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Lista 7 - C IV

a)

$$(1+8x^2)y'' + 2y = 0$$
$$y(0) = 2 \quad , y'(0) = -1$$

$P(x) = 1 + 8x^2 \neq 0$

$8x^2 \neq -1$

$x^2 \neq -1/8$

$x \neq \pm \sqrt{1/8}\, i$

$x \neq \pm 1/\sqrt{8}\, i$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

Substituindo na EDO:

$$(1+8x^2)\left[\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}\right] + 2\left[\sum_{n=0}^{\infty} a_n x^n\right] = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} 8n(n-1)a_n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$K = n-2$

$$\sum_{K=0}^{\infty} (K+2)(K+1)a_{K+2} x^n + \sum_{n=0}^{\infty} 8n(n-1)a_n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$n = K$

$$\sum_{n=0}^{\infty}\left[(n+2)(n+1)a_{n+2} + ((8n^2-8n)+2)a_n\right]x^n = 0$$

$$(n+2)(n+1)a_{n+2} + (8n^2-8n+2)a_n = 0$$

$$a_{n+2} = \frac{-(8n^2-8n+2)a_n}{(n+2)(n+1)} \quad ; n = 1, 2, 3, \dots$$

$y(0) = 2 = a_0$; $y'(0) = -1 = a_1$

$$a_{n+2} = \frac{-(8n^2-8n+2)\,a_n}{(n+2)(n+1)}$$

$$a_0 = 2;\quad a_1 = -1$$

$n=0$

$$a_2 = -\frac{(2)}{2}a_0 = -1(2) = -2$$

$n=1$

$$a_3 = -\frac{(2)}{(3)(2)}a_1 = -\frac{1(-1)}{3} = \frac{1}{3}$$

$n=2$

$$a_4 = -\frac{(18)}{(4)(3)}a_2 = -\frac{18(-2)}{12} = +\frac{18}{6} = 3$$

$$\therefore\ y = 2 - 1x - 2x^2 + \frac{1}{3}x^3 + 3x^4$$

$(-1/\sqrt{2},\ +1/\sqrt{2})$

$$(2+x)y'' + (2+x)y' + y = 0$$

$$y(-1) = -2,\ y'(-1) = 3$$

$f(x) = 2+x \neq 0$

$x \neq -2 \rightarrow$ ponto singular

$$y = \sum_{n=0}^{\infty} a_n (x-x_0)^n$$

$$t = x+1$$

$$y = \sum_{n=0}^{\infty} a_n t^n$$

$$(2+(t-1))y'' + (2+(t-1))y' + y = 0$$

$$(t+1)y'' + (t+1)y' + y = 0$$

Como:

$$y = \sum_{n=0}^{\infty} a_n t^n$$

$$y' = \sum_{n=1}^{\infty} n a_n t^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n t^{n-2}$$

$$(1+t)\left[\sum_{n=2}^{\infty} n(n-1)a_n t^{n-2}\right] + (1+t)\left[\sum_{n=1}^{\infty} n a_n t^{n-1}\right] + \dots$$

$$\dots + \left[\sum_{n=0}^{\infty} a_n t^n\right] = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n t^{n-2} + \sum_{n=1}^{\infty} n(n-1)a_n t^{n-1} + \sum_{n=1}^{\infty} n a_n t^{n-1} + \sum_{n=0}^{\infty} n a_n t^n + \sum_{n=0}^{\infty} a_n t^n = 0$$

$K = n-2; \quad K = n-1$

$$\sum_{K=0}^{\infty} (K+2)(K+1)a_{K+2} t^K + \sum_{K=0}^{\infty} (K+1)(K)a_{K+1} t^K + \sum_{K=0}^{\infty} (K+1)a_{K+1} t^K + \sum_{n=0}^{\infty} n a_n t^n + \dots$$

$$\dots + \sum_{n=0}^{\infty} a_n t^n = 0 \quad ; \quad n = K$$

$$\sum_{n=0}^{\infty} \left[(n+2)(n+1)a_{n+2} + [(n+1)(n) + (n+1)]a_{n+1} + (n+1)a_n \right] t^n = 0$$

$$(n+2)(n+1)a_{n+2} + (n+1)(n+1)a_{n+1} + (n+1)a_n = 0$$

↳ Relação de recorrência

$$a_{n+2} = -\frac{(n+1)(n+1)a_{n+1}}{(n+2)(n+1)} - \frac{\cancel{(n+1)}a_n}{(n+2)\cancel{(n+1)}} \qquad p/\ n = 1, 2, 3 \dots$$

$$a_{n+2} = -\frac{(n+1)}{(n+2)}a_{n+1} - \frac{1}{(n+2)}a_n$$

$$y(t=0) = y(x_0 = -1) = -2 = a_0 \ ; \ y'(t=0) = y'(-1) = 3 = a_1$$

$$a_0 = -2 \ ; \ a_1 = 3$$

$n = 0$

$$a_2 = -\frac{(1)}{2}a_0 - \frac{1}{2}a_1 = -\frac{1}{2}(-2) - \frac{1}{2}(3) = 1 - \frac{3}{2} = \frac{2-3}{2} = -\frac{1}{2}$$

$n = 1$

$$a_3 = -\frac{(2)}{(3)}a_2 - \frac{1}{(3)}a_1 = -\frac{2}{3}\left(\frac{-1}{2}\right) - \frac{1}{3}(3) = \frac{+1}{3} - 1 = \frac{1-3}{3} = \frac{-2}{3}$$

$n = 2$

$$a_4 = -\frac{(3)}{(4)}a_3 - \frac{1}{(4)}a_2 = -\frac{3}{4}\left(\frac{-2}{3}\right) - \frac{1}{4}\left(\frac{-1}{2}\right) = \frac{+2}{4} + \frac{1}{8} = \frac{4+1}{8} = \frac{5}{8}$$

$$\therefore y = -2 + 3(x+1) - \frac{1}{2}(x+1)^2 - \frac{2}{3}(x+1)^3 + \frac{5}{8}(x+1)^4 + \dots$$

$(-2, 0)$