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$$x^2y''+xy'-4y=6x-4,\ y_1=x^2$$

$$x^2y''+xy'-4y=0$$
$$y_1=x^2;\ y_1'=2x;\ y_1''=2$$
$$x^2(2)+x(2x)-4(x^2)=0\ \checkmark$$

Assim: $\div x^2$

$$y''+\frac{1}{x}y'-\frac{4}{x^2}y=0$$

Pela fórmula de Abel:

$$y_2=y_1\int\frac{e^{-\int p(x)dx}}{(y_1)^2}dx \Rightarrow$$

$$p(x)=1/x \Rightarrow -\int 1/x\,dx=-\ln|x| \Rightarrow e^{-\ln|x|} \Rightarrow x^{-1} \Rightarrow \int\frac{x^{-1}}{(x^2)^2}dx \Rightarrow \int x^{-5}dx$$

$$\Rightarrow \frac{x^{-4}}{-4} \Rightarrow \frac{-1}{4x^4}+C \Rightarrow y_2=(x^2)\left[\frac{-1}{4x^4}+C\right]$$

$$y_h=\frac{-1}{4x^2}+x^2C$$

Assim, a solução particular:

$$y_p=Ax^2+Bx+C$$
$$y_p'=2Ax+B$$
$$y_p''=2A$$

$$x^2(2A)+x(2Ax+B)-4(Ax^2+Bx+C)=6x-4+x^2\cdot 0$$
$$2Ax^2+2Ax^2+Bx-4Ax^2-4Bx-4C=6x-4$$
$$x^2(4A-4A)+x(-3B)-4C=x^2\cdot 0+6x-4$$
$$A=0;\ -3B=6\Rightarrow B=-2;\ C=1$$

$Y_p = -2x + 1 \Rightarrow Y(x) = Y_h + Y_p$

$\therefore Y(x) = -\dfrac{1}{4x^2} + x^2 C - 2x + 1$

Verificando

$y'(x) = 2Cx + \dfrac{1}{2x^3} - 2 \qquad C = 0$

$y''(x) = 2C - \dfrac{3}{2x^4}$

$x^2\left[\dfrac{-3}{2x^4}\right] + x\left[\dfrac{1}{2x^3} - 2\right] - 4\left[\dfrac{-1}{4x^2} - 2x + 1\right] = 6x - 4$

$\cancel{\dfrac{-3}{2x^2}} + \cancel{\dfrac{1}{2x^2}} - 2x + \cancel{\dfrac{1}{x^2}} + 8x - 4 = 6x - 4$

$6x - 4 = 6x - 4 \checkmark$ é solução

$$y'' + 5y' - 6y = 22 + 18x - 18x^2$$

$$y'' + 5y' - 6y = 0$$

$r^2 + 5r - 6 = 0$

$r = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(-6)}}{2} \qquad r_1 = \dfrac{-5 + 7}{2} = 1$

$r = \dfrac{-5 \pm \sqrt{25 + 24}}{2} \qquad r_2 = \dfrac{-5 - 7}{2} = -6$

$Y_h = C_1 e^{x} + C_2 e^{-6x}$

Solução particular:

$y_p = Ax^2 + Bx + C \Rightarrow 2A + 5(2Ax + B) - 6(Ax^2 + Bx + C) = 22 + 18x - 18x^2$

$y_p' = 2Ax + B \Rightarrow 2A + 10Ax + 5B - 6Ax^2 - 6Bx - 6C = -18x^2 + 18x + 22$

$y_p'' = 2A \Rightarrow x^2(-6A) + x(10A - 6B) + (2A + 5B - 6C) = -18x^2 + 18x + 22$

$$\begin{cases} -6A = -18 \Rightarrow \boxed{A = 3} \\ 10A - 6B = 18 \Rightarrow 30 - 6B = 18 \Rightarrow \boxed{B = 2} \\ 2A + 5B - 6C = 22 \Rightarrow 6 + 10 - 6C = 22 \Rightarrow \boxed{C = -1} \end{cases}$$

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$$y_p = 3x^2 + 2x - 1$$

$$\therefore y(x) = y_h + y_p$$

$$y(x) = C_1 e^{x} + C_2 e^{-6x} + 3x^2 + 2x - 1$$

Verificando

$C_1 = 0;\ C_2 = 0$

$y'(x) = 6x + 2$

$y''(x) = 6$

$$(6) + 5(6x+2) - 6(3x^2+2x-1) = 22 + 18x - 18x^2$$

$$6 + 30x + 10 - 18x^2 - 12x + 6 =$$

$$22 + 18x - 18x^2 = 22 + 18x - 18x^2 \checkmark$$

$$\therefore y(x) = C_1 e^{x} + C_2 e^{-6x} + 3x^2 + 2x - 1 \text{ é solução!}$$

$$y'' - 4y' - 5y = -6xe^{-x}$$

$$y'' - 4y' - 5y = 0$$

$r^2 - 4r - 5 = 0$

$r = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2}$ $\quad r_1 = \dfrac{4+6}{2} = 5$

$r = \dfrac{4 \pm \sqrt{16 + 20}}{2}$

$r_2 = \dfrac{4-6}{2} = -1$

$$y_h = C_1 e^{-x} + C_2 e^{3x}$$

Assim, a solução particular:

$y_p = (Ax^2 + Bx)e^{-x}$

$y_p' = (-Ax^2 + (-B+2A)x + B)e^{-x}$

$y_p'' = (Ax^2 + (B-4A)x - 2B + 2A)e^{-x}$

Substituindo na eq.:

$$y'' - 4y' - 5y = -6xe^{-x}$$

$$\boxed{e^{-x}\left[(Ax^2 + (B-4A)x - 2B + 2A) - 4(-Ax^2 + (-B+2A)x + B) - 5(Ax^2 + Bx)\right]}$$
$$= -6xe^{-x}$$

$$[Ax^2 + (B-4A)x - 2B + 2A + 4Ax^2 + (+4B - 8A)x - 4B - 5Ax^2 - 5Bx = -6x$$

$$x^2(A + \cancel{4A} - 5A) + x(B - 4A + 4B - 8A - 5B) + (-2B + 2A - 4B) = -6x$$

$$\begin{cases} -12A = -6 & \text{(I)} \\ 2A - 6B = 0 & \text{(II)} \end{cases}$$

(I) $A = -6/-12$

$\boxed{A = 1/2}$

(II) $2(1/2) - 6B = 0$

$-6B = -1$

$\boxed{B = 1/6}$

Assim:

$$y_p = \left(\tfrac{1}{2}x^2 + \tfrac{1}{6}x\right)e^{-x}$$

$$y(x) = y_h + y_p$$

$$\therefore y(x) = C_1 e^{-x} + C_2 e^{5x} + \left(x^2/2 + x/6\right)e^{-x}$$

Verificando, considerando $C_1 = C_2 = 0$

$$y(x) = (x^2/2 + x/6)e^{-x}$$

$$y'(x) = (-3x^2 + 5x + 1)e^{-x}/6$$

$$y''(x) = (3x^2 - 11x + 4)e^{-x}/6$$

Substituindo na EDO:

$$y'' - 4y' - 5y = -6x e^{-x}$$

$$e^{-x}\left[\frac{(3x^2 - 11x + 4)}{6} - \frac{4(-3x^2 + 5x + 1)}{6} - 5\left(\frac{x^2}{2} + \frac{x}{6}\right)\right] =$$

$$e^{-x}\left[\frac{\cancel{3x^2} - 11x + \cancel{4}}{6} + \frac{\cancel{12x^2} - 20x - \cancel{4}}{6} \frac{-\cancel{15x^2} - 5x}{6}\right] =$$

$$e^{-x}\left[\frac{-36x}{6}\right] = -6x e^{-x} \checkmark$$

$$\therefore y(x) = (x^2/2 + x/6)e^{-x} \text{ é solução.}$$

$$y'' - y' + y = e^x(2 + x)\,\text{sen}\,x$$

$$y'' - y' + y = 0$$

$r^2 - r + 1 = 0$

$$r = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2}$$

$$r = \frac{1 \pm \sqrt{1-4}}{2}$$

$$r = \frac{1 \pm \sqrt{-3}}{2}$$

$$r = \frac{1 \pm \sqrt{3}\,i}{2}$$

$$r_1 = \frac{1 + \sqrt{3}\,i}{2}$$

$$r_2 = \frac{1 - \sqrt{3}\,i}{2}$$

$$y_h = C_1 e^{+1/2\,x} \cos(\sqrt{3}/2\,x) + C_2 e^{+1/2\,x} \operatorname{sen}(\sqrt{3}/2\,x)$$

Assim, uma solução particular:

$$y'' - y' + y = 2e^x \operatorname{sen} x + x e^x \operatorname{sen} x$$

$$\begin{cases} (1)\quad y'' - y' + y = 2e^x \operatorname{sen} x \\ (2)\quad y'' - y' + y = x e^x \operatorname{sen} x \end{cases}$$

$$y(x) = y_h + y_{p_1} + y_{p_2}$$

$$y(x) = C_1 e^{x/2} \cos(\sqrt{3}/2\,x) + C_2 e^{x/2} \operatorname{sen}(\sqrt{3}/2\,x) + \dots$$

$$\dots + e^x(-2\cos x)$$

(I) $y'' - y' + y = 2e^x \operatorname{sen} x$

Propor:

$$y_p = e^x(B\cos x + C\operatorname{sen} x)$$
$$y_p' = e^x(B\cos x - B\operatorname{sen} x + C\cos x + C\operatorname{sen} x)$$
$$y_p'' = e^x(-2B\operatorname{sen} x + 2C\cos x)$$

Substituindo:

$$\cancel{e^x}\left[(-2B\operatorname{sen} x + 2C\cos x) - (B\cos x - B\operatorname{sen} x + C\cos x + C\operatorname{sen} x) + (B\cos x + C\operatorname{sen} x)\right]$$

$$= 2\cancel{e^x}\operatorname{sen} x$$

$$\operatorname{sen} x\left[-2B + B - \cancel{C} + \cancel{C}\right] + \cos x\left[2C - \cancel{B} - C + \cancel{B}\right] = 2\operatorname{sen} x$$

$$\begin{cases} -B = 2 \rightarrow \boxed{B = -2} \\ C = 0 \end{cases}$$

$$\therefore y_p = e^x(-2\cos x)$$

$$y_p' = 2e^x(\operatorname{sen} x - \cos x)$$
$$y_p'' = 4e^x\operatorname{sen} x$$

Substituindo:

$$[4e^x\operatorname{sen} x] - [2e^x(\operatorname{sen} x - \cos x)] + -2e^x(\cos x) =$$
$$e^x[4\operatorname{sen} x - 2\operatorname{sen} x + \cancel{\cos x} - \cancel{\cos x}] = 2e^x\operatorname{sen} x \checkmark$$

(II) $y'' - y' + y = xe^x \operatorname{sen} x$