DTU Compute
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Wave simulation with PINNs

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1 Introduction

For our project we focused on utilizing a Physics-Informed Neural Network (PINN) to simulate a snapshot of a small wave. PINNs integrate domain-specific knowledge directly into neural network architectures, allowing for accurate predictions even with limited or noisy data. PINNs excel in various fields like fluid dynamics, materials science, and structural mechanics, offering computational efficiency, robustness, and the ability to handle multi-physics problems.

Our design utilizes a PINN to solve the Laplace equation within a square. This PDE is part of the set of equations that govern the motion of a small amplitude wave. The analytical solution ϕ is already known, and therefore allows for comparison with our model. The boundary value problem and solution is described using the following set of equations:

$$\Phi = \phi, z = 0, \quad \forall 0 \leq x \leq L \tag{1}$$

$$\delta_{xx}\Phi + \delta_{zz}\Phi = 0, \forall z \in (-h, 0)x \in (0, L) \tag{2}$$

$$\begin{pmatrix} n_x \\ n_z \end{pmatrix} \begin{pmatrix} \delta_x \\ \delta_z \end{pmatrix} \Phi = 0, \forall [x, z] \in \Omega \tag{3}$$

$$\phi = -\frac{Hc}{2}\frac{\cosh(k(z+h))}{\sinh kh}\sin\omega t - kx \tag{4}$$

The directional derivatives of ϕ is the local velocity. This derivative of the solution can be seen in figure 1.

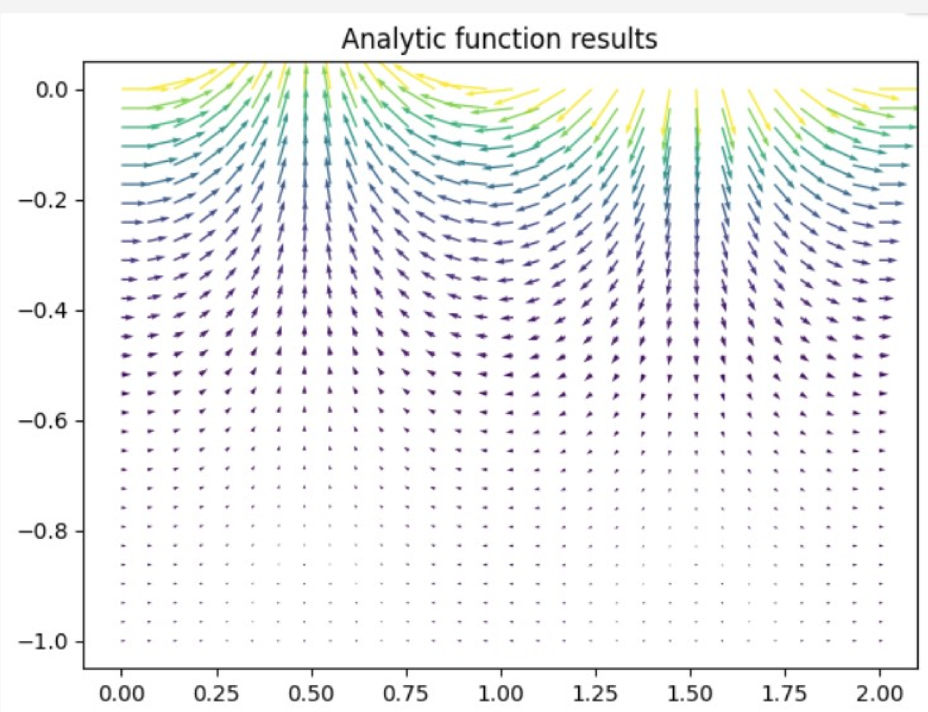

Figure 1: The directional derivatives of the analytic result.

2 Our model

A 4-layer feed-forward neural network is used for construction of the PINN.

The network is trained on a total of 250 data points randomly distributed on the surface.

Boundary conditions were implemented in the loss function along with the PDE loss.

One boundary was set on the free surface, based on function (1), another boundary was set to represent the no-flux condition through the bottom (4), and a third boundary was added to impose the periodicity of the wave, ensuring that the points in the beginning and end are equal.

The **total loss** is calculated based on the multiple components present in this problem:

$L = L_{PDE} + L_{FreeSurface}$
$+ L_{Periodic} + L_{BottomNormalVelocity}$

Multi-part loss presents some challenges, since the losses may have different magnitudes. It was decided to introduce adjustable weights using **SoftAdapt** which is a framework for dynamically adjusting the weights of a loss function based on the live performance statistics of the different component losses.

3 Initial results

The solution from the PINN for the scalar velocity field can be seen in figure 2 along with the analytical solution.

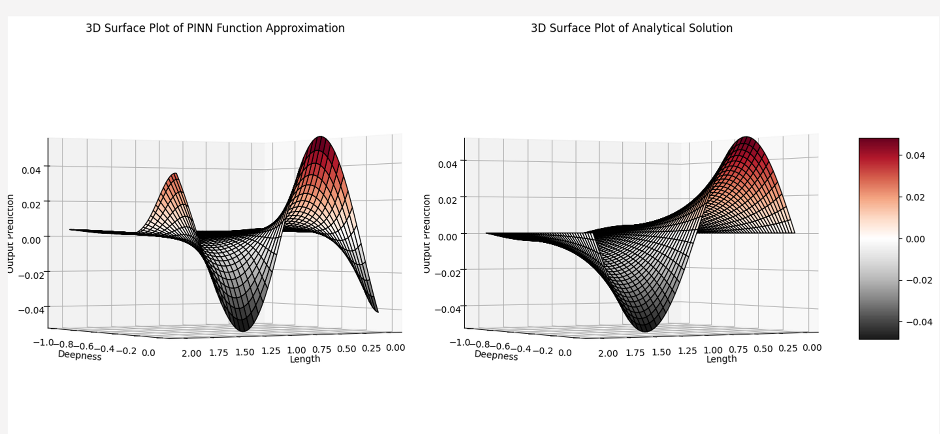

Figure 2: Scalar velocity potential comparison between the PINN and the analytical solution

For comparison with a purely data-driven approach a network is constructed by using only function approximation in the loss function. This model is trained on 250 points randomly distributed in the area.

In figure 3 we can see the difference between this purely data-driven approach and our PINN using. The PINN is clearly superior as it can more accurately reproduce the surface behaviour as well as the reduced velocity near the bottom. From figure 3 it can be seen that one aspect where the data-driven model performs better is adhering to the periodic boundary.

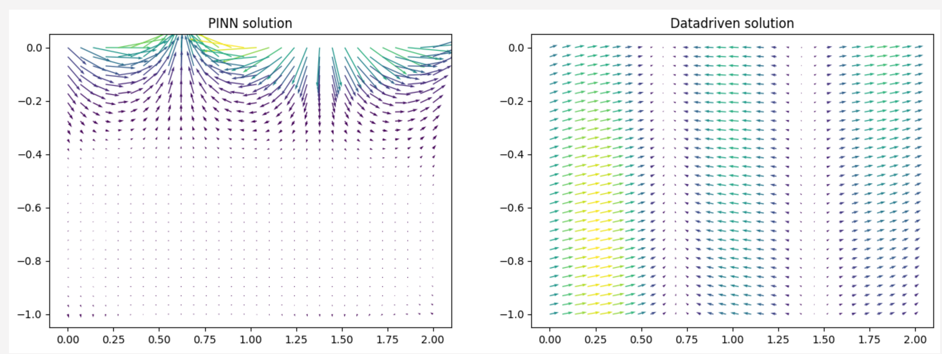

Figure 3: The resulting velocity fields from the PINN (left) and the data-driven approach (right).

4 Model including body

We include a body S in the model:

$$\begin{pmatrix} n_x \\ n_z \end{pmatrix} \begin{pmatrix} \delta_x \\ \delta_z \end{pmatrix} \Phi = 0, \forall [x, z] \in \partial(S) \tag{5}$$

We chose to place the body near the surface, without penetrating it. That way the surface boundary is not invalidated. This allows us to investigate the effect of the body near the surface.

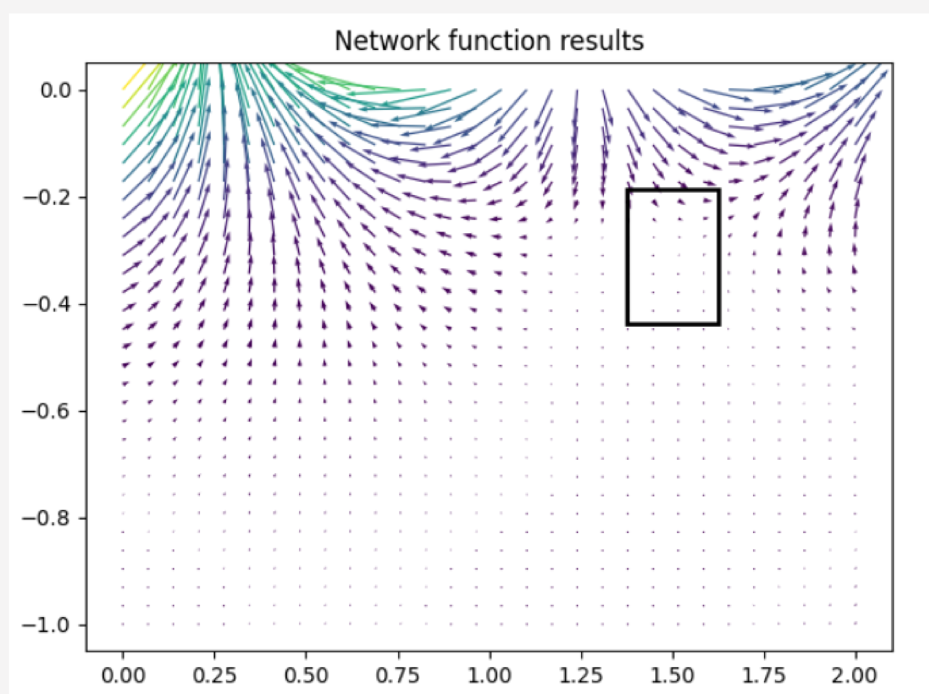

Figure 4: Resulting velocity field of our model with the body included