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Lista 9 - C IV

a)

II) $y'' - 5y' + 6y = 10e^{t}\cos t$
$y(0) = 2$, $y'(0) = 1$

$$\mathcal{L}\{y'' - 5y' + 6y\} = \mathcal{L}\{10e^{t}\cos t\}$$
$$\mathcal{L}\{y''\} - 5\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = 10\mathcal{L}\{e^{t}\cos t\}$$
$$s^2 Y(s) - sY(0) - Y'(0) - 5[sY(s) - Y(0)] + 6Y(s) = 10\left(\frac{s-1}{(s-1)^2 + 1^2}\right)$$

$$Y(s)(s^2 - 5s + 6) - 2s + 9 = \frac{10(s-1)}{(s-1)^2 + 1}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{10(s-1)}{((s-1)^2 + 1^2)(s^2 - 5s + 6)} + \frac{2s}{(s^2 - 5s + 6)} + \frac{-9}{(s^2 - 5s + 6)}\right\}$$
I II III

I:

$$\frac{10(s-1)}{(s-2)(s-3)(s^2-2s+2)} = \frac{A}{(s-2)} + \frac{B}{(s-3)} + \frac{Cs + D}{(s^2 - 2s + 2)}$$

$$10s - 10 = (s-3)(s^2-2s+2)A + (s-2)(s^2-2s+2)B + (Cs+D)(s-2)(s-3)$$

p/ $s = 3$

$$10(3) - 10 = (3-2)(3^2 - 2(3) + 2)B$$
$$\boxed{B = 4}$$

p/ $s = 2$

$$10(2) - 10 = (2-3)(2^2 - 2(2) + 2)A$$
$$\boxed{A = -5}$$

p/ $s = 0$

$$10(0) - 10 = (0-3)(2)A + (-2)(2)B + D(-2)(-3)$$
$$\boxed{D = -4}$$

p/ $s = 1$

$$10(1) - 10 = (1-3)(1-2+2)A + (1-2)(1-2+2)B + (C+D)(1-2)(1-3)$$
$$\boxed{C = 1}$$

$$\frac{10(s-1)}{((s-1)^2+1)(s^2-5s+6)} = \frac{-5}{(s-2)} + \frac{4}{(s-3)} + \frac{s-4}{(s^2-2s+2)}$$

$$-5\,\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) = -5e^{2t}$$

$$4\,\mathcal{L}^{-1}\left(\frac{1}{(s-3)}\right) = 4e^{3t}$$

$$\frac{s-4}{(s-1)^2+1} = \frac{s-1}{(s-1)^2+1} - 3\,\frac{1}{(s-1)^2+1}$$

$$\mathcal{L}^{-1}\left(\frac{s-1}{(s-1)^2+1}\right) = e^{t}\cos t$$

$$-3\,\mathcal{L}^{-1}\left(\frac{1}{(s-1)^2+1}\right) = -3e^{t}\operatorname{sen} t$$

Para (I): $-5e^{2t}+4e^{3t}+e^{t}\cos t-3e^{t}\operatorname{sen} t$

Assim em (II):

$$\frac{2s}{(s-3)(s-2)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} = \frac{6}{(s-3)} - \frac{4}{(s-2)}$$

$$2s = (s-2)A + (s-3)B$$

p/ $s = 2$

$2(2) = (2-3)B$

$\boxed{B=-4}$

p/ $s = 3$

$2(3) = (3-2)A$

$\boxed{A=6}$

$$6\,\mathcal{L}^{-1}\left(\frac{1}{(s-3)}\right) = 6e^{3t}$$

$$-4\,\mathcal{L}^{-1}\left(\frac{1}{(s-2)}\right) = -4e^{2t}$$

Para (II): $6e^{3t}-4e^{2t}$

$$\frac{-9}{(s-3)(s-2)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} = \frac{-9}{(s-3)} + \frac{9}{(s-2)}$$

$$-9 = A(s-2) + B(s-3)$$

p/ $s=2$

$$-9 = B(2-3)$$

$$\boxed{B=9}$$

p/ $s=3$

$$-9 = A(3-2)$$

$$\boxed{A=-9}$$

$$-9\mathcal{L}^{-1}\left(\frac{1}{(s-3)}\right) = -9e^{3t}$$

$$9\mathcal{L}^{-1}\left(\frac{1}{(s-2)}\right) = 9e^{2t}$$

Para (III): $-9e^{3t} + 9e^{2t}$

Logo:

$$y(t) = -5e^{2t} + 4e^{3t} + e^{t}\cos t - 3e^{t}\operatorname{sen} t + 6e^{3t} - 4e^{2t} - 9e^{3t} + 9e^{2t}$$

$$= e^{3t}(4+6-9) + e^{2t}\underbrace{\cancel{(-5-4+9)}}_{0} + e^{t}(\cos t - 3\operatorname{sen} t)$$

$$\therefore y(t) = e^{3t} + e^{t}(\cos t - 3\operatorname{sen} t)$$

I)

$$y'' + 4y' + 13y = 10e^{-t} - 36e^{t}$$

$$y(0) = 0 \; ; \; y'(0) = -16$$

$$\mathcal{L}\{y'' + 4y' + 13y\} = \mathcal{L}\{10e^{-t} - 36e^{t}\}$$

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 13\mathcal{L}\{y\} = 10\mathcal{L}\{e^{-t}\} - 36\mathcal{L}\{e^{t}\}$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 4[s\mathcal{L}\{y\} - y(0)] + 13\mathcal{L}\{y\} = 10\mathcal{L}\{e^{-t}\} - 36\mathcal{L}\{e^{t}\}$$

$$s^2 Y(s) + 16 + 4s\,Y(s) + 13\,Y(s) = \frac{10}{s+1} - \frac{36}{s-1}$$

$$Y(s) = \mathcal{L}^{-1}\left\{ \underbrace{\frac{10}{(s+1)(s^2+4s+13)}}_{I} - \underbrace{\frac{36}{(s-1)(s^2+4s+13)}}_{II} - \underbrace{\frac{16}{(s^2+4s+13)}}_{III} \right\}$$

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$$\frac{10}{(s+1)(s^2+4s+13)} = \frac{A}{(s+1)} + \frac{Bs+C}{(s^2+4s+13)} = \frac{1}{(s+1)} + \frac{-s-3}{(s^2+4s+13)}$$

$$10 = A(s^2+4s+13) + (s+1)(Bs+C)$$

$$10 = s^2(A+B) + s(4A+B+C) + 13A + C$$

$$\begin{cases} \text{I} & A+B=0 \\ \text{II} & 4A+B+C=0 \\ \text{III} & 13A+C=10 \end{cases} \rightarrow B=-A \qquad \boxed{B=-1}$$

II:

$$4A(-A) + (10-13A) = 0$$

$$4A - A + 10 - 13A = 0$$

$$\boxed{A=1}$$

(III)

$$13(1) + C = 10$$

$$\boxed{C=-3}$$

$$s^2+4s+13$$

$$s^2+4s=-13$$

$$s^2+4s+2^2=-13+2^2$$

$$(s+2)^2+9=0$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s+1)}\right) - e^{-t}$$

$$\frac{-s-3+1-1}{(s+2)^2+9} = -\frac{s+2}{(s+2)^2+9} - 1 \cdot \frac{1}{(s+2)^2+9}$$

$$-\mathcal{L}^{-1}\left(\frac{s+2}{(s+2)^2+9}\right) = -e^{-2t}\cos(3t)$$

$$-\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2+9}\right) = \frac{1}{3}e^{-2t}\operatorname{sen}(3t)$$

Para (I): $e^{-t} - e^{-2t}\cos(3t) - \frac{1}{3}e^{-2t}\operatorname{sen}(3t)$

6m:

$$\frac{-36}{(s-1)(s^2+4s+13)} = \frac{A}{(s-1)} + \frac{Bs+C}{(s^2+4s+13)} = \frac{-2}{(s-1)} + \frac{2s+10}{(s+2)^2+9}$$

$$-36 = s^2(A+B) + s(4A+C-B) + 13A - C$$

$$\begin{cases} \text{I} & A+B=0 \\ \text{II} & 4A-B+C=0 \\ \text{III} & 13A-C=-36 \end{cases} \rightarrow B=-A$$

$\text{II} + \text{I}$:

$$\begin{aligned} 4A - B + C &= 0 \\ + \; 13A \quad\;\; - C &= -36 \\ \hline 17A - B &= -36 \end{aligned}$$

$$\boxed{B = 36 + 17A}$$

$B = -A$

$\boxed{B = 2}$

(I): $A + (36 + 17A) = 0$

$A + 36 + 17A = 0$

$18A = -36$

$\boxed{A = -2}$

(III): $13(-2) - C = -36$

$-26 - C = -36$

$-C = -10$

$\boxed{C = 10}$

$$-2\mathcal{L}^{-1}\left(\frac{1}{(s-1)}\right) = -2e^{t}$$

$$\frac{2s+10}{(s+2)^2+9} = 2 \cdot \frac{s+2}{(s+2)^2+9} + 6 \cdot \frac{1}{(s+2)^2+9}$$

$$2\mathcal{L}^{-1}\left(\frac{s+2}{(s+2)^2+9}\right) = 2e^{-2t}\cos(3t)$$

$$6\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2+9}\right) = 6\frac{1}{3}e^{-2t}\,\text{sen}(3t)$$

Assim: $-2e^{t} + 2e^{-2t}\cos(3t) + 2e^{-2t}\,\text{sen}(3t)$

$$\frac{-16}{(s^2+4s+13)} = -16 \cdot \frac{1}{(s+2)^2+9}$$

$$-16\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2+9}\right) = \frac{1}{3}e^{-2t}\,\text{sen}(3t) = -\frac{16}{3}e^{-2}\,\text{sen}(3t)$$

Portanto:

$$y(t) = e^{-t} - e^{-2t}\cos 3t - \frac{1}{3}e^{-2t}\,\text{sen}\,3t - 2e^{t} + 2e^{-2t}\cos 3t + 2e^{-2t}\,\text{sen}\,3t - \frac{16}{3}e^{-2t}\,\text{sen}\,3t$$

$$y(t) = e^{-t} - 2e^{t} + e^{-2t}\left(-\cos(3t) + 2\cos(3t)\right) + e^{-2t}\,\text{sen}\,3t\left(2 - \frac{1}{3} - \frac{16}{3}\right)$$

$$\therefore y(t) = e^{-t} - 2e^{t} + e^{-2t}\cos 3t - \frac{11}{3}e^{-2t}\,\text{sen}\,3t$$