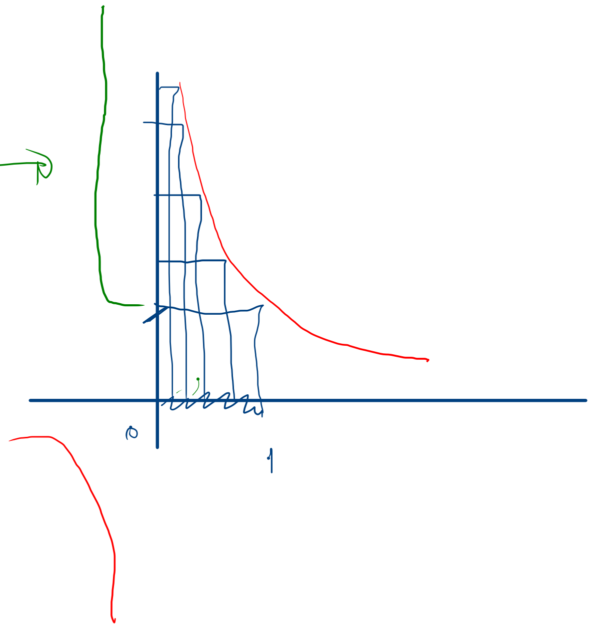
$g(x) = x^{-\beta}$ $\qquad$ $\beta > 0$

$g(0) = \left(\frac{1}{0}\right)^{\beta} = \infty^{\beta} = \infty$

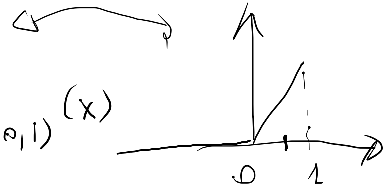
ABUSO

$\big(g(1), g(0)\big) =$

$(1, +\infty)$



ESERCIZIO (nuovo)



Sia X una v.a. continua con densità $f_X(x) = 2x \, \mathbb{1}_{(0,1)}(x)$

Sia $Y = X^3 + 1$

Trovare la densità continua di Y.

RISPOSTA

Oss. $f(x) = x^3 + 1$ è crescente; quindi Y assume valori in $(f(0), f(1)) = (0^3+1, 1^3+1) = (1,2)$

$$F_Y(y) = \begin{cases} 0 & \text{per } y \le 1 \\ \circledast & \text{per } y \in (1,2) \\ 1 & \text{per } y \ge 2 \end{cases}$$

$$\circledast = P(X^3 + 1 \le y) = P(X^3 \le y-1) = P\Big(X \le \underbrace{(y-1)^{1/3}}_{\in(0,1)}\Big) = \int_{-\infty}^{(y-1)^{1/3}} f_X(x)\,dx = \int_0^{(y-1)^{1/3}} 2x\,dx = \Big[x^2\Big]_0^{(y-1)^{1/3}} = (y-1)^{2/3}$$

Infine $\boxed{f_Y(y) = \frac{2}{3}(y-1)^{\frac{2}{3}-1} \, \mathbb{1}_{(1,2)}(y).}$