Chapter Twenty-One

Cost Curves

Types of Cost Curves

- A **total cost curve** is the graph of a firm's total cost function.
- A **variable cost curve** is the graph of a firm's variable cost function.
- An **average total cost curve** is the graph of a firm's average total cost function.

Types of Cost Curves

- An **average variable cost curve** is the graph of a firm's average variable cost function.
- An **average fixed cost curve** is the graph of a firm's average fixed cost function.
- A **marginal cost curve** is the graph of a firm's marginal cost function.

Types of Cost Curves

- **How are these cost curves related to each other?**
- **How are a firm's long-run and short-run cost curves related?**

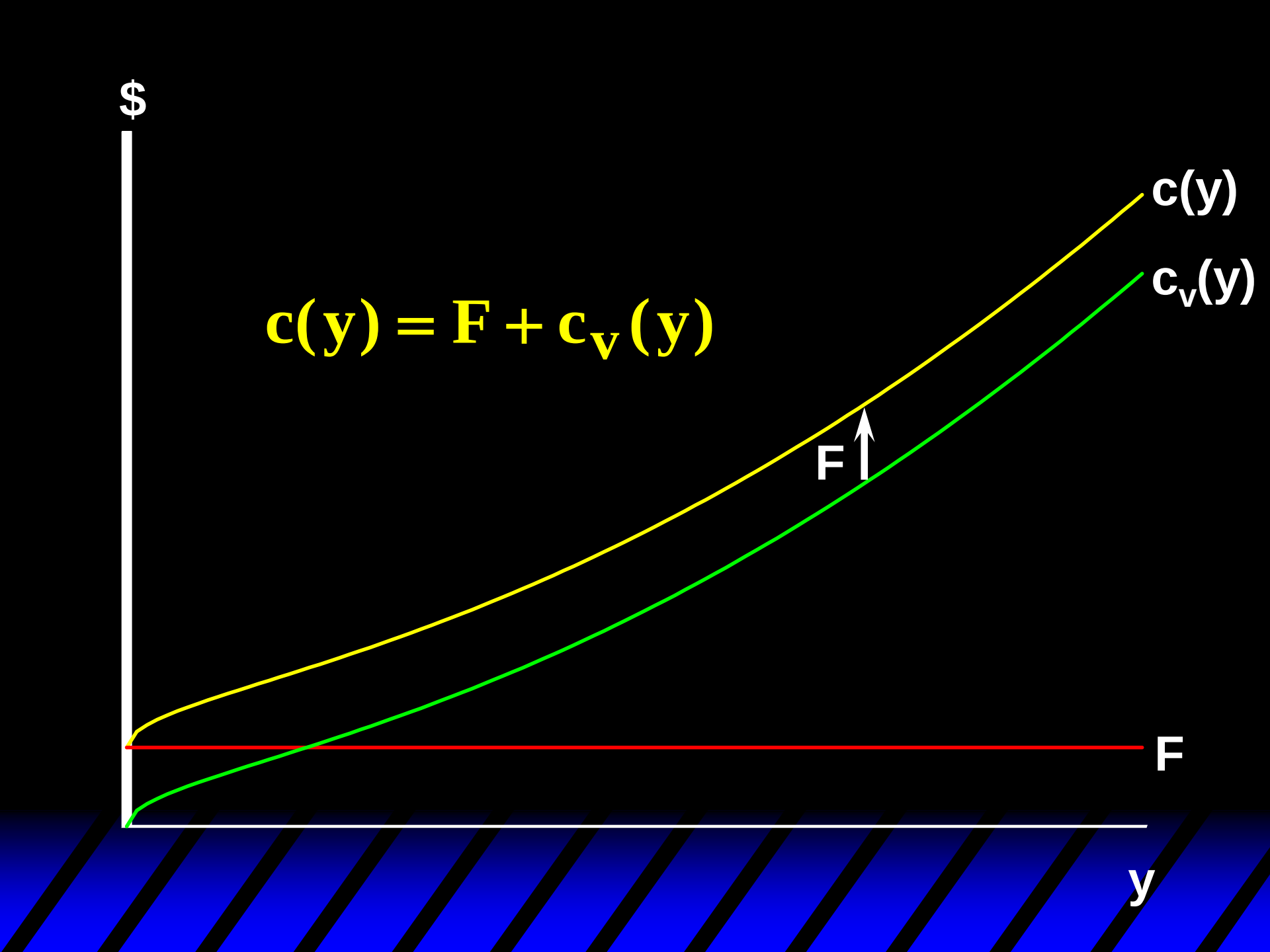
Fixed, Variable & Total Cost Functions

- **F is the total cost to a firm of its short-run fixed inputs. F, the firm's fixed cost, does not vary with the firm's output level.**
- **$c_v(y)$ is the total cost to a firm of its variable inputs when producing y output units. $c_v(y)$ is the firm's variable cost function.**
- **$c_v(y)$ depends upon the levels of the fixed inputs.**

Fixed, Variable & Total Cost Functions

- **c(y) is the total cost of all inputs, fixed and variable, when producing y output units. c(y) is the firm's total cost function;**

$$c(y) = F + c_v(y).$$


$
c(y)
$c_v(y)$
$c(y) = F + c_v(y)$
F
F
y

Av. Fixed, Av. Variable & Av. Total Cost Curves

- The firm's total cost function is

$$c(y) = F + c_V(y).$$

For y > 0, the firm's average total cost function is

$$AC(y) = \frac{F}{y} + \frac{c_V(y)}{y}$$

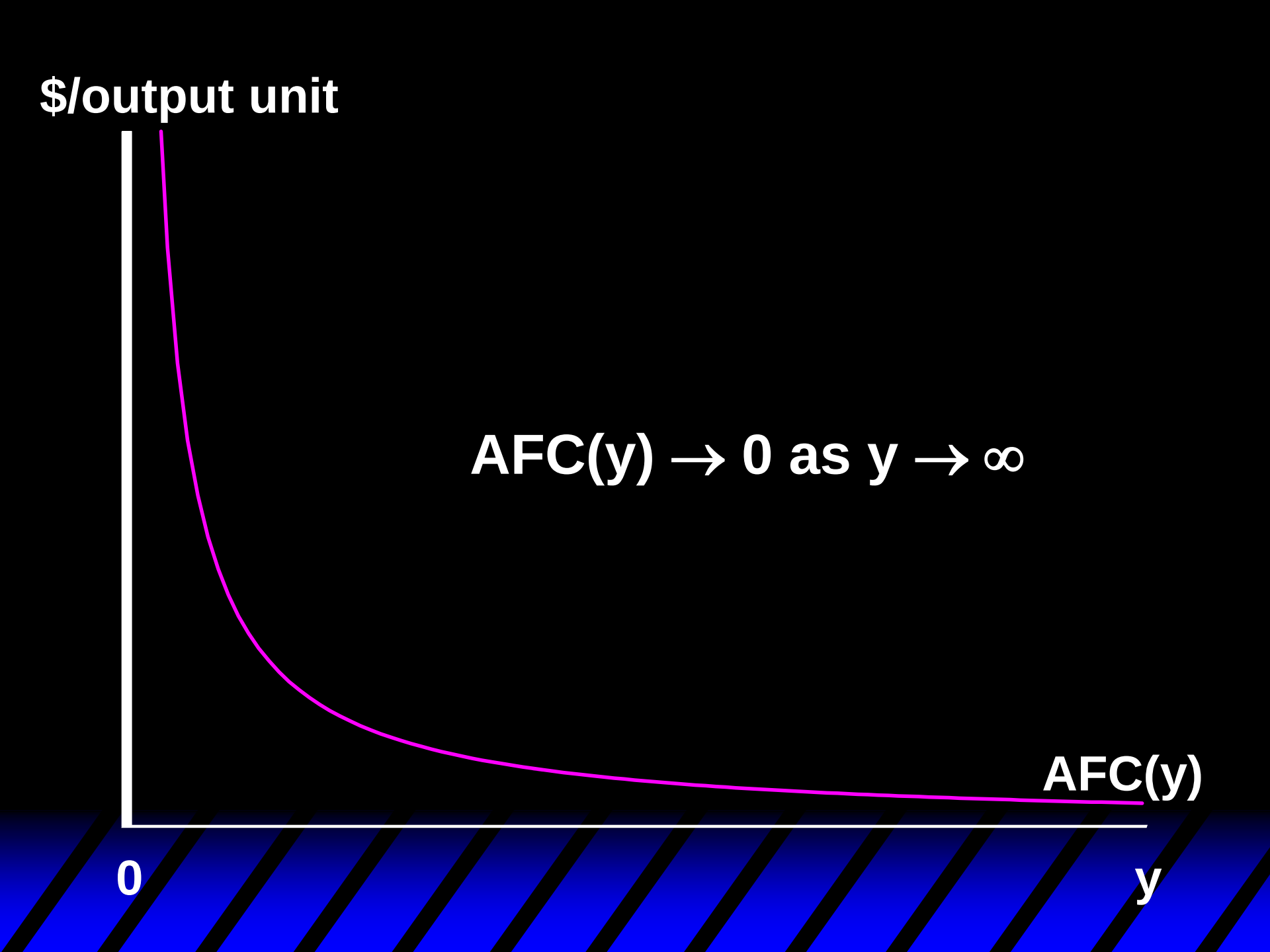
$$= AFC(y) + AVC(y).$$

Av. Fixed, Av. Variable & Av. Total Cost Curves

- **What does an average fixed cost curve look like?**

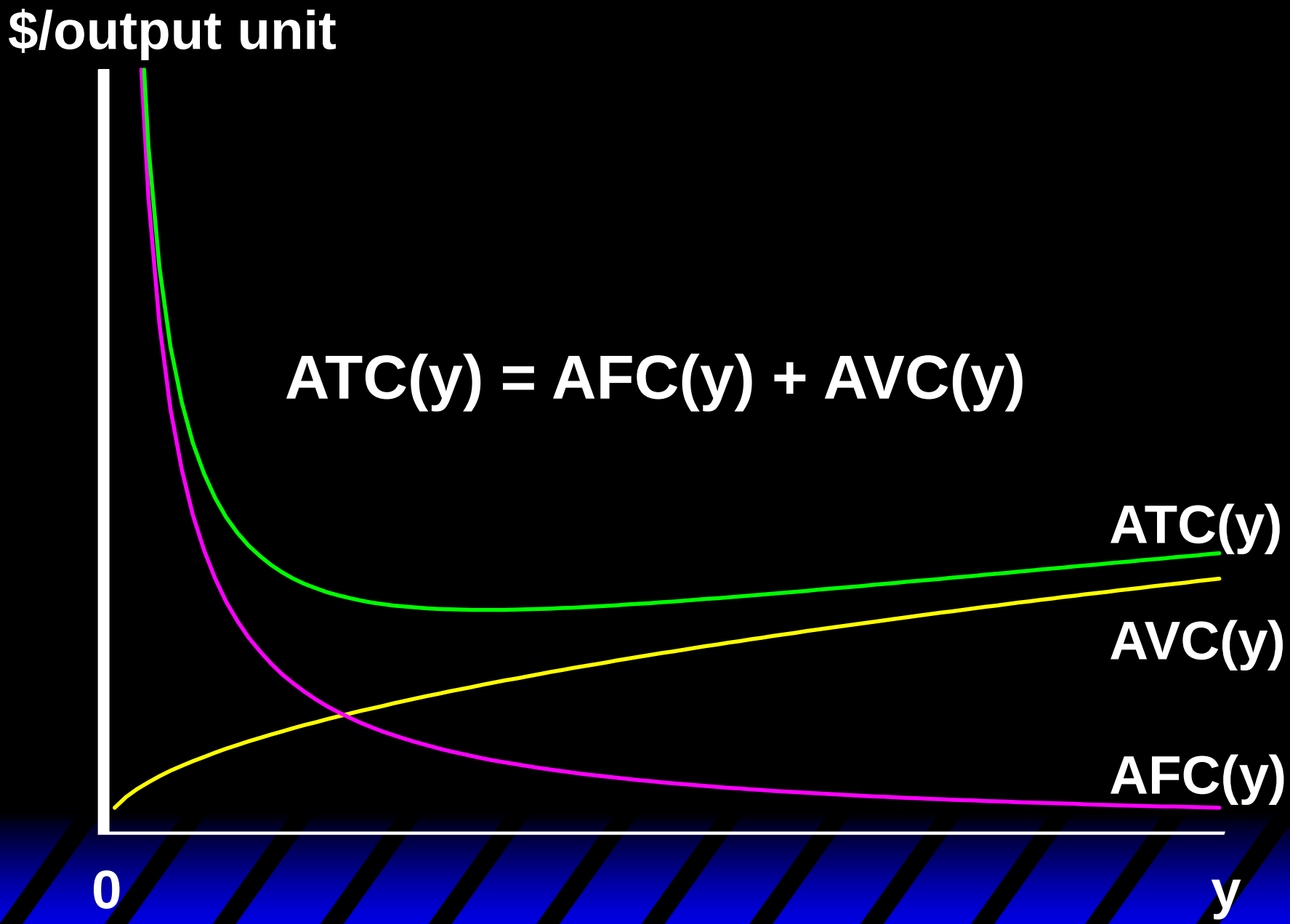
$$AFC(y) = \frac{F}{y}$$

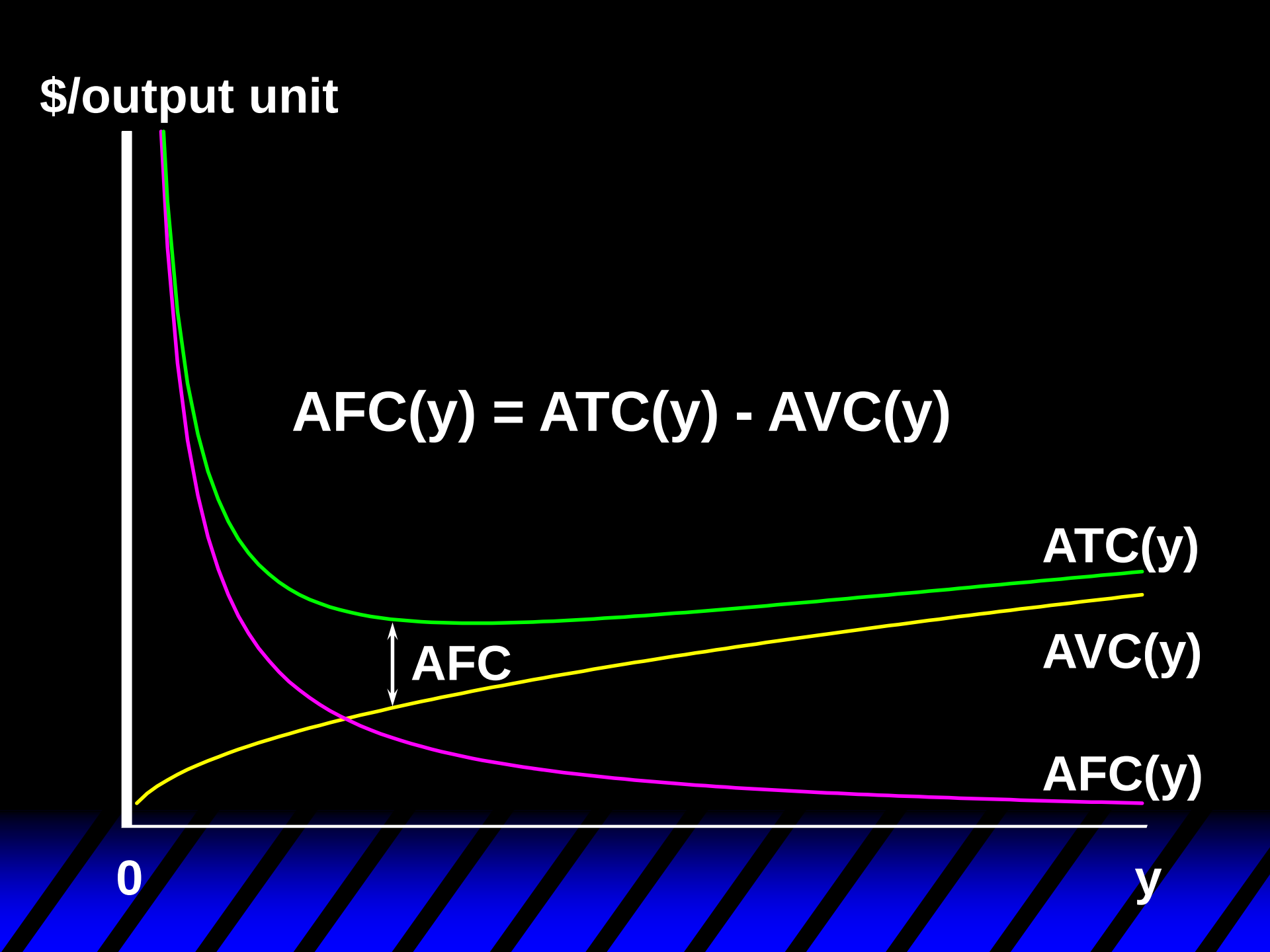
- **AFC(y) is a rectangular hyperbola so its graph looks like ...**

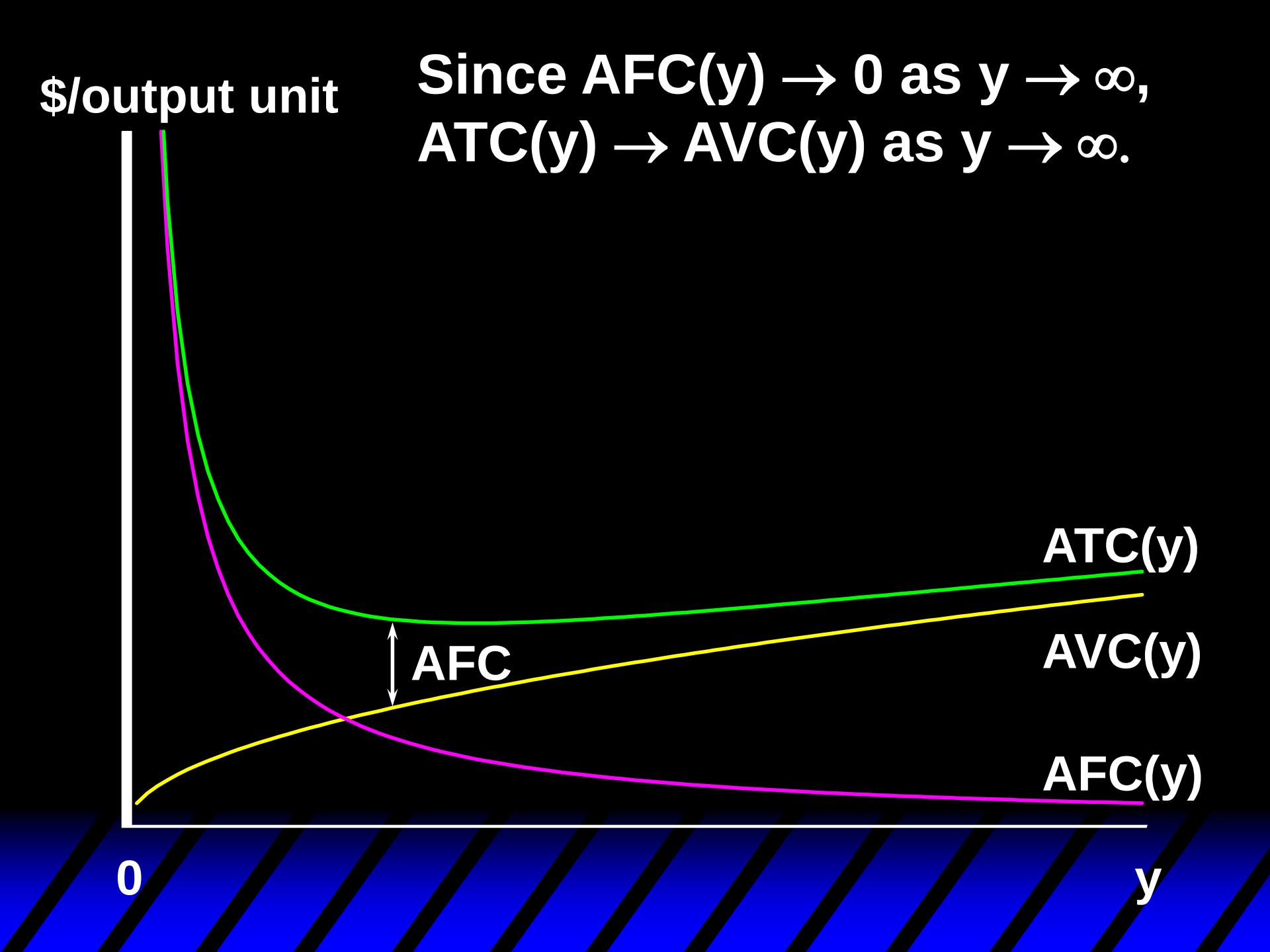

$/output unit
AFC(y) → 0 as y → ∞
AFC(y)
0
y

Av. Fixed, Av. Variable & Av. Total Cost Curves

- **In a short-run with a fixed amount of at least one input, the Law of Diminishing (Marginal) Returns must apply, causing the firm's average variable cost of production to increase eventually.**

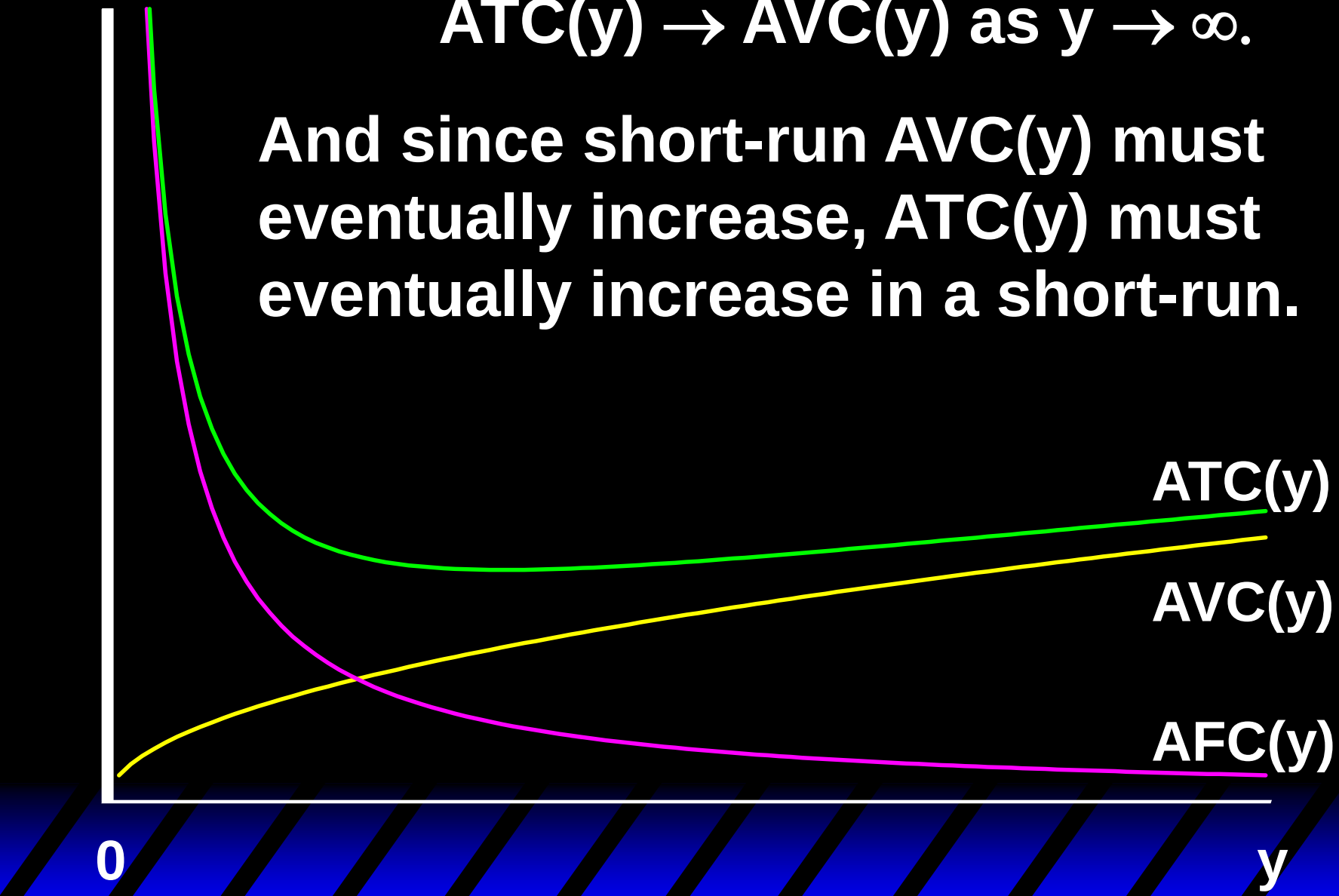

$/output unit
ATC(y) = AFC(y) + AVC(y)
ATC(y)
AVC(y)
AFC(y)
0
y


$/output unit
AFC(y) = ATC(y) - AVC(y)
ATC(y)
AVC(y)
AFC
AFC(y)
0
y


$/output unit
Since AFC(y) → 0 as y → ∞,
ATC(y) → AVC(y) as y → ∞.
ATC(y)
AVC(y)
AFC
AFC(y)
0
y

Since AFC(y) $\rightarrow$ 0 as y $\rightarrow \infty$,
ATC(y) $\rightarrow$ AVC(y) as y $\rightarrow \infty$.

And since short-run AVC(y) must eventually increase, ATC(y) must eventually increase in a short-run.



Marginal Cost Function

- **Marginal cost is the rate-of-change of variable production cost as the output level changes. That is,**

$$\mathbf{MC(y)} = \frac{\partial\, \mathbf{c_v(y)}}{\partial\, \mathbf{y}}.$$

Marginal Cost Function

- The firm's total cost function is
$$c(y) = F + c_v(y)$$
and the fixed cost F does not change with the output level y, so
$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}.$$
- MC is the slope of both the variable cost and the total cost functions.

Marginal and Variable Cost Functions

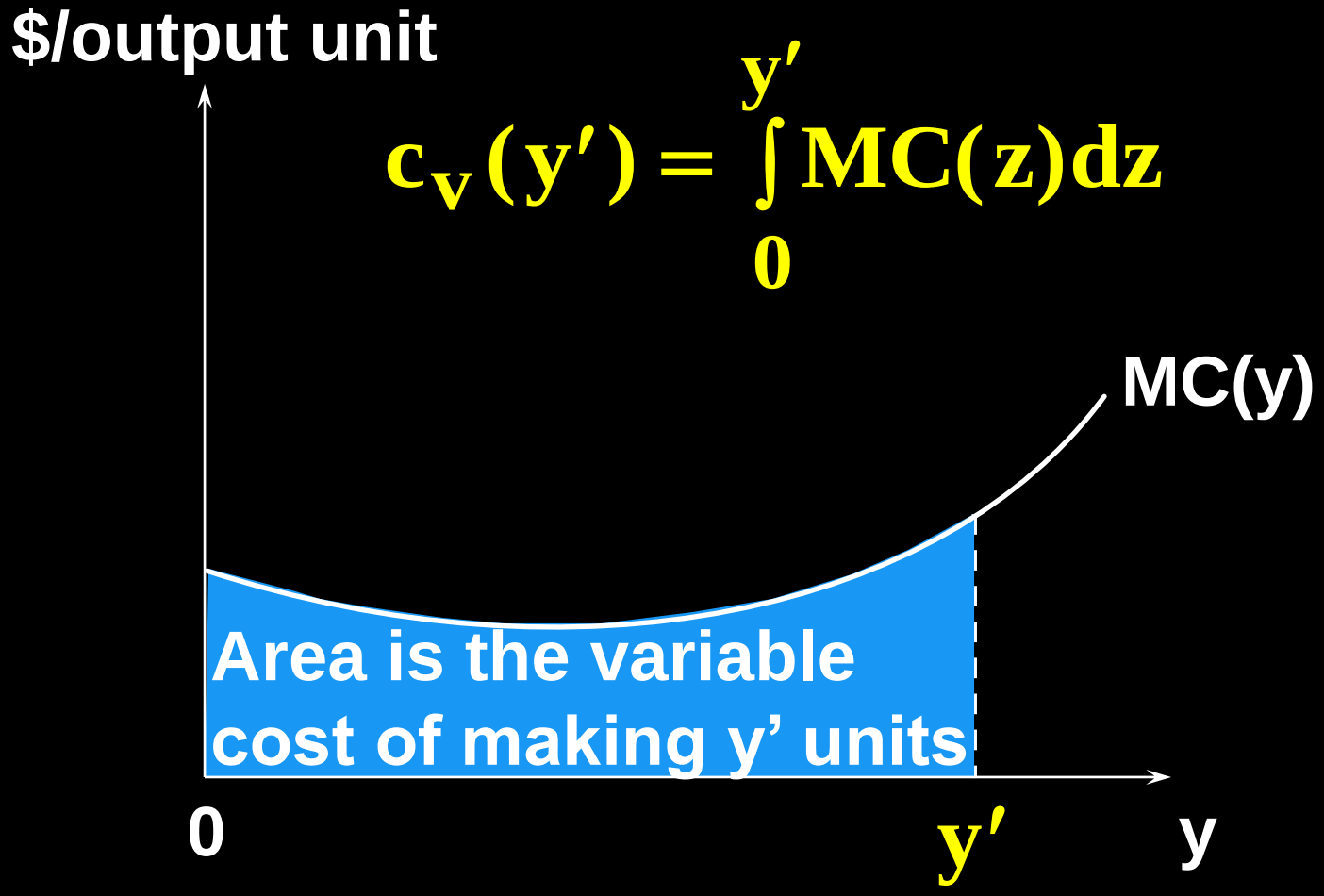
- Since MC(y) is the derivative of $c_v(y)$, $c_v(y)$ must be the integral of MC(y). That is,

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

$$\Rightarrow \quad c_v(y) = \int_0^y MC(z)dz.$$

Marginal and Variable Cost Functions

$$c_v(y') = \int_0^{y'} MC(z)dz$$



Marginal & Average Cost Functions

- **How is marginal cost related to average variable cost?**

Marginal & Average Cost Functions

Since $AVC(y) = \frac{c_V(y)}{y}$,

$$\frac{\partial AVC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c_V(y)}{y^2}.$$

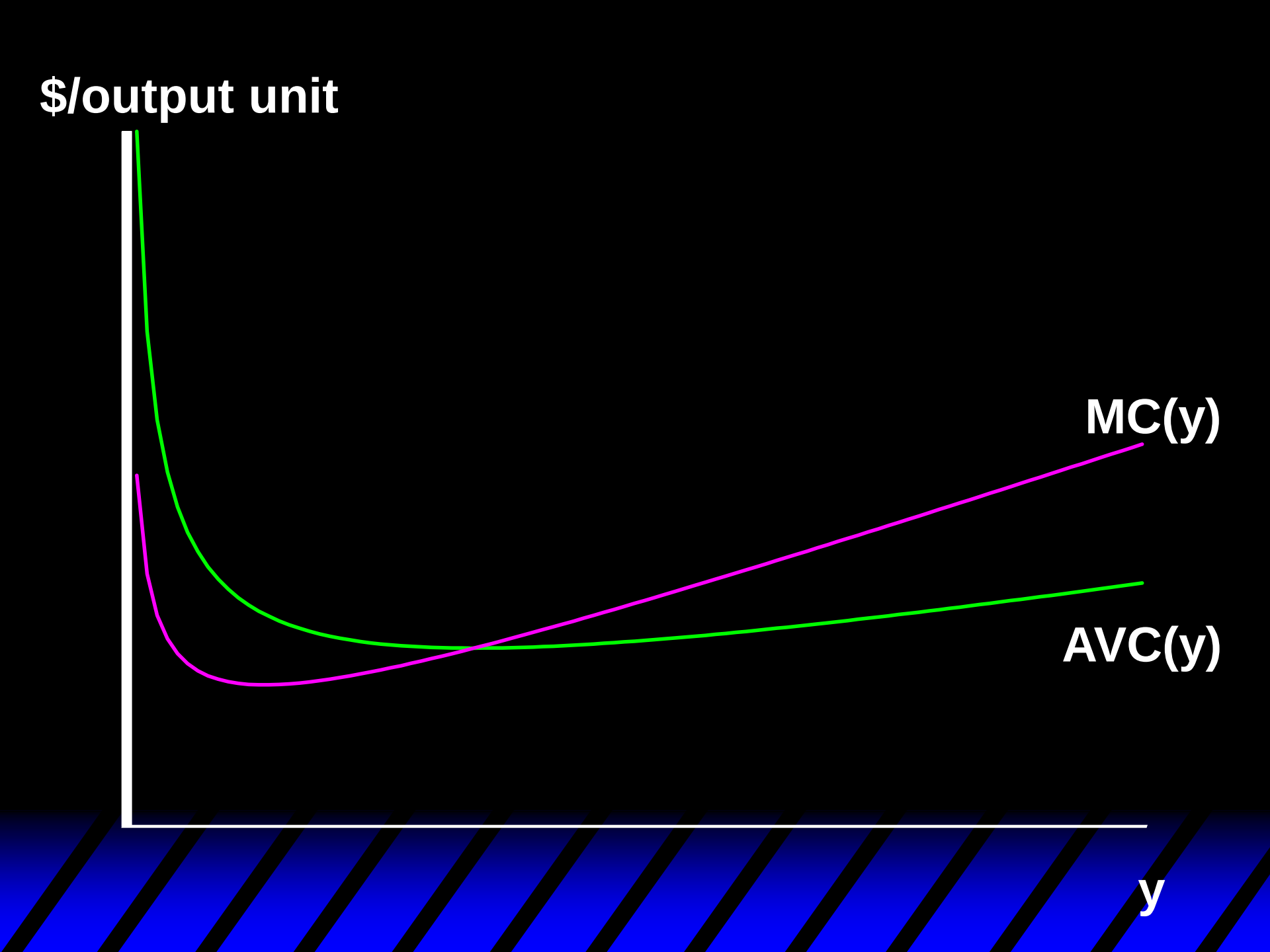
Therefore,

$$\frac{\partial AVC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad y \times MC(y) \begin{matrix} > \\ = \\ < \end{matrix} c_V(y).$$

$$\frac{\partial AVC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c_V(y)}{y} = AVC(y).$$

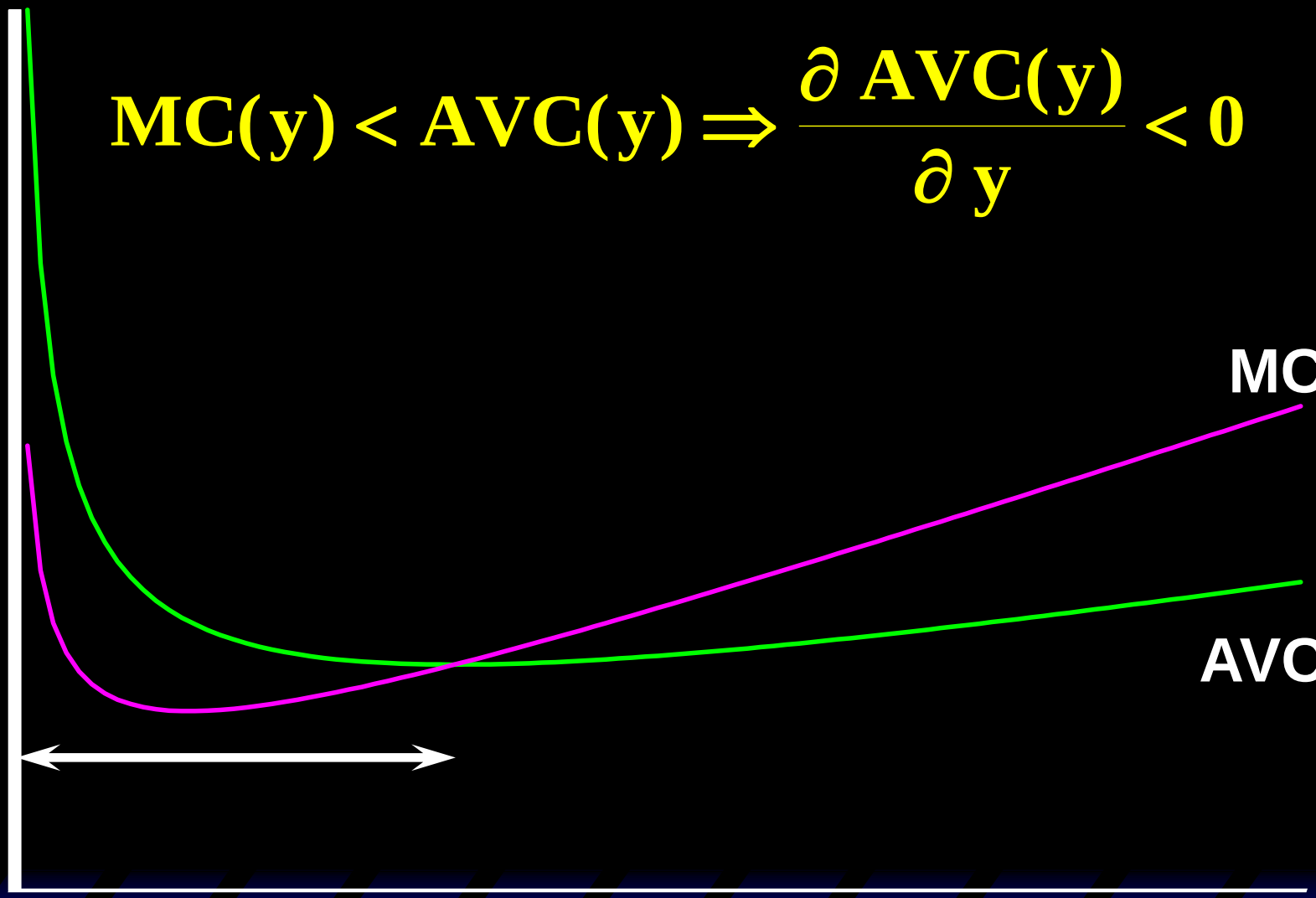
Marginal & Average Cost Functions

$$\frac{\partial \mathbf{AVC(y)}}{\partial \mathbf{y}} \begin{matrix} > \\ = \\ < \end{matrix} \mathbf{0} \quad \textbf{as} \quad \mathbf{MC(y)} \begin{matrix} > \\ = \\ < \end{matrix} \mathbf{AVC(y).}$$


$/output unit
MC(y)
AVC(y)
y

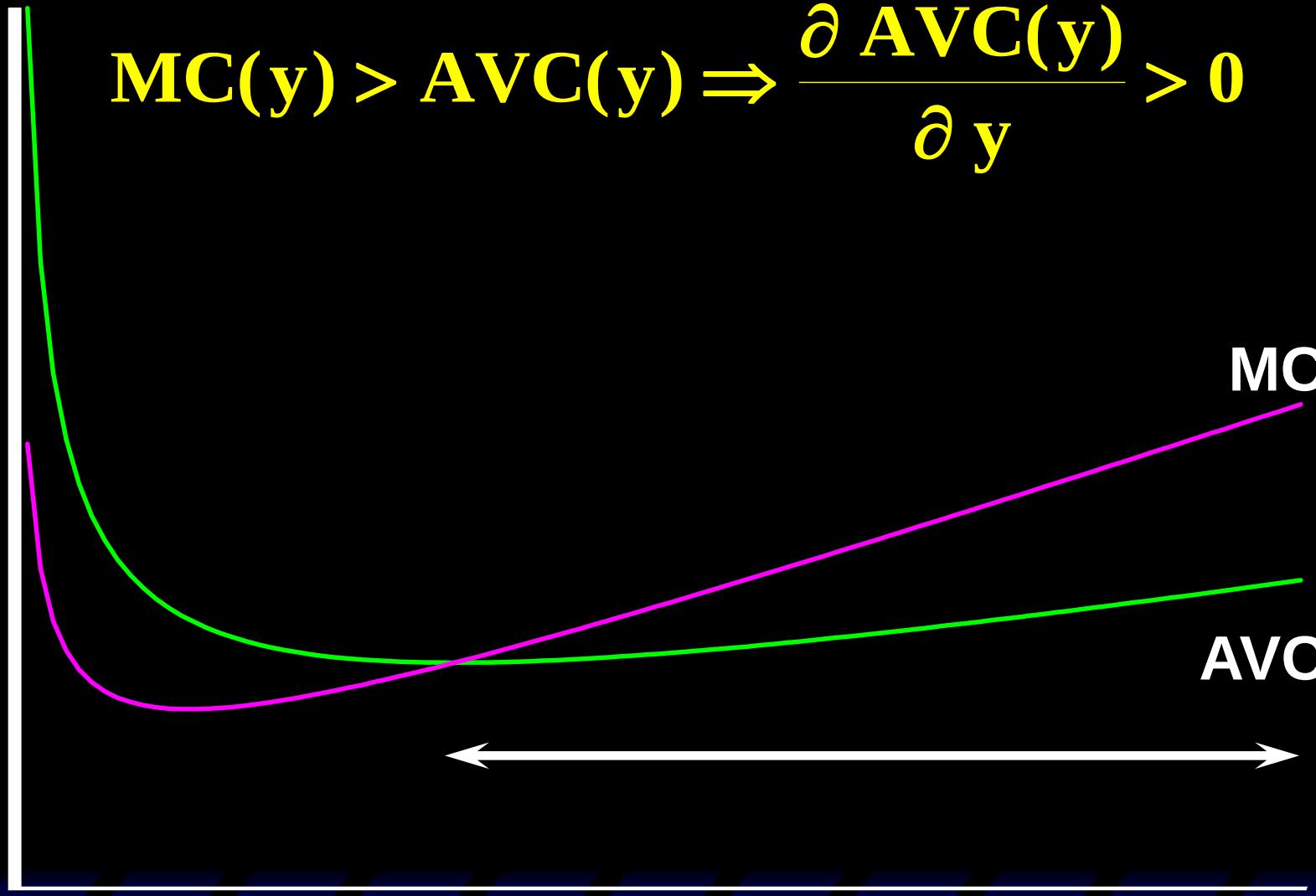
$/output unit

$$MC(y) < AVC(y) \Rightarrow \frac{\partial\, AVC(y)}{\partial\, y} < 0$$



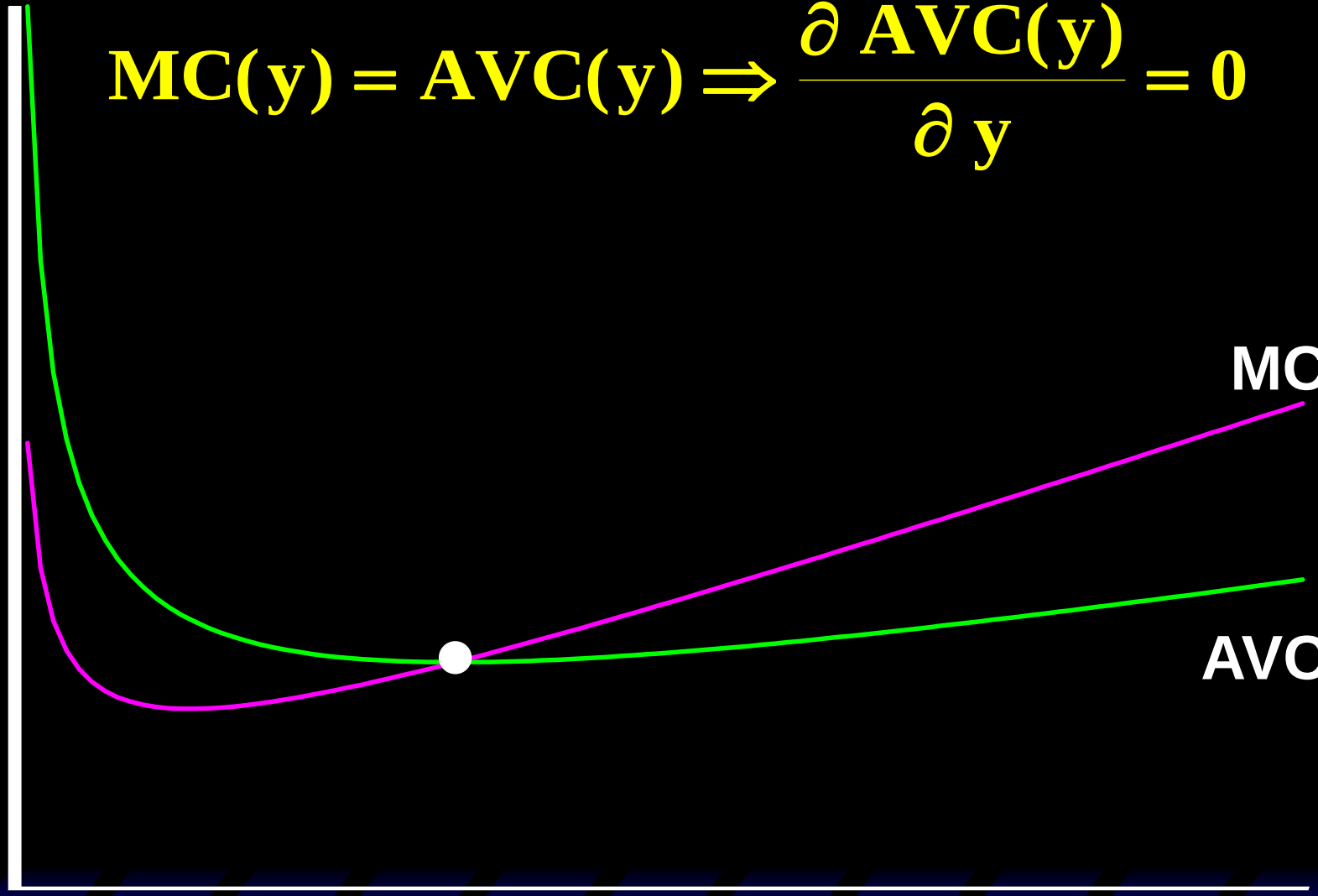
$/output unit

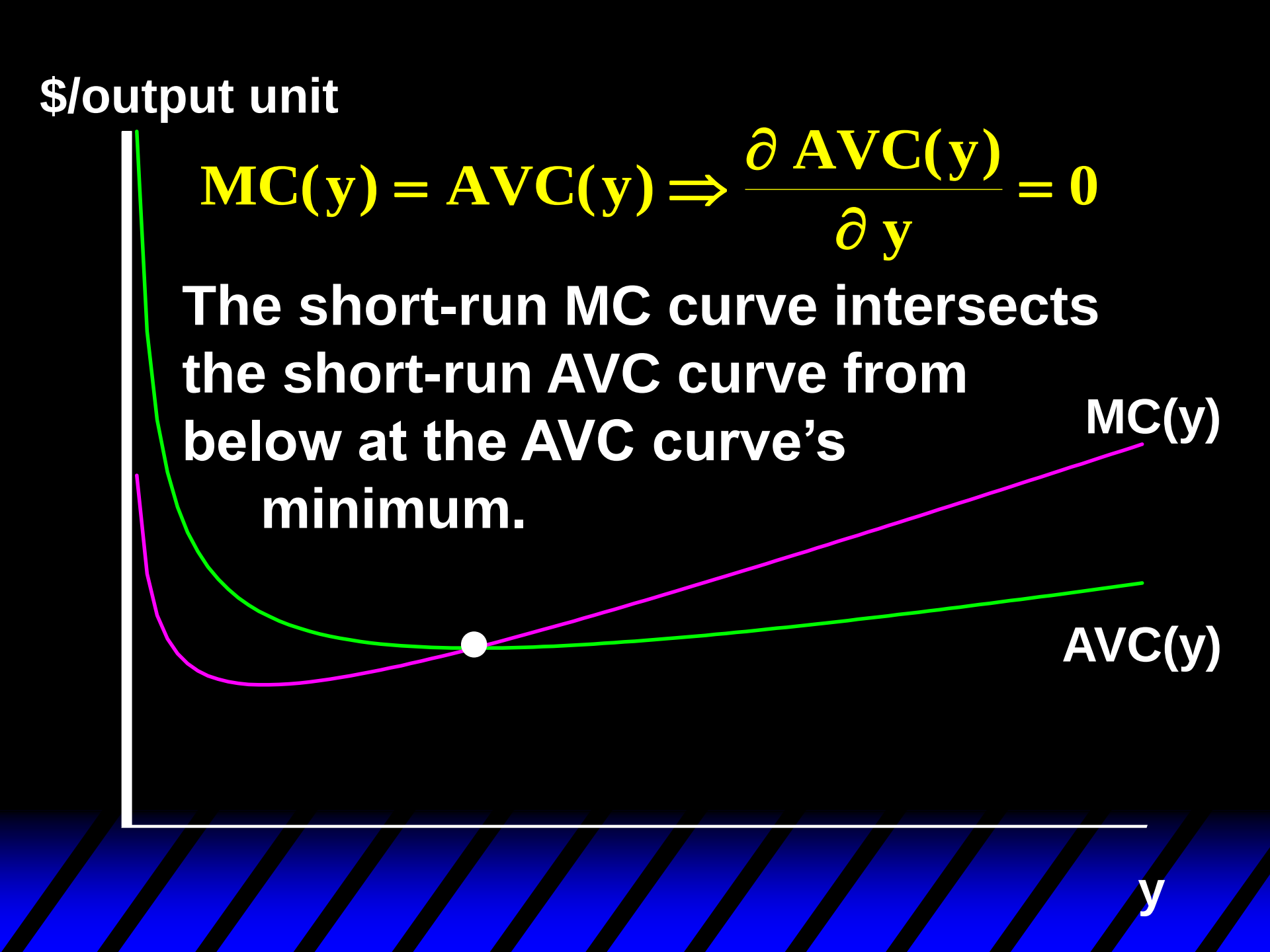
$$MC(y) > AVC(y) \Rightarrow \frac{\partial\, AVC(y)}{\partial\, y} > 0$$



$/output unit

$$MC(y) = AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} = 0$$




$/output unit
$$MC(y) = AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} = 0$$
The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.
MC(y)
AVC(y)
y

Marginal & Average Cost Functions

Similarly, since $\mathbf{ATC(y) = \frac{c(y)}{y}}$,

$$\frac{\partial \mathbf{ATC(y)}}{\partial \mathbf{y}} = \frac{\mathbf{y \times MC(y) - 1 \times c(y)}}{\mathbf{y^2}}.$$

Marginal & Average Cost Functions

Similarly, since $ATC(y) = \frac{c(y)}{y}$,

$$\frac{\partial ATC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c(y)}{y^2}.$$

Therefore,

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad y \times MC(y) \begin{matrix} > \\ = \\ < \end{matrix} c(y).$$

Marginal & Average Cost Functions

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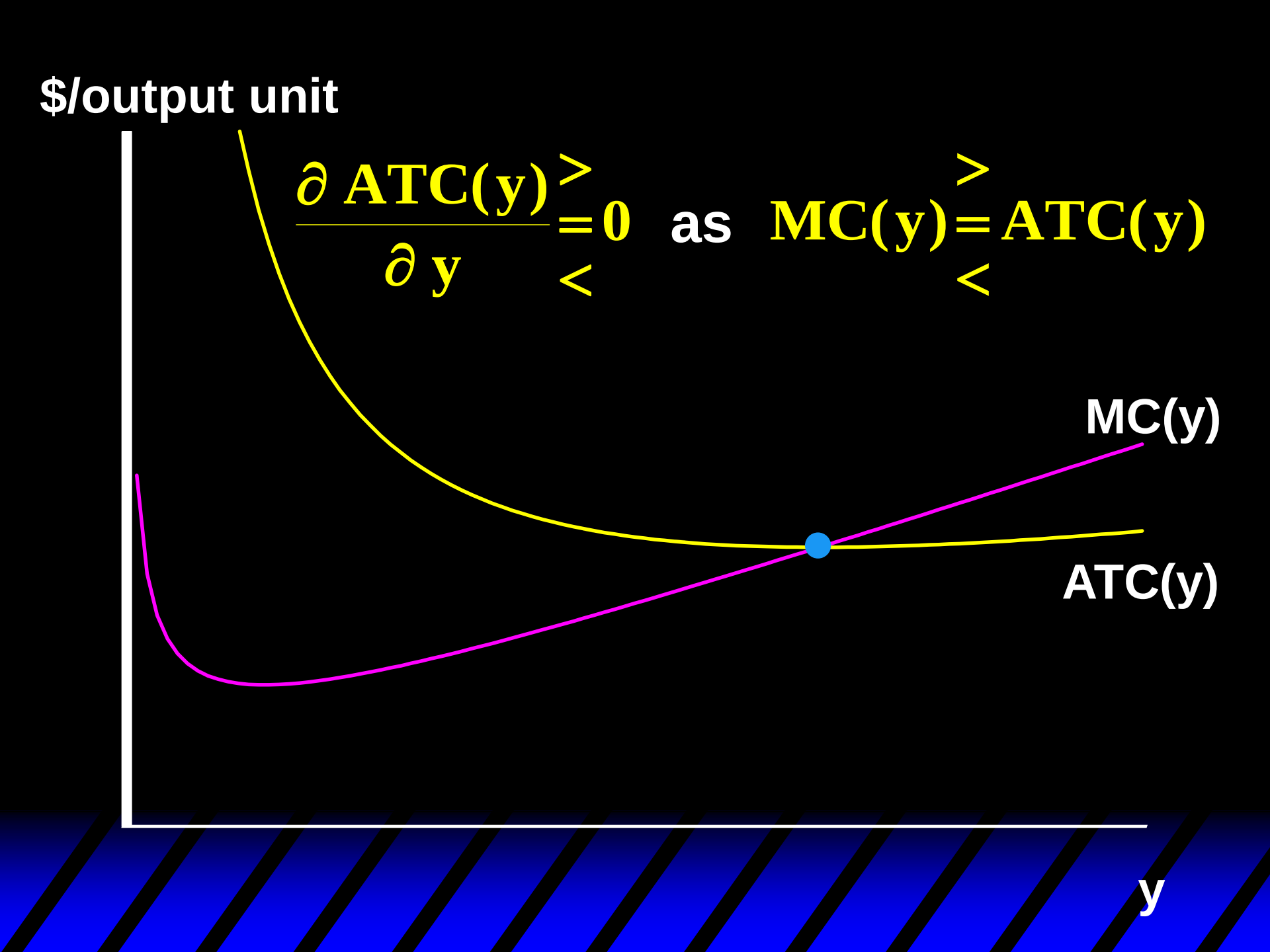
$$\frac{\partial ATC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c(y)}{y^2}.$$

Therefore,

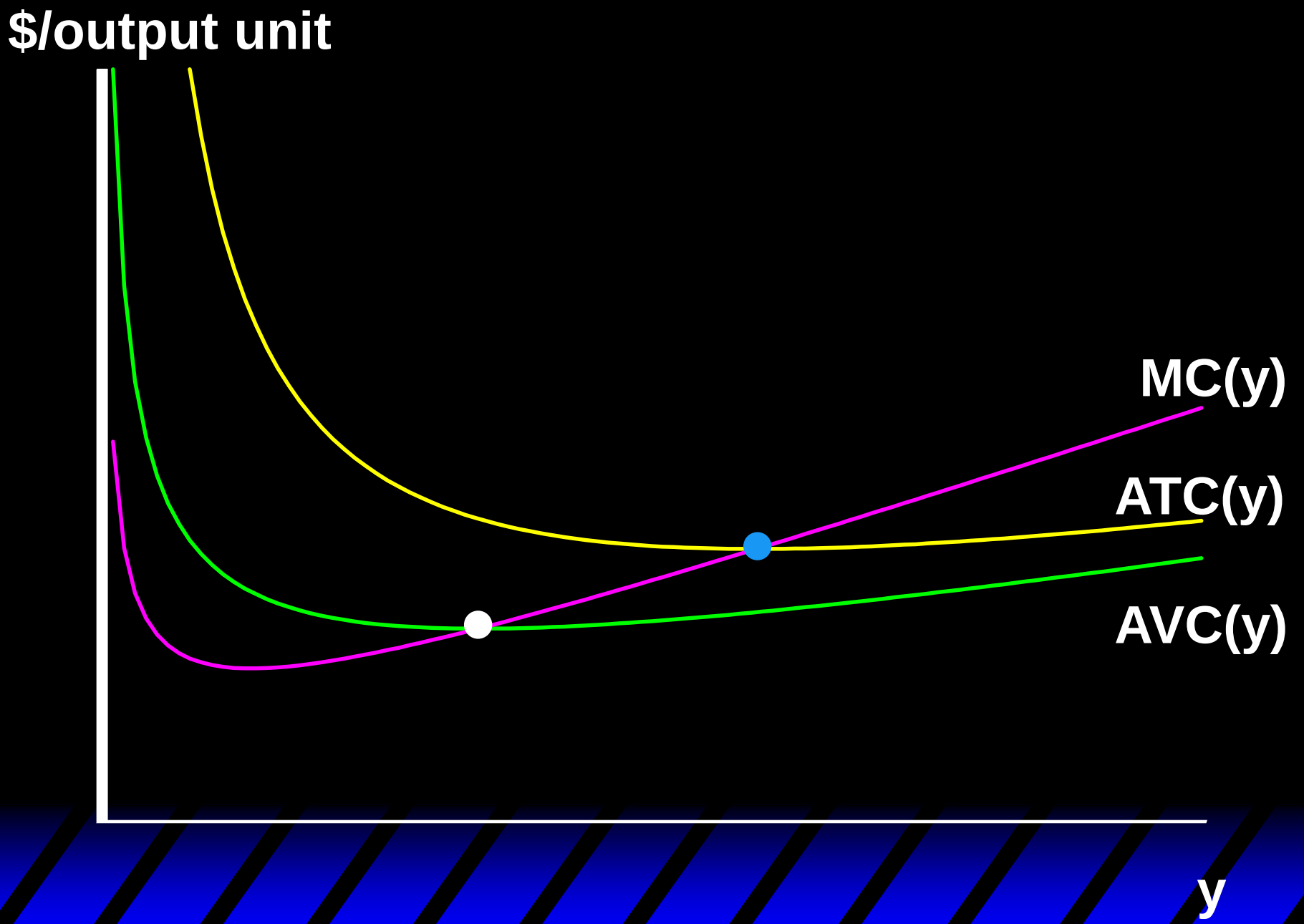
$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad y \times MC(y) \begin{matrix} > \\ = \\ < \end{matrix} c(y).$$

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c(y)}{y} = ATC(y).$$


$/output unit
$\frac{\partial ATC(y)}{\partial y} \gtrless 0$ as $MC(y) \gtrless ATC(y)$
MC(y)
ATC(y)
y

Marginal & Average Cost Functions

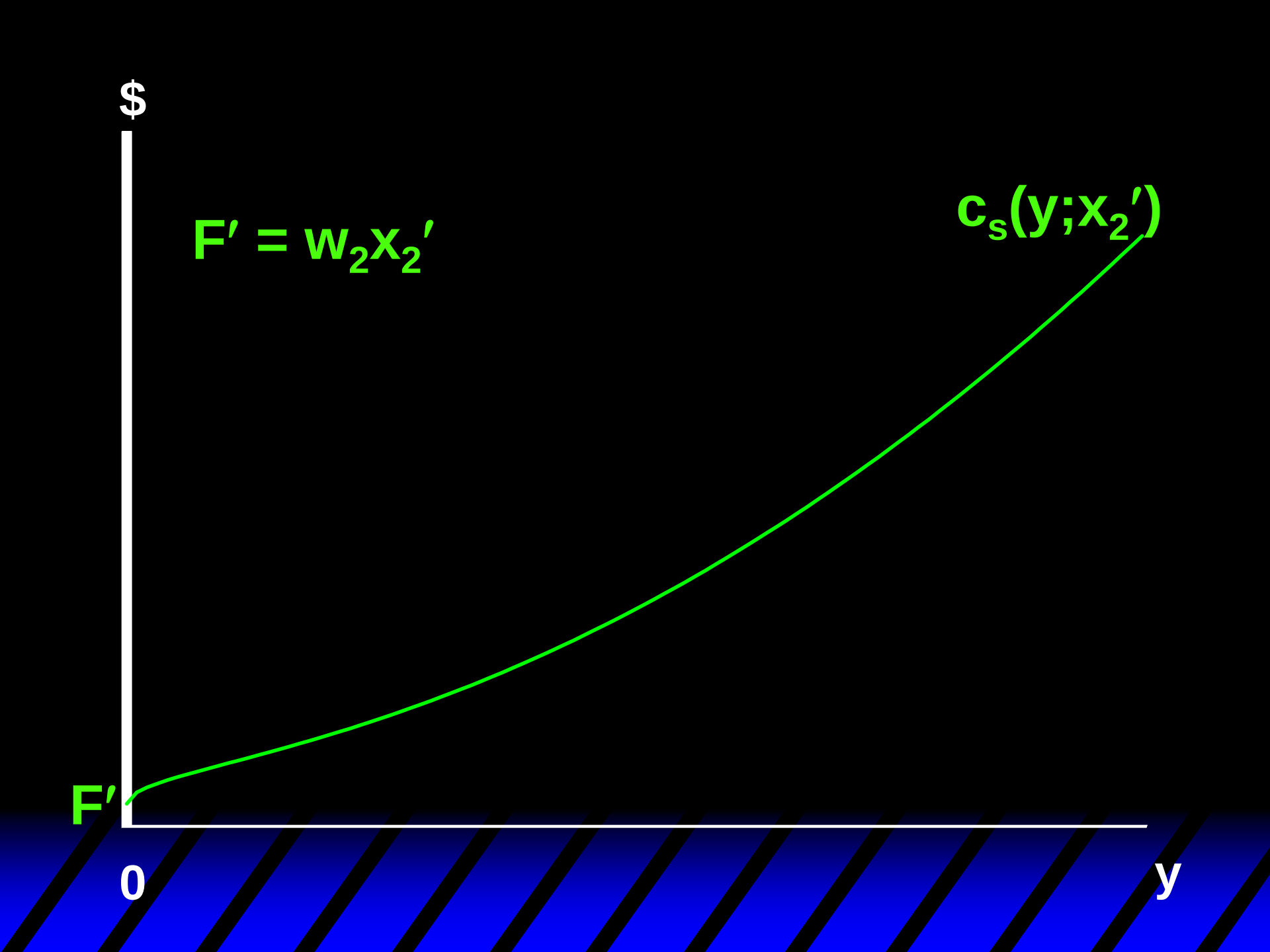
- **The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.**
- **And, similarly, the short-run MC curve intersects the short-run ATC curve from below at the ATC curve's minimum.**

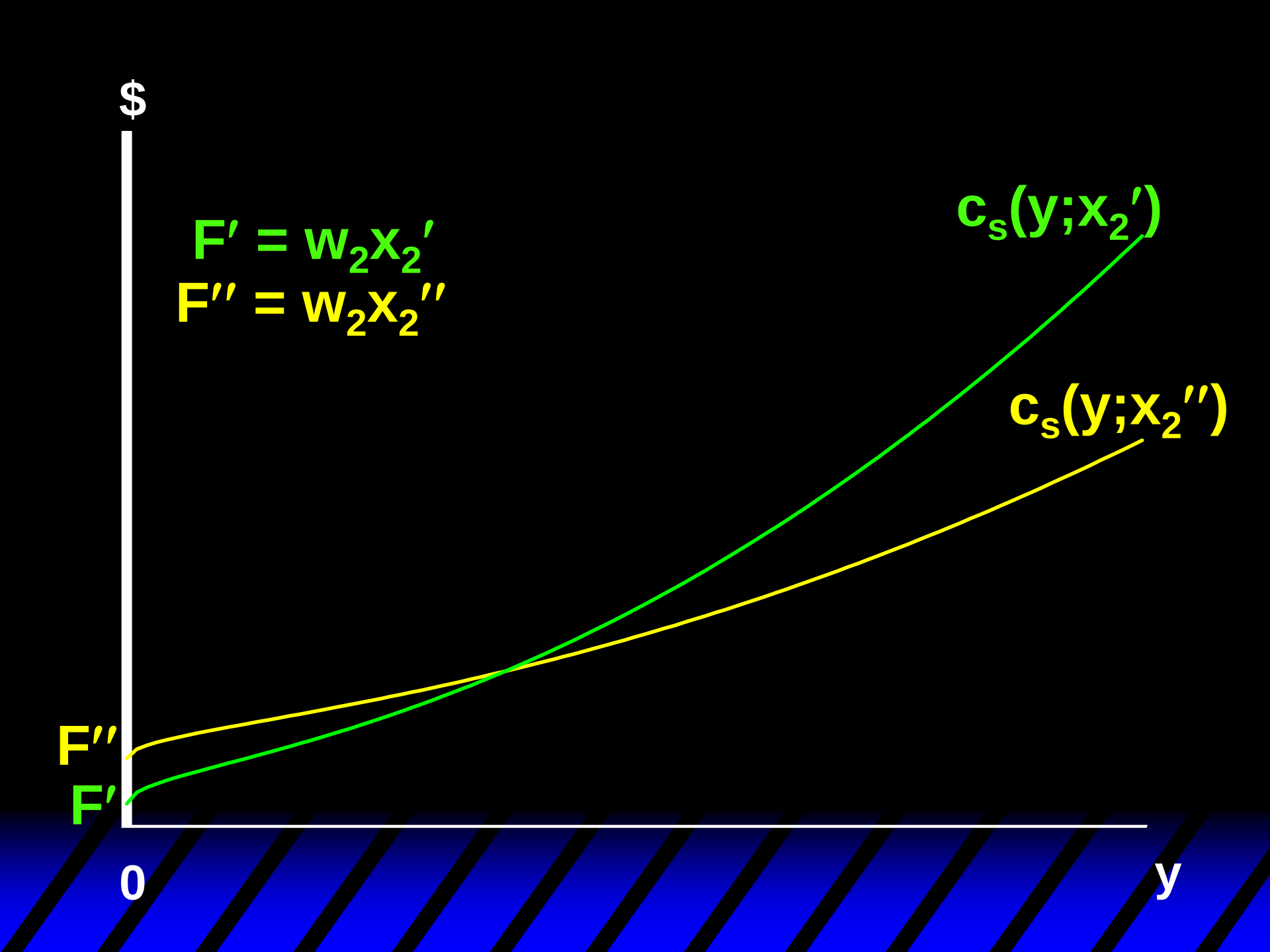

$/output unit
MC(y)
ATC(y)
AVC(y)
y

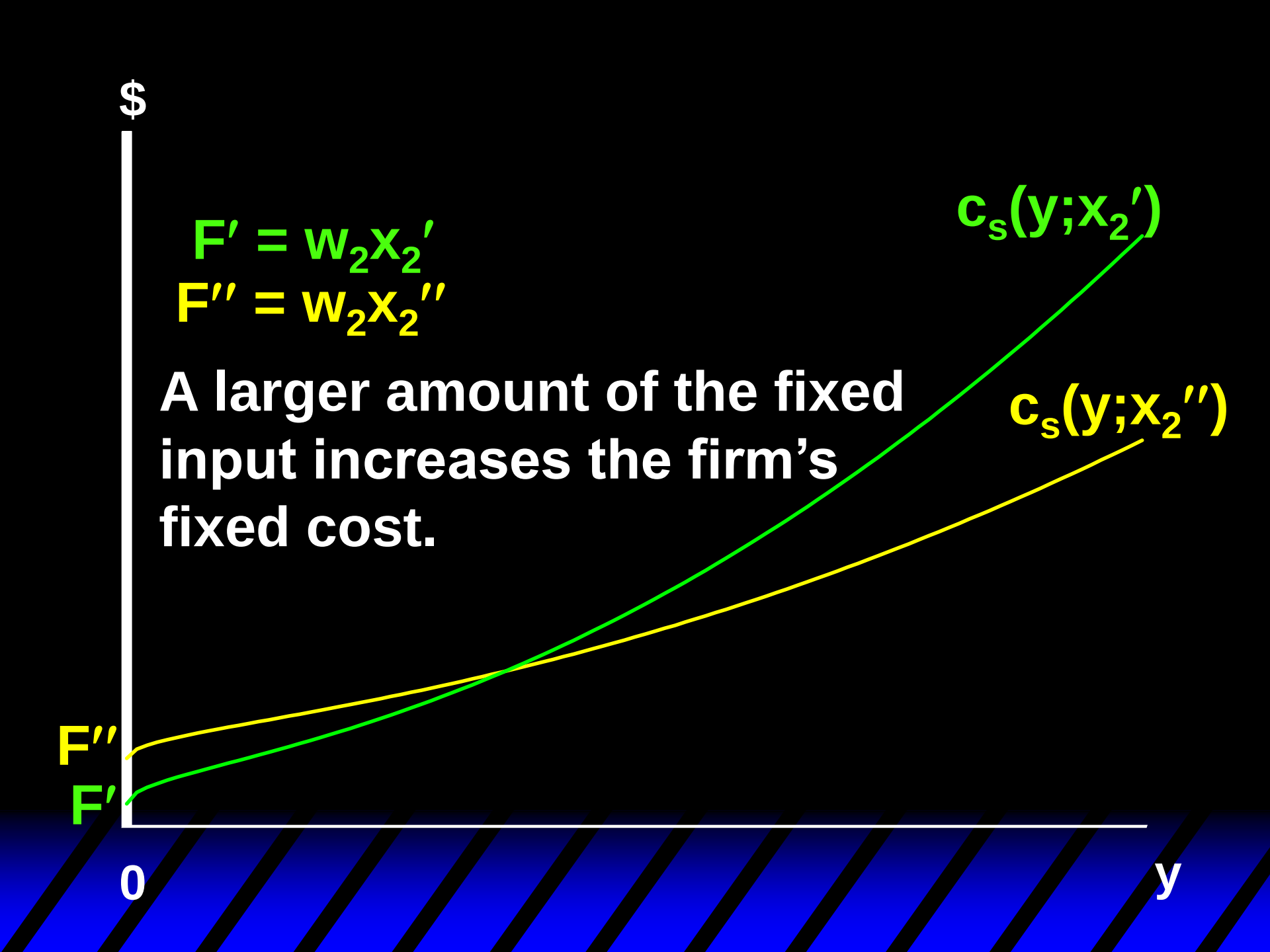
Short-Run & Long-Run Total Cost Curves

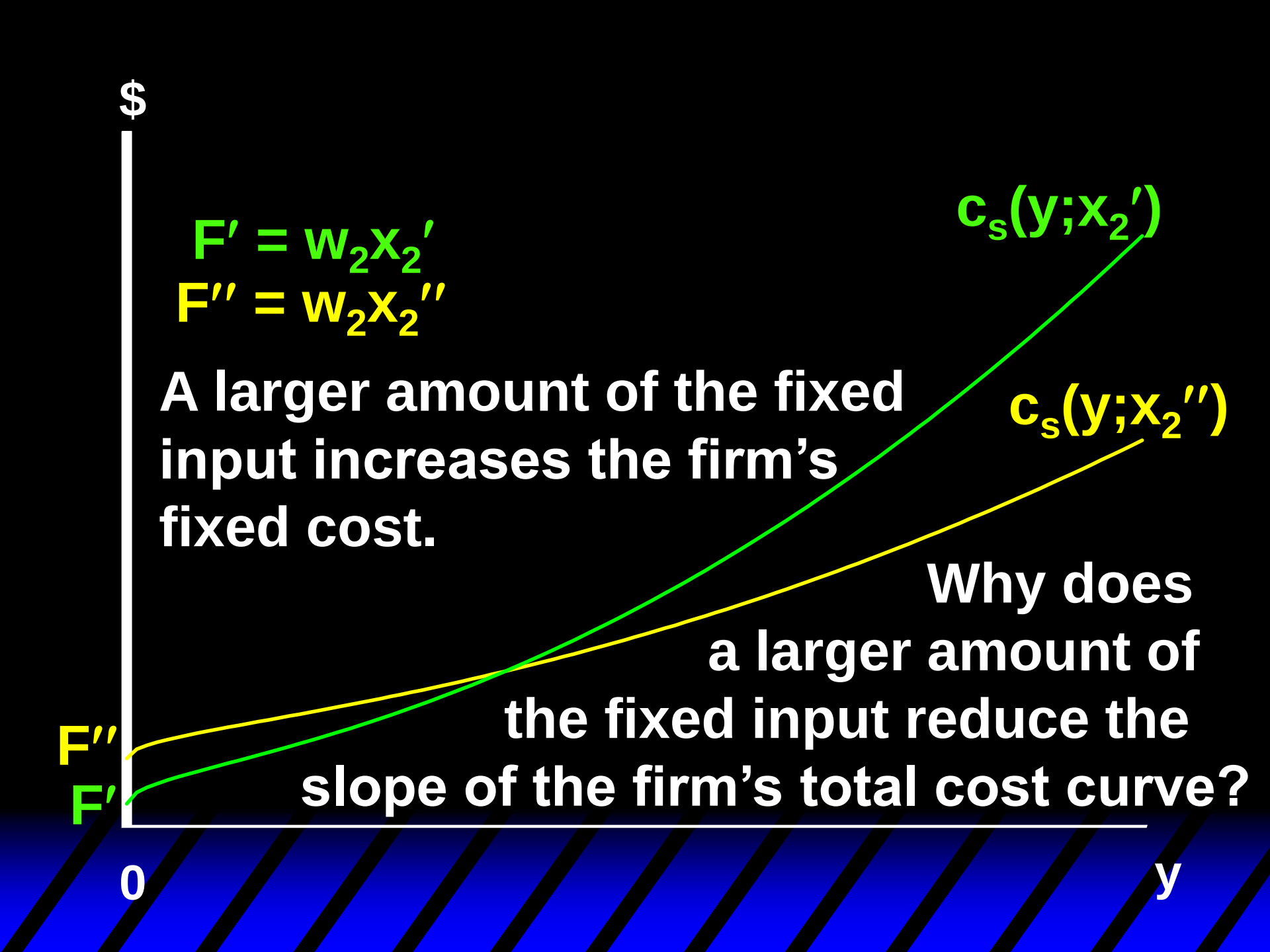
- A firm has a different short-run total cost curve for each possible short-run circumstance.
- Suppose the firm can be in one of just three short-runs;

$x_2 = x_2'$
or $x_2 = x_2''$ $\qquad x_2' < x_2'' < x_2'''$.
or $x_2 = x_2'''$.


$
F′ = w2x2′
cs(y;x2′)
F′
0
y


$
$F' = w_2x_2'$
$F'' = w_2x_2''$
$c_s(y;x_2')$
$c_s(y;x_2'')$
F''
F'
0
y


$
$F' = w_2x_2'$
$F'' = w_2x_2''$
A larger amount of the fixed input increases the firm's fixed cost.
$c_s(y;x_2')$
$c_s(y;x_2'')$
F''
F'
0
y


$
$F' = w_2x_2'$
$F'' = w_2x_2''$
$c_s(y;x_2')$
$c_s(y;x_2'')$
A larger amount of the fixed input increases the firm's fixed cost.
Why does a larger amount of the fixed input reduce the slope of the firm's total cost curve?
F''
F'
0
y

Short-Run & Long-Run Total Cost Curves

MP_1 is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives MP_1 extra output units. Therefore, the extra amount of input 1 needed for 1 extra output unit is

Short-Run & Long-Run Total Cost Curves

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Short-Run & Long-Run Total Cost Curves

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Each unit of input 1 costs w_1, so the firm's extra cost from producing one extra unit of output is

Short-Run & Long-Run Total Cost Curves

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Each unit of input 1 costs w_1, so the firm's extra cost from producing one extra unit of output is $$MC = \frac{w_1}{MP_1}.$$

Short-Run & Long-Run Total Cost Curves

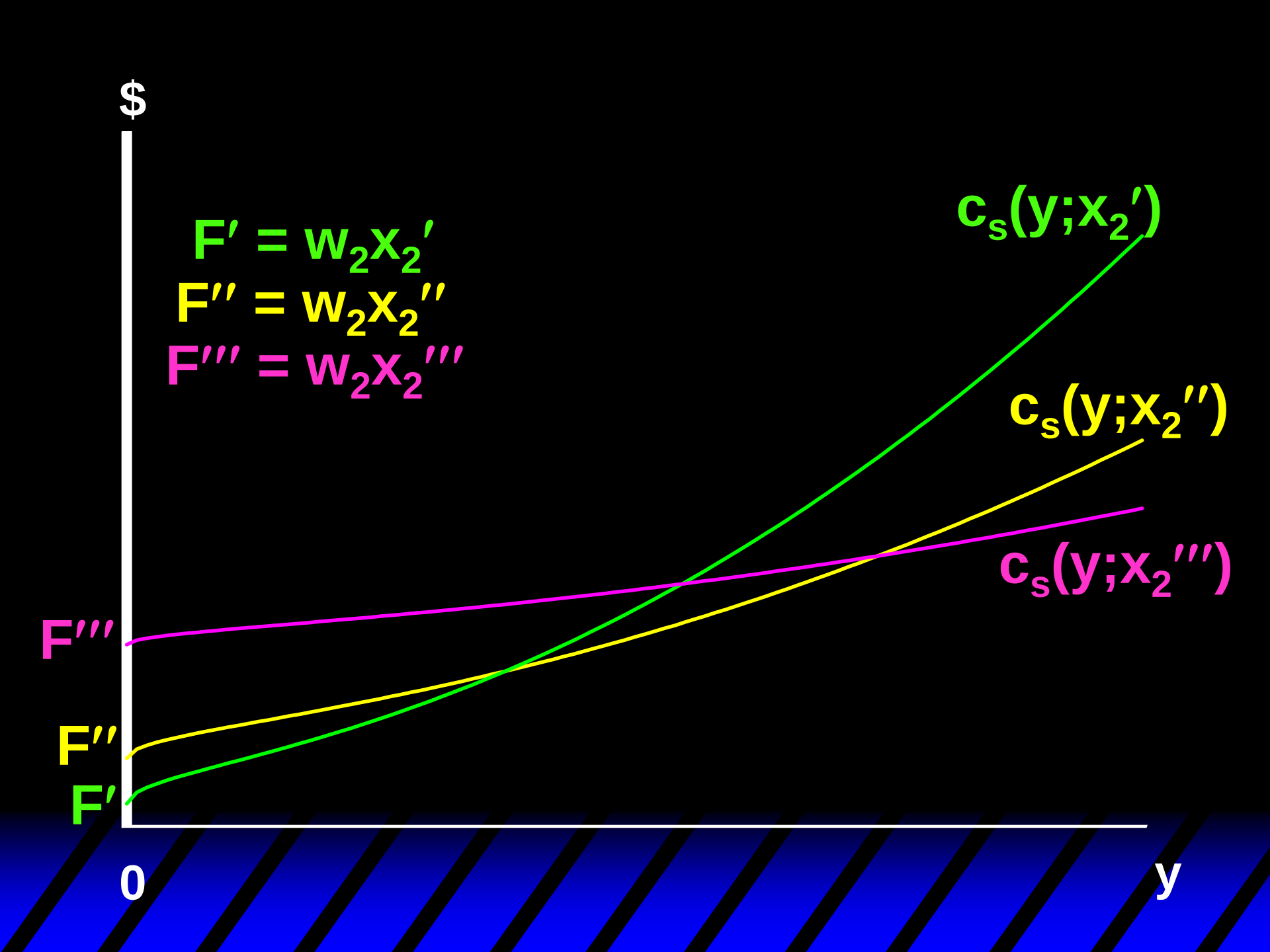
$$MC = \frac{w_1}{MP_1}$$ **is the slope of the firm's total cost curve.**

Short-Run & Long-Run Total Cost Curves

$MC = \frac{w_1}{MP_1}$ **is the slope of the firm's total cost curve.**

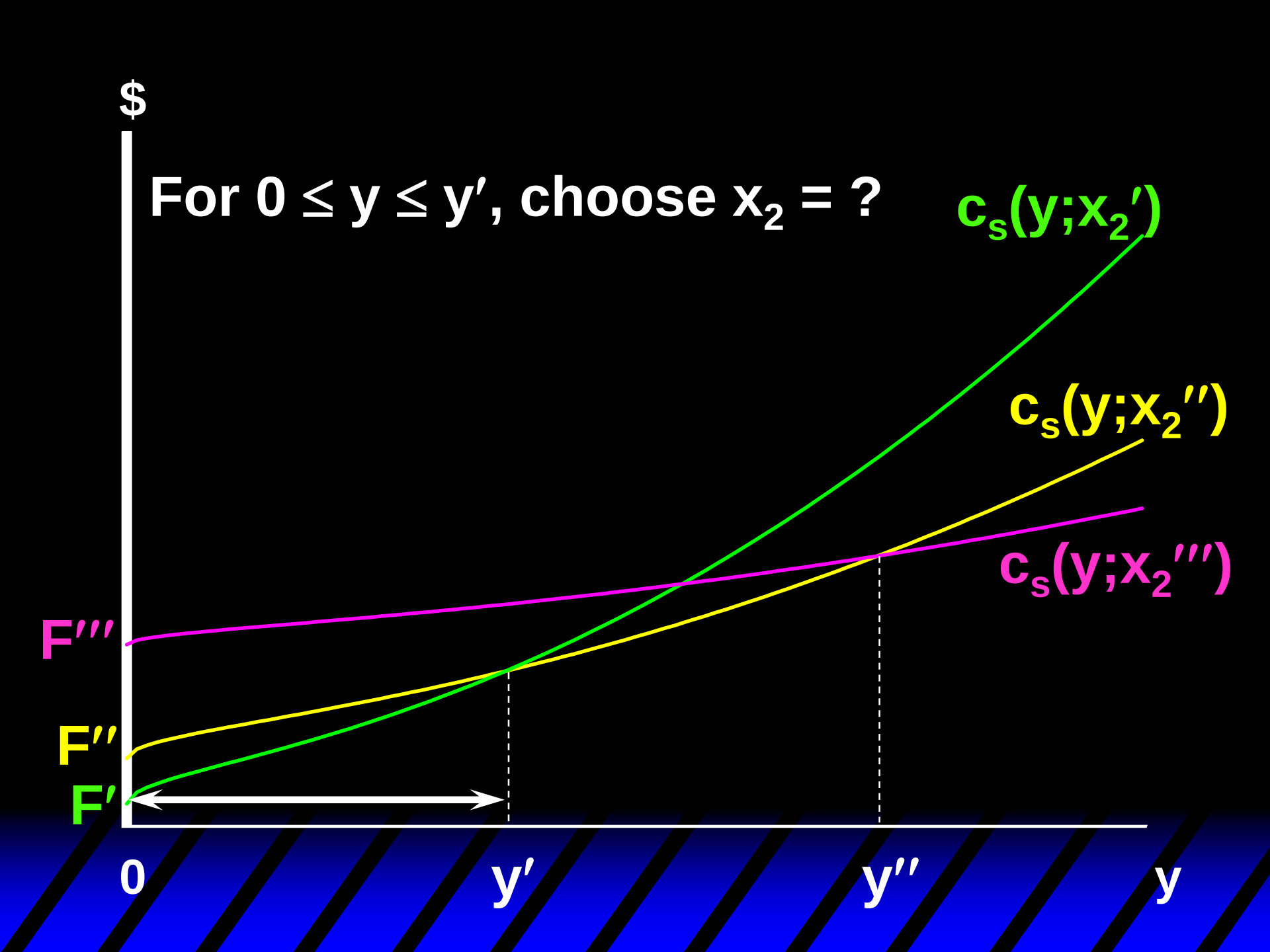
If input 2 is a complement to input 1 then MP_1 is higher for higher x_2.
Hence, MC is lower for higher x_2.

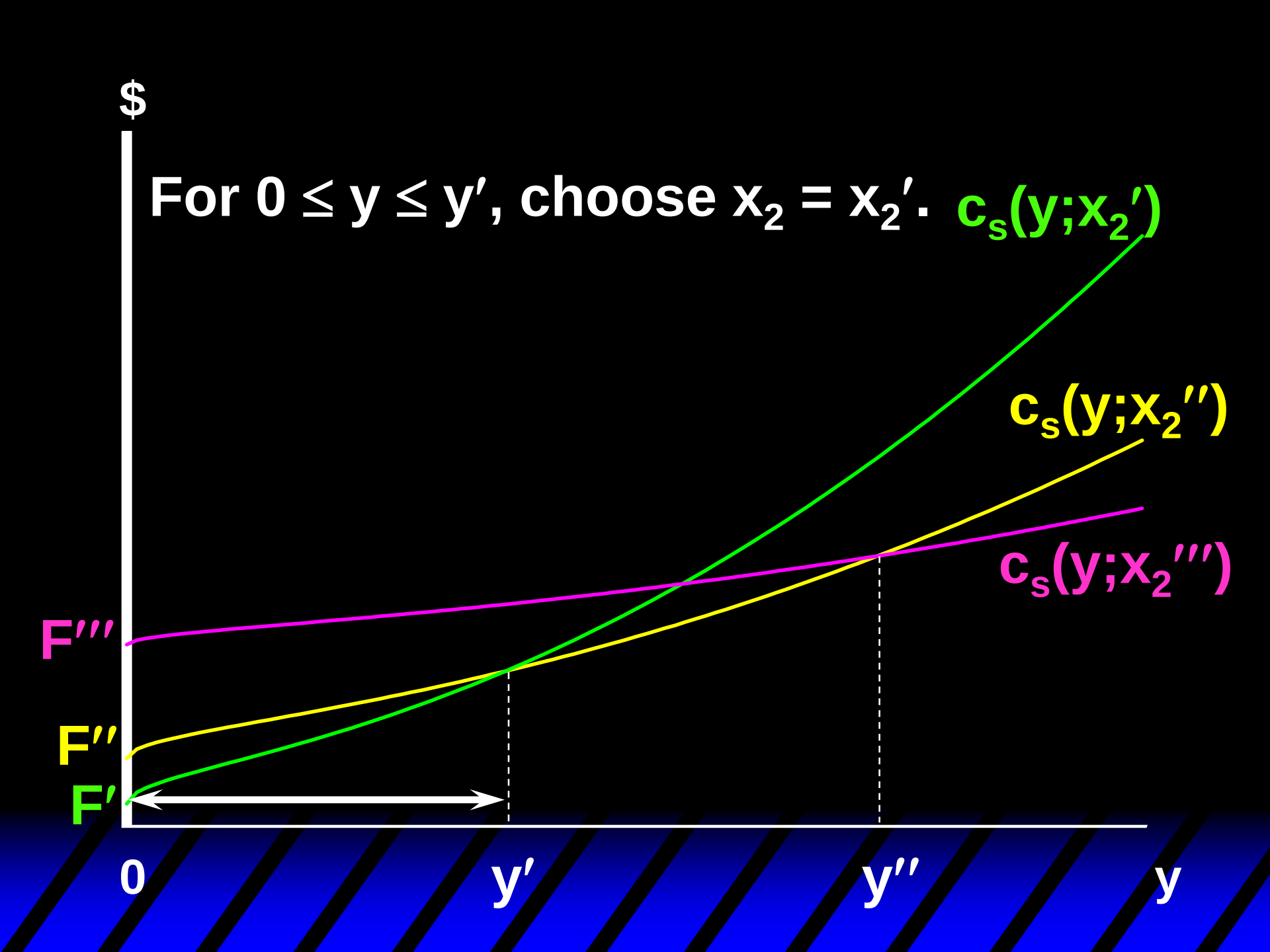
That is, a short-run total cost curve starts higher and has a lower slope if x_2 is larger.

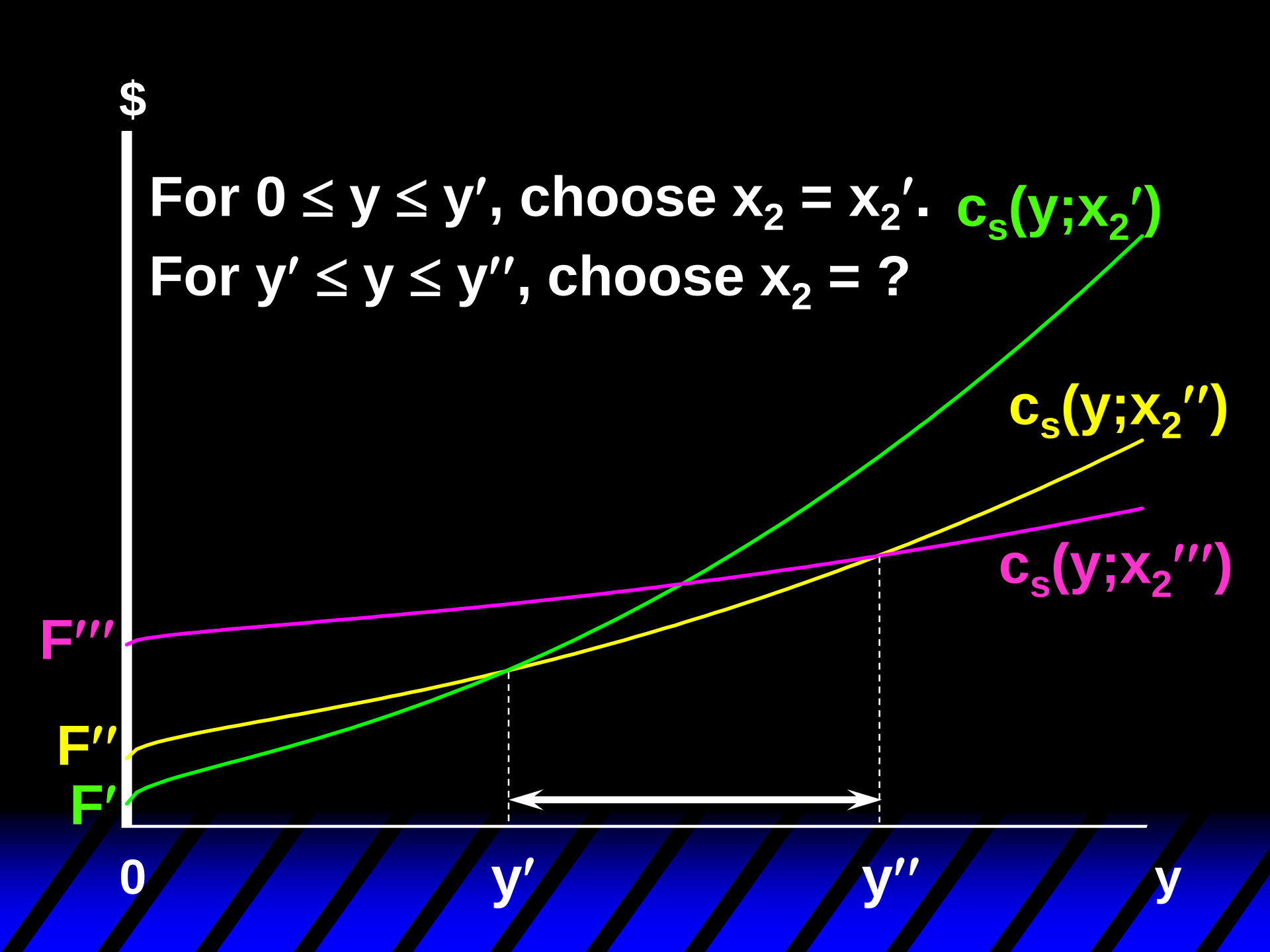

$
$F' = w_2x_2'$
$F'' = w_2x_2''$
$F''' = w_2x_2'''$
$c_s(y;x_2')$
$c_s(y;x_2'')$
$c_s(y;x_2''')$
F'''
F''
F'
0
y

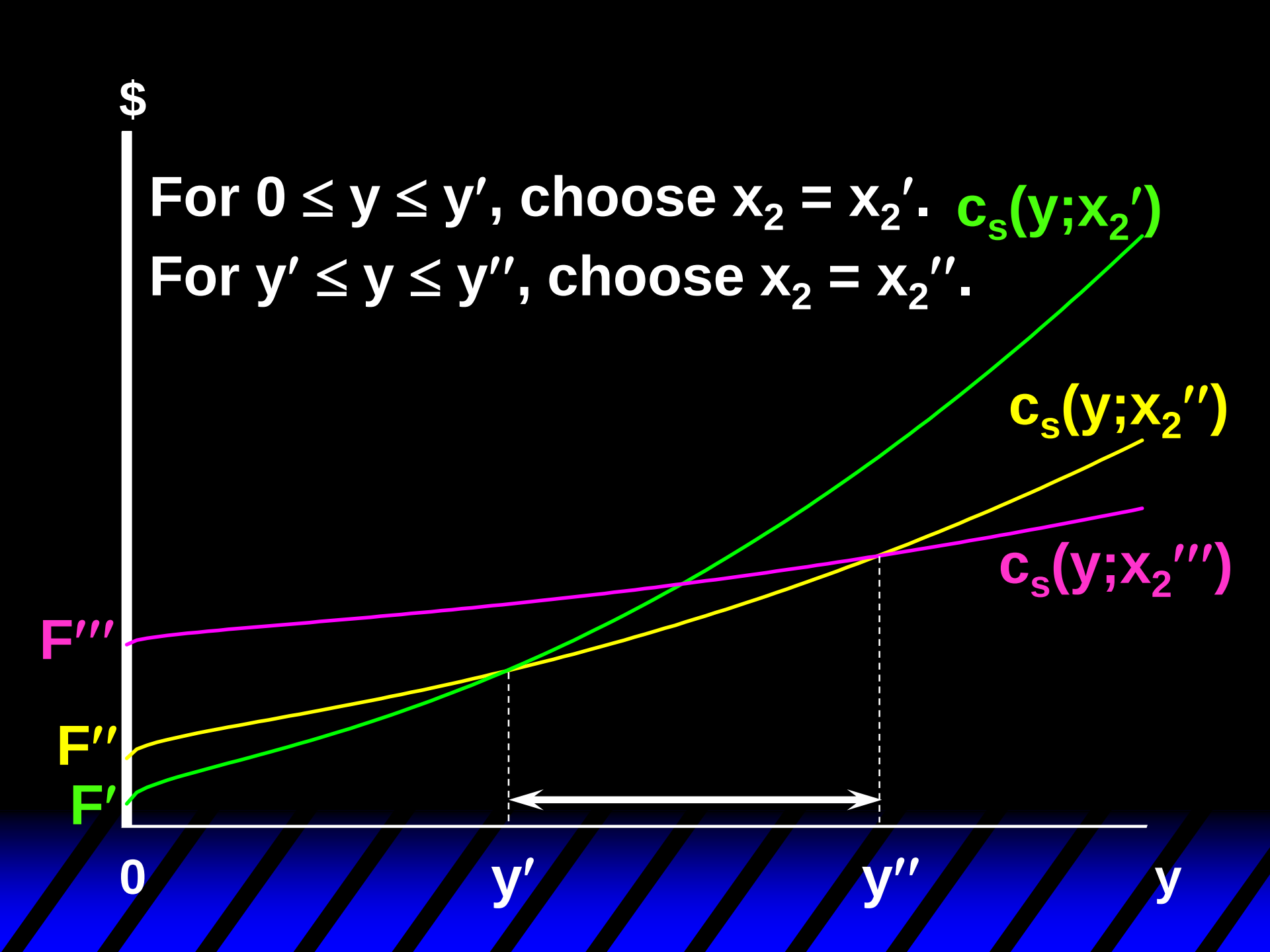
Short-Run & Long-Run Total Cost Curves

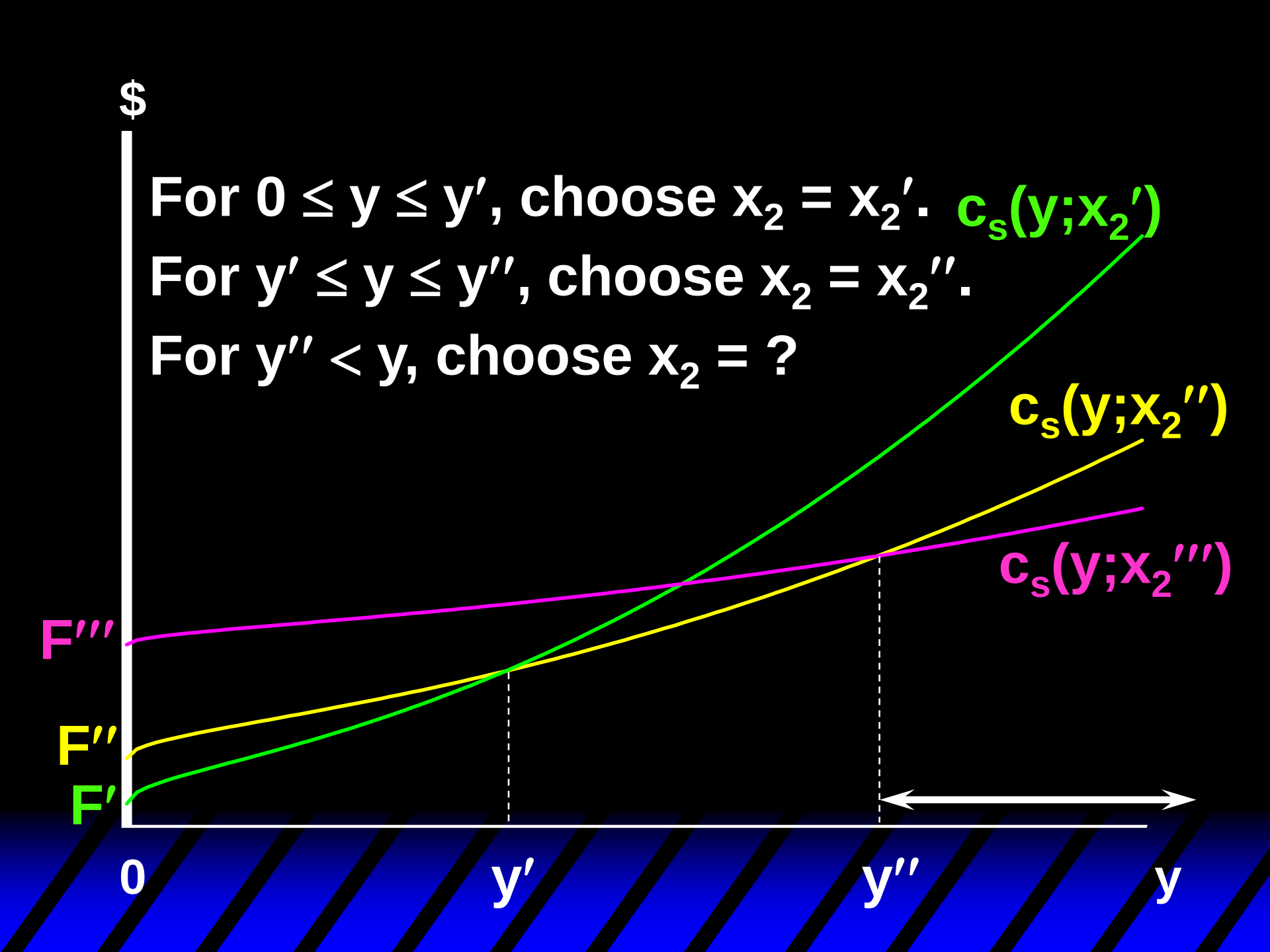
- The firm has three short-run total cost curves.
- In the long-run the firm is free to choose amongst these three since it is free to select x_2 equal to any of x_2', x_2'', or x_2'''.
- How does the firm make this choice?

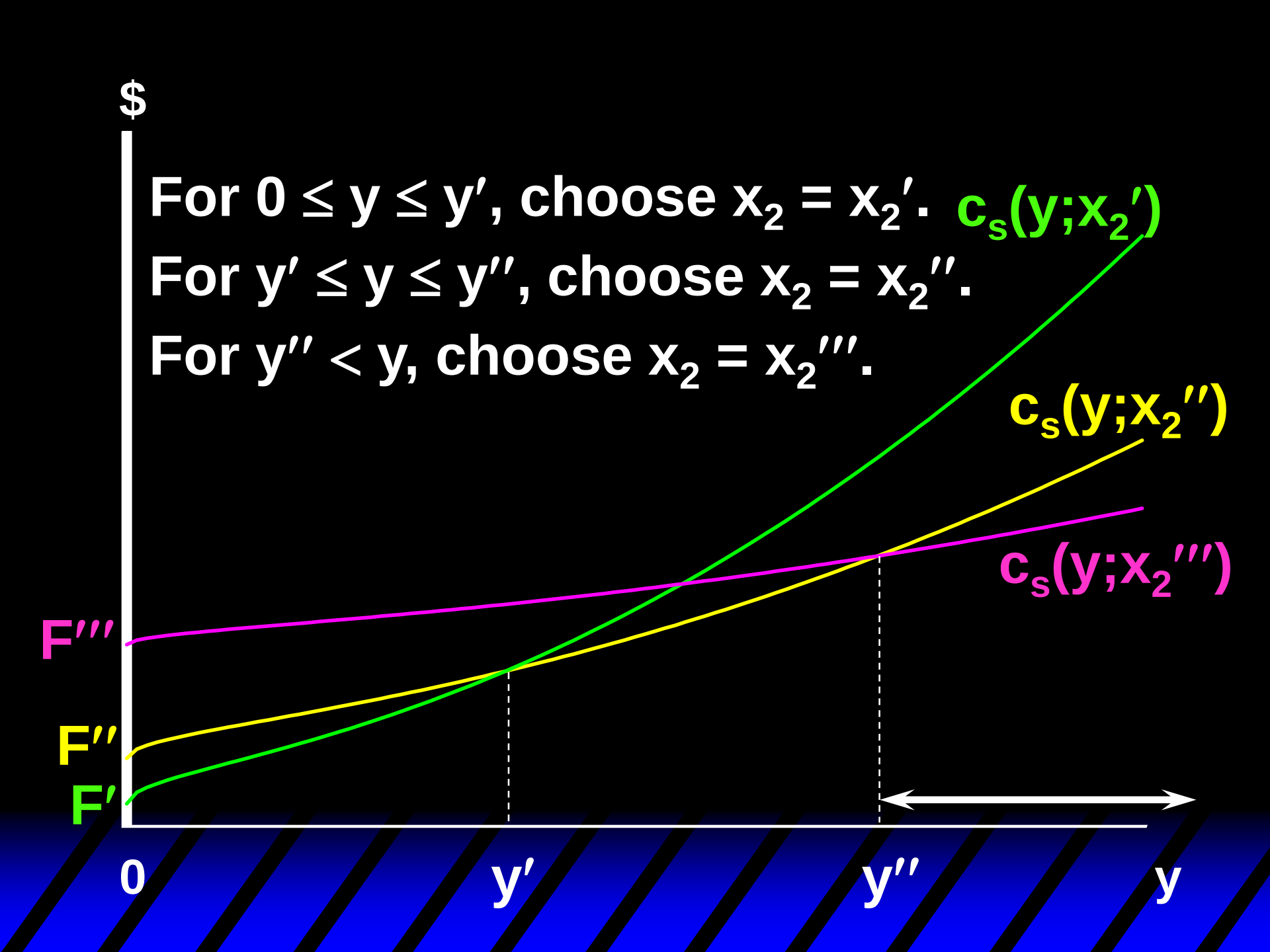

$
For 0 ≤ y ≤ y′, choose x2 = ?
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y

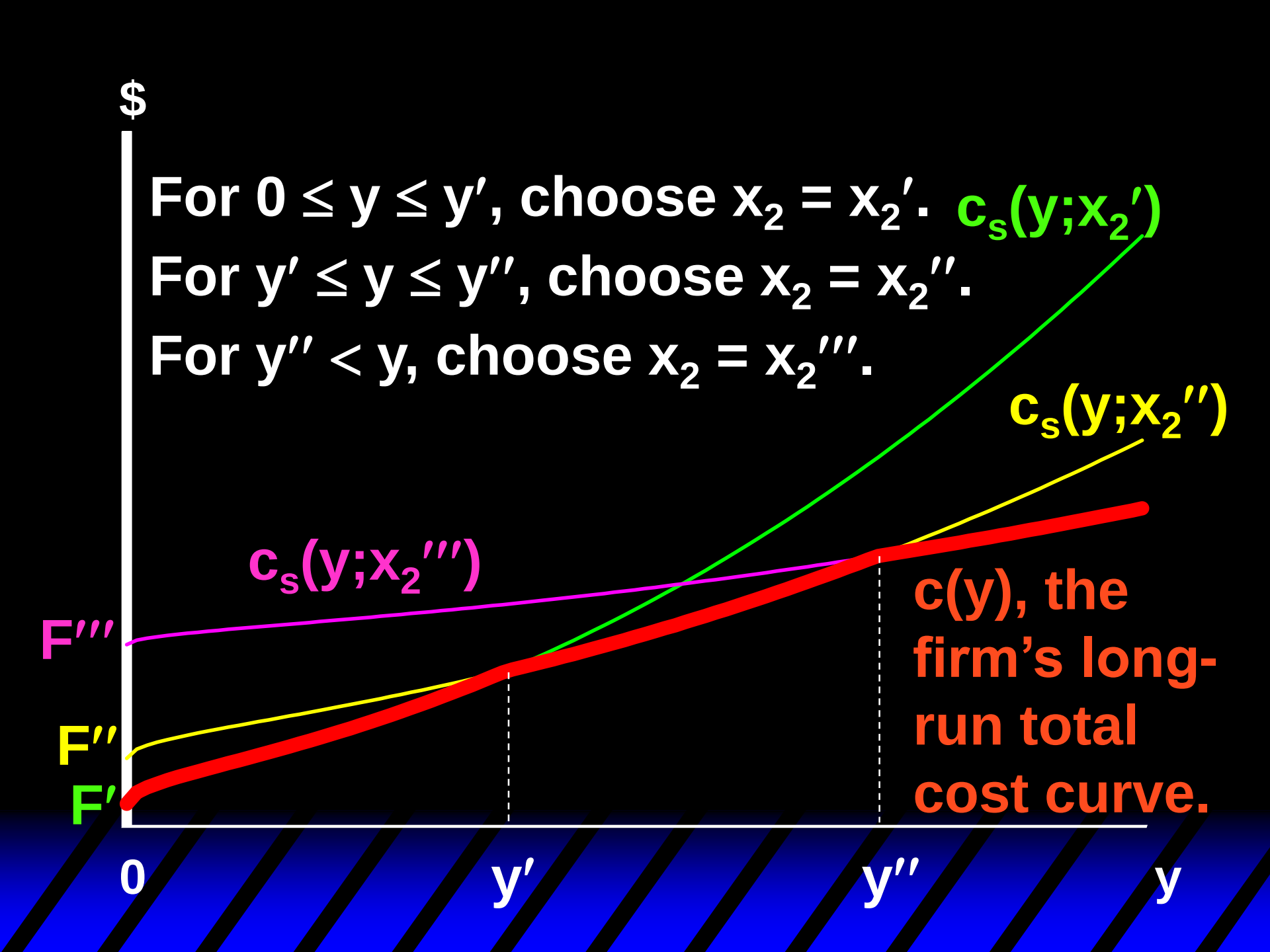

$
For 0 ≤ y ≤ y′, choose x2 = x2′.
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y


$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = ?
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y


$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = x2″.
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y


$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = x2″.
For y″ < y, choose x2 = ?
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y


$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = x2″.
For y″ < y, choose x2 = x2‴.
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y

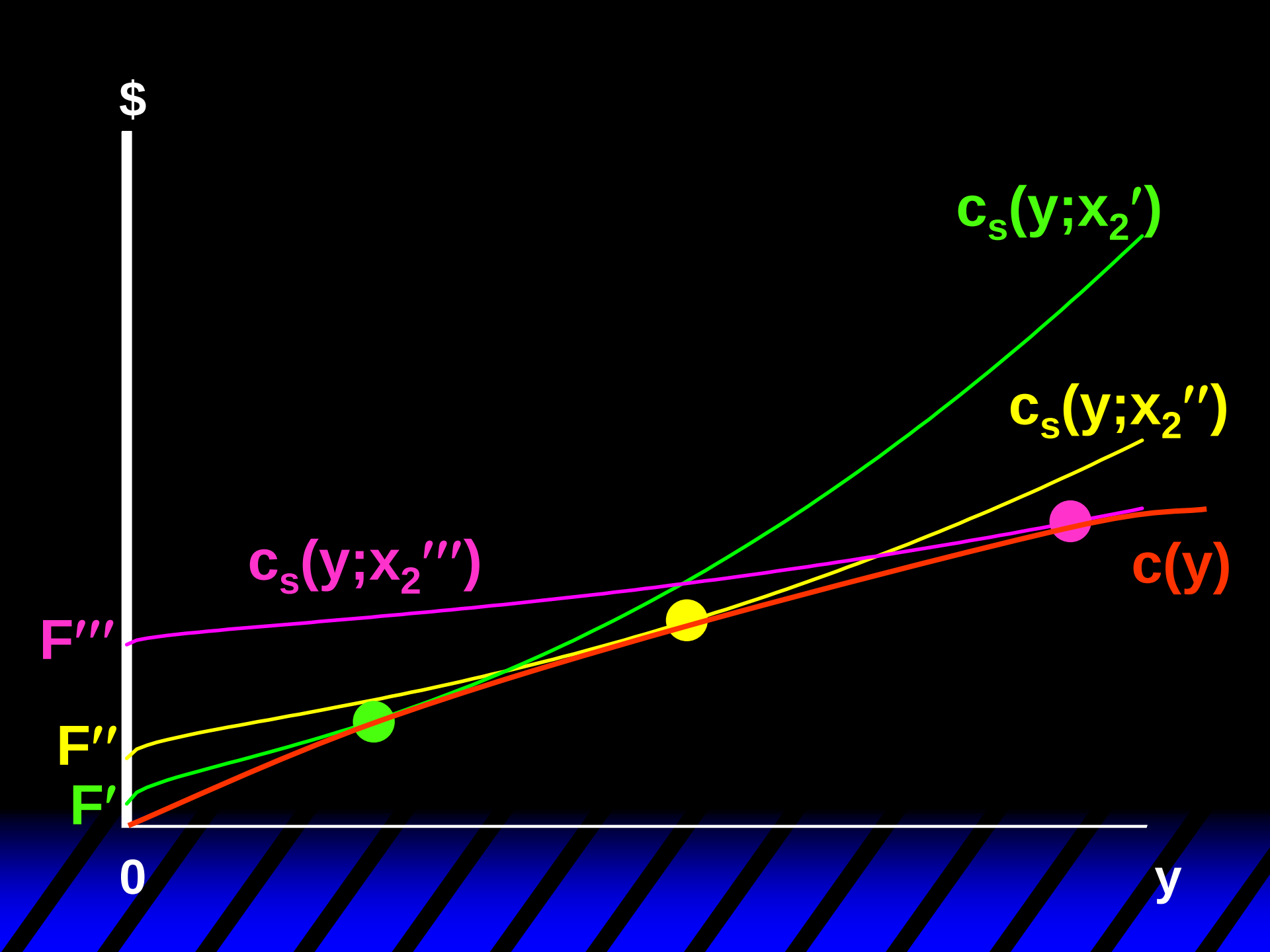

$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = x2″.
For y″ < y, choose x2 = x2‴.
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
c(y), the firm's long-run total cost curve.
0
y′
y″
y

Short-Run & Long-Run Total Cost Curves

- **The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves. The long-run total cost curve is the lower envelope of the short-run total cost curves.**

Short-Run & Long-Run Total Cost Curves

- **If input 2 is available in continuous amounts then there is an infinity of short-run total cost curves but the long-run total cost curve is still the lower envelope of all of the short-run total cost curves.**


$
$c_s(y;x_2')$
$c_s(y;x_2'')$
$c_s(y;x_2''')$
c(y)
F‴
F″
F′
0
y

Short-Run & Long-Run Average Total Cost Curves

- **For any output level y, the long-run total cost curve always gives the lowest possible total production cost.**
- **Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.**
- **The long-run av. total cost curve must be the lower envelope of all of the firm's short-run av. total cost curves.**

Short-Run & Long-Run Average Total Cost Curves

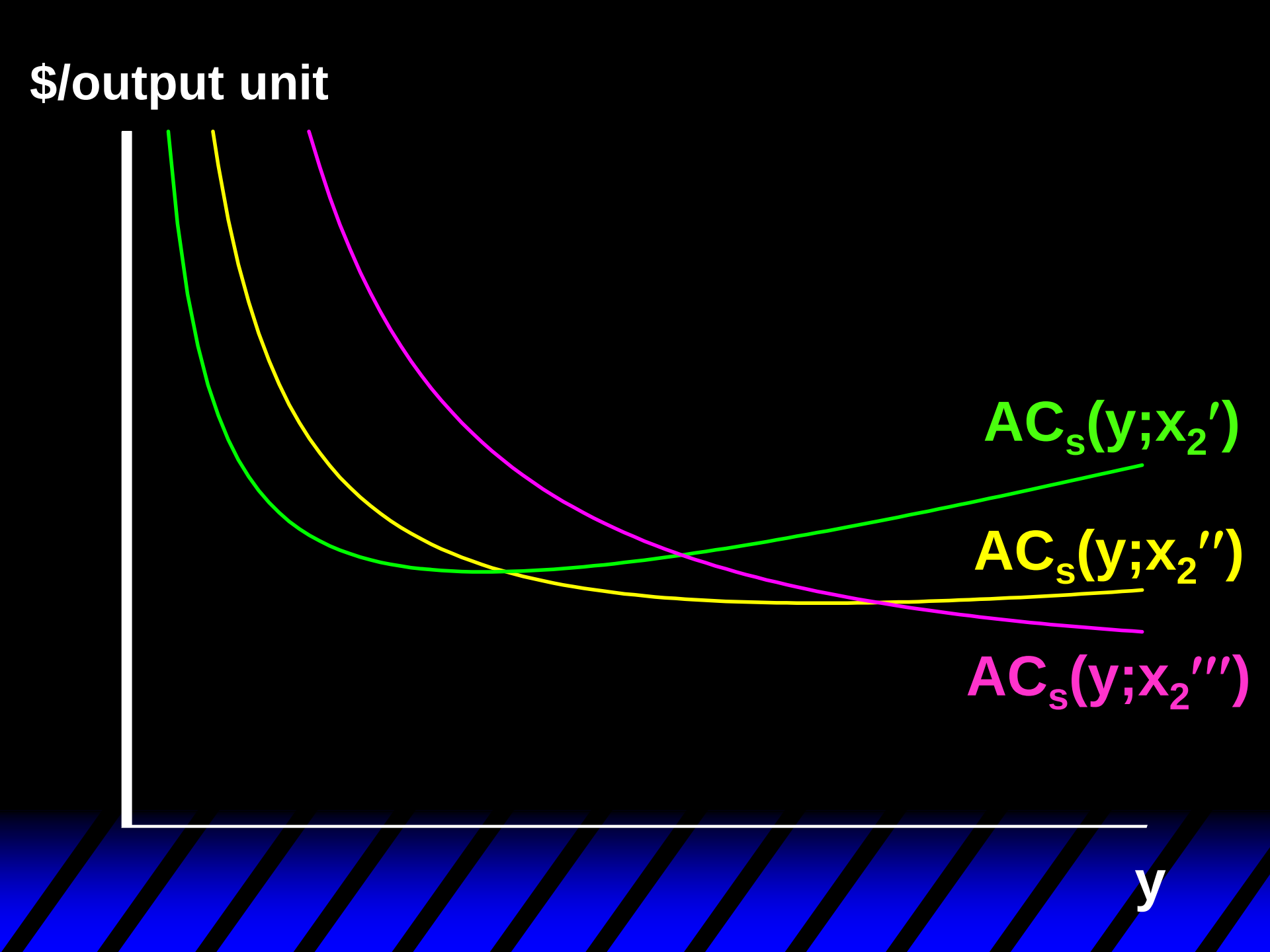
- **E.g. suppose again that the firm can be in one of just three short-runs;**

  $x_2 = x_2'$

  or $x_2 = x_2''$ $\qquad (x_2' < x_2'' < x_2''')$

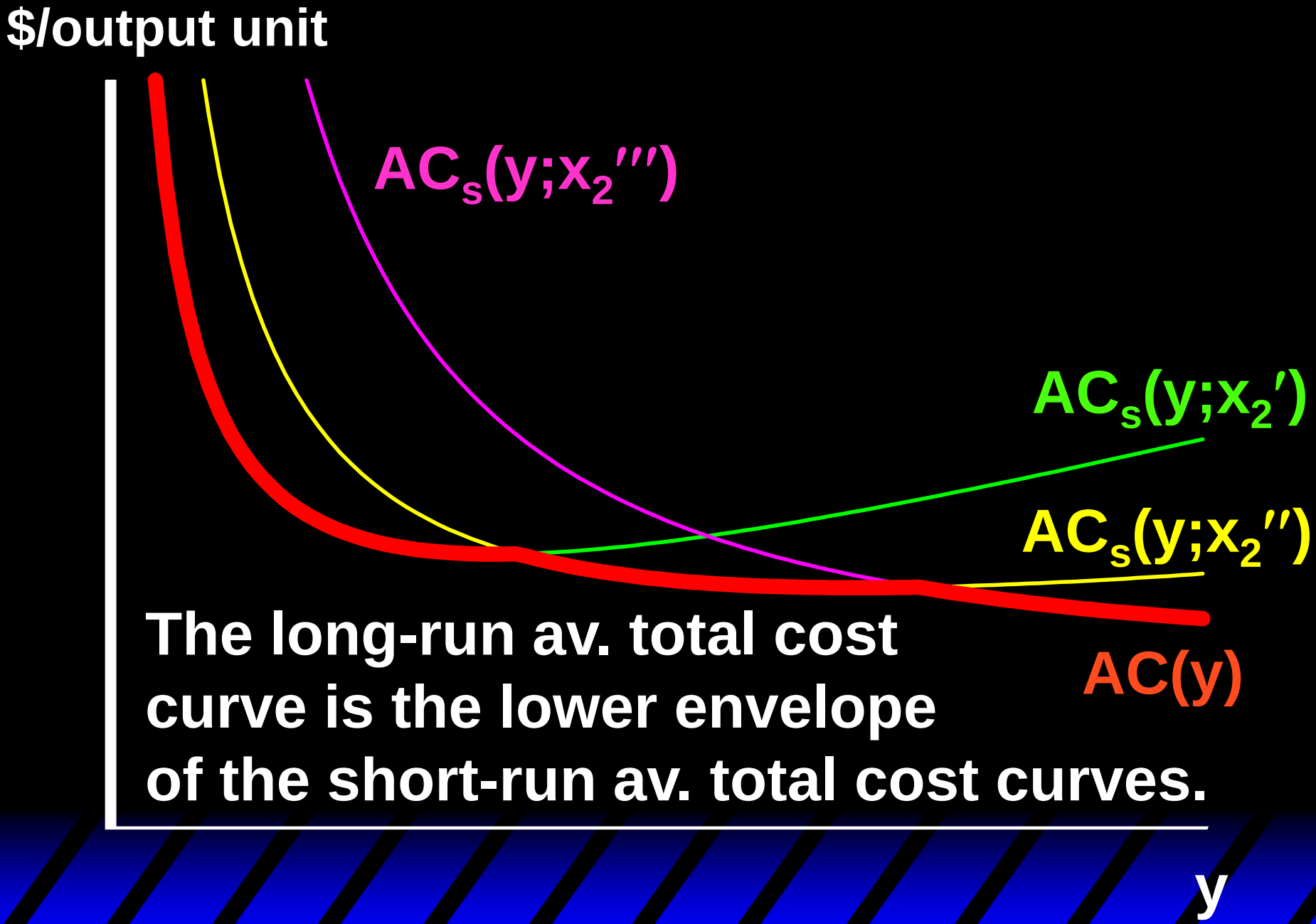
  or $x_2 = x_2'''$

  then the firm's three short-run average total cost curves are ...


$/output unit
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$AC_s(y;x_2''')$
y

Short-Run & Long-Run Average Total Cost Curves

- **The firm's long-run average total cost curve is the lower envelope of the short-run average total cost curves ...**


$/output unit
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
AC(y)
The long-run av. total cost curve is the lower envelope of the short-run av. total cost curves.
y

Short-Run & Long-Run Marginal Cost Curves

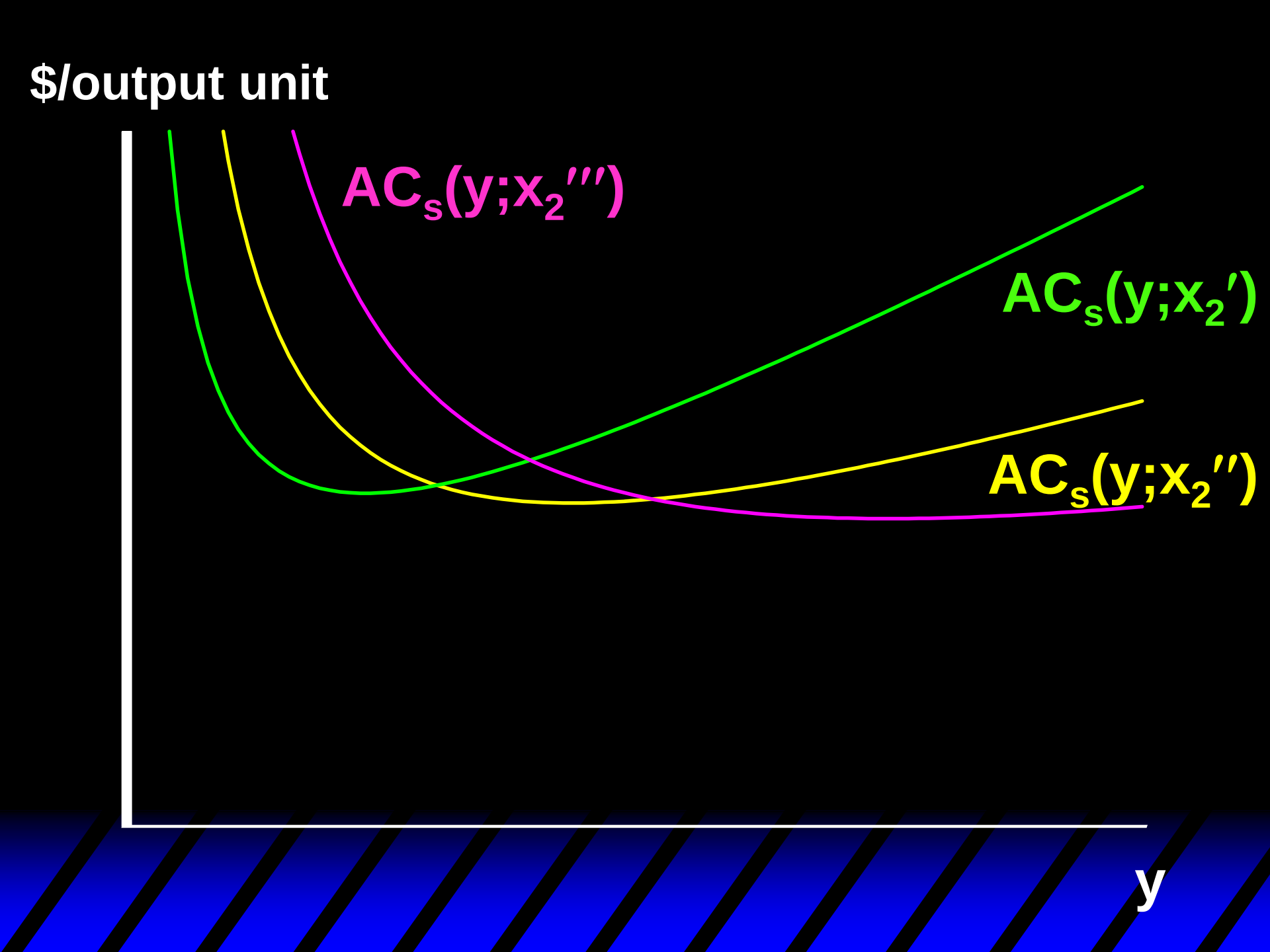
- **Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**

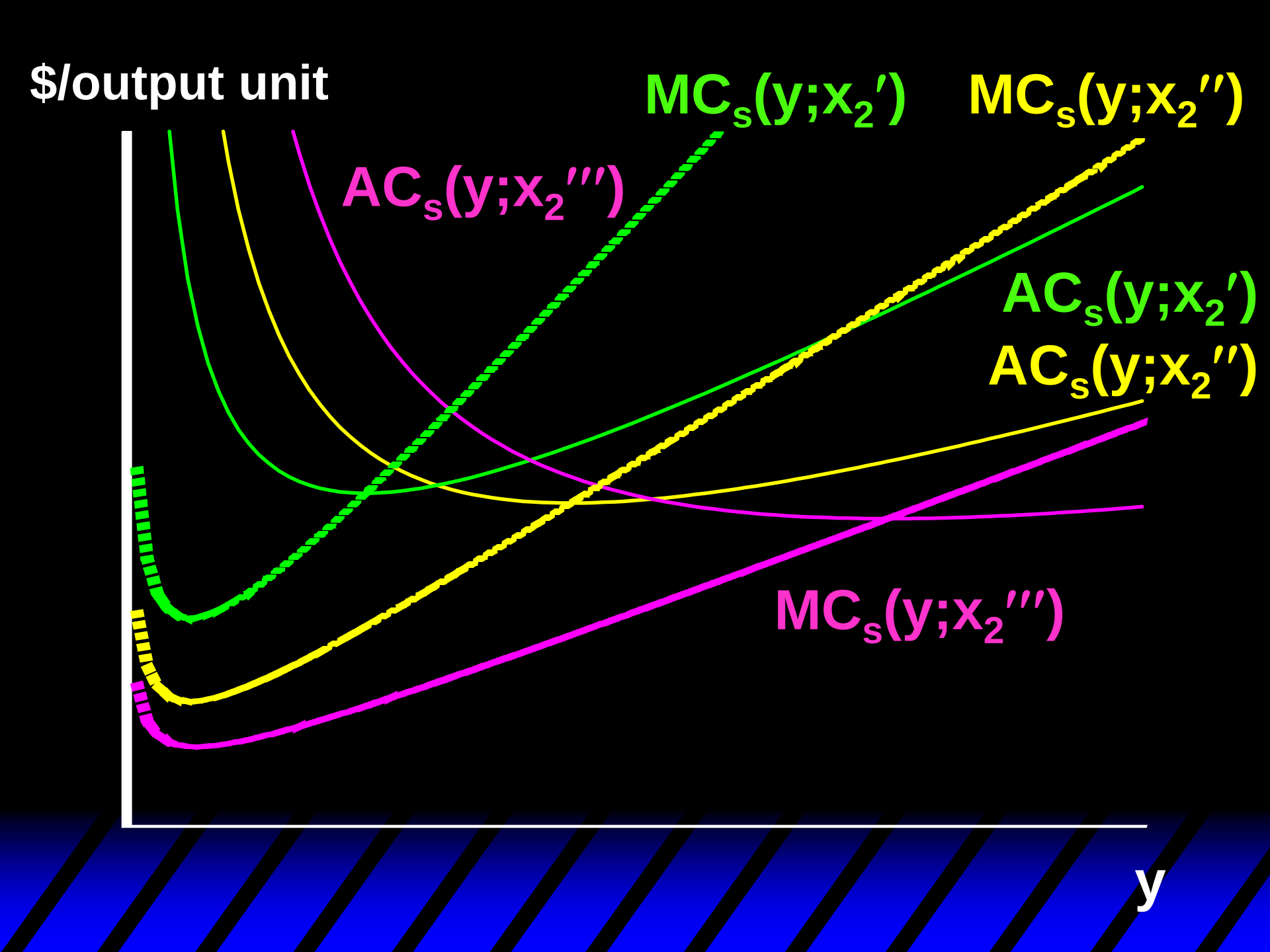
Short-Run & Long-Run Marginal Cost Curves

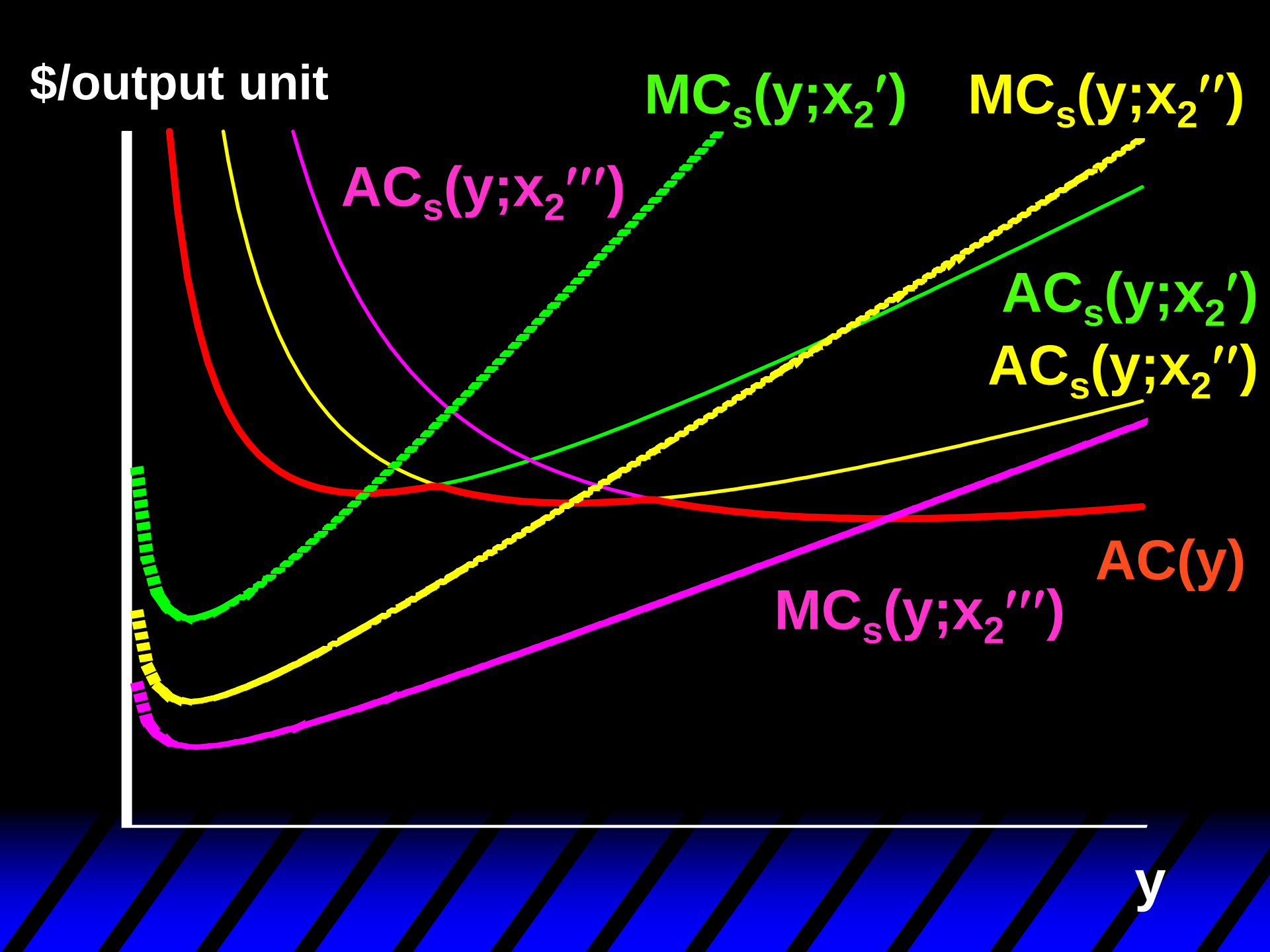
- **Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**
- **A: No.**

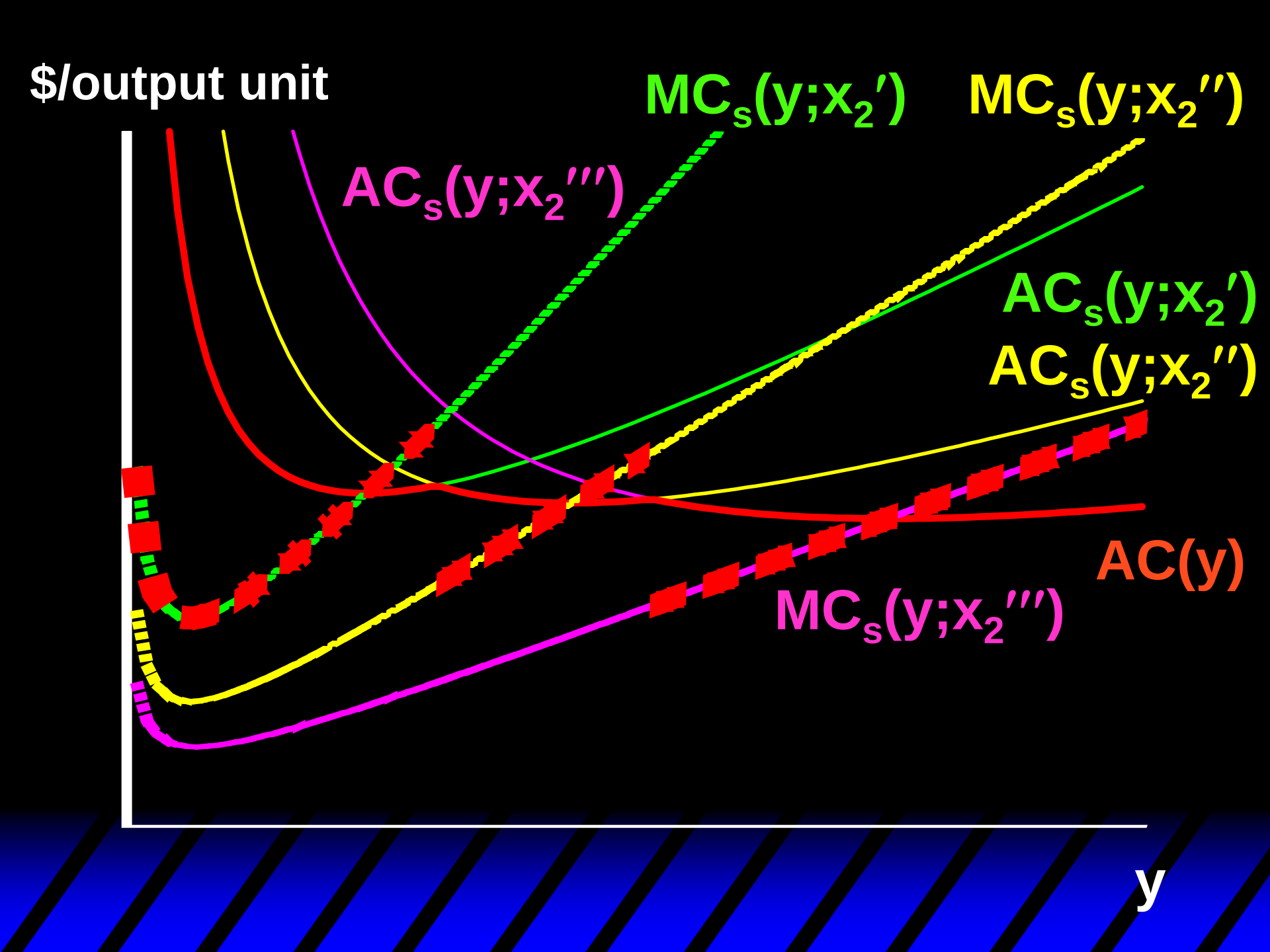
Short-Run & Long-Run Marginal Cost Curves

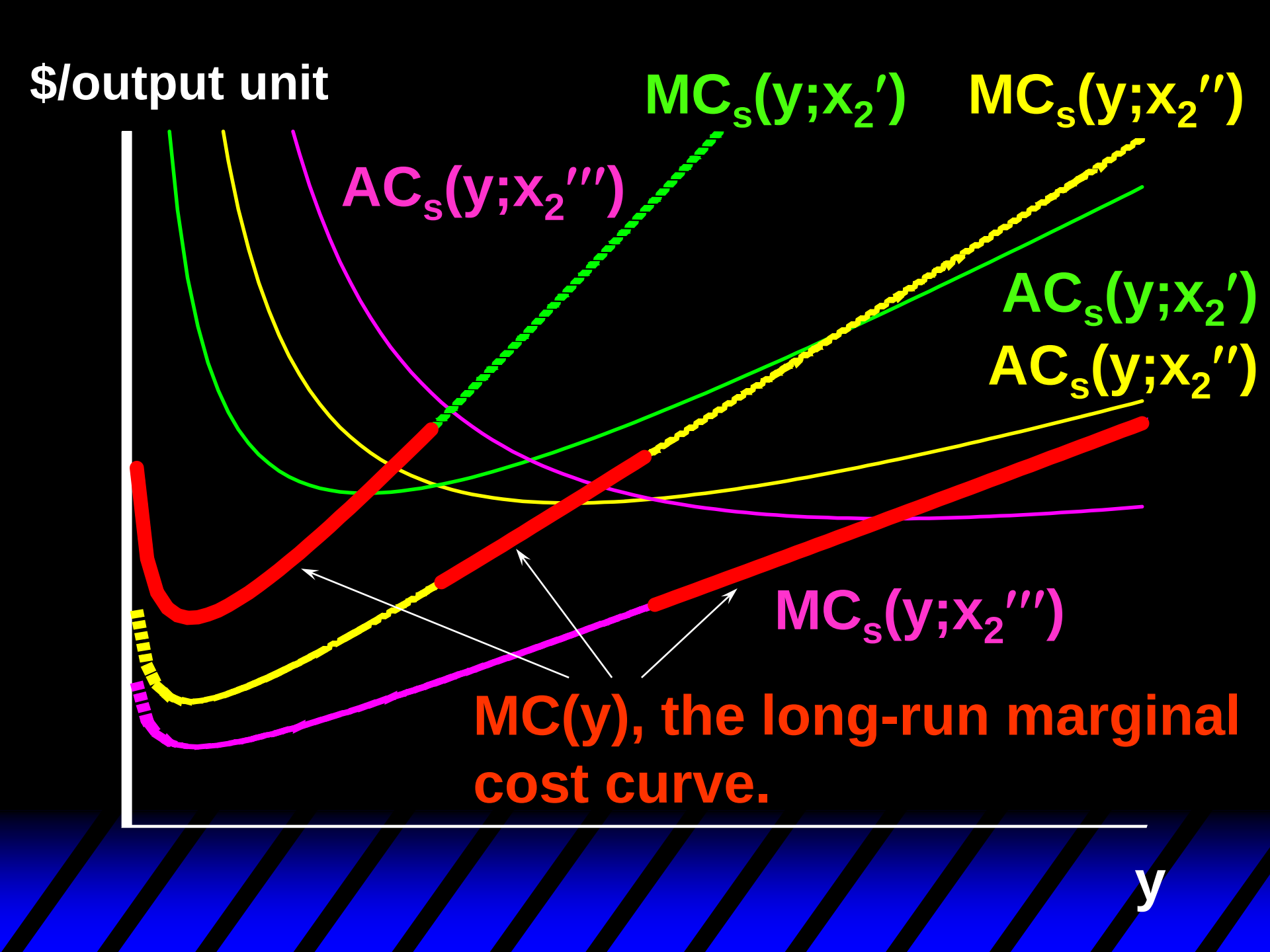
- **The firm's three short-run average total cost curves are ...**


$/output unit
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
y


$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$MC_s(y;x_2''')$
y

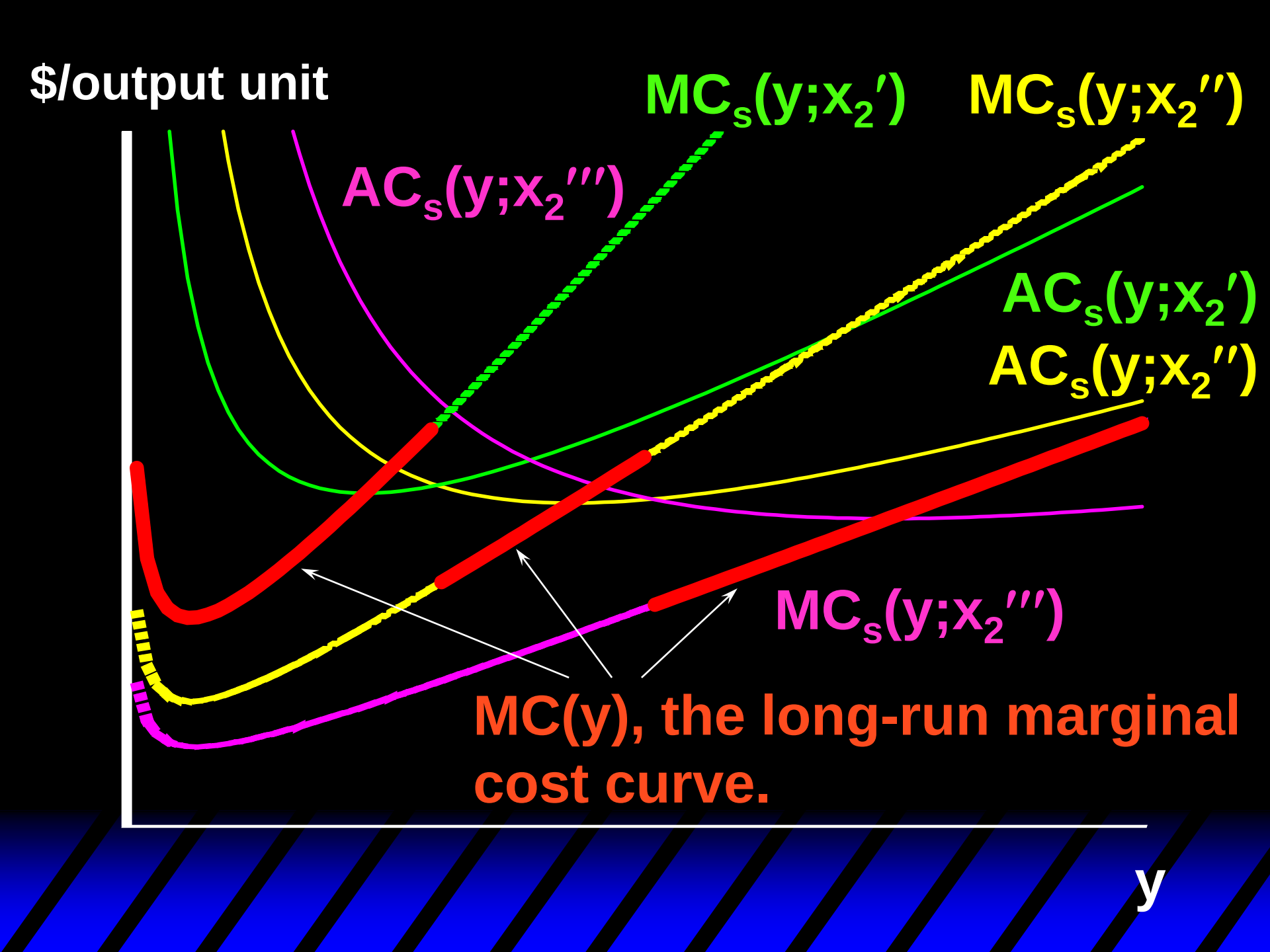

$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$AC(y)$
$MC_s(y;x_2''')$
y


$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$AC(y)$
$MC_s(y;x_2''')$
y


$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$MC_s(y;x_2''')$
MC(y), the long-run marginal cost curve.
y

Short-Run & Long-Run Marginal Cost Curves

- **For any output level y > 0, the long-run marginal cost of production is the marginal cost of production for the short-run chosen by the firm.**


$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$MC_s(y;x_2''')$
MC(y), the long-run marginal cost curve.
y

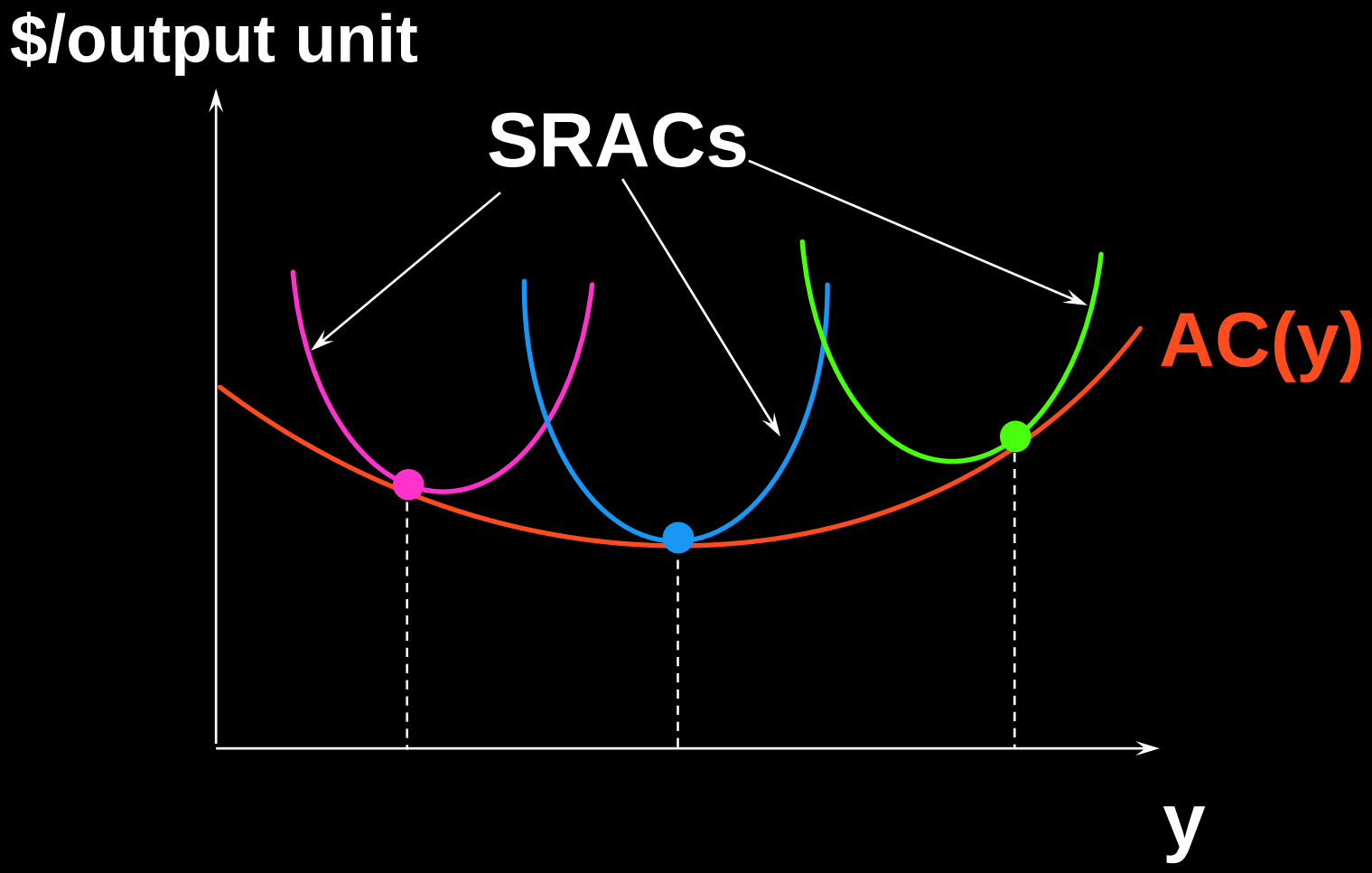
Short-Run & Long-Run Marginal Cost Curves

- For any output level $y > 0$, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.
- This is always true, no matter how many and which short-run circumstances exist for the firm.

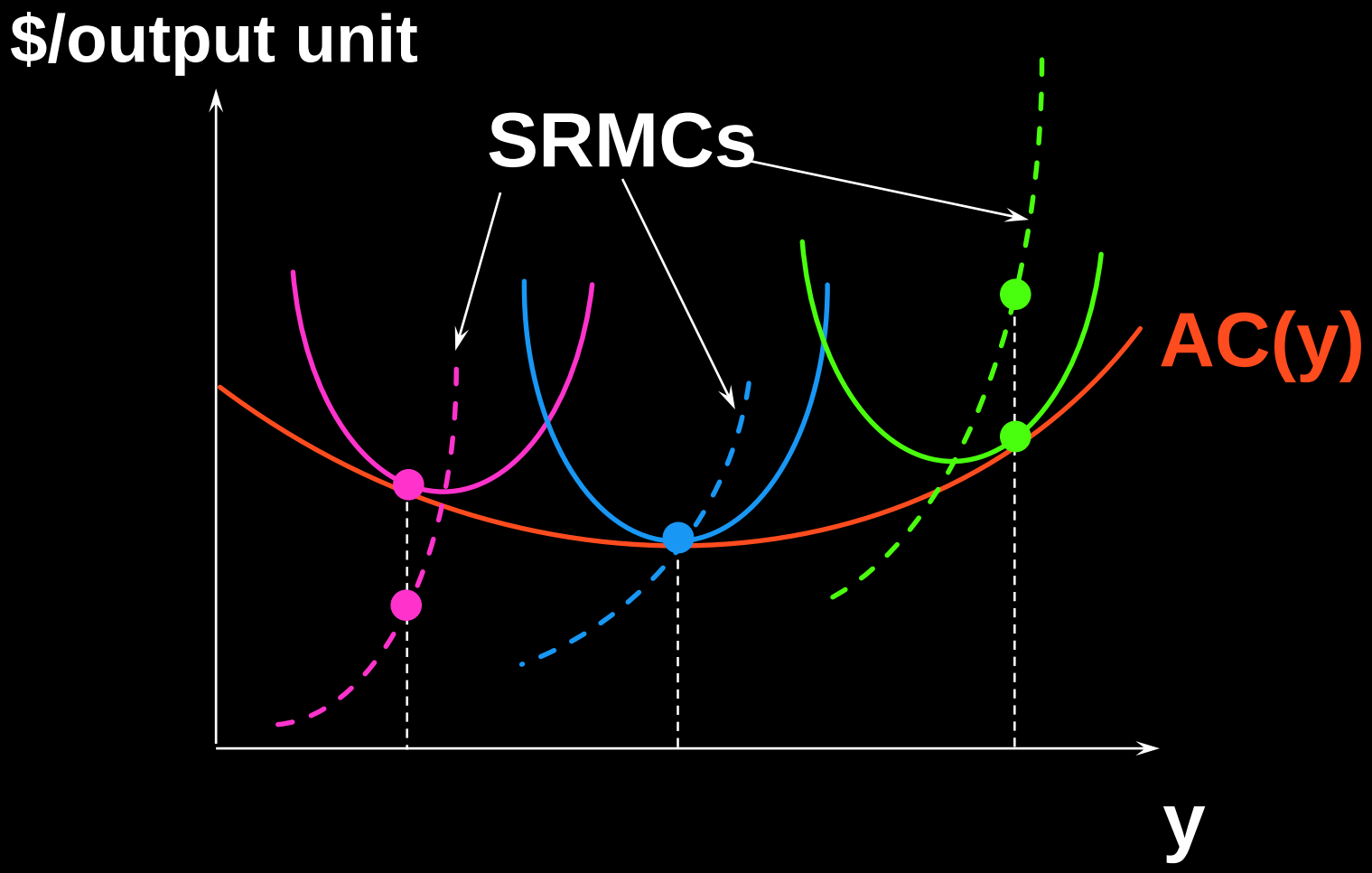
Short-Run & Long-Run Marginal Cost Curves

- For any output level $y > 0$, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.
- So for the continuous case, where x_2 can be fixed at any value of zero or more, the relationship between the long-run marginal cost and all of the short-run marginal costs is ...

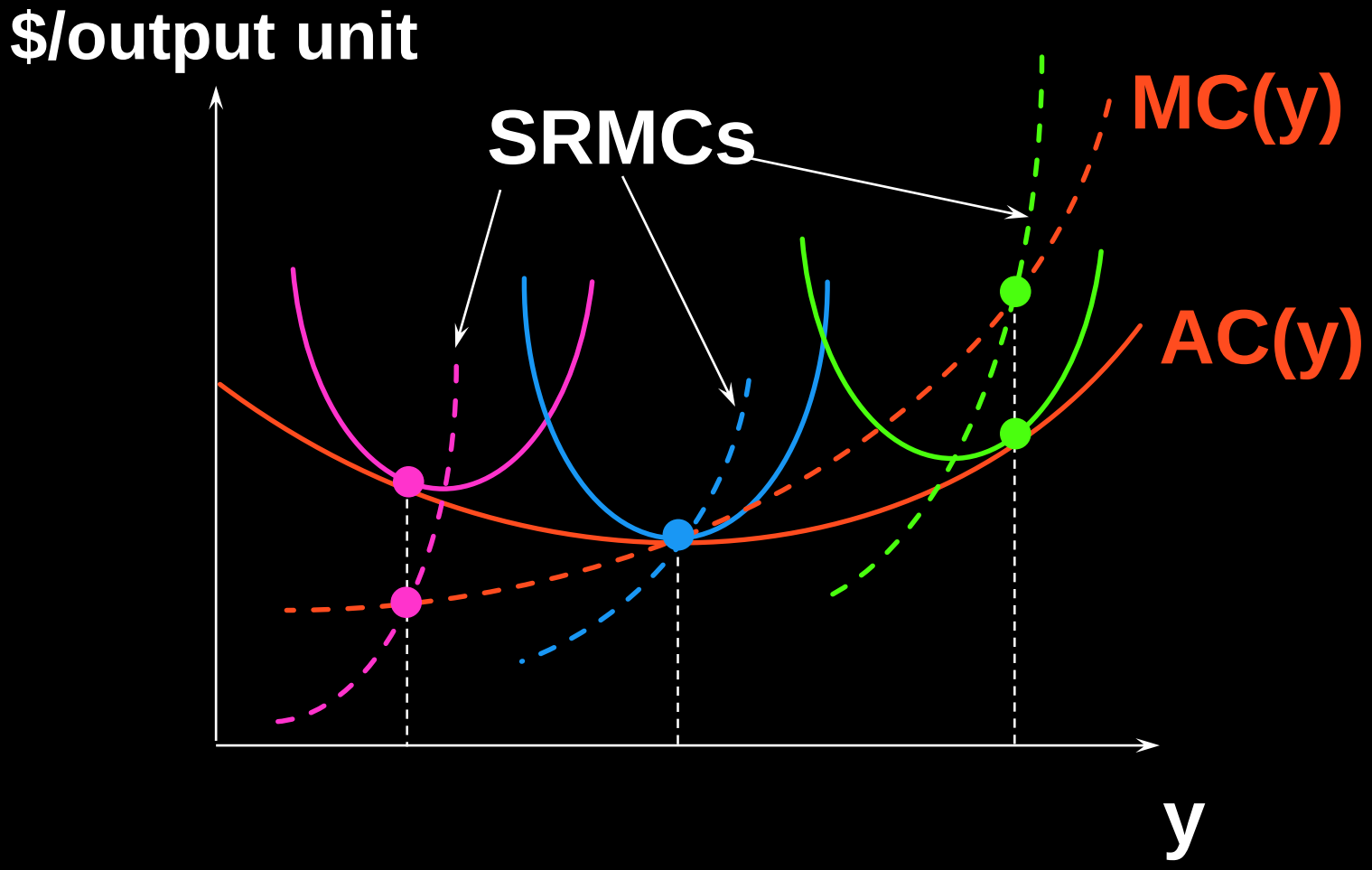
Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



- For each y > 0, the long-run MC equals the MC for the short-run chosen by the firm.