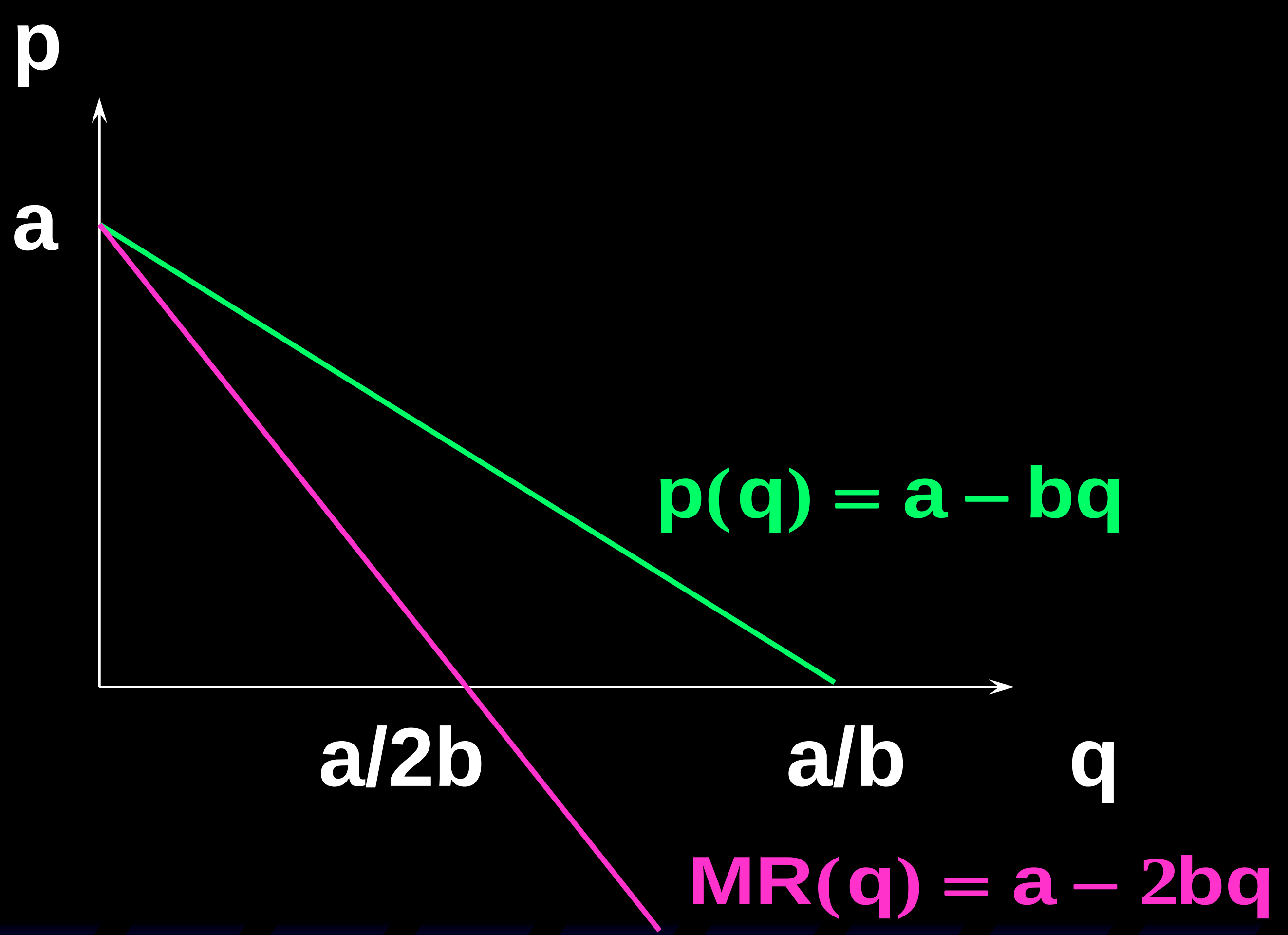
An example with linear inverse demand.

$$p(q) = a - bq.$$

Then $R(q) = p(q)q = (a - bq)q$

and $MR(q) = a - 2bq.$

Marginal Revenue and Own-Price Elasticity of Demand



Marginal Revenue and Own-Price Elasticity of Demand