Nonlinear control and aerospace applications - Lab session 8

Exercise 1

A satellite is traveling on an Earth orbit. At time $t = 0$, the satellite has position $\mathbf{r}(0)$ and velocity $\mathbf{v}(0)$ given by

$$\mathbf{r}(0) = \begin{bmatrix} 6.54e6 \\ 0 \\ 0 \end{bmatrix} m, \qquad \mathbf{v}(0) = \begin{bmatrix} 0 \\ 7.88e3 \\ 0 \end{bmatrix} m/s \ .$$

All coordinates are expressed in an inertial frame with origin at the Earth CoM.

1. Using the free restricted two body equation, simulate the motion of the satellite on the orbit for the time interval $[0, 2e4]\ s$, and produce the following plots (on separate figures):
   (a) a plot of the position vector in function of time;
   (b) a 3D plot of the satellite trajectory around the Earth (use the Matlab commands "sphere" and "mesh"; see the help of these commands);
   (c) a plot of the total mechanical energy and the angular momentum (per unit mass) in function of time.
2. Repeat Step 1 considering different values of the initial velocity second component.
3. Repeat Step 1b using the orbit equation instead of the free restricted two body equation.

Exercise 2

Consider an Earth orbit with the following orbital elements:

$$a = 40e3\,km,\ e = 0.5,\ i = 50^o,\ \Omega = 30^o,\ \omega = 100^o.$$

1. Plot the orbit in the GE frame, together with a sphere of radius $6.38e3\,km$, representing the Earth.
2. Take the initial condition $x(0) = (\mathbf{r}_{GE}(0), \mathbf{v}_{GE}(0))$, where $\mathbf{r}_{GE}(0)$ and $\mathbf{v}_{GE}(0)$ are the position and velocity in the GE frame, corresponding to the true anomaly value $\theta = 0\,rad$.
   (a) Integrate the free restricted two body equation in the time interval $[0, 2P]$, where P is the orbit period.
   (b) Plot the orbit obtained from this integration in the GE frame, together with the orbit previously obtained and the Earth sphere.