Solução da Lista 2 de Cálculo IV

1) $xy\dfrac{dy}{dx} + 4x^2 + y^2 = 0$ $\qquad y(2) = 7 \qquad x > 0$

$$y = ux \qquad y' = u'x + u$$

$$x \cdot ux\,(u'x + u) + 4x^2 + u^2x^2 = 0$$

$$uu'x^3 + 2u^2x^2 + 4x^2 = 0 \qquad \text{se } x \neq 0$$

$$uu'x + 2u^2 + 4 = 0$$

$$u\, u'\, x = -(2u^2+4)$$

$$-\frac{u}{2u^2+4}\frac{du}{dx} = \frac{1}{x}$$

$$-\int \frac{u\,du}{2u^2+4} = \int \frac{dx}{x} = \ln|x| + c'$$

$$v = 2u^2+4 \qquad dv = 4u\,du$$

$$-\frac{1}{4}\int \frac{dv}{v} = \ln|x| + c'$$

$$-\frac{1}{4}\ln|v| = \ln|x| + c'$$

$$-\frac{1}{4}\ln\left|\frac{2y^2}{x^2}+4\right| = \ln|x| + c'$$

Como $x > 0$

$$\frac{2y^2}{x^2}+4 > 0$$

$$\ln\left(\frac{2y^2}{x^2}+4\right)^{-1/4} = \ln x + c'$$

$$\left(\frac{2y^2}{x^2}+4\right)^{-1/4} = c\,x$$

$$\frac{2y^2}{x^2}+4 = d\,x^{-4}$$

$$2y^2 + 4x^2 = dx^{-2}$$

$$\boxed{2x^2y^2 + 4x^4 = d}$$

$$y(2) = 7$$

$$2 . 4 . 49 + 4 . 16 = d$$

$$d = 456$$

Verificação

$2x^2y^2 + 4x^4 = d$, derivando:

$$4xy^2 + 4x^2yy' + 16x^3 = 0$$

$$xyy' + y^2 + 4x^2 = 0$$

Ok

2) $\cos(x)\, y' + \operatorname{sen}(x)\, y = 2(\cos(x))^3 \operatorname{sen}(x) - 1$

$y(\pi/4) = 3\sqrt{2}$

$$y' + \operatorname{tg}(x)\, y = 2\cos^2(x)\operatorname{sen}(x) - 1$$

$0 \leq x < \pi/2$

$\Downarrow$

$\cos(x) \neq 0$

A homogênea associada:

$$y_1' + \operatorname{tg}(x)\, y_1 = 0$$

$$\int \frac{dy_1}{y_1} = -\int \frac{\operatorname{sen} x}{\cos x}\, dx \qquad u = \cos x \quad du = -\operatorname{sen} x\, dx$$

$$\ln|y_1| = \int \frac{du}{u} = \ln|u| + C = \ln|\cos(x)| + C$$

$$\boxed{y_1 = A\cos(x)}$$

$$y = \cos(x)\, v$$

$$y' = -\text{sen}(x)\, v + \cos(x)\, v' \quad \text{na EDO}$$

$$-\text{sen}(x)\cancel{\cos(x)}\,v + \cos^2(x)\, v' + \text{sen}(x)\cancel{\cos(x)}\,v = 2\cos^3(x)\,\text{sen}(x) - 1$$

$$v' = \frac{2\cos^3(x)\,\text{sen}(x) - 1}{\cos^2(x)} = 2\,\text{sen}\,x\cos x - \frac{1}{\cos^2 x}$$

$$v = \int \text{sen}\,2x\, dx - \int \sec^2 x\, dx = -\frac{1}{2}\cos 2x - \text{tg}\, x + c$$

$$\frac{y}{\cos x} = -\frac{1}{2}\cos(2x) - tg(x) + c$$

$$\boxed{y = -\frac{1}{2}\cos x \cos 2x - \operatorname{sen} x + c\cos x}$$

$y\left(\frac{\pi}{4}\right) = 3\sqrt{2}$ no BDD

$$3\sqrt{2} = -\frac{1}{2}\cos\frac{\pi}{4}\,\cos\frac{\pi}{2} - \operatorname{sen}\frac{\pi}{4} + c\cos\frac{\pi}{4}$$

$\cos\frac{\pi}{2} = 0$

$$3\sqrt{2} = -\frac{\sqrt{2}}{2} + c\frac{\sqrt{2}}{2} \rightarrow \frac{7}{2}\sqrt{2} = \frac{c\sqrt{2}}{2} \rightarrow \boxed{c = 7}$$

Verificação:

$$y' = -\frac{1}{2}\left(-\text{sen}\,x\cos 2x - 2\cos x\,\text{sen}\,2x\right) - \cos x - C\,\text{sen}\,x$$

no EDO

$$\cancel{\frac{1}{2}\text{sen}\,x\cos x\cos 2x} + \cos^2 x\,\text{sen}\,2x - \cancel{\cos^2 x} - \cancel{C\,\text{sen}\,x\cos x}$$

$$\cancel{-\frac{1}{2}\text{sen}\,x\cos x\cos 2x} - \cancel{\text{sen}^2 x} + \cancel{C\cos x\,\text{sen}\,x} = 2\cos^2 x\,\text{sen}\,x - \cancel{1}$$

$$\cos x\,\underbrace{\text{sen}\,2x} = 2\cos^2 x\,\text{sen}\,x$$

$$2\,\text{sen}\,x\cos^2 x = 2\,\text{sen}\,x\cos^2 x$$

OK

3) $\dfrac{dy}{dx} - \dfrac{3y}{x+1} = (x+1)^4$

$y_1' - \dfrac{3}{x+1} y_1 = 0$

$\displaystyle\int \frac{dy_1}{y_1} = \int \frac{3\,dx}{x+1}$

$\ln|y_1| = 3\ln|x+1| + C'$

$y_1 \sim (x+1)^3$

$y = (x+1)^3 v$

$y' = 3(x+1)^2 v + (x+1)^3 v'$

$\cancel{3(x+1)^2 v} + (x+1)^3 v'$

$\cancel{-\dfrac{3}{(x+1)} (x+1)^3 v} = (x+1)^4$

$v' = (x+1)$ pour $x \neq -1$

$$\int dv = \int (x+1)\,dx$$

$$v = \frac{(x+1)^2}{2} + C$$

$$v = \frac{y}{(x+1)^3} = \frac{(x+1)^2}{2} + C$$

$$\boxed{y = \frac{(x+1)^5}{2} + C(x+1)^3}$$

Verificação:

$$y' = \frac{5}{2}(x+1)^4 + 3C(x+1)^2$$

no EDO

$$\frac{5}{2}(x+1)^4 + \cancel{3C(x+1)^2}$$

$$-\frac{3}{x+1}\left(\frac{(x+1)^5}{2} + \cancel{C(x+1)^3}\right)$$

$$= (x+1)^4$$

$$\frac{5}{2}(x+1)^4 + 3c(x+1)^2 - 3c(x+1)^2 - \frac{3}{2}(x+1)^4$$

$$= (x+1)^4 \quad //$$

4) $(2xy - 9x^2)\,dx + (2y + x^2 + 1)\,dy = 0$

$$M = 2xy - 9x^2$$

$$N = (2y + x^2 + 1)$$

$M_y = N_x$ e contínuas

$$M_y = 2x \qquad N_x = 2x$$

<u>Exata</u>

$$\frac{\partial f}{\partial x} = M = 2xy - 9x^2$$

$$f = x^2 y - 3x^3 + h(y)$$

$$\left\{ \begin{aligned} \frac{\partial f}{\partial y} &= N = \cancel{x^2} + \frac{\partial h}{\partial y} \\ &= 2y + \cancel{x^2} + 1 \\ \frac{\partial h}{\partial y} &= 2y + 1 \end{aligned} \right.$$

$$h = y^2 + y + c$$

$$f = x^2 y - 3x^3 + y^2 + y + c = cte$$

$$x^2 y - 3x^3 + y^2 + y = d$$

Verificação:

$$2xy + x^2 y' - 9x^2 + 2yy' + y' = 0$$

$$(x^2 + 2y + 1)\, y' + (2xy - 9x^2) = 0$$

$$(2xy - 9x^2)\, dx + (x^2 + 2y + 1)\, dy = 0$$

OK

$$8xy + 4x^2y' - 9x^2 + 2yy' + y' = 0$$

$$(4x^2 + 2y + 1)y' + (8xy - 9x^2) = 0$$

$$(8xy - 9x^2)dx + (4x^2 + 2y + 1)dy = 0$$