

Nonlinear control and aerospace applications - Lab session 5

Exercise 1

Consider two reference frames $F1 = \{O, \mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3\}$ and $F2 = \{O, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. Suppose that $F2$ is rotated wrt $F1$ through a proper Euler rotation 323, with angles $\psi = \pi/3$, $\theta = -\pi/6$, $\phi = \pi/4$. Let the following vector be given:

$$\mathbf{r} = -\mathbf{b}_1 + 4\mathbf{b}_2 + 2\mathbf{b}_3.$$

1. Compute the components of \mathbf{r} in $F1$.
2. Supposing to be an observer in $F2$, plot the following vectors:
 - (a) the vector \mathbf{r} ;
 - (b) the rotated vector $\mathbf{r}' = \mathbf{T}_{323}\mathbf{r}$, where \mathbf{T}_{323} is the cosine matrix associated with the above rotation.
3. Compute the norms of \mathbf{r} and \mathbf{r}' .
4. Derive the general expression of the matrix \mathbf{T}_{323} as a function of ψ , θ , ϕ (Matlab symbolic toolbox).
5. Compute the axis of the rotation produced by \mathbf{T}_{323} .

Exercise 2

1. Plot the vector $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j} + 2\mathbf{k}$ and the following rotated vectors:
 - (a) $\mathbf{r}_1 = \mathbf{T}_1(\pi/2)\mathbf{r}$;
 - (b) $\mathbf{r}_2 = \mathbf{T}_3(\pi/2)\mathbf{T}_1(\pi/2)\mathbf{r}$;
 - (c) $\mathbf{r}_3 = \mathbf{T}_3(\pi/2)\mathbf{r}$;
 - (d) $\mathbf{r}_2 = \mathbf{T}_1(\pi/2)\mathbf{T}_3(\pi/2)\mathbf{r}$.
2. Verify that $\mathbf{T}_3(\pi/2)\mathbf{T}_1(\pi/2)$ is an extrinsic rotation $x - z$ or an intrinsic rotation $z - x'$, where x' denotes the rotated x axis.
3. Verify that $\mathbf{T}_1(\pi/2)\mathbf{T}_3(\pi/2)$ is an extrinsic rotation $z - x$ or an intrinsic rotation $x - z'$, where z' denotes the rotated z axis.
4. For each of the 4 rotations above, compute the eigenvalues and the axis of rotation.

Exercise 3

Repeat Steps 1-3 of Exercise 1 using quaternions instead of rotation matrices.

Exercise 4

Let the quaternions $\mathbf{q} = (0.866, 0.4319, 0.216, 0.1296)$ and $\mathbf{p} = (0.9659, 0.183, 0, 0.183)$ be given. Compute:

1. The reciprocals \mathbf{q}^{-1} and \mathbf{p}^{-1} .
2. The dot products $\mathbf{q} \cdot \mathbf{p}$ and $\mathbf{p} \cdot \mathbf{q}$.
3. The quaternion products $\mathbf{q} \otimes \mathbf{p}$ and $\mathbf{p} \otimes \mathbf{q}$.
4. The quaternion products $\mathfrak{I} \otimes \mathbf{p}$ and $\mathbf{p} \otimes \mathfrak{I}$, where \mathfrak{I} is the identity element.

Exercise 5

1. Compute the quaternion corresponding to the DCM

$$\mathbf{T} = \begin{bmatrix} 0.2276 & -0.9354 & 0.2706 \\ 0.7571 & -0.004773 & -0.6533 \\ 0.6124 & 0.3536 & 0.7071 \end{bmatrix}.$$

2. Consider the quaternion

$$q = (0.588, 0.03378, -0.2566, 0.7663).$$

Compute the corresponding DCM and Euler angles 313.