# Chapter Eighteen

## Technology

# Technologies

- A technology is a process by which inputs are converted to an output.
- *E.g.* labor, a computer, a projector, electricity, and software are being combined to produce this lecture.

# Technologies

- Usually several technologies will produce the same product -- a blackboard and chalk can be used instead of a computer and a projector.
- Which technology is “best”?
- How do we compare technologies?

# Input Bundles

- $x_i$ denotes the amount used of input i; *i.e.* the level of input i.
- An input bundle is a vector of the input levels; $(x_1, x_2, \ldots , x_n)$.
- *E.g.* $(x_1, x_2, x_3) = (6, 0, 9 \times 3)$.

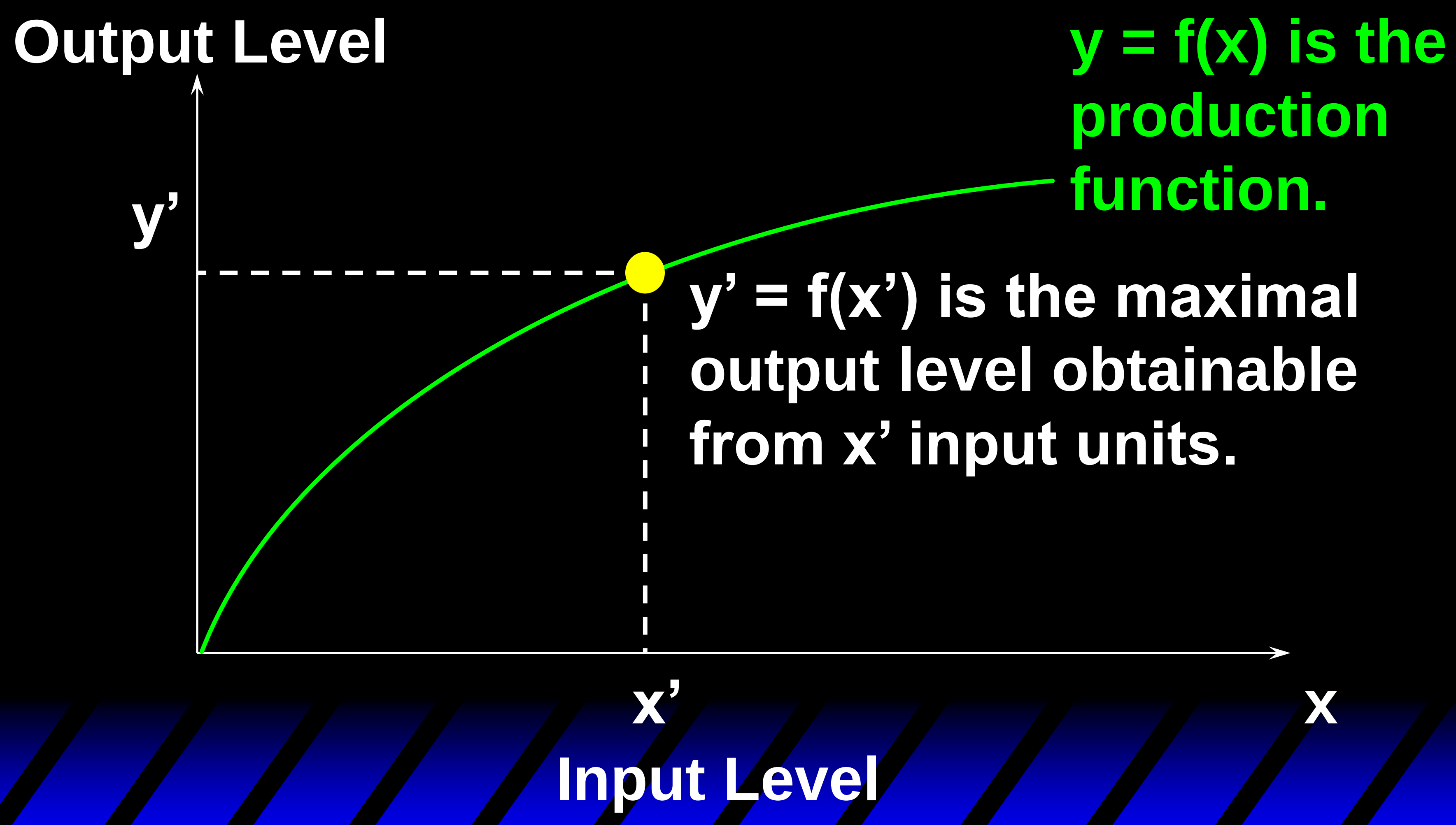
# Production Functions

- y denotes the output level.
- The technology's **production function** states the **maximum** amount of output possible from an input bundle.

$$y = f(x_1, \cdots, x_n)$$

# Production Functions

## One input, one output



# Technology Sets

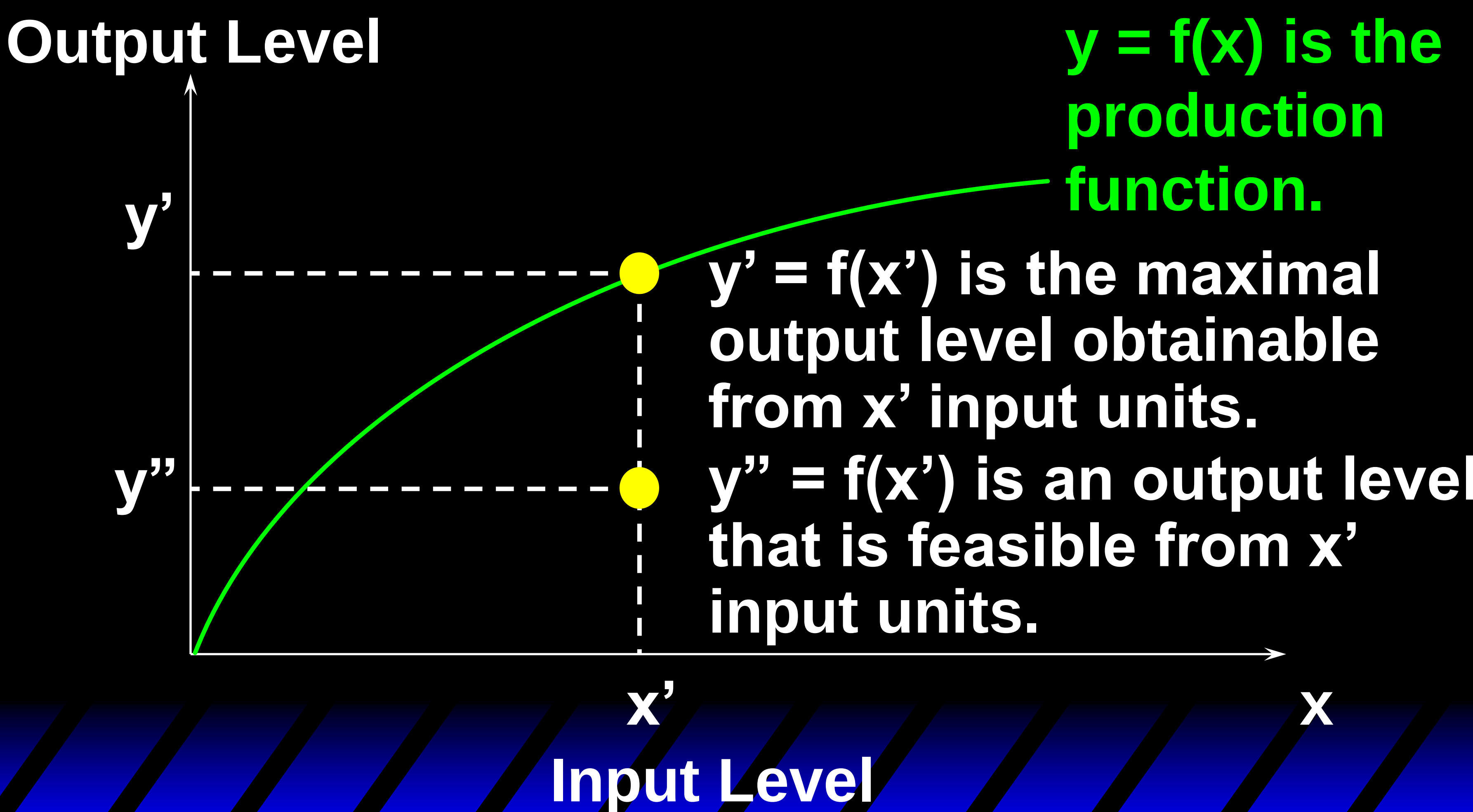
- A **production plan** is an input bundle and an output level; $(x_1, \ldots, x_n, y)$.
- A production plan is **feasible** if

$$y \le f(x_1, \cdots, x_n)$$

- The collection of all feasible production plans is the **technology set**.

# Technology Sets

## One input, one output



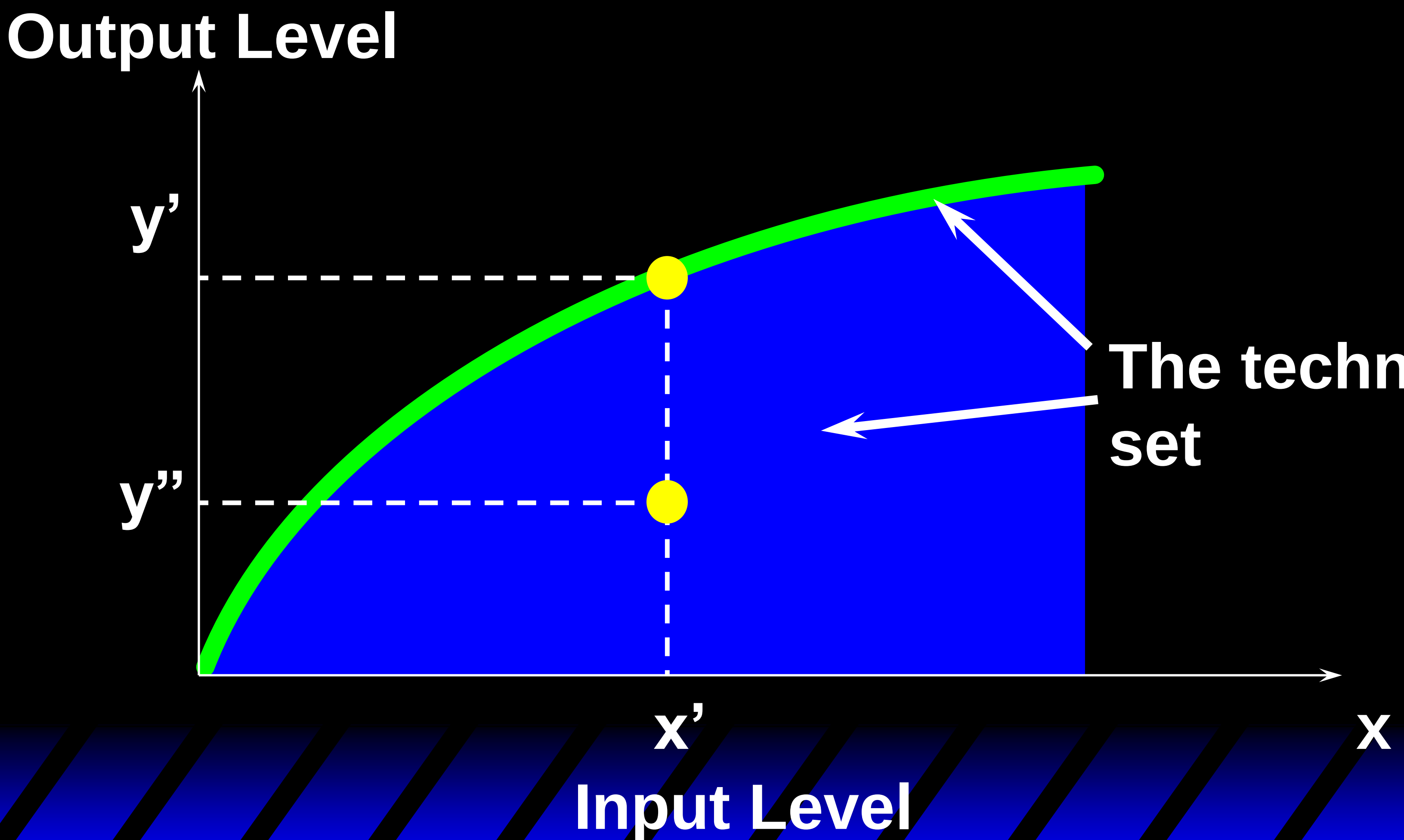
# Technology Sets

The **technology set** is

$$T = \{(x_1,\cdots,x_n,y) \mid y \le f(x_1,\cdots,x_n) \text{ and } x_1 \ge 0,\ldots,x_n \ge 0\}.$$

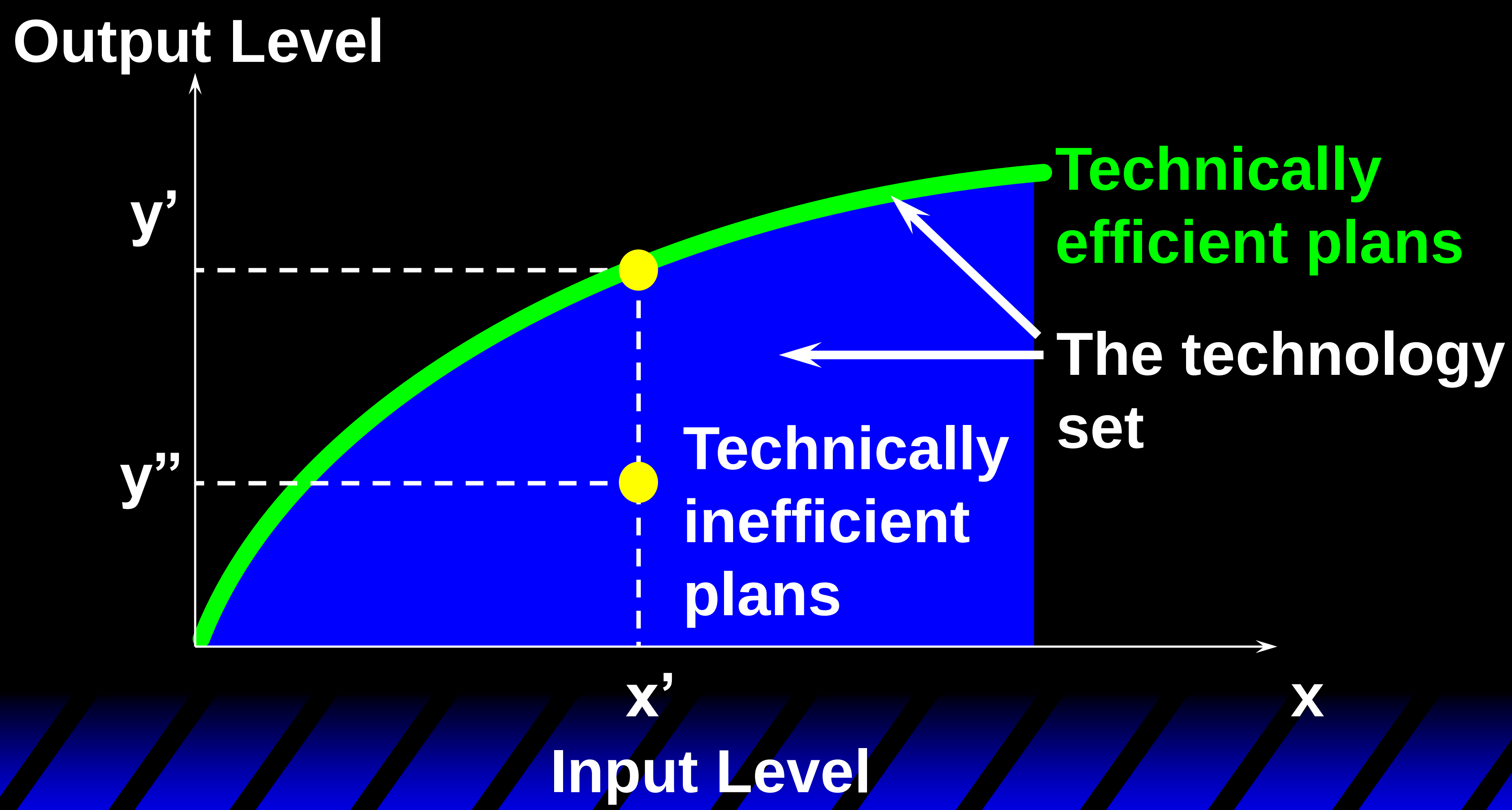
# Technology Sets

## One input, one output



# Technology Sets

## One input, one output



# Technologies with Multiple Inputs

- What does a technology look like when there is more than one input?
- The two input case: Input levels are $x_1$ and $x_2$. Output level is y.
- Suppose the production function is

$$y = f(x_1, x_2) = 2x_1^{1/3}x_2^{1/3}.$$

# Technologies with Multiple Inputs

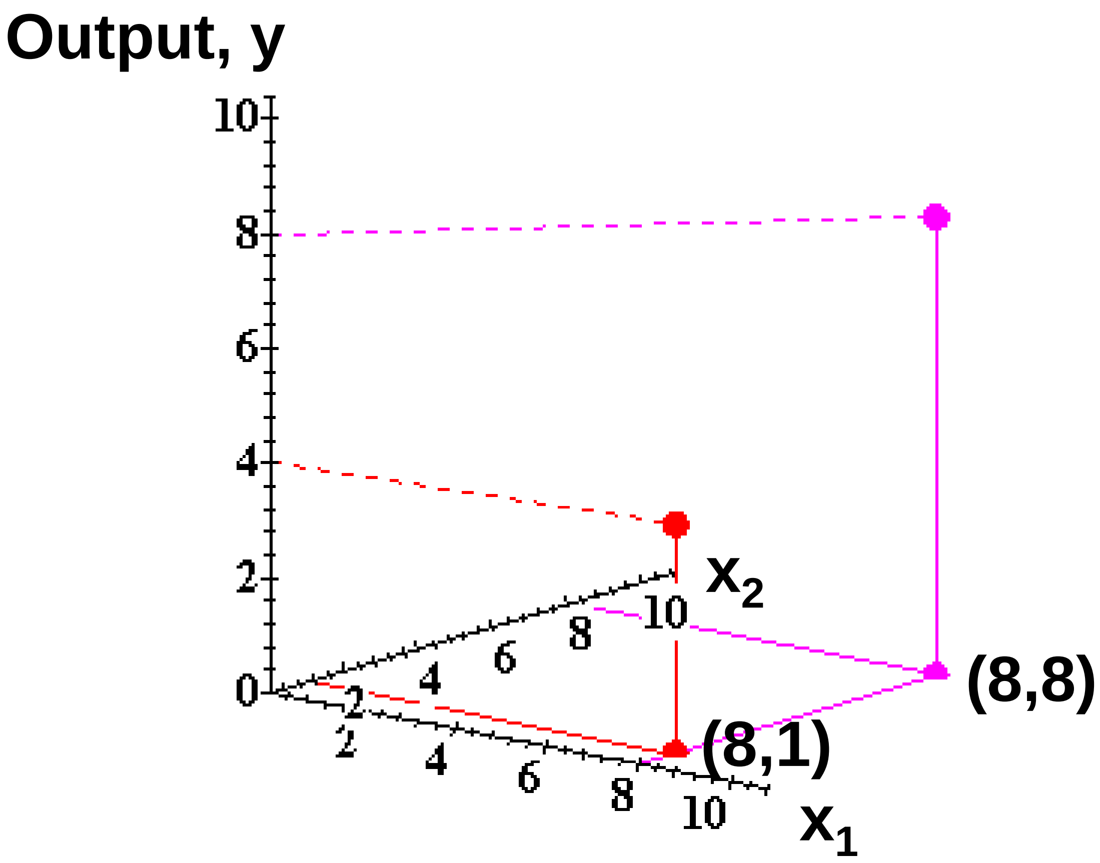
- ***E.g.*** **the maximal output level possible from the input bundle $(x_1, x_2) = (1, 8)$ is**

$$y = 2x_1^{1/3}x_2^{1/3} = 2 \times 1^{1/3} \times 8^{1/3} = 2 \times 1 \times 2 = 4.$$

- **And the maximal output level possible from $(x_1, x_2) = (8,8)$ is**

$$y = 2x_1^{1/3}x_2^{1/3} = 2 \times 8^{1/3} \times 8^{1/3} = 2 \times 2 \times 2 = 8.$$

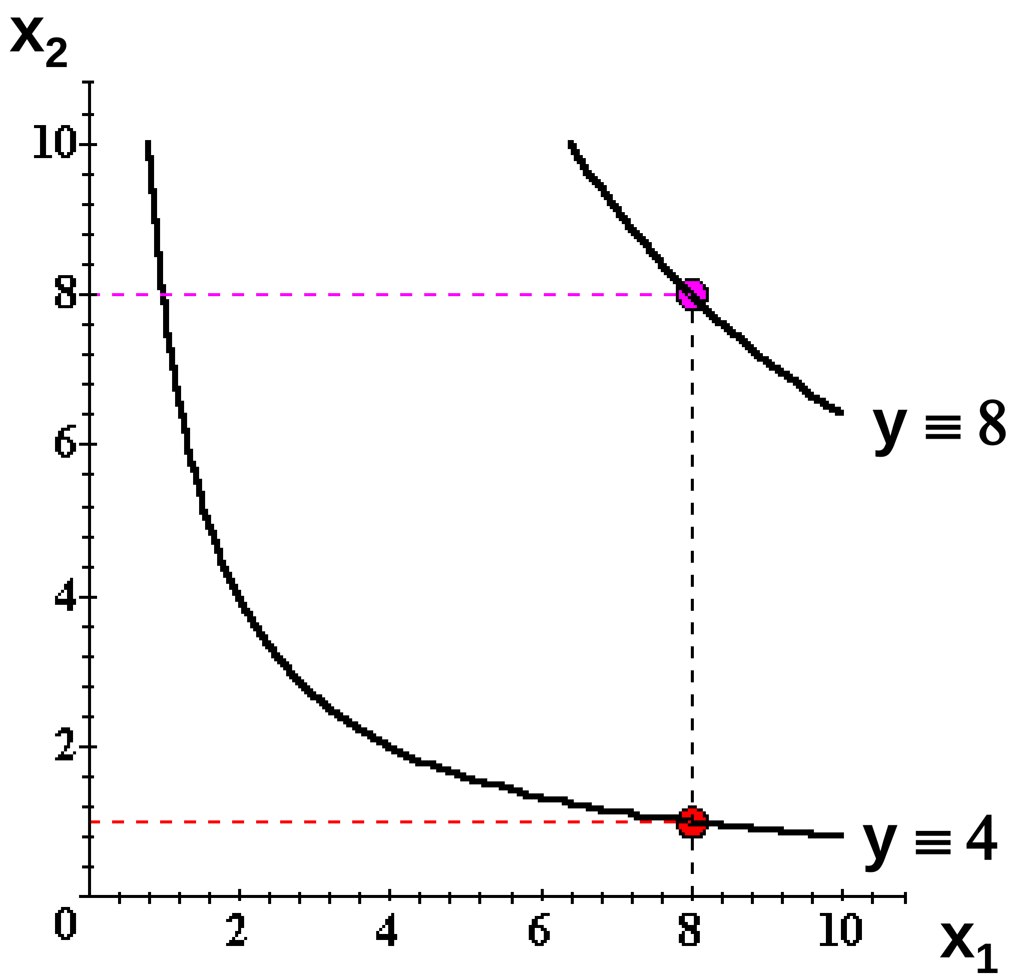
# Technologies with Multiple Inputs



# Technologies with Multiple Inputs

- The y output unit **isoquant** is the set of all input bundles that yield at most the same output level y.

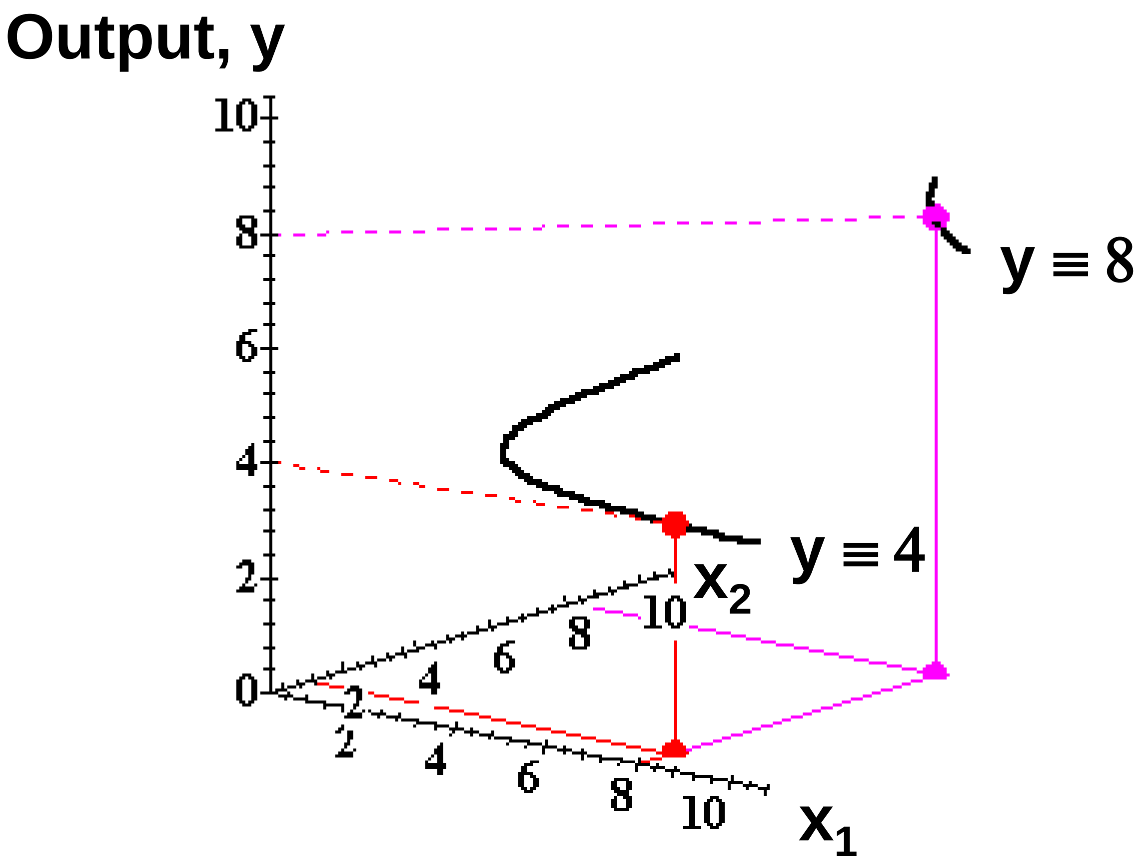
# Isoquants with Two Variable Inputs



# Isoquants with Two Variable Inputs

- **Isoquants can be graphed by adding an output level axis and displaying each isoquant at the height of the isoquant's output level.**

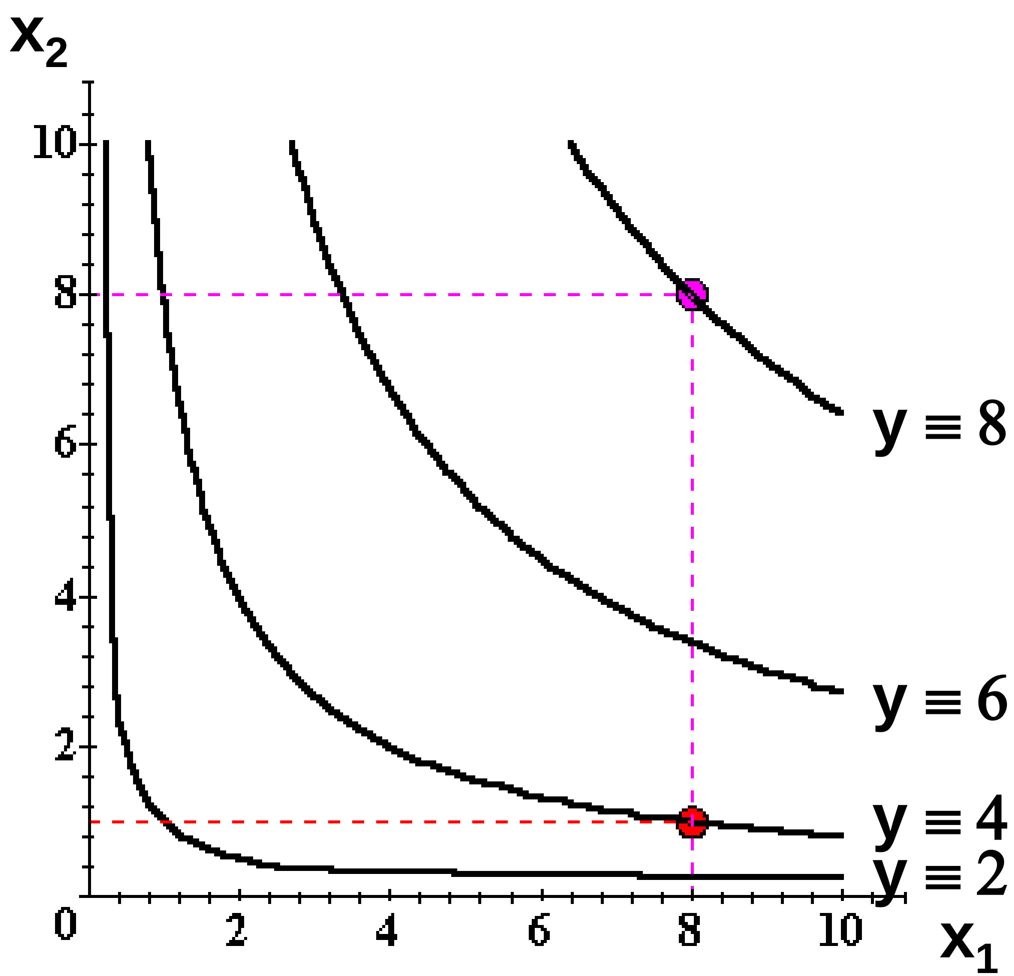
# Isoquants with Two Variable Inputs



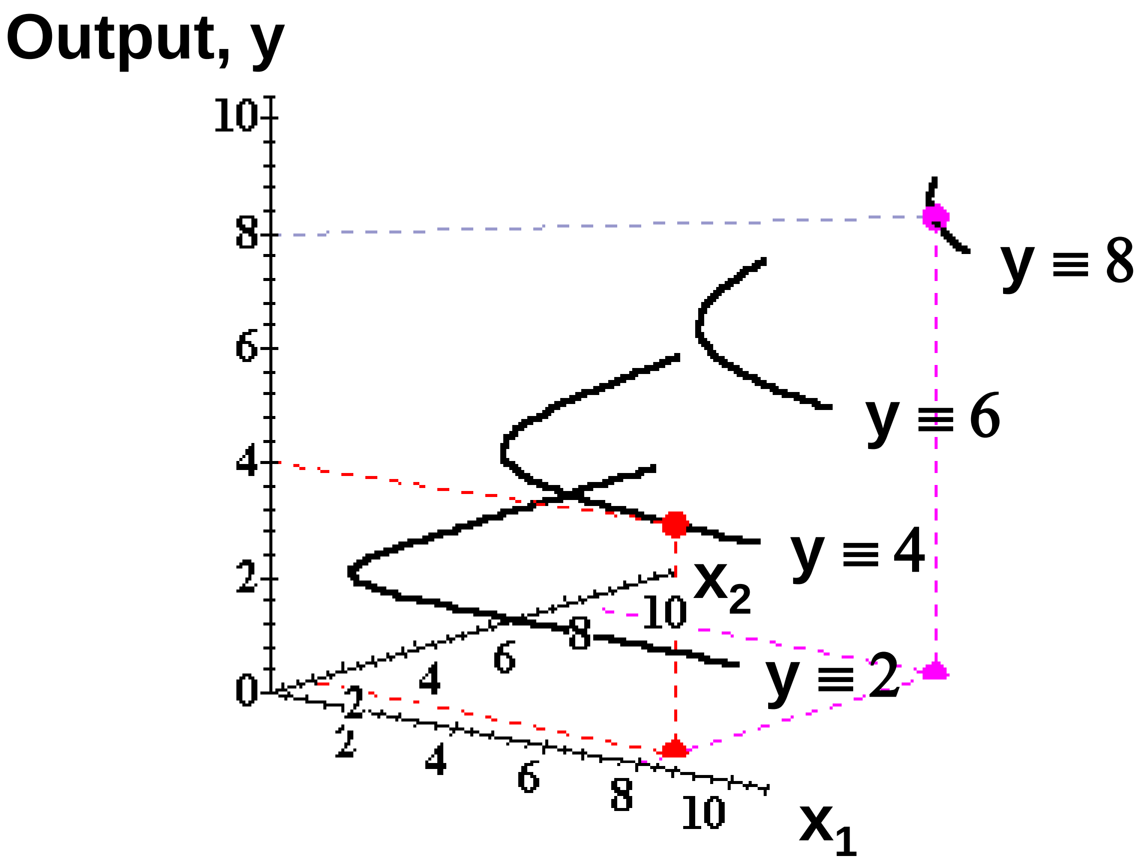
# Isoquants with Two Variable Inputs

- More isoquants tell us more about the technology.

# Isoquants with Two Variable Inputs



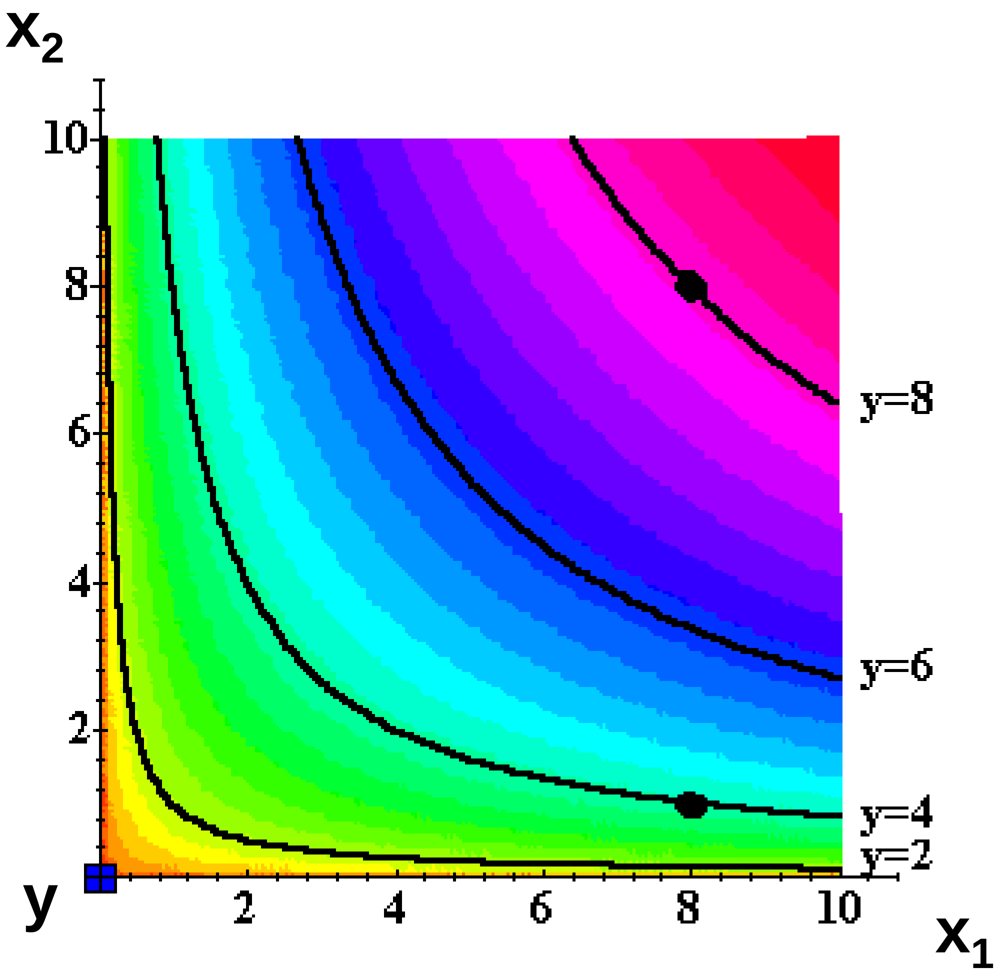
# Isoquants with Two Variable Inputs



# Technologies with Multiple Inputs

- The complete collection of isoquants is the **isoquant map.**
- **The isoquant map is equivalent to the production function** -- each is the other.
- E.g. $$y = f(x_1, x_2) = 2x_1^{1/3}x_2^{1/3}$$

# Technologies with Multiple Inputs



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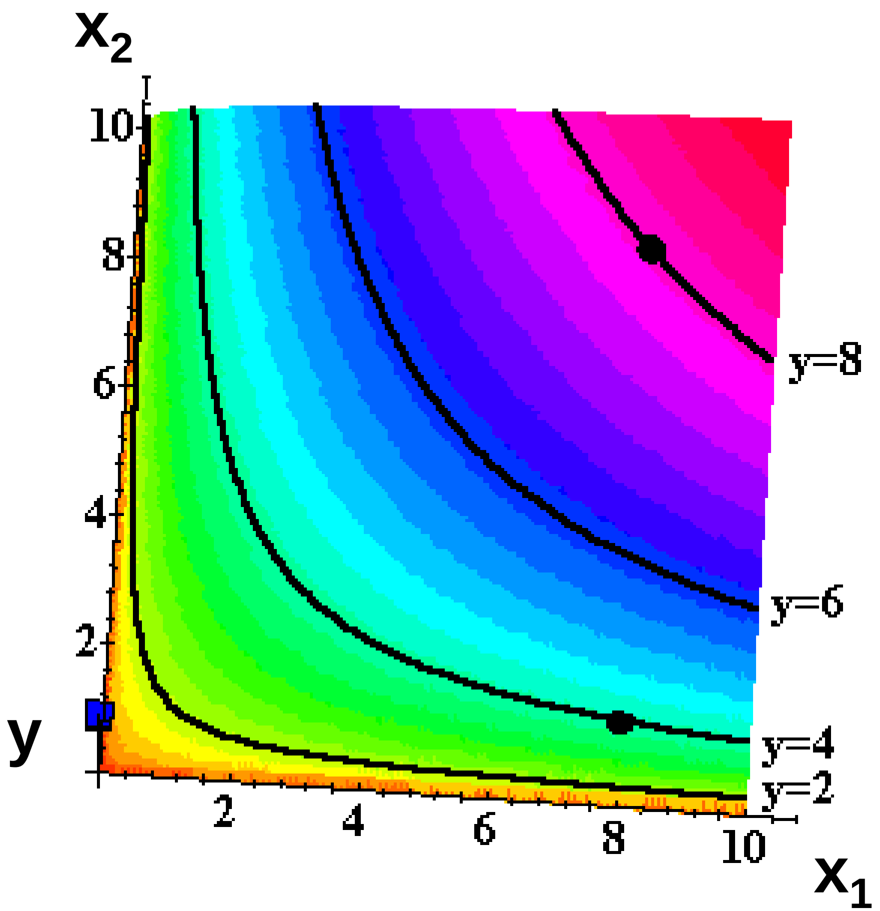

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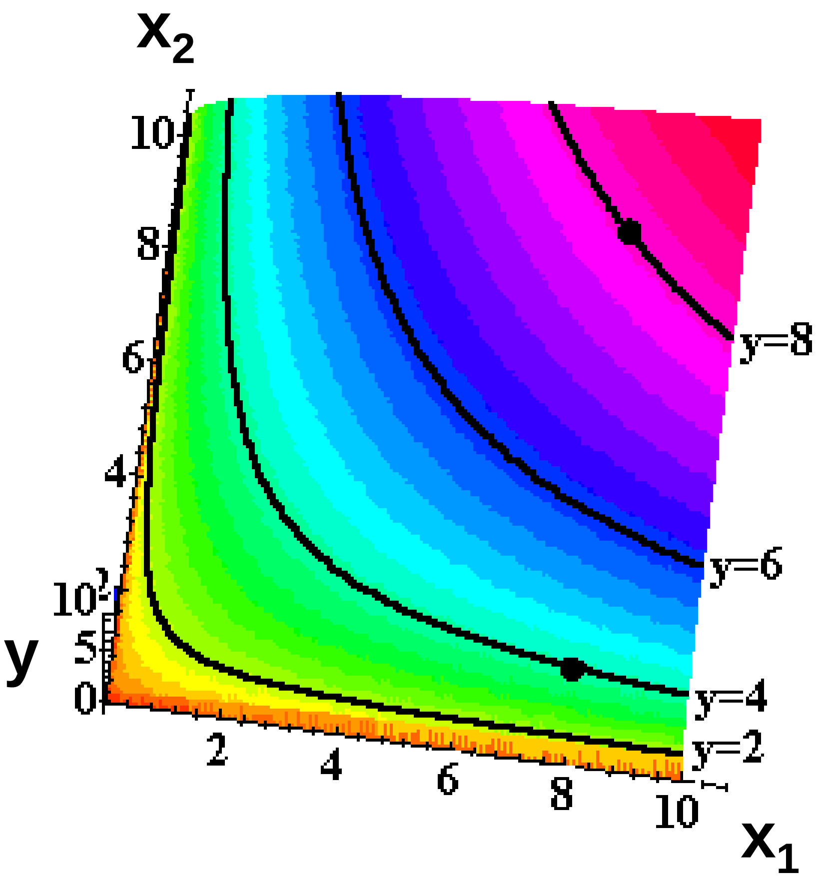

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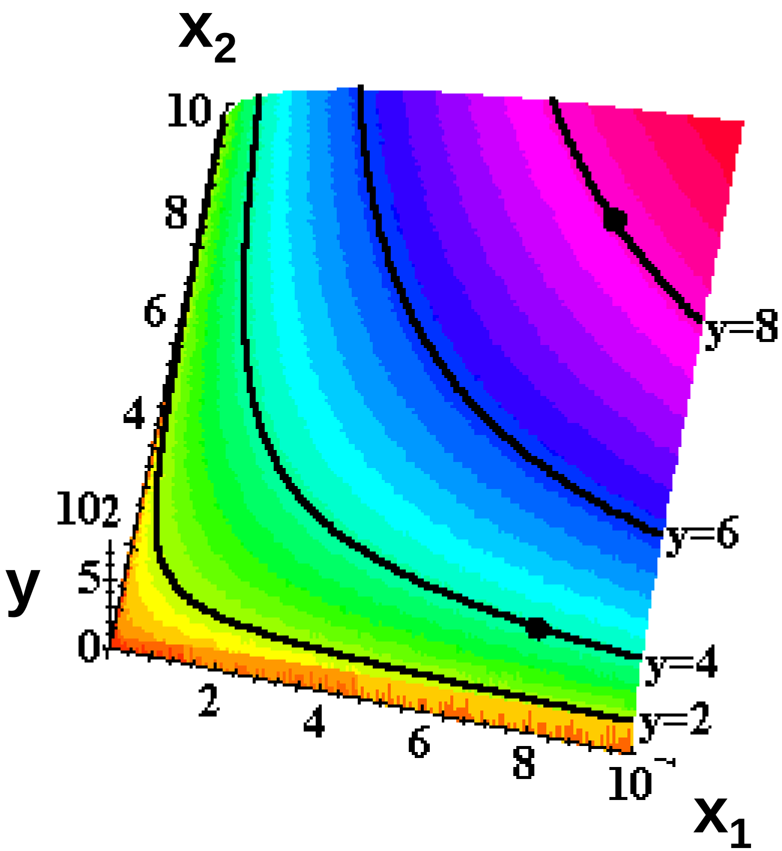

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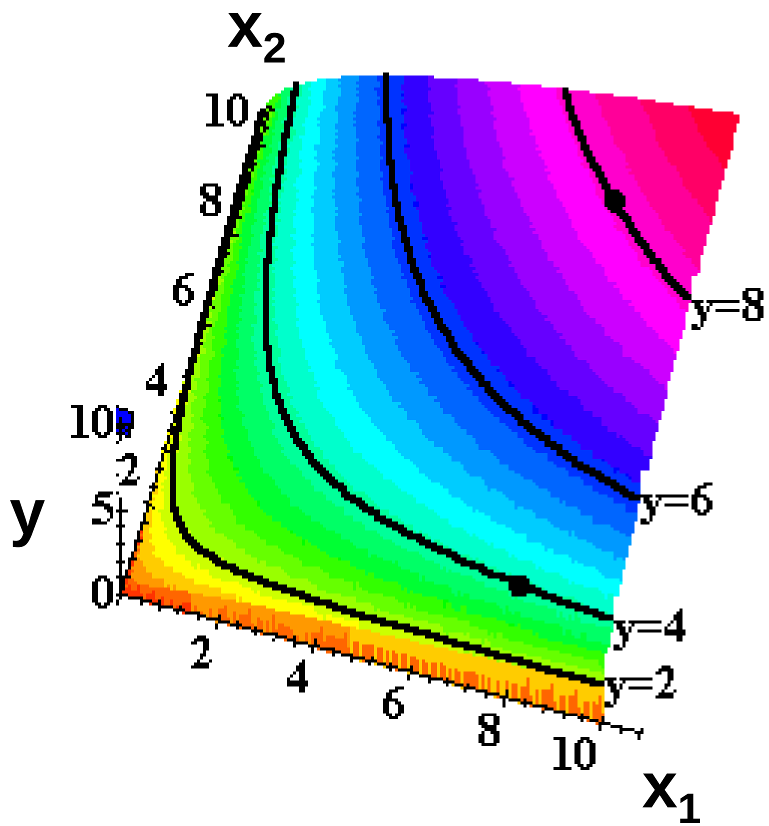

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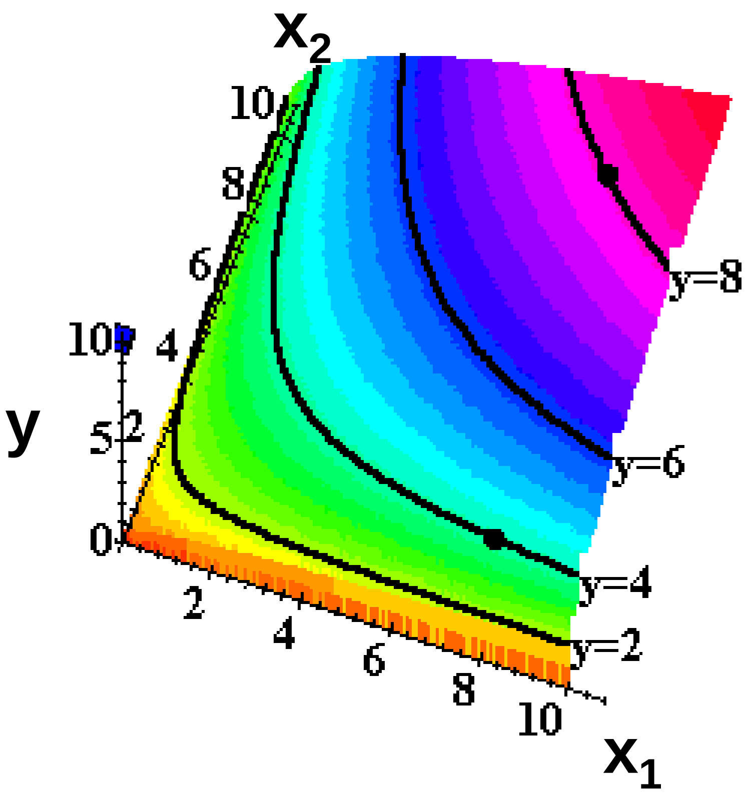

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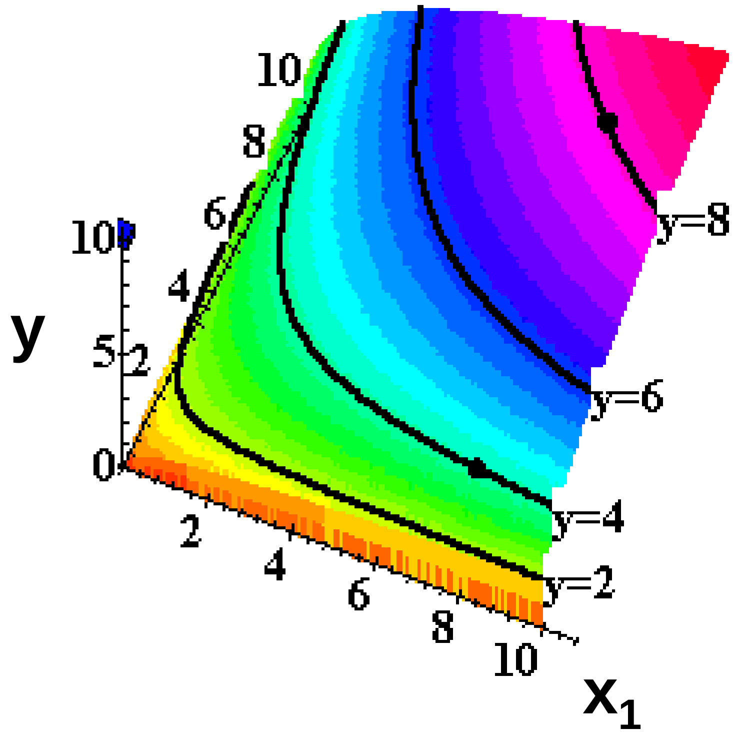

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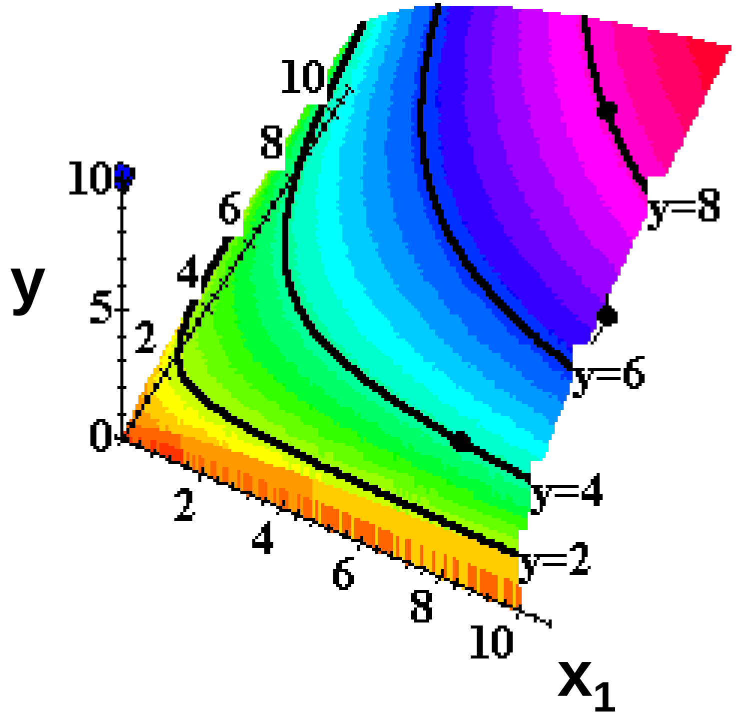

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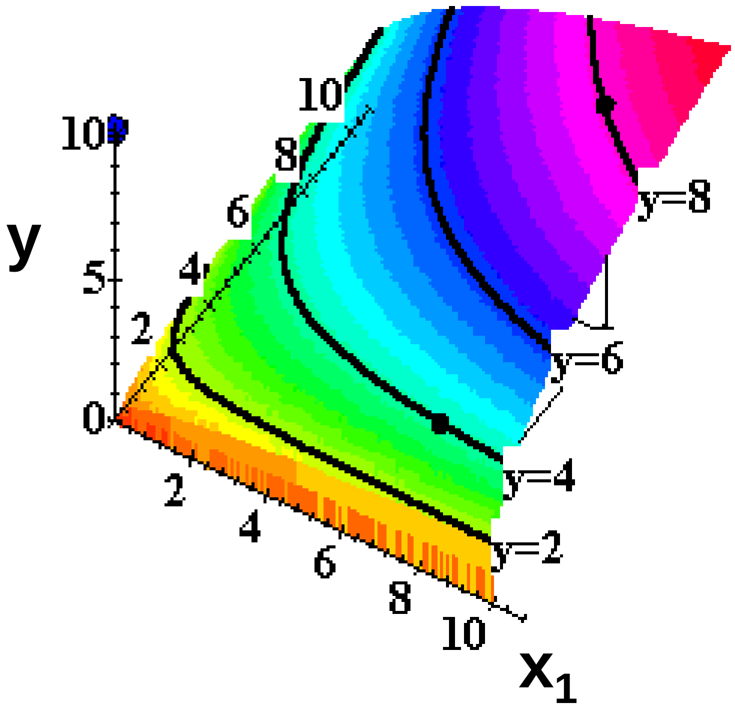

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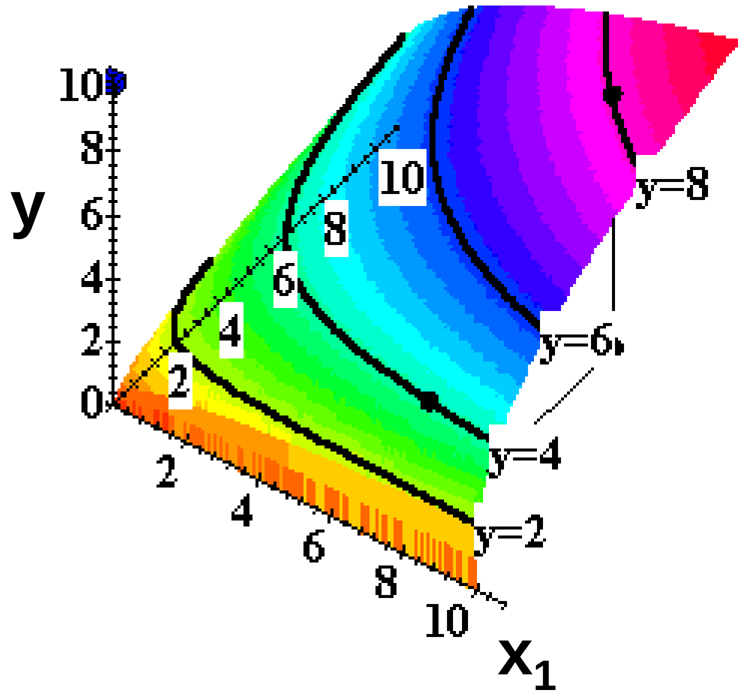

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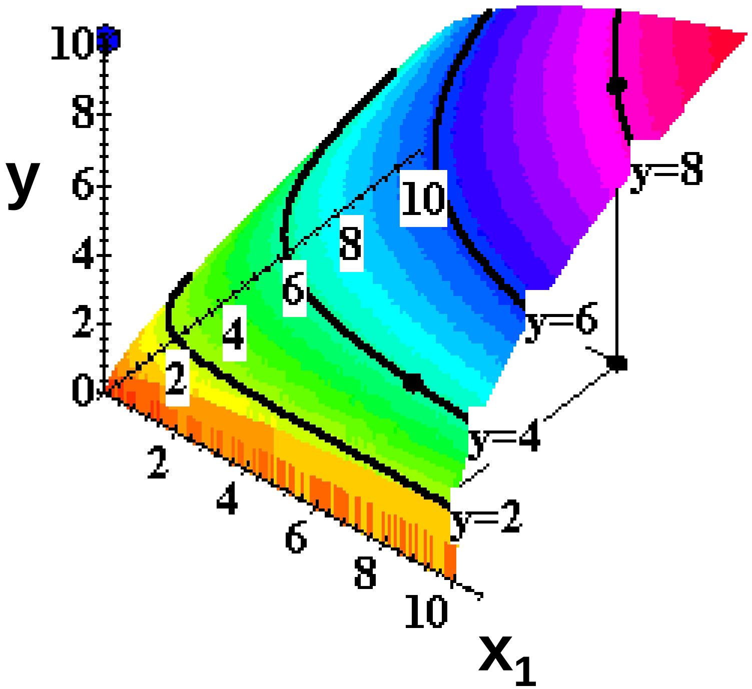

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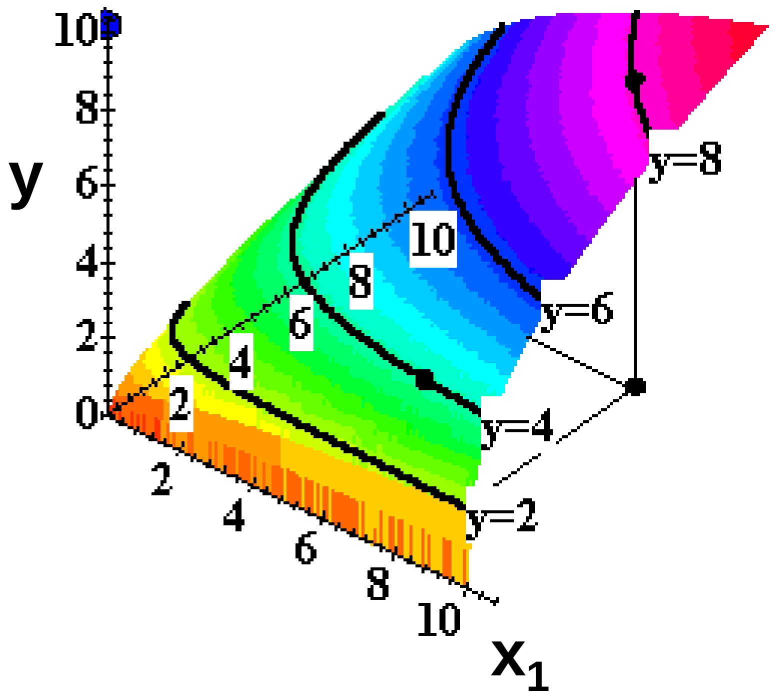

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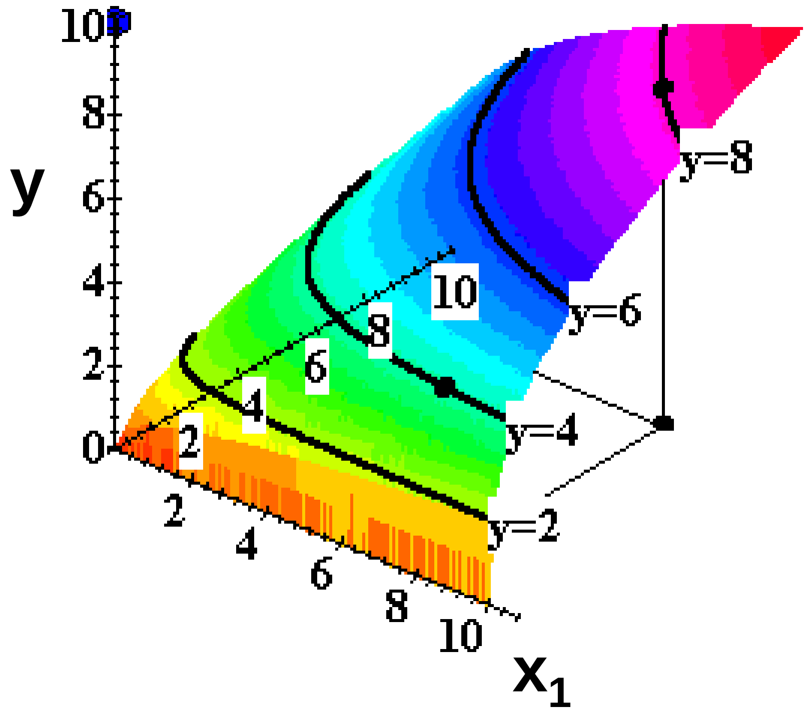

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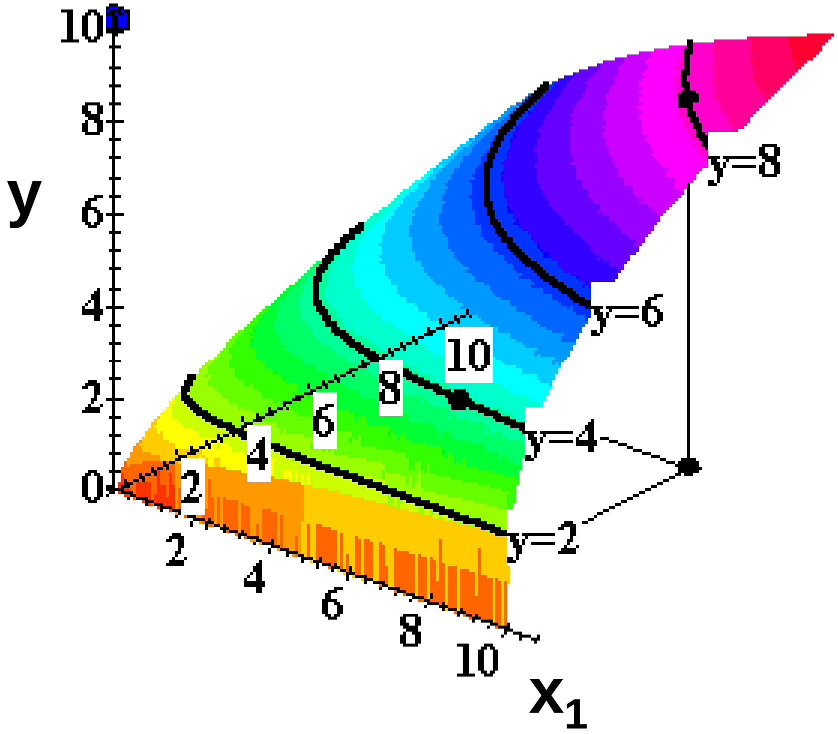

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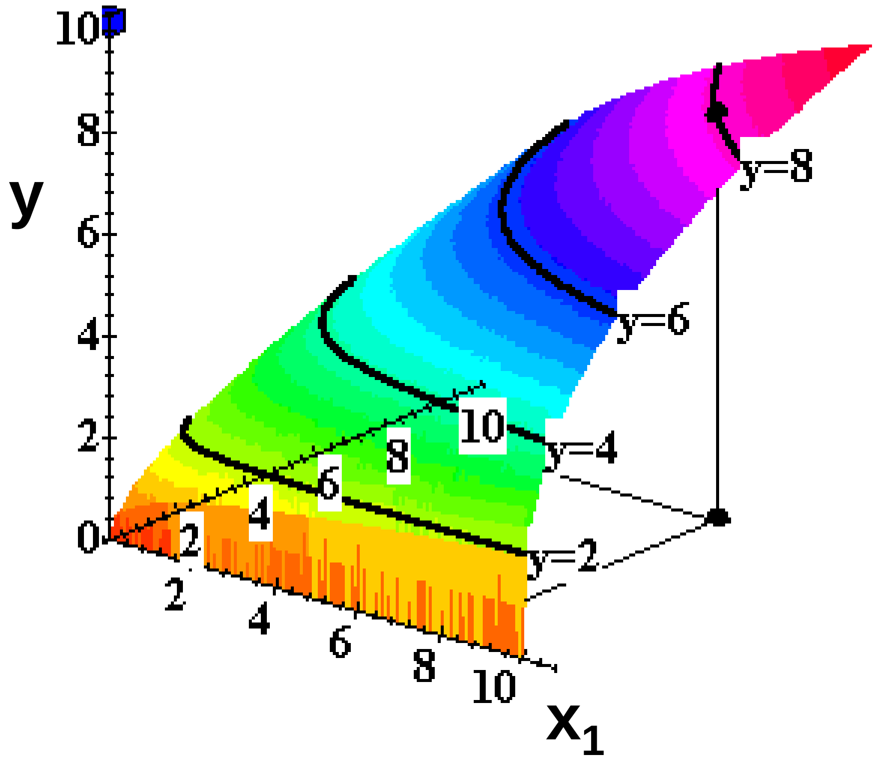

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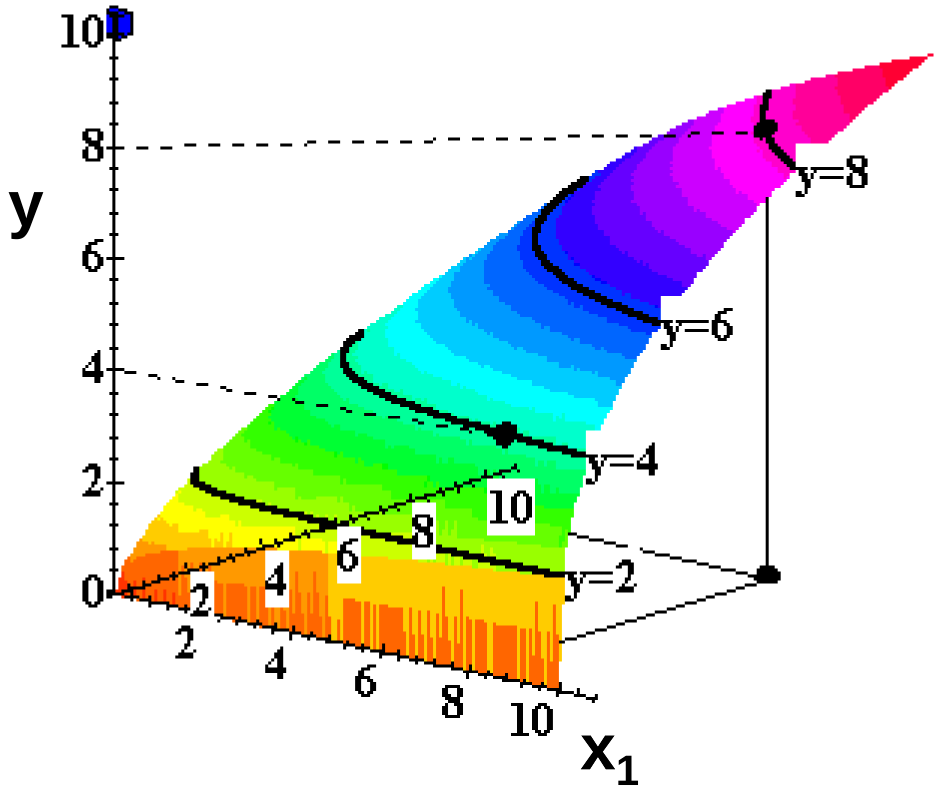
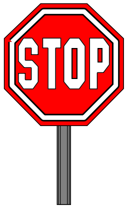


# Cobb-Douglas Technologies

- A Cobb-Douglas production function is of the form

$$y = A x_1^{a_1} x_2^{a_2} \times \cdots \times x_n^{a_n}.$$

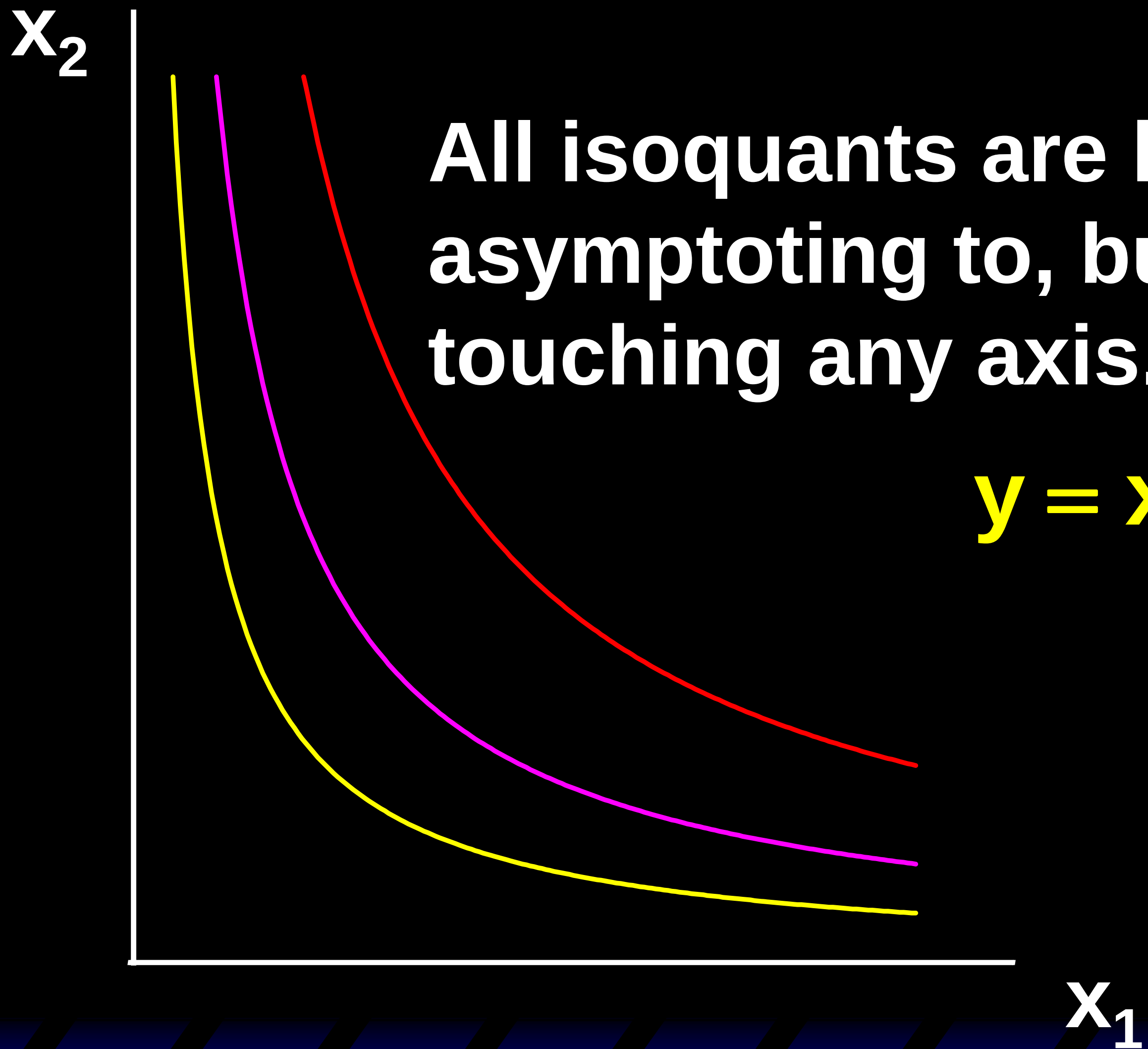
- E.g.

$$y = x_1^{1/3} x_2^{1/3}$$

with

$$n = 2,\ A = 1,\ a_1 = \frac{1}{3} \text{ and } a_2 = \frac{1}{3}.$$

# Cobb-Douglas Technologies




**All isoquants are hyperbolic, asymptoting to, but never touching any axis.**

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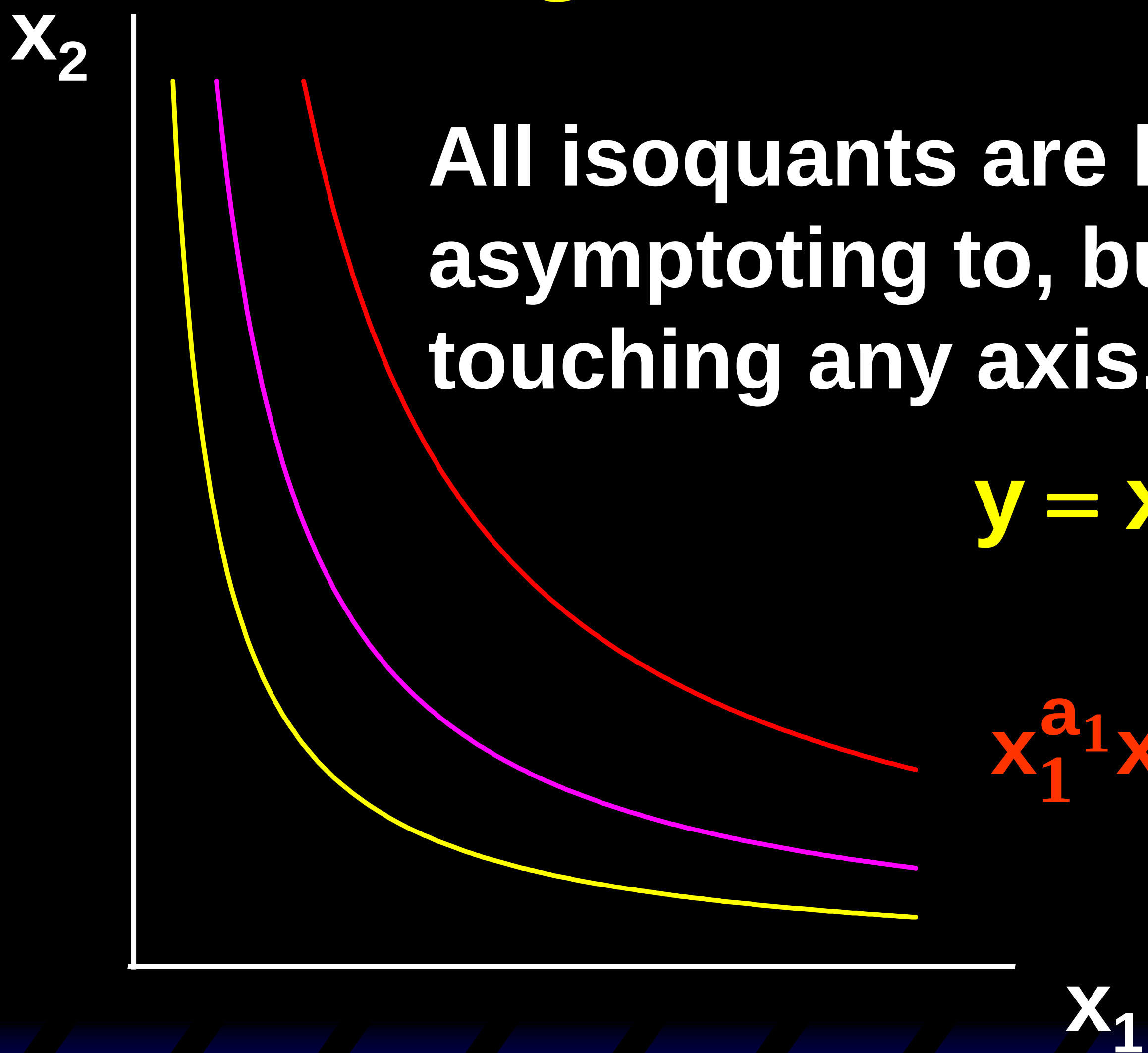
# Cobb-Douglas Technologies




**All isoquants are hyperbolic, asymptoting to, but never touching any axis.**

$$y = x_1^{a_1} x_2^{a_2}$$

$$x_1^{a_1} x_2^{a_2} = y''$$

# Cobb-Douglas Technologies

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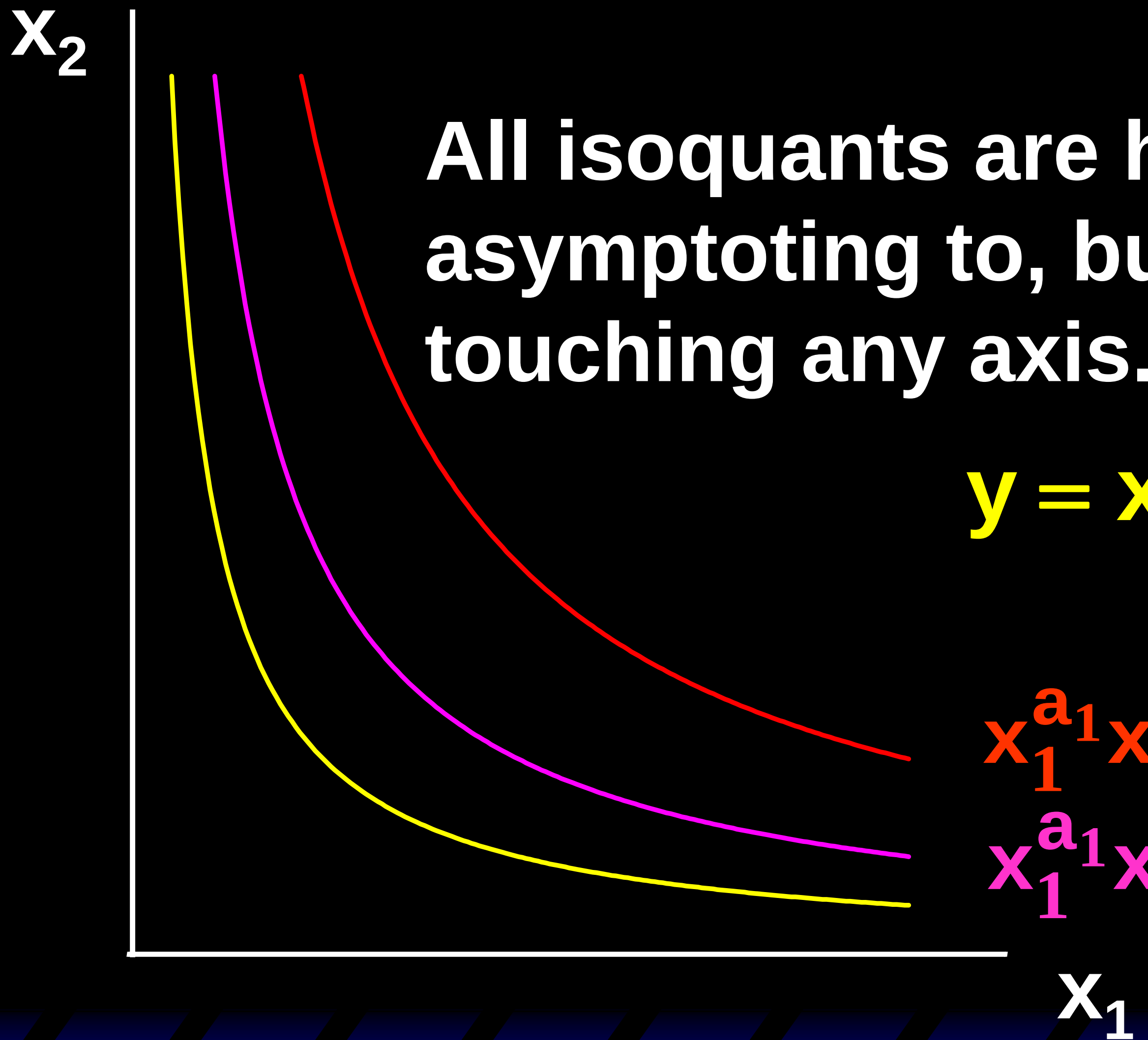

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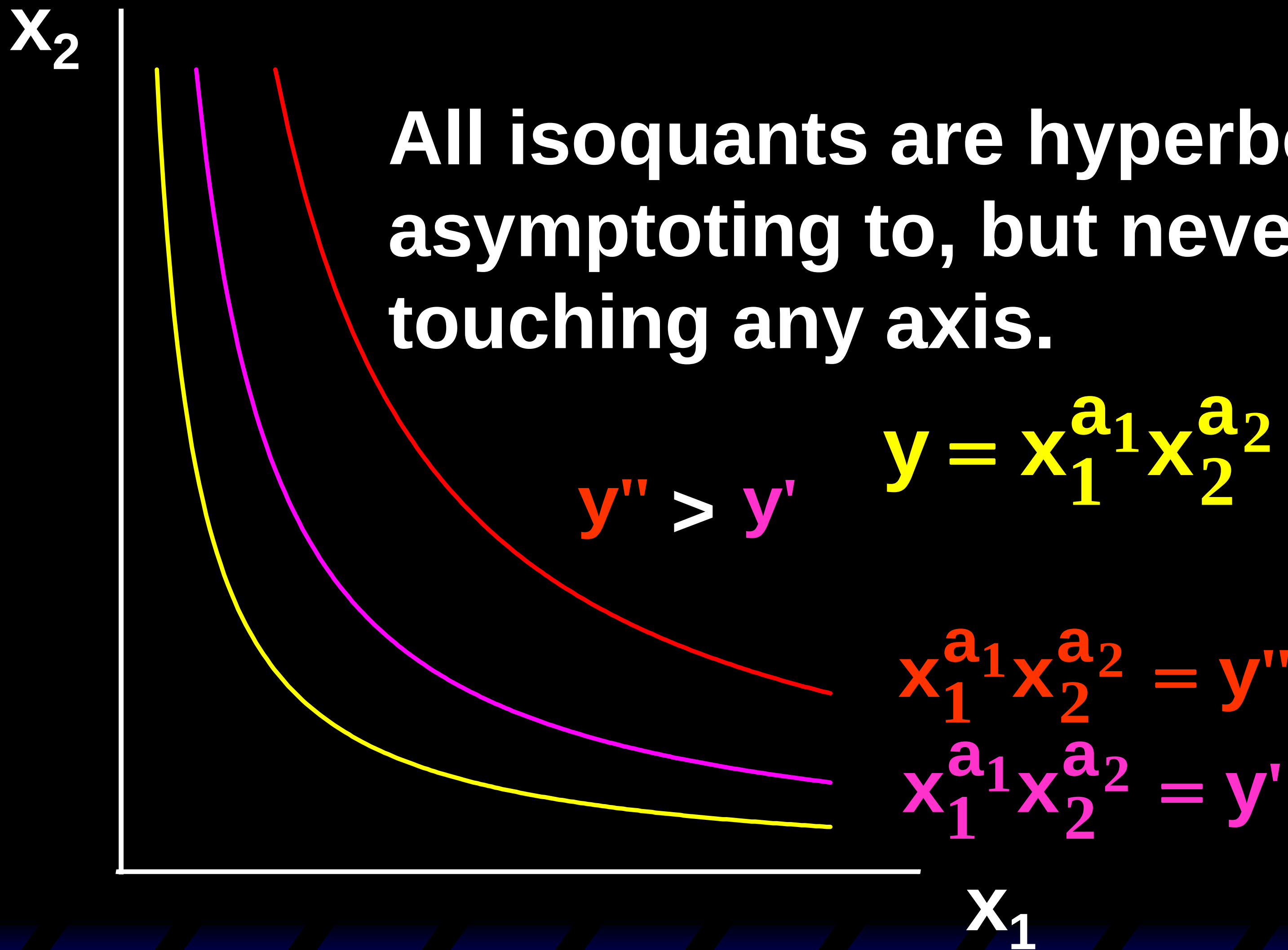



**All isoquants are hyperbolic, asymptoting to, but never touching any axis.**

$$y = x_1^{a_1} x_2^{a_2}$$

# Fixed-Proportions Technologies

- A fixed-proportions production function is of the form

$$y = \min\{a_1 x_1, a_2 x_2, \cdots, a_n x_n\}.$$

- E.g.

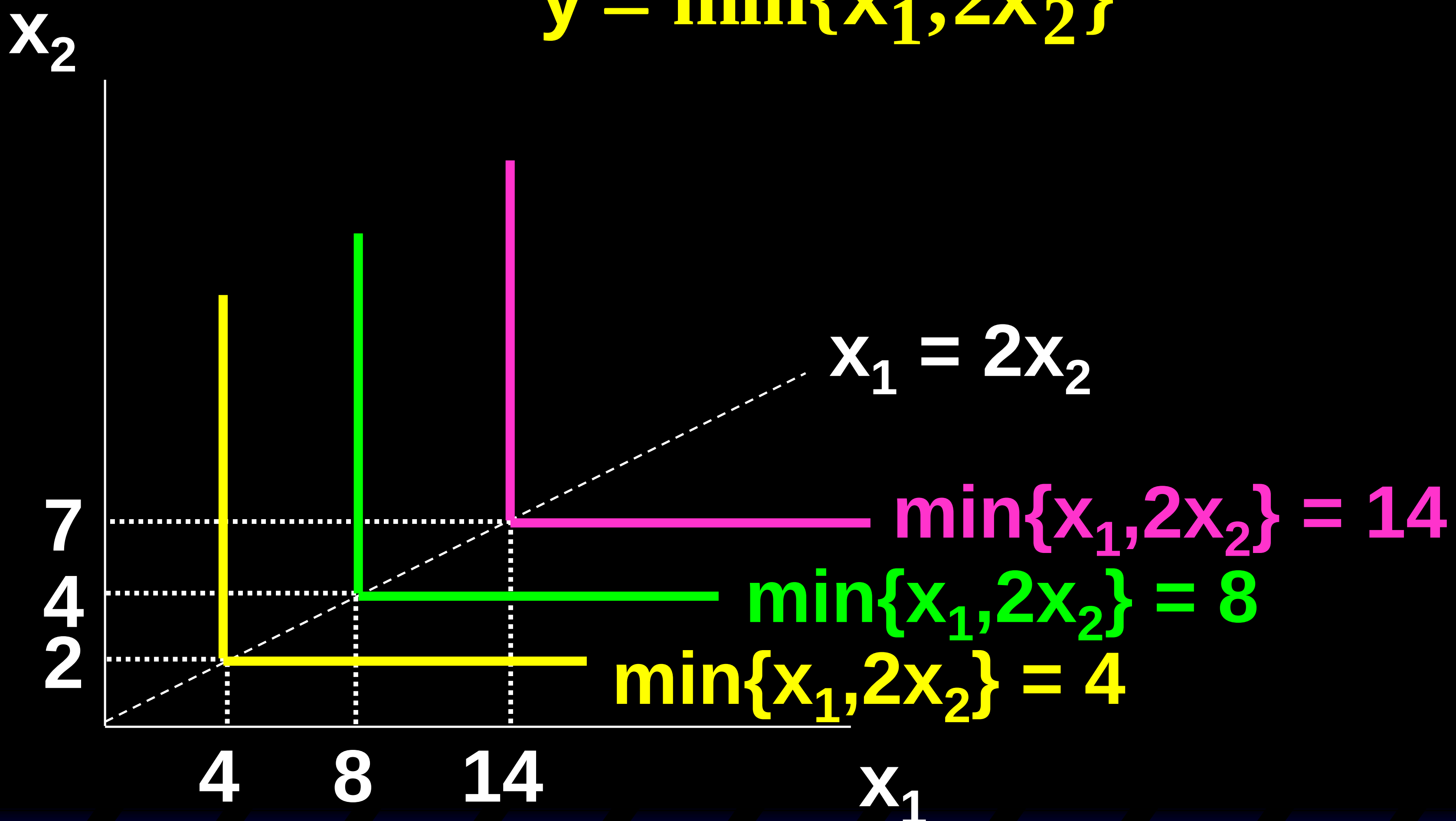
$$y = \min\{x_1, 2x_2\}$$

with

$$n = 2,\ a_1 = 1 \text{ and } a_2 = 2.$$

# Fixed-Proportions Technologies

$$y = \min\{x_1, 2x_2\}$$



# Perfect-Substitutes Technologies

- A perfect-substitutes production function is of the form

$$y = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n.$$

- E.g.

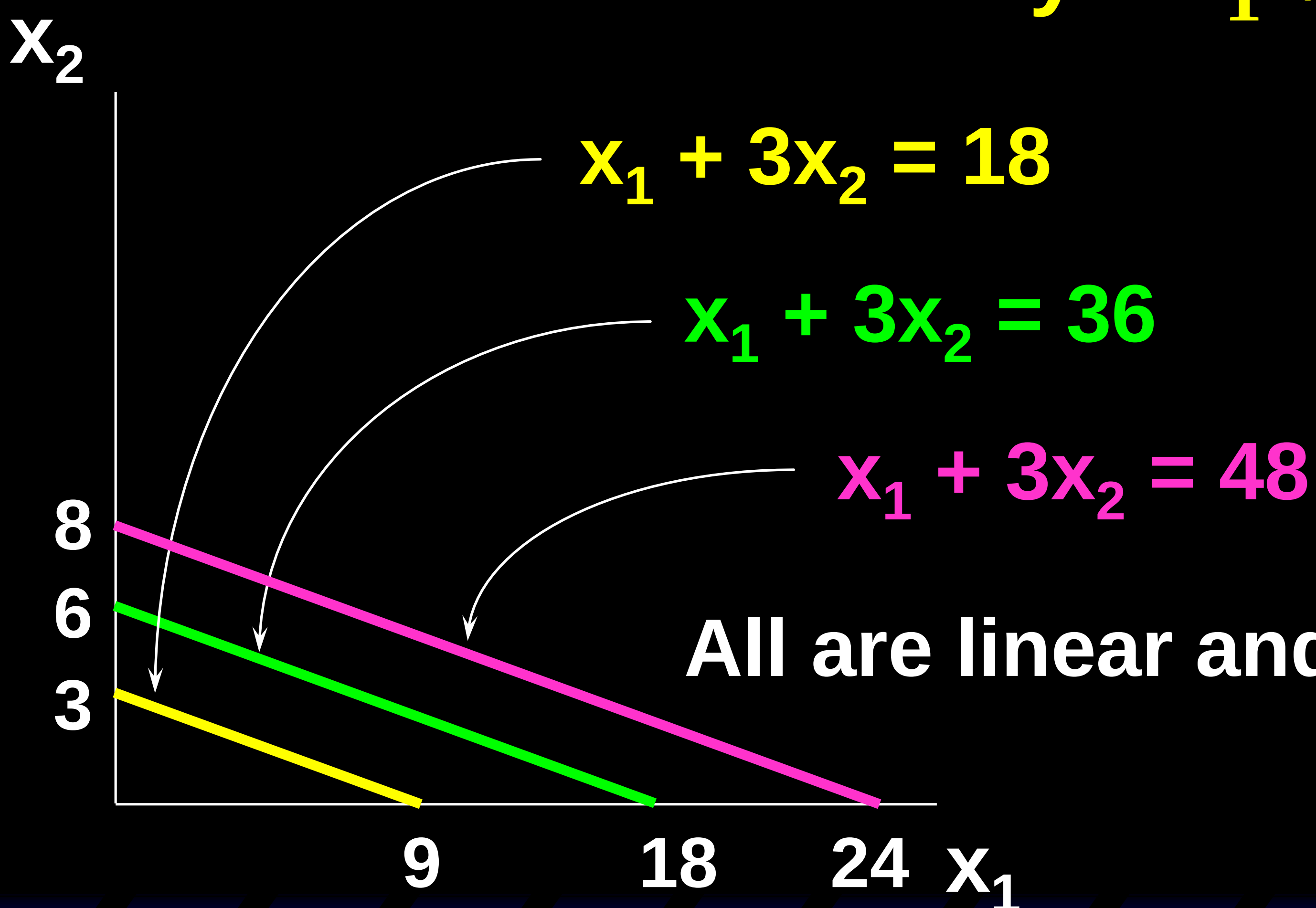
$$y = x_1 + 3x_2$$

with

$$n = 2, a_1 = 1 \text{ and } a_2 = 3.$$

# Perfect-Substitution Technologies

$$y = x_1 + 3x_2$$




All are linear and parallel

# Marginal (Physical) Products

$$y = f(x_1, \cdots, x_n)$$

- The marginal product of input i is the rate-of-change of the output level as the level of input i changes, holding all other input levels fixed.
- That is,

$$MP_i = \frac{\partial y}{\partial x_i}$$

# Marginal (Physical) Products

E.g. if

$$y = f(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

then the marginal product of input 1 is

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and the marginal product of input 2 is

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and the marginal product of input 2 is

$$MP_2 = \frac{\partial y}{\partial x_2} = \frac{2}{3} x_1^{1/3} x_2^{-1/3}.$$

# Marginal (Physical) Products

Typically the marginal product of one input depends upon the amount used of other inputs. E.g. if

$MP_1 = \frac{1}{3}x_1^{-2/3}x_2^{2/3}$ then,

if $x_2 = 8$, $MP_1 = \frac{1}{3}x_1^{-2/3}8^{2/3} = \frac{4}{3}x_1^{-2/3}$

and if $x_2 = 27$ then

$$MP_1 = \frac{1}{3}x_1^{-2/3}27^{2/3} = 3x_1^{-2/3}.$$

# Marginal (Physical) Products

- The marginal product of input i is **diminishing** if it becomes smaller as the level of input i increases. That is, if

$$\frac{\partial MP_i}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\partial y}{\partial x_i}\right) = \frac{\partial^2 y}{\partial x_i^2} < 0.$$

# Marginal (Physical) Products

E.g. if $y = x_1^{1/3} x_2^{2/3}$ then

$MP_1 = \frac{1}{3} x_1^{-2/3} x_2^{2/3}$ and $MP_2 = \frac{2}{3} x_1^{1/3} x_2^{-1/3}$

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so

$$\frac{\partial MP_1}{\partial x_1} = -\frac{2}{9} x_1^{-5/3} x_2^{2/3} < 0$$

# Marginal (Physical) Products

E.g. if $y = x_1^{1/3} x_2^{2/3}$ then

$$MP_1 = \frac{1}{3} x_1^{-2/3} x_2^{2/3} \quad \text{and} \quad MP_2 = \frac{2}{3} x_1^{1/3} x_2^{-1/3}$$

so

$$\frac{\partial MP_1}{\partial x_1} = -\frac{2}{9} x_1^{-5/3} x_2^{2/3} < 0$$

and

$$\frac{\partial MP_2}{\partial x_2} = -\frac{2}{9} x_1^{1/3} x_2^{-4/3} < 0.$$

# Marginal (Physical) Products

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and

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Both marginal products are diminishing.

# Returns-to-Scale

- Marginal products describe the change in output level as a **single** input level changes.
- **Returns-to-scale** describes how the output level changes as **all** input levels change in **direct proportion** (*e.g.* all input levels doubled, or halved).

# Returns-to-Scale

**If, for any input bundle $(x_1,\ldots,x_n)$,**

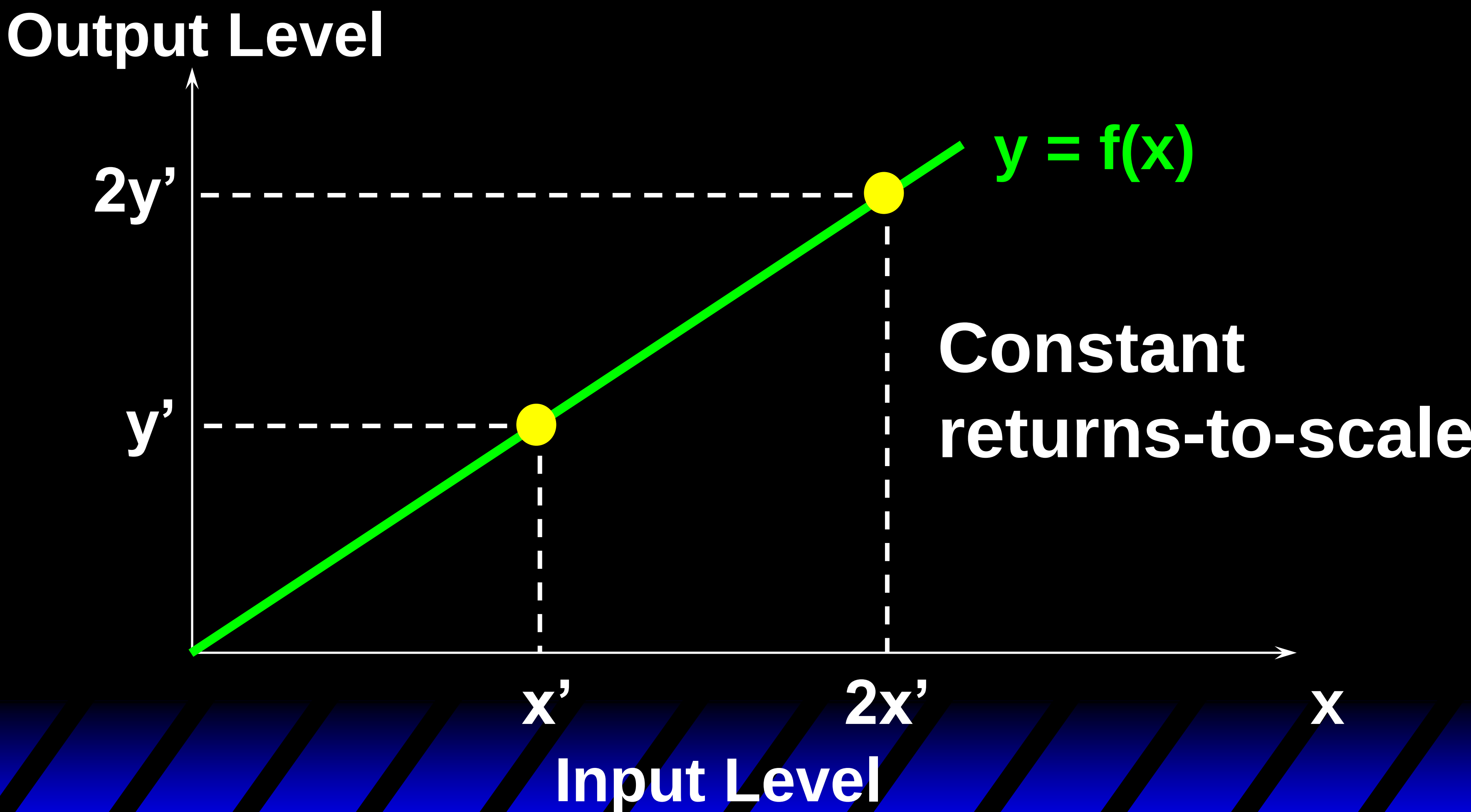
$$f(kx_1, kx_2, \cdots, kx_n) = kf(x_1, x_2, \cdots, x_n)$$

**then the technology described by the production function f exhibits constant returns-to-scale.**

***E.g.* (k = 2) doubling all input levels doubles the output level.**

# Returns-to-Scale

## One input, one output



# Returns-to-Scale

**If, for any input bundle ($x_1,\ldots,x_n$),**

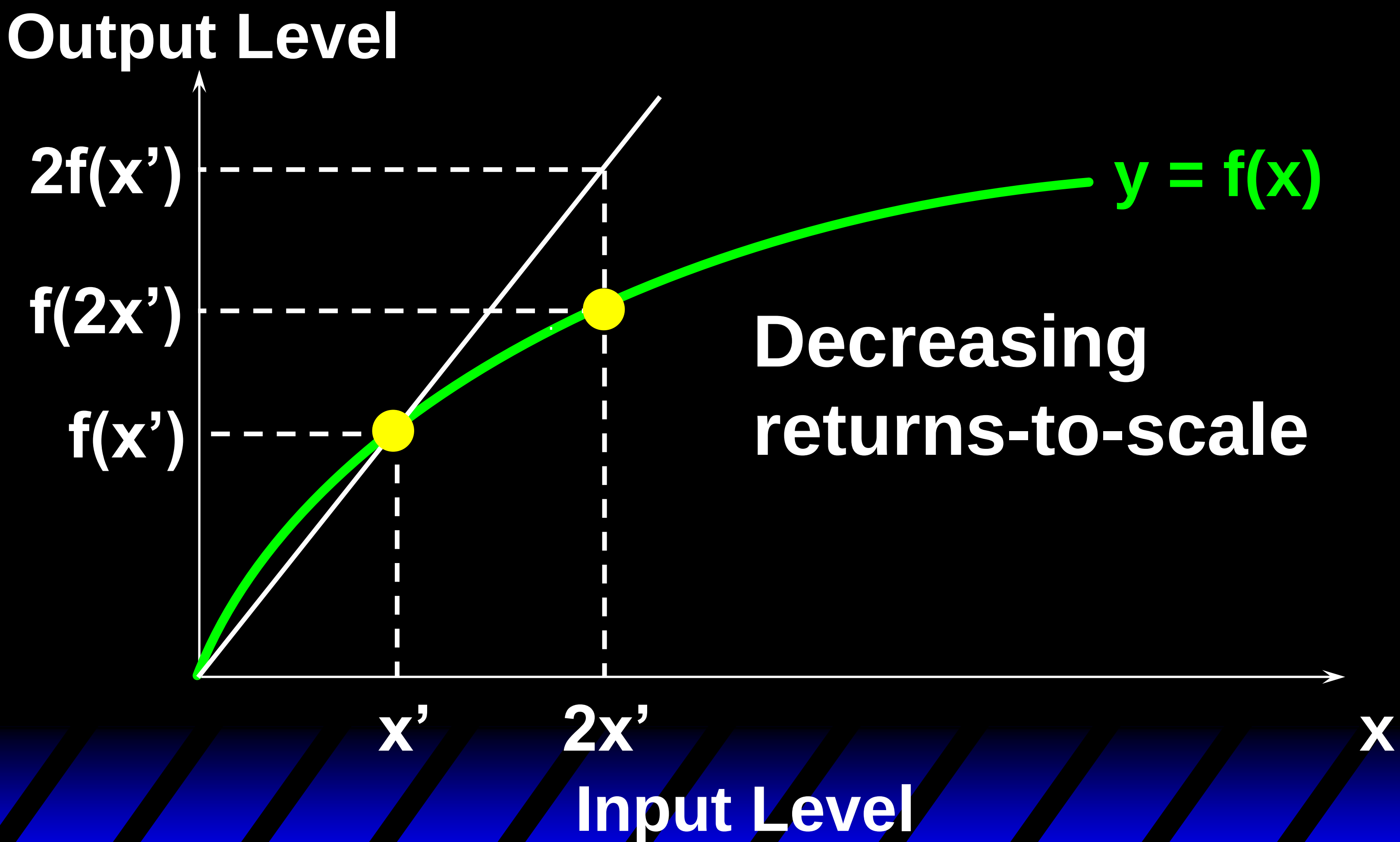
$$f(kx_1, kx_2, \cdots, kx_n) < kf(x_1, x_2, \cdots, x_n)$$

**then the technology exhibits diminishing returns-to-scale.**

***E.g.*. (k = 2) doubling all input levels less than doubles the output level.**

# Returns-to-Scale

## One input, one output



# Returns-to-Scale

If, for any input bundle $(x_1,\ldots,x_n)$,

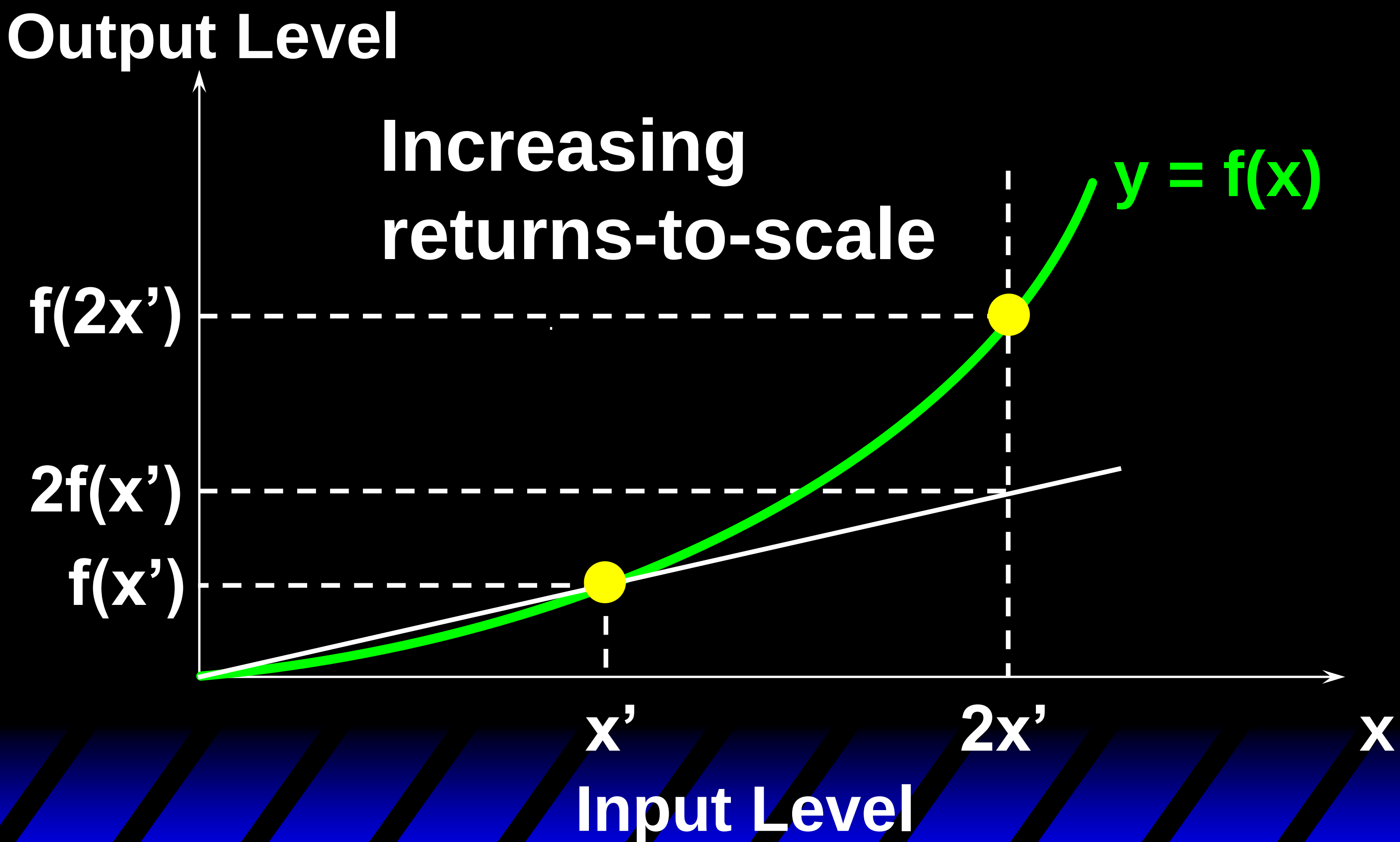
$$f(kx_1, kx_2, \cdots, kx_n) > kf(x_1, x_2, \cdots, x_n)$$

then the technology exhibits **increasing returns-to-scale**.

*E.g.* (k = 2) doubling all input levels more than doubles the output level.

# Returns-to-Scale

## One input, one output

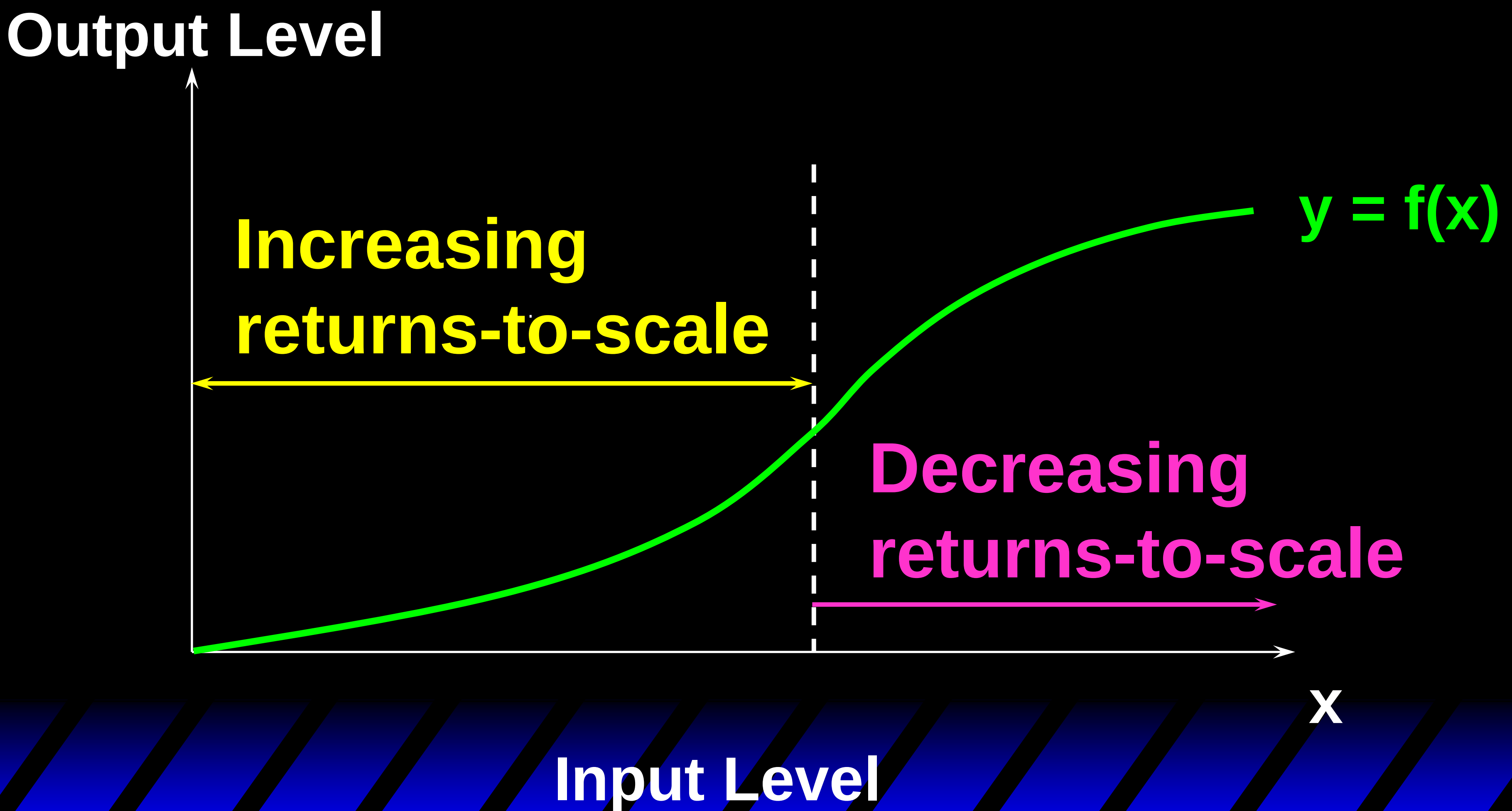


# Returns-to-Scale

- A single technology can ‘locally’ exhibit different returns-to-scale.

# Returns-to-Scale

## One input, one output



# Examples of Returns-to-Scale

The perfect-substitutes production function is

$$y = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n.$$

Expand all input levels proportionately by k. The output level becomes

$$a_1(kx_1) + a_2(kx_2) + \cdots + a_n(kx_n)$$

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$$a_1(kx_1) + a_2(kx_2) + \cdots + a_n(kx_n)$$
$$= k(a_1 x_1 + a_2 x_2 + \cdots + a_n x_n)$$
$$= ky.$$

The perfect-substitutes production function exhibits constant returns-to-scale.

# Examples of Returns-to-Scale

The perfect-complements production function is

$$y = \min\{a_1 x_1, a_2 x_2, \cdots, a_n x_n\}.$$

Expand all input levels proportionately by k. The output level becomes

$$\min\{a_1 (kx_1), a_2 (kx_2), \cdots, a_n (kx_n)\}$$

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# Examples of Returns-to-Scale

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# Examples of Returns-to-Scale

The Cobb-Douglas production function is

$$y = x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}.$$

Expand all input levels proportionately by k. The output level becomes

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**The Cobb-Douglas technology's returns-to-scale is**

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The Cobb-Douglas technology's returns-to-scale is

constant if $a_1 + \ldots + a_n = 1$

increasing if $a_1 + \ldots + a_n > 1$

decreasing if $a_1 + \ldots + a_n < 1$.

# Returns-to-Scale

- **Q: Can a technology exhibit increasing returns-to-scale even if all of its marginal products are diminishing?**

# Returns-to-Scale

- **Q: Can a technology exhibit increasing returns-to-scale even if all of its marginal products are diminishing?**
- **A: Yes.**
- **E.g.** $y = x_1^{2/3} x_2^{2/3}$.

# Returns-to-Scale

$$y = x_1^{2/3} x_2^{2/3} = x_1^{a_1} x_2^{a_2}$$

$$a_1 + a_2 = \frac{4}{3} > 1$$

so this technology exhibits increasing returns-to-scale.

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# Returns-to-Scale

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But $MP_1 = \frac{2}{3} x_1^{-1/3} x_2^{2/3}$ diminishes as $x_1$ increases and

$MP_2 = \frac{2}{3} x_1^{2/3} x_2^{-1/3}$ diminishes as $x_1$ increases.

# Returns-to-Scale

- **So a technology can exhibit increasing returns-to-scale even if all of its marginal products are diminishing.  Why?**

# Returns-to-Scale

- A marginal product is the rate-of-change of output as **one** input level increases, holding all other input levels fixed.
- Marginal product diminishes because the other input levels are fixed, so the increasing input's units have each less and less of other inputs with which to work.

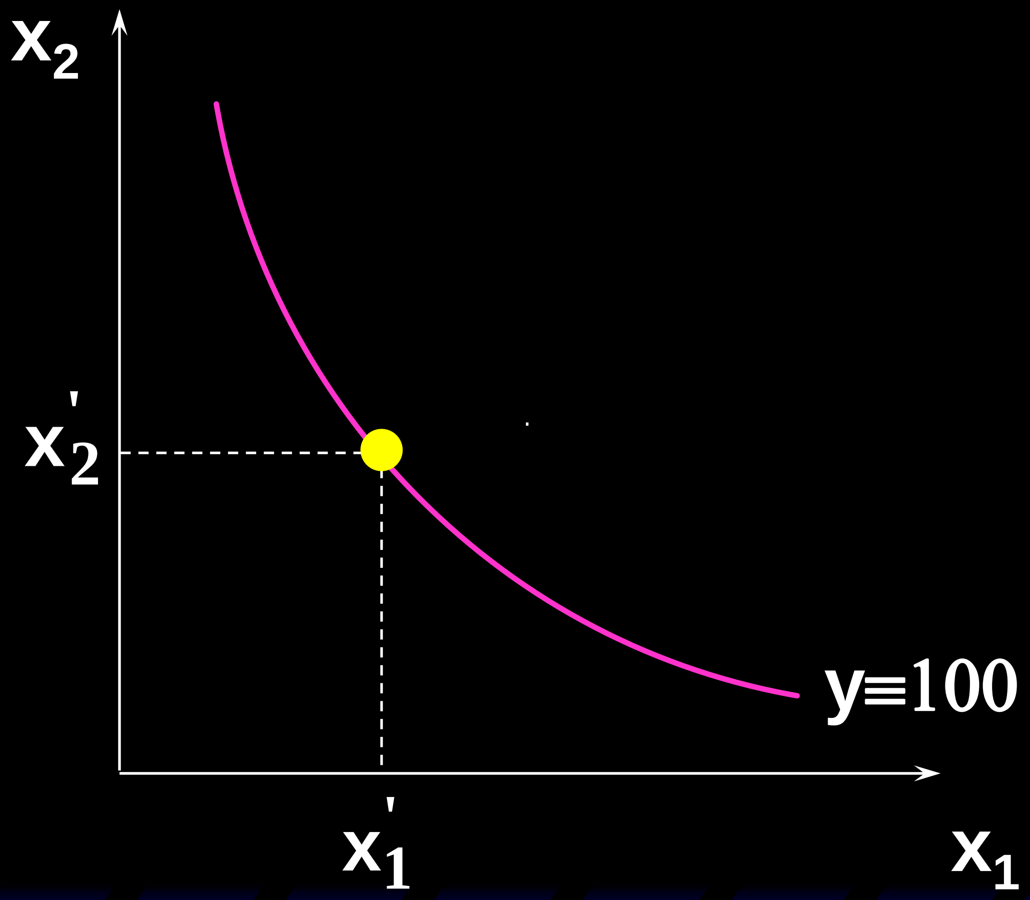
# Returns-to-Scale

- When all input levels are increased proportionately, there need be no diminution of marginal products since each input will always have the same amount of other inputs with which to work.  Input productivities need not fall and so returns-to-scale can be constant or increasing.

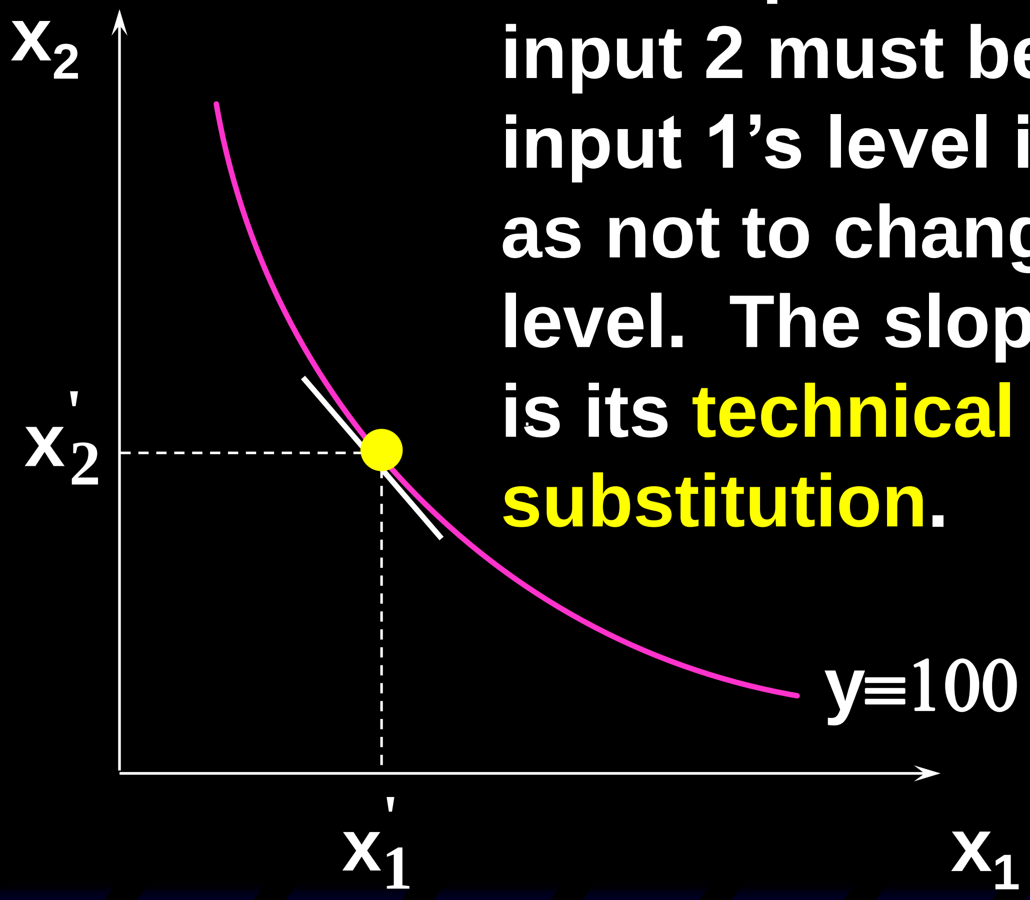
# Technical Rate-of-Substitution

- At what rate can a firm substitute one input for another without changing its output level?

# Technical Rate-of-Substitution



# Technical Rate-of-Substitution




**The slope is the rate at which input 2 must be increased as input 1's level is decreased so as not to change the output level. The slope of an isoquant is its technical rate-of-substitution.**

# Technical Rate-of-Substitution

- **How is a technical rate-of-substitution computed?**

# Technical Rate-of-Substitution

- How is a technical rate-of-substitution computed?
- The production function is $y = f(x_1, x_2)$.
- A small change $(dx_1, dx_2)$ in the input bundle causes a change to the output level of

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2.$$

# Technical Rate-of-Substitution

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2.$$

**But dy = 0 since there is to be no change to the output level, so the changes $dx_1$ and $dx_2$ to the input levels must satisfy**

$$0 = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2.$$

# Technical Rate-of-Substitution

$$0 = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2$$

rearranges to

$$\frac{\partial y}{\partial x_2} dx_2 = -\frac{\partial y}{\partial x_1} dx_1$$

so

$$\frac{dx_2}{dx_1} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2}.$$

# Technical Rate-of-Substitution

$$\frac{dx_2}{dx_1} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2}$$

is the rate at which input 2 must be increased as input 1 is decreased so as to keep the output level constant. It is the slope of the isoquant.

# Technical Rate-of-Substitution; A Cobb-Douglas Example

$$y = f(x_1, x_2) = x_1^a x_2^b$$

so $\dfrac{\partial y}{\partial x_1} = a x_1^{a-1} x_2^b$ and $\dfrac{\partial y}{\partial x_2} = b x_1^a x_2^{b-1}$.

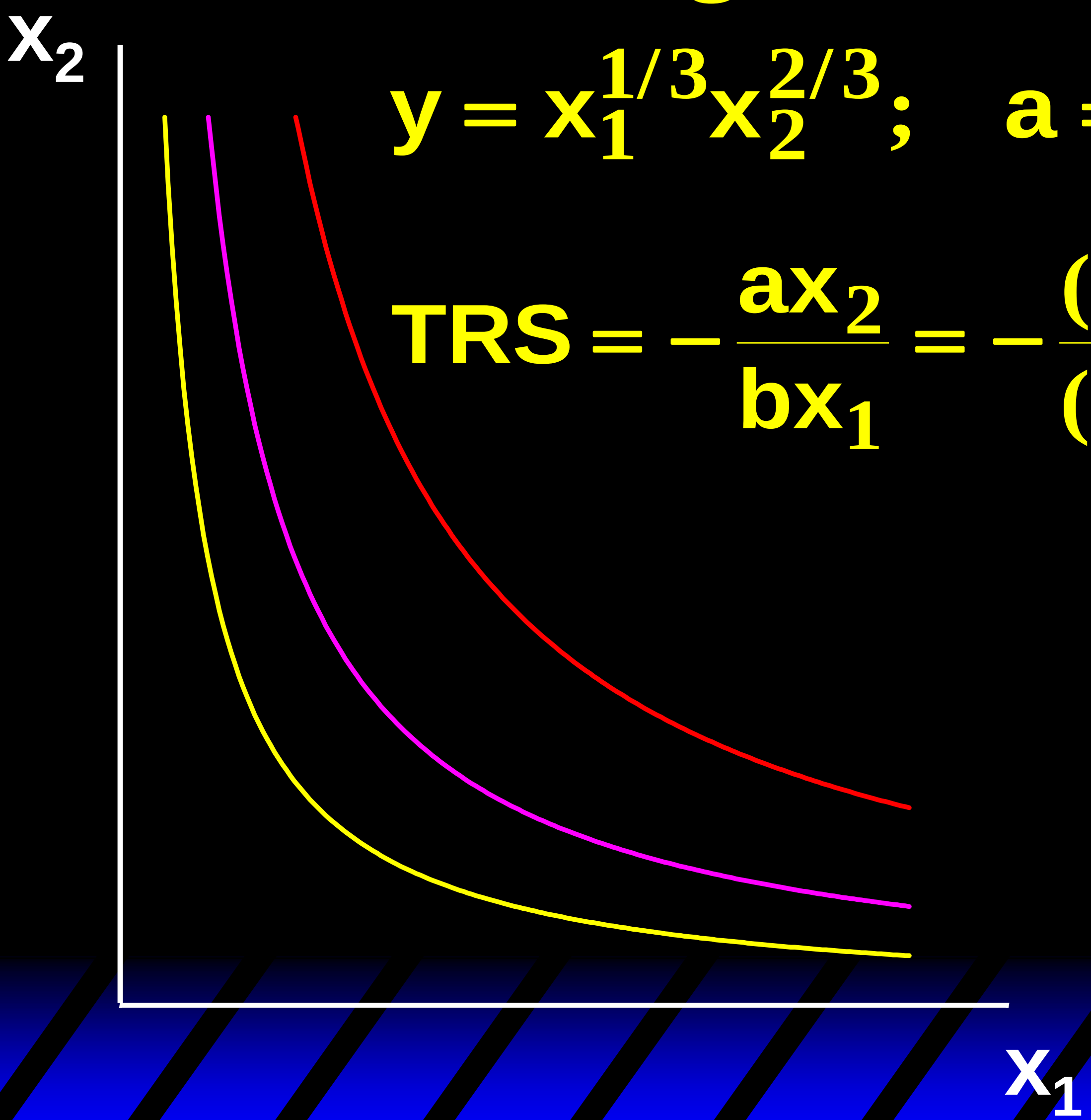
The technical rate-of-substitution is

$$\frac{dx_2}{dx_1} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2} = -\frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

# Technical Rate-of-Substitution; A Cobb-Douglas Example

$$y = x_1^{1/3} x_2^{2/3}; \quad a = \frac{1}{3} \text{ and } b = \frac{2}{3}$$

$$TRS = -\frac{ax_2}{bx_1} = -\frac{(1/3)x_2}{(2/3)x_1} = -\frac{x_2}{2x_1}$$



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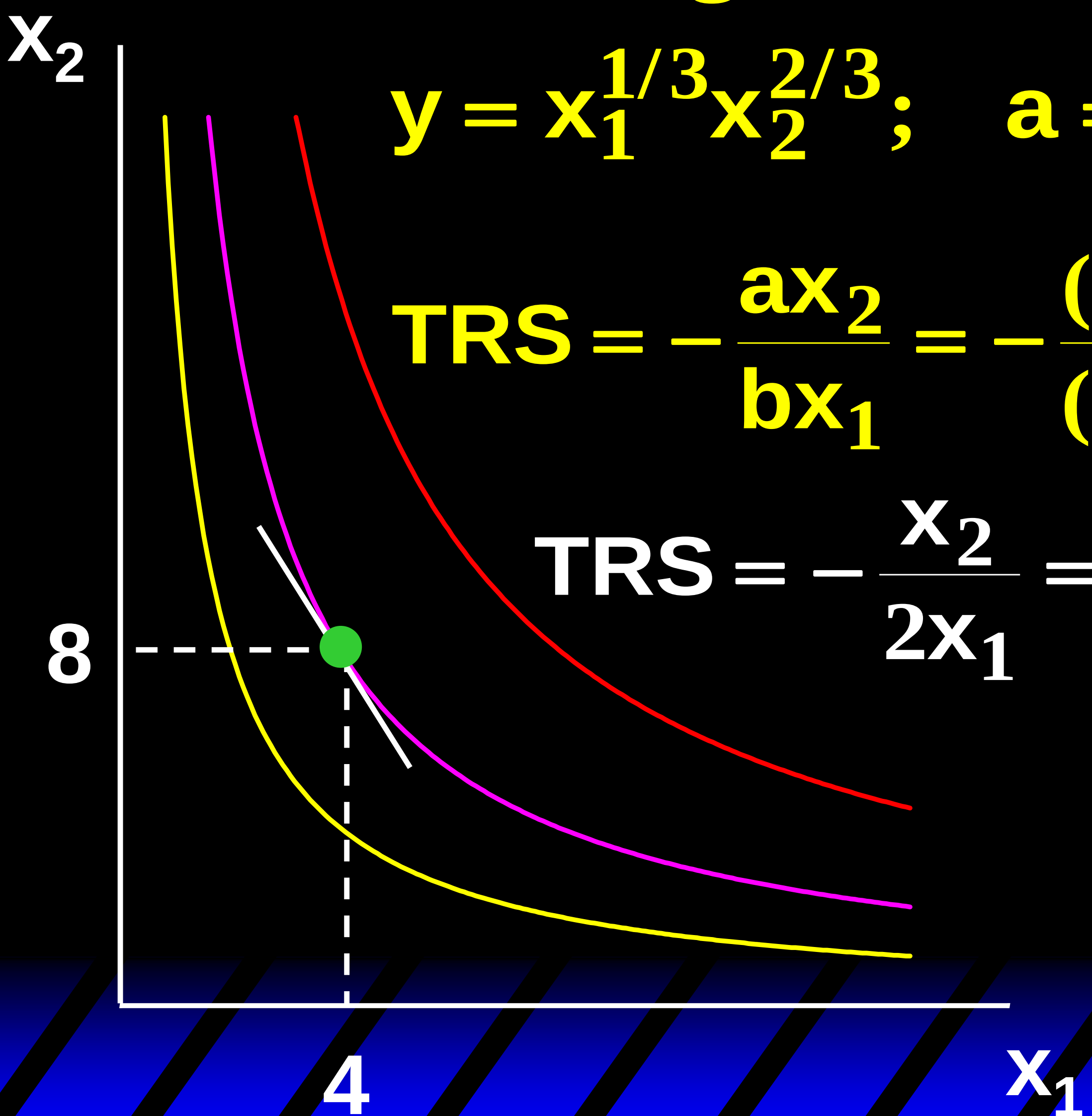
$$\text{TRS} = -\frac{x_2}{2x_1} = -\frac{8}{2 \times 4} = -1$$



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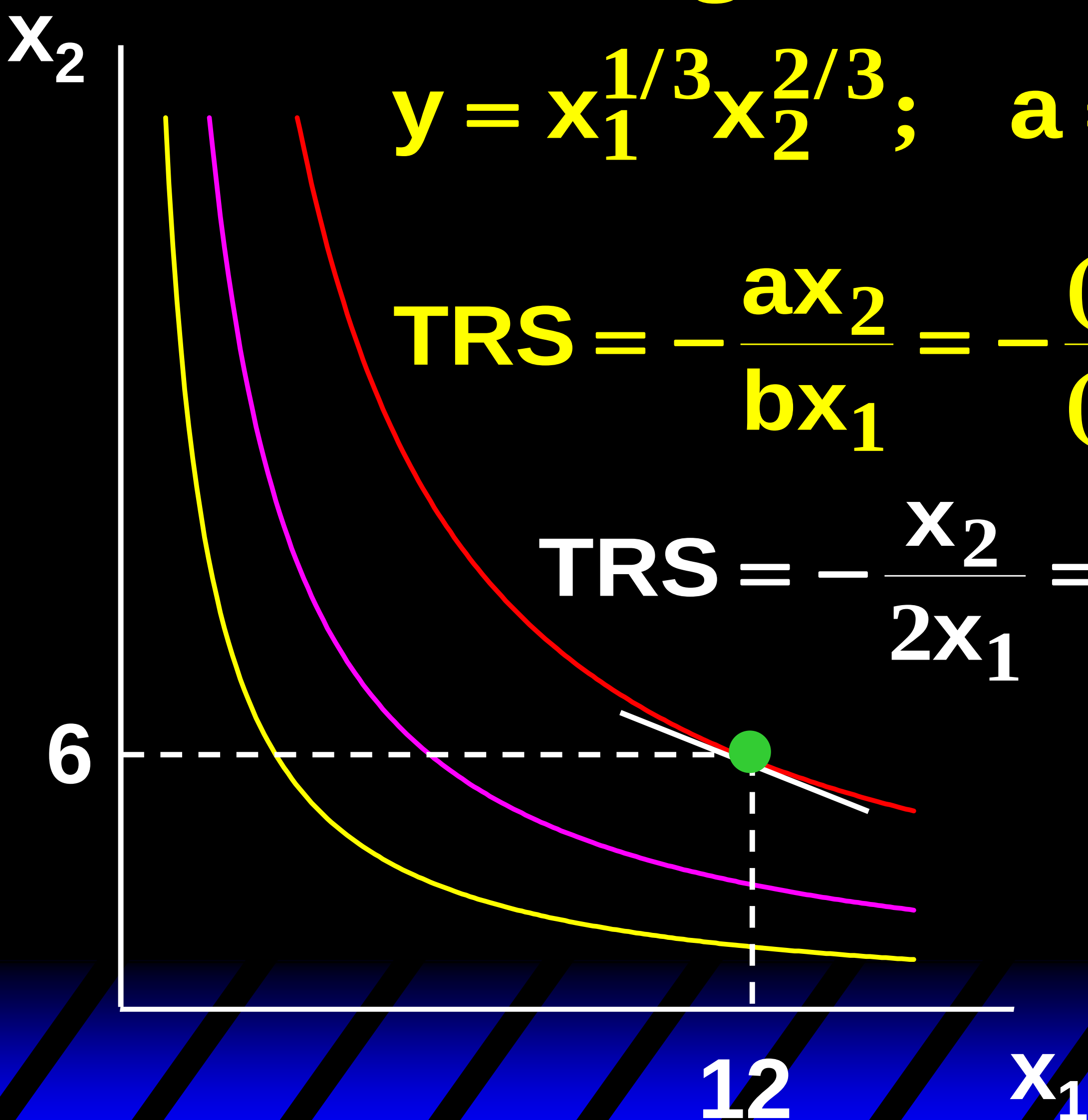
$$TRS = -\frac{x_2}{2x_1} = -\frac{6}{2 \times 12} = -\frac{1}{4}$$

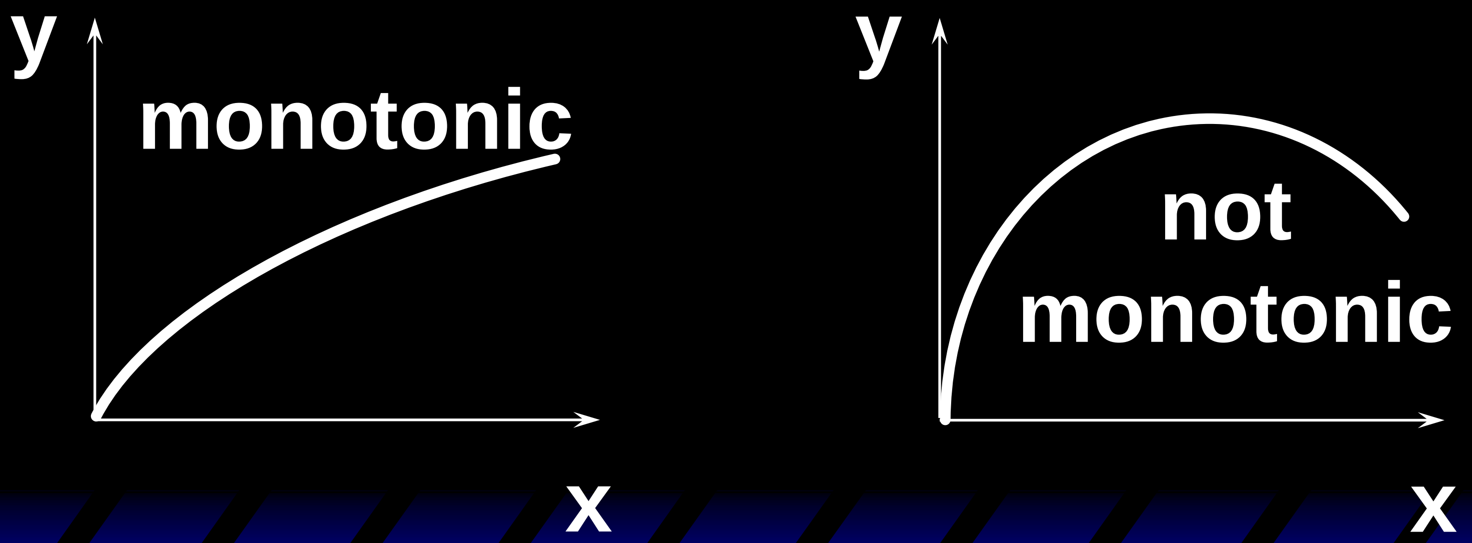


# Well-Behaved Technologies

- A **well-behaved** technology is
  - **monotonic**, and
  - **convex**.

# Well-Behaved Technologies - Monotonicity

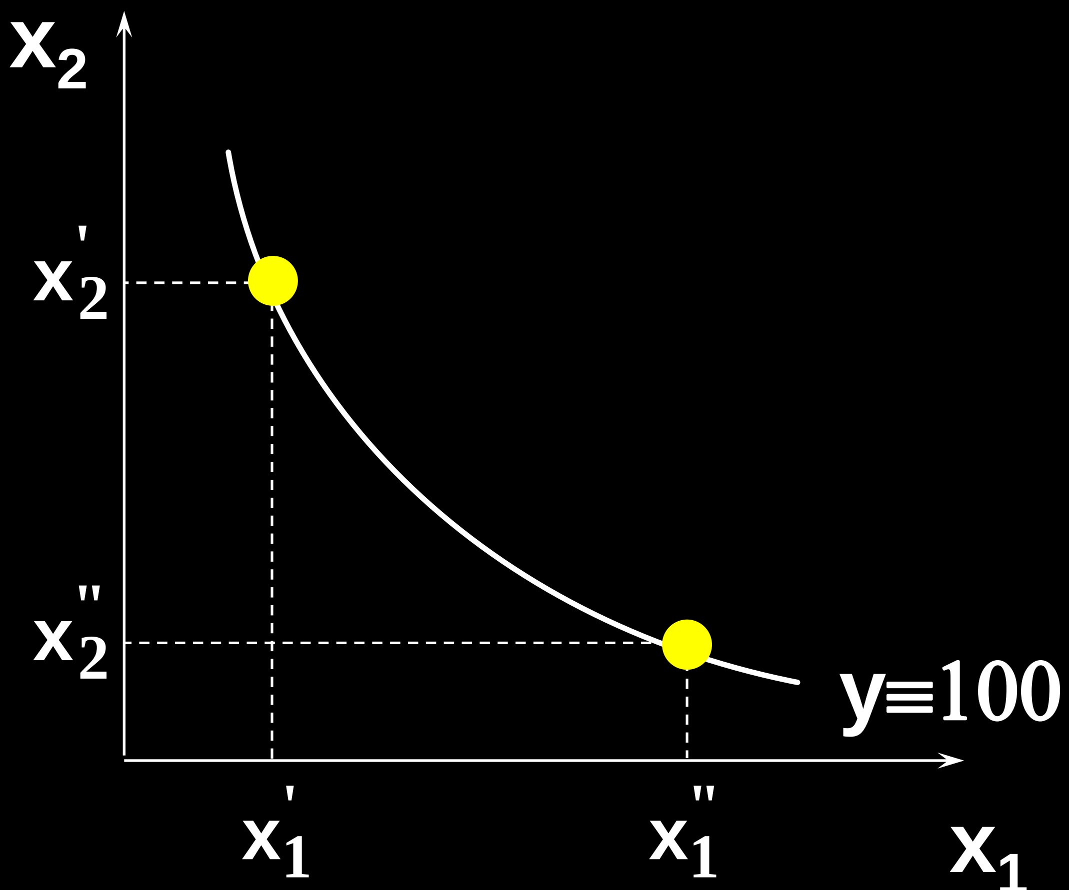
- **Monotonicity: More of any input generates more output.**



# Well-Behaved Technologies - Convexity

- **Convexity**: If the input bundles x' and x'' both provide y units of output then the mixture $tx' + (1-t)x''$ provides at least y units of output, for any $0 < t < 1$.

# Well-Behaved Technologies - Convexity



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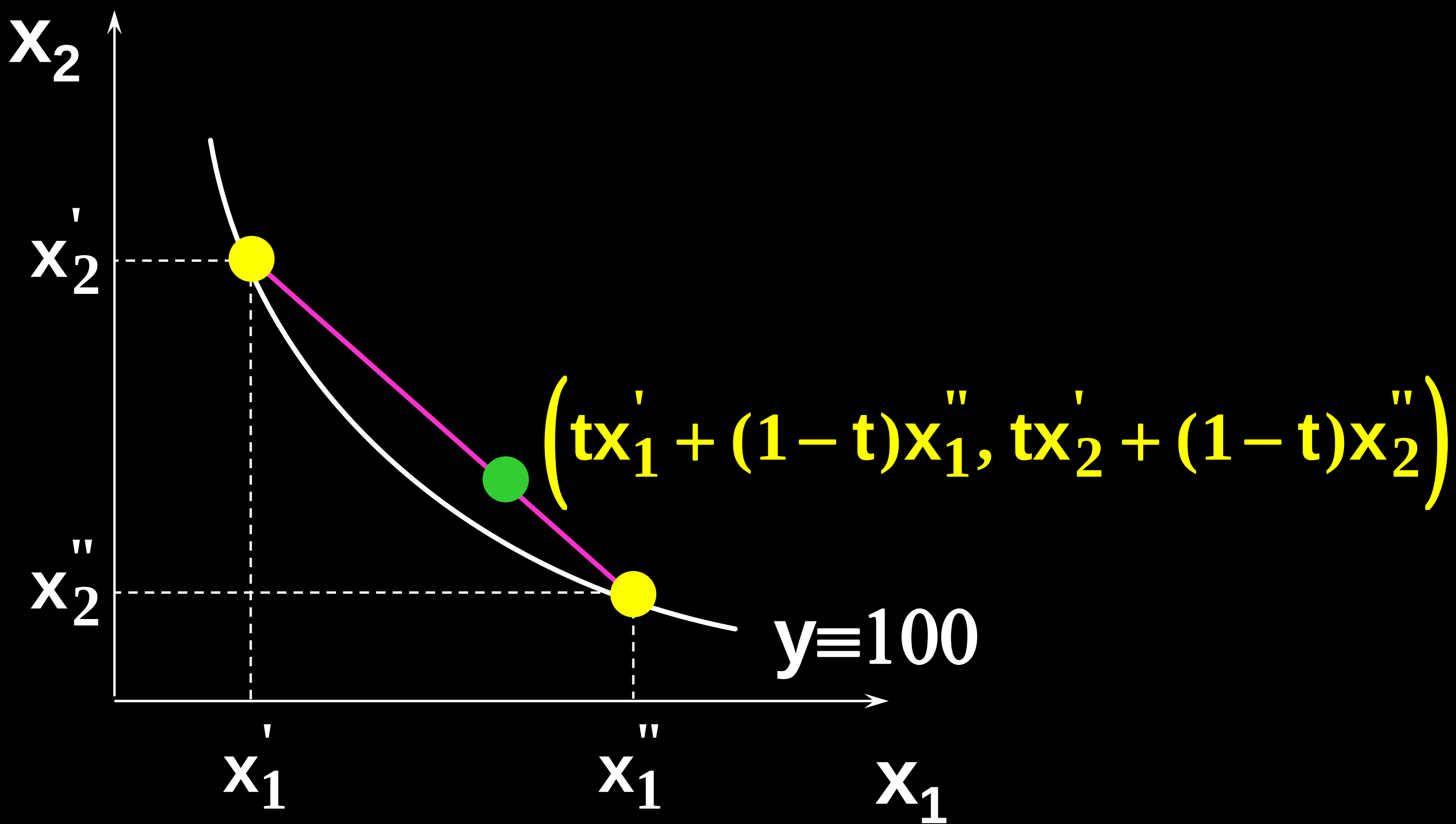

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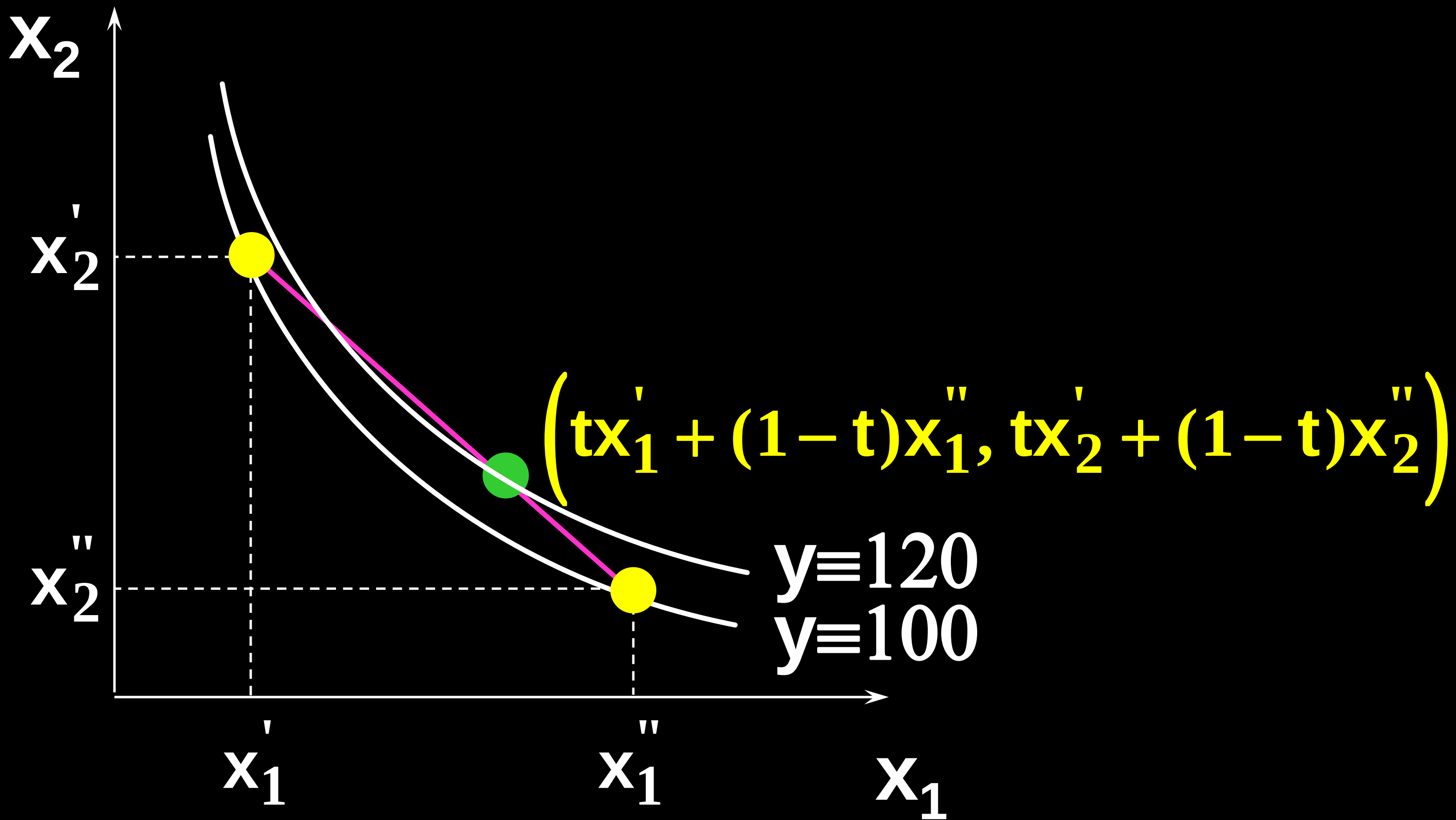

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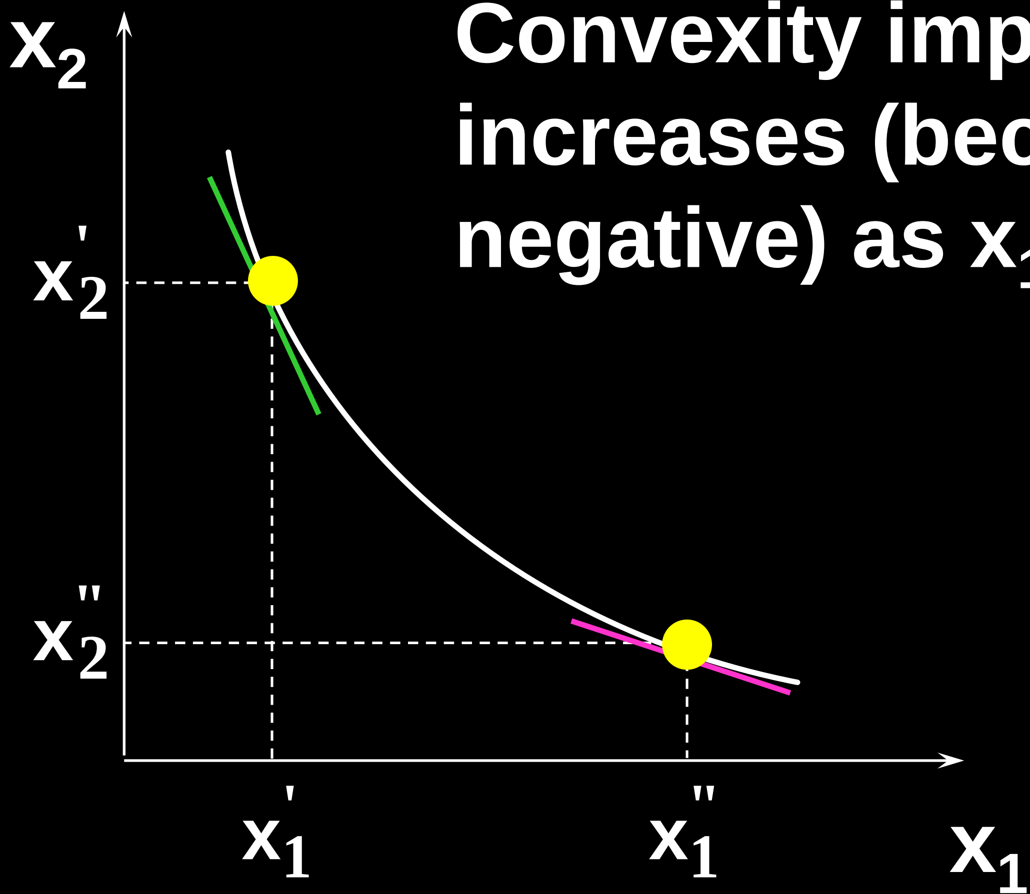
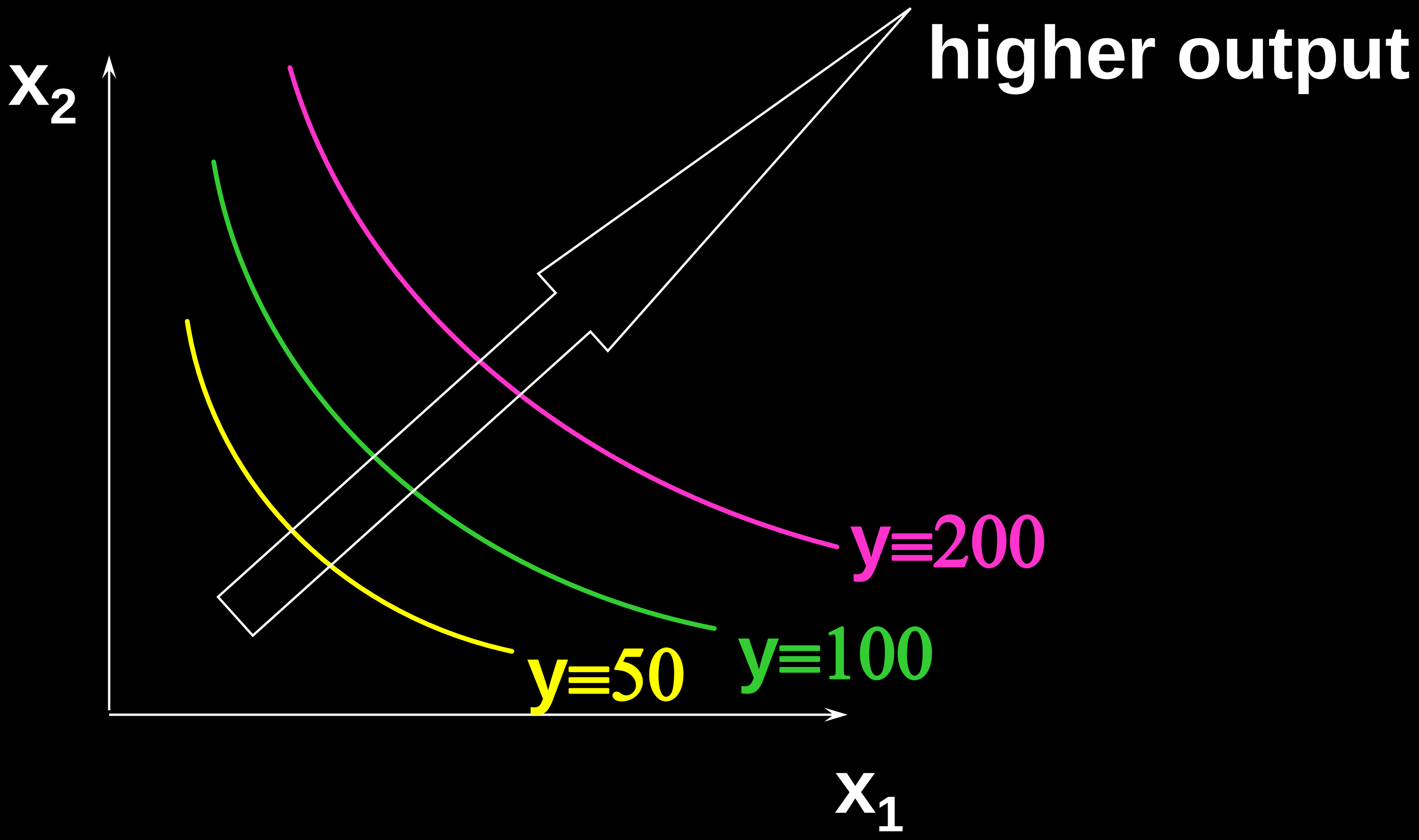



**Convexity implies that the TRS increases (becomes less negative) as $x_1$ increases.**

# Well-Behaved Technologies



# The Long-Run and the Short-Runs

- **The long-run is the circumstance in which a firm is unrestricted in its choice of all input levels.**
- **There are many possible short-runs.**
- **A short-run is a circumstance in which a firm is restricted in some way in its choice of at least one input level.**

# The Long-Run and the Short-Runs

- Examples of restrictions that place a firm into a short-run:
  - temporarily being unable to install, or remove, machinery
  - being required by law to meet affirmative action quotas
  - having to meet domestic content regulations.

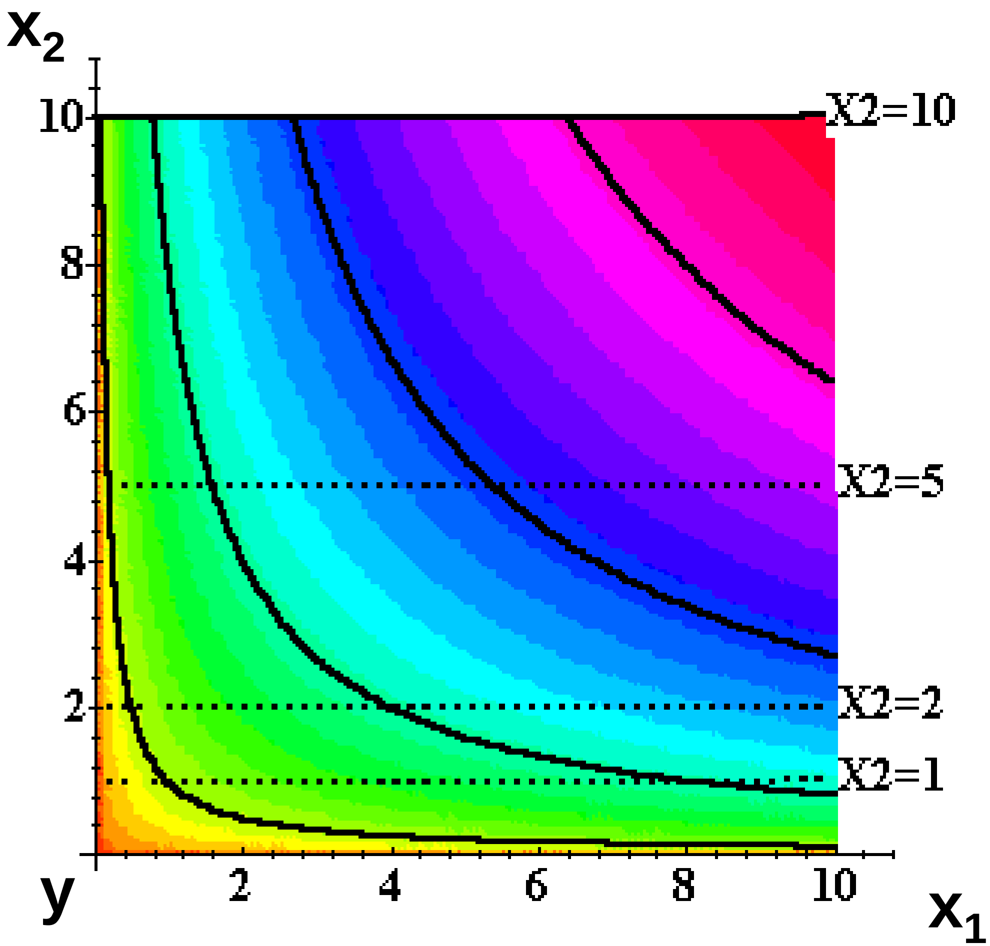
# The Long-Run and the Short-Runs

- **A useful way to think of the long-run is that the firm can choose as it pleases in which short-run circumstance to be.**

# The Long-Run and the Short-Runs

- **What do short-run restrictions imply for a firm's technology?**
- **Suppose the short-run restriction is fixing the level of input 2.**
- **Input 2 is thus a fixed input in the short-run. Input 1 remains variable.**

# The Long-Run and the Short-Runs



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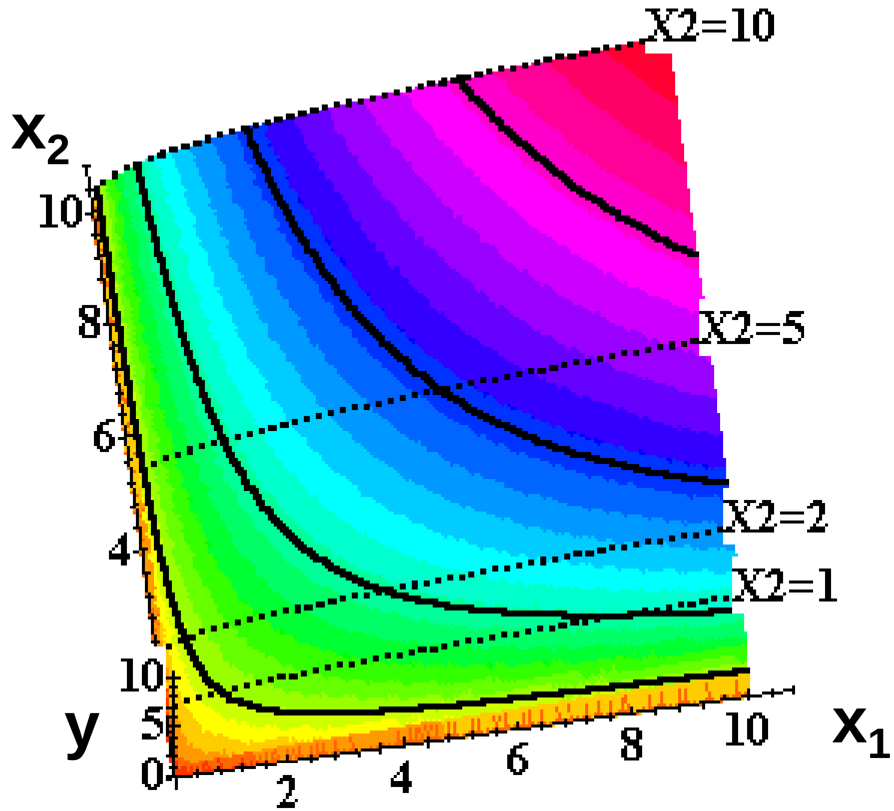

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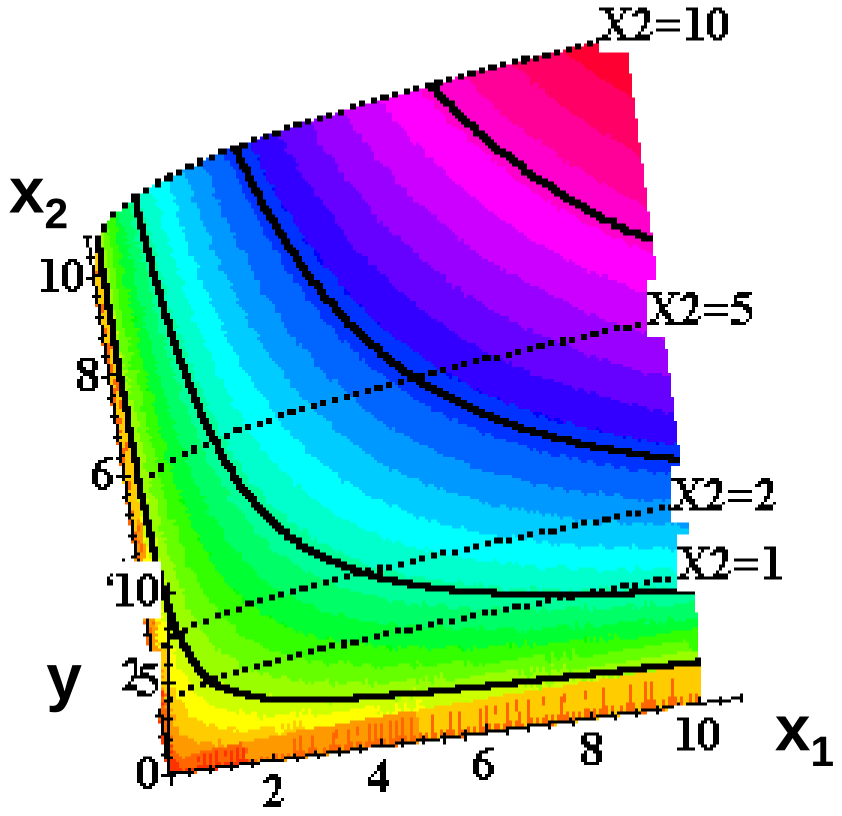

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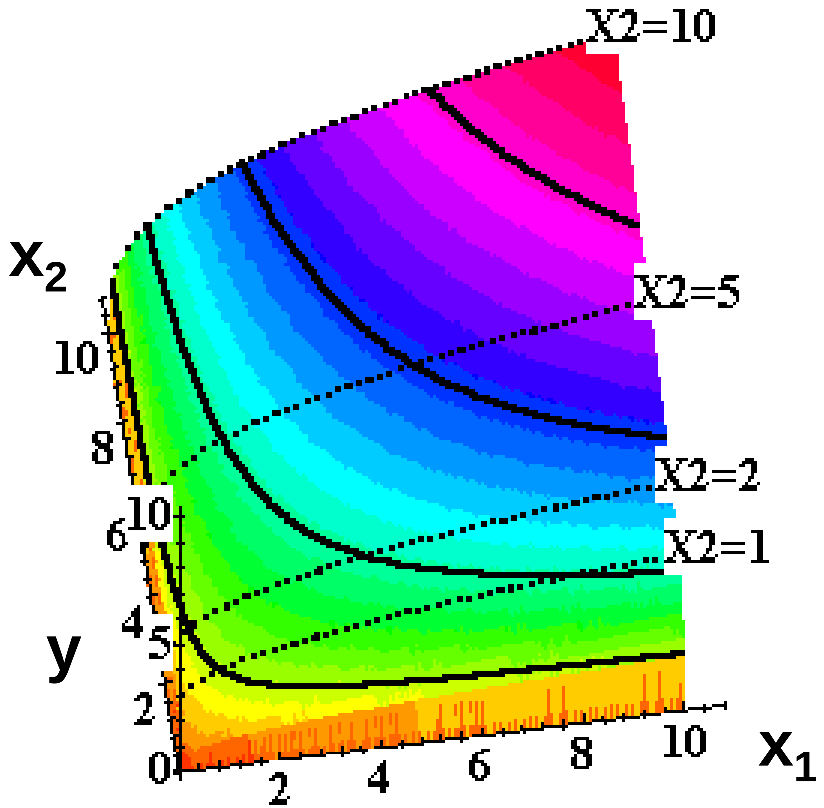

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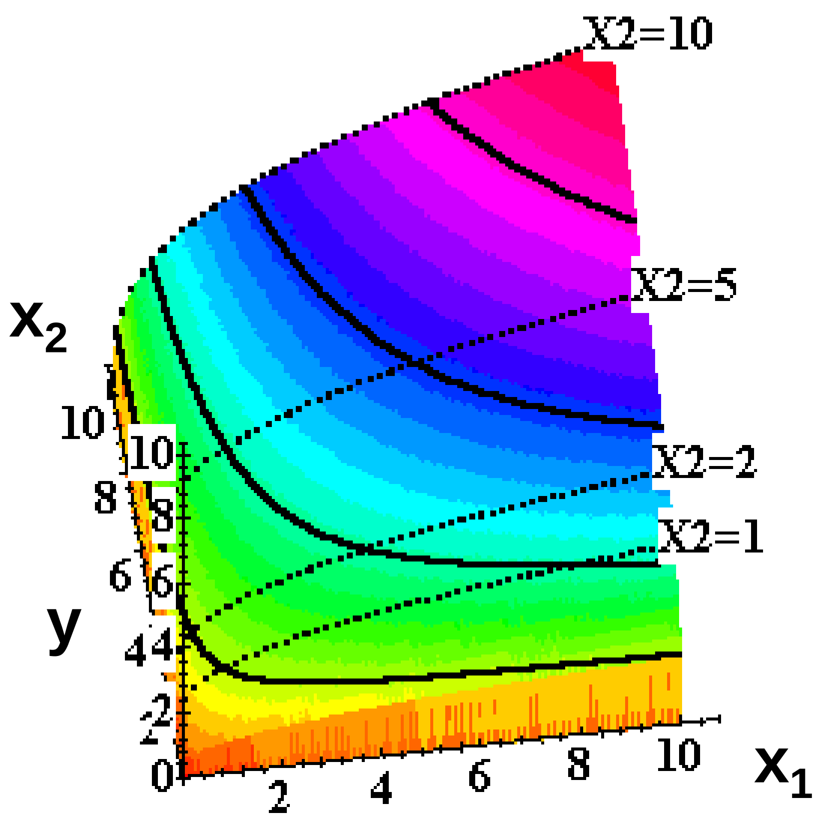

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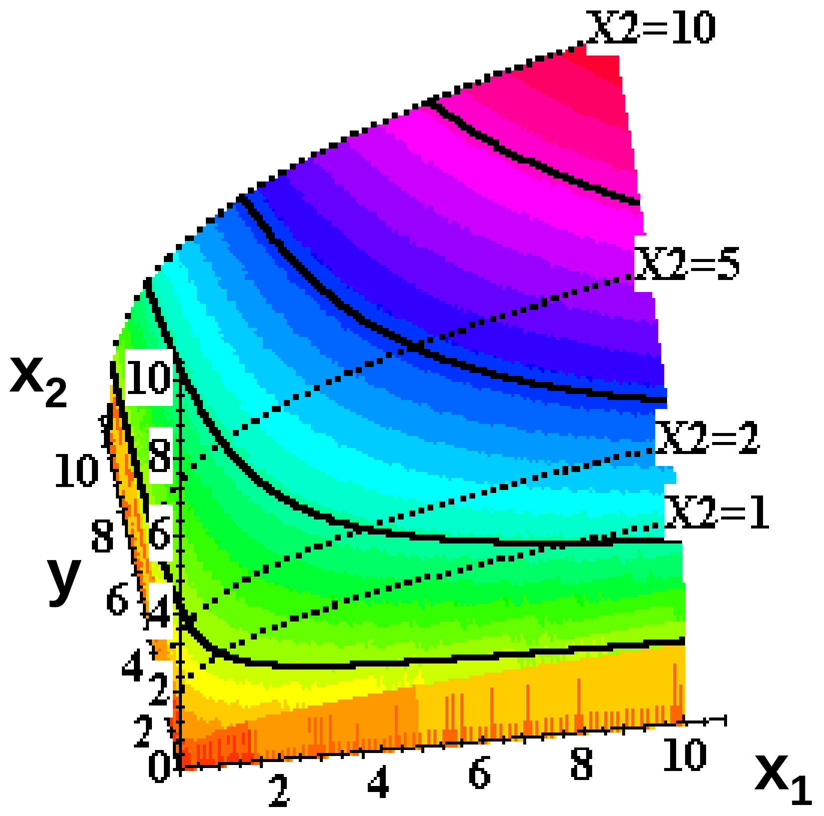

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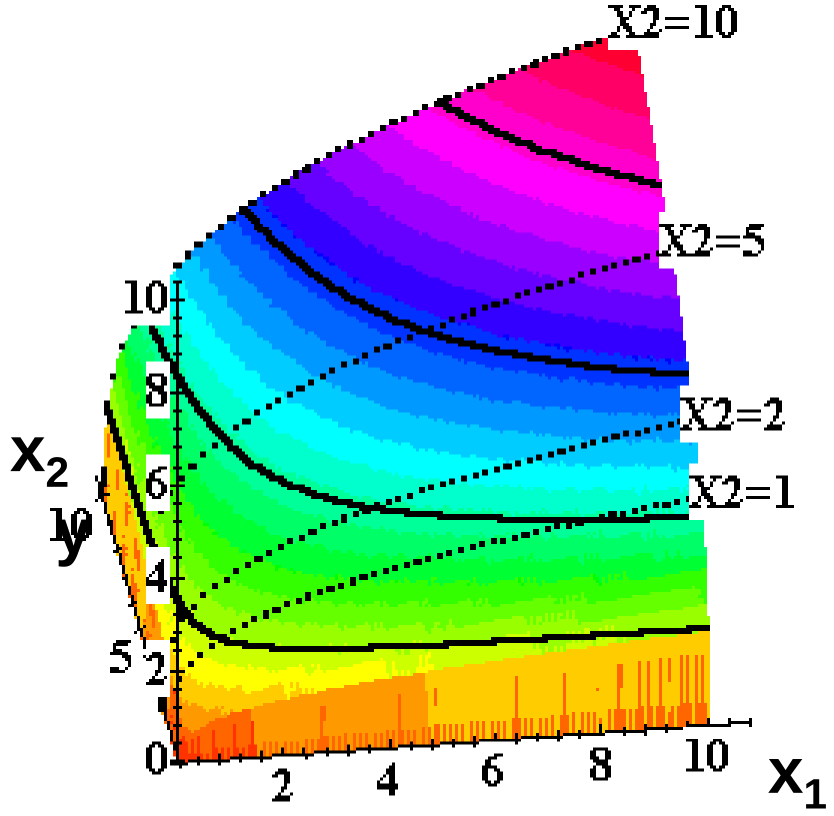

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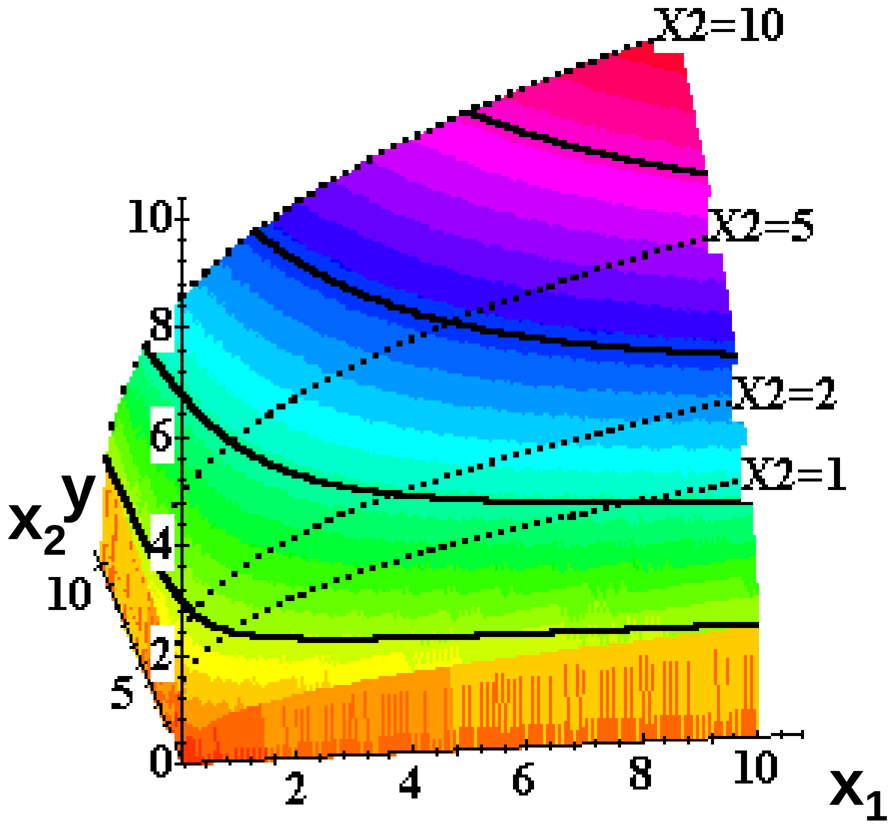

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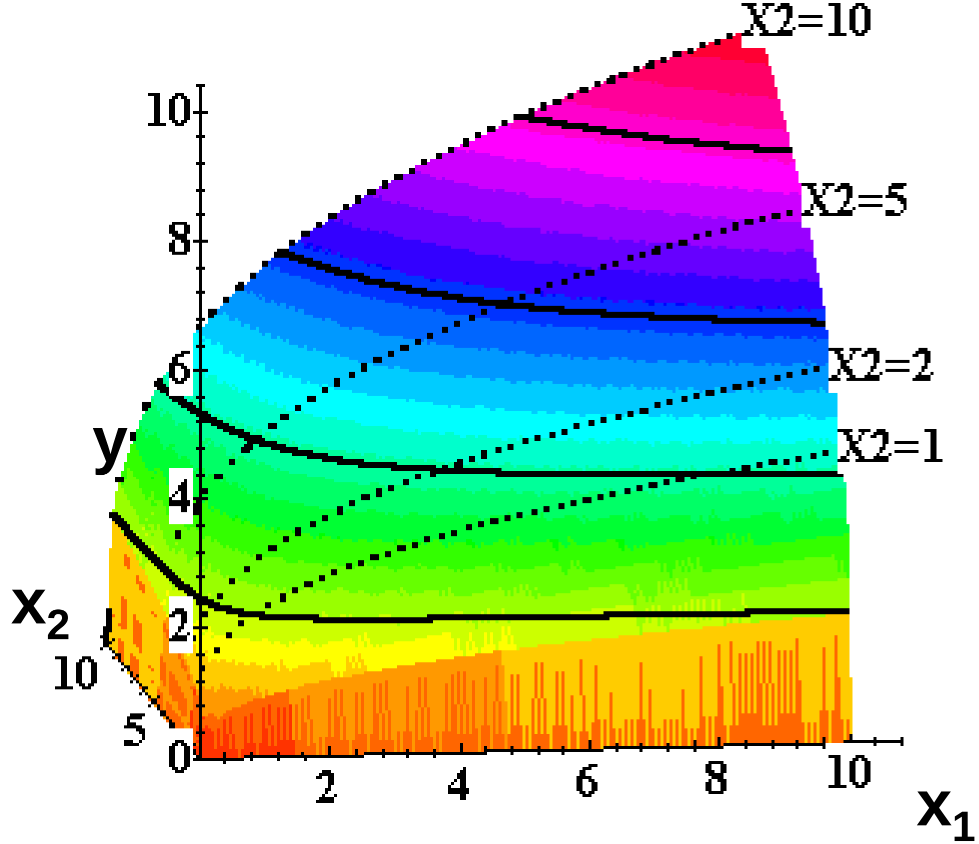

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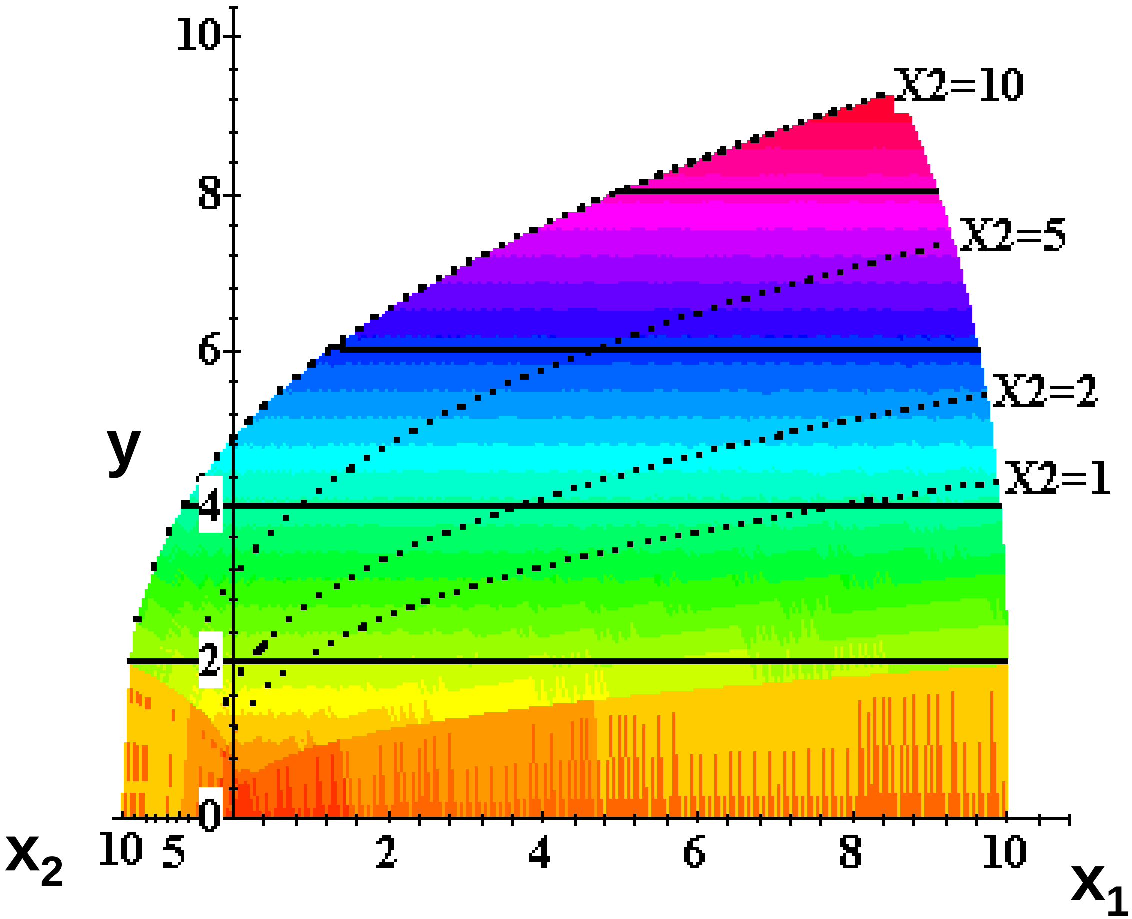

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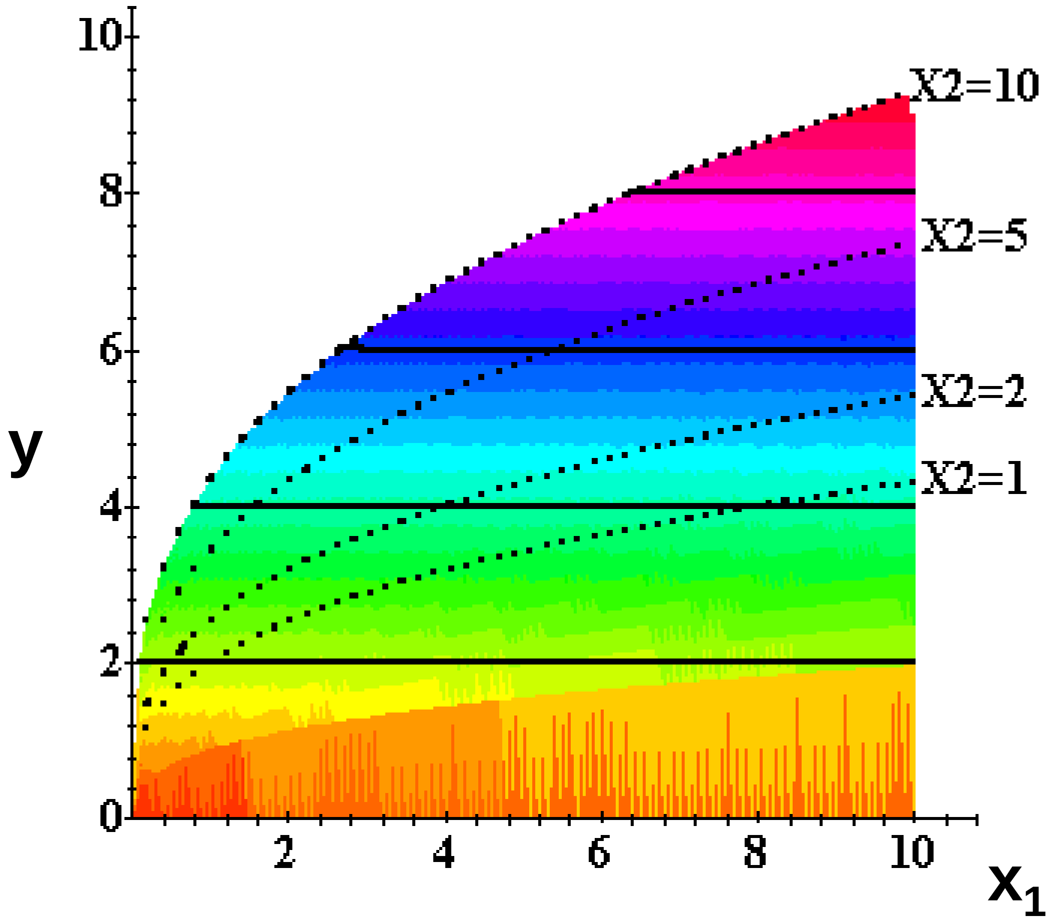

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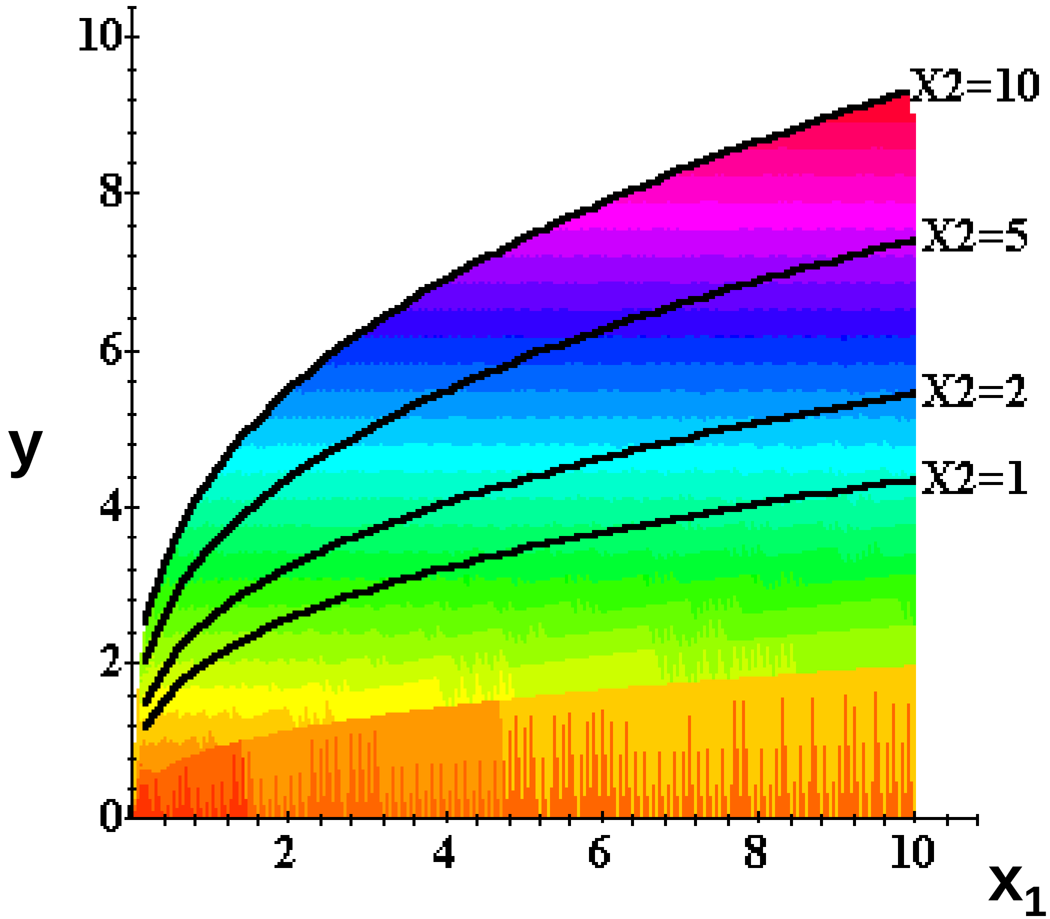

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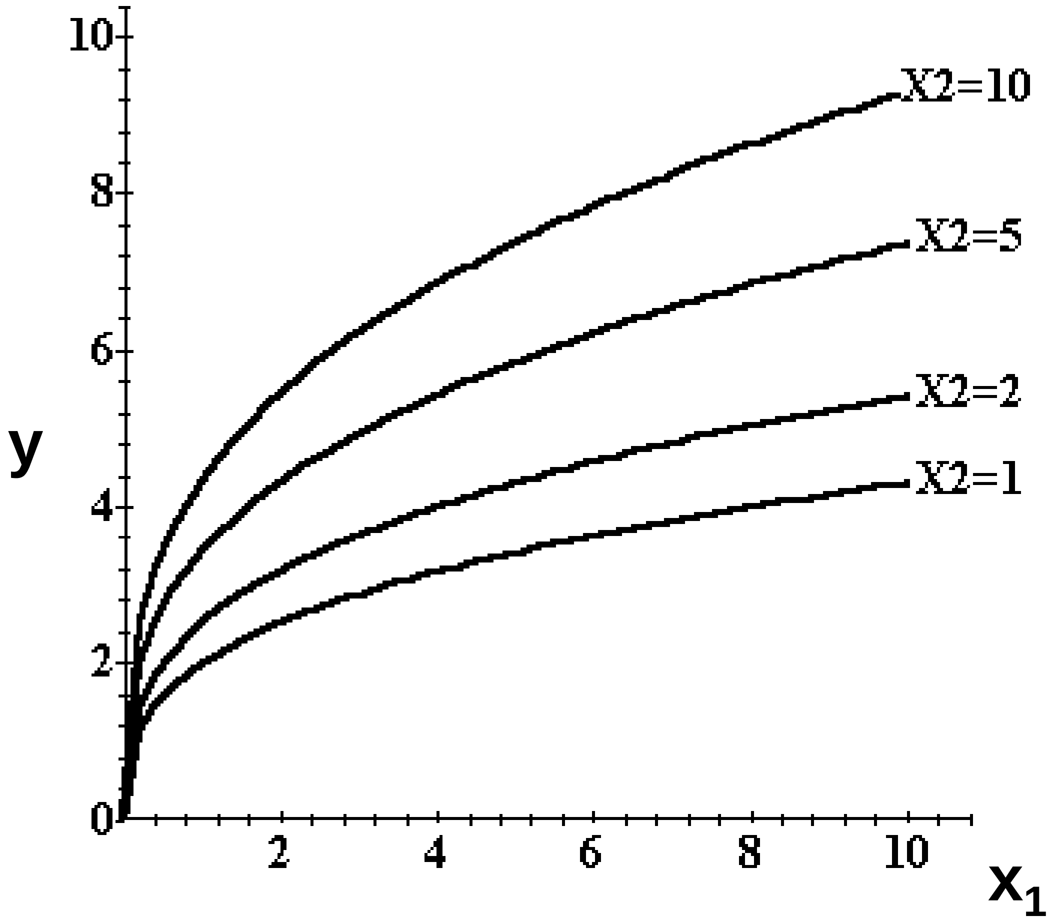



**Four short-run production functions.**

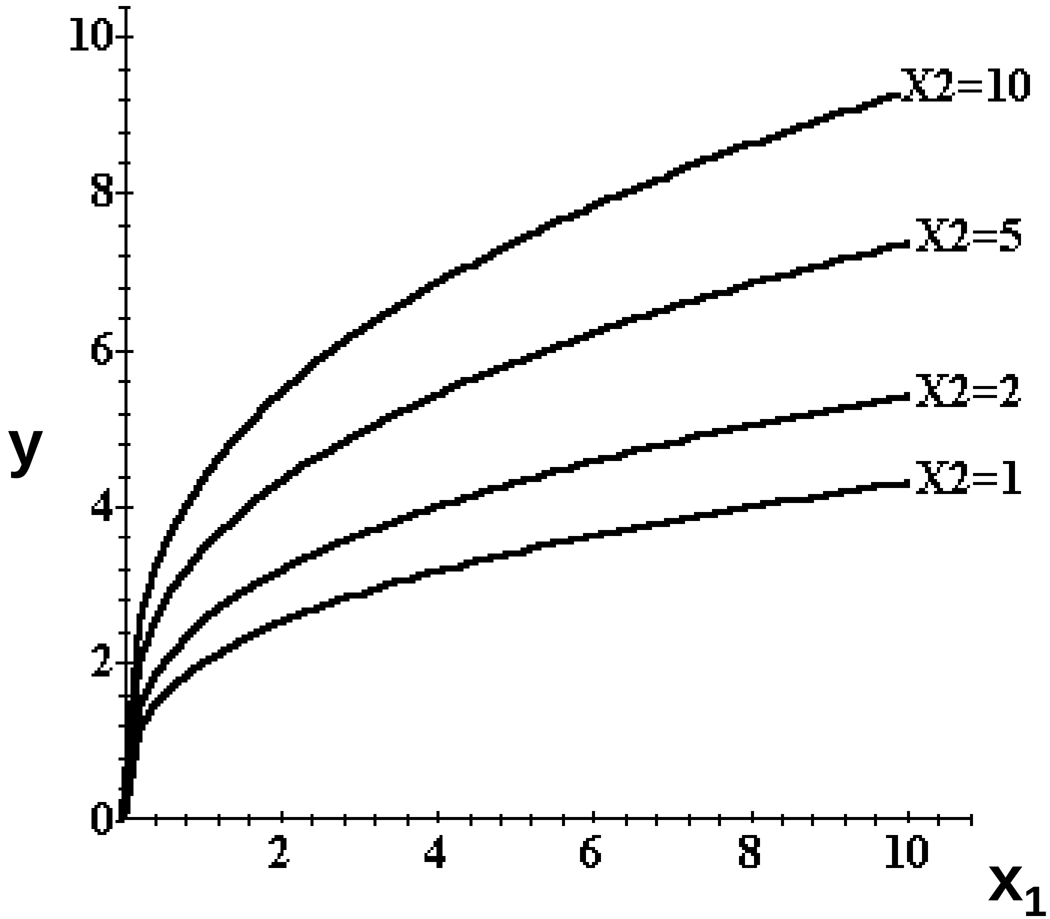
# The Long-Run and the Short-Runs

$y = x_1^{1/3} x_2^{1/3}$ is the long-run production function (both $x_1$ and $x_2$ are variable).

The short-run production function when $x_2 \equiv 1$ is $y = x_1^{1/3} 1^{1/3} = x_1^{1/3}$.

The short-run production function when $x_2 \equiv 10$ is $y = x_1^{1/3} 10^{1/3} = 2 \cdot 15 x_1^{1/3}$.

# The Long-Run and the Short-Runs




$y = x_1^{1/3} 10^{1/3}$

$y = x_1^{1/3} 5^{1/3}$

$y = x_1^{1/3} 2^{1/3}$

$y = x_1^{1/3} 1^{1/3}$

**Four short-run production functions.**