A09

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a) $\sum_{K=1}^{\infty} \frac{4^{K+2}}{7^{K-1}}$

$\frac{4^3}{7^0} + \frac{4^4}{7^1} + \frac{4^5}{7^2} + \frac{4^6}{7^3} \rightarrow$ série geométrica

$a = 4^3$

$r = \frac{\frac{4^4}{7}}{4^3} = \frac{4^4}{7} \cdot \frac{1}{4^3} = \frac{4}{7}$

$|r| < 1 \rightarrow$ Converge

$\sum_{K=1}^{\infty} \frac{4^{K+2}}{7^{K-1}} = \frac{a}{1-r}$

$" = \frac{4^3}{1 - 4/7} = \frac{4^3}{3/7} = \frac{7}{3} 4^3$

$\therefore S_n = \frac{7}{3} 4^3$

b) $\sum_{K=1}^{\infty} \frac{1}{9K^2 + 3K - 2} = \sum_{K=1}^{\infty} \frac{1}{(K-1/3)(K+2/3)}$

$\lim_{n \to +\infty} \frac{1}{9n^2 + 3n - 2} \Rightarrow \frac{1/n^2}{9 + 3/n - 2/n^2} \Rightarrow \frac{0}{9} = 0$, pode convergir ou divergir

Assim:

$9K^2 + 3K - 2$

$K = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$K = \frac{-3 \pm \sqrt{9 - 4(9)(-2)}}{2(9)}$

$K = \frac{-3 \pm \sqrt{81}}{18}$ $\quad K_1 = \frac{-3+9}{18} \rightarrow K_1 = \boxed{1/3}$

$K = \frac{-3 \pm 9}{18}$ $\quad K_2 = \frac{-3-9}{18} \rightarrow K_2 = \boxed{-2/3}$

$$a_n = \frac{1}{(n-1/3)(n+2/3)} = \frac{A}{(n-1/3)} + \frac{B}{(n+2/3)} = \frac{A(n+2/3)+B(n-1/3)}{(n-1/3)(n+2/3)}$$

$$1 = A(n+2/3) + B(n-1/3)$$

$n = 1/3$

$$1 = A(1/3+2/3) + B(\underbrace{1/3-1/3}_{0})$$

$$\boxed{A = 1}$$

$n = -2/3$

$$1 = A(\underbrace{-2/3+2/3}_{0}) + B(-\tfrac{2}{3}-1/3)$$

$$\boxed{B = -1}$$

$$a_n = \frac{1}{(n-1/3)} - \frac{1}{(n+2/3)}$$

$$\sum_{K=1}^{\infty} \frac{1}{(K-1/3)(K+2/3)} = \sum_{K=1}^{\infty}\left(\frac{1}{(K-1/3)} - \frac{1}{(K+2/3)}\right)$$

$$S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = \left(\frac{1}{(1-1/3)} - \cancel{\frac{1}{(1+2/3)}}\right) + \left(\cancel{\frac{1}{(2-1/3)}} - \cancel{\frac{1}{(2+2/3)}}\right) + \dots + \left(\cancel{\frac{1}{(n-1/3)}} - \frac{1}{(n+2/3)}\right)$$

$$S_n = \left(\frac{1}{(1-1/3)} - \frac{1}{(n+2/3)}\right)$$

$$\lim_{n\to+\infty}\left(\frac{1}{1-1/3}\right) - \lim_{n\to+\infty}\left(\frac{1}{n+2/3}\right) \Rightarrow \frac{1}{1-1/3} - 0$$

$$\lim_{n\to+\infty} S_n = \frac{1}{2/3} = 3/2 \to \text{Converge}$$

Assim

$$\therefore \sum_{K=1}^{\infty} \frac{1}{9K^2+3K-2} = \boxed{3/2}$$

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