Soluções do trabalho 3 de Cálculo IV

a) $xy' + 2y\ln(ay) = axy$

$$u = \ln(ay)$$

$$u' = \frac{1}{\cancel{a}y}\cancel{a}y'$$

$$x(u'\cancel{y}) + 2\cancel{y}u = ax\cancel{y}$$

$$u' + \frac{2u}{x} = a$$

$$u_1' + \frac{2u_1}{x} = 0$$

$$\int \frac{du_1}{u_1} = -\int \frac{2}{x}\,dx$$

$$\ln|u_1| = -2\ln|x| + c$$

$$u_1 = \frac{1}{x^2}$$

$$u = \frac{v}{x^2}$$

$$u' = \frac{v'}{x^2} - \frac{2v}{x^3}$$

$$\frac{v'}{x^2} - \cancel{\frac{2v}{x^3}} + \cancel{\frac{2v}{x^3}} = a$$

$$v' = ax^2$$

$$\int dv = \int ax^2 dx$$

$$v = \frac{a}{3}x^3 + C = x^2 u = x^2 \ln(ay)$$

$$\ln(ay) = \frac{ax}{3} + \frac{C}{x^2}$$

$$y = \frac{e^{\frac{ax}{3} + \frac{C}{x^2}}}{a}$$

b) Verificação

$$y = \frac{e^{\frac{ex}{3} + \frac{c}{x^2}}}{a}$$

$$y' = \frac{\left(\frac{e}{3} - \frac{2c}{x^3}\right) e^{\frac{ex}{3} + \frac{c}{x^2}}}{a}$$

na EDO:

$$\frac{x}{e}\left(\frac{e}{3} - \frac{2c}{x^3}\right) e^{\frac{ex}{3} + \frac{c}{x^2}} + \frac{2}{e}\left(e^{\frac{ex}{3} + \frac{c}{x^2}} \right)\left(\frac{ex}{3} + \frac{c}{x^2}\right) = exy$$

$$\left(\frac{x}{3} - \cancel{\frac{2c}{ax^2}} + \frac{2x}{3} + \cancel{\frac{2c}{ax^2}}\right) e^{\frac{ax}{3} + \frac{c}{x^2}} = axy$$

$$\frac{\cancel{3}x}{\cancel{3}} e^{\frac{ax}{3} + \frac{c}{x^2}} = axy \qquad \text{ok}$$

c) $y(x_0) = y_0$ para $x_0 = 1$ $y_0 = 1$ $a = 1$

$$1 = e^{1/3 + c} \longrightarrow c = -1/3$$

$$y = e^{\frac{x}{3}} - \frac{1}{3x^2}$$

d)