Chapter Five

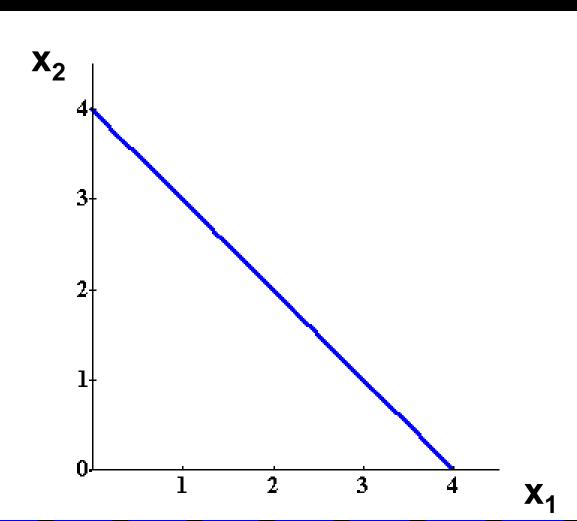
Choice

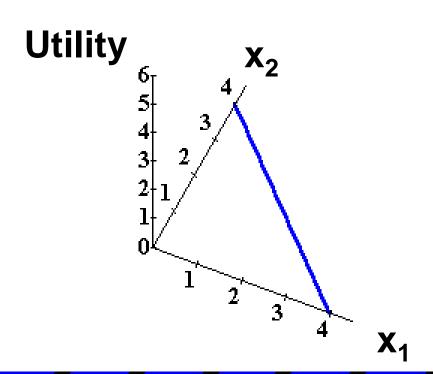
Economic Rationality

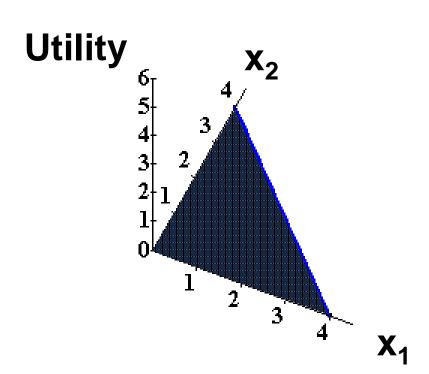
The principal behavioral postulate is that a decisionmaker chooses its most preferred alternative from those available to it.

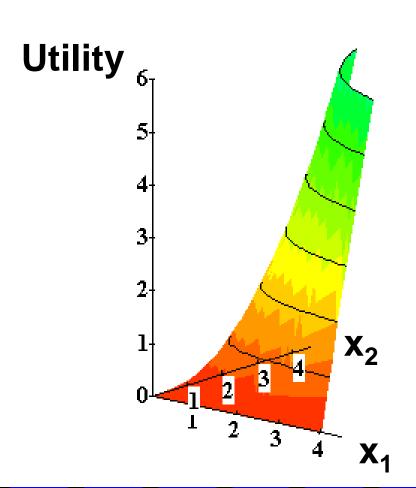
The available choices constitute the choice set.

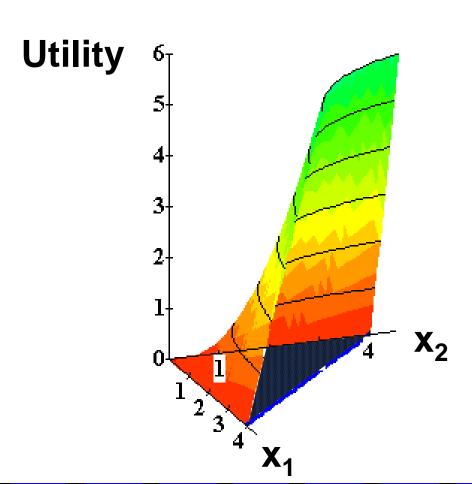
How is the most preferred bundle in the choice set located?

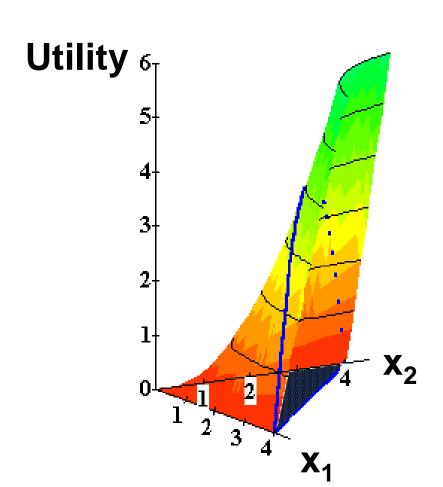


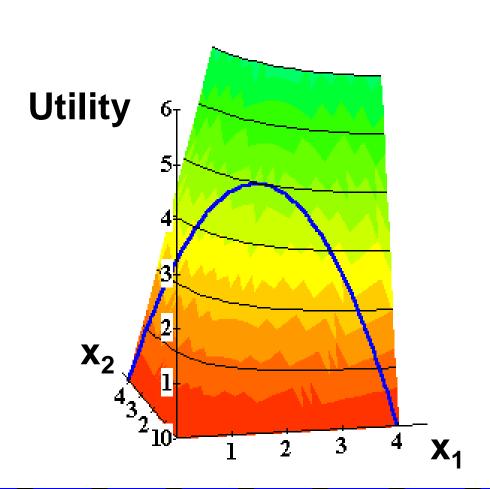


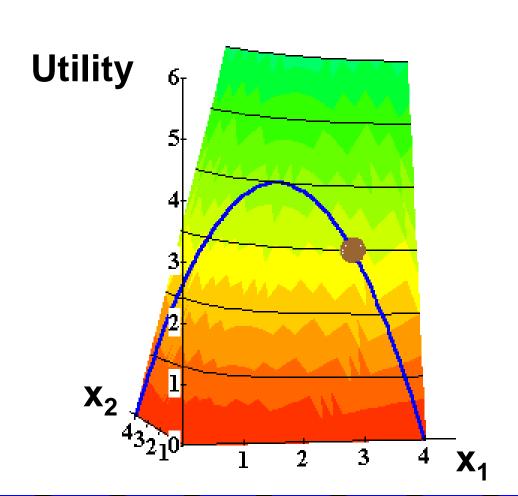


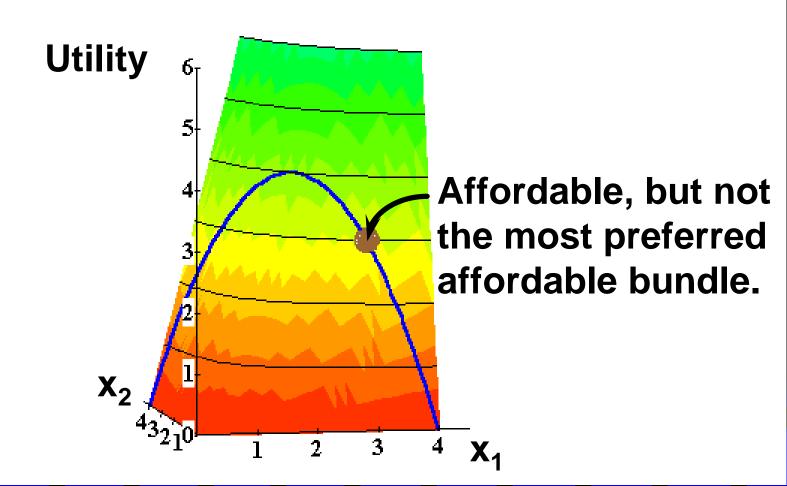


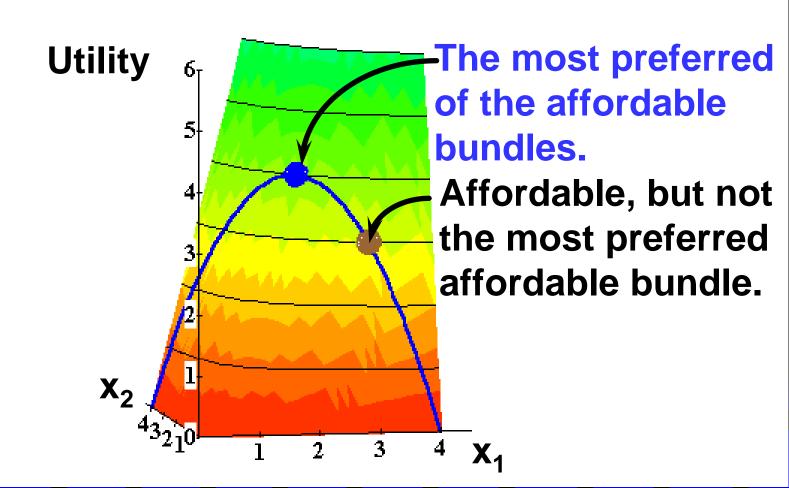


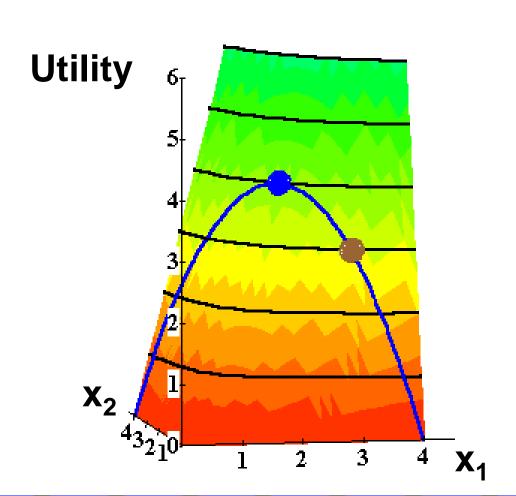


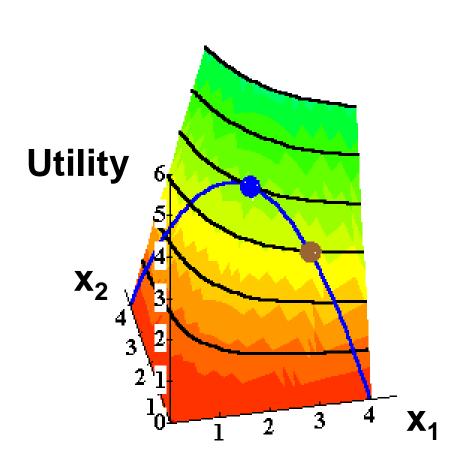


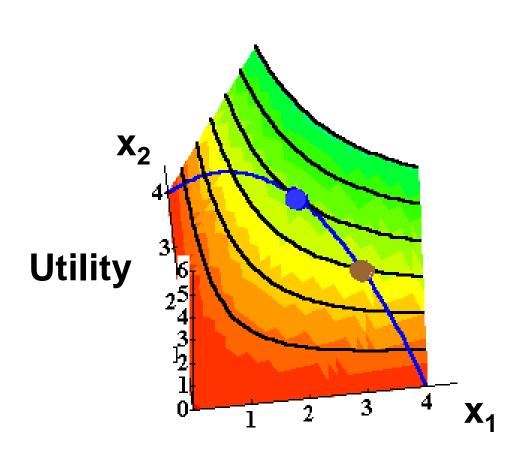


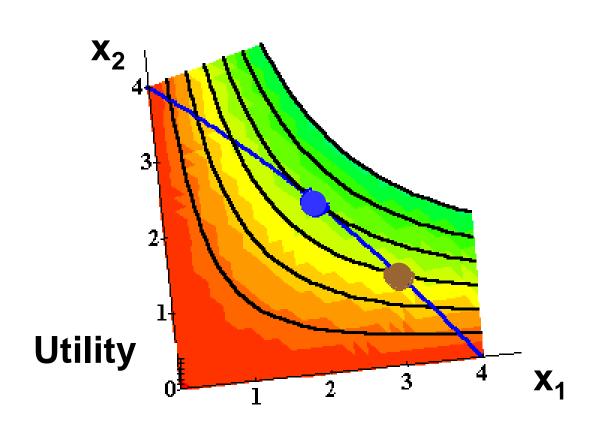


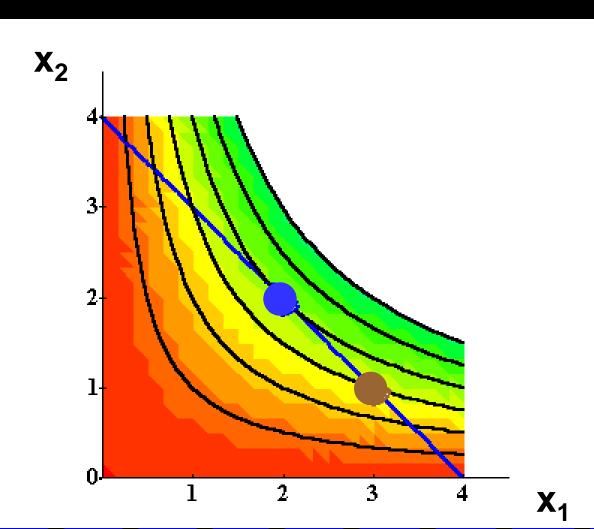


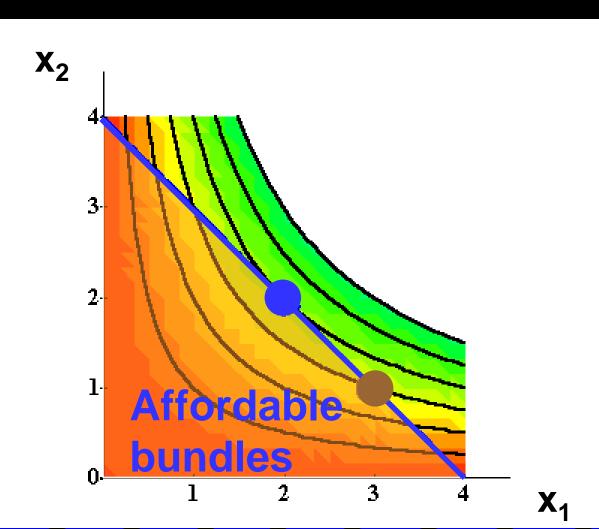


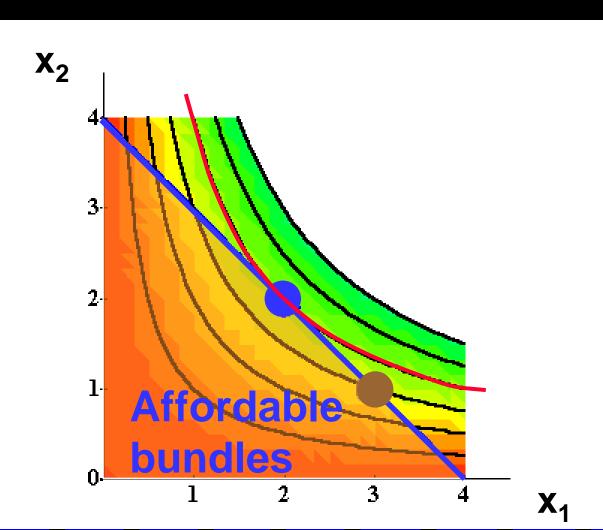


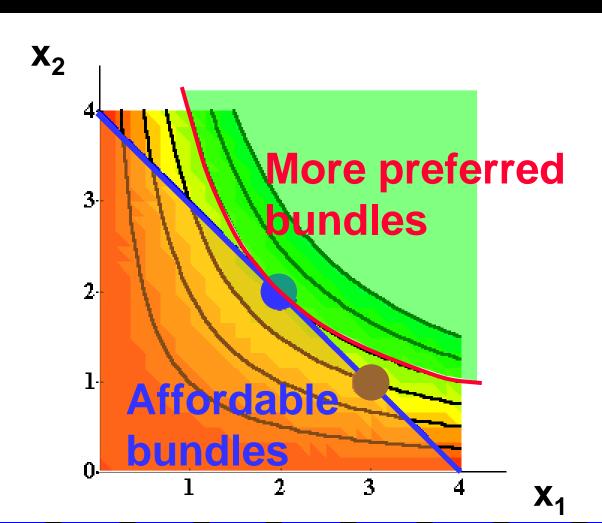




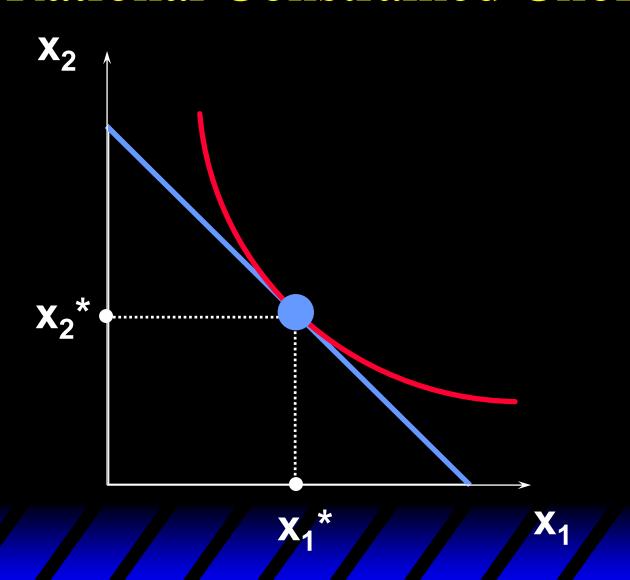


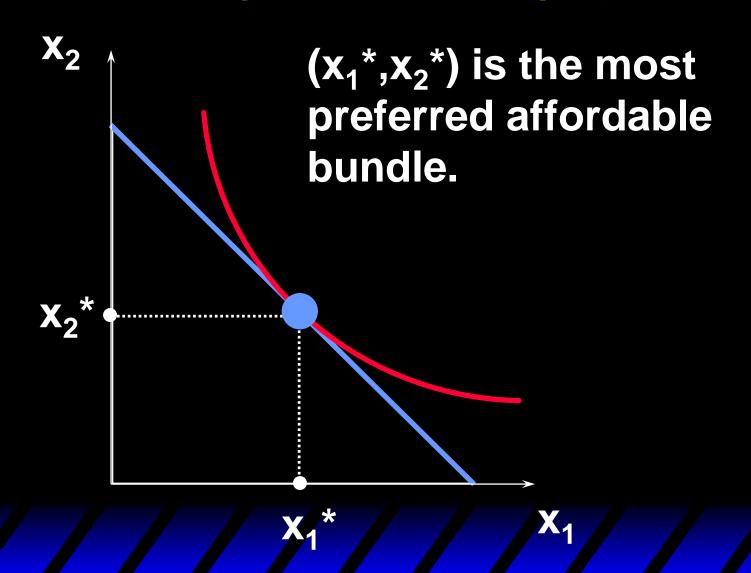








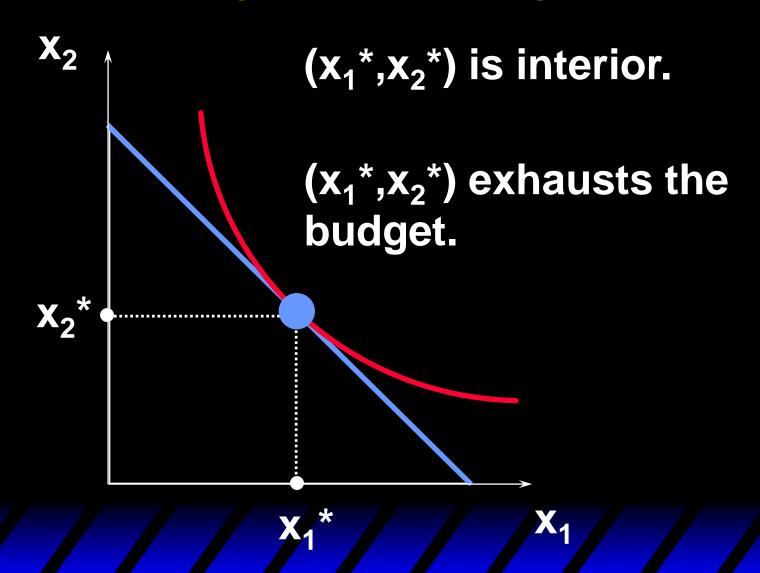


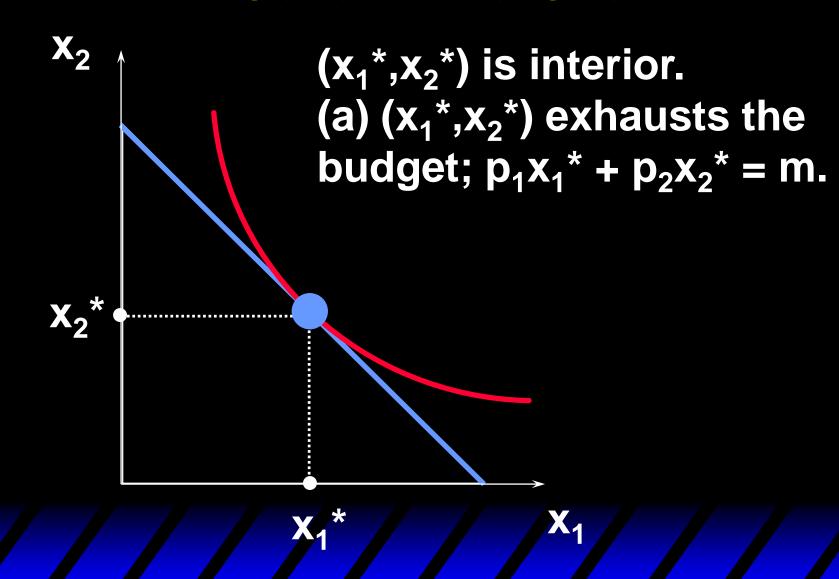


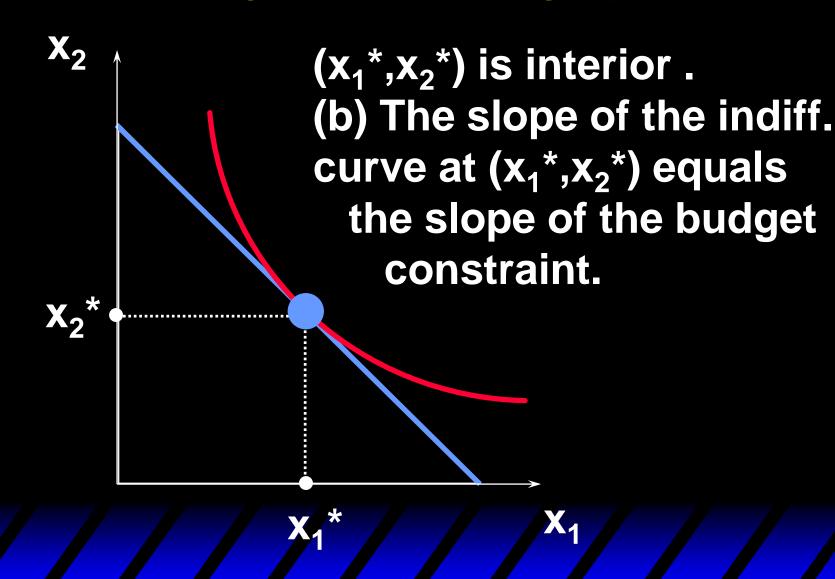
The most preferred affordable bundle is called the consumer's ORDINARY DEMAND at the given prices and budget.

Ordinary demands will be denoted by $x_1^*(p_1,p_2,m)$ and $x_2^*(p_1,p_2,m)$.

When $x_1^* > 0$ and $x_2^* > 0$ the demanded bundle is INTERIOR. If buying (x_1^*, x_2^*) costs \$m then the budget is exhausted.







- (x₁*,x₂*) satisfies two conditions:
 - (a) the budget is exhausted; $p_1x_1^* + p_2x_2^* = m$
- (b) the slope of the budget constraint, $-p_1/p_2$, and the slope of the indifference curve containing (x_1^*,x_2^*) are equal at (x_1^*,x_2^*) .

Computing Ordinary Demands

How can this information be used to locate (x_1^*,x_2^*) for given p_1 , p_2 and m?

Suppose that the consumer has Cobb-Douglas preferences.

$$U(\mathbf{x}_1, \mathbf{x}_2) = \mathbf{x}_1^{\mathbf{a}} \mathbf{x}_2^{\mathbf{b}}$$

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$$\mathbf{U}(\mathbf{x}_1,\mathbf{x}_2) = \mathbf{x}_1^{\mathbf{a}}\mathbf{x}_2^{\mathbf{b}}$$

Then
$$MU_1 = \frac{\partial U}{\partial x_1} = ax_1^{a-1}x_2^b$$

$$\mathbf{MU}_2 = \frac{\partial \mathbf{U}}{\partial \mathbf{x}_2} = \mathbf{b} \mathbf{x}_1^{\mathbf{a}} \mathbf{x}_2^{\mathbf{b} - 1}$$

So the MRS is

$$\text{MRS} = \frac{\text{d}x_2}{\text{d}x_1} = -\frac{\partial \text{U}/\partial x_1}{\partial \text{U}/\partial x_2} = -\frac{\text{a}x_1^{a-1}x_2^b}{\text{b}x_1^ax_2^{b-1}} = -\frac{\text{a}x_2}{\text{b}x_1}.$$

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At
$$(x_1^*, x_2^*)$$
, MRS = $-p_1/p_2$ so

$$-\frac{ax_2^*}{bx_1^*} = -\frac{p_1}{p_2} \qquad \Rightarrow \quad x_2^* = \frac{bp_1}{ap_2}x_1^*. \tag{A}$$

(x₁*,x₂*) also exhausts the budget so

$$p_1x_1^* + p_2x_2^* = m.$$
 (B)

So now we know that

$$\mathbf{x}_2^* = \frac{\mathsf{bp}_1}{\mathsf{ap}_2} \mathbf{x}_1^* \tag{A}$$

$$p_1x_1^* + p_2x_2^* = m.$$
 (B)

So now we know that

Substitute
$$x_2^* = \frac{bp_1}{ap_2}x_1^*$$
 (A)
 $p_1x_1^* + p_2x_2^* = m$. (B)

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 (A) Substitute $p_1x_1^* + p_2x_2^* = m$. (B) and get $p_1x_1^* + p_2\frac{bp_1}{ap_2}x_1^* = m$.

This simplifies to

$$x_1^* = \frac{am}{(a+b)p_1}.$$

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Substituting for x₁* in

$$p_1x_1^* + p_2x_2^* = m$$

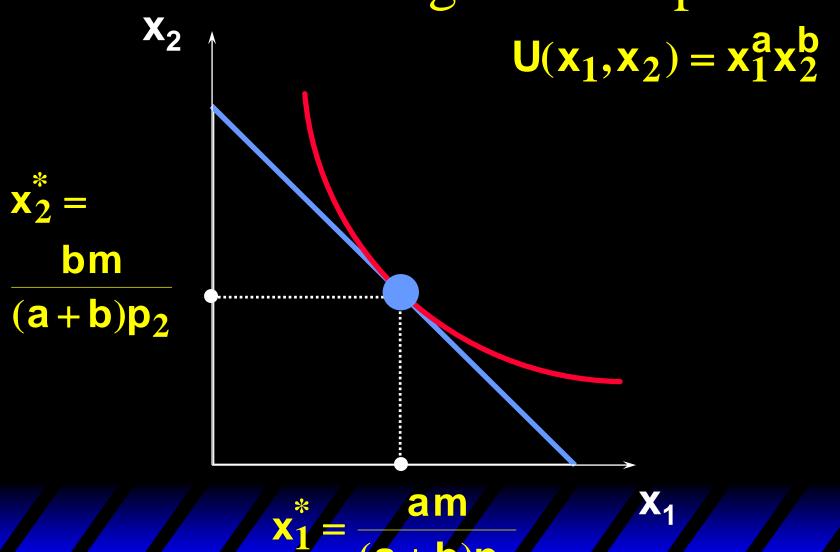
then gives

$$x_2^* = \frac{bm}{(a+b)p_2}.$$

So we have discovered that the most preferred affordable bundle for a consumer with Cobb-Douglas preferences

$$\mathbf{U}(\mathbf{x}_1, \mathbf{x}_2) = \mathbf{x}_1^{\mathbf{a}} \mathbf{x}_2^{\mathbf{b}}$$

is
$$(x_1^*, x_2^*) = \left(\frac{am}{(a+b)p_1}, \frac{bm}{(a+b)p_2}\right).$$



Rational Constrained Choice

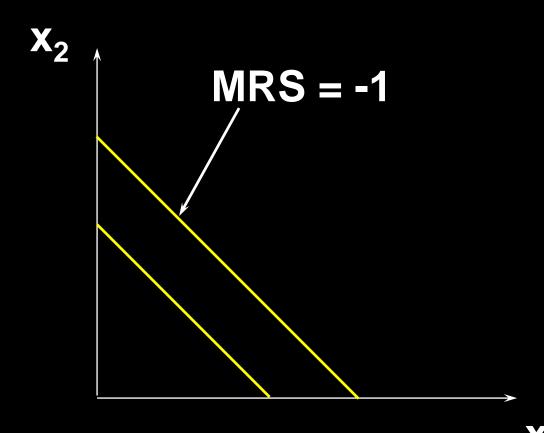
When $x_1^* > 0$ and $x_2^* > 0$ and (x_1^*, x_2^*) exhausts the budget, and indifference curves have no 'kinks', the ordinary demands are obtained by solving:

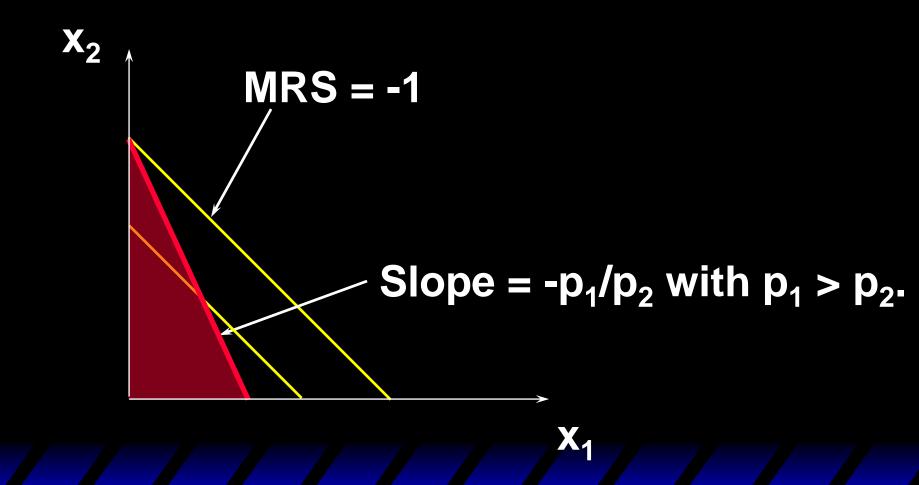
(a)
$$p_1x_1^* + p_2x_2^* = y$$

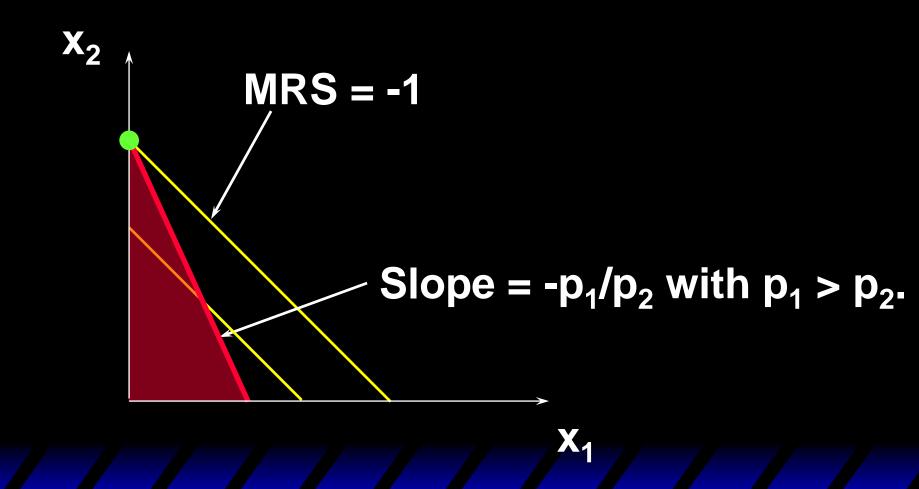
(b) the slopes of the budget constraint, $-p_1/p_2$, and of the indifference curve containing (x_1^*,x_2^*) are equal at (x_1^*,x_2^*) .

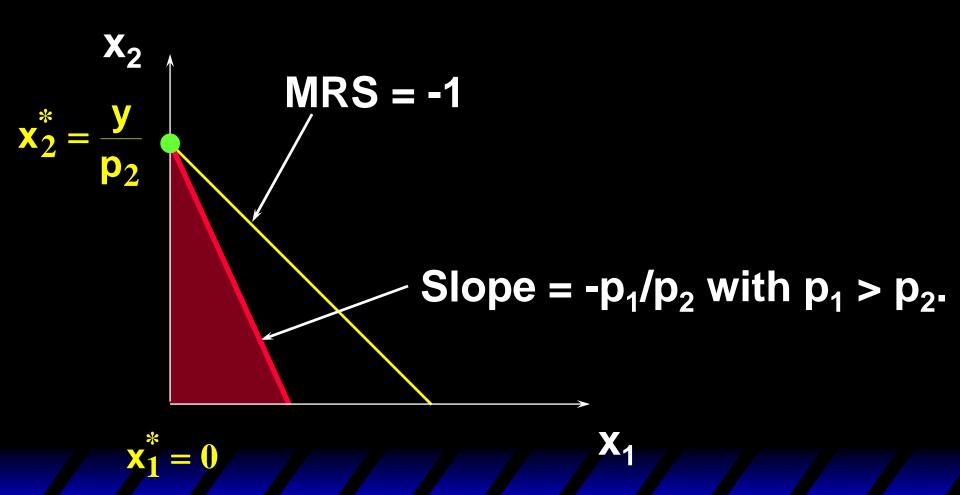
Rational Constrained Choice

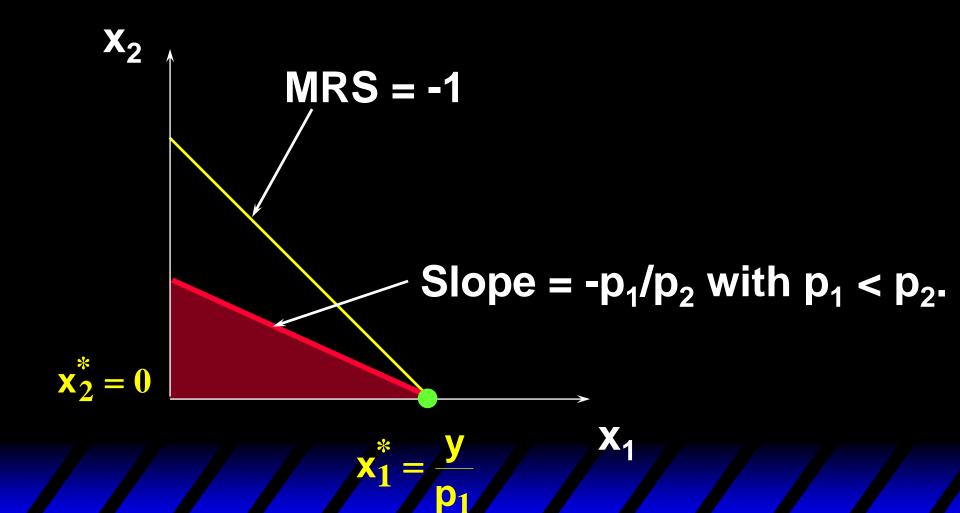
But what if $x_1^* = 0$? Or if $x_2^* = 0$? If either $x_1^* = 0$ or $x_2^* = 0$ then the ordinary demand (x₁*,x₂*) is at a corner solution to the problem of maximizing utility subject to a budget constraint.









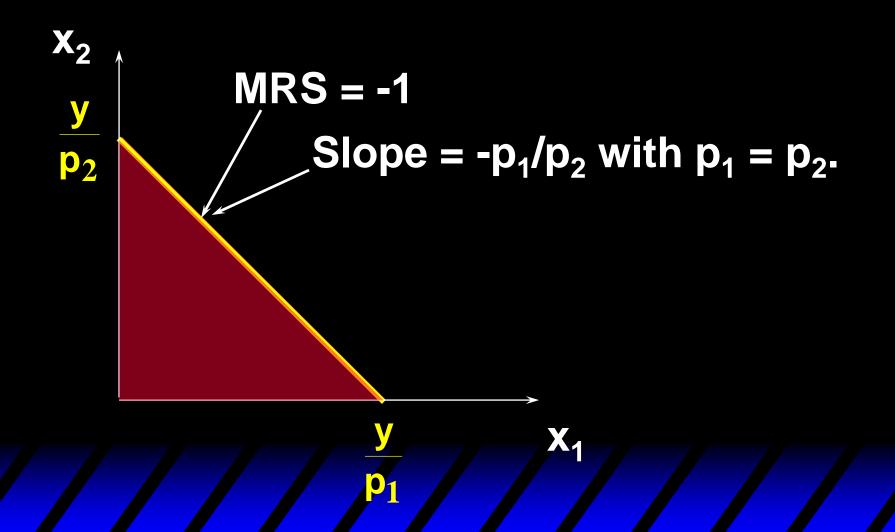


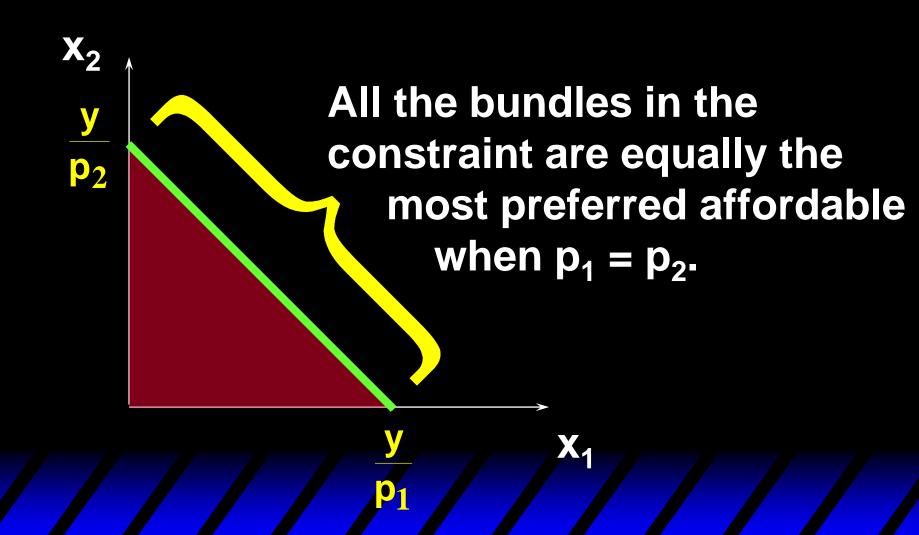
So when $U(x_1,x_2) = x_1 + x_2$, the most preferred affordable bundle is (x_1^*,x_2^*) where

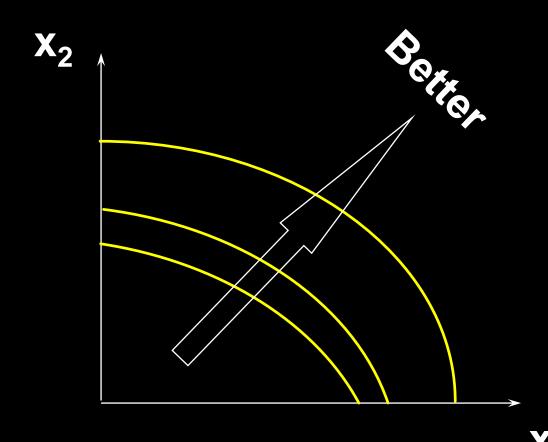
$$(x_1^*, x_2^*) = \left(\frac{y}{p_1}, 0\right)$$
 if $p_1 < p_2$

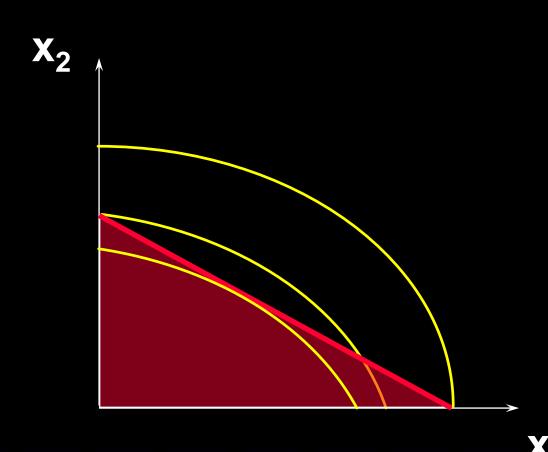
and

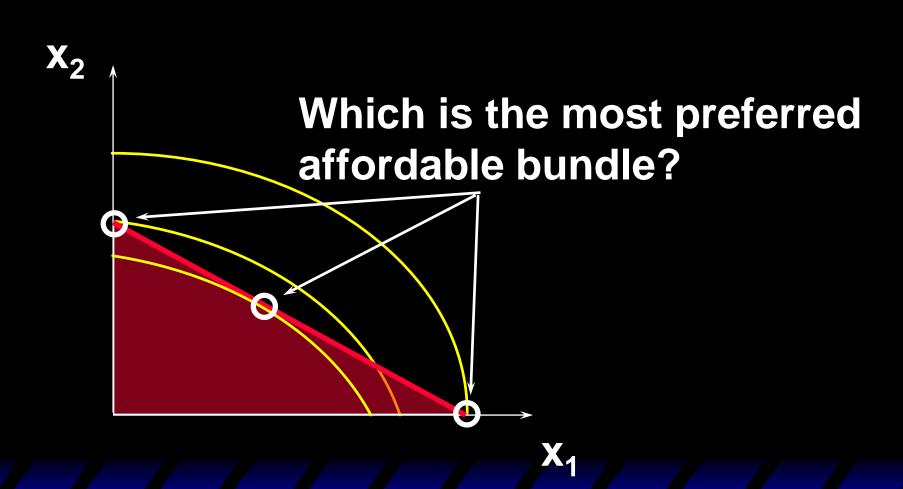
$$(x_1^*, x_2^*) = \left(0, \frac{y}{p_2}\right)$$
 if $p_1 > p_2$.

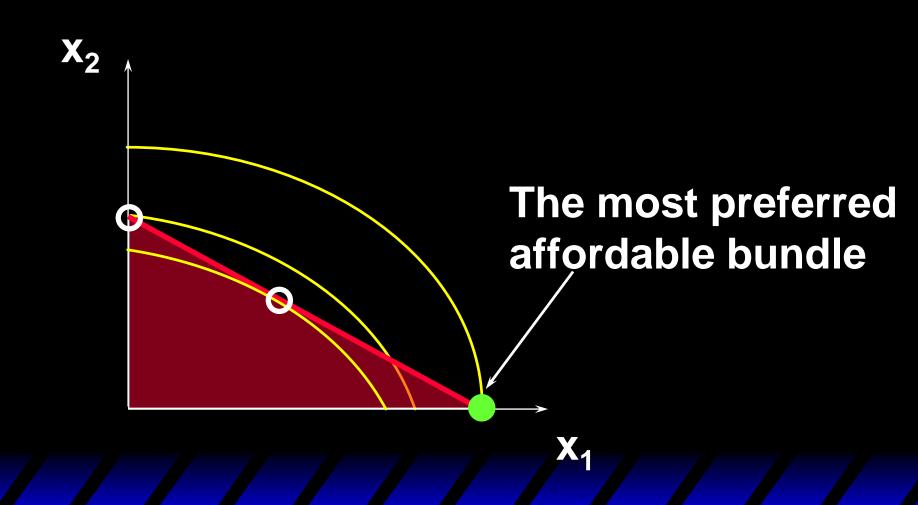




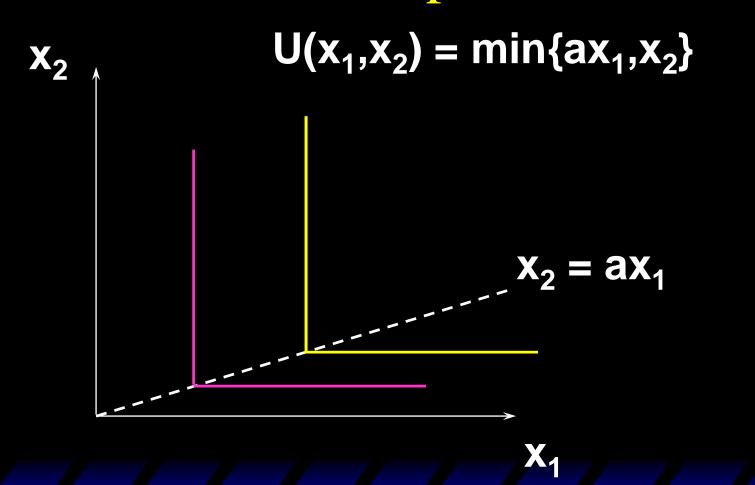


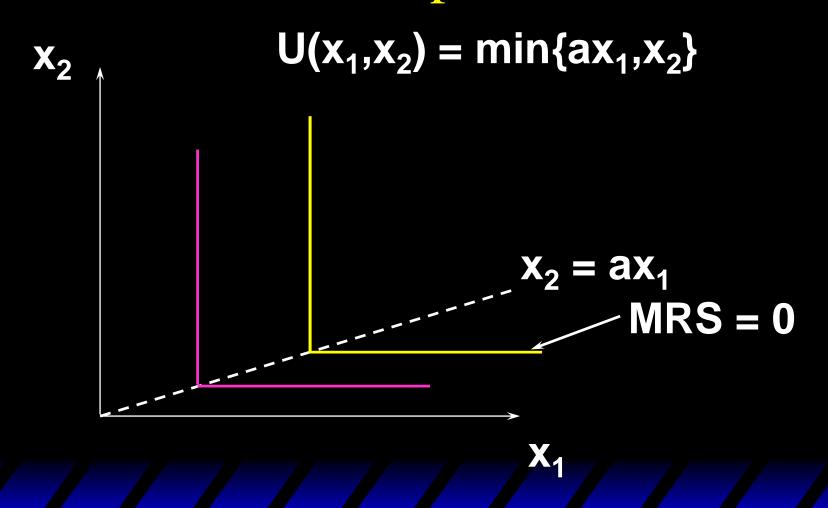


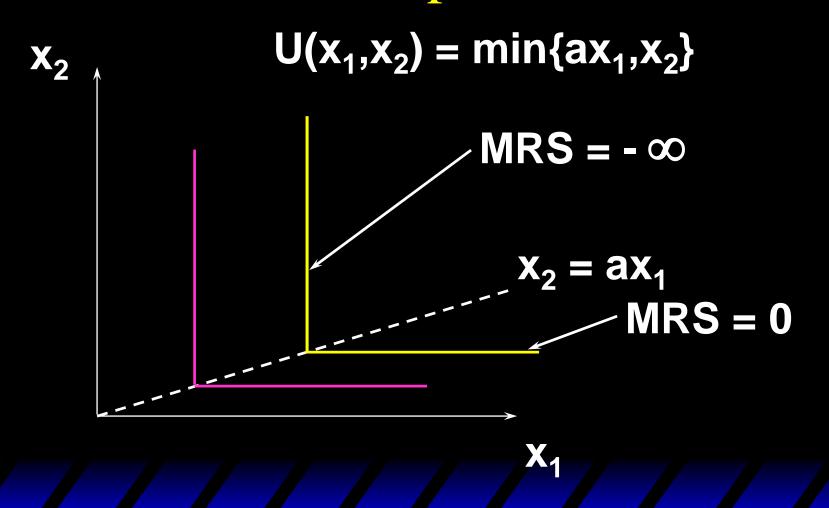


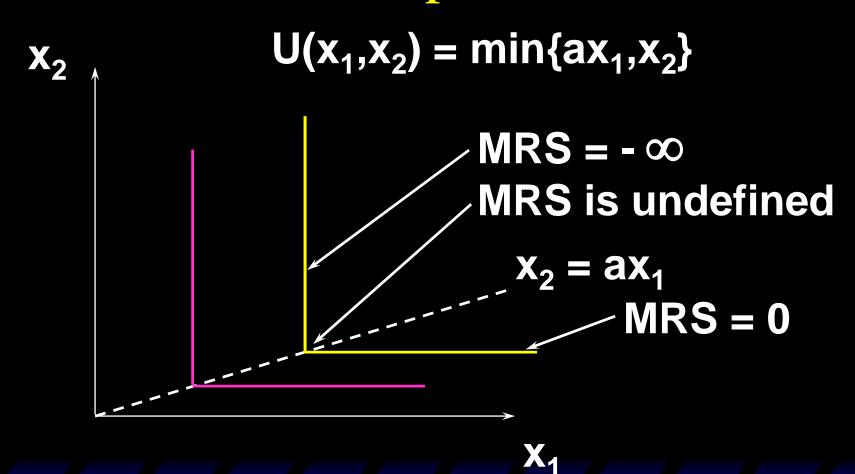


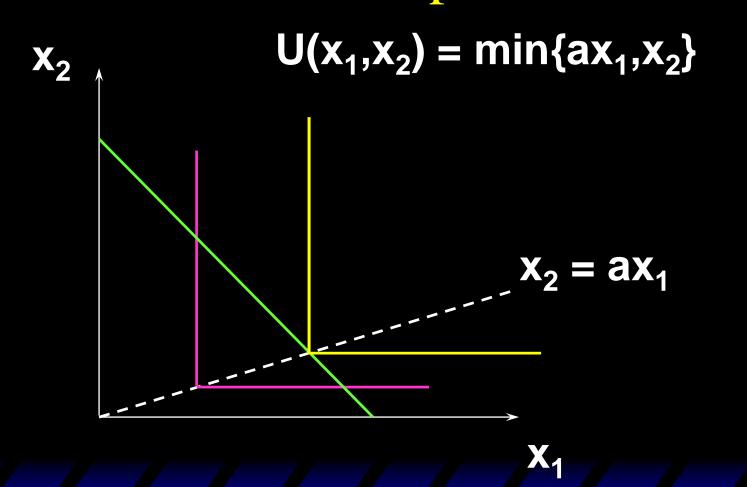
Notice that the "tangency solution" is not the most preferred affordable bundle. The most preferred affordable bundle

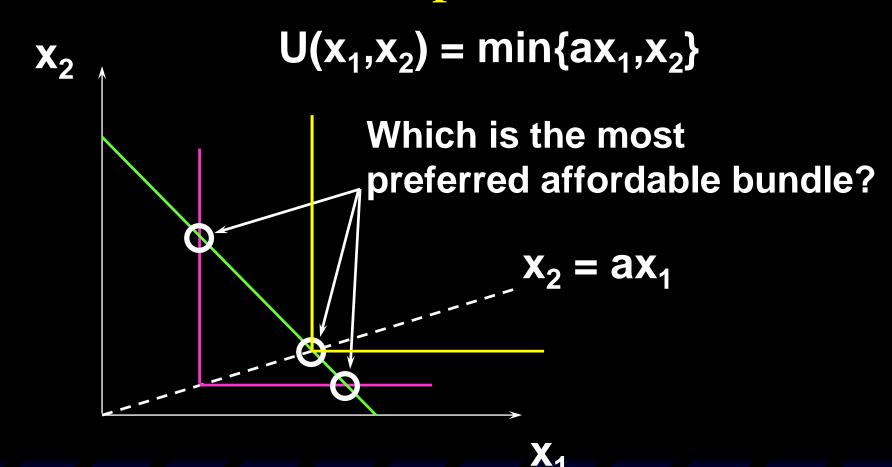


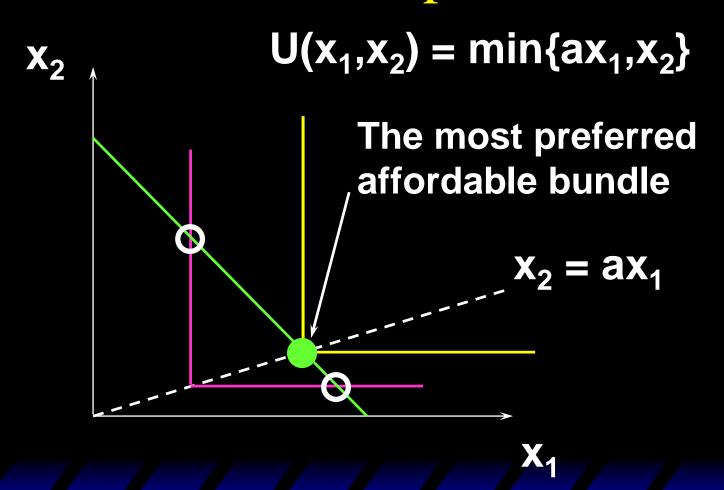


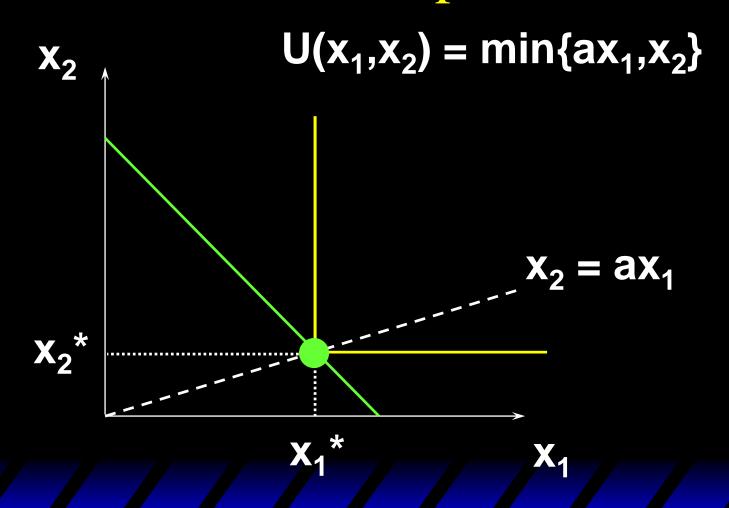


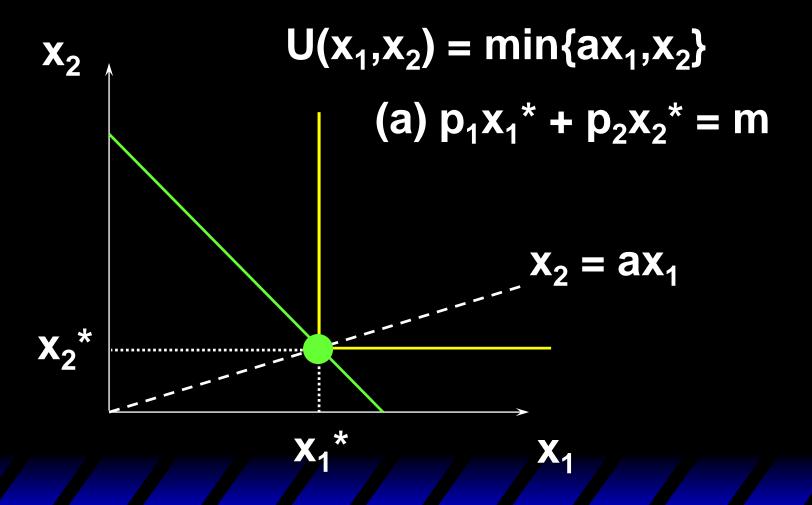


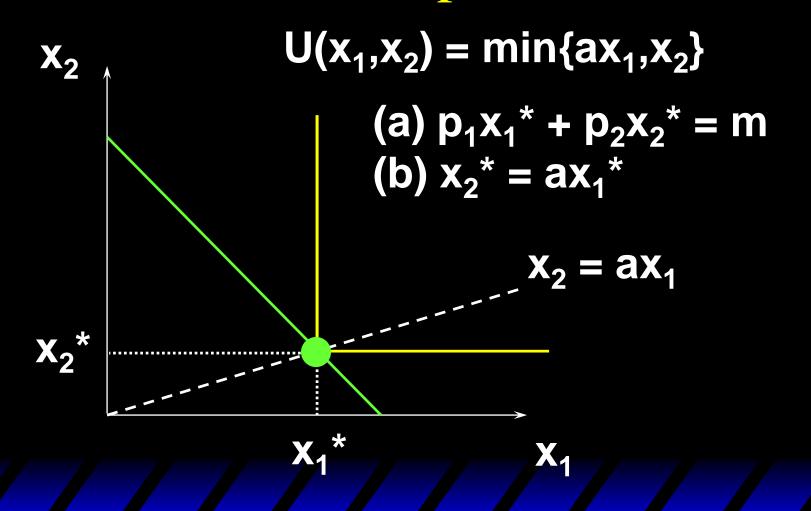












(a)
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A bundle of 1 commodity 1 unit and a commodity 2 units costs $p_1 + ap_2$; $m/(p_1 + ap_2)$ such bundles are affordable.

