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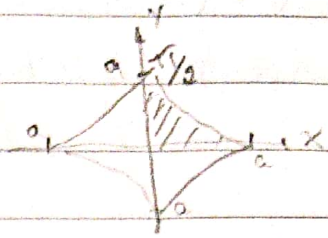
1.

a) (Falso)

$$\begin{cases} x = a\cos^3 t \rightarrow x'(t) = -3a\cos^2 t \,\text{sen}\, t \\ y = a\,\text{sen}^3 t \rightarrow y'(t) = 3a\,\text{sen}^2 t \cos t \end{cases}$$



A área pode ser obtida calculando apenas o primeiro quadrante e em seguida multiplica-se por 4 para obter-se a área total. $t \in [0, \pi/2]$

$$A = \frac{1}{2}\int_0^{\pi/2} [-y(t)x'(t) + x(t)y'(t)]\,dt$$

$$[(-a\,\text{sen}^3 t)(-3a\cos^2 t\,\text{sen}\,t) + (a\cos^3 t)(3a\,\text{sen}^2 t\cos t)]$$

$$3a^2\cos^2 t\,\text{sen}^4 t + 3a^2\,\text{sen}^2 t\cos^4 t$$

$$= 3a^2\cos^2 t \cdot \text{sen}^2 t$$

. $\cos^2 t = \frac{1}{2}(1+\cos 2t)$

. $\text{sen}^2 t = \frac{1}{2}(1-\cos 2t)$

$$3a^2[\tfrac{1}{2}(1-\cos 2t)][\tfrac{1}{2}(1+\cos 2t)]$$

$$\frac{3a^2}{2}[(1-\cos^2 2t)]$$

$$A = \frac{3a^2}{2}\int_0^{\pi/2}(1-\cos^2 2t)\,dt$$

Entretanto, é apenas no primeiro quadrante, multiplicando por 4:

$$A = 6a^2\int_0^{\pi/2}(1-\cos^2 2t)\,dt \rightarrow \text{área Total}$$

b) (Verdadeiro)

$$\begin{cases} x = u\cos v \\ y = u\,\text{sen}\,v \\ z = u^2 \end{cases}$$

$0 \le u \le 2\sqrt{2}$; $0 \le v \le \pi/4$; $P = (2,2,8)$

$u^2 = 8$

$\boxed{u = 2\sqrt{2}}$

$2 = u\cos v$

$v = \arccos(\sqrt{2}/2)$

$\boxed{v = \pi/4}$

u - Curva: $v = \pi/4$

$\vec{r}(u, \pi/4) = (u\cos(\pi/4), u\,\text{sen}(\pi/4), u^2)$

$= (u\sqrt{2}/2, u\sqrt{2}/2, u^2)$

u - Curva: Parábola (plano paralelo (YZ)

• v - Curva: $u = 2\sqrt{2}$

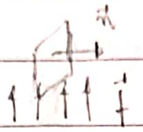
$\vec{r}(2\sqrt{2}, v) = (2\sqrt{2}\cos(v), 2\sqrt{2}\,\text{sen}(v), (2\sqrt{2})^2)$

$= (2\sqrt{2}\cos(v), 2\sqrt{2}\,\text{sen}\,v, 8)$

v - Curva: Circunferência plano paralelo (xy)

e) (Falso)

De acordo com: $\vec{f}\cdot\vec{n} = |\vec{f}||\vec{n}|\cdot\cos\theta$, assim se $\vec{f} \perp \vec{n}$ $(\theta = \pi/2)$, $\vec{f}\cdot\vec{n} = 0$. Logo, o campo $\vec{f}$ deve ser ortogonal à superfície em todos os pontos de S, então o fluxo de $\vec{f}$ em S é nulo.



$\vec{f}\cdot\vec{n} = 0 \to \phi = 0$.

d) (Falso); $\vec{r}(u,v) = (u, v, 1)$

$\frac{\partial\vec{r}}{\partial u} = (1,0,0)$; $\frac{\partial\vec{r}}{\partial v} = (0,1,0)$

$\vec{n} = \left(\frac{\partial\vec{r}}{\partial u} \times \frac{\partial\vec{r}}{\partial v}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,1)$

$\vec{F}(\vec{r}(u,v))\cdot\vec{n} = (f(u,v), g(u,v), 0)\cdot(0,0,1) = (0+0+0) \to \phi = 0$

Logo, $\vec{F}(x,y,z) \perp \vec{n}$ e ϕ é nulo em todos os pontos de S.

2.

De acordo com a definição, integral de linha de um campo conservativo $\vec{F}(x,y)$ independe do caminho, nesse caso a região é simplesmente conexa.

$$\int_C \vec{F}\cdot d\vec{r} = \phi(B) - \phi(A)$$

Se

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

$g = x(x^2+y^2)^{-1}$

$$\frac{\partial g}{\partial x} = 1(x^2+y^2)^{-1} + x(-1)(x^2+y^2)^{-2}(2x) = +\frac{1}{x^2+y^2} - \frac{2x}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$f = -y(x^2+y^2)^{-1}$

$$\frac{\partial f}{\partial y} = -1(x^2+y^2)^{-1} + (y)(-1)(x^2+y^2)^{-2}(2y) = \frac{-1}{x^2+y^2} + \frac{2y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{\partial g}{\partial x}(x,y) = \frac{\partial f}{\partial y}(x,y), \text{ para cada } (x,y) \neq (0,0)$$

Assim $\vec{F}(x,y)$ é conservativo em uma região simplesmente conexa (fora da origem)

$$\vec{F} = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j}$$

$$\begin{cases} \partial\phi/\partial x = -y/x^2+y^2 \\ \partial\phi/\partial y = x/x^2+y^2 \end{cases}$$

$$\vec{F}(x,y) = \frac{1}{x^2+y^2}(-y, x)$$

$$\frac{\partial \phi}{\partial x} = \frac{-y}{x^2+y^2}$$

$$\int \partial\phi = \int \frac{-y}{x^2+y^2}\,\partial x$$

Pela tabela de integrais (Cálculo B - Diva)

(17)

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\operatorname{arctg}(u/a) + C$$

$u = x/y$

$\frac{du}{\partial x} = 1/y \rightarrow \partial x = y\,du$

$$\int \frac{-y^2}{y^2u^2+y^2}\,du$$

$$-\int \frac{1}{u^2+1}\,du$$

$$= -\operatorname{arctg}(u) + C$$

$$\phi = -\operatorname{arctg}(x/y) + C(y)$$

$$\frac{\partial \phi}{\partial y}(-\operatorname{arctg}(x/y) + C(y))$$

$$\frac{\partial \phi}{\partial y} = \frac{x}{y^2+x^2} + \frac{\partial C}{\partial y}$$

$$\cancel{\frac{x}{y^2+x^2}} + \frac{\partial C}{\partial y} = \cancel{\frac{x}{y^2+x^2}}$$

$$\int \partial C = \int 0\,dy$$

$$C = 0 + K$$

Assim:

$$\phi = -\operatorname{arctg}(x/y) + K$$

$A = (1,1)$

$B = (4,1)$

$$\int_c \vec{F}\cdot d\vec{r} = \phi(B) - \phi(A)$$

$$(-\operatorname{arctg}(4/1) + K) - (-\operatorname{arctg}(1/1) + K)$$

$$\therefore (-\operatorname{arctg}(4) + \operatorname{arctg}(1))$$

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3.

$z^2 = 4 - x^2 - y^2 \quad ; \quad z = \sqrt{\frac{x^2+y^2}{3}}$

$\frac{x^2+y^2}{3} = 4 - x^2 - y^2$

$x^2 + y^2 + 3x^2 + 3y^2 = 12$

$\boxed{x^2+y^2 = 3}$ //

a)

$$\begin{cases} x = x \\ y = y \\ z = \sqrt{4-x^2-y^2} \end{cases}$$

$(x,y) \in D: x^2+y^2 \leq 3$

$\vec{f}(x,y) = (x, y, \sqrt{4-x^2-y^2})$

b)

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ z = \sqrt{4-r^2} \end{cases}$$

$0 \leq r \leq 1$

$0 \leq \theta \leq 2\pi$

$x^2+y^2 = r^2$

$r^2 = 3$

$r = \sqrt{3}$

(I) $z^2 = 4 - x^2 - y^2$

$4 - z^2 = x^2 + y^2$

(II) $z = \sqrt{\frac{(4-z^2)}{3}}$

$z^2 = \frac{(4-z^2)}{3}$

$3z^2 = 4 - z^2$

$\boxed{z = 1}$ //

c)

$x^2+y^2+z^2 = 4$

$$\begin{cases} x = \rho\,\text{sen}\,\phi\cos\theta \\ y = \rho\,\text{sen}\,\phi\,\text{sen}\,\theta \\ z = \rho\cos\phi \end{cases}$$

$(\rho\,\text{sen}\,\phi\cos\theta)^2 + (\rho\,\text{sen}\,\phi\,\text{sen}\,\theta)^2 + (\rho\cos\phi)^2 = 4$

$\rho^2 = 4$

$\boxed{\rho = 2}$ //

$x^2+y^2 = 3$

$\rho^2 [\text{sen}^2\phi\cos^2\theta + \text{sen}^2\phi\,\text{sen}^2\theta] = 3$

$\rho^2 [\text{sen}^2\phi\,(\cos^2\theta + \text{sen}^2\theta)] = 3$

$\rho^2\,\text{sen}^2\phi = 3$

$(2)^2 = \text{sen}^2\phi = 3$

$\boxed{\phi = \pi/3}$ //

Assim

$$\begin{cases} x = 2\,\text{sen}\,\phi\cos\theta \\ y = 2\,\text{sen}\,\phi\,\text{sen}\,\theta \\ z = 2\cos\phi \end{cases}$$

$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \pi/3$

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d)

Utilizando a parametrização do item a):

$$\begin{cases} x = x \\ y = y \\ z = \sqrt{4-x^2-y^2} \end{cases} \qquad A(S) = \iint \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

$$\frac{\partial z}{\partial x} = (1/2)(4-x^2-y^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{4-x^2-y^2}}$$

$$\frac{\partial z}{\partial y} = (1/2)(4-x^2-y^2)^{-1/2}(-2y) = \frac{-y}{\sqrt{4-x^2-y^2}}$$

$$A(S) = \iint \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2-y^2}}\right)^2 + \left(\frac{-y}{\sqrt{4-x^2-y^2}}\right)^2}\, dA$$

$$" = \iint \sqrt{1 + \frac{x^2}{4-x^2-y^2} + \frac{y^2}{4-x^2-y^2}}\, dA \Rightarrow \sqrt{1 + \frac{x^2+y^2}{4-x^2-y^2}} \Rightarrow \sqrt{\frac{4}{4-x^2-y^2}}$$

$$A(S) = \iint \sqrt{\frac{4}{4-x^2-y^2}}\, dA$$

Passando para coordenadas polares:

$$\begin{cases} x = r\cos\theta & 0 \le r \le \sqrt{3} \\ y = r\,\text{sen}\,\theta & 0 \le \theta \le 2\pi \\ dA = r\,dr\,d\theta \end{cases}$$

$$A(S) = \iint \sqrt{\frac{4}{4-[(r^2(\cos^2\theta + \text{sen}^2\theta)]}}\, dA \Rightarrow \sqrt{\frac{4}{4-r^2}} \Rightarrow A(S) = \int_0^{2\pi}\int_0^{\sqrt{3}} \frac{2}{\sqrt{4-r^2}}\, r\,dr\,d\theta$$

$$A(S) = 2\int_0^{2\pi}\int_0^{\sqrt{3}} \frac{r}{\sqrt{4-r^2}}\, dr\,d\theta \Rightarrow u = 4-r^2 \Rightarrow \partial u/\partial r = -2r \Rightarrow -\partial u/2 = r\,dr$$

$$A(S) = 2\int_0^{2\pi}\int \frac{1}{u^{1/2}}\cdot\frac{-du}{2}\, d\theta \Rightarrow 2\int_0^{2\pi}\left[-\frac{1}{2}\left(\frac{u^{1/2}}{1/2}\right)\right] d\theta \Rightarrow 2\int_0^{2\pi}\left[-\sqrt{4-r^2}\right]_0^{\sqrt{3}} d\theta$$

$$A(S) = 2\int_0^{2\pi} -[\sqrt{4-3} - (\sqrt{4}-0)]\, d\theta \Rightarrow 2\int_0^{2\pi} -[1-2]\, d\theta \Rightarrow 2[\theta]_0^{2\pi} \Rightarrow 2[2\pi - 0]$$

$$A(S) = 4\pi \text{ u.a.}$$

$$\boxed{\therefore 4\pi \text{ u.a.}}$$

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4.

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, ds \ ; \ \vec{F}(x,y,z) = (z + \operatorname{sen}(x),\ x + y^2,\ y + e^z)$$

$$\vec{\nabla} \times \vec{F} = \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right)\hat{i} + \left(\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}\right)\hat{j} + \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)\hat{k}$$

$$(1-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k}$$

$$\vec{\nabla} \times \vec{F} = (1,1,1)$$

$$\begin{cases} x = u\cos v & 0 \le u \le \sqrt{2} \\ y = u \operatorname{sen} v & 0 \le v \le 2\pi \\ z = 1 \end{cases}$$

$$C: \begin{cases} \text{I} \ \ x^2 + y^2 + z^2 = 3 \\ \text{II} \ \ 2z = x^2 + y^2 \end{cases}$$

$$(2z) + z^2 = 3$$

$$z^2 + 2z + 1 - 1 = 3$$

$$(z+1)^2 = 4$$

$$z + 1 = 2$$

$$\boxed{z = 1}$$

I:

$$x^2 + y^2 + (1)^2 = 3$$

$$x^2 + y^2 + 1 = 3$$

$$\boxed{x^2 + y^2 = 2}$$

$$\vec{r}(u,v) = (u\cos v,\ u \operatorname{sen} v,\ 1)$$

$$\frac{\partial \vec{r}}{\partial u} = (\cos v,\ \operatorname{sen} v,\ 0) \ ; \ \frac{\partial \vec{r}}{\partial v} = (-u\operatorname{sen} v,\ u\cos v,\ 0)$$

$$\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos v & \operatorname{sen} v & 0 \\ -u\operatorname{sen} v & u\cos v & 0 \end{vmatrix}$$

$$= u\cos^2 v\,\hat{k} + u\operatorname{sen}^2 v\,\hat{k}$$

$$= (0, 0, u)$$

$$(\vec{\nabla} \times \vec{F}) \cdot (0,0,u) = u$$

Assim:

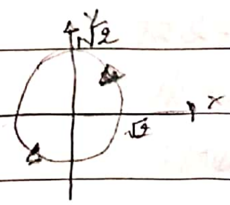
$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, ds = \int_0^{2\pi}\int_0^{\sqrt{2}} u\,du\,dv \Rightarrow \int_0^{2\pi} \left[\frac{u^2}{2}\right]_0^{\sqrt{2}} dv \Rightarrow \int_0^{2\pi} \left[\frac{(\sqrt{2})^2}{2} - 0\right] dv$$

$$\Rightarrow \int_0^{2\pi} 1\,dv \Rightarrow [v]_0^{2\pi} \Rightarrow [2\pi - 0] = 2\pi$$

$$\boxed{\therefore\ 2\pi}$$

Por outro lado, em:

Plano xy



$$\oint_C \vec{F} \cdot d\vec{r} = \oint_C \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

$$x^2 + y^2 = 2$$

$$\begin{cases} x = \sqrt{2}\cos t & ; \ t \in [0, 2\pi] \\ y = \sqrt{2}\operatorname{sen} t & \vec{r}(t) = (\sqrt{2}\cos t,\ \sqrt{2}\operatorname{sen} t,\ 1) \\ z = 1 & \vec{r}\,'(t) = (-\sqrt{2}\operatorname{sen} t,\ \sqrt{2}\cos t,\ 0) \end{cases}$$

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$\vec{F}(x,y,z) = (z + \text{sen}(x), x + y^2, y + e^z)$

$\vec{r}(t) = (\sqrt{2}\cos t, \sqrt{2}\,\text{sen}\,t, 1)$

$\vec{F}(\vec{r}(t)) = (1 + \text{sen}(\sqrt{2}\cos t), \sqrt{2}\cos t + 2\,\text{sen}^2 t, \sqrt{2}\,\text{sen}\,t + e^1)$

$r'(t) = (-\sqrt{2}\,\text{sen}\,t, \sqrt{2}\cos t, 0)$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)$:

$(1 + \text{sen}(\sqrt{2}\cos t), \sqrt{2}\cos t + 2\,\text{sen}^2 t, \sqrt{2}\,\text{sen}\,t + e) \cdot (-\sqrt{2}\,\text{sen}\,t, \sqrt{2}\cos t, 0)$

$= -\sqrt{2}\,\text{sen}\,t[1 + \text{sen}(\sqrt{2}\cos t)] + \sqrt{2}\cos t[\sqrt{2}\cos t + 2\,\text{sen}^2 t] + 0$

$$\oint_0^{2\pi} -\sqrt{2}\,\text{sen}\,t[1 + \text{sen}(\sqrt{2}\cos t)] + \sqrt{2}\cos t[\sqrt{2}\cos t + 2\,\text{sen}^2 t]\,dt$$

$$= \underbrace{\int_0^{2\pi} -\sqrt{2}\,\text{sen}\,t[1 + \text{sen}(\sqrt{2}\cos t)]\,dt}_{\text{I}} + \underbrace{\int_0^{2\pi} 2\sqrt{2}\cos t\,\text{sen}^2 t\,dt}_{\text{II}} + \underbrace{\int_0^{2\pi} 2\cos^2 t\,dt}_{\text{III}}$$

(I) $\int_0^{2\pi} -\sqrt{2}\,\text{sen}\,t[1 + \text{sen}(\sqrt{2}\cos t)]\,dt \Rightarrow \int [1 + \text{sen}(u)]\,du \Rightarrow [u + (-\cos(u))]$

$\Rightarrow [\sqrt{2}\cos t - \cos(\sqrt{2}\cos t)]\Big|_0^{2\pi} \Rightarrow [\sqrt{2}\cos(2\pi) - \cos(\sqrt{2}\cos(2\pi)) - (\sqrt{2}\cos(0) \ldots$

$u = \sqrt{2}\cos t$ $\quad \ldots - (\cos(\sqrt{2}\cos(0))] \Rightarrow [\sqrt{2} - \cos(\sqrt{2}) - (\sqrt{2} - \cos(\sqrt{2}))]$

$du = -\sqrt{2}\,\text{sen}\,t\,dt \quad \Rightarrow \quad \sqrt{2} - \cos(\sqrt{2}) - \sqrt{2} + \cos(\sqrt{2}) = 0$ //

(II) $\int_0^{2\pi} 2\sqrt{2}\cos t\,\text{sen}^2 t\,dt \Rightarrow 2\sqrt{2}\int^{2\pi} \cos t\,\text{sen}^2 t\,dt \Rightarrow 2\sqrt{2}\int u^2\,du \Rightarrow -2\sqrt{2}\,[u^3/3]$

$\Rightarrow 2\sqrt{2}/3\,[\text{sen}^3 t]\Big|_0^{2\pi} \Rightarrow 2\sqrt{2}/3\,[\text{sen}^3(2\pi) - \text{sen}^3(0)] \Rightarrow 2\sqrt{2}/3\,[0 - 0]$

$u = \text{sen}\,t \qquad = 0$ //

$du = -\cos t\,dt$

$-du = \cos t\,dt$

(III) $\int_0^{2\pi} 2\cos^2 t\,dt \Rightarrow 2\int_0^{2\pi} \frac{1}{2}(1 + \cos 2t)\,dt \Rightarrow \int^{2\pi} 1 + \cos 2t\,dt \Rightarrow 1/2\int 1 + \cos(u)\,du$

$\cos^2 t = \frac{1}{2}(1 + \cos 2t) \quad \Rightarrow 1/2\,[u + \text{sen}(u)] \Rightarrow 1/2\,[2t + \text{sen}(2t)]\Big|_0^{2\pi}$

$u = 2t \quad \Rightarrow 1/2\,[2(2\pi) + \text{sen}(4\pi) - 0] \Rightarrow 1/2\,[4\pi + 0] = 2\pi$

$du = 2\,dt$ $\quad \boxed{\therefore\ 2\pi}$ //

$du/2 = dt$

Assim:

$$\oint_C \vec{F}\,d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \vec{n}\,dS$$ //

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