A 06

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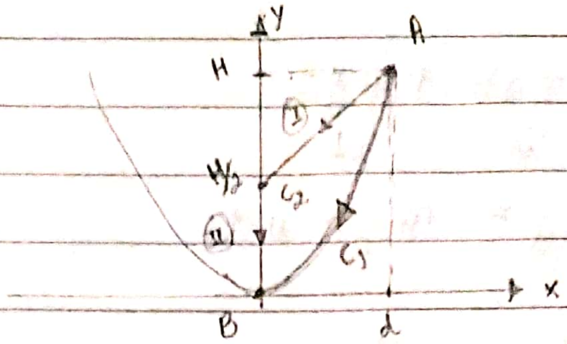
$\vec{F} = -mg\hat{j}$

$A = (d, H)$

$B = (0, 0)$



a) C_2: I $(d, H) \to (0, H/2)$

$y = mx + n$

$m = \frac{\Delta y}{\Delta x} = \frac{(H/2 - H)}{(0 - d)} = \frac{-H/2}{-d} \to m = \frac{H}{2d}$

$n = H/2$

Assim a eq. reta será:

$y = \frac{H}{2d}x + \frac{H}{2}$

$\begin{cases} x = t & t \in [0, d] \\ y = \frac{H}{2d}t + \frac{H}{2} \end{cases}$

. Invertendo fica: $t \to 0 + d - t$

$\begin{cases} x = d - t & ; t \in [0, d] \\ y = H - \frac{Ht}{2d} \end{cases}$

Assim:

$\vec{r}(t) = (d - t, H - Ht/2d)$

$\vec{r}\,'(t) = (-1, -H/2d)$

Sabe-se que:

$W = \int_{T_0}^{T_1} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

$W = \int_0^d -mg\hat{j} \cdot (-1, -\frac{H}{2d})\, dt$

$W = \int_0^d \frac{mgH}{2d}\, dt \to \frac{mgH}{2d}[t]\Big|_0^d$

$W = \frac{mgH}{2d}[d - 0] \to \frac{mgHd}{2d}$

$W_{I} = \frac{mgH}{2}$

Por outro lado, em C_2: II $(0, H/2) \to (0, 0)$

$\begin{cases} x = 0 & ; t \in [0, H/2] \\ y = t \end{cases}$

. Invertendo o sentido fica, $t \to 0 + \frac{H}{2} - t$

$\begin{cases} x = 0 & ; t \in [0, H/2] \\ y = \frac{H}{2} - t \end{cases}$

$\vec{r}(t) = (0, \frac{H}{2} - t)$

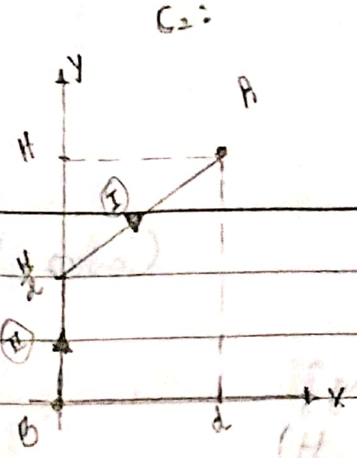
$\vec{r}\,'(t) = (0, -1)$

$(-mg\hat{j}) \cdot (0, -1) = mg$

$W = \int_0^{H/2} mg\, dt \to mg[t]\Big|_0^{H/2} \to mg[H/2 - 0]$

$W_{II} = \frac{mgH}{2}$

tilibra

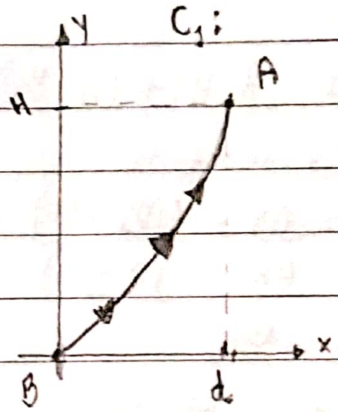


Assim em C_2:

$$W_{Total} = W_{I} + W_{II}$$

$$W_{Total} = \frac{mgH}{2} + \frac{mgH}{2}$$

$$W_{Total} = mgH \text{ u.t.}$$



Em C_1: $(d, H) \to (0,0)$

$$\begin{cases} x = t & ; t \in [0, d] \\ y = t^2 & 0 \le x \le d \end{cases}$$

. Invertendo $t \to 0 + d - t$

$$\begin{cases} x = d - t & ; t \in [0, d] \\ y = d^2 - 2dt + t^2 \end{cases}$$

$$\vec{r}(t) = (d - t, d^2 - 2dt + t^2)$$

$$\vec{r}\,'(t) = (-1, -2d + 2t)$$

Assim:

$$W = \int_0^d -mg\vec{j} \cdot (-1, -2d + 2t)\, dt$$

$$W = \int_0^d 2mgd - 2mgt \, dt$$

$$W = 2mgd[t]\Big|_0^d - 2mg[t^2/2]\Big|_0^d$$

$$= 2mgd[d - 0] - mg[d^2 - 0^2]$$

$$= 2mgd^2 - mgd^2$$

$$W = mgd^2 \text{ u.t.}$$

Logo o trabalho (W) em C_1 e C_2 será dado por

C_1:

$$W_{C_1} = mgd^2 \text{ u.t.}$$

C_2:

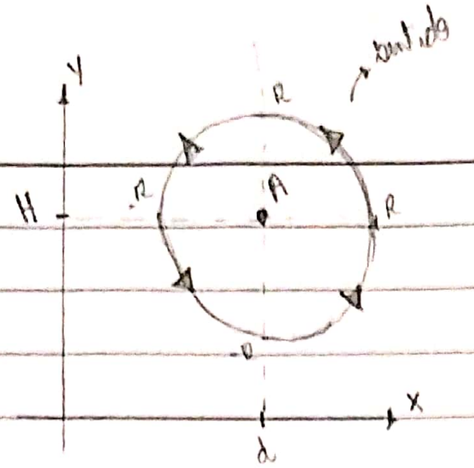
$$W_{C_2} = \frac{mgH}{2} + \frac{mgH}{2}$$

$$W_{C_2} = mgH \text{ u.t.}$$

OBS: u.t. (unidades de trabalho)

Exemplo:

b)



centro: $A(\overset{x_0}{d}, \overset{y_0}{H})$

$$(x-x_0)^2+(y-y_0)^2=R^2$$

$$\begin{cases} x = x_0 + R\cos t \\ y = y_0 + R\,\text{sen}\,t \end{cases} ;\ t \in [0, 2\pi]$$

Substituindo:

$$\begin{cases} x = d + R\cos t \\ y = H + R\,\text{sen}\,t \end{cases} ;\ t \in [0, 2\pi]$$

$$\vec{r}(t) = (d + R\cos t,\ H + R\,\text{sen}\,t)$$

$$\vec{r}\,'(t) = (-R\,\text{sen}\,t,\ R\cos t)$$

Assim:

$$\oint_C \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$$\vec{F} = -mg\hat{j}$$

$$(-mg\hat{j}) \cdot (-R\,\text{sen}\,t,\ R\cos t) = -mgR\cos t$$

Fazendo a substituição:

$$\oint_0^{2\pi} -mgR\cos t\,dt \rightarrow -mgR\,[\text{sen}\,t]\Big|_0^{2\pi}$$

$$\Rightarrow -mgR\,[\overset{0}{\cancel{\text{sen}(2\pi)}} - \overset{0}{\cancel{\text{sen}(0)}}] = 0$$

Logo, em um caminho fechado:

$$\oint_C \vec{F} \cdot d\vec{r} = 0$$