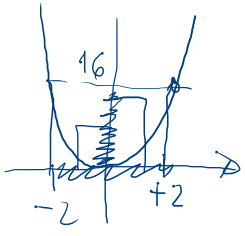
ESERCIZIO

Sia $X$ una v.a. con distribuzione $U(-2,2)$.

Trovare la funzione di distribuzione di $Y = X^4$.

RISPOSTA




$$P(0 \le Y \le 16) = 1$$

$$F_Y(y) = \begin{cases} 0 & \text{per } y \le 0 \\ (*) & \text{per } 0 < y < 16 \\ 1 & \text{per } y \ge 16 \end{cases}$$

$[-\sqrt[4]{y}, \sqrt[4]{y}] \subset [-2,2]$ perché $y \in [0,16]$

$y > 0$

$$\{X^4 \le y\} = \{-\sqrt[4]{y} \le X \le \sqrt[4]{y}\}$$

$$(*) = P(Y \le y) = P(X^4 \le y) = P\left(-\sqrt[4]{y} \le X \le \sqrt[4]{y}\right) = \int_{-\sqrt[4]{y}}^{\sqrt[4]{y}} \underbrace{f_X(x)}_{= \frac{1}{2-(-2)} = \frac{1}{4}} dx = \frac{1}{4}\left(\sqrt[4]{y} - (-\sqrt[4]{y})\right) = \frac{2}{4}\sqrt[4]{y} = \frac{\sqrt[4]{y}}{2}$$

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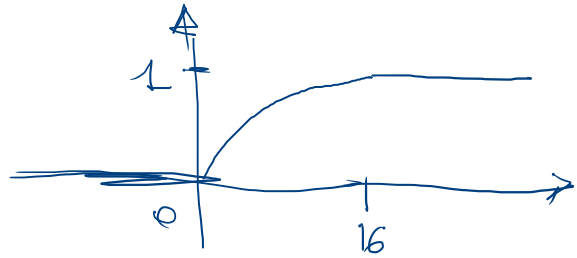


Se volessimo trovare la densità continua, si avrebbe

$$f_Y(y) = \frac{1}{2} \cdot \frac{1}{4} y^{\frac{1}{4}-1} 1_{(0,16)}(y) = \frac{1}{8} y^{-3/4} 1_{(0,16)}(y)$$