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Trabalho 8 - C IV

a)

$$2x^2(3+x)y'' + x(1+5x)y' + (1+x)y = 0$$
$$x^2(6+2x)y'' + x(1+5x)y' + (1+x)y = 0$$

$A(x) = (6+2x)$

$B(x) = (1+5x)$

$C(x) = (1+x)$

Pelo Teorema:

$$Ly = x^2(\alpha_0 + \alpha_1 x + \alpha_2 x^2)y'' + x(\beta_0 + \beta_1 x + \beta_2 x^2)y' + (\gamma_0 + \gamma_1 x + \gamma_2 x^2)y$$

$\alpha_0 = 6, \quad \alpha_1 = 2, \quad \alpha_2 = 0$

$\beta_0 = 1, \quad \beta_1 = 5, \quad \beta_2 = 0$

$\gamma_0 = 1, \quad \gamma_1 = 1, \quad \gamma_2 = 0$

Assim:

(I) $P_0(n) = 6n(n-1) + 1n + 1$

(II) $P_1(n) = 2n(n-1) + 5n + 1$

$P_2(n) = 0$

Em (I):

$P_0(n) = 6n^2 - 6n + 1n + 1 \Rightarrow 6n^2 - 5n + 1$

$$n = \frac{5 \pm \sqrt{(-5)^2 - 4(6)(1)}}{2(6)}$$

$$n = \frac{5 \pm \sqrt{25 - 24}}{12}$$

$$n_1 = \frac{5+1}{12} = \frac{6}{12} = \frac{1}{2}$$

$$n_2 = \frac{5-1}{12} = \frac{4}{12} = 1/3$$

Para $n_1 > n_2$

$\frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = 1/6 \neq$ inteiro $\Rightarrow$ Caso 1

$P_0 = (n - 1/2)(n - 1/3)$

Em ⓐ: $P_1(r) = 2r^2 - 2r + 5r + 1$
$= 2r^2 + 3r + 1$

$r = \dfrac{-3 \pm \sqrt{(3)^2 - 4(2)(1)}}{2(2)}$ $r_1 = \dfrac{-3+1}{4} = \dfrac{-2}{4} = -\dfrac{1}{2}$

$r = \dfrac{-3 \pm \sqrt{9-8}}{4}$

$r_2 = \dfrac{-3-1}{4} = \dfrac{-4}{4} = -1$

Para, $P_1(r)$:

$P_1(r) = (r+1)(r + 1/2)$

Logo, para o Caso 1:

$Y_1 = x^{r_1} \sum_{n=0}^{\infty} a_n(r_1) x^n \qquad Y_2 = x^{r_2} \sum_{n=0}^{\infty} a_n(r_2) x^n$

$a_0(r) = 1; \qquad a_1(r) = \dfrac{-P_1(r)}{P_0(r+1)}$

$a_n(r) = \dfrac{-(P_1(n+r-1)a_{n-1}(r) + P_0(n+r-2)a_{n-2}(r))}{P_0(n+r)} \quad n \geq 2$

Em $P_0(r)$ e para $r_1 = 1/2$

$P_0(r) = (r - 1/2)(r - 1/3)$

$\dfrac{1}{2} + 1 = \dfrac{1+2}{2} = \dfrac{3}{2} = r+1$

$P_0(3/2) = (3/2 - 1/2)(3/2 - 1/3)$
$= (1)(7/6)$

$P_0(r+1) = 7/6$

$P_1(r) = (r+1)(r+1/2)$

$P_1(1/2) = (1/2+1)(1/2+1/2)$
$(3/2)(1)$

$\boxed{P_1(r) = 3/2}$

$a_1(r) = -\dfrac{3/2}{7/6}$

$a_1(r) = -\dfrac{3}{2} \cdot \dfrac{6}{7}$

$\boxed{a_1(r) = -\dfrac{9}{7}}$

$a_n(r) = -\dfrac{(P_1(n-1/2)a_{n-1})}{P_0(n+1/2)} \Rightarrow -\dfrac{(\cancel{n}(n+1/2))a_{n-1}}{(\cancel{n}(n+1/6))} \Rightarrow -\dfrac{(n+1/2)a_{n-1}}{(n+1/6)}$

$a_n(r) = -\dfrac{(n+1/2)\,a_{n-1}}{(n+1/6)}$

para $n = 1, 2, 3, \ldots$; Temos:

$n = 1$

$$a_1(n) = -\frac{(1 + 1/2)\, a_0}{(1 + 1/6)} \Rightarrow -\frac{(3/2)\, a_0}{7/6} \Rightarrow -9/7\, a_0 \Rightarrow a_1(n) = -9/7$$

Logo, é válido para $n \geq 1$. Assim,

$n = 2$

$$a_2(n) = -\frac{(2 + 1/2)\, a_1}{(13/6)} \Rightarrow -\frac{(15)}{13} a_1 \Rightarrow -\frac{15}{13}\left[\frac{-9}{7}\right] = +\frac{135}{91}$$

$$y_1 = x^{1/2}\left(1 - 9/7\, x + 135/91\, x^2 + \ldots\right)$$

Em $P_0(n)$ e para $n_2 = 1/3$

$$P_0(n) = (n - 1/2)(n - 1/3)$$

$$n + 1 = \frac{1}{3} + 1 = \frac{1+3}{3} = \frac{4}{3}$$

$$P_0(4/3) = (4/3 - 1/2)(4/3 - 1/3)$$

$$(5/6)(1)$$

$$P_0(n+1) = 5/6$$

$$P_1(n) = (n + 1)(n + 1/2)$$

$$P_1(1/3) = (1/3 + 1)(1/3 + 1/2)$$

$$= (4/3)(5/6)$$

$$(20/18)$$

$$\boxed{P_1(n) = (10/9)}$$

$$a_1(n) = -\frac{10/9}{5/6}$$

$$a_1(n) = -\frac{10}{9} \cdot \frac{6}{5}$$

$$a_1(n) = -\frac{4}{3}$$

$$a_n(n) = -\frac{(P_1(n - 2/3)\, a_{n-1})}{P_0(n + 1/3)} \Rightarrow -\frac{((n + 1/3)\cancel{(n - 1/6)})}{n\cancel{(n - 1/6)}} \Rightarrow -\frac{(n + 1/3)\, a_{n-1}}{n}$$

$$a_n(n) = -\frac{(n + 1/3)\, a_{n-1}}{n}$$

para $n = 1, 2, 3, \ldots$; Temos:

$n = 1$

$$a_1(n) = -\frac{(1 + 1/3)\, a_0}{1} = -4/3$$

Logo, é válido para $n \geq 1$. Assim:

p/ $n = 2$

$$a_2(n) = -\frac{(2 + 1/3)\, a_1}{2} \Rightarrow -\frac{(7/3)\, a_1}{2} \Rightarrow -\frac{(7/3)(-4/3)}{2} \Rightarrow (7/3)(2/3)$$

$a_2(n) = 14/9$. Logo, $y_2 = x^{1/3}\left(1 - \frac{4}{3}x + \frac{14}{9}x^2 + \ldots\right)$

Logo, a solução geral pode ser escrita como:

$$y = C_1 \cdot y_1 + C_2 y_2$$

$$\therefore y = C_1\left[x^{1/2} - 9/7\, x^{3/2} + 135/91\, x^{5/2} + \dots\right] + C_2\left[x^{1/3} - \frac{4}{3} x^{4/3} + \frac{14}{9} x^{7/3} + \dots\right]$$

GeoGebra Calculadora Gráfica

ENTRAR...

eq1: $y\,x = x^{(1/2)} - 9/7\,x^{(3/2)} + 135/91\,x^{(5/2)} + x^{(1/3)} - 4/3\,x^{(4/3)} + 14/9\,x^{(7/3)}$

Entrada...