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Film flow thickness along the outer surface of rotating cones

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ABSTRACT

A thin liquid film flow goes fully upward along the outer surface of a rotating cone, when the cone is immersed in the liquid, turned upside down and rotated. We derive equations for the velocities and the film thickness of the rising film flow by using the boundary layer theory, and solve them analytically and numerically. The film thickness is obtained analytically in the centrifugal zone, while it is numerically in the Coriolis zone where two different branches with a turning point are found. It is proven that the upper branch is unstable for normalized film thickness $\delta^+ > 3$ from Rayleigh's criterion.

1. Introduction

In the atomization process of a liquid, there are two types of atomizers. One is an atomizer that uses a jet flow, while the other is a centrifugal atomizer that uses the centrifugal force. In the former devices, it is common to use a liquid jet driven through a nozzle at high pressure generated by a compressor. However, the system based on the liquid jet tends to become large because fans, pumps, etc. as well as compressors are necessary for operating it. On the other hand, the latter centrifugal atomizers are quite compact and make it easier to control the characteristics of the atomization such as droplet diameter, quantity of the mist flow composed of the atomized droplets, etc. In the centrifugal atomizers, the droplets are generated by breaking the film flow released from a rotating device or by thinning it not to keep its filmwise condition, where the film thickness as well as the corresponding droplet diameter and the quantity can be adjusted by changing the rotation rate. Therefore, it is important to control the film thickness on the centrifugal atomizers by knowing the relation between the thickness, the rotation rate, etc. for various practical applications.

There are many types of centrifugal atomizers. It is well known, for example, that the liquid rises along the inner surface of a rotating hollow cone due to the centrifugal force. The fluid migrates up the internal wall of the cone under the centrifugal force generating the thin liquid film flow. Bruin [1] has analytically obtained velocity distributions and the film thickness in a liquid film over a rotating conical inner surface. He used the boundary layer theory assuming the film thickness δ is much smaller than the representative radius r_0 such that $r_0 \gg \delta$, where he did not show the representative length scale explicitly. Makarytchev el al.[2] have modified Bruin's model by introducing a better normalization considering the multiplier $\sin\beta$, where β is a half tip angle of cone. They obtained a simpler form of velocity profiles. It should be noted that the results of both Bruin and Makarytchev et al. were the same in a centrifugal zone in which the centrifugal force is dominant, while the results are different from each other in a Coriolis zone in which the Coriolis force is also dominant in addition to the centrifugal force. This is because their normalizations were different from each other. Bruin and Makarytchev et al. obtained the film thickness in the centrifugal zone, but did not explicitly obtain it in the Coriolis zone. This is mainly because the equation for the film thickness in the Coriolis zone is a nonlinear equation and difficult to solve. Furthermore, Makarytchev et al. [3] have studied the structure and regimes of liquid film flow in spinning cone columns. They classified the flow regimes such as the inner inlet-dependent, the intermediate Coriolis, and the outer centrifugal zones using a radial length scale r_0, where they proposed a normalization of the radial scale r_0 at which the thickness of the film becomes equal to the Ekman thickness. Not only for theoretical observation but Makarytchev et al. [4] have also performed an experiment to measure the film thickness. They used a method of the intensity of induced fluorescence illustrated by an ultra violet light source, and obtained the film thickness of wavy liquid film flow on rotating conical inner surfaces, where the film thickness is larger than the thickness in the centrifugal and Coriolis zones. In addition, Symons and Bizard [5] also have performed an experiment to measure the fluid flow thickness within a rotating cone by using an optical

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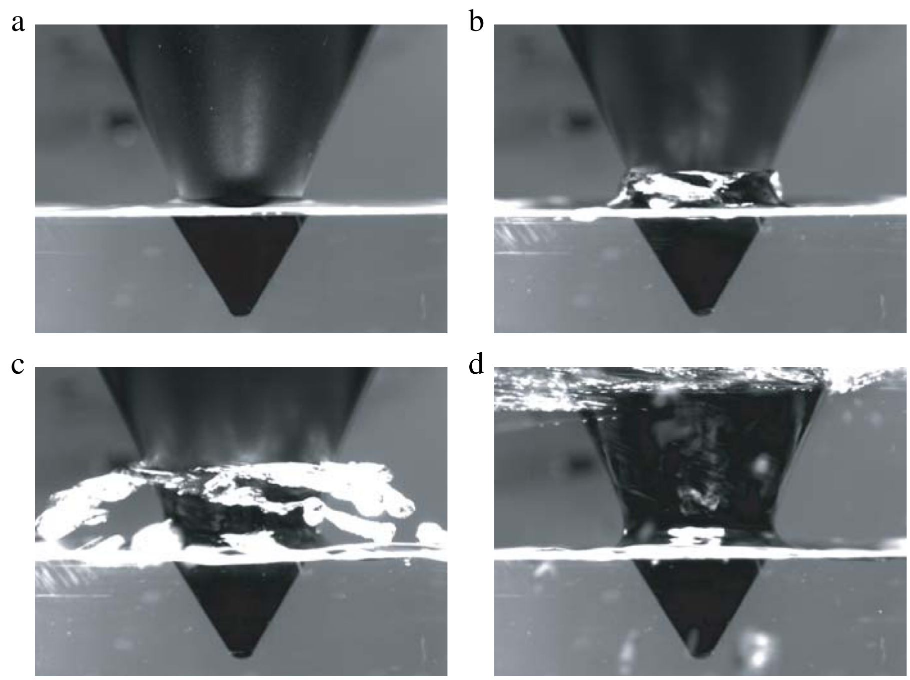

Fig. 1. Visualization photographs of rising film flow.

Nomenclature

E:	Ekman number
Fr:	Froude number
g:	Gravitational acceleration
P:	Pressure
Q_0:	Volumetric flow rate
Q_0^+:	Dimensionless volumetric flow rate
Ro:	Rossby number
r:	Coordinate along the cone surface from apex
r_0:	Distance from apex, representative length scale
s:	Coordinate perpendicular to the cone surface

Greek symbols

β:	Half tip angle of cone
δ:	Film thickness (perpendicular to the cone surface)
δ_E:	Ekman thickness
δ^+:	Dimensionless film thickness
η:	Dimensionless coordinate in the r direction
θ:	Polar angle from the axis in spherical coordinate system
ν:	Kinematic viscosity
ρ:	Density
σ:	Dimensionless coordinate in the s direction
ϕ:	Tangential angle in spherical coordinate system
ω:	Rotation rate of the cone

method. They used a high viscosity fluid and examined the effect of gravity in the centrifugal zone.

Contrary to the film flow within the rotating hollow cone, Adachi et al. [6] have found an unique flow phenomena. Namely, a thin liquid film flow rising along the outer surface of the rotating cone is generated when the cone is immersed and rotates in the liquid by turning the top upside down. Fig. 1 shows the flow visualization of the phenomena with a high-speed video camera, where the rotation rate of the cone is gradually changed from 0 to 6000 rpm. Fig. 1(a) shows an initial state where the cone is in a state of rest. Once the cone begins to rotate, water is deformed and lifted up at the vicinity of the cone surface as seen in Fig. 1(b). However, the water does not go up anymore at that time because the rotation rate is small. When the rotation rate is further increased, the deformation of the water becomes larger as seen in Fig. 1(c), and the water surface winding around the cone rises higher due to the larger degree of the deformation caused by the lifting up. After that, a mass of water is scattered out in radial and tangential directions. Subsequently, a thin liquid film flow is generated and rises along the outer surface on the cone as seen in Fig. 1(d). We call the flow phenomena a pumping-up mechanism caused by the thin liquid film flow. Indeed, it is comprehensible that the liquid rises along the inner surface of a rotating hollow cone due to the centrifugal force but there is only the research of Adachi et al. [6] on the phenomena that the liquid rises along the outer surface of the rotating cone and does not separate from the surface. Recently, Adachi [7] has applied the pumping-up mechanism into oxygen mass transfer and showed that the mist flow composed of atomized water droplets generated by the mechanism is effective for aeration.

In this paper, we focus on the liquid film flow over the outer surface of a rotating cone and derive the equations for the velocity profiles and the film thickness of the rising film flow by using the boundary layer theory. Particular attention is paid to the film thickness. We propose an appropriate nondimensional parameter to express the film flow thickness in both the centrifugal and the Coriolis zones.

2. Mathematical formulation

We consider a laminar steady-state flow around the cone as shown in Fig. 2. Since the water is lifted up in the vicinity of the cone surface when the cone rotates, the water level sinks and the lifting up of the film flow is interrupted. Therefore, in order to generate the rising film flow steadily, we need to supply water. If we supply the water with the flow rate Q_0 shown in Fig. 2(b), the flow rate of the rising film flow balances with the water supply

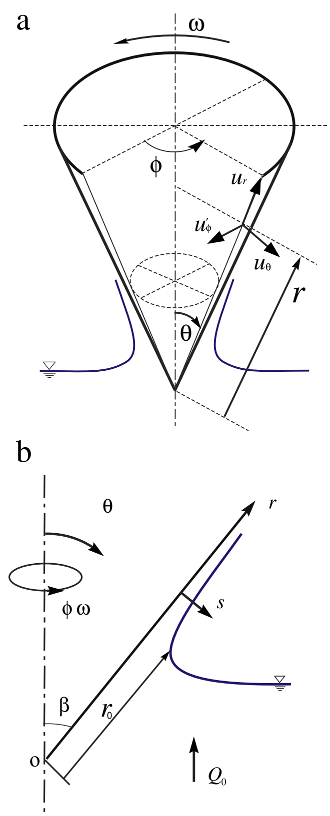

Fig. 2. Physical model and co-ordinates.

after the elapse of enough time. Eventually, the system is in a steady state.

The cone rotates at angular velocity of ω. Spherical coordinates are suited for this model. The liquid rises along the cone surface from a horizontal plane at a radius r_0. In order to derive simple equations for this model, we apply order estimates as commonly used in the boundary layer theory, because the representative length scale r_0 in the radial direction is much larger than the thickness δ of the film flow as $\delta \ll r_0$. The condition implies that the velocity in the θ direction is much smaller than both the velocities in the radial and tangential directions from the continuity equation. Introducing a coordinate system which rotates at angular velocity, the velocity in the tangential direction with respect to the new coordinate system becomes

$$u'_\phi = \omega r \sin\theta - u_\phi. \tag{1}$$

With these assumptions, a stationary axisymmetric liquid flow over an outer surface in the spherical coordinate system of rotating with constant angular velocity ω is described by the Navier–Stokes equations and the continuity equation, where the boundary layer approximation is applied, as

$$u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u'^2_\phi}{r} = -2u'_\phi\omega\sin\beta + \omega^2 r\sin^2\beta - g\cos\beta + \frac{\nu}{r^2}\frac{\partial^2 u_r}{\partial\theta^2}, \tag{2}$$

$$-\frac{u'^2_\phi}{r}\cot\beta = -2u'_\phi\omega\cos\beta + \omega^2 r\sin\beta\cos\beta + g\sin\beta - \frac{1}{\rho r}\frac{\partial p}{\partial\theta}, \tag{3}$$

$$u_r\frac{\partial u'_\phi}{\partial r} + \frac{u_\theta}{r}\frac{\partial u'_\phi}{\partial\theta} + \frac{u'_\phi u_r}{r} = 2u_r\omega\sin\beta + \frac{\nu}{r^2}\frac{\partial^2 u'_\phi}{\partial\theta^2}, \tag{4}$$

$$\frac{\partial u_r}{\partial r} + \frac{2}{r}u_r + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} = 0. \tag{5}$$

According to Bruin [1] and Makarytchev et al. [2], we use the following dimensionless parameters to nondimensionalize Eqs. (2)–(5),

$$Ro = \frac{u_{av}}{\omega\sin\beta r_0},\quad Fr = \frac{\omega^2 r\sin\beta}{g}, \tag{6}$$

$$U = \frac{u_r}{\omega r\sin\beta},\quad V = \frac{u_\theta}{(\omega\nu\sin\beta)^{1/2}},\quad W = \frac{u'_\phi}{\omega r\sin\beta}, \tag{7}$$

$$\sigma = \frac{s}{\delta_E},\quad \eta = \frac{r}{r_0}, \tag{8}$$

$$P = \frac{p - p_0}{\rho(\omega r\sin\beta)^{1/2}\omega r\sin\beta}, \tag{9}$$

$$\delta^+ = \frac{\delta}{\delta_E},\quad Q_0^+ = \frac{Q_0}{2\pi\sin\beta r^2(\omega\nu\sin\beta)^{1/2}} \tag{10}$$

where the Ekman thickness δ_E is defined as (Makarytchev et al. [2])

$$\delta_E = \left(\frac{\nu}{\omega\sin\beta}\right)^{\frac{1}{2}}. \tag{11}$$

It should be noted that the radial length scale r_0 and the mean velocity u_{av} are not defined explicitly. In addition, the trigonometric functions of θ are replaced by the same function of the angle β. A coordinate system s instead of θ is introduced as

$$ds = rd\theta \tag{12}$$

where s-coordinate is perpendicular to the cone surface forward to the outer direction. Then, we obtain the following simplified equations,

$$U^2 + \eta U\frac{\partial U}{\partial\eta} + V\frac{\partial U}{\partial\sigma} - W^2 - 1 + 2W = -\frac{\cot\beta}{Fr} + \frac{\partial^2 U}{\partial\sigma^2}, \tag{13}$$

$$2UW + \eta U\frac{\partial W}{\partial\eta} + V\frac{\partial W}{\partial\sigma} - 2U = \frac{\partial^2 W}{\partial\sigma^2}, \tag{14}$$

$$-W^2 - 1 + 2W = \frac{\tan\beta}{Fr} - \tan\beta\frac{\partial P}{\partial\sigma}, \tag{15}$$

$$\eta\frac{\partial U}{\partial\eta} + 3U + \frac{\partial V}{\partial\sigma} = 0. \tag{16}$$

Boundary conditions are given as

$$U = V = W = 0 \quad \text{at } \sigma = 0, \tag{17}$$

$$\frac{\partial U}{\partial\sigma} = \frac{\partial W}{\partial\sigma} = 0 \text{ and } P = 0 \quad \text{at } \sigma = \delta^+, \tag{18}$$

$$\int_0^{\delta^+} U d\sigma = Q_0^+. \tag{19}$$

In addition, the following transformation as $(U, V, W) \rightarrow (U_0, V_0, W_0)$ is introduced.

$$U_0 = \frac{u_r}{u_{av}} = \frac{\eta U}{Ro},\quad V_0 = \frac{u_\theta}{u_{av}} = \frac{EV}{Ro},\quad W_0 = \frac{u'_\phi}{u_{av}} = \frac{\eta W}{Ro} \tag{20}$$

where E is the Ekman number defined as $E = \delta_E/r_0$. Substituting Eq. (20) into Eqs. (13)–(16), we obtain the equations for (U_0, V_0, W_0) with the Rossby number Ro as

$$U_0^2 + \eta^2 U_0 \frac{\partial}{\partial\eta}\left(\frac{U_0}{\eta}\right) + \frac{\eta V_0}{E}\frac{\partial U_0}{\partial\sigma} - W_0^2 - \frac{\eta^2}{Ro^2} + \frac{2\eta W_0}{Ro} = -\frac{\eta^2 \cot\beta}{Ro^2 Fr} + \frac{\eta}{Ro}\frac{\partial^2 U_0}{\partial\sigma^2}, \tag{21}$$

$$2U_0W_0 + \eta U_0 \frac{\partial}{\partial\eta}\left(\frac{W_0}{\eta}\right) + \frac{\eta V_0}{E}\frac{\partial W_0}{\partial\sigma} - \frac{2\eta U_0}{Ro} = \frac{\eta}{Ro}\frac{\partial^2 W_0}{\partial\sigma^2}, \tag{22}$$

$$-W_0^2 - \frac{\eta^2}{Ro^2} + \frac{2\eta W_0}{Ro} = \frac{\eta^2\tan\beta}{Ro^2 Fr} - \frac{\eta^2\tan\beta}{Ro^2}\frac{\partial P}{\partial\sigma}, \tag{23}$$

$$\eta\frac{\partial}{\partial\eta}\left(\frac{U_0}{\eta}\right) + \frac{3U_0}{\eta} + \frac{\partial V_0}{\partial\sigma} = 0. \tag{24}$$

In the case of fast rotation and/or sufficiently large distance from the cone tip, the Rossby number is much less than unity as $Ro \ll 1$. Then, keeping the terms with Ro^2 in Eqs. (21)–(24) and inverse-transforming $(U_0, V_0, W_0) \to (U, V, W)$, we obtain the following linearized equations for the dimensionless velocity components U, V, W and the pressure P as

$$-1 = -\frac{\cot\beta}{Fr} + \frac{\partial^2 U}{\partial\sigma^2}, \tag{25}$$

$$-2U = \frac{\partial^2 W}{\partial\sigma^2}, \tag{26}$$

$$-1 = \frac{\tan\beta}{Fr} - \tan\beta\frac{\partial P}{\partial\sigma}, \tag{27}$$

$$\eta\frac{\partial U}{\partial\eta} + 3U + \frac{\partial V}{\partial\sigma} = 0. \tag{28}$$

In this case, the terms of the Coriolis force have been omitted, while the terms of the centrifugal force are maintained. It should be noted that the above set of equations are similar to the model of Bruin [1] and Makarytchev et al. [2], but the sign of the σ direction is reversed because the film flow is on the outer surface in this study while it was on the inner surface in their studies.

Now our interest is in the film flow thickness. In order to derive the film thickness, we solve Eq. (25) for the velocity component U, in advance, under the boundary conditions as

$$U = \left(1 - \frac{\cot\beta}{Fr}\right)\left(\delta^+\sigma - \frac{1}{2}\sigma^2\right). \tag{29}$$

Using Eq. (19), we obtain the dimensionless film thickness, in the centrifugal zone in which the centrifugal force is dominant, as

$$\delta^+ = \left(\frac{3Q_0^+}{1 - \frac{\cot\beta}{Fr}}\right)^{\frac{1}{3}}. \tag{30}$$

If the Froude number tends to infinity, the film thickness obtained from Eq. (30) is called the Nusselt model in Makarytchev et al. [3,4], which is identical to that obtained by simple adaptation of the classic Nusselt formula for thickness of the liquid film in laminar flow down a wall under the action of gravity.

Next, we treat the case of $Ro < 1$. Then, keeping the Ro and Ro^2 terms in this case, we obtain the following linearized equations for the dimensionless velocity components U, V, W and the pressure P as

$$-1 + 2W = -\frac{\cot\beta}{Fr} + \frac{\partial^2 U}{\partial\sigma^2}, \tag{31}$$

$$-2U = \frac{\partial^2 W}{\partial\sigma^2}, \tag{32}$$

$$-1 + 2W = \frac{\tan\beta}{Fr} - \tan\beta\frac{\partial P}{\partial\sigma}, \tag{33}$$

$$\eta\frac{\partial U}{\partial\eta} + 3U + \frac{\partial V}{\partial\sigma} = 0 \tag{34}$$

where the terms of the Coriolis force are added as well as the terms of the centrifugal force. The dimensionless radial velocity is obtained as

$$U = \frac{1}{2}\left(1 - \frac{\cot\beta}{Fr}\right)\left(-\sinh\sigma\sin\sigma + \frac{\sinh 2\delta^+\cosh\sigma\sin\sigma + \sin 2\delta^+\sinh\sigma\cos\sigma}{\cosh 2\delta^+ + \cos 2\delta^+}\right). \tag{35}$$

Using Eq. (19) also here, we obtain a nonlinear equation for the film thickness in the Coriolis zone in which the Coriolis force as well as the centrifugal force is dominant. The thickness of the film is determined by solving the nonlinear equation as

$$\frac{\sinh 2\delta^+ - \sin 2\delta^+}{\cosh 2\delta^+ + \cos 2\delta^+} = \frac{4Q_0^+}{\left(1 - \frac{\cot\beta}{Fr}\right)}. \tag{36}$$

In addition, the tangential velocity is given for the later stage as

$$W = \frac{1}{2}\left(1 - \frac{\cot\beta}{Fr}\right)\left(1 - \cosh\sigma\cos\sigma + \frac{\sinh 2\delta^+\sinh\sigma\cos\sigma - \sin 2\delta^+\cosh\sigma\sin\sigma}{\cosh 2\delta^+ + \cos 2\delta^+}\right). \tag{37}$$

3. Results

The film thickness is analytically obtained from Eq. (30) in the centrifugal zone of $Ro \ll 1$, while it is obtained by solving the nonlinear equation (36) by using Newton method in the Coriolis zone of $Ro < 1$. The obtained film thickness δ^+ is shown in Fig. 3 for both cases of $Ro < 1$ and $Ro \ll 1$, where we propose $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right)$ from Eqs. (30) and (36) for the most acceptable parameter which expresses a function of the film thickness. By using such parameter, we can treat the effect of each tip angle β and Froude number Fr in a unified manner. To our knowledge, no one has ever used the parameter to express the film thickness. The deviation between these two lines can be seen around $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right) \sim 0.1$ where the deviation is about 5%. This means that the centrifugal zone is defined as the parameter space for $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right) < 0.1$. Then, we can use the simple form of the film thickness from Eq. (30) only for $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right) < 0.1$. On the other hand, we should use the numerical solutions from Eq. (36) for $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right) > 0.1$. Since the representative length r_0 and the mean velocity u_{av} are not defined explicitly, the Rossby number Ro is not defined explicitly too. However, the parameter of $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right)$ can be calculated from the experimental conditions, which is convenient to summarize the results.

Particular attention should be paid to the film thickness from Eq. (36). We can see that there are two branches of the film thickness connecting at the turning point of $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right) = 0.272$. Therefore, the Coriolis zone is defined as the parameter space between 0.1 and 0.272 of $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right)$. The lower branch exists between $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right) = 0$ and the turning point, while the upper branch between the turning point and $Q_0^+/\left(1 - \frac{\cot\beta}{Fr}\right) \sim 0.25$ obtained analytically from Eq. (36) where the film thickness δ^+ tends to infinity.

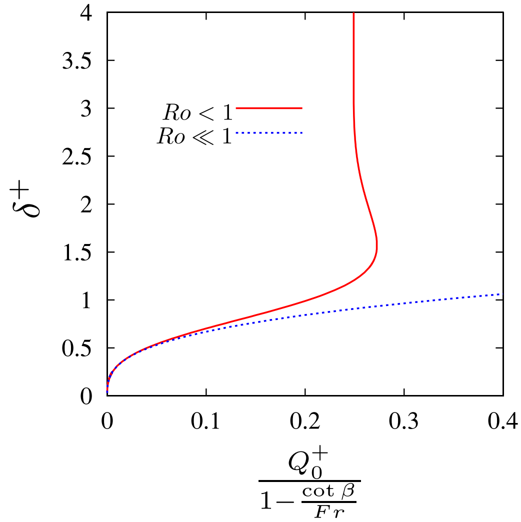

Fig. 3. Film thickness of δ^+ as a function of $Q_0^+/(1-\frac{\cot\beta}{Fr})$.

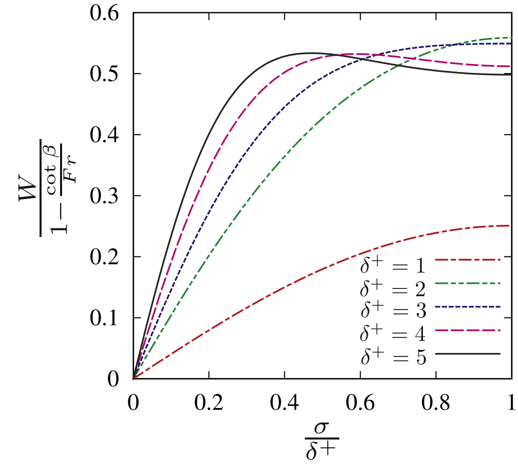

Fig. 4. The tangential velocity profiles along the normalized σ direction.

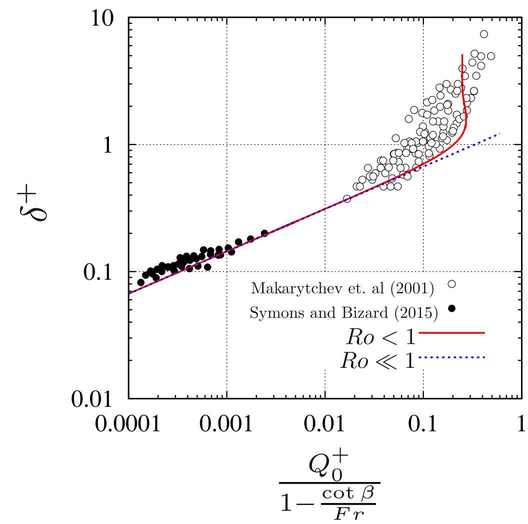

Fig. 5. Comparison of the film thickness δ^+ for $Q_0^+/(1-\frac{\cot\beta}{Fr})$.

In the above observation, we have found the new solution branches in the Coriolis zone. It should be noted that the lower branch behaves in a similar tendency to the thickness of the centrifugal zone from Eq. (30), while the upper branch tends to infinity. Apparently, the upper branch seems to be unrealistic because the boundary layer approximation is broken down if the thickness tends to be thick enough compared with the representative length r_0. However, in the case that the film thickness is comparable with one of Ekman layer, there is a possibility that the upper branch is realistic. In order to clarify the possibility, here we consider the hydrodynamical stability problem of the rotating fluid, and examine the stability of the upper branch as the nonlinear equilibrium solution from Eq. (36). To verify the instability for the upper branch, we use the Rayleigh's criterion for instability of an inviscid fluid (Drazin and Reid [8]). The criterion says that if the square of angular momentum per unit mass of a fluid element decreases with the distance from the fixed rotating axis anywhere, it implies flow instability. In this case, the film thickness for the upper branch is greater than unity. This means that the film flow is outside the Ekman boundary layer. Therefore, it is possible to treat the film flow of the upper branch as the inviscid fluid. Considering the scale difference of $r_0 \gg \delta$, the difference of the square of angular momentum can be replaced with the difference of the tangential velocities. Then, the Rayleigh's criterion can be modified in this case as follows. If the tangential velocity W decreases along σ anywhere, the flow becomes unstable. Fig. 4 shows the tangential velocity profiles along σ calculated form Eq. (37). We can see that the velocity W monotonically increases for the smaller δ^+, while it starts to have the local maximum value around for $\delta^+ > 3$. This implies that the W decreases along σ and the film flow becomes unstable from the modified Rayleigh's criterion. Therefore, we conclude that the film flow is unstable and is not in a realistic situation for $\delta^+ > 3$.

Eqs. (29), (30), (35) and (36) are invariant under the transformation as $(\delta^+, \sigma) \to (-\delta^+, -\sigma)$. This means that the film thickness obtained for the outer surface of rotating cones can be compared with the thickness for the inner surface of rotating hollow cones. So, we compare our results with experimental results for the inner surface of Symons and Bizard [5] and Makarytchev et al. [4] in reference. Fig. 5 shows the comparisons, where the results of Symons and Bizard [5] are located for the smaller values of $Q_0^+/(1-\frac{\cot\beta}{Fr})$, while the results of Makarytchev et al. [4] for the larger range of $Q_0^+/(1-\frac{\cot\beta}{Fr})$. Therefore, the results of Symons and Bizard [5] are considered to be in the centrifugal zone. The Froude number is limited to the finite value under their experimental conditions, which means that the results include the effect of gravity. We can see that the filled circles shows good agreement with our solutions also in such conditions. On the other hand, The results of Makarytchev et al. [4] were for wavy film flow regime and the thickness is relatively large. They modeled the film thickness as a wavy layer on top of a laminar sub-layer represented by the Nusselt model. However, we will see that our thickness lines derived from the theoretical basis are quite close to their experimental results shown by the open circles, although we cannot compare properly because we treat a steady-state film flow and omit the nonlinear terms in our study.

4. Conclusions

In this paper, we have focused on the liquid film flow over the outer surface of a rotating cone and derived the equations for the velocity profiles and the film thickness of the rising film flow by using the boundary layer theory. Particular attention has been paid to the film thickness. The film thickness has been obtained analytically in the centrifugal zone, while it has been obtained as a solution of the nonlinear equation in the Coriolis zone. We have proposed an appropriate parameter $Q_0^+/\left(1-\frac{\cot\beta}{Fr}\right)$ to express the film thickness. In addition, it has been found that the film thickness in the Coriolis zone is composed of the upper and lower branches with the turning point. Therefore, the film thickness is a double-valued function at the vicinity for $Q_0^+/\left(1-\frac{\cot\beta}{Fr}\right) < 0.272$. It has been proven that the upper branch for $\delta^+ > 3$ is unstable and unrealistic by using the modified Rayleigh's criterion.

Acknowledgments

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