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Questão 1.

a) j^j

$$z = a + jb = r.e^{j\theta}$$

$$a = r\cos\theta$$
$$b = r\,\text{sen}\,\theta$$
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

~~a = (1) cos(~~

$$j = \sqrt{-1}$$
$$j^j = \sqrt{-1}^{\sqrt{-1}}$$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{0 + (j^j)^2}$$
$$a = 1\cos(-\pi/2) + j\,\text{sen}(\pi/2)$$
$$z = 0 + j(-1)$$

b) $e^{j(\pi/4)}$

$r = 1$

$a = r\cos\theta$ | $b = r\,\text{sen}\,\theta$

$a = (1)\cos(\pi/4)$ | $b = (1)\,\text{sen}(\pi/4)$

$a = 0{,}7071$ | $b = 0{,}7071$

$$\therefore z = \sqrt{2}/2 + j\sqrt{2}/2$$

c) $\cos(\sqrt{j})$

$r = 1/2$

$$\cos(y) = \frac{e^{jy} + e^{-jy}}{2}$$

$$\cos(\sqrt{j}) = \frac{e^{j\sqrt{j}} + e^{-j(\sqrt{j})}}{2} = \frac{1}{2}\left(e^{j\sqrt{j}} - e^{-j(\sqrt{j})}\right)$$

$\theta = \tan^{-1}(b/a) = \theta = \sqrt{j} = \sqrt{-1}$; $a = \frac{1}{2}\cos(\sqrt{-1})$; $b = \frac{1}{2}\,\text{sen}(\sqrt{-1})$

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Questão 2.

a) $x(t) = 2\cos(t)$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

$$P_x = \lim_{T\to\infty} \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} [2\cos(t)^2]\,dt \Rightarrow \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 4\cos^2(t)\,dt$$

$$\Rightarrow \frac{4}{\pi}\int_{-\pi/2}^{\pi/2} \cos^2(t)\,dt \rightarrow \cos^2(t) = \frac{1}{2}(1+\cos(2t)) \Rightarrow \frac{4}{\pi}\int_{-\pi/2}^{\pi/2} \frac{1}{2}(1+\cos(2t))\,dt$$

$$\Rightarrow \frac{2}{\pi}\int_{-\pi/2}^{\pi/2}(1+\cos(2t))\,dt \Rightarrow \frac{2}{\pi}\left[\int_{-\pi/2}^{\pi/2} 1\,dt + \int_{-\pi/2}^{\pi/2}\cos(2t)\,dt\right] \Rightarrow \frac{2}{\pi}\left[[t]_{-\pi/2}^{\pi/2} + \dots\right.$$

$$\left.\dots + \int_{-\pi/2}^{\pi/2}\cos(2t)\,dt\right] \Rightarrow u = 2t \Rightarrow du = 2dt \Rightarrow du/2 = dt \Rightarrow \int_{-\pi/2}^{\pi/2}\cos(u)\frac{du}{2}$$

$$\Rightarrow \frac{\operatorname{sen}(u)}{2} \Rightarrow \left[\frac{\operatorname{sen}(2t)}{2}\right]\Bigg|_{-\pi/2}^{\pi/2} \Rightarrow \frac{2}{\pi}\left[t\Big|_{-\pi/2}^{\pi/2} + \frac{\operatorname{sen}(2t)}{2}\Bigg|_{-\pi/2}^{\pi/2}\right] \Rightarrow$$

$$\Rightarrow \frac{2}{\pi}\left[\left(\pi/2 + \pi/2\right) + \left(\cancel{\frac{\operatorname{sen}(\pi)}{2}} + \cancel{\frac{\operatorname{sen}(\pi)}{2}}\right)\right] \Rightarrow \frac{2}{\cancel{\pi}}[\cancel{\pi}] \Rightarrow 2$$

$$\therefore P_x = 2 \text{ u.p.}$$

A energia, pode ser obtida por:

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$\Rightarrow E_x = \int_{-\infty}^{\infty} 4\cos^2(t)\,dt \Rightarrow 4\int_{-\infty}^{\infty}\cos^2(t)\,dt \Rightarrow 4\int_{-\infty}^{\infty}\frac{1}{2}(1+\cos(2t))\,dt \Rightarrow 2\int_{-\infty}^{\infty}(1+\cos(2t))\,dt$$

$$u = 2t$$

$$\Rightarrow 2\left[\int_{-\infty}^{\infty} 1\,dt + \int_{-\infty}^{\infty}\cos(2t)\,dt\right] \Rightarrow du/2 = dt \Rightarrow 2\left[t\Big|_{-\infty}^{\infty} + \operatorname{sen}(2t)/2\Big|_{-\infty}^{\infty}\right] \Rightarrow$$

$\Rightarrow$ ~~$\therefore E_x$ tende ao infinito~~

Com o intervalo de $[-\pi, \pi] \Rightarrow 2\left[t + \frac{\operatorname{sen}(2t)}{2}\right]\Bigg|_{-\pi}^{\pi} \Rightarrow 2[2\pi + 0]$

$$E_x = 4\pi \text{ u.e.}$$

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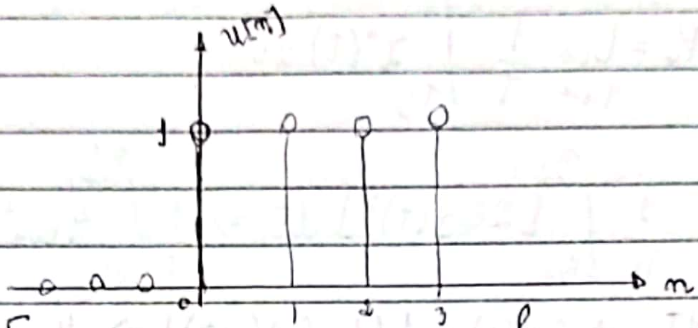
b) $x[n] = u[n]$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



$E_x \longrightarrow P_x$

$E_x = \sum_{n=\infty}^{\infty} |x[n]|^2$, $E_x < \infty \rightarrow$ Tipo energia

$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$, $0 < P_x < \infty \rightarrow$ Tipo potência

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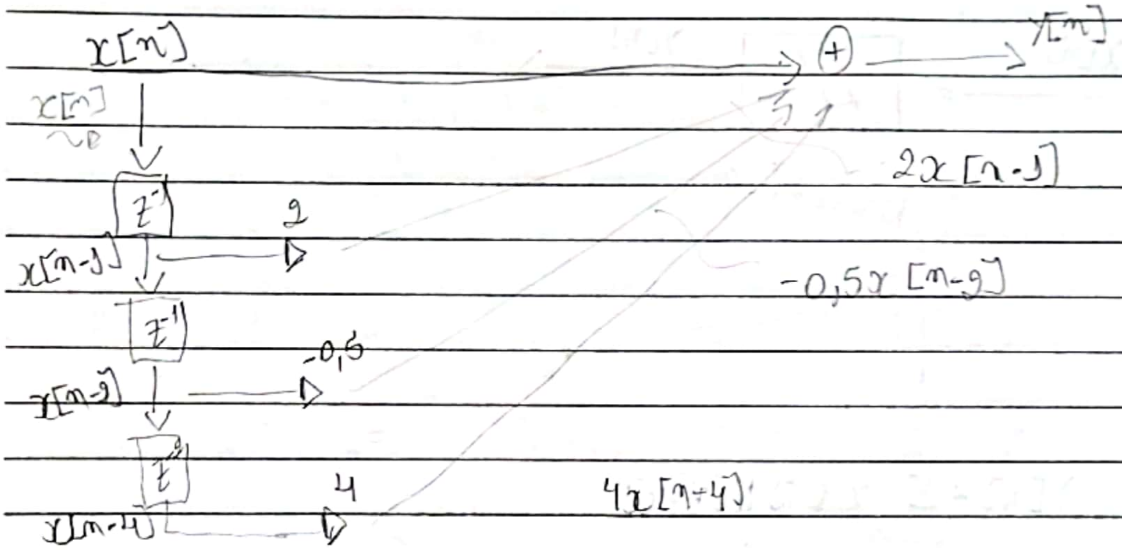
Questão 3.

$$y[n] = x[n] + 2x[n-1] - 0{,}5x[n-2] + 4x[n-4]$$



	x[n]	x[n-1]	x[n-2]	x[n-4]	y[n] = x[n] + 2x[n-1] - 0,5 x[n-2] + 4x[n-4]
0	1	0	0	0	y[0] = 1
1	0	1	0	0	y[1] = 2
2	0	0	1	0	y[2] = -0,5
3	0	0	0	0	y[3] = 0
4	0	0	0	1	y[4] = 4

h[t]

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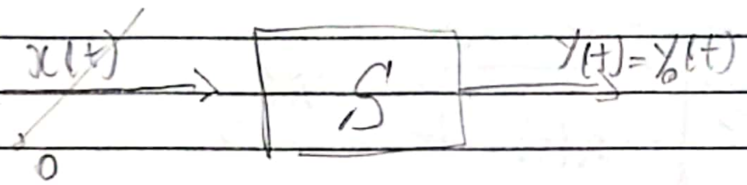
Questão 4.

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = x(t) + 2\frac{d}{dt}x(t) + \frac{d^2}{dt^2}x(t)$$



$$\sum_{k=0}^{N} a_k \frac{d^{N-k}}{dt^{N-k}} y_0(t) = 0$$

$$y_0(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \dots + C_n e^{\lambda_n t}$$

$$y_0''(t) + 3y_0'(t) + 2y_0(t) = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-3 \pm \sqrt{1}}{2} \quad \lambda_1 = -1, \quad \lambda_2 = -2$$

$$y_0(t) = C_1 e^{-t} + C_2 e^{-2t}$$

$y(0) = 1$ e $y'(0) = 2$

$$\begin{cases} \text{(I)} \; y(0) = C_1 + C_2 = 1 \\ \text{(II)} \; y'(0) = -C_1 - 2C_2 = 2 \end{cases}$$

(I) $\to C_1 = 1 - C_2$

(II) $\to -(1 - C_2) - 2C_2 = 2$

$-1 - C_2 = 2$

$\boxed{C_2 = -3}$

(I): $C_1 = 1 + 3$

$\boxed{C_1 = 4}$

$\therefore y_0(t) = 4e^{-t} - 3e^{-2t}$

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~~7.~~ Questão 5.

$$h(t) = e^{-t}u(t)$$
$$x(t) = e^{-2t}u(t)$$

$$x(t) \longrightarrow \boxed{h(t)} \xrightarrow{\;y(t)\;}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau) \cdot e^{-2(t-\tau)}\,u(t-\tau)\,d\tau$$

$$y(t) = \int^{t} e^{-\tau} \cdot e^{-2t+2\tau}\,d\tau \Rightarrow \int^{t} e^{-2t+\tau}\,d\tau \qquad t-\tau > 0 \Rightarrow \tau < t \qquad u = -2t+\tau,\ du = d\tau$$

$$\Rightarrow \int e^{u}\,du \Rightarrow e^{u} \Rightarrow e^{-2t+\tau} \Rightarrow \left[e^{-2t+\tau}\right]\Big|_0^t$$

$$\Rightarrow \left[(e^{-2t+t}) - (e^{-2t+0})\right] = (e^{-t} - e^{-2t})\,u(t)$$

$$\therefore\ y(t) = (e^{-t} - e^{-2t})\,u(t)$$

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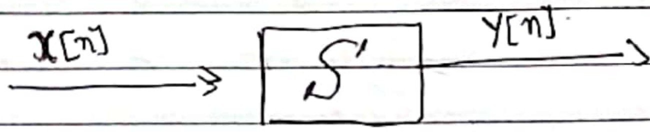
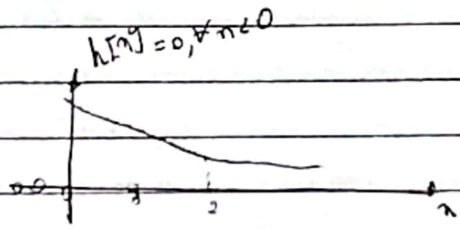
Questão 6.

$$h[n] = (0,5)^n u[n]$$
$$x[n] = u[n] - u[n-5]$$





$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} (u[k] - u[k-5]).(0,5)^{n-k}\, u[n-k]$$

$n - 5 \geq 0 \Rightarrow 1; \quad n \geq 5$

$n - k \geq 0 \Rightarrow 1$

$k \leq n$

$$y[n] = \sum_{k=0}^{\infty} (\overset{1}{\cancel{u[k]}} - \underset{0}{\cancel{u[-5]}})(0,5)^{n-k}\, u[n-k]$$

$$y[n] = \sum_{k=0}^{n} (0,5)^{n-k} \qquad n \geq 0$$

$$y[n] = (0,5)^n \sum_{k=0}^{n} (0,5)^{-k}$$

$$y[n] = (0,5)^n \sum_{k=0}^{n} \frac{1}{(0,5)^k} . u[n]$$