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$$(1-x+x^2)y'' - (1-4x)y' + 2y = 0$$
$$y(1) = 6 \quad , \quad y'(1) = 7$$

$P(x) = 1 - x + x^2 \neq 0 \rightarrow$ pontos ordinários

$x^2 - x + 1$

$x = \dfrac{+1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2}$ $\qquad x = \dfrac{1 \pm \sqrt{3}i}{2} \rightarrow$ Raízes complexas, $x \in \mathbb{R}$ $P(x) \neq 0$

Proposto, $x_0 = 1 \rightarrow t = x - 1$

$$y = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

$$x = t + 1$$
$$y = \sum_{n=0}^{\infty} a_n (t)^n$$

$$y' = \sum_{n=1}^{\infty} n a_n t^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n t^{n-2}$$

Substituindo na EDO:

$$(1-(t+1)+(t+1)^2)\left[\sum_{n=2}^{\infty} n(n-1)a_n t^{n-2}\right] - (1-4(t+1))\left[\sum_{n=1}^{\infty} n a_n t^{n-1}\right] + \dots$$
$$\dots + 2\left[\sum_{n=0}^{\infty} a_n t^n\right] = 0$$

$$(t^2+t+1)y'' + (+4t+3)y' + 2y = 0$$
$$\sum_{n=0}^{\infty} n(n-1)a_n t^n + \sum_{n=1}^{\infty} n(n-1)a_n t^{n-1} + \sum_{n=2}^{\infty} n(n-1)a_n t^{n-2} + \sum_{n=0}^{\infty} 4n a_n t^n +$$
$$\sum_{n=1}^{\infty} 3n a_n t^{n-1} + \sum_{n=0}^{\infty} 2a_n t^n = 0$$

$$k = n-1 \quad ; \quad k = n-2$$
$$\sum_{n=0}^{\infty} n(n-1)a_n t^n + \sum_{k=0}^{\infty} (k+1)(k)a_{k+1} t^k + \sum_{k=0}^{\infty} (k+2)(k+1)a_{k+2} t^k + \sum_{n=0}^{\infty} 4n a_n t^n$$
$$+ \sum_{k=0}^{\infty} 3(k+1)a_{k+1} t^k + \sum_{n=0}^{\infty} 2a_n t^n = 0$$

$k = n$

$$\sum_{n=0}^{\infty} n(n-1)a_n t^n + \sum_{n=0}^{\infty} (n+1)(n)a_{n+1} t^n + \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} t^n + \sum_{n=0}^{\infty} 4n a_n t^n$$

$$+ \sum_{n=0}^{\infty} 3(n+1)a_{n+1} t^n + \sum_{n=0}^{\infty} 2a_n t^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+2)(n+1)a_{n+2} + ((n+1)(n) + 3(n+1))a_{n+1} + (n(n-1) + 4n + 2) \cdot \ldots a_n \right] t^n = 0$$

$$(n+2)(n+1)a_{n+2} + (n^2 + 4n + 3)a_{n+1} + (n^2 + 3n + 2)a_n = 0$$

$\hookrightarrow$ Relação de recorrência

$$a_{n+2} = -\frac{(n^2+4n+3)a_{n+1}}{(n+2)(n+1)} - \frac{(n^2+3n+2)a_n}{(n+2)(n+1)} \quad ; n = 0, 1, 2, \ldots$$

Como:

$$y(t=0) = y(x_0 = 1)$$
$$y'(t=0) = y'(x_0 = 1)$$

$$y(t=0) = a_0 = 6$$
$$y'(t=0) = a_1 = 7$$

$n = 0$

$$a_2 = -\frac{(3)a_1}{(2)} - \frac{(2)a_0}{2} \Rightarrow -\frac{3}{2}(7) - 6 = \frac{-21-12}{2} = \frac{-33}{2}$$

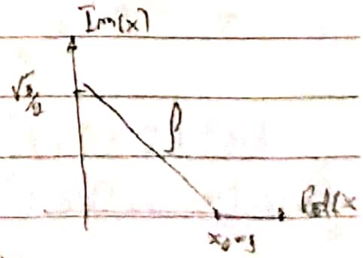
$n = 1$

$$a_3 = -\frac{(8)a_2}{(3)(2)} - \frac{(6)a_1}{(3)(2)} \Rightarrow -\frac{8}{6}\left(\frac{-33}{2}\right) - 7 = 22 - 7 = 15$$

Assim:

$$y = 6 + 7t - \frac{33}{2}t^2 + 15t^3 + \ldots$$

$$\therefore y = 6 + 7(x-1) - \frac{33}{2}(x-1)^2 + 15(x-1)^3 + \ldots$$



$\rho = \sqrt{(\sqrt{3})^2 + 1}$

$\rho = \sqrt{7}/2$

$$(x_0 - \rho, x_0 + \rho)$$
$$(1 - \sqrt{7}/2, 1 + \sqrt{7}/2) \rightarrow \text{convergência}$$

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GeoGebra Calculadora Gráfica

eq1: $y\,x = 6 + 7(x-1) - 33/2\,(x-1)^2 + 15(x-1)^3$

Entrada...