

Lecture 7 Intangible Assets and the time value of money



- **Definition of Intangible Assets**
- Categories of Intangible Assets
- Amortization
- Goodwill
- Time Value of Money

Receivables don't have physical substance but do not belong to intangible assets.

Definition of Intangible Assets

- Long term assets with **no physical substances** and differ from property, plant and equipment
 - patents, copyrights, leaseholds, trademarks, and brand names
 - goodwill 商誉 → 不列在 Intangible Assets 里.

AICPA(Accounting Principles Board) Opinion No. 17:

1. Determining an initial carrying amount
2. Accounting for that amount after acquisition under normal business conditions—that is, through periodic write-off or amortization—in a manner similar to depreciation
3. Accounting for that amount if the value declines substantially and permanently. 需要计提减值

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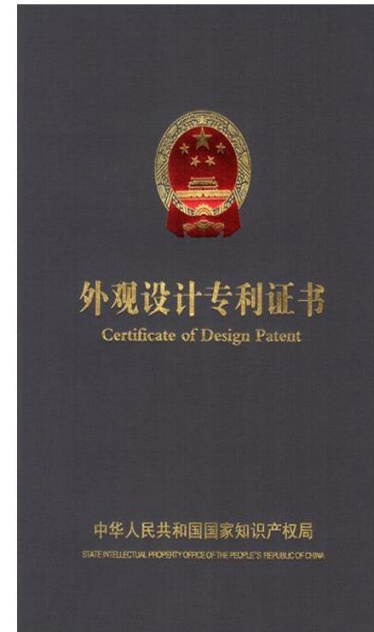
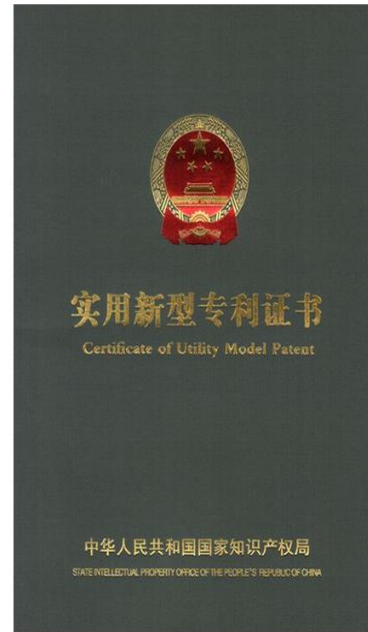
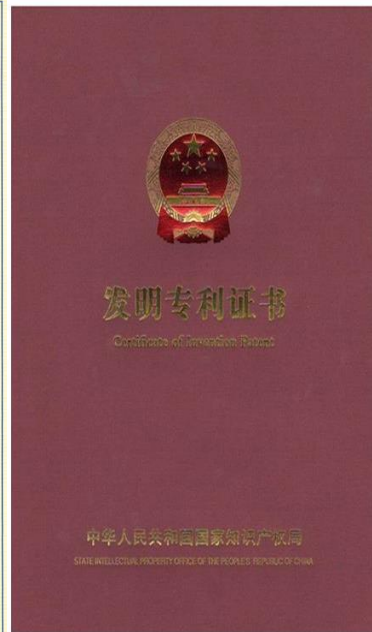
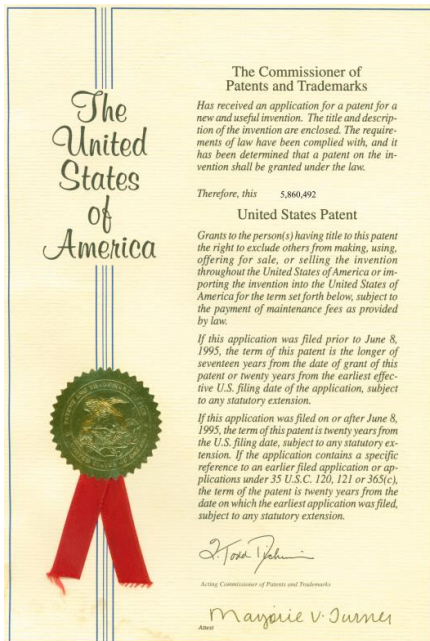


- Definition of Intangible Assets
- **Categories of Intangible Assets**
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Categories of Intangible Assets

● Patents, Trademarks, Copyrights and Franchises

- A patent is a monopoly license granted by the 专利 government giving the owner control of ***the use or sale*** of an invention for a period of 20 years.



Categories of Intangible Assets

● Patents, Trademarks, Copyrights and Franchises

- A trademark(trade name) is registered with the Federal Trade Commission and can only be used by the entity that owns the trademark or by another entity that has secured permission from the owner.



● Patents, Trademarks, Copyrights and Franchises

- A copyright is a protection of printed or recorded material. A copyright is granted for a period of time equal to the life of the writer or artist.

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Categories of Intangible Assets

- Patents, Trademarks, Copyrights and Franchises
 - A franchise is a right granted by a company or a *连锁经营权* government unit to conduct a certain type of business in a specific geographic area.
- Goodwill *只会在公司兼并过程中产生*
 - Represents an amount that a company has paid to acquire certain favorable intangible attributes as part of an acquisition of another company.

Acquiring company is willing to pay more amount of money than market value of acquired company.

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↳ Represent the positive expectation of acquired company.

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Amortization

摊销

- Amortization describes the systematic write-off to expense of the cost of an intangible asset over its useful life.
 - the process of allocating the cost of an intangible asset from the balance sheet to the income statement as an expense
- straight-line method

Balance Sheet			Income Statement		
Assets =	Liabilities	Owners' Equity	Net Income =	Revenue	Expenses
<u>-intangible Assets</u> ↓					<u>-Amortization Expenses</u> ↑

Amortization

● Journal Entry:

Dr. Amortization Expense	xxx
Cr. Intangible Assets	xxx

Soda Bottling Company purchases a patent on a unique bottle cap for 18,000. The entry to record the patent would be:

● Journal Entry:

Dr. Intangible Asset---Patent	18000
Cr. Cash	18000

Amortization

● Journal Entry:

Dr. Amortization Expense	xxx
Cr. Intangible Assets	xxx

Although the patent for the bottle cap will last for 17 years, the product using the cap will be sold only for the next 6 years. The entry to record the annual amortization would be:

● Journal Entry:

Dr. Amortization (Expense)	3000
Cr. Intangible Asset---Patent	3000

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Goodwill

- Goodwill results from the purchase of one firm by another for a price that is **greater than** the fair market value of the net assets acquired



Why would one firm be willing to pay more for a business than the fair market value?

Means that the acquiring company believes the future of the acquired one will be good.

- Market value of the company
 - Net assets ~ Owner's Equity
 - Net Identifiable Assets(Book Value) ~ Net Assets - Intangible Assets - Goodwill
原因: hard to measure the intangible assets.
 - Future Profit and Cashflows
- Goodwill is a general term that encompasses a wide variety of favorable attributes expected to permit the acquiring company to operate at a greater-than-normal (abnormal) level of profitability

Large amount of goodwill means: High risk

Goodwill

Assume that two similar restaurants are offered for sale and that the normal return on the fair market value of the net identifiable assets of restaurants of this type is 15 percent a year. The relative earning power of the two restaurants during the past five years is as follows:

	Mandarin Coast	Golden Dragon
Fair market value of net identifiable assets	1,000,000	1,000,000
Normal rate of return on net assets	15%	15%
Normal earnings, computed as 15% of net identifiable assets	150,000	150,000
Average actual net income for past five years	150,000	200,000
Earnings in excess of normal	0	50,000

Goodwill

If the buyer of Golden Dragon pays 1,300,000 to purchase the business, the 300,000 of goodwill would be recorded. 300,000 will be included in the buyer's balance sheet.

	Mandarin Coast	Golden Dragon
Fair market value of net identifiable assets	1,000,000	1,000,000
Normal rate of return on net assets	15%	15%
Normal earnings, computed as 15% of net identifiable assets	150,000	150,000
Average actual net income for past five years	150,000	200,000
Earnings in excess of normal	0	50,000

- Positive attributes in goodwill:
 - Favorable reputation
 - Positive market share
 - Positive advertising image
 - Reputation for high quality and loyal employees
 - Superior management
 - Manufacturing and other operating efficiency...

Goodwill

- Goodwill is recorded only when the buyer has purchased the other company.
- Internally generated goodwill is not recorded in the accounting records. *The balance sheet records the goodwill for other companies*



Notice: Balance sheets do not indicate a company's current market value.

Notice: When the recorded amount of goodwill is no longer recoverable, an impairment loss must be recorded by reducing the asset amount and including a loss in the income statement of the same accounting period.

Even though the goodwill gets recovered in the future, the company can't take it back.

Goodwill was identified as a topic for harmonization efforts when the FASB (Financial Accounting Standards Board) and the IASB (International Accounting Standards Boards) agreed to work toward convergence of reporting requirements in 2002, an effort that continues today and will undoubtedly continue to years to come. U.S. GAAP requires capitalization of goodwill but no amortization. Instead, goodwill is reviewed annually and its value is adjusted if subject to impairment. Until March 2004, international standards required goodwill to be capitalized and amortized over its estimated useful life (20 years or less). In 2004, the IASB changed international standards for goodwill to be consistent with the U.S. GAAP approach by requiring an impairment test rather than amortization for goodwill.



Goodwill

Cruisers, Inc., purchased a business by paying 1,000,000 in cash and assuming a note payable liability of 100,000. The fair market value of the net assets acquired was 800,000, assigned as follows: Inventory, 250,000; Land, 150,000; Buildings, 400,000; and Notes Payable, 100,000.

Balance Sheet			Income Statement			
Assets	=	Liabilities	Owners' Equity	Net Income	Revenue	Expenses
Inventory +250,000		Notes Payable +100, 000				
Land +150,000						
Buildings + 400,000						
Goodwill + 300,000						
Cash -1,000,000						

Goodwill

- The journal entry:

Dr. Inventory	250,000
Land	150,000
Buildings	450,000
Goodwill	300,000
Cr. Notes Payable	100,000
Cash	1,000,000

Goodwill

Three years after the business was acquired, the fair value of the resulting goodwill of 300,000 was determined to be only 180,000. The effects on the financial statement of the impairment loss would be:

Balance Sheet				Income Statement		
Assets	=	Liabilities	Owners' Equity	Net Income	Revenue	Expenses
Goodwill -120,000						Goodwill Impairment Loss -120,000

- The journal entry:

Dr. Goodwill Impairment Loss	120,000
Cr. Goodwill	120,000

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Time Value of Money



Why do we put our money into the bank?



- The Time Value of Money (TVM) is the concept that money available at the present time is worth more than the identical sum in the future due to its potential earnings capacity.

Time Value of Money



- Future Value: the amount accumulated when interest on an investment is compounded for a given number of periods.

- **Compounding:** calculate interest for a period on the sum of the principal and interest accumulated at the beginning of the period

- $$FV_{r,n} = 1 * (1 + r\%)^n$$

Time Value of Money

Simple Interests

Year	Principal at beginning of the year	Interest Earned at 10%	Principal at the End of the year	Amount of Money at the end of the year
1	1000	100	1000	1100
2	1000	100	1000	1200
3	1000	100	1000	1300
4	1000	100	1000	1400

Compounding Interests

Year	Principal at beginning of the year	Interest Earned at 10%	Principal at the End of the year	Amount of Money at the end of the year
1	1000	100	1100	1100
2	1100	110	1210	1210
3	1210	121	1331	1310
4	1331	133	1464	1464

Factors for calculating the Future Value of 1

期数	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.1
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.21
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.191	1.225	1.2597	1.295	1.331
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641
5	1.051	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.828	1.9487
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.999	2.1719	2.3579
10	1.1046	1.219	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531
12	1.1268	1.2682	1.4258	1.601	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975
15	1.161	1.3459	1.558	1.8009	2.0789	2.3966	2.759	3.1722	3.6425	4.1772

Time Value of Money



- Present Value: the current value of a future sum of money or stream of cash flows given a specified rate of return.

➤ Discounting: the opposite direction of compounding

➤
$$PV_{r,n} = \frac{1}{(1+r\%)^n}$$

- Annuity:

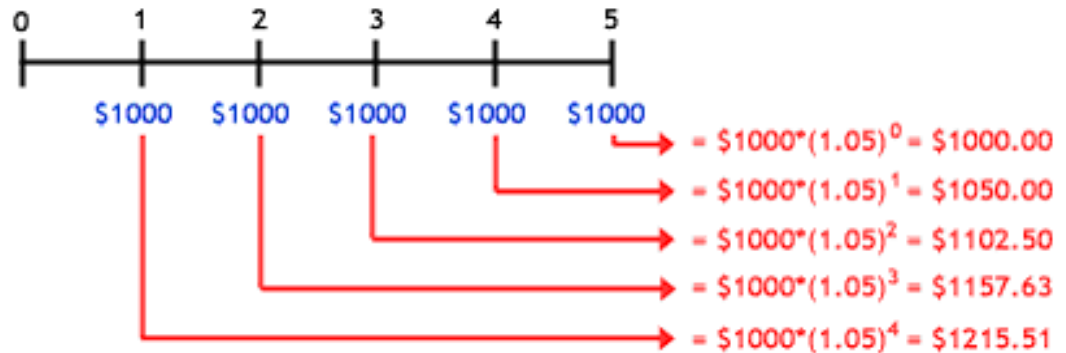
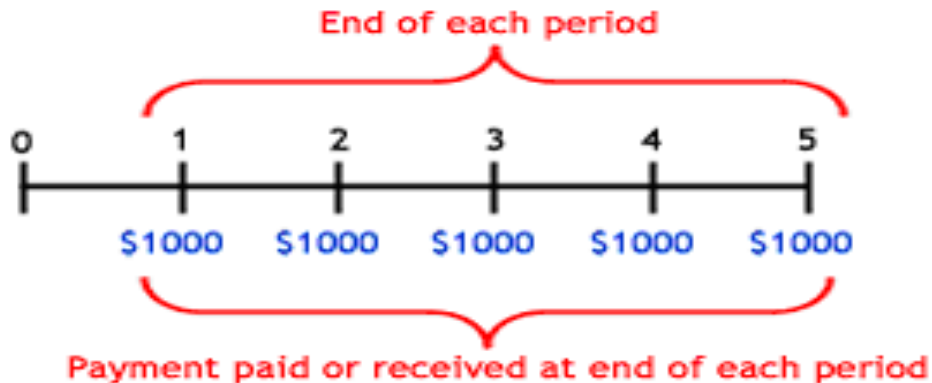
- A financial product that pays out a fixed stream of payments
- Payments at equal intervals
- When the investment is made at the end of each compounding period, a usual practice, the annuity is in arrears.

- Future Value of Annuity:

- The sum of the future value of each individual investments

Future Value of Annuity

- Ordinary Annuity: payment is made at the end of each period



Future Value of an Ordinary Annuity = \$5525.64

Future Value of Annuity

● Future Value of Ordinary Annuity:

- Year 1: $FV = 1 * (1 + 5\%)^0$
- Year 2: $FV = 1 * (1 + 5\%)^1 + 1 * (1 + 5\%)^0$
- Year 3: $FV = 1 * (1 + 5\%)^2 + 1 * (1 + 5\%)^1 + 1 * (1 + 5\%)^0$
- Year 4: $FV = 1 * (1 + 5\%)^3 + 1 * (1 + 5\%)^2 + 1 * (1 + 5\%)^1 + (1 + 5\%)^0$
- Year 5: $FV = 1 * (1 + 5\%)^4 + 1 * (1 + 5\%)^3 + 1 * (1 + 5\%)^2 + (1 + 5\%)^1 + (1 + 5\%)^0$
- Year n: $FV = 1 * (1 + 5\%)^{n-1} + 1 * (1 + 5\%)^{n-2} + 1 * (1 + 5\%)^{n-3} + \dots + (1 + 5\%)^2 + (1 + 5\%)^1 + (1 + 5\%)^0$

- Future Value of Ordinary Annuity :

$$\blacktriangleright FV(A, i, n) = A \left[\frac{(1+i)^n - 1}{i} \right]$$

where A = Cash flow per period

i = Interest Rate

n = Number of payments

Future Value of Annuity

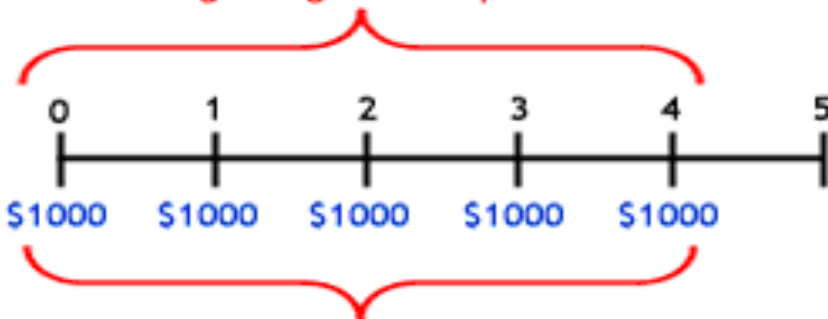
FUTURE VALUE OF ORDINARY ANNUITY (annuity in arrears -- end of period payments)

Periods	RATE PER PERIOD							
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.00250	2.00500	2.00750	2.01000	2.01500	2.02000	2.02500	2.03000
3	3.00751	3.01502	3.02256	3.03010	3.04522	3.06040	3.07563	3.09090
4	4.01503	4.03010	4.04523	4.06040	4.09090	4.12161	4.15252	4.18363
5	5.02506	5.05025	5.07556	5.10101	5.15227	5.20404	5.25633	5.30914
6	6.03763	6.07550	6.11363	6.15202	6.22955	6.30812	6.38774	6.46841
7	7.05272	7.10588	7.15948	7.21354	7.32299	7.43428	7.54743	7.66246
8	8.07035	8.14141	8.21318	8.28567	8.43284	8.58297	8.73612	8.89234
9	9.09053	9.18212	9.27478	9.36853	9.55933	9.75463	9.95452	10.15911
10	10.11325	10.22803	10.34434	10.46221	10.70272	10.94972	11.20338	11.46388
11	11.13854	11.27917	11.42192	11.56683	11.86326	12.16872	12.48347	12.80780
12	12.16638	12.33556	12.50759	12.68250	13.04121	13.41209	13.79555	14.19203
13	13.19680	13.39724	13.60139	13.80933	14.23683	14.68033	15.14044	15.61779
14	14.22979	14.46423	14.70340	14.94742	15.45038	15.97394	16.51895	17.08632
15	15.26537	15.53655	15.81368	16.09690	16.68214	17.29342	17.93193	18.59891
16	16.30353	16.61423	16.93228	17.25786	17.93237	18.63929	19.38022	20.15688
17	17.34429	17.69730	18.05927	18.43044	19.20136	20.01207	20.86473	21.76159

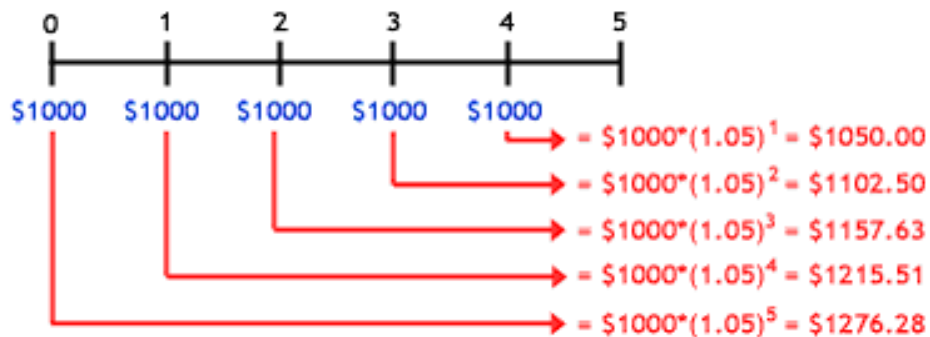
Future Value of Annuity

- Annuity Due: payment is made at the beginning of each period

Beginning of each period



Payment paid or received at the beginning of each period



Future Value of an Annuity Due = \$5801.92

Future Value of Annuity

● Future Value of Annuity Due:

- Year 1: $FV = 1 * (1 + 5\%)^1$
- Year 2: $FV = 1 * (1 + 5\%)^2 + 1 * (1 + 5\%)^1$
- Year 3: $FV = 1 * (1 + 5\%)^3 + 1 * (1 + 5\%)^2 + 1 * (1 + 5\%)^1$
- Year 4: $FV = 1 * (1 + 5\%)^4 + 1 * (1 + 5\%)^3 + 1 * (1 + 5\%)^2 + (1 + 5\%)^1$
- Year 5: $FV = 1 * (1 + 5\%)^5 + 1 * (1 + 5\%)^4 + 1 * (1 + 5\%)^3 + (1 + 5\%)^2 + (1 + 5\%)^1$
- Year n: $FV = 1 * (1 + 5\%)^n + 1 * (1 + 5\%)^{n-1} + 1 * (1 + 5\%)^{n-2} + ... + (1 + 5\%)^2 + (1 + 5\%)^1$

● Future Value of Annuity Due:

$$\blacktriangleright FV(A, i, n) = A \left[\frac{(1+i)^n - 1}{i} \right] * (1 + i)$$

where A = Cash flow per period

i = Interest Rate

n = Number of payments

Future Value of Annuity

FUTURE VALUE OF ANNUITY DUE (annuity in advance -- beginning of period payments)

Periods	RATE PER PERIOD							
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	1.00250	1.00500	1.00750	1.01000	1.01500	1.02000	1.02500	1.03000
2	2.00751	2.01502	2.02256	2.03010	2.04522	2.06040	2.07563	2.09090
3	3.01503	3.03010	3.04523	3.06040	3.09090	3.12161	3.15252	3.18363
4	4.02506	4.05025	4.07556	4.10101	4.15227	4.20404	4.25633	4.30914
5	5.03763	5.07550	5.11363	5.15202	5.22955	5.30812	5.38774	5.46841
6	6.05272	6.10588	6.15948	6.21354	6.32299	6.43428	6.54743	6.66246
7	7.07035	7.14141	7.21318	7.28567	7.43284	7.58297	7.73612	7.89234
8	8.09053	8.18212	8.27478	8.36853	8.55933	8.75463	8.95452	9.15911
9	9.11325	9.22803	9.34434	9.46221	9.70272	9.94972	10.20338	10.46388
10	10.13854	10.27917	10.42192	10.56683	10.86326	11.16872	11.48347	11.80780
11	11.16638	11.33556	11.50759	11.68250	12.04121	12.41209	12.79555	13.19203
12	12.19680	12.39724	12.60139	12.80933	13.23683	13.68033	14.14044	14.61779
13	13.22979	13.46423	13.70340	13.94742	14.45038	14.97394	15.51895	16.08632
14	14.26537	14.53655	14.81368	15.09690	15.68214	16.29342	16.93193	17.59891
15	15.30353	15.61423	15.93228	16.25786	16.93237	17.63929	18.38022	19.15688
16	16.34429	16.69730	17.05927	17.43044	18.20136	19.01207	19.86473	20.76159
17	17.38765	17.78579	18.19472	18.61475	19.48938	20.41231	21.38635	22.41444

Present Value of Annuity

- Annuity:

- A financial product that pays out a fixed stream of payments
- Payments at equal intervals
- When the investment is made at the end of each compounding period, a usual practice, the annuity is in arrears.

- Present Value of Annuity:

- The sum of the present value of each of the annuity payment amounts

Present Value of Annuity

The objective of using present value in an accounting measurement is to capture, to the extent possible, the economic difference between sets of estimated future cash flows. Without present value, a 1,000 cash flow due tomorrow and a 1,000 cash flow due in 10 years appear the same. Because present value distinguishes between cash flows that otherwise might appear similar, a measurement based on the present value of estimated future cash flows provides more relevant information than a measurement based on the undiscounted sum of those cash flows.

Which investment project will you choose?

Suppose the interest rate (risk-free rate) is 5%

	Project A	Project B
Initial Investment	1200	1200
1 st year estimated cashflow	700	300
2 nd year estimated cashflow	400	800
3 rd year estimated cashflow	800	500
4 th year estimated cashflow	200	500
5 th year estimated cashflow	400	400

2198.50

2168.02

Which investment project will you choose?

Suppose the interest rate (risk-free rate) is 5%

$$\text{Project A: } \frac{400}{(1+5\%)^5} + \frac{200}{(1+5\%)^4} + \frac{800}{(1+5\%)^3} + \frac{400}{(1+5\%)^2} + \frac{700}{(1+5\%)^1}$$

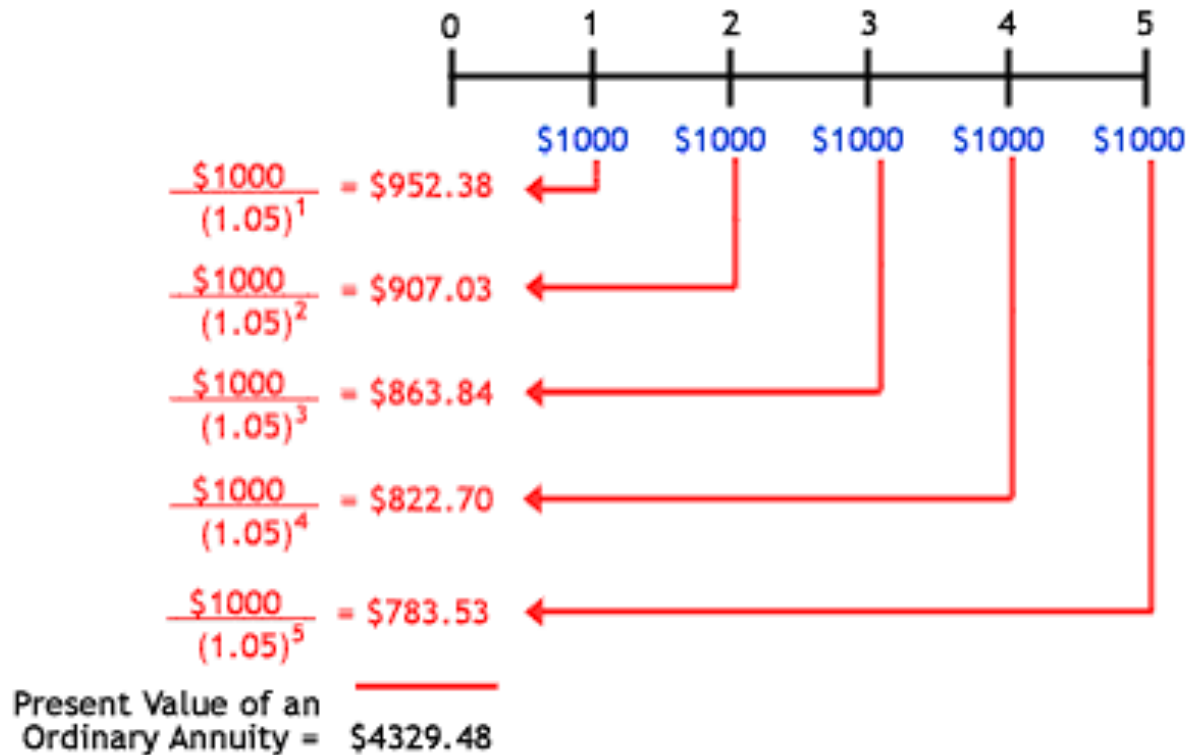
$$- 1200 = \mathbf{998}$$

$$\text{Project B: } \frac{400}{(1+5\%)^5} + \frac{500}{(1+5\%)^4} + \frac{500}{(1+5\%)^3} + \frac{800}{(1+5\%)^2} + \frac{300}{(1+5\%)^1}$$

$$- 1200 = \mathbf{968}$$

Present Value of Annuity

- Ordinary Annuity: payment is made at the end of each period



Present Value of Annuity

● Present Value of ordinary annuity:

$$\text{➤ Year 1: FV} = 1 * \frac{1}{(1+5\%)^1}$$

$$\text{➤ Year 2: FV} = 1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1}$$

$$\text{➤ Year 3: FV} = 1 * \frac{1}{(1+5\%)^3} + 1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1}$$

$$\text{➤ Year 4: FV} = 1 * \frac{1}{(1+5\%)^4} + 1 * \frac{1}{(1+5\%)^3} + 1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1}$$

$$\text{➤ Year n: FV} = 1 * \frac{1}{(1+5\%)^n} + 1 * \frac{1}{(1+5\%)^{n-1}} + 1 * \frac{1}{(1+5\%)^{n-2}} + \dots$$
$$1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1}$$

- Present Value of Ordinary Annuity :

$$\text{➤ } PV(A, i, n) = A \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

where A = Cash flow per period

i = Interest Rate

n = Number of payments

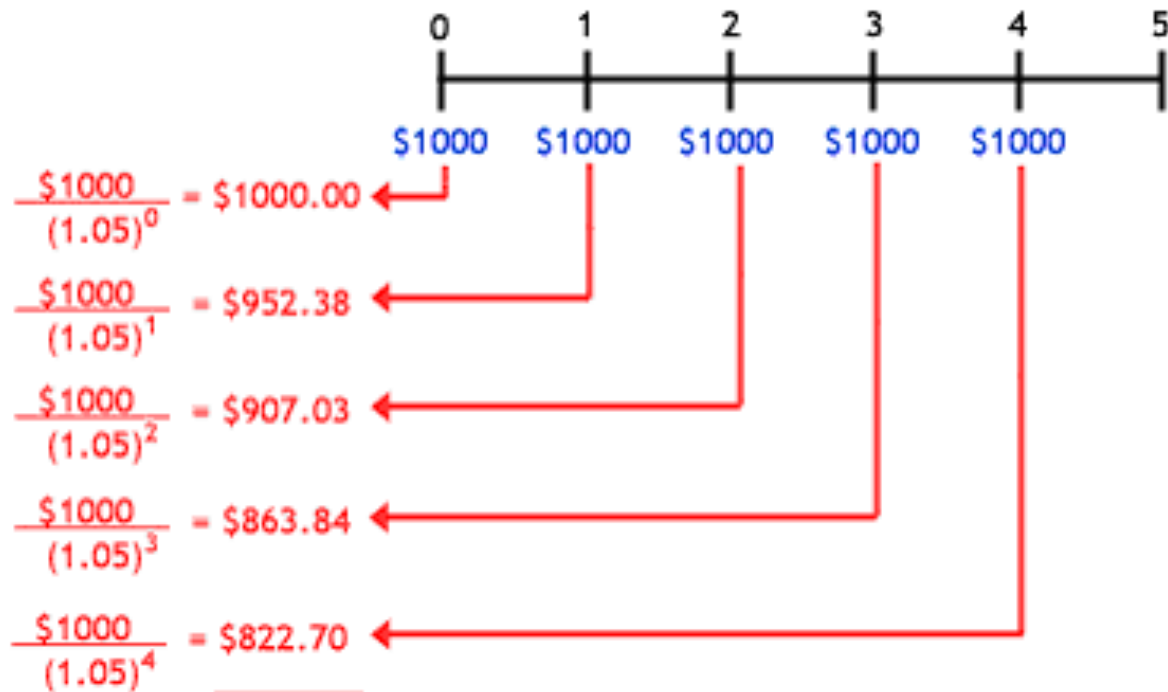
Present Value of Annuity

PRESENT VALUE OF ORDINARY ANNUITY (annuity in arrears -- end of period payments)

Periods	RATE PER PERIOD							
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88388	2.85602	2.82861
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971
6	5.94785	5.89638	5.84560	5.79548	5.69719	5.60143	5.50813	5.41719
7	6.93052	6.86207	6.79464	6.72819	6.59821	6.47199	6.34939	6.23028
8	7.91074	7.82296	7.73661	7.65168	7.48593	7.32548	7.17014	7.01969
9	8.88852	8.77906	8.67158	8.56602	8.36052	8.16224	7.97087	7.78611
10	9.86386	9.73041	9.59958	9.47130	9.22218	8.98259	8.75206	8.53020
11	10.83677	10.67703	10.52067	10.36763	10.07112	9.78685	9.51421	9.25262
12	11.80725	11.61893	11.43491	11.25508	10.90751	10.57534	10.25776	9.95400
13	12.77532	12.55615	12.34235	12.13374	11.73153	11.34837	10.98318	10.63496
14	13.74096	13.48871	13.24302	13.00370	12.54338	12.10625	11.69091	11.29607
15	14.70420	14.41662	14.13699	13.86505	13.34323	12.84926	12.38138	11.93794
16	15.66504	15.33993	15.02431	14.71787	14.13126	13.57771	13.05500	12.56110
17	16.62348	16.25863	15.90502	15.56225	14.90765	14.29187	13.71220	13.16612
18	17.57953	17.17277	16.77918	16.39827	15.67256	14.99203	14.35336	13.75351

Present Value of Annuity

- Annuity Due: payment is made at the beginning of each period



Present Value of
an Annuity Due = \$4545.95

Present Value of Annuity

● Present Value of Annuity Due:

$$\text{➤ Year 1: FV} = 1 * \frac{1}{(1+5\%)^1} + 1 * \frac{1}{(1+5\%)^0}$$

$$\text{➤ Year 2: FV} = 1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1} + 1 * \frac{1}{(1+5\%)^0}$$

$$\text{➤ Year 3: FV} = 1 * \frac{1}{(1+5\%)^3} + 1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1} + 1 * \frac{1}{(1+5\%)^0}$$

$$\text{➤ Year 4: FV} = 1 * \frac{1}{(1+5\%)^4} + 1 * \frac{1}{(1+5\%)^3} + 1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1} + 1 * \frac{1}{(1+5\%)^0}$$

$$\text{➤ Year n: FV} = 1 * \frac{1}{(1+5\%)^n} + 1 * \frac{1}{(1+5\%)^{n-1}} + 1 * \frac{1}{(1+5\%)^{n-2}} + \dots + 1 * \frac{1}{(1+5\%)^2} + 1 * \frac{1}{(1+5\%)^1} + 1 * \frac{1}{(1+5\%)^0}$$

- Present Value of Annuity Due:

$$\text{➤ } PV(A, i, n) = A \left[\frac{1 - (1+i)^{-n}}{i} \right] * (1 + i)$$

where A = Cash flow per period

i = Interest Rate

n = Number of payments

Present Value of Annuity

PRESENT VALUE OF ANNUITY DUE (annuity in advance -- beginning of period payments)

Periods	RATE PER PERIOD							
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.99751	1.99502	1.99256	1.99010	1.98522	1.98039	1.97561	1.97087
3	2.99252	2.98510	2.97772	2.97040	2.95588	2.94156	2.92742	2.91347
4	3.98506	3.97025	3.95556	3.94099	3.91220	3.88388	3.85602	3.82861
5	4.97512	4.95050	4.92611	4.90197	4.85438	4.80773	4.76197	4.71710
6	5.96272	5.92587	5.88944	5.85343	5.78264	5.71346	5.64583	5.57971
7	6.94785	6.89638	6.84560	6.79548	6.69719	6.60143	6.50813	6.41719
8	7.93052	7.86207	7.79464	7.72819	7.59821	7.47199	7.34939	7.23028
9	8.91074	8.82296	8.73661	8.65168	8.48593	8.32548	8.17014	8.01969
10	9.88852	9.77906	9.67158	9.56602	9.36052	9.16224	8.97087	8.78611
11	10.86386	10.73041	10.59958	10.47130	10.22218	9.98259	9.75206	9.53020
12	11.83677	11.67703	11.52067	11.36763	11.07112	10.78685	10.51421	10.25262
13	12.80725	12.61893	12.43491	12.25508	11.90751	11.57534	11.25776	10.95400
14	13.77532	13.55615	13.34235	13.13374	12.73153	12.34837	11.98318	11.63496
15	14.74096	14.48871	14.24302	14.00370	13.54338	13.10625	12.69091	12.29607
16	15.70420	15.41662	15.13699	14.86505	14.34323	13.84926	13.38138	12.93794
17	16.66504	16.33993	16.02431	15.71787	15.13126	14.57771	14.05500	13.56110