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Lista 10 - C IV

a)

$$y'' + 2y' + y = \begin{cases} 4e^t & , 0 \leq t < 1 \\ 0 & , t \geq 1 \end{cases}$$

$$y(0) = 0 \quad ; \quad y'(0) = 0$$

Usando o Teorema da função degrau:

$$f(t) = 4e^t + u(t-1)(0 - 4e^t)$$
$$f(t) = 4e^t - 4u(t-1)e^t \cdot e \cdot e^{-1}$$
$$f(t) = 4e^t - 4u(t-1)e^{t-1} \cdot e$$
$$y'' + 2y' + y = 4e^t - 4u(t-1)e^{t-1} \cdot e$$

Aplicando a transformada de Laplace:

$$\mathcal{L}\{y'' + 2y' + y\} = \mathcal{L}\{4e^t - 4u(t-1)e^{t-1} \cdot e\}$$
$$s^2\mathcal{L}\{y\} + 2s\mathcal{L}\{y\} + \mathcal{L}\{y\} = \frac{4}{s-1} + e^{-s}\left(\frac{-4e}{s-1}\right)$$
$$Y(s)(s^2 + 2s + 1) = \frac{4}{s-1} - \frac{4e^{-s+1}}{s-1}$$
$$Y(s) = \frac{4}{(s-1)(s^2+2s+1)} - \frac{4e^{-s+1}}{(s-1)(s^2+2s+1)}$$

$s^2 + 2s + 1$

$$s = \frac{-2 \pm \sqrt{4 - 4(1)(1)}}{2}$$

$$s = \frac{-2}{2}$$

$$\boxed{s = -1}$$

$$Y(s) = \underbrace{\frac{4}{(s-1)(s+1)^2}}_{\text{I}} - \underbrace{\frac{4e^{-s+1}}{(s-1)(s+1)^2}}_{\text{II}}$$

(I): $\dfrac{4}{(s-1)(s+1)^2} = \dfrac{A}{(s-1)} + \dfrac{B}{(s+1)^2} + \dfrac{C}{(s+1)}$

$= \dfrac{A(s+1)^2 + B(s-1) + C(s-1)(s+1)}{(s-1)(s+1)^2}$

$4 = A(s+1)^2 + B(s-1) + C(s-1)(s+1)$

p/ $s=1$	p/ $s=-1$	p/ $s=0$
$4 = A(1+1)^2$	$4 = B(-1-1)$	$4 = A - B - C$
$4 = A4$	$4 = -2B$	$4 = 1+2-C$
$\boxed{A=1}$	$B=-2$	$\boxed{C=-1}$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left(\dfrac{4}{(s-1)(s+1)^2} - \dfrac{4e^{-s+1}}{(s-1)(s+1)^2}\right)$

$y = \mathcal{L}^{-1}\left\{\left(\dfrac{1}{(s-1)} - \dfrac{2}{(s-1)^2} - \dfrac{1}{(s+1)}\right) - e^{-s+1}\left(\dfrac{1}{(s-1)} - \dfrac{2}{(s-1)^2} - \dfrac{1}{(s+1)}\right)\right\}$

$y = e^{t} - 2e^{-t}t - e^{-t} + e\,u(t-1)\left(e^{-(t-1)} + 2e^{-(t-1)}(t-1) - e^{t-1}\right)$

$y(t) = e^{t} - e^{-t} - 2te^{-t} + u(t-1)\left(-e^{-t+2} + 2e^{-t+2} - e^{t}\right)$

$y'' + y = \operatorname{sen}(3t) + 2\delta(t - \pi/2)$

$y(0) = 1 \,;\; y'(0) = -1$

Pela Transformada:

$\mathcal{L}\{y'' + y\} = \mathcal{L}\{\operatorname{sen}(3t) + 2\delta(t - \pi/2)\}$

$s^2 Y(s) + 1 - s + Y(s) = \dfrac{3}{s^2+9} + 2e^{-\frac{\pi}{2}s}$

$Y(s)(s^2+1) = s - 1 + \dfrac{3}{s^2+9} + 2e^{-\frac{\pi}{2}s}$

$Y(s) = \dfrac{s}{s^2+1}^{(I)} - \dfrac{1}{s^2+1}^{(II)} + \dfrac{3}{(s^2+9)(s^2+1)}^{(III)} + \dfrac{2e^{-\frac{\pi}{2}s}}{(s^2+1)}^{(IV)}$

(III):

$\dfrac{3}{(s^2+1)(s^2+9)} = \dfrac{As+B}{s^2+1} + \dfrac{Cs+D}{s^2+9} = \dfrac{(As+B)(s^2+9) + (Cs+D)(s^2+1)}{(s^2+1)(s^2+9)}$

$$3 = (As+B)(s^2+9) + (Cs+D)(s^2+1)$$

$$3 = As^3 + 9As + Bs^2 + 9B + Cs^3 + Cs + Ds^2 + D$$

$$3 = s^3(A+C) + s^2(B+D) + s(9A+C) + 9B + D$$

$$\begin{cases} \text{I} & A+C=0 \\ \text{II} & B+D=0 \\ \text{III} & 9A+C=0 \\ \text{IV} & 9B+D=3 \end{cases}$$

$$\begin{aligned} \text{II}: &\ B+D=0 \\ \text{IV}: - &\ 9B+D=3 \\ \hline &-8B=-3 \\ &\boxed{B=+3/8} \end{aligned}$$

$$\begin{aligned} \text{I}: &\ A+C=0 \\ \text{III}: - &\ 9A+C=0 \\ \hline &-8A=0 \\ &\boxed{A=0} \end{aligned}$$

$0+C=0$

$\boxed{C=0}$

$\text{II}: \frac{3}{8} + D = 0$

$\boxed{D=-3/8}$

$$\Rightarrow \frac{3}{(s^2+1)(s^2+9)} = \frac{3/8}{(s^2+1)} - \frac{3/8}{(s^2+9)}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\left(\frac{s}{s^2+1} - \frac{1}{s^2+1} + \frac{3}{8}\left(\frac{1}{(s^2+1)} - \frac{1}{(s^2+9)}\right) + \frac{2e^{-\pi/2 s}}{(s^2+1)}\right)\right\}$$

$$Y(t) = \cos t - \operatorname{sen} t + \frac{3}{8}\left(\operatorname{sen} t - \frac{\operatorname{sen} 3t}{3}\right) + 2u(t-\pi/2)\operatorname{sen}(t-\pi/2)$$

$$\therefore Y(t) = \cos t - \frac{5}{8}\operatorname{sen} t - \frac{1}{8}\operatorname{sen} 3t + 2u(t-\pi/2)\operatorname{sen}(t-\pi/2)$$