A08

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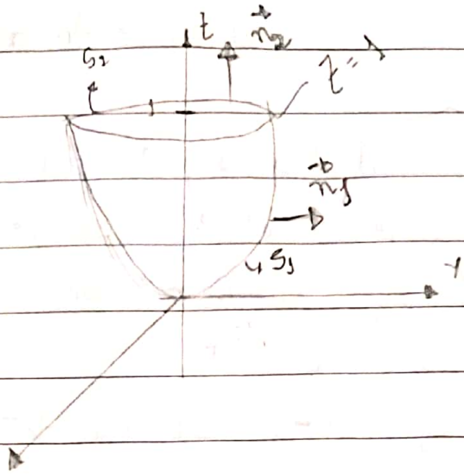

$$\begin{cases} S_1: z = x^2 + y^2 \;;\; 0 \leq z \leq 1 \\ S_2: x^2 + y^2 \leq 1 \;,\; z = 1 \end{cases}$$

$\vec{F}(x,y,z) = (0, z, -y)$

$$\phi = \iint_S \vec{F} \cdot \vec{n}\, dS = \iint_{S_1} \vec{F} \cdot \vec{n}_1 dS + \iint_{S_2} \vec{F} \cdot \vec{n}_2\, dS$$

Em S_1: Paraboloide

$$\begin{cases} x = t\cos\theta \\ y = t\sin\theta \\ z = t^2 \end{cases}$$

$\vec{r}_1(\theta, t) = (t\cos\theta, t\sin\theta, t^2)$

$\dfrac{\partial \vec{r}_1}{\partial \theta} = (-t\sin\theta, t\cos\theta, 0)$; $\dfrac{\partial \vec{r}_1}{\partial t} = (\cos\theta, \sin\theta, 2t)$

$\theta \in [0, 2\pi]$

$t \in [0, 1]$

$$\frac{\partial \vec{r}_1}{\partial \theta} \times \frac{\partial \vec{r}_1}{\partial t} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -t\sin\theta & t\cos\theta & 0 \\ \cos\theta & \sin\theta & 2t \end{vmatrix} \begin{aligned} & 2t^2\cos\theta\, \hat{i} + 0\hat{j} - t\sin^2\theta\, \hat{k} \\ & = 0\hat{i} + 2t^2\sin\theta\, \hat{j} - t\cos^2\theta\, \hat{k} \end{aligned}$$

$\vec{n}_1 = (2t^2\cos\theta, 2t^2\sin\theta, -t)$ ↳ indica que aponta para fora

$\vec{F}(\vec{r}_1(\theta,t)) = (0, -t, -2t^2\sin\theta)$

$\vec{F}(\vec{r}_1(\theta,t)) \cdot \vec{n} = (0, -t, -2t^2\sin\theta) \cdot (2t\cos\theta, 2t^2\sin\theta, -t)$

$= 0 - 2t^3\sin\theta + 2t^3\sin\theta$

$\vec{F}(\vec{r}_1(\theta,t)) \cdot \vec{n} = 0 \to \vec{F} \perp \vec{n}$

Assim:

$$\phi_1 = \iint_{S_1} \vec{F}(\vec{r}_1(\theta,t)) \cdot \vec{n}_1\, dt\, d\theta = \iint_{S_1} 0\, dt\, d\theta \to \phi_1 = 0$$

Em S_2: disco

$$\begin{cases} x = t\cos\theta \\ y = t\sin\theta \\ z = 1 \end{cases}$$

$\vec{r}_2(\theta, t) = (t\cos\theta, t\sin\theta, 1)$

$\dfrac{\partial \vec{r}_2}{\partial \theta} = (-t\sin\theta, t\cos\theta, 0)$; $\dfrac{\partial \vec{r}_2}{\partial t} = (\cos\theta, \sin\theta, 0)$

$t \in [0, 1]$

$\theta \in [0, 2\pi]$

$$\frac{\partial \vec{r}_2}{\partial \theta} \times \frac{\partial \vec{r}_2}{\partial t} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -t\sin\theta & t\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \end{vmatrix} \begin{aligned} & 0\hat{i} + 0\hat{j} + (-t\sin^2\theta)\hat{k} \\ & = 0\hat{i} \quad 0\hat{j} \quad -t\cos^2\theta\, \hat{k} \end{aligned}$$

$\vec{n}_2 = (0, 0, -t)$

$\vec{n}_2 = (0,0,-t) = -\vec{n}_2 = (0,0,t)$

↳ sentido negativo, aponta p/ dentro da superfície

$\vec{F}(\vec{n}_2(\theta,t)) = (0, 1, -t\,\text{sen}\,\theta)$

$\vec{F}(\vec{n}_2(\theta,t)) \cdot \vec{n}_2 = (0, 1, -t\,\text{sen}\,\theta) \cdot (0,0,t)$

$\qquad '' \qquad = 0 + 0 - t^2\,\text{sen}\,\theta$

Em S_2:

$$\phi_2 = \int_0^{2\pi}\int_0^1 -t^2\,\text{sen}\,\theta\, dt\, d\theta \to -\int_0^{2\pi} \text{sen}\,\theta \left[\frac{t^3}{3}\right]_0^1 d\theta \to -\int_0^{2\pi} \text{sen}\,\theta \left[\frac{1^3}{3} - \frac{0^3}{3}\right] d\theta$$

$$\to -\int_0^{2\pi} \text{sen}\,\theta \left[\frac{1}{3}\right] d\theta \to -\frac{1}{3}\left[-\cos\theta\right]_0^{2\pi} \to +\frac{1}{3}\left[\cos(2\pi) - \cos(0)\right] \to +\frac{1}{3}\left[1-1\right]$$

$$\phi_2 = 0$$

Logo:

$$\phi = \iint_{S_1} \vec{F} \cdot \vec{n}\, ds + \iint_{S_2} \vec{F} \cdot \vec{n}\, ds$$

$$\phi = 0 + 0$$

$$\boxed{\phi = 0}$$

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