Chapter Ten

Intertemporal Choice

Intertemporal Choice

- Persons often receive income in "lumps"; *e.g.* monthly salary.
- How is a lump of income spread over the following month (saving now for consumption later)?
- Or how is consumption financed by borrowing now against income to be received at the end of the month?

Present and Future Values

Begin with some simple financial arithmetic.

Take just two periods; 1 and 2.

Let r denote the interest rate per period.

Future Value

E.g., if r = 0.1 then $100 saved at the start of period 1 becomes $110 at the start of period 2.

The value next period of $1 saved now is the **future value** of that dollar.

Future Value

Given an interest rate r the future value one period from now of $1 is

$$FV = 1 + r.$$

Given an interest rate r the future value one period from now of $m is

$$FV = m(1 + r).$$

Present Value

Suppose you can pay now to obtain $1 at the start of next period.

What is the most you should pay?

$1?

No. If you kept your $1 now and saved it then at the start of next period you would have $(1+r) > $1, so paying $1 now for $1 next period is a bad deal.

Present Value

Q: How much money would have to be saved now, in the present, to obtain \$1 at the start of the next period?

A: \$m saved now becomes \$m(1+r) at the start of next period, so we want the value of m for which

$$m(1+r) = 1$$

That is, $m = 1/(1+r)$,

the present-value of \$1 obtained at the start of next period.

Present Value

The present value of $1 available at the start of the next period is

$$PV = \frac{1}{1+r}.$$

And the present value of $m available at the start of the next period is

$$PV = \frac{m}{1+r}.$$

Present Value

E.g., if r = 0.1 then the most you should pay now for $1 available next period is

$$\mathbf{PV} = \frac{1}{1+0\cdot 1} = \$0\cdot 91.$$

And if r = 0.2 then the most you should pay now for $1 available next period is

$$\mathbf{PV} = \frac{1}{1+0\cdot 2} = \$0\cdot 83.$$

The Intertemporal Choice Problem

- Let m_1 and m_2 be incomes received in periods 1 and 2.
- Let c_1 and c_2 be consumptions in periods 1 and 2.
- Let p_1 and p_2 be the prices of consumption in periods 1 and 2.

The Intertemporal Choice Problem

The intertemporal choice problem:
Given incomes m_1 and m_2, and given consumption prices p_1 and p_2, what is the most preferred intertemporal consumption bundle (c_1, c_2)?

For an answer we need to know:

- **the intertemporal budget constraint**
- **intertemporal consumption preferences.**

The Intertemporal Budget Constraint

To start, let's ignore price effects by supposing that

$$p_1 = p_2 = \$1.$$

The Intertemporal Budget Constraint

Suppose that the consumer chooses not to save or to borrow.

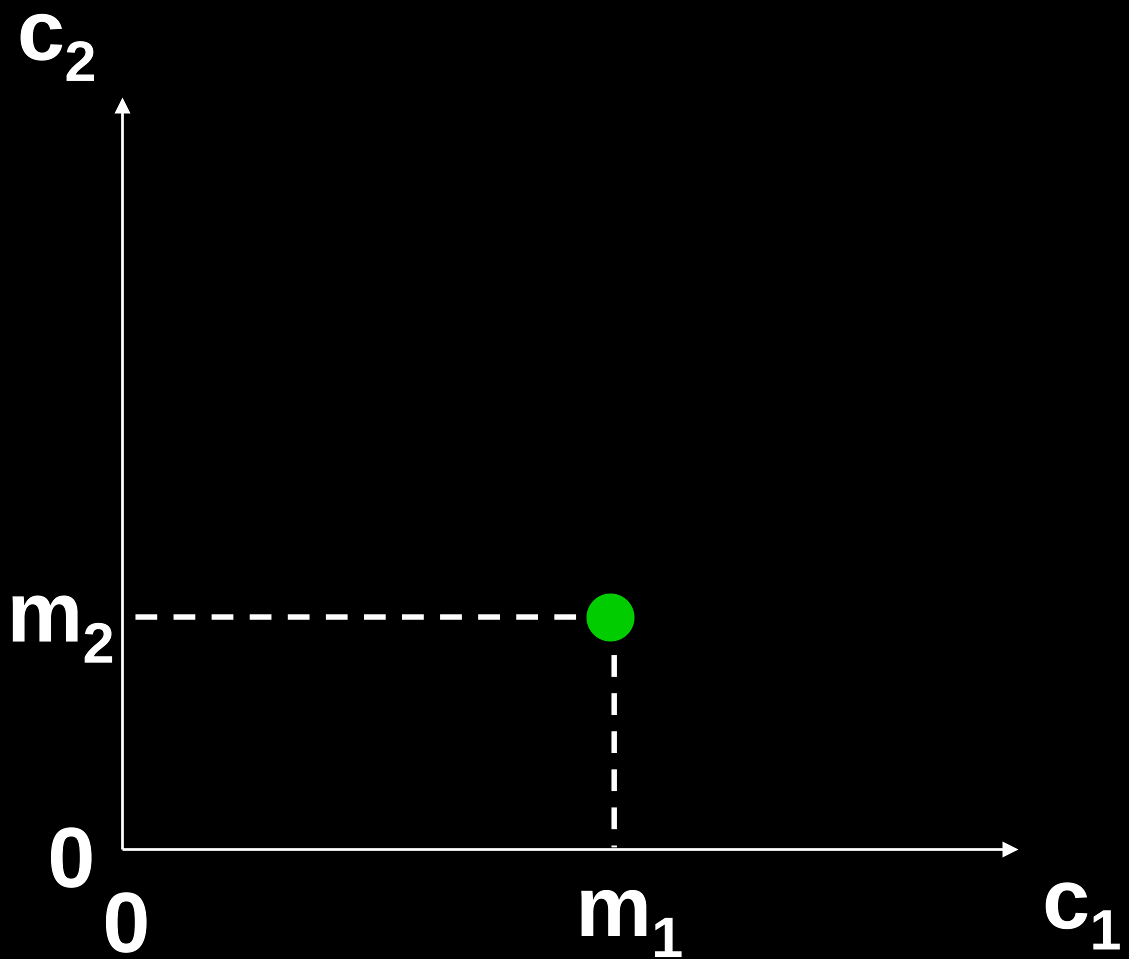
Q: What will be consumed in period 1?

A: $c_1 = m_1$.

Q: What will be consumed in period 2?

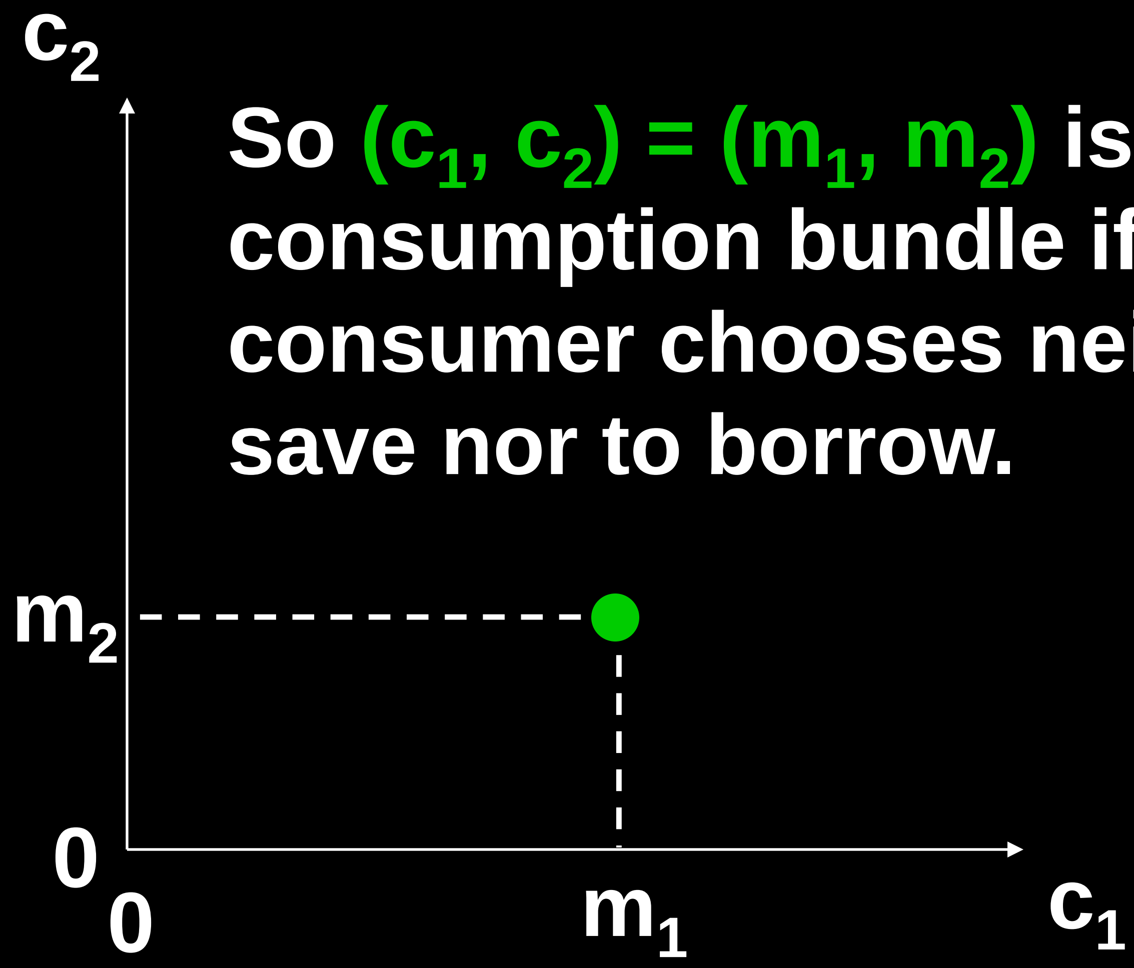
A: $c_2 = m_2$.

The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

So $(c_1, c_2) = (m_1, m_2)$ is the consumption bundle if the consumer chooses neither to save nor to borrow.



The Intertemporal Budget Constraint

Now suppose that the consumer spends nothing on consumption in period 1; that is, $c_1 = 0$ and the consumer saves

$$s_1 = m_1.$$

The interest rate is r.

What now will be period 2's consumption level?

The Intertemporal Budget Constraint

Period 2 income is m_2.

Savings plus interest from period 1 sum to $(1 + r)m_1$.

So total income available in period 2 is

$$m_2 + (1 + r)m_1.$$

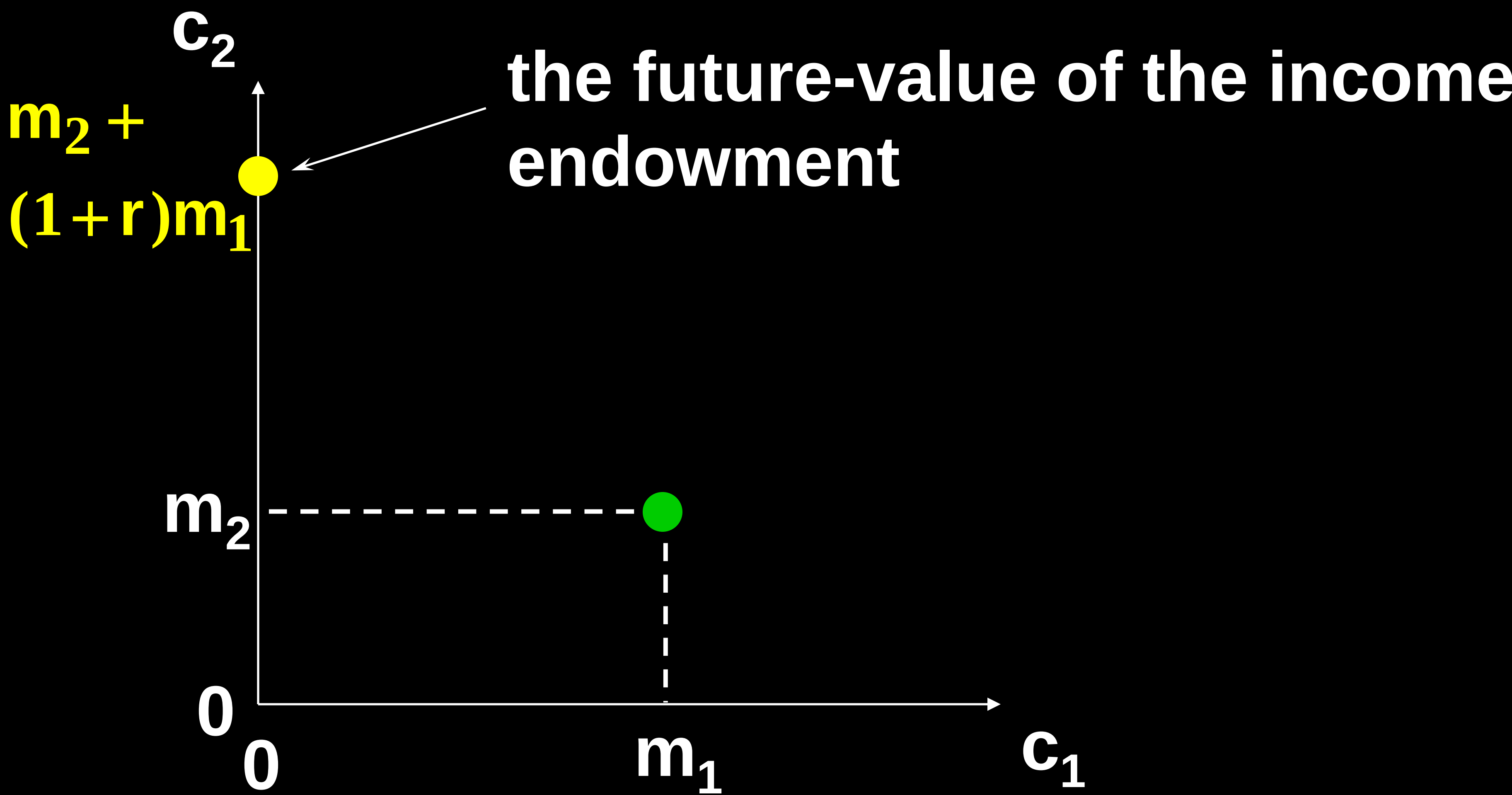
So period 2 consumption expenditure is

The Intertemporal Budget Constraint

- Period 2 income is m_2.
- Savings plus interest from period 1 sum to $(1 + r)m_1$.
- So total income available in period 2 is $m_2 + (1 + r)m_1$.
- So period 2 consumption expenditure is

$$c_2 = m_2 + (1 + r)m_1$$

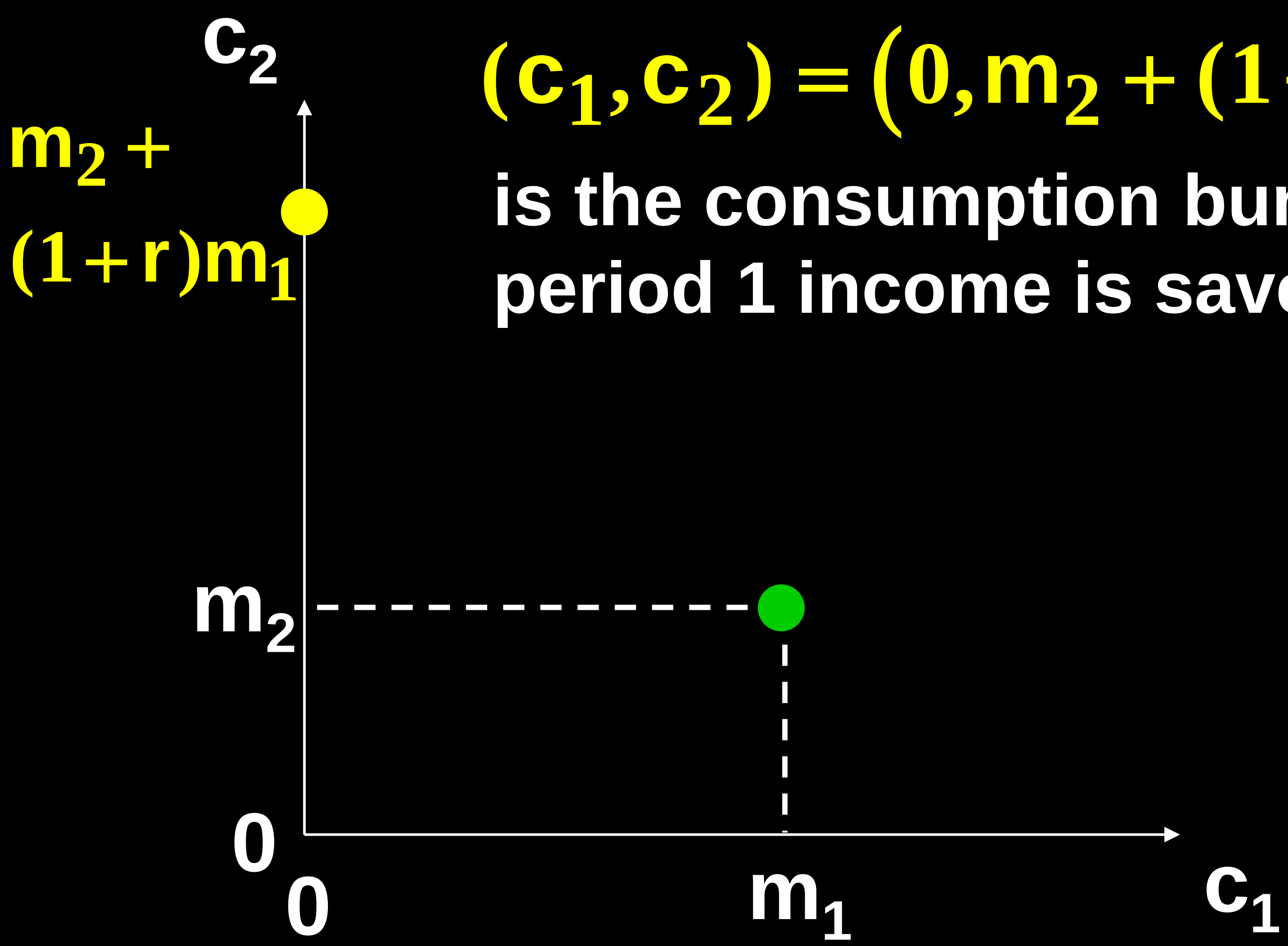
The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

$$(c_1, c_2) = \left(0, m_2 + (1+r)m_1\right)$$

is the consumption bundle when all period 1 income is saved.



The Intertemporal Budget Constraint

Now suppose that the consumer spends everything possible on consumption in period 1, so $c_2 = 0$.

What is the most that the consumer can borrow in period 1 against her period 2 income of \$$m_2$?

Let b_1 denote the amount borrowed in period 1.

The Intertemporal Budget Constraint

Only $\$m_2$ will be available in period 2 to pay back $\$b_1$ borrowed in period 1.

So $\quad b_1(1 + r) = m_2.$

That is, $\quad b_1 = m_2 / (1 + r).$

So the largest possible period 1 consumption level is

The Intertemporal Budget Constraint

Only \$$m_2$ will be available in period 2 to pay back \$$b_1$ borrowed in period 1.

So $b_1(1 + r) = m_2$.

That is, $b_1 = m_2 / (1 + r)$.

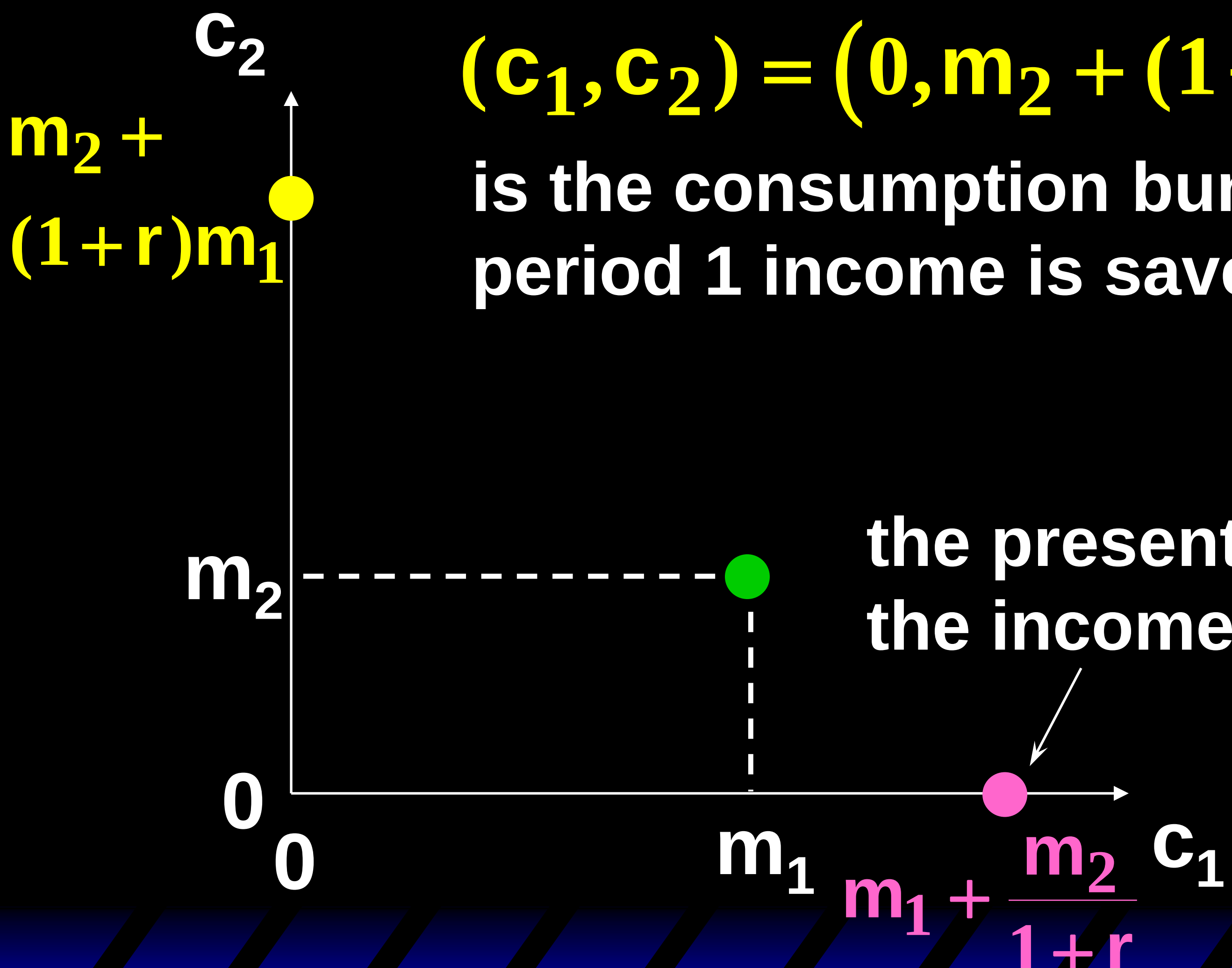
So the largest possible period 1 consumption level is

$$c_1 = m_1 + \frac{m_2}{1+r}$$

The Intertemporal Budget Constraint

$$(c_1, c_2) = \left(0, m_2 + (1+r)m_1\right)$$

is the consumption bundle when all period 1 income is saved.



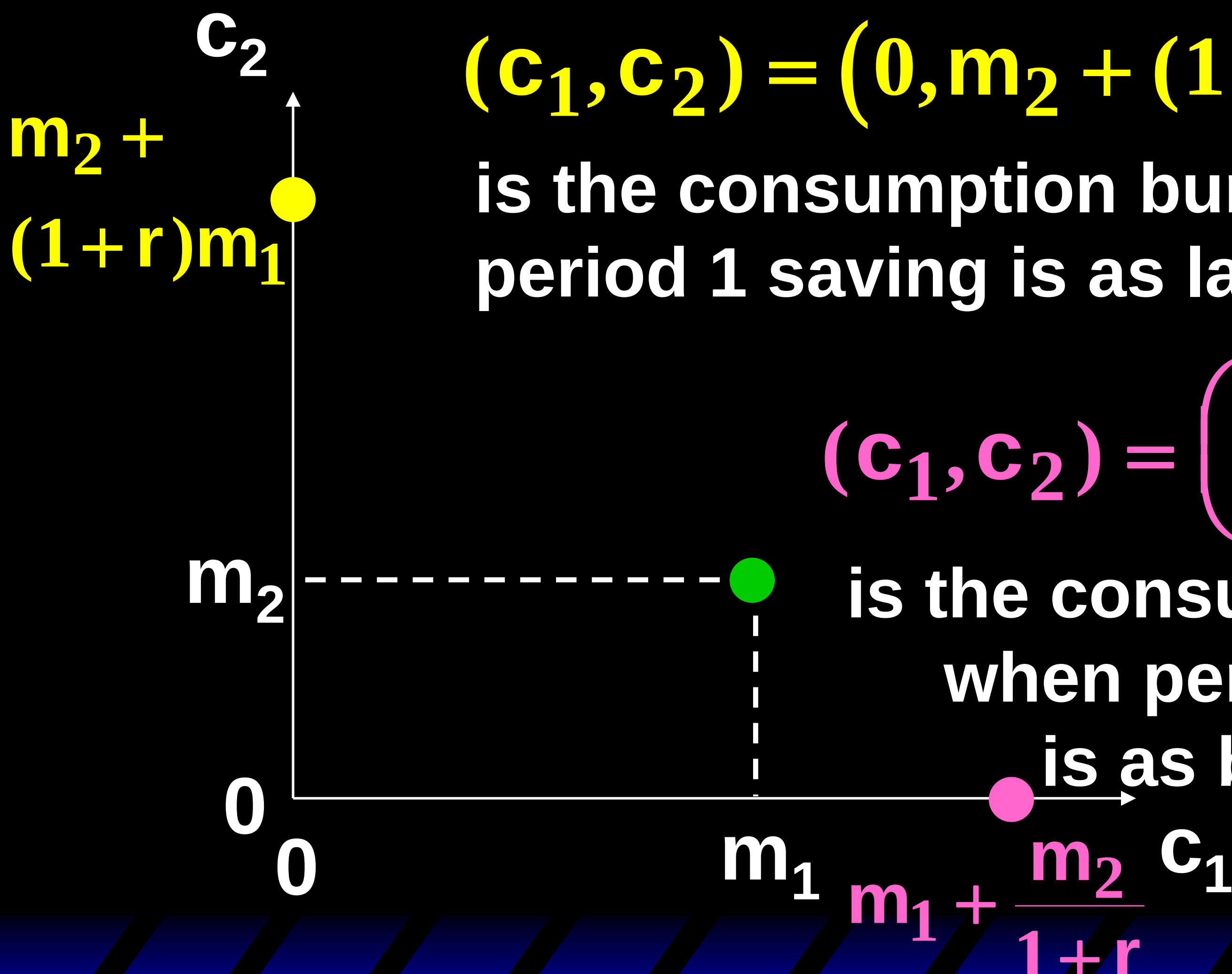
The Intertemporal Budget Constraint

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when period 1 saving is as large as possible.

$$(c_1, c_2) = \left(m_1 + \frac{m_2}{1+r}, 0\right)$$

is the consumption bundle when period 1 borrowing is as big as possible.



The Intertemporal Budget Constraint

Suppose that c_1 units are consumed in period 1. This costs $\$c_1$ and leaves m_1- c_1 saved. Period 2 consumption will then be

$$c_2 = m_2 + (1 + r)(m_1 - c_1)$$

The Intertemporal Budget Constraint

Suppose that c_1 units are consumed in period 1. This costs \$$c_1$ and leaves m_1- c_1 saved. Period 2 consumption will then be

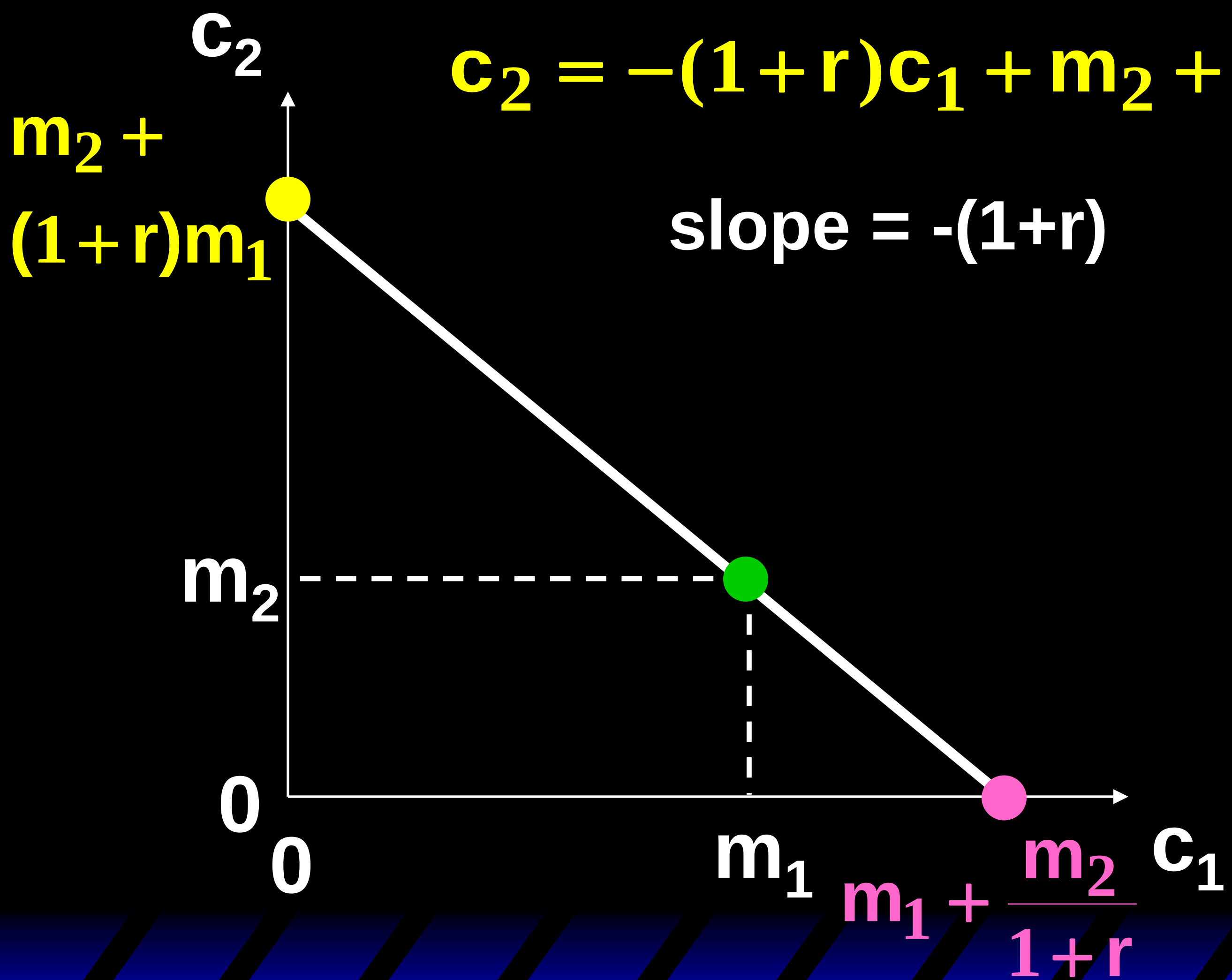
$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

which is

$$c_2 = \underbrace{-(1+r)}_{\text{slope}}c_1 + \underbrace{m_2 + (1+r)m_1}_{\text{intercept}}.$$

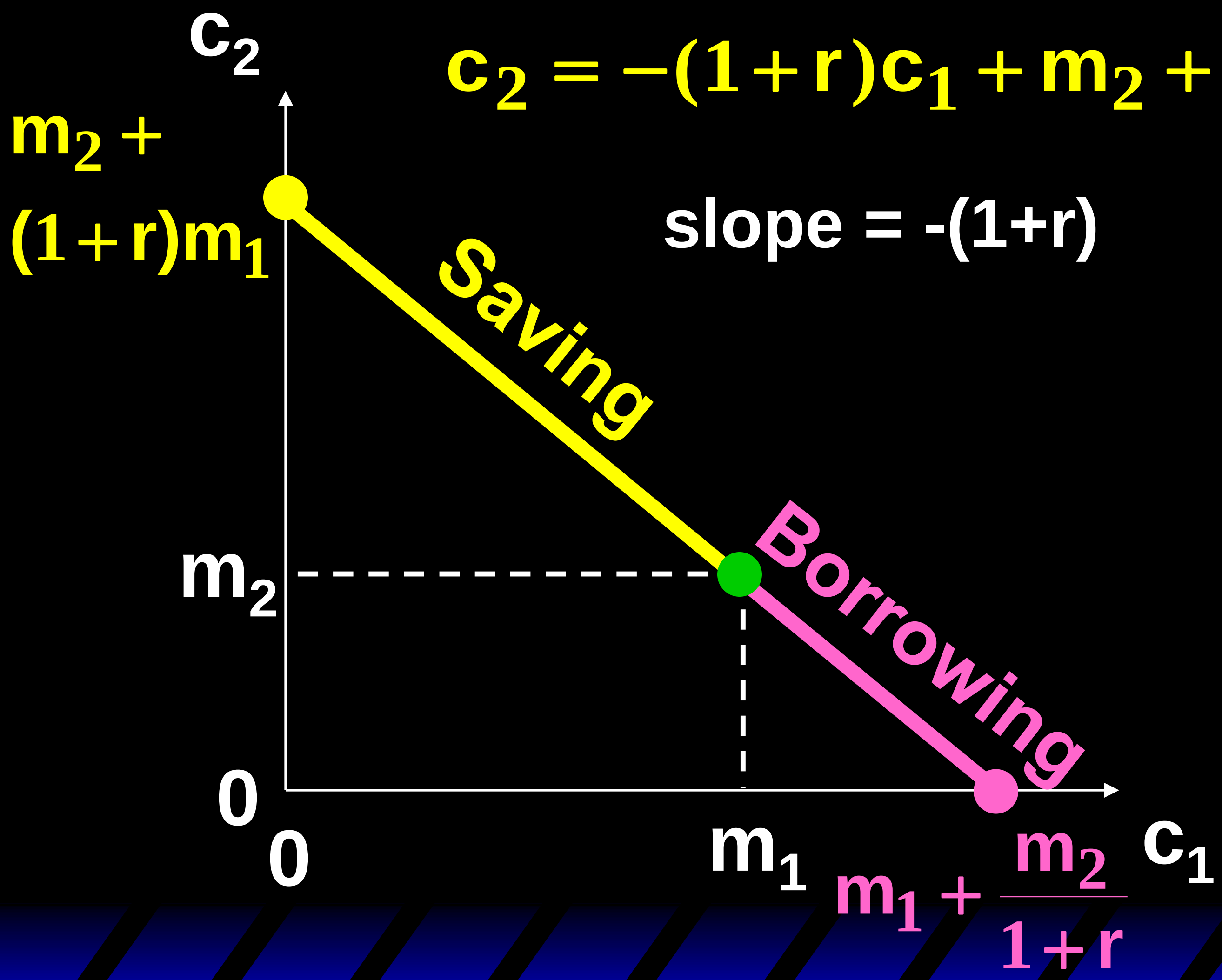
The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$



The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$



The Intertemporal Budget Constraint

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

is the "future-valued" form of the budget constraint since all terms are in period 2 values. This is equivalent to

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

which is the "present-valued" form of the constraint since all terms are in period 1 values.

The Intertemporal Budget Constraint

- Now let's add prices p_1 and p_2 for consumption in periods 1 and 2.
- How does this affect the budget constraint?

Intertemporal Choice

- Given her endowment (m_1,m_2) and prices p_1, p_2 what intertemporal consumption bundle (c_1^*,c_2^*) will be chosen by the consumer?
- Maximum possible expenditure in period 2 is $m_2 + (1+r)m_1$ so maximum possible consumption in period 2 is $$c_2 = \frac{m_2 + (1+r)m_1}{p_2}.$$

Intertemporal Choice

Similarly, maximum possible expenditure in period 1 is

$$m_1 + \frac{m_2}{1+r}$$

so maximum possible consumption in period 1 is

$$c_1 = \frac{m_1 + m_2/(1+r)}{p_1}.$$

Intertemporal Choice

Finally, if c_1 units are consumed in period 1 then the consumer spends p_1c_1 in period 1, leaving $m_1 - p_1c_1$ saved for period 1. Available income in period 2 will then be

$$m_2 + (1+r)(m_1 - p_1c_1)$$

so

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1).$$

Intertemporal Choice

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1)$$

rearranged is

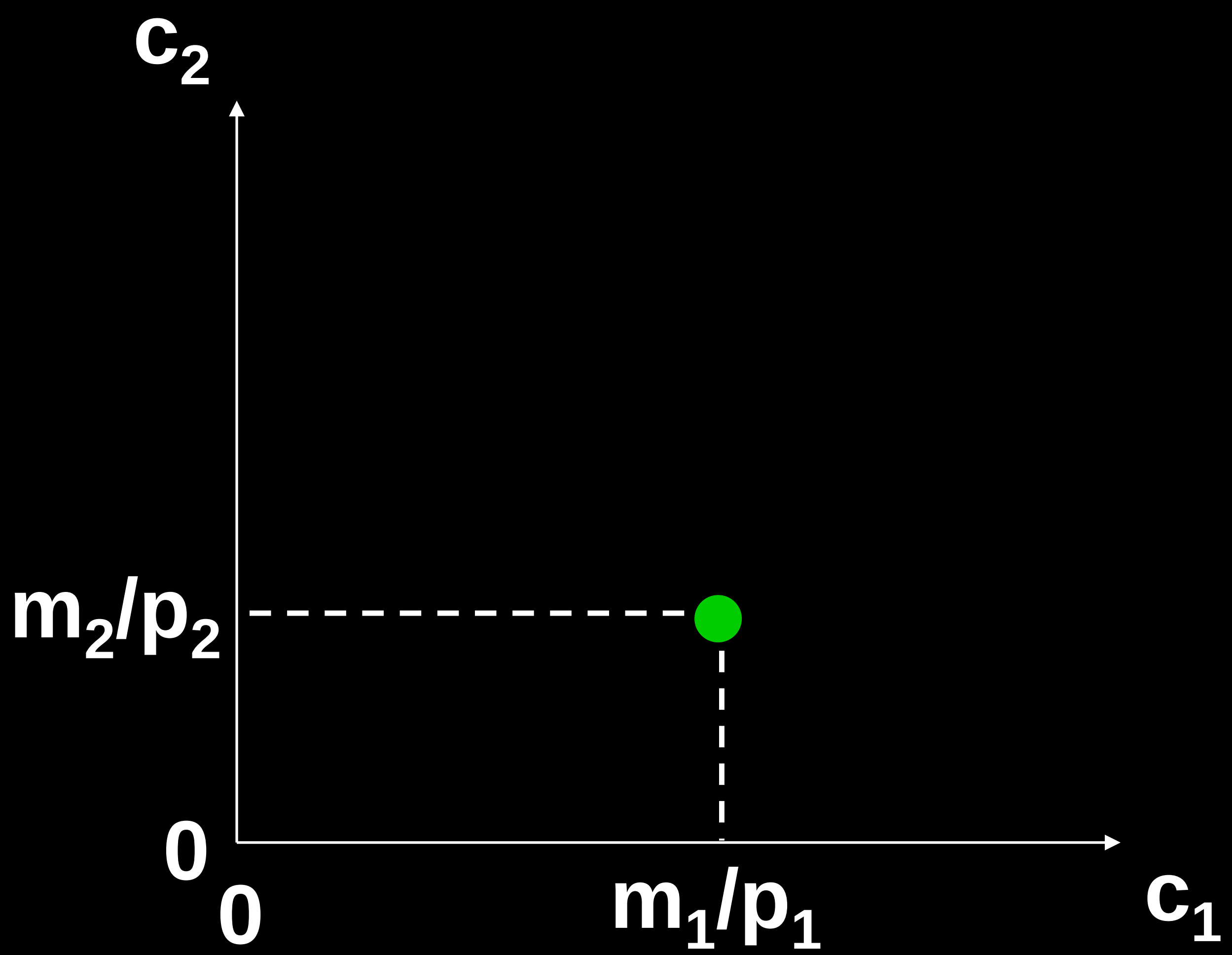
$$(1+r)p_1c_1 + p_2c_2 = (1+r)m_1 + m_2.$$

This is the "future-valued" form of the budget constraint since all terms are expressed in period 2 values. Equivalent to it is the "present-valued" form

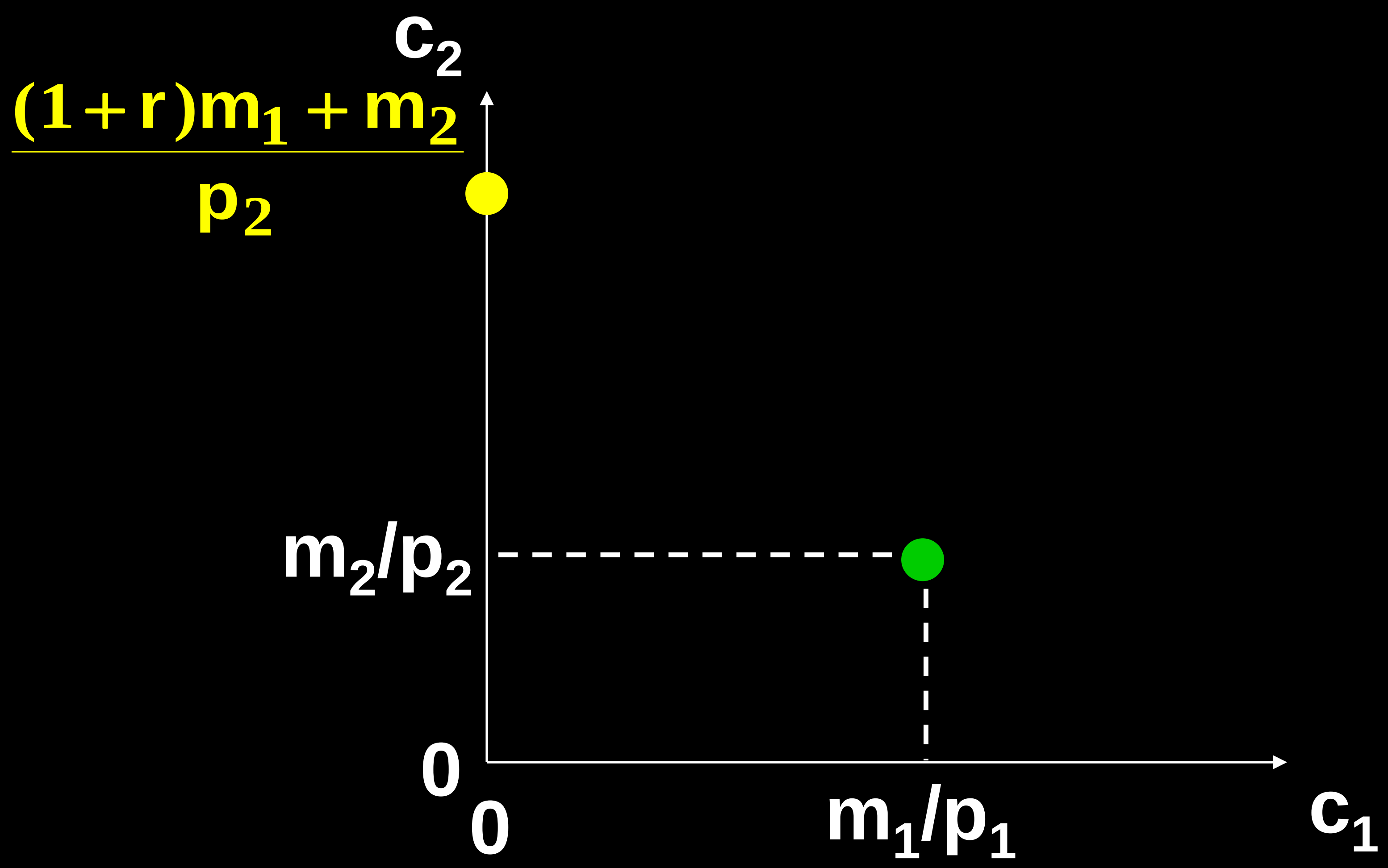
$$p_1c_1 + \frac{p_2}{1+r}c_2 = m_1 + \frac{m_2}{1+r}$$

where all terms are expressed in period 1 values.

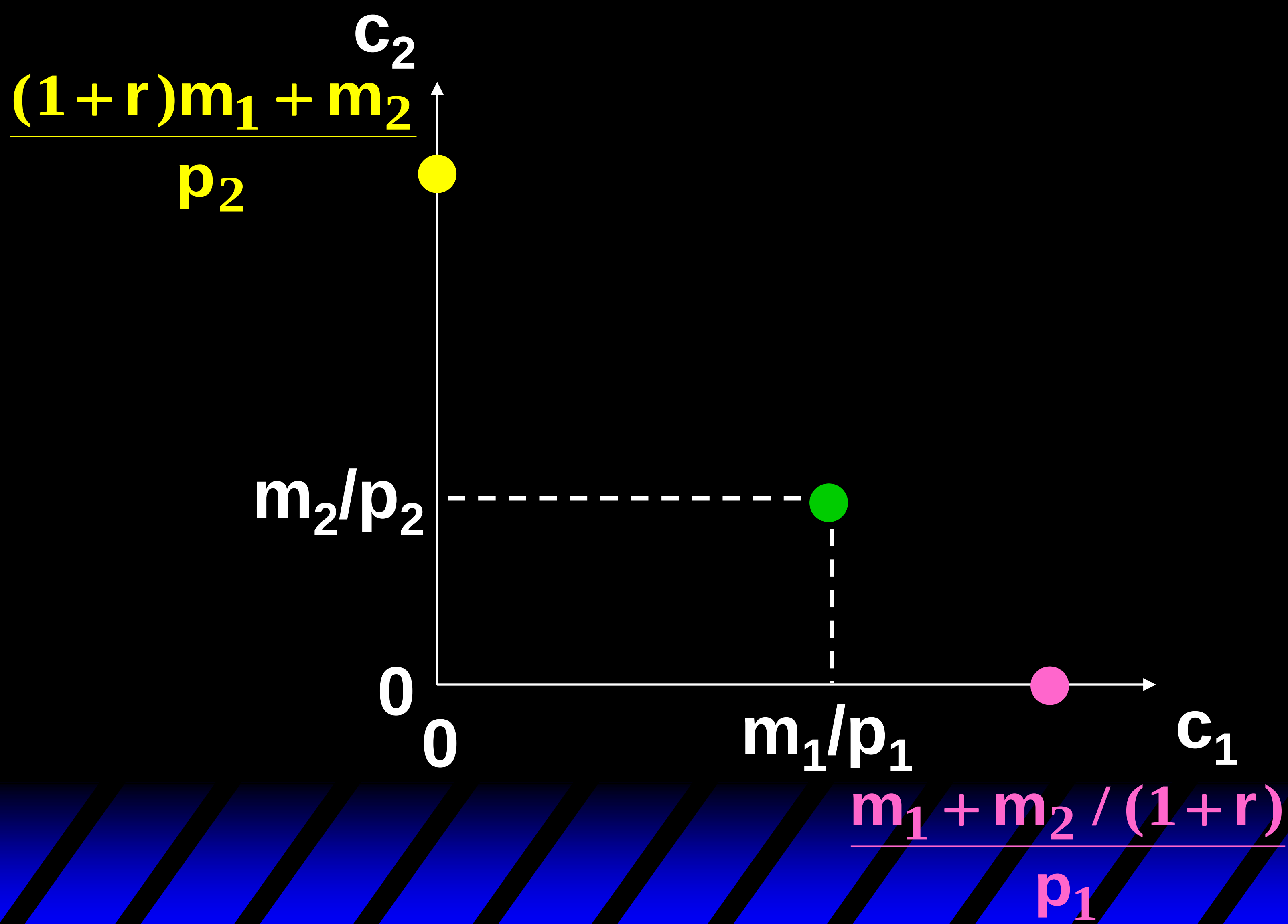
The Intertemporal Budget Constraint



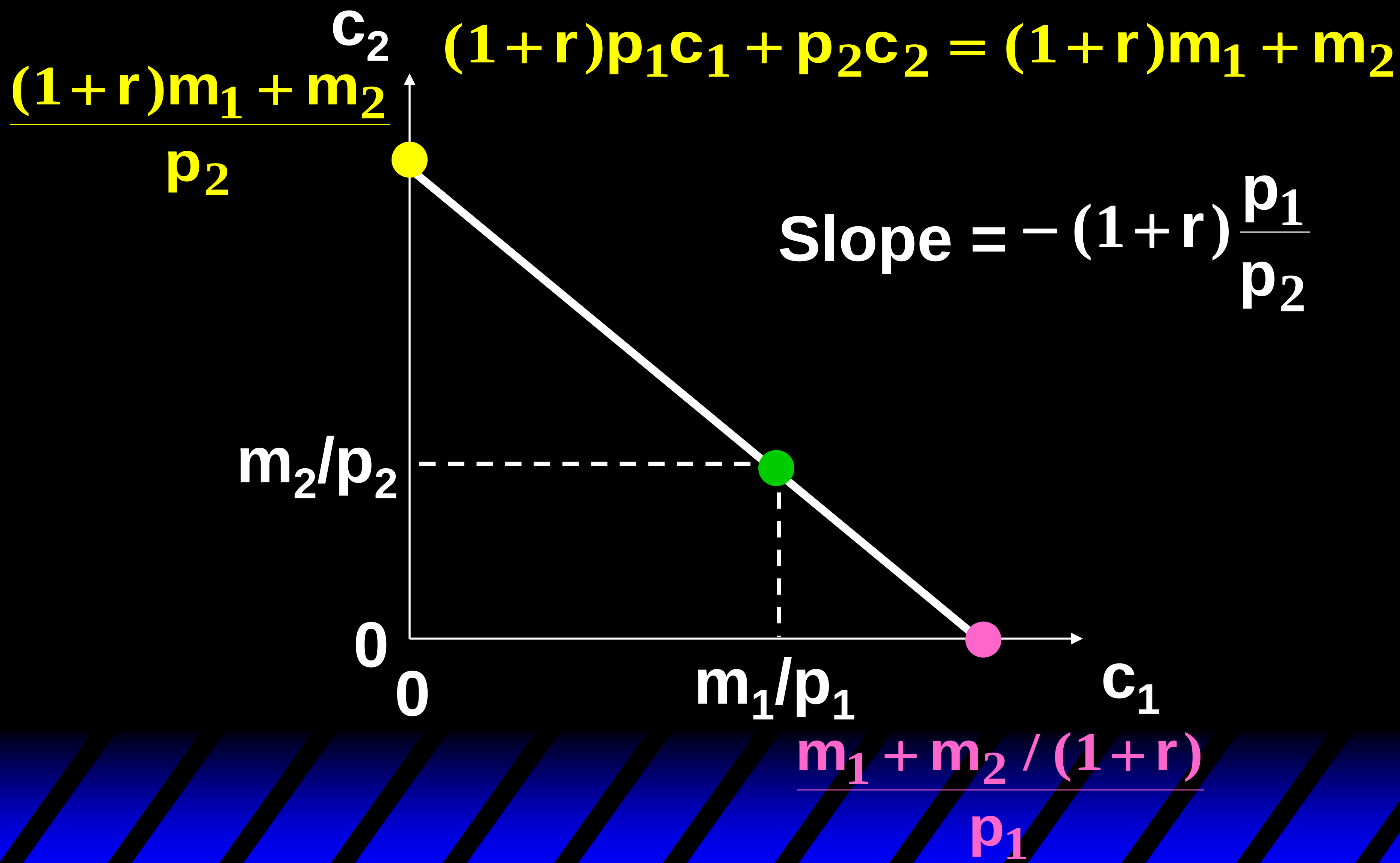
The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

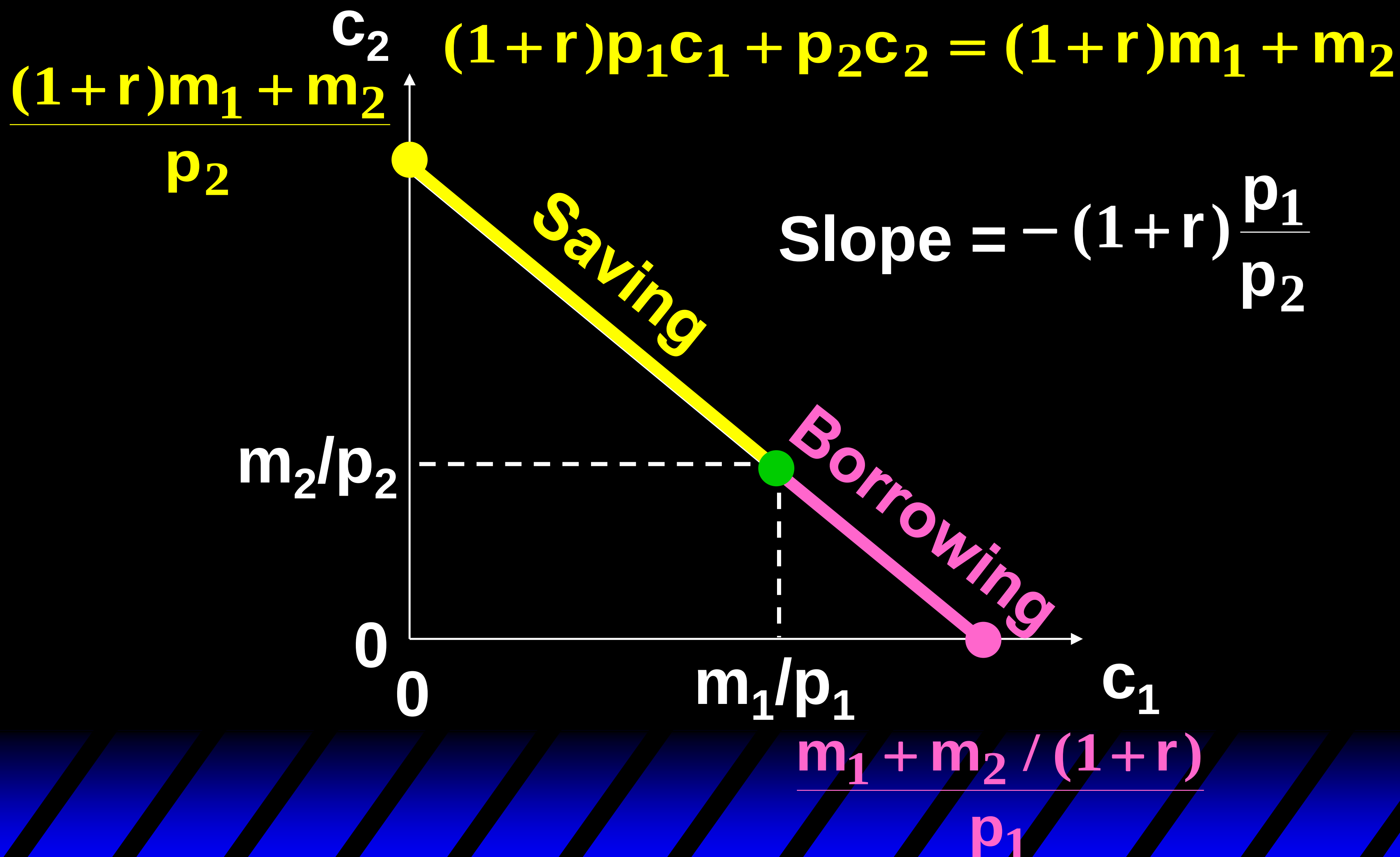


The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

$$(1+r)p_1c_1 + p_2c_2 = (1+r)m_1 + m_2$$



Price Inflation

Define the inflation rate by π where

$$p_1(1+\pi) = p_2.$$

For example,

π = 0.2 means 20% inflation, and

π = 1.0 means 100% inflation.

Price Inflation

We lose nothing by setting $p_1=1$ so that $p_2 = 1+\pi$.

Then we can rewrite the budget constraint

$$p_1 c_1 + \frac{p_2}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

as

$$c_1 + \frac{1+\pi}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

Price Inflation

$$c_1 + \frac{1+\pi}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

rearranges to

$$c_2 = -\frac{1+r}{1+\pi} c_1 + (1+\pi)\left(\frac{m_1}{1+r} + m_2\right)$$

so the slope of the intertemporal budget constraint is

$$-\frac{1+r}{1+\pi}.$$

Price Inflation

When there was no price inflation (p_1=p_2=1) the slope of the budget constraint was -(1+r).

Now, with price inflation, the slope of the budget constraint is -(1+r)/(1+ π). This can be written as

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

ρ is known as the real interest rate.

Real Interest Rate

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

gives

$$\rho = \frac{r-\pi}{1+\pi}.$$

For low inflation rates ($\pi \approx 0$), $\rho \approx r - \pi$. For higher inflation rates this approximation becomes poor.

Real Interest Rate

r	0.30	0.30	0.30	0.30	0.30
π	0.0	0.05	0.10	0.20	1.00
$r - \pi$	0.30	0.25	0.20	0.10	-0.70
ρ	0.30	0.24	0.18	0.08	-0.35

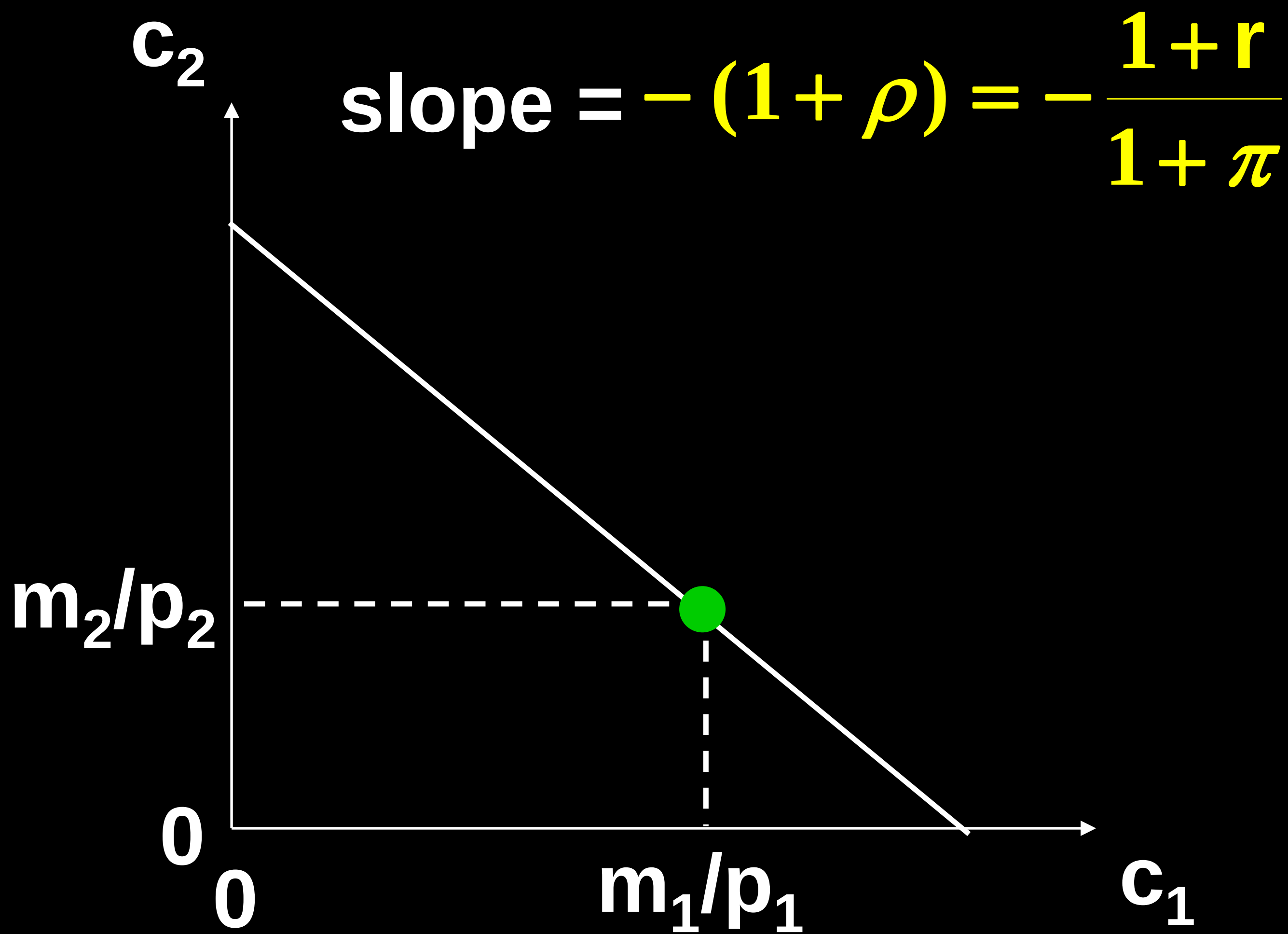
Comparative Statics

The slope of the budget constraint is

$$-(1+\rho) = -\frac{1+r}{1+\pi}.$$

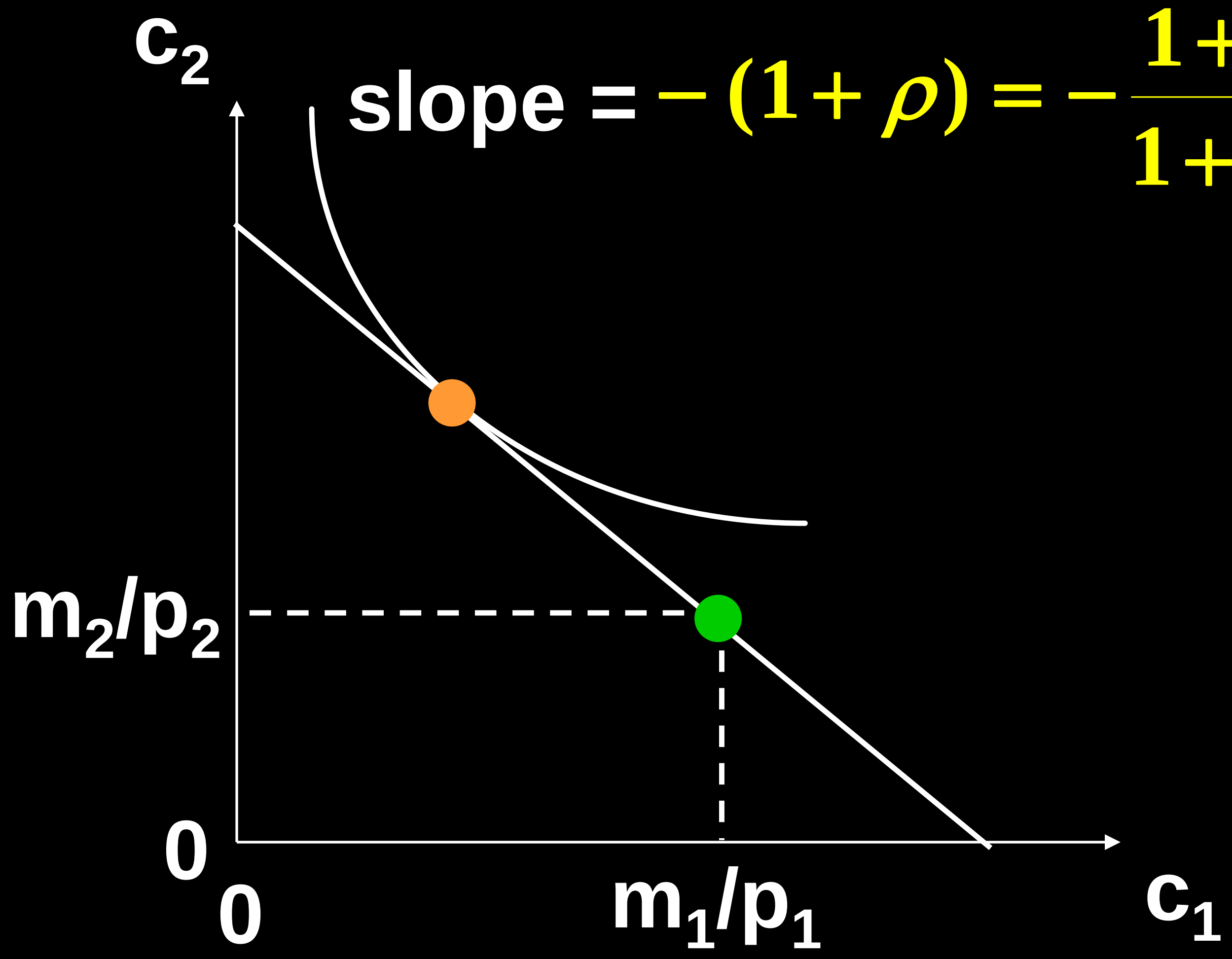
The constraint becomes flatter if the interest rate r falls or the inflation rate π rises (both decrease the real rate of interest).

Comparative Statics



Comparative Statics

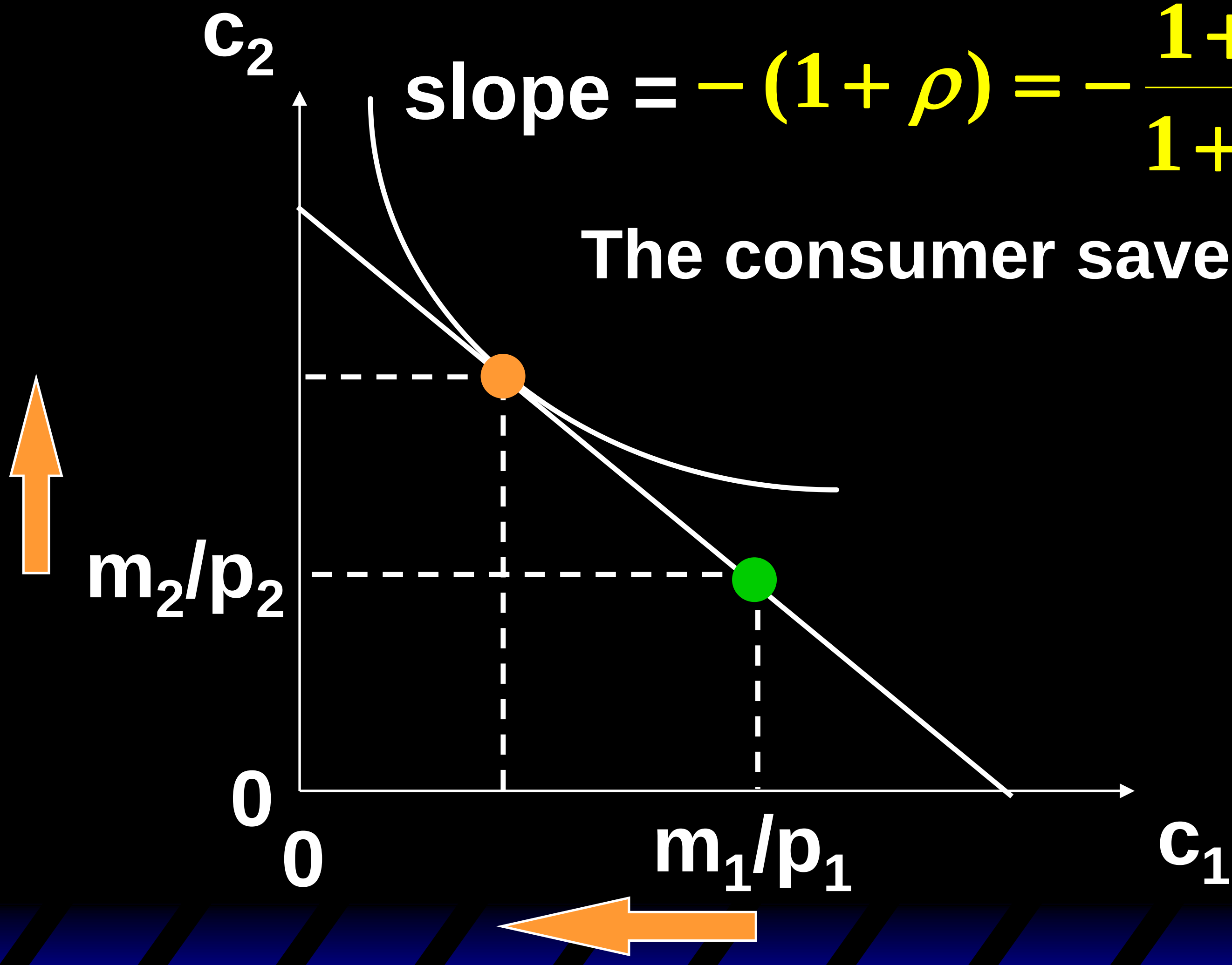
$$\text{slope} = -(1+\rho) = -\frac{1+r}{1+\pi}$$



Comparative Statics

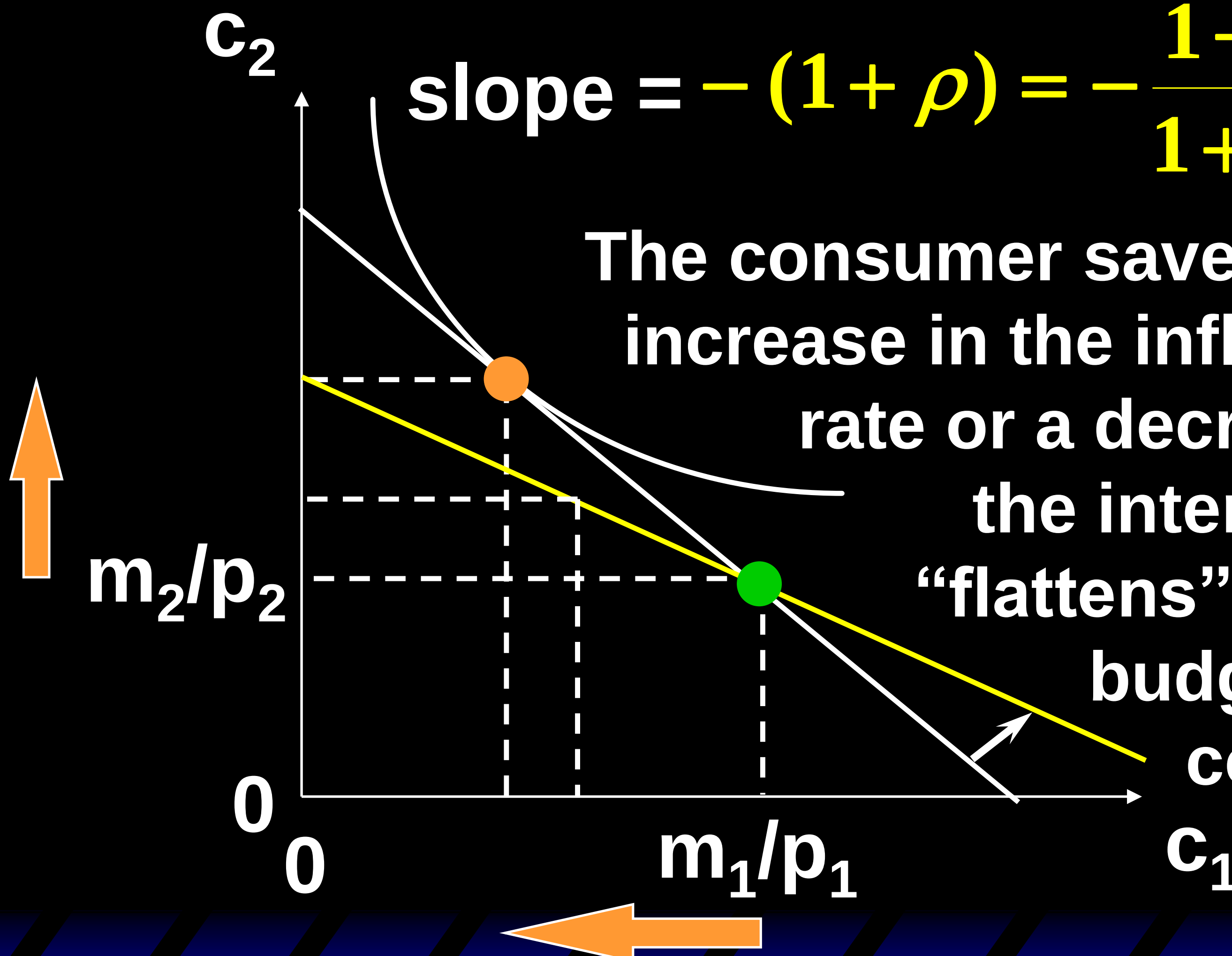
slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$

The consumer saves.



Comparative Statics

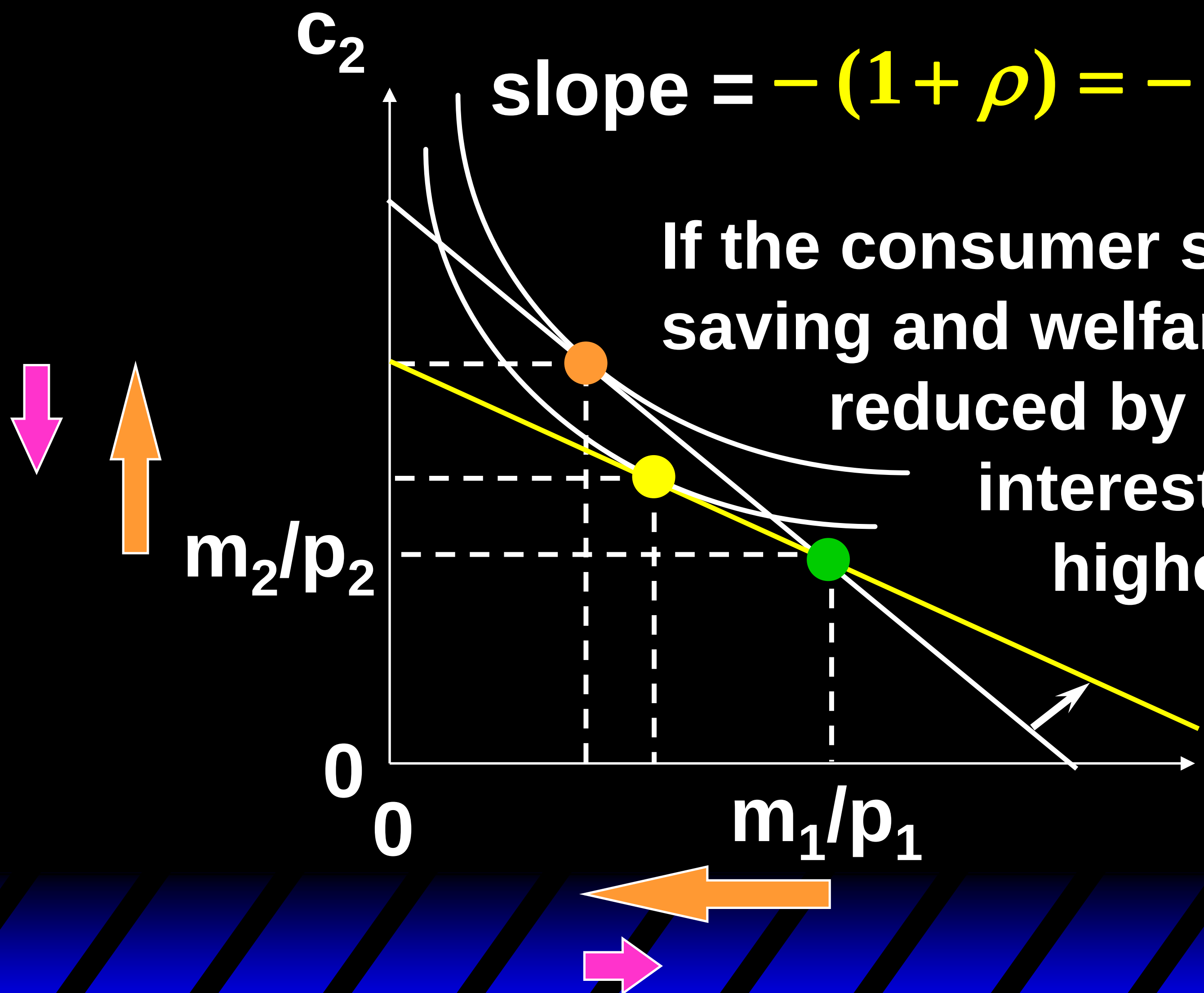
slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



The consumer saves. An increase in the inflation rate or a decrease in the interest rate "flattens" the budget constraint.

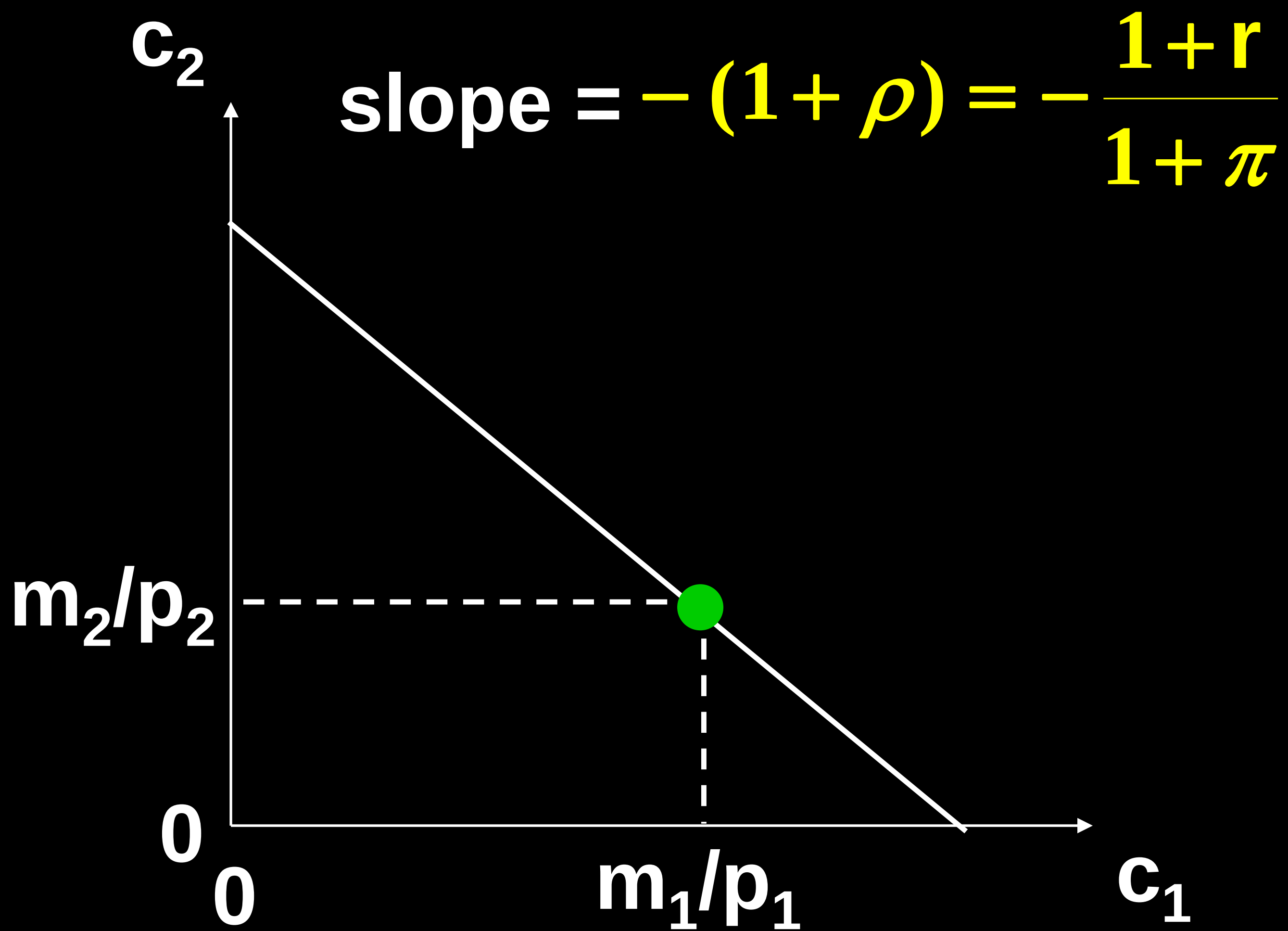
Comparative Statics

slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



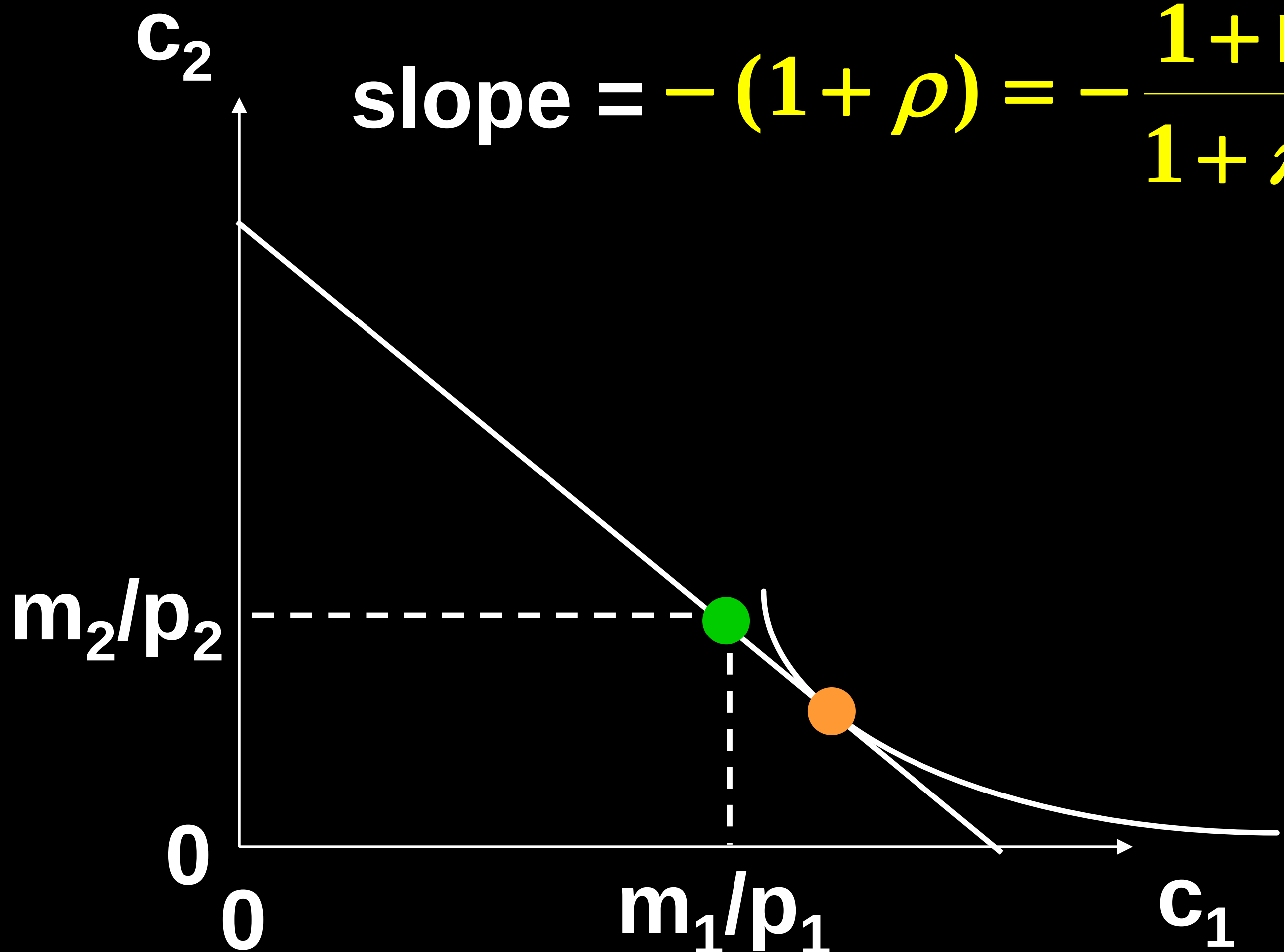
If the consumer saves then saving and welfare are reduced by a lower interest rate or a higher inflation rate.

Comparative Statics



Comparative Statics

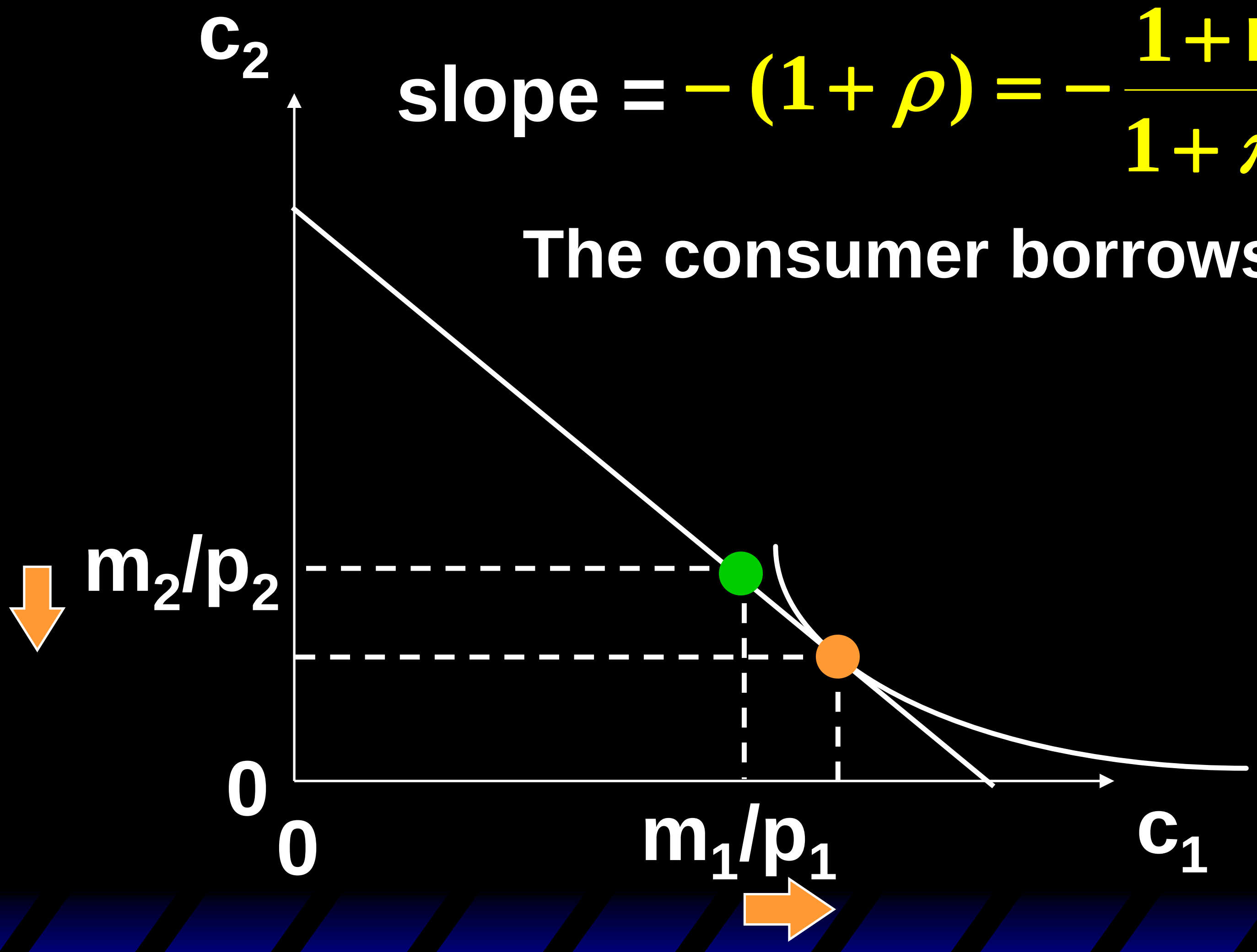
slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



Comparative Statics

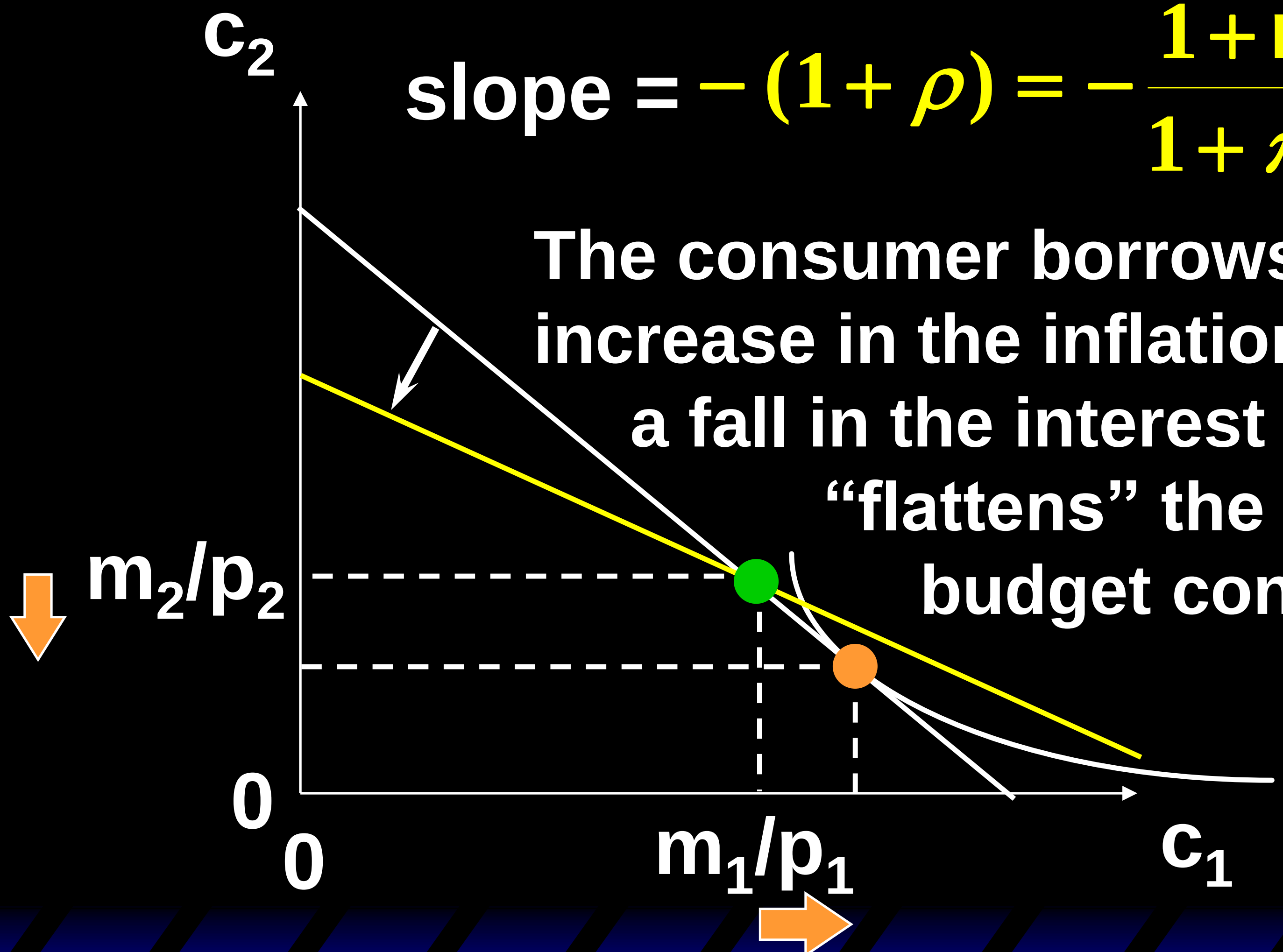
slope = $-(1+\rho) = -\dfrac{1+r}{1+\pi}$

The consumer borrows.



Comparative Statics

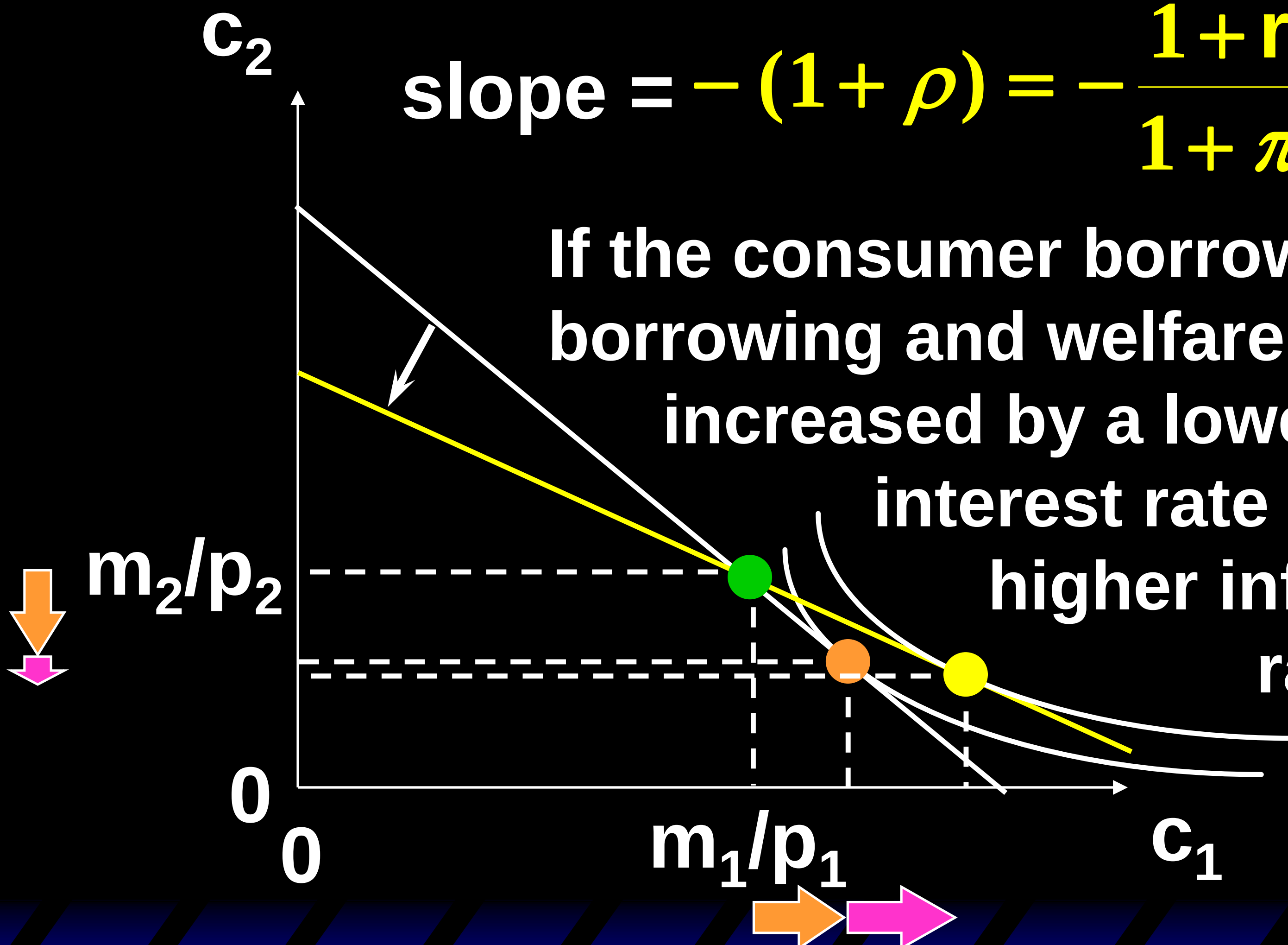
slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



The consumer borrows. A increase in the inflation rate or a fall in the interest rate "flattens" the budget constraint.

Comparative Statics

slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



If the consumer borrows then borrowing and welfare are increased by a lower interest rate or a higher inflation rate.

Summary

	r increase		**r decrease**	
	Borrower	Saver	Borrower	Saver
Price of C1 relative to C2	More expensive	More expensive	cheaper	cheaper
Budget Line	steeper	steeper	flatter	flatter
SE of C1	<0	<0	>0	>0
IE of C1	<0	>0	>0	<0
TE of C1	<0	?	>0	?
SE of C2	>0	>0	<0	<0
IE of C2	<0	>0	>0	<0
TE of C2	?	>0	?	<0
Overall	?	Saver	Borrower	?

Valuing Securities

A **financial security** is a financial instrument that promises to deliver an income stream.

E.g.; a security that pays

$m_1 at the end of year 1,
$m_2 at the end of year 2, and
$m_3 at the end of year 3.

What is the most that should be paid now for this security?

Valuing Securities

The security is equivalent to the sum of three securities;

- the first pays only $\$m_1$ at the end of year 1,
- the second pays only $\$m_2$ at the end of year 2, and
- the third pays only $\$m_3$ at the end of year 3.

Valuing Securities

The PV of $\$m_1$ paid 1 year from now is

$$m_1 / (1+r)$$

The PV of $\$m_2$ paid 2 years from now is

$$m_2 / (1+r)^2$$

The PV of $\$m_3$ paid 3 years from now is

$$m_3 / (1+r)^3$$

The PV of the security is therefore

$$m_1 / (1+r) + m_2 / (1+r)^2 + m_3 / (1+r)^3.$$

Valuing Bonds

A **bond** is a special type of security that pays a fixed amount \$x for T years (its **maturity date**) and then pays its **face value** \$F.

What is the most that should now be paid for such a bond?

Valuing Bonds

End of Year	1	2	3	…	T-1	T
Income Paid	\$x	\$x	\$x	\$x	\$x	\$F
Present -Value	$\frac{\$x}{1+r}$	$\frac{\$x}{(1+r)^2}$	$\frac{\$x}{(1+r)^3}$	…	$\frac{\$x}{(1+r)^{T-1}}$	$\frac{\$F}{(1+r)^T}$

$$\text{PV} = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots + \frac{x}{(1+r)^{T-1}} + \frac{F}{(1+r)^T}.$$

Valuing Bonds

Suppose you win a State lottery. The prize is $1,000,000 but it is paid over 10 years in equal installments of $100,000 each. What is the prize actually worth?

Valuing Bonds

$$\text{PV} = \frac{\$100{,}000}{1+0\cdot 1} + \frac{\$100{,}000}{(1+0\cdot 1)^2} + \ldots + \frac{\$100{,}000}{(1+0\cdot 1)^{10}}$$

$$= \$614{,}457$$

is the actual (present) value of the prize.

Valuing Consols

A **consol** is a bond which never terminates, paying $x per period forever.

What is a consol's present-value?

Valuing Consols

End of Year	1	2	3	…	t	…
Income Paid	\$x	\$x	\$x	\$x	\$x	\$x
Present-Value	$\frac{\$x}{1+r}$	$\frac{\$x}{(1+r)^2}$	$\frac{\$x}{(1+r)^3}$	…	$\frac{\$x}{(1+r)^t}$	…

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots + \frac{x}{(1+r)^t} + \ldots.$$

Valuing Consols

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots$$

$$= \frac{1}{1+r}\left[x + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots\right]$$

$$= \frac{1}{1+r}[x + PV].$$

Solving for PV gives

$$PV = \frac{x}{r}.$$

Valuing Consols

E.g. if r = 0.1 now and forever then the most that should be paid now for a console that provides $1000 per year is

$$\mathbf{PV = \frac{x}{r} = \frac{\$1000}{0 \cdot 1} = \$10{,}000.}$$