Solução do Trabalho 13 de Cálculo IV

d) $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$ $\quad u(x,0) = e \quad u(-\pi,t) = u(\pi,t)$

$$\left.\frac{\partial u}{\partial x}\right|_{x=-\pi} = \left.\frac{\partial u}{\partial x}\right|_{x=\pi}$$

$u = XT$

$kTX'' = XT'$

$\dfrac{X''}{X} = \dfrac{T'}{kT} = -\lambda^2$

1) $-\lambda^2 < 0$

$T = Ce^{-k\lambda^2 t}$

$X = A\cos\lambda x + B\,\mathrm{sen}\,\lambda x$

$$u(x,t) = e^{-\kappa l^2 t}\left(D\cos lx + E\,\text{sen}\, lx\right)$$

$$u(-\pi,t) = u(\pi,t)$$

$$e^{-\kappa l^2 t}\left(D\cos l\pi + E\,\text{sen}\, l\pi\right) = e^{-\kappa l^2 t}\left(D\cos(-l\pi) + E\,\text{sen}(-l\pi)\right)$$

$$\cancel{D\cos \pi l} + E\,\text{sen}\, l\pi = \cancel{D\cos l\pi} - E\,\text{sen}\, l\pi$$

$$2E\,\text{sen}\, l\pi = 0$$

$E = 0$ ou

$l\pi = n\pi$

$l = n$ inteiro

$$\left.\frac{\partial u}{\partial x}\right|_{x=\pi} = e^{-\kappa l^2 t}\left(-D l \operatorname{sen} l\pi + E l \cos l\pi\right)$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=-\pi} = e^{-\kappa l^2 t}\left(-D l \operatorname{sen}(-\pi l) + E l \cos(-\pi l)\right)$$

$$-D l \operatorname{sen} l\pi \cancel{+ E l \cos l\pi} = +D l \operatorname{sen} l\pi \cancel{+ E l \cos l\pi}$$

$$2 D l \operatorname{sen} l\pi = 0$$

$D = 0$ ou $l\pi = n\pi$

$l = n$ n inteiro

Portanto a) $D=0$ e $\lambda = n$ inteiro, $C \neq 0$

b) $C=0$ e $\lambda = n$ inteiro, $D \neq 0$

a) $u_n(x,t) = e^{-n^2 k t}\,(C_n \sin nx)$

b) $u_n(x,t) = e^{-n^2 k t}\,(D_n \cos nx)$

2) $\lambda = 0$ $\quad X'' = 0 \quad T' = 0$

$$X = Ax + B \qquad T = C$$

$$u = Dx + E$$

$$u(\pi, t) = u(-\pi, t) \Rightarrow D\pi + \cancel{E} = -D\pi + \cancel{E}$$

$$D = 0$$

$$\underline{u = E = cte}$$

3) $\lambda^2 > 0$ temos sol. trivial

Portanto a solução é combinação linear das sol. obtidas e assim

$$u(x,t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n e^{-kn^2 t} \cos nx + B_n e^{-kn^2 t} \sin nx$$

Com a condição de contorno

$$u(x,0) = a$$

$$A_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e \, dx = \frac{e}{\pi}\left(\pi - (-\pi)\right) = 2e$$

$$B_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e \operatorname{sen} nx \, dx = -\frac{e}{\pi} \left.\frac{\cos nx}{n}\right|_{-\pi}^{\pi}$$

0

impar

$$= -\frac{e}{\pi n}\cancel{\left(\cos n\pi - \cos(-n\pi)\right)} = 0$$

$$A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} a \cos nx \, dx = \frac{a}{\pi}\,\frac{\sin nx}{n}\Big|_{-\pi}^{\pi}$$

$$= 0$$

Pertanto $u(x,t) = \underline{a}$