Constellation Design - a quick overview

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Constellation Design for Earth Observation

A.k.a. Tell me what **you** need and I'll (probably) make it happen

Goal: design a satellite constellation that satisfies specific observation objectives

The Challenges:

- Meeting observation frequency requirements (how many times per day would you like to observe a target?)
- Minimizing station-keeping and satellite count.
- Designing specific orbits (e.g., repeating groundtracks).

What we'll go through:

- Basics of orbital mechanics relevant to constellation design.
- How to compute repeating groundtracks.
- Using Python interactively to evaluate design trade-offs.

We can pick any orbit we desire

As long as we know what we want

The six keplerian elements (KE) define the shape (a, e) and orientation (i, Ω, ω) of the orbit, while ν (true anomaly) defines the position at epoch of our satellite (SC)

Among all KE possible choices, certain "special" combinations that define families of orbits for Earth observation

Family	**Characteristic KE**	**Advantages**	**Disadvantages**
Polar	$i \approx 90\,\mathrm{deg}$	- Covers all latitudes - Great polar monitoring	- Inefficient mid-latitude coverage
Sun-Synchronous	$i = f\left(a, e\right)$	- Constant solar time (lighting conditions) - Covers most latitudes	- No polar imaging - Requires delicate SK
Critically-inclined	$i \approx \{63.43, 116.57\}\;\;\mathrm{deg}$	- Overlaps high-latitude targets - Stable orbit for apsis precession	- Frequent low-latitude gaps - Can't image outside lat bounds

Orbital elements and groundtrack

How to pass from the first to the latter

Keplerian elements define a geometric, closed-form orbital representation

- In an inertial frame (ECI), an orbit is (quasi) inertial; in the ECEF frame, the Earth is fixed, making it easy for plotting groundtracks as the SC's motion accounts for Earth's rotation.
  - Earth's angular speed [rad/s]?
  - One revolution per sideral day (86164.09 s)
- ***r*** and ***V*** rotated by α_g and corrected by $\omega_\oplus$

$$\boldsymbol{V}_{rot,SC} = [0, 0, \omega_\oplus] \wedge \boldsymbol{r}_{IJK}$$

$$\boldsymbol{r}_{ECEF} = R_z(\alpha_g)\,\boldsymbol{r}_{ECI}$$

$$\boldsymbol{V}_{ECEF} = R_z(\alpha_g)\left(\boldsymbol{V}_{ECI} - \boldsymbol{V}_{rot,SC}\right)$$

$$La = \sin^{-1}\left(\frac{z_{ECEF}}{\|\boldsymbol{r}_{ECEF}\|}\right)$$

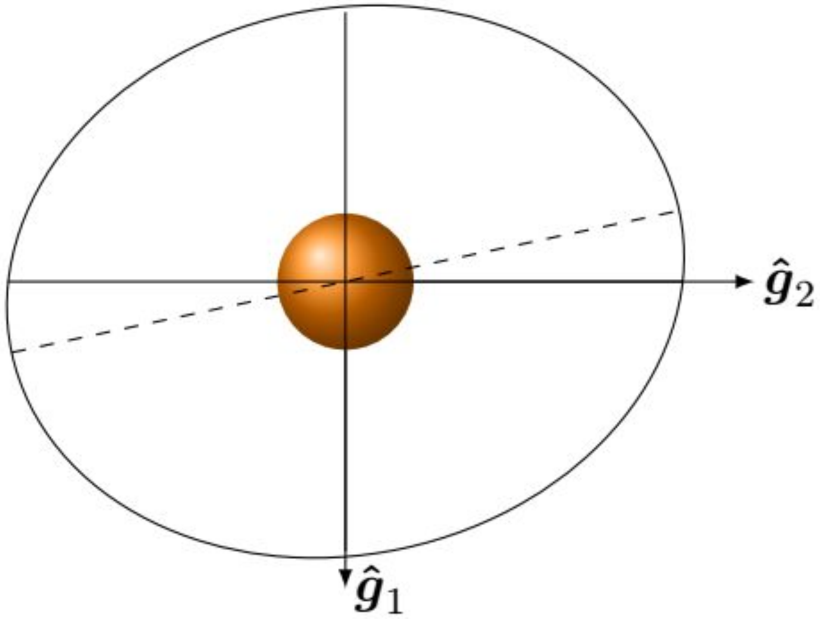
$$Lo = \operatorname{atan2}(y_{ECEF}, x_{ECEF})$$

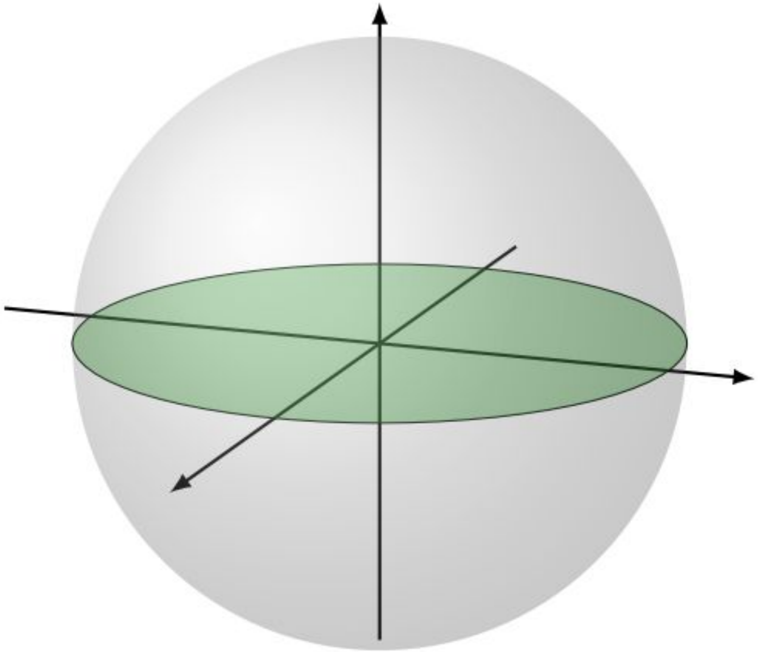
Algorithm 1 Ground Track Plot

procedure $\textsc{GroundTrackPlot}(Kepler, \alpha_{g0}, nTf, timestep)$
  $a, e, i, \Omega, \omega, \nu \leftarrow Kepler$
  $n \leftarrow \textsc{MeanMotion}(a)$
  $E \leftarrow \textsc{EccentricAnomalyFromEnu}(e, \nu)$
  $MA \leftarrow \textsc{MeanAnomalyFromEccentricAnomaly}(e, E)$
  $T \leftarrow \textsc{Period}(a)$
  $time \leftarrow \textsc{CreateArray}(0, nTf \times T, timestep)$
  for $j \leftarrow 0$ to $\textsc{Length}(time) - 1$ **do**
    $\alpha_g \leftarrow \alpha_{g0} + \textsc{OmegaEarth} \times time[j]$ ← Quick digression **QD**
    $M \leftarrow MA + n \times time[j]$
    $\nu \leftarrow \textsc{MeanToTrueAnomaly}(M, e, 1e-10)$
    $r_{ECI}, v_{ECI} \leftarrow \textsc{StateVectorsFromOrbitalElements}(a, e, i, \boxed{\Omega, \omega}, \nu)$ **P**
    $r_{ECEF}, v_{ECEF} \leftarrow \textsc{EciToEcefState}(r_{ECI}, v_{ECI}, \alpha_g)$
    $Lat[j], Lon[j] \leftarrow \textsc{EcefToLatLon}(r_{ECEF})$

Quick trick: sketch to guess a_g!

Share your birth date and time—I'll read your stars (this time scientifically)





Kepler inverse time problem

A small digression: true anomaly from time

$$\nu \stackrel{?}{=} 2\tan^{-1}\left[\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)\right]$$

Start by defining guesses

$$n = \sqrt{\frac{\mu}{a^3}}$$

$$E_0 = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\nu_0}{2}\right)\right]$$

$$M_0 = E_0 - e\sin E_0$$

Iterative procedure $\forall t \in [0, T]$

$$\alpha_g = \alpha_{g,0} + \omega_{\oplus} \cdot t$$

$$M = M_0 + n * t$$

iteratively

$$f(E) = E - e\sin(E) - M$$

$$f'(E) = 1 - e\cos(E)$$

$$E_{j+1} = E_j - \frac{f(E_j)}{f'(E_j)}$$

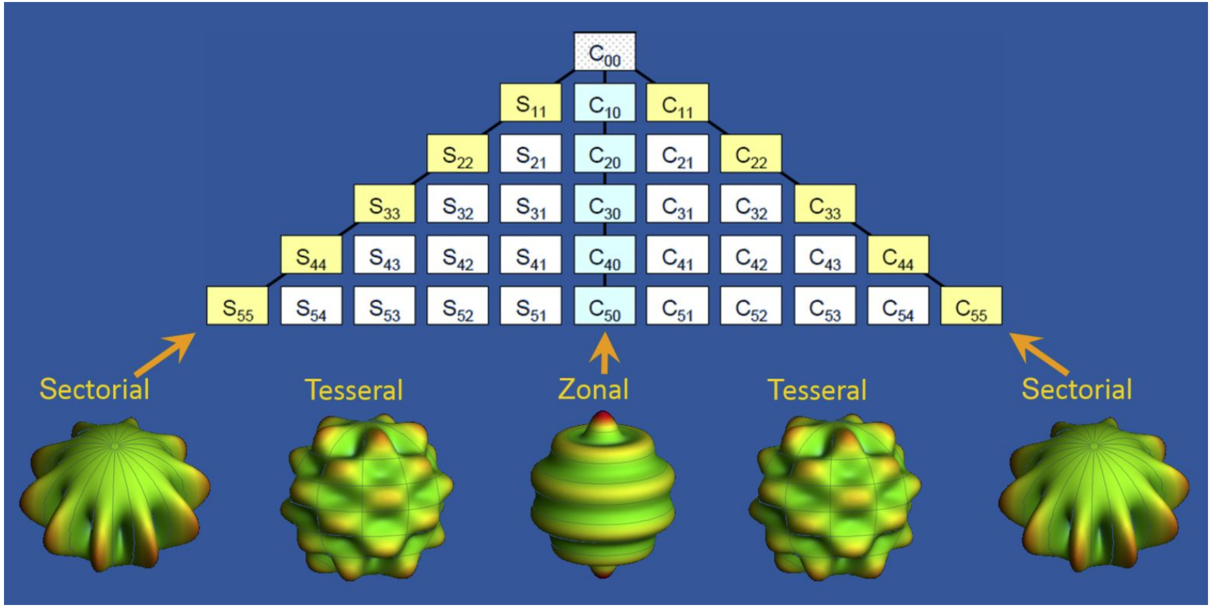
$$\nu = 2\tan^{-1}\left[\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E_{convergence}}{2}\right)\right]$$

LEO perturbations (1/5)

What makes us deviate from perfect geometrical motion

Earth is not a perfect sphere: its varying geopotential is represented by thousands of coefficients in Earth Gravity Models like EGM2008

- The most well-known coefficient is $C_{20} = - J_2$
- The J_2 term induces an average effect per revolution on Ω and ω (semi-analytical formulae)



Perturbations (2/5)

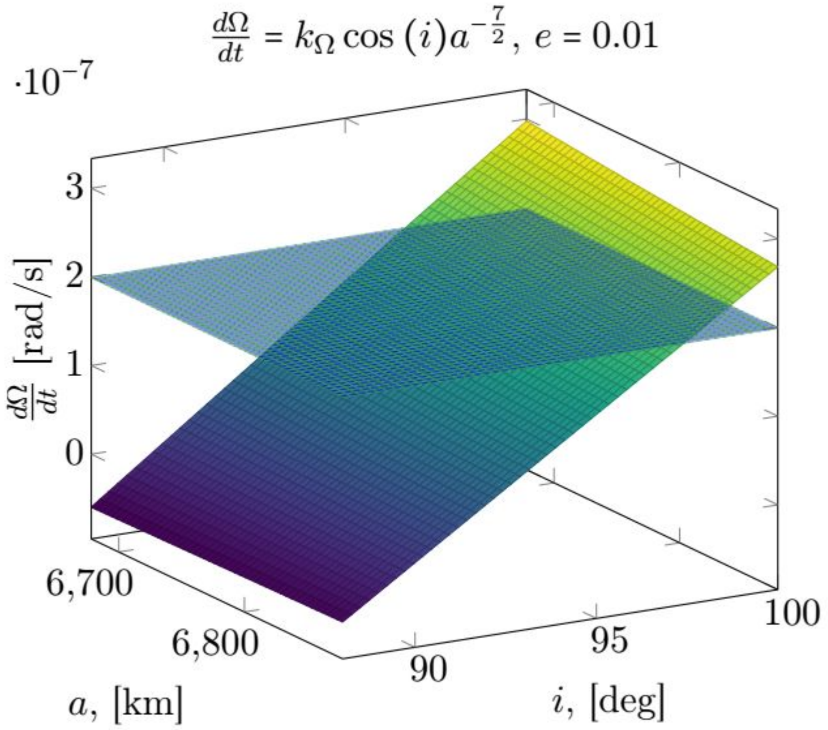
Variation of the Right Ascension of the Ascending Node (RAAN, Ω)

$$\frac{d\Omega}{dt}\,[\text{rad/period}] = -3\pi J_2 \left[\frac{R_\oplus}{a\left(1-e^2\right)}\right]^2 \cos(i),$$

$$\frac{d\Omega}{dt}\,[\text{rad/s}] = -3\pi J_2 \left[\frac{R_\oplus}{a\left(1-e^2\right)}\right]^2 \cos(i)\frac{1}{T},$$

$$= \underbrace{-\frac{3}{2}J_2\left(\frac{R_\oplus}{1-e^2}\right)^2\sqrt{\mu}}_{k_\Omega}\frac{\cos(i)}{a^{\frac{7}{2}}},$$

$$= k_\Omega \cos(i) a^{-\frac{7}{2}}$$



Perturbations (3/5)

Variation of the argument of periapsis (ω)

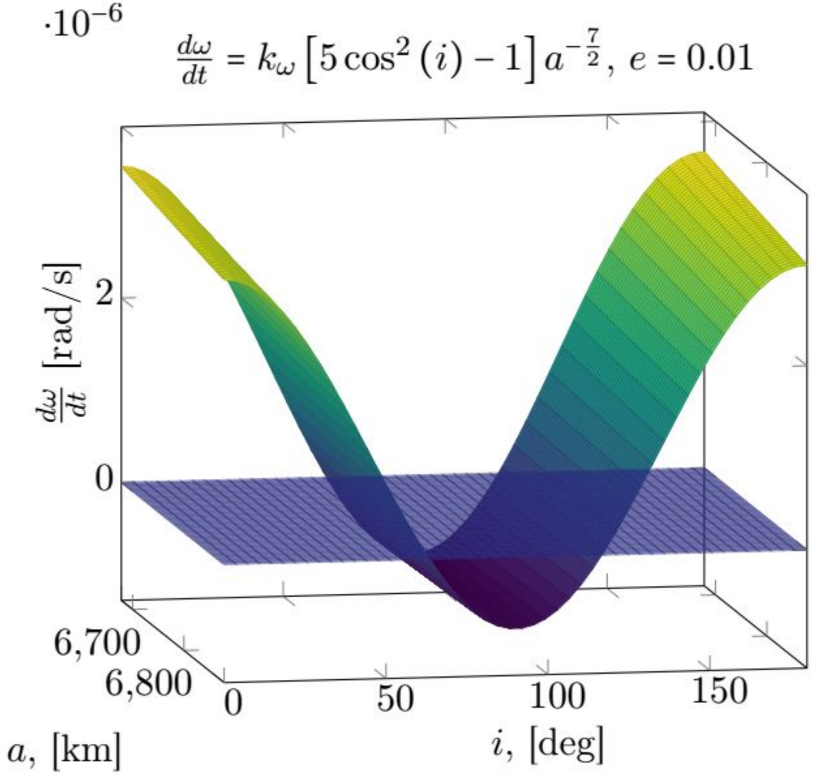
$$\frac{d\omega}{dt}\,[\text{rad/period}] = \frac{3\pi J_2}{2}\left[\frac{R_\oplus}{a\left(1-e^2\right)}\right]^2\left(5\cos^2(i)-1\right),$$

$$\frac{d\omega}{dt}\,[\text{rad/s}] = \frac{3\pi J_2}{2}\left[\frac{R_\oplus}{a\left(1-e^2\right)}\right]^2\left(5\cos^2(i)-1\right)\frac{1}{T},$$

$$= \underbrace{\frac{3}{4}J_2\left(\frac{R_\oplus}{1-e^2}\right)^2\sqrt{\mu}}_{k_\omega}\,\frac{\left[5\cos^2(i)-1\right]}{a^{\frac{7}{2}}},$$

$$= k_\omega\left(5\cos^2(i)-1\right)a^{-\frac{7}{2}}$$

$$i \approx \{63.43, 116.57\}\ \text{deg}$$

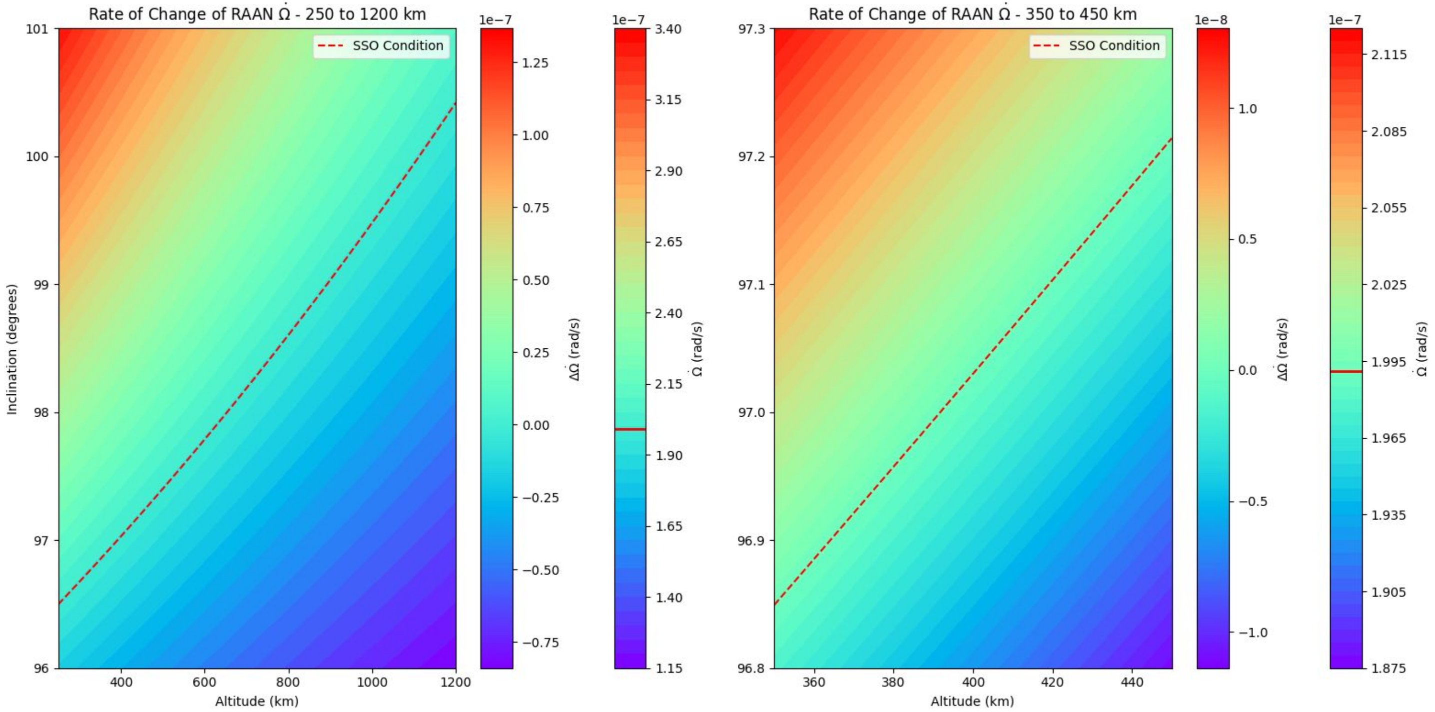


Perturbations (4/5)

Selecting the correct SSO inclination

$$i_{SSO} = \cos^{-1}\left[\frac{\omega_{\oplus\odot}}{k_\Omega} a^{\frac{7}{2}}\right]$$

The sun-synchronous condition means that the orbital plane **precesses** at the same rate of the Sun-Earth joining line (one full revolution in one tropical year, 365.2425 days)



Perturbations (5/5)

A quick recap

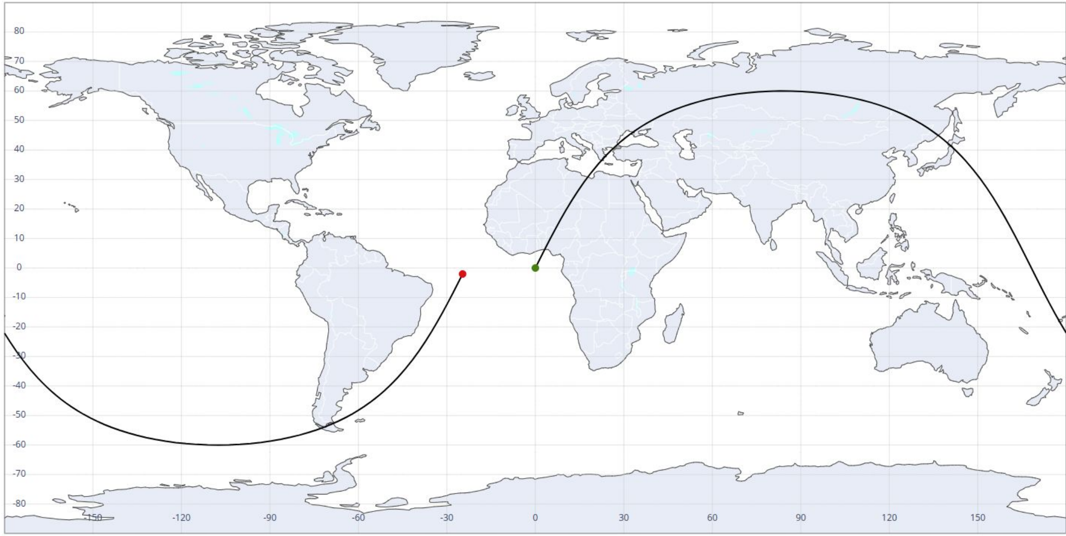
$$\frac{d\Omega}{dt} = k_\Omega \cos(i) a^{-\frac{7}{2}} \qquad \frac{d\omega}{dt} = k_\omega \left[5\cos^2(i) - 1\right] a^{-\frac{7}{2}}$$

Eccentricity	**Inclination range**	**RAAN**	**Argument of periapsis**
e = 0	*i* = 0 deg (prograde)	non-existent	non-existent
e ≠ 0	*i* < 63.43 deg (prograde)	Retrogresses	Precesses
	i = 63.43 deg (critical, prograde)	Retrogresses	Null
	63.43 < *i* < 90 deg (prograde)	Retrogresses	Retrogresses
	i = 90 deg (polar)	Null	Retrogresses
	90 < *i* < 116.57 deg (retrograde)	Precesses	Retrogresses
	i = 116.57 deg (critical, retrograde)	Precesses	Null
	i > 116.57 deg (retrograde)	Precesses	Precesses

Repeating groundtracks (1/3)

We'll soar back over the same point

- Example:
  - h = 500 km
  - Circular
  - i = 60 deg
- Apparent longitudinal retrogression (the SC "absorbs" Earth's rotation)
  ♦ $\Delta Lo = 2\pi/T_{sid} \cdot T = \omega_{\oplus} \cdot T \approx 23.72$ deg



- Repeating groundtrack:
  - After N_T integer orbits (true period T)...
  - ... and for every integer desired R revisit period (in days) (nodal period T_N)...
  - ... we aim to pass over the same {La, Lo} pair
  - Thus, for a repeating groundtrack, the following relation must hold: $N_T T = R T_N$, or, equivalently, there must be an integer ratio: $\tau = N_T/R = T_N/T$
- In the ideal case, formula ♦ is easily solvable. However, in the real scenario, not only the Earth moves beneath the SC, but the orbital plane and the orbit itself are perturbed

Repeating groundtracks (2/3)

Brace yourselves - here's a slide full of math neither of us will remember later!

HP: quasi-circular e ≈ 0

$$\text{Unperturbed} \quad N_T \cdot T = R \cdot T_N \implies N_T \frac{2\pi}{n} = R \frac{2\pi}{\omega_\oplus}$$

$$\text{Perturbed} \quad N_T \frac{2\pi}{n + \dot{M}_{J_2} + \dot{\omega}} = R \frac{2\pi}{\omega_\oplus - \dot{\Omega}}$$

Ideal nodal period = sideral day 86164.09 s

$$\tau = \frac{N_T}{R} = \frac{T_N}{T} = \frac{n + \dot{M}_{J_2} + \dot{\omega}}{\omega_\oplus - \dot{\Omega}}$$

$$\begin{cases} \dot{\Omega} = -\frac{3}{2} J_2 R_\oplus^2 \sqrt{\mu} \left[\frac{\cos(i)}{a^{\frac{7}{2}} (1-e^2)^2} \right] \\ \dot{\omega} = \frac{3}{4} J_2 R_\oplus^2 \sqrt{\mu} \left[\frac{5\cos^2(i) - 1}{a^{\frac{7}{2}} (1-e^2)^2} \right] \\ \dot{M} = n + \frac{3}{4} J_2 R_\oplus^2 \sqrt{\mu} \left[\frac{3\cos^2(i) - 1}{a^{\frac{7}{2}} (1-e^2)^{\frac{3}{2}}} \right] = n + \dot{M}_{J_2} \end{cases}$$

$$\implies \tau \left(\omega_\oplus - \dot{\Omega} \right) \approx n + \frac{3}{4} \underbrace{J_2 R_\oplus^2 \sqrt{\mu}}_{k} \left[\frac{8\cos^2(i) - 2}{a^{\frac{7}{2}}} \right]$$

$$= n + \frac{3}{2} k \left[\frac{4\cos^2(i) - 1}{a^{\frac{7}{2}}} \right] = f(a, i)$$

$$\textbf{if SSO} \implies \tau \left(\omega_\oplus - \omega_{\oplus\odot} \right) = n - \frac{3}{2} \frac{k}{a^{\frac{7}{2}}} + \frac{8}{3} \frac{a^{\frac{7}{2}} \omega_{\oplus\odot}^2}{k} = f(a)$$

Repeating groundtracks (3/3)

Our last step - let's plan it together (interactive)

- We are free to chose our desired location (pick a city or give a Lat/Lon pair)
- We will:
  - Select our orbit family (SSO, polar, critical, or custom inclined)
  - Try to find the correct altitude to obtain a repeating groundtrack
  - Fail (unless you're ***super*** lucky)
  - See dynamically the search space for the revolutions number to revisit period ratio
  - Narrow down the search space by picking our ratio
- At this point, we'll have our orbit (a, i) with well-defined perturbations ($d\Omega$, $d\omega$)
  - The code will use a simple bisection method to find the repeating groundtrack

Before we start: the code is setup to find the α_g for today at our local time (converted to UTC time), therefore 2025-11-25T22:30:00Z (that's an $\alpha_g \approx 27.5$ deg, a to-Sun angle ≈ 244.7 deg, and a relative angle to the Sun from us ≈ 282.3 deg, supposing we're talking at 2.30 PM) - we're close to the winter solstice!)

References & contacts

- Google Scholar
- luigi.mascolo@planet.com
- GitHub
- High-Fidelity Constellation Architect for the Operational Deployment and Maintenance of Co-Planar Multi-Satellite Systems (paper)