中级微观经济学

技术

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一、判断题

1. ✓

$f(x,y)=\min\{2x+y,\ x+2y\}$

$f(tx,ty)=\min\{2tx+ty,\ tx+2ty\}=t\min\{2x+y,\ x+2y\}=tf(x,y)$

具有规模报酬不变

2. ✓

$f(x,y)=\min\{12x,3y\}$　　$f(x,y)=f(x',y)$，令选择 $0<t<1$，$f(tx+(1-t)x',y)$.

二、选择题

1. C

B是拟凹函数 ×　A是 ×　D ×　选C

三、计算题

1. $y=x_1^{\frac14}x_2^{\frac14}$

$MP_1=\frac{dy}{dx_1}=\frac14 x_1^{-\frac34}x_2^{\frac14}$　　$\frac{\partial MP_1}{\partial x_1}=-\frac{3}{16}x_1^{-\frac74}x_2^{\frac14}<0$，递减

$MP_2=\frac{dy}{dx_2}=\frac14 x_1^{\frac14}x_2^{-\frac34}$　　$\frac{\partial MP_2}{\partial x_2}=-\frac{3}{16}x_1^{\frac14}x_2^{-\frac74}<0$，递减

边际替代率：$-\frac{dx_2}{dx_1}=\frac{MP_1}{MP_2}=\frac{x_1^{-\frac74}x_2^{\frac14}}{x_1^{\frac14}x_2^{-\frac74}}=\frac{x_2^2}{x_1^2}$

$x_1\uparrow$，边际替代率 $\downarrow$

$x_2\uparrow$，边际替代率 $\uparrow$

规模报酬：$f(x_1,x_2)=x_1^{\frac14}x_2^{\frac14}$

$f(kx_1,kx_2)=(kx_1)^{\frac14}(kx_2)^{\frac14}=k^{\frac12}f(x_1,x_2)<kf(x_1,x_2)$　　规模报酬是减少的

2. $y=(x_1^\rho+x_2^\rho)^{\frac1\rho}$

$MP_1=\frac{dy}{dx_1}=\frac1\rho(x_1^\rho+x_2^\rho)^{\frac1\rho-1}\cdot\rho x_1^{\rho-1}=(x_1^\rho+x_2^\rho)^{\frac1\rho-1}x_1^{\rho-1}$

$$\frac{\partial MP_1}{\partial x_1}=(x_1^\rho+x_2^\rho)^{\frac1\rho-1}(\rho-1)x_1^{\rho-2}+x_1^{\rho-1}(x_1^\rho+x_2^\rho)^{\frac1\rho-2}\left(\frac1\rho-1\right)\cdot\rho x_1^{\rho-1}$$

$$=(x_1^\rho+x_2^\rho)^{\frac1\rho-1}(\rho-1)x_1^{\rho-2}+x_1^{2\rho-2}(1-\rho)(x_1^\rho+x_2^\rho)^{\frac1\rho-2}$$

$$=(\rho-1)x_1^{\rho-2}\left[(x_1^\rho+x_2^\rho)^{\frac1\rho-1}-x_1^\rho(x_1^\rho+x_2^\rho)^{\frac1\rho-2}\right]=(\rho-1)x_1^{\rho-2}x_2^\rho(x_1^\rho+x_2^\rho)^{\frac1\rho-2}$$

$$\begin{cases} 当\ \rho \geq 1 时, \dfrac{\partial MP_1}{\partial x_1} \geq 0 \\ 当\ \rho < 1 时, \dfrac{\partial MP_1}{\partial x_1} < 0 \end{cases}$$

同理 $\dfrac{dy}{dx_2} = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}-1} x_2^{\rho-1}$

$$\frac{\partial MP_2}{\partial x_2} = (\rho-1)x_2^{\rho-2}(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-1} + x_2^{\rho-1}\left(\frac{1}{\rho}-1\right)(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-2}\rho x_2^{\rho-1}$$

$$= (\rho-1)x_2^{\rho-2}(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-1} + (1-\rho)x_2^{2\rho-2}(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-2}$$

$$= (\rho-1)x_2^{\rho-2}\left[(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-1} - x_2^\rho(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-2}\right]$$

$$= (\rho-1)x_2^{\rho-2}x_1^\rho(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-2}$$

$$当\begin{cases} \rho \geq 1 时, \dfrac{\partial MP_2}{\partial x_2} \geq 0 \\ \rho < 1 时, \dfrac{\partial MP_2}{\partial x_2} < 0 \end{cases}$$

边际替代率 $-\dfrac{dx_2}{dx_1} = \dfrac{MP_1}{MP_2} = \dfrac{(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-1}x_1^{\rho-1}}{(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}-1}x_2^{\rho-1}} = \left(\dfrac{x_1}{x_2}\right)^{\rho-1}$

$$\begin{cases} 当\ \rho > 1 时, x_1 \uparrow, 替代率 \uparrow \\ 当\ \rho = 1 时, 替代率不变 \\ 当\ \rho < 1 时, x_1 \uparrow, 替代率 \downarrow \end{cases}$$

规模报酬: $f(x_1, x_2) = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}}$

$$f(kx_1, kx_2) = (k^\rho x_1^\rho + x_2^\rho k^\rho)^{\frac{1}{\rho}} = k^{\rho\cdot\frac{1}{\rho}}(x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} = k(x_1^\rho+x_2^\rho)^{\frac{1}{\rho}} = kf(x_1,x_2)$$

∴ 生产函数规模报酬不变.