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Lista 2 C IV

a) $$xy\frac{dy}{dx} + 4x^2 + y^2 = 0; \quad y(2) = 7, \; x > 0$$

$$xy\frac{dy}{dx} = -(4x^2+y^2)$$

$$\frac{dy}{dx} = \frac{-(4x^2+y^2)}{xy}$$

$$xy\,dy = -(4x^2+y^2)\,dx$$

$$(xy)dy + (4x^2+y^2)dx = 0$$

$$\frac{\partial M}{\partial y} = x \neq \frac{\partial N}{\partial x} = 8x$$

Assim, pelo fator integrante:

$$-\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{M} = h(y) \Rightarrow I(y) = e^{\int h(y)dy}$$

$$\frac{-(x-8x)}{xy} = \frac{x(-1+8)}{xy} = \frac{7}{y}$$

$h(y) = \frac{7}{y}$ → função apenas em 'y'

Assim:

$$I(y) = e^{\int h(y)dy} \Rightarrow \int \frac{7}{y}dy \Rightarrow 7\ln|y| \Rightarrow e^{7\ln|y|} = e^{\ln|y^7|}$$

$I(y) = y^7$ → fator integrante

$$(xy)\,dy + (4x^2 + y^2)\,dx = 0 \quad \times y^7$$

$$y^7(xy)\,dy + (4x^2+y^2)\,dx\,y^7 = 0$$

$$\frac{\partial M}{\partial y} = 8y^7x = \frac{\partial N}{\partial x} = 8y^7x$$

Assim

$$\int M(x,y)\,dx \Rightarrow \int y^8 x\,dx \Rightarrow \frac{y^8x^2}{2} + g(y)$$

$$\frac{\partial}{\partial y}\left(\int M(x,y)\,dx\right) \Rightarrow \frac{\partial}{\partial y}\left(\frac{y^8x^2}{2} + g(y)\right) \Rightarrow \frac{8y^7x^2}{2} + g'(y)$$

$$\Rightarrow 4y^7x^2 + g'(y)$$

Assim:

$$\cancel{4y^7x^2} + g'(y) = \cancel{4y^7x^2} + y^9$$

$$g'(y) = y^9$$

Integrando:

$$g(y) = \int g'(y)\,dy \Rightarrow \int y^9\,dy \Rightarrow \frac{y^{10}}{10}$$

$$g(y) = \frac{y^{10}}{10}$$

$$u(x,y) = \int M(x,y)\,dx = C$$

$$\frac{y^8x^2}{2} + g(y) = C$$

$$\therefore \frac{y^8x^2}{2} + \frac{y^{10}}{10} = C$$

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PVI: $y(2) = 7$, $x > 0$

$$\frac{y^8 x^2}{2} + \frac{y^{10}}{10} = \mathbb{C}$$

$$\mathbb{C} = \frac{(7)^8 (2)^2}{2} + \frac{(7)^{10}}{10} \text{ //}$$

$$\cos(x)\frac{dy}{dx} + \operatorname{sen}(x)y = 2(\cos(x))^3 \operatorname{sen}(x) - 1$$

$$y(\pi/4) = 3\sqrt{2}, \quad 0 \leq x < \pi/2$$

$$\cos(x)\,y' + \operatorname{sen}(x)y = 2(\cos(x))^3 \operatorname{sen}(x) - 1$$

$P(x).y' + Q(x).y = R(x) \Rightarrow$ EDO Linear

$$I = e^{\int \operatorname{sen}(x)\,dx}$$

$\Rightarrow \int \operatorname{sen}(x)dx = -\cos x + \mathbb{C} \Rightarrow I = e^{-\cos(x)+\mathbb{C}}$

$$y(x) = \frac{1}{I(x)} \cdot \left(\int I(x).B(x)\,dx + \mathbb{C} \right)$$

$$y(x) = \frac{1}{e^{-\cos(x)}} \cdot \left(\int \left(e^{-\cos(x)} \cdot 2\cos^3(x)\operatorname{sen}x - 1\right) dx \right)$$

b) $\frac{dy}{dx} - \frac{3y}{x+1} = (x+1)^4$

$y' - \frac{3y}{x+1} = (x+1)^4$

$y'(x) + p(x)\,y = g(x)$

$p(x) = -\frac{3}{x+1}$

$\int \frac{-3}{x+1}\,dx \Rightarrow -3\int \frac{1}{x+1}\,dx \Rightarrow -3\ln|x+1| \Rightarrow e^{-3\ln|x+1|} \Rightarrow (x+1)^{-3}$

$I(x) = \frac{1}{(x+1)^3}$

Assim:

$(I(x)\cdot y)' = I(x)\cdot g(x)$

$\left(\frac{1}{(x+1)^3}\cdot y\right)' = \frac{1}{\cancel{(x+1)^3}}\cdot (x+1)^{\cancel{4}}$

$\int\left(\frac{1}{(x+1)^3}\cdot y\right)' = \int x+1$

$\frac{1}{(x+1)^3}\cdot y = \frac{x^2}{2} + x + C$

$y = \frac{x^2(x+1)^3}{2} + x(x+1)^3 + C(x+1)^3$

$\therefore y = \frac{x^2(x+1)^3}{2} + x(x+1)^3 + C(x+1)^3$

$$(2xy - 9x^2)dx + (2y + x^2 + 1)dy = 0$$

$$\frac{\partial M}{\partial y} = 2x \quad = \quad \frac{\partial N}{\partial x} = 2x \Rightarrow \text{Eq. exata}$$

$$\int M(x,y)\,dx \Rightarrow \int (2xy - 9x^2)\,dx \Rightarrow x^2y - 3x^3 + g(y)$$

$$\frac{\partial}{\partial y}\left(\int M(x,y)\,dx\right) \Rightarrow \frac{\partial}{\partial y}\left(x^2y - 3x^3 + g'(y)\right)$$

$$\Rightarrow x^2 + g'(y)$$

$$\cancel{x^2} + g'(y) = 2y + \cancel{x^2} + 1$$

$$g'(y) = 2y + 1$$

$$\int 2y + 1\,dy \Rightarrow y^2 + y$$

Assim:

$$u(x,y) = \int M(x,y)\,dx = C$$

$$u(x,y) = x^2y - 3x^3 + g(y)$$

$$u(x,y) = x^2y - 3x^3 + y^2 + y$$

$$x^2y - 3x^3 + y^2 + y = C$$

$$y + x^2y + y^2 - 3x^3 = C \Rightarrow \text{forma implícita}$$