一. A：有工作； B_1 强推， B_2 中推， B_3 弱推。

$P(A|B_1)=0.8$ $P(A|B_2)=0.4$ $P(A|B_3)=0.1$

$P(B_1)=0.7$, $P(B_2)=0.2$; $P(B_3)=0.1$

(1) $P(A)=\sum P(A|B_i)P(B_i) = 0.8\times0.7+0.4\times0.2+0.1\times0.1$

$= 0.65$.

(2) $P(B_1|A) = \frac{P(A,B_1)}{P(A)} = \frac{P(A|B_1)P(B_1)}{P(A)} = \frac{0.8\times0.7}{0.65} = 0.8615$

(3) $P(B_3|\bar{A}) = \frac{P(\bar{A},B_3)}{P(\bar{A})} = \frac{P(\bar{A}|B_3)P(B_3)}{P(\bar{A})} = \frac{0.9\times0.1}{1-0.65} = 0.2571$

二. (1) $E[X] = 600\times\frac{1}{6} = 100$

$E[Y] = 200\times\frac{1}{2} = 100$

(2) $P(80\le X\le 120) = \sum_{x_i=80}^{120} P(X=x_i)$

$P(X=x_i) = C_{600}^{x_i}\left(\frac{1}{6}\right)^{x_i}\left(\frac{5}{6}\right)^{600-x_i} = C_{600}^{x_i}\frac{5^{600-x_i}}{6^{600}}$

$P(80\le X\le 120) = \sum_{i=80}^{120} C_{600}^{i}\frac{5^{600-i}}{6^{600}}$

(3) $X=\sum_{k=1}^{600} X_k$. $X_k \sim Ber(\frac{1}{6})$ 独立，同分布，期望方差存在。

故满足假设。

$E[X_k]=\frac{1}{6}$, $D(X_k)=\frac{1}{6}\times\frac{5}{6}=\frac{5}{36}$.

$E[X]=100$, $D(X)=600\times\frac{5}{36}=\frac{500}{6}$

②

$P(80 \le X \le 120) = P\left(\frac{80-E[X]}{\sqrt{D(X)}} \le Z \le \frac{120-E[X]}{\sqrt{D(X)}}\right)$, $Z \sim N(0,1)$

$= P\left(-2\sqrt{\frac{6}{5}} \le Z \le 2\sqrt{\frac{6}{5}}\right)$

$= \Phi\left(2\sqrt{\frac{6}{5}}\right) - \Phi\left(-2\sqrt{\frac{6}{5}}\right)$

$= 1 - 2\Phi\left(-2\sqrt{\frac{6}{5}}\right)$ (或 $2\Phi\left(2\sqrt{\frac{6}{5}}\right)-1$)

(4) $Y = \sum_{k=1}^{200} Y_k$ $Y_k \sim Ber(\frac{1}{2})$. 独立同分布. 期望方差存在.

故 Y 也可以使用中心极限定理得 $Y \sim N(100, 200\times\frac{1}{4})$.

即求 $P(X-Y>0)$ 令 $W = X-Y$.

$W \sim N(0, \frac{500}{6}+50)$

$P(W>0) = \Phi(0) = 0.5$

三. (1) $\int_1^2\int_0^1 C(xy+1)\,dx\,dy = \int_1^2 \left[\left(\frac{1}{2}Cx^2y + Cx\right)\Big|_0^1\right] dy = \int_1^2 \left(\frac{1}{2}Cy + C\right) dy$

$= \frac{1}{4}Cy^2 + Cy\Big|_1^2 = \frac{7}{4}C = 1$

~~$= 6 - \frac{11}{4} = 1$~~

$\Rightarrow C = \frac{4}{7}$

(2) $f_X(x)=\int_1^2 \frac{4}{7}(xy+1)dy=\frac{2}{7}xy^2+\frac{4}{7}y\Big|_{y=1}^{y=2}=\frac{6}{7}x+\frac{4}{7}$, $0\le x\le 1$

$f_Y(y)=\int_0^1 \frac{4}{7}(xy+1)dx=\frac{2}{7}yx^2+\frac{4}{7}x\Big|_{x=0}^{x=1}=\frac{2}{7}y+\frac{4}{7}$, $1\le y\le 2$

$\because f(x,y)\neq f_X(x)\cdot f_Y(y)$ $\therefore$ 不独立.

(3) $f(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{\frac{4}{7}(xy+1)}{\frac{2}{7}y+\frac{4}{7}}=\frac{2(xy+1)}{y+2}$, $0\le x\le 1$, $1\le y\le 2$

(4) $P(X<2/3|Y=3/2)=\int_0^{2/3} f(x|\frac{3}{2})dx$

$=\int_0^{2/3}\frac{2(\frac{3}{2}x+1)}{2+\frac{3}{2}}dx=\int_0^{\frac{2}{3}}\frac{6x+4}{7}dx=\frac{4}{7}$

四. 第三章书后35题.

五 (1) $f(x,y)=\begin{cases}\frac{1}{\pi} & x^2+y^2\le 1\\ 0 & \text{其它.}\end{cases}$

~~(2) $f_X(x)=\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{1}{\pi}dy=\frac{2}{\pi}\sqrt{1-x^2}\quad -1\le x\le 1$~~

~~$f_Y(y)=\frac{2}{\pi}\sqrt{1-y^2},\ -1\le y\le 1$~~

~~$E[X]=E[Y]=0$~~

~~$E[X^2]=\int_{-1}^{1}\frac{2}{\pi}x^2\sqrt{1-x^2}\,dx=$~~

(2) $E[XY]=\frac{1}{\pi}\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}xy\,dx\,dy$

$=\frac{1}{\pi}\int_{-1}^{1}\left(-\frac{1}{2}yx^2\Big|_{x=-\sqrt{1-y^2}}^{x=\sqrt{1-y^2}}\right)dy$

$=0$

(3) $\rho=\frac{E[XY]}{\sigma(X)\sigma(Y)}=0$

因此 X.Y 不相关.

X.Y 不独立. (可以说明当Y改变的时候, X服从Y=y在圆上的弦的均匀分布, 也可以通过求边缘密度或条件密度函数证明不独立)

六.

$X=\sum_{i=1}^{300} X_i$ ~~$=1\times0.7+2\times0.2=1.1$~~

$E[X_i]=1\times0.7+2\times0.2=1.1$

$D[X_i]=1\times0.7+2^2\times0.2-1.1^2=0.29$

$E[X]=300\times1.1=330$

$D[X]=300\times0.29=\sqrt{87}$

$X\sim N(330,87)$

$P(X\geq L)\leq 0.05$

$\Phi\left(\frac{L-330}{\sqrt{87}}\right)=0.95 \quad L=\Phi^{-1}(0.95)\cdot\sqrt{87}+330$

七. (1) $f_{X_1}(x)=f_{X_2}(x)=\begin{cases}\theta e^{-\theta x} & x>0\\ 0 & \text{其它}\end{cases}$

$E[Y_1]=E[X_1]=\frac{1}{\theta}=1 \Rightarrow \theta=1$

$f(x,y)=\begin{cases}f_{X_1}(x)f_{X_2}(y)=e^{-(x+y)} & x>0,\ y>0\\ 0 & \text{其它}\end{cases}$

(2) $P(X_1+X_2<1)$ $f_{X_1+X_2}(z)$

$\int_0^1 e^{-x}e^{-(z-x)}dx=e^{-z},\ z>0$

$P(Z<1)=1e^{-z}$ $1-2e^{-1}$

(3) ~~X 之概率密度~~.

$$P(U<z) = P(X_1<z)P(X_2<z)$$

$$= F_{X_1}(z)F_{X_2}(z)$$

$$= (1-e^{-z})(1-e^{-z}) = (1-e^{-z})^2$$