Comparison between our results and Guangzhao Zhou's results of stable flow

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Calculation conditions

All conditions and settings are from Guangzhao Zhou's article.

Here we foucs on differences between calculating from front to back versus from back to front.

If we calculate from back to front, thus we can use the large-a limiting value as our initial value:

$$h_f \simeq \left(c_2 \frac{\nu}{g} \frac{q}{a_{int} \sin \alpha} \right)^{\frac{1}{3}}$$

If we calculate from front to back, due to Watson's article and Guangzhao Zhou's article, we have: *(r_0 means the initial radius of cylinder, and $\alpha \neq 0$)*

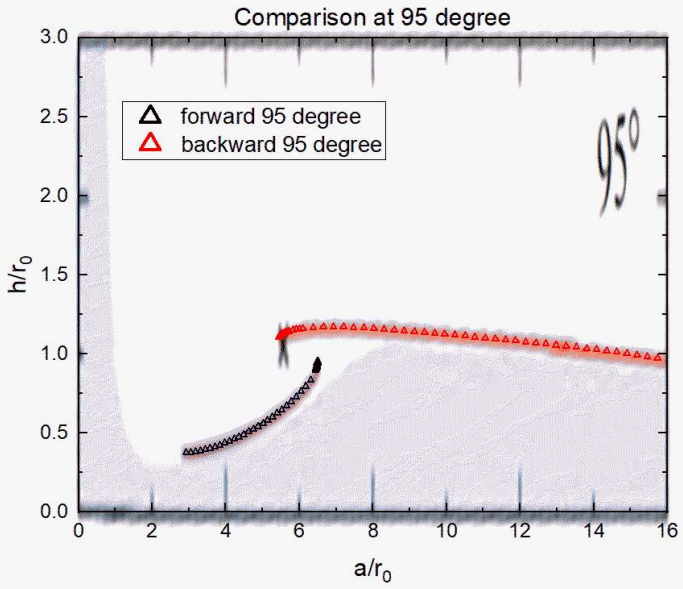
$$\begin{aligned} r_m &\simeq 1.426 r_{bl} \\ \frac{r_{bl}}{r_0} &\simeq 0.3155 \left(\frac{Q}{\nu r_0} \right)^{\frac{1}{3}} \\ \frac{h_m}{r_0} &\simeq 2.308 \left(\frac{\nu r_0}{Q} \right)^{\frac{1}{3}} \end{aligned}$$

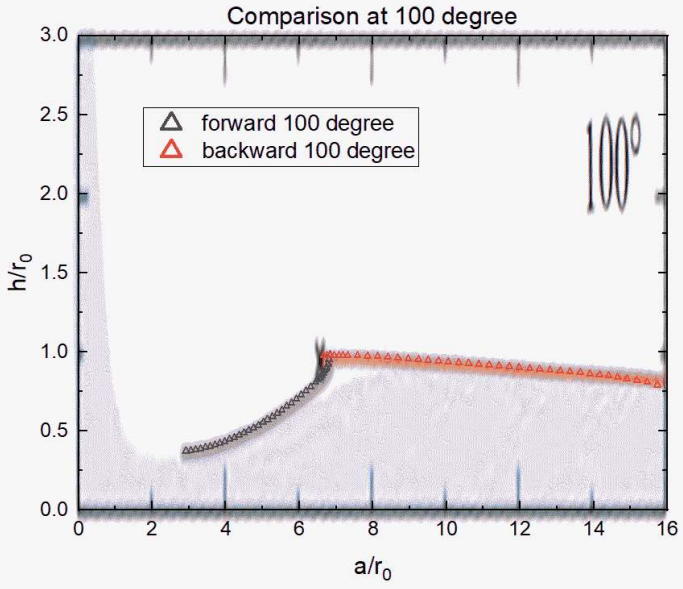
The viscous fluid's properties and other conditions are as follows:

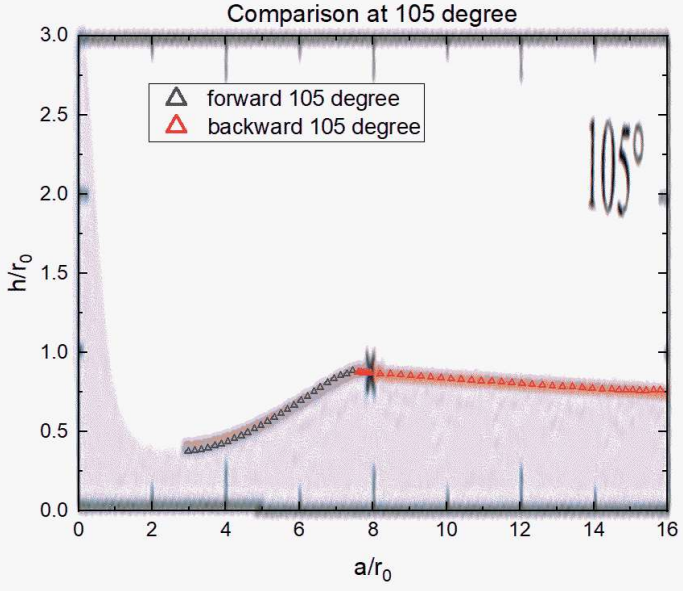
$$\begin{aligned} \rho &= 0.97 \times 10^3 [\mathrm{kg\ m^{-3}}] \\ \nu &= 5.0 \times 10^{-5} [\mathrm{m^2 s^{-1}}] \\ g &= 9.8 [\mathrm{m\ s^{-2}}] \\ r_0 &= 2.5 \times 10^{-3} [\mathrm{m}] \\ Q &= 3.0 \times 10^{-5} [\mathrm{m^3 s^{-1}}] \\ Re_j &\simeq 152.8 \end{aligned}$$

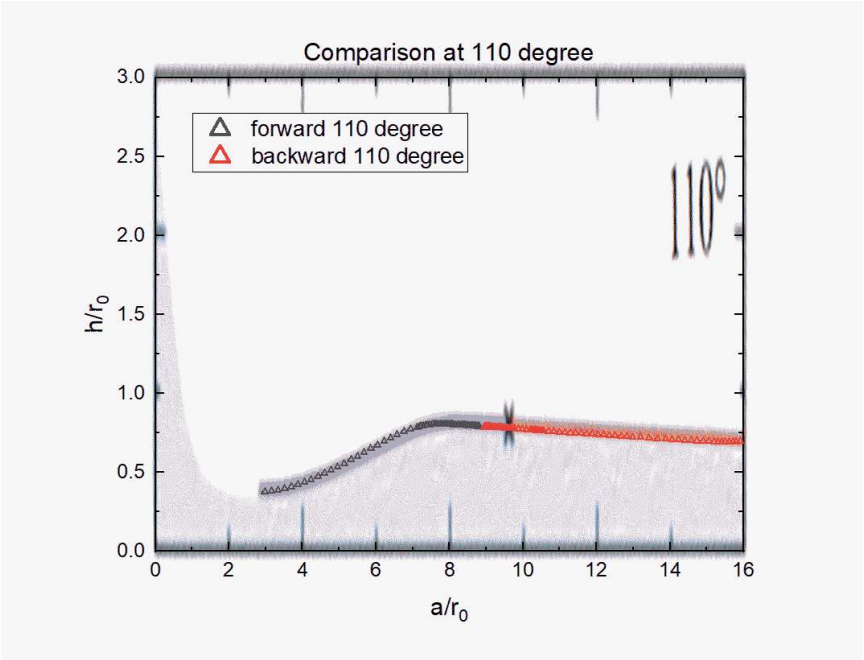
Results and comparison

The background results are Guangzhao Zhou's results.

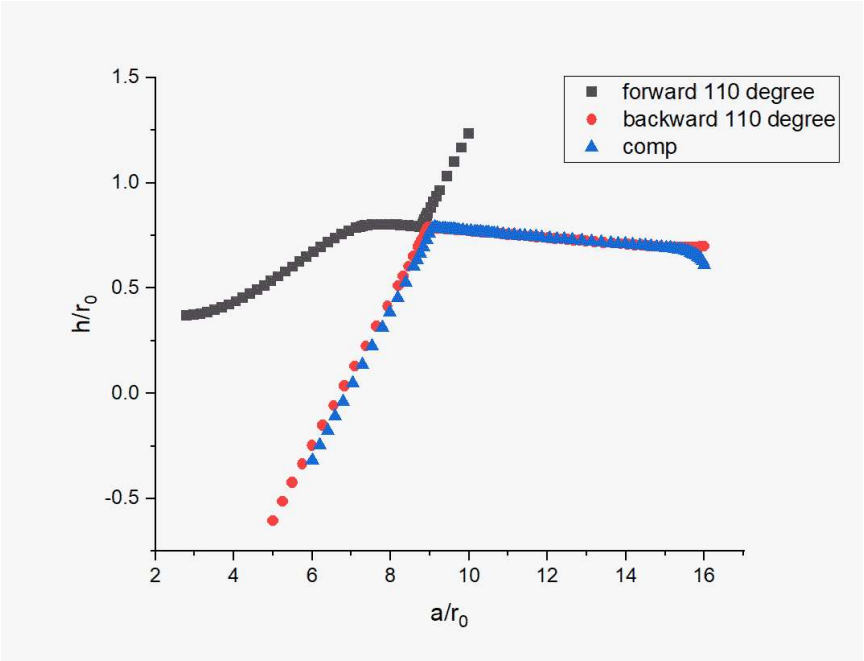

Comparison at 95 degree
forward 95 degree
backward 95 degree
95°
h/r0
a/r0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10
12
14
16


Comparison at 100 degree
forward 100 degree
backward 100 degree
100°
h/r0
a/r0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10
12
14
16


Comparison at 105 degree
forward 105 degree
backward 105 degree
105°
h/r0
a/r0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10
12
14
16



The original data and their connection are as below:



Codes (Matlab based)

Forward calculation

```
clear
clc

%Settings

nu=50e-6;%kinematic viscosity \nu=\mu/\rho
g=9.8;
sc=deg2rad(20);
rint=2.5e-3;

q0=3e-5/(2*pi);
Q=q0*2*pi;

xbl=rint*0.3155*(Q/(nu*rint))^(1/3);

xint=1.426*xbl;
xend=10*rint;

c0=q0;

C=c0;
c1=27/35;
c2=3;

EXT=c2*nu*C;

hint=2.308*rint*((nu*rint)/Q)^(1/3);
kk=hint/rint;

%Calculation

yprime = @(x,y) ((x.^3).*(y.^3).*g.*sin(sc)+2.*c1.*y.*C.^2-EXT.*(x.^2))./
((x.^3).*(y.^3).*g.*cos(sc)-2.*c1.*x.*C.^2);

xspan = [xint xend];

[x,y]=ode45(yprime,xspan,hint);

%Output and plot

v = C./(x.*y); %u_a^*

rx=x/rint;
ry=y/rint;
ResultMatrix=[rx ry];

figure()
subplot(2,1,1)
```

```
plot(x/rint,y/rint,'-o')
ylim([0,3])
xlabel('a/r_0')
ylabel('h/r_0')

subplot(2,1,2)
plot(x/rint,v,'-o')
xlabel('a/r_0')
ylabel('u_a^*')
```

Backward calculation

```
clear
clc

%Settings

g=9.8;
sc=deg2rad(20);
rint=2.5e-3;
q0=3e-5/(2*pi);
%H0=4.8e-3;

xint=16*rint;
xend=6*rint;
aint=24*rint; %this value is used to estimate the hint,
%thus the larger this value, the accurate the calculation it will be.

c0=q0;

nu=50e-6;%kinematic viscosity \nu=\mu/\rho

C=c0;
c1=27/35;
c2=3;

EXT=c2*nu*C;
hint=(C*c2*nu/(g*sin(sc)*aint))^(1/3);

%Calculation

yprime = @(x,y) ((x.^3).*(y.^3).*g.*sin(sc)+2.*c1.*y.*C.^2-EXT.*(x.^2))./
((x.^3).*(y.^3).*g.*cos(sc)-2.*c1.*x.*C.^2);

xspan = [xint xend];

[x,y]=ode45(yprime,xspan,hint);

%Output and plot

v = C./(x.*y); %u_a^*

rx=x/rint;
ry=y/rint;
ResultMatrix=[rx ry];

figure()
subplot(2,1,1)
plot(x/rint,y/rint,'-o')
ylim([0,3])
xlabel('a/r_0')
ylabel('h/r_0')

subplot(2,1,2)
```

```
plot(x/rint,v,'-o')
xlabel('a/r_0')
ylabel('u_a^*')
```