Solução da Lista 12 de Cálculo IV

1) $y'' + 3y = t \qquad 0 \leq t \leq 1 \qquad y'(0) = y'(1) = 0$

Condições de Neumann

Propomos $y(t) = \dfrac{A_0}{2} + \displaystyle\sum_{n=1}^{\infty} A_n \cos\frac{n\pi t}{L}$

$y' = \displaystyle\sum_{n=1}^{\infty} -\frac{n\pi}{L} A_n \operatorname{sen}\frac{n\pi t}{L} \qquad y'' = \displaystyle\sum_{n=1}^{\infty} -\frac{n^2\pi^2}{L^2} \cos\frac{n\pi t}{L} A_n$

$$f(t) = t \qquad L = 1$$

$$A_0 = \frac{2}{L}\int_0^L f(t)\,dt \qquad A_n = \frac{2}{L}\int_0^L f(t)\cos\frac{n\pi t}{L}\,dt$$

$$A_0 = 2\int_0^1 t\,dt = 2\,\frac{t^2}{2}\Big|_0^1 = 1$$

$$A_n = 2\int_0^1 t\cos n\pi t\,dt = 2\left[\cancelto{0}{\frac{t}{n\pi}\sin n\pi t\Big|_0^1} - \frac{1}{n\pi}\int_0^1 \sin n\pi t\,dt\right]$$

$$= 2\left[+\frac{1}{n^2\pi^2}\cos n\pi t\Big|_0^1\right] = \frac{2}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$u = t \qquad dv = \cos n\pi t\,dt$$

$$du = dt \qquad v = \frac{\sin n\pi t}{n\pi}$$

$$\sum_{n=1}^{\infty} -A_n n^2\pi^2 \cos n\pi t + 3\left(\frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos n\pi t \right)$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\left((-1)^n - 1\right) \cos n\pi t$$

$$\sum_{n=1}^{\infty}\left(\cos n\pi t \left[A_n \{ 3 - n^2\pi^2 \} - \frac{2}{n^2\pi^2}\left((-1)^n - 1\right)\right]\right) + \left(\frac{3A_0}{2} - \frac{1}{2}\right) = 0$$

$$\rightarrow \quad A_0 = 1/3$$

$$A_n = \frac{2\left((-1)^n - 1\right)}{n^2\pi^2\left(3 - n^2\pi^2\right)}$$

$$y(t) = \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2\left((-1)^n - 1\right)}{n^2\pi^2\left(3 - n^2\pi^2\right)} \cos n\pi t$$

$$y'(t) = \sum_{n=1}^{\infty} -\frac{2}{n\pi\left(3 - n^2\pi^2\right)} \left((-1)^n - 1\right) \operatorname{sen} n\pi t$$

$$y'(0) = 0 \qquad y'(1) = 0 \quad !$$

d) $f(t) = \cos 2\pi a t$

$$\mathcal{F}f = \int_{-\infty}^{\infty} e^{-2\pi i s t} \cos 2\pi a t \, dt = \int_{-\infty}^{\infty} e^{-2\pi i s t} \frac{e^{2\pi i a t} + e^{-2\pi i a t}}{2} \, dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{-2\pi i t (s-a)} + e^{-2\pi i t (s+a)} \, dt$$

$$= \frac{1}{2} \left(\delta(s-a) + \delta(s+a) \right)$$