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Trabalho 5 - C IV

a)

$$xy'' - (2x+8)y' + (x+8)y = -e^x$$

$y_1 = e^x$

$$\Rightarrow y_p = Ae^x \quad \text{solução particular}$$

$$y_p' = Ae^x$$

$$y_p'' = Ae^x$$

Substituindo:

$$x[Ae^x] - (2x+8)[Ae^x] + (x+8)[Ae^x] = -e^x$$

$$Ae^x[\cancel{x} - \cancel{2x} - \cancel{8} + \cancel{x} + \cancel{8}] = -e^x$$

$$Ae^x = -e^x$$

$$A = -1$$

$$\boxed{\therefore y_p = -e^x}$$

Assim, para a solução homogênea:

$$xy'' - (2x+8)y' + (x+8)y = 0 \quad \div x$$

$$y'' - (8/x + 2)y' + (8/x + 1)y = 0$$

Pela fórmula de Abel:

$$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{(y_1(x))^2}$$

$$P(x) = -(8/x + 2) \Rightarrow -\int -(8/x + 2)dx \Rightarrow 8\ln|x| + 2x \Rightarrow e^{8\ln|x| + 2x} = x^8 \cdot e^{2x}$$

$$y_2(x) = e^x \int \frac{x^8 \cdot \cancel{e^{2x}}}{\cancel{(e^x)^2}} dx \Rightarrow e^x \int x^8 dx$$

$$y_h(x) = e^x \left[\frac{x^9}{9} + C\right]$$

Assim:

$$y(x) = y_h + y_p \Rightarrow y(x) = e^x\left[\frac{x^9}{9} + C\right] - e^x$$

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b)

$y_h = \frac{e^x x^9}{9}$; $y_h' = \frac{e^x(x^9+9x^8)}{9}$; $y_h'' = \left[\frac{x^9+18x^8+72x^7}{9}\right]e^x$

$$y'' - (8/x + 2)y' + (8/x + 1)y = 0$$

$$\cancel{e^x}\left[\frac{x^9+18x^8+72x^7}{9}\right] - \left(\frac{8}{x}+2\right)\left[\frac{\cancel{e^x}(x^9+9x^8)}{9}\right] + (8/x+1)\left[\frac{\cancel{e^x}x^9}{9}\right] = 0$$

$$\frac{\cancel{x^9}+\cancel{18x^8}+\cancel{72x^7}}{9} - \frac{\cancel{8x^8}}{9} - \frac{\cancel{72x^7}}{9} - \frac{\cancel{2x^9}}{9} - \frac{\cancel{18x^8}}{9} + \frac{\cancel{8x^8}}{9} + \frac{\cancel{x^9}}{9} = 0 \checkmark$$

$y_p = -e^x$; $y_p' = -e^x$; $y_p'' = -e^x$

$$x(-e^x) - (2x+8)(-e^x) + (x+8)(-e^x) = -e^x$$

$$-e^x(\cancel{x} - 2x - \cancel{8} + \cancel{x} + \cancel{8}) = -e^x$$

$$-e^x = -e^x \checkmark$$

$\therefore y(x) = e^x\left[\frac{x^9}{9} + C\right] - e^x$ é solução.

e)

$y(x_0) = y_0 \Rightarrow y(6) = 5$

$y'(x_0) = y_1 \Rightarrow y'(6) = 6$

$y = \frac{e^x x^9}{9} + e^x C - e^x$; $y' = x^9 e^x + x^8 e^x + Ce^x - e^x$

$$\begin{cases} 5 = \frac{e^6(6)^9}{9} + e^6 C - e^6 \Rightarrow e^6 C = 5 - \frac{e^6(6)^9}{9} + e^6 \Rightarrow C = \frac{5}{e^6} - \frac{(6)^9}{9} + 1 \\ \therefore C = 5/e^6 + 1 - 6^9/9 \end{cases}$$

$$\therefore y(x) = e^x\left[\frac{x^9}{9} + 5/e^6 + 1 - 6^9/9\right] - e^x$$

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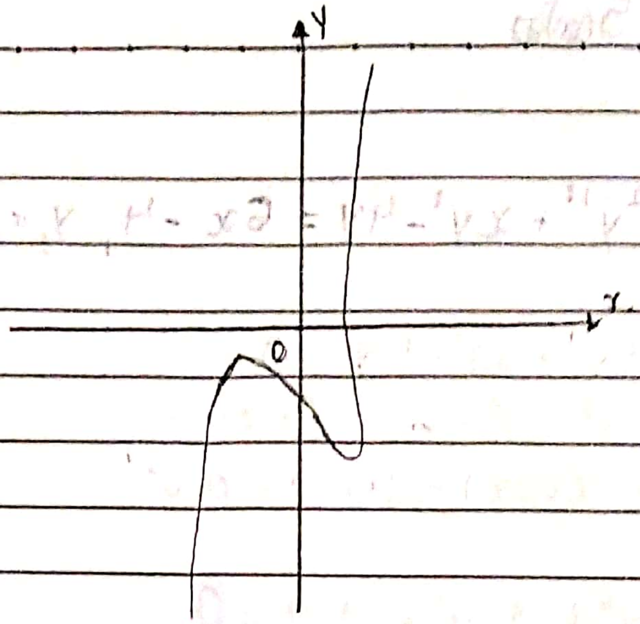
d)



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