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Lista 11 - C IV

a)

$$f(t) = e^t \quad , -1 < t \leq 1$$

Pela definição:

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt = \int_{-1}^{1} e^t dt = \left[e^t\right]\Big|_{-1}^{1} = \left[e - e^{-1}\right]$$ //

Assim, em a_n:

$$a_n = \int_{-1}^{1} e^t \cos n\pi t\,dt = \left[\frac{e^t}{n\pi}\,\text{sen}(n\pi t)\right]\Big|_{-1}^{1} - \frac{1}{n\pi}\int_{-1}^{1} e^t\,\text{sen}(n\pi t)\,dt$$

$u = e^t$ | $dv = \cos n\pi t\,dt$ | $u = e^t$ $dv = \text{sen}(n\pi t)dt$

$du = e^t dt$; $v = \frac{\text{sen}\, n\pi t}{n\pi}$; $du = e^t dt$ $v = -\frac{\cos(n\pi t)}{n\pi}$

$$a_n = \left[\frac{e^1}{n\pi}\text{sen}(n\pi) - \left(\frac{e^{-1}}{n\pi}\text{sen}(n\pi)\right)\right] \Rightarrow -\frac{1}{n\pi}\left[-e^t\frac{\cos(n\pi t)}{n\pi} + \frac{1}{n\pi}\int_{-1}^{1} e^t\cos(n\pi t)\,dt\right]$$

$\cancel{\quad}^{\,0}$

$$= \left[\frac{e^t}{n^2\pi^2}\cos(n\pi t)\right]\Big|_{-1}^{1} - \frac{1}{n^2\pi^2}a_n \Rightarrow a_n(1 + 1/n^2\pi^2) = \left[\frac{e^t\cos(n\pi t)}{n^2\pi^2}\right]\Big|_{-1}^{1}$$

$$\Rightarrow a_n = \left[\frac{e^t\cos(n\pi t)}{1+n^2\pi^2}\right]\Big|_{-1}^{1} = \frac{1}{1+n^2\pi^2}\left(e\cos(n\pi) - e^{-1}\cos(n\pi)\right) \Rightarrow \frac{\cos(n\pi)(e-e^{-1})}{1+n^2\pi^2}$$

$$a_n = \frac{(-1)^n(e-e^{-1})}{1+n^2\pi^2}$$ //

Por outro lado, em b_n:

$$b_n = \int_{-1}^{1} e^t\,\text{sen}(n\pi t)\,dt = \left[-e^t\frac{\cos(n\pi t)}{n\pi}\right]\Big|_{-1}^{1} + \frac{1}{n\pi}\int_{-1}^{1} e^t\cos(n\pi t)\,dt$$

$u = e^t$ | $dv = \text{sen}(n\pi t)\,dt$

$du = e^t dt$; $v = -\frac{\cos(n\pi t)}{n\pi}$

$$\Rightarrow b_n = \frac{-1}{n\pi}\left(e\cos(n\pi) - e^{-1}\cos(n\pi)\right) + \frac{1}{n\pi}a_n \Rightarrow b_n = \frac{-(-1)^n}{n\pi}(e-e^{-1}) + \ldots$$

$$\ldots + \frac{1}{n\pi}\,\frac{(-1)^n(e-e^{-1})}{1+n^2\pi^2} \Rightarrow b_n = \frac{(e-e^{-1})}{n\pi}(-1)^n\left(\frac{1}{1+n^2\pi^2} - 1\right)$$

$$b_n = \frac{(-1)^n (e - e^{-1})}{\cancel{n\pi}} \frac{n^2\pi^2}{1+n^2\pi^2}$$

$$\therefore b_n = \frac{(-1)^{n+1}(e-e^{-1})\, n\pi}{1+n^2\pi^2}$$

$$f(t) = \begin{cases} \frac{e+e^{-1}}{2}, & t = \pm 1 \\ e^t & -1 < t < 1 \end{cases}$$

II)

$$f(t) = \begin{cases} 0, & -\pi < t \leq 0 \\ \pi, & 0 < t \leq \pi \end{cases}$$

Para o caso em que:

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nt) + b_n \operatorname{sen}(nt)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt \;;\quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\,dt;\quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\operatorname{sen}(nt)\,dt$$

Assim, para a_0:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\cancel{\pi}}\int_0^{\pi} \cancel{\pi}\,dt = \pi$$

Em a_n:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\,dt = \frac{1}{\cancel{\pi}}\int_0^{\pi} \cancel{\pi}\cos(nt)\,dt \Rightarrow \cancel{\left[\frac{\operatorname{sen}(nt)}{n}\right]_0^{\pi}} = 0 \quad n \geq 1$$

Por fim, em b_n:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\operatorname{sen}(nt)\,dt = \frac{1}{\cancel{\pi}}\int_0^{\pi} \cancel{\pi}\operatorname{sen}(nt)\,dt = \left[-\frac{\cos(nt)}{n}\right]_0^{\pi} = -\frac{1}{n}\left((-1)^n - 1\right)$$

$$\therefore b_n = \frac{1}{n}\left(1-(-1)^n\right)$$

$$F(t) = \begin{cases} \pi/2, & t = -\pi, 0, \pi \\ 0, & -\pi < t < 0 \\ \pi, & 0 < t < \pi \end{cases}$$

Logo, a solução geral será:

$$F(t) = \pi/2 + \sum_{n=1}^{\infty} 1/n \left(1-(-1)^n\right)\operatorname{sen}(nt)$$