Corso di Laurea in Ingegneria Informatica (GI-ZZ) - Politecnico di Torino
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CONTROLLI AUTOMATICI (18AKSOA)

Lab activity on frequency domain loop-shaping control system design (Part I): Problems P1-P4

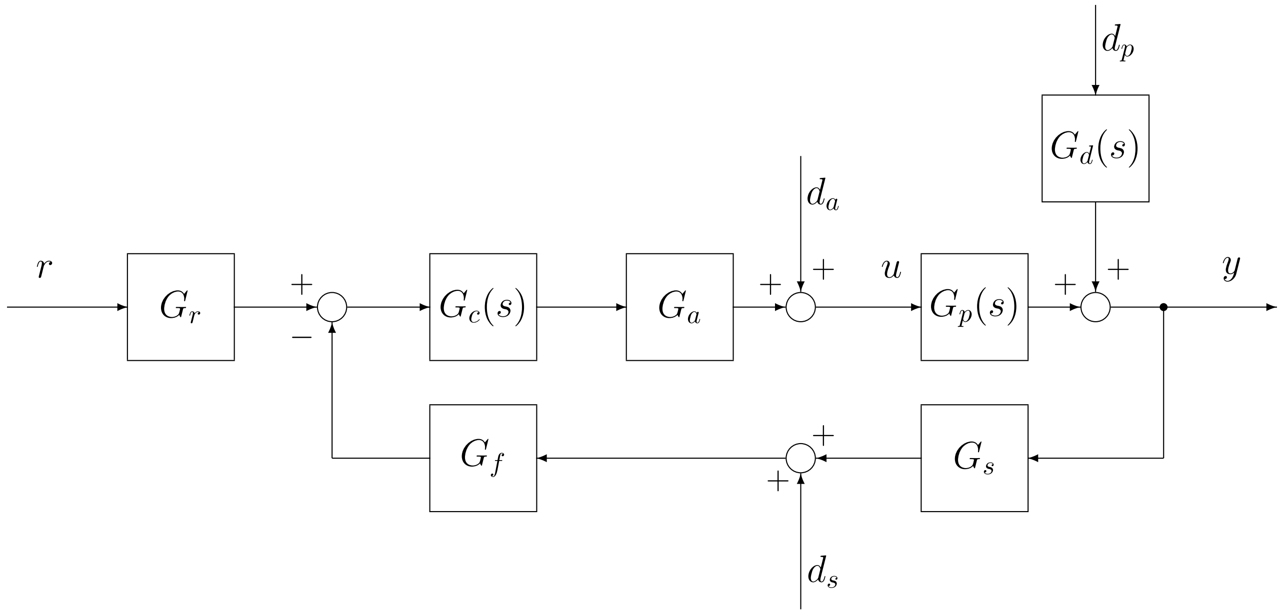
Diego Regruto

Main learning objectives

Upon successful completion of this homework, students will

1. Be able to design a cascade controller G_c through loop-shaping design techniques (generalized steady-state gain K_c, phase-lead, phase-lag).
2. Be able to derive a simulink model for the simulation of the designed feedback control system.
3. Be able to provide accurate graphical and numerical documentation of time domain performance of the designed feedback control system.

Consider the feedback control system below.



For problem $P1$ to problem $P4$ presented below, students are asked to:

(a) Design a cascade controller G_c and a feedback controller G_f trying to meet all required specifications, through loop-shaping design techniques.

(b) Check, through time simulation of the feedback control system, that all required specifications are really met.

(c) Provide accurate graphical and numerical documentation of time domain performance of the designed feedback control system, even if requirements are not satisfied.

Problem 1 — Given

$$G_p(s) = \frac{25}{s^3 + 3.3s^2 + 2s}$$

$G_s = 1$
$G_a = 0.095$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ |\ D_{a0}\ | \leq 5.5 \cdot 10^{-3};$
$d_p(t) = a_p \sin(\omega_p t), \quad |\ a_p\ | \leq 2 \cdot 10^{-2},\ \ \omega_p \leq 0.02$ rad s^{-1}.
$d_s(t) = a_s \sin(\omega_s t), \quad |\ a_s\ | \leq 10^{-1},\ \ \omega_s \geq 40$ rad s^{-1}.

Specifications
Steady-state gain of the feedback control system: $K_d = 1$
Steady-state output error when the reference is a ramp $(R_0 = 1)$: $|\ e_r^\infty\ | \leq 1.5 \cdot 10^{-1}$
Steady-state output error in the presence of d_a: $|\ e_{d_a}^\infty\ | \leq 1.5 \cdot 10^{-2}$
Steady-state output error in the presence of d_p: $|\ e_{d_p}^\infty\ | \leq 5 \cdot 10^{-4}$.
Steady-state output error in the presence of d_s: $|\ e_{d_s}^\infty\ | \leq 5 \cdot 10^{-4}$.
Rise time: $t_r \leq 3$ s
Settling time: $t_{s,\,5\%} \leq 12$ s
Step response overshoot: $\hat{s} \leq 10\%$

Problem 2 — Given

$$G_p(s) = \frac{40}{s^2 + 3s + 4.5}$$

$G_s = 1$
$G_a = -0.09$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ |\ D_{a0}\ | \leq 8.5 \cdot 10^{-3};$
$d_p(t) = D_{p0}t;\ |\ D_{p0}\ | \leq 3 \cdot 10^{-3};$
$d_s(t) = a_s \sin(\omega_s t), \quad |\ a_s\ | \leq 10^{-2},\ \ \omega_s \geq 50$ rad s^{-1}.

Specifications
Steady-state gain of the feedback control system: $K_d = 1$
Steady-state output error when the reference is a ramp $(R_0 = 1)$: $|\ e_r^\infty\ | \leq 3.5 \cdot 10^{-1}$
Steady-state output error in the presence of d_a: $|\ e_{d_a}^\infty\ | \leq 1.75 \cdot 10^{-2}$
Steady-state output error in the presence of d_p: $|\ e_{d_p}^\infty\ | \leq 1 \cdot 10^{-3}$
Steady-state output error in the presence of d_s: $|\ e_{d_s}^\infty\ | \leq 2 \cdot 10^{-4}$.
Rise time: $t_r \leq 2.5$ s
Settling time: $t_{s,\,5\%} \leq 10$ s
Step response overshoot: $\hat{s} \leq 8\%$

Problem 3 — Given

$$G_p(s) = \frac{100}{s^2 + 5.5s + 4.5}$$

$G_s = 1$
$G_a = 0.014$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ |\ D_{a0}\ | \leq 1.5 \cdot 10^{-3};$
$d_p(t) = a_p \sin(\omega_p t), \quad |\ a_p\ | \leq 16 \cdot 10^{-2},\ \ \omega_p \leq 0.03$ rad s^{-1}.
$d_s(t) = a_s \sin(\omega_s t), \quad |\ a_s\ | \leq 2 \cdot 10^{-1},\ \ \omega_s \geq 60$ rad s^{-1}.

Specifications
Steady-state gain of the feedback control system: $K_d = 1$
Steady-state output error when the reference is a ramp $(R_0 = 1)$: $|\ e_r^\infty\ | \leq 1.5 \cdot 10^{-1}$
Steady-state output error in the presence of d_a: $|\ e_{d_a}^\infty\ | \leq 4.5 \cdot 10^{-3}$
Steady-state output error in the presence of d_p: $|\ e_{d_p}^\infty\ | \leq 2 \cdot 10^{-3}$.
Steady-state output error in the presence of d_s: $|\ e_{d_s}^\infty\ | \leq 8 \cdot 10^{-4}$.
Rise time: $t_r \leq 2$ s
Settling time: $t_{s,\,5\%} \leq 8$ s
Step response overshoot: $\hat{s} \leq 12\%$

Problem 4 — Given

$$G_p(s) = \frac{-30}{s^3 + 3s^2 + 2s}$$

$G_s = 1$
$G_a = 0.006$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ |\ D_{a0}\ | \leq 2.5 \cdot 10^{-3};$
$d_p(t) = D_{p0}t;\ |\ D_{p0}\ | \leq 8.5 \cdot 10^{-3};$
$d_s(t) = a_s \sin(\omega_s t), \quad |\ a_s\ | \leq 5 \cdot 10^{-2},\ \ \omega_s \geq 40$ rad s^{-1}.

Specifications
Steady-state gain of the feedback control system: $K_d = 1$
Steady-state output error when the reference is a ramp $(R_0 = 1)$: $|\ e_r^\infty\ | \leq 2.5 \cdot 10^{-1}$
Steady-state output error in the presence of d_a: $|\ e_{d_a}^\infty\ | \leq 1 \cdot 10^{-2}$
Steady-state output error in the presence of d_p: $|\ e_{d_p}^\infty\ | \leq 1.5 \cdot 10^{-3}$
Steady-state output error in the presence of d_s: $|\ e_{d_s}^\infty\ | \leq 5 \cdot 10^{-4}$.
Rise time: $t_r \leq 3.5$ s
Settling time: $t_{s,\,5\%} \leq 14$ s
Step response overshoot: $\hat{s} \leq 15\%$

Some results of given problems

Problem P1

$\mid e_r^\infty \mid \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 0, \quad \mid K_c \mid \geq 5.614$
$\mid e_{d_a}^\infty \mid \leq 1.5 \cdot 10^{-2} \Rightarrow \nu \geq 0, \quad \mid K_c \mid \geq 3.8596$
$\zeta \geq 0.59$
$T_{po} \leq 1.049 = 0.41$ dB
$S_{po} \leq 1.361 = 2.7$ dB
$t_r \leq 3 \Rightarrow \omega_c \geq 0.66$ rad/s
$t_s \leq 12 \Rightarrow \omega_c \geq 0.31$ rad/s
$\mid e_{d_p}^\infty \mid \leq 5 \cdot 10^{-4} \Rightarrow M_S^{LF} \approx -32$ dB, $\omega_c \geq 0.25$ rad/s.
$\mid e_{d_s}^\infty \mid \leq 5 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -46$ dB, $\omega_c \leq 1.4$ rad/s.

Problem P2

$\mid e_r^\infty \mid \leq 3.50 \cdot 10^{-1} \Rightarrow \nu \geq 1, \quad \mid K_c \mid \geq 3.5714$
$\mid e_{d_a}^\infty \mid \leq 1.75 \cdot 10^{-2} \Rightarrow \nu \geq 0$, since $\nu \geq 1 \Rightarrow \mid e_{d_a}^\infty \mid = 0$ and no constraints on $\mid K_c \mid$ can be derived.
$\mid e_{d_p}^\infty \mid \leq 1.00 \cdot 10^{-3} \Rightarrow \nu \geq 1, \quad \mid K_c \mid \geq 3.75$
$\zeta \geq 0.63$
$T_{po} \leq 1.024 = 0.21$ dB
$S_{po} \leq 1.33 = 2.5$ dB
$t_r \leq 2.5 \Rightarrow \omega_c \geq 0.81$ rad/s
$t_s \leq 10 \Rightarrow \omega_c \geq 0.33$ rad/s
$\mid e_{d_s}^\infty \mid \leq 2 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -34$ dB, $\omega_c \leq 3.5$ rad/s.

Problem P3

$\mid e_r^\infty \mid \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 1, \quad \mid K_c \mid \geq 21.429$
$\mid e_{d_a}^\infty \mid \leq 4.5 \cdot 10^{-3} \Rightarrow \nu \geq 0$, since $\nu \geq 1 \Rightarrow \mid e_{d_a}^\infty \mid = 0$ and no constraints on $\mid K_c \mid$ can be derived.
$\zeta \geq 0.56$
$T_{po} \leq 1.078 = 0.65$ dB
$S_{po} \leq 1.39 = 2.9$ dB
$t_r \leq 2 \Rightarrow \omega_c \geq 0.972$ rad/s
$t_s \leq 8 \Rightarrow \omega_c \geq 0.498$ rad/s
$\mid e_{d_p}^\infty \mid \leq 2 \cdot 10^{-3} \Rightarrow M_S^{LF} \approx -38$ dB, $\omega_c \geq 0.54$ rad/s.
$\mid e_{d_s}^\infty \mid \leq 8 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -48$ dB, $\omega_c \leq 1.9$ rad/s.

Problem P4

$\mid e_r^\infty \mid \leq 2.5 \cdot 10^{-1} \Rightarrow \nu \geq 0, \quad \mid K_c \mid \geq 44.4$
$\mid e_{d_a}^\infty \mid \leq 1.0 \cdot 10^{-2} \Rightarrow \nu \geq 0, \quad \mid K_c \mid \geq 41.6$
$\mid e_{d_p}^\infty \mid \leq 1.5 \cdot 10^{-3} \Rightarrow \nu \geq 0, \quad \mid K_c \mid \geq 62.963$
$\zeta \geq 0.52$
$T_{po} \leq 1.13 = 1.1$ dB
$S_{po} \leq 1.45 = 3.2$ dB
$t_r \leq 3.5 \Rightarrow \omega_c \geq 0.55$ rad/s
$t_s \leq 14 \Rightarrow \omega_c \geq 0.32$ rad/s
$\mid e_{d_s}^\infty \mid \leq 5 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -40$ dB, $\omega_c \leq 2$ rad/s.

Useful Matlab commands

Following is a list of commands which are useful for this homework. If you type `help control`, you get the complete list of commands included in the Control System Toolbox of Matlab. Use `help` in MATLAB for more information on how to use any of these commands.

- `help`: Matlab help documentation.
- `figure`: Create a new figure or redefine the current figure, see also subplot, axis.
- `hold`: Hold the current graph, see also figure.
- `axis`: Set the scale of the current plot, see also plot, figure.
- `plot`: Draw a plot, see also figure, axis, subplot.
- `xlabel/ylabel`: Add a label to the horizontal/vertical axis of the current plot, see also title, text, gtext.
- `title`: Add a title to the current plot.
- `text`: Add a piece of text to the current plot, see also title, xlabel, ylabel, gtext.
- `subplot`: Divide the plot window up into pieces, see also plot, figure.
- `abs`: returns the absolute value of of a complex number.
- `angle`: returns the phase angles, in radians, of a complex number.
- `squeeze`: Remove singleton dimensions.
- `bode`: Draw the Bode plot, see also logspace, margin, nyquist1.
- `nyquist`: Draw the Nyquist plot.
- `nyquist1`: Draw the Nyquist plot, see also nyquist. Note this command was written to replace the MATLAB standard command nyquist to get more accurate Nyquist plots.
- `grid`: Draw the grid lines on the current plot.
- `logspace`: Provides logarithmically spaced vector.
- `dcgain`: Computes the steady-state (D.C. or low frequency) gain of LTI models.
- `tf`: Creation of transfer functions or conversion to transfer function. `s = tf('s')` specifies the transfer function $H(s) = s$ (Laplace variable).
- `zpk`: Create zero-pole-gain models or convert to zero-pole-gain format.
- `minreal`: Minimal realization and pole-zero cancellation.
- `tfdata`: Quick access to transfer function data. `[num,den] = tfdata(sys)` returns the numerator(s) and denominator(s) of the transfer function `sys`.
- `nichols` : Draws the Nichols plot of the frequency response of LTI models.
- `myngridst` : Draws the constant magnitude loci related to T_{po} (complementary sensitivity resonance peak) and S_{po} (sensitivity resonance peak) on the Nichols plane. This is not a native matlab command. This matlab function is provided by the instructor and should be copied in the working directory.