Solução da Lista 13 de Cálculo IV

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

$u(0,t) = 0 = u(c,t)$

$u(x,0) = 20$

$u = XT$

$$\frac{1}{Tk}\frac{dT}{dt} = -\lambda^2 = \frac{1}{X}\frac{d^2X}{dx^2}$$

1) $\lambda^2 > 0$

$$X'' - \lambda^2 X = 0$$

$$r = \pm \lambda$$

$$X = Ae^{\lambda x} + Be^{-\lambda x}$$

$$X(0) = 0 = A + B$$

$$X(L) = 0 = Ae^{\lambda L} - Ae^{-\lambda L}$$

$$= 2A \operatorname{senh} \lambda L$$

2) $\lambda^2 = 0$

$$X = Ax + B$$

$$X(0) = B = 0$$

$$X(L) = 0 = AL \rightarrow A = 0$$

3) $-\lambda^2 < 0$

$X'' + X\lambda^2 = 0$

$r^2 + \lambda^2 = 0$

$r_\pm = \pm \lambda i$

$X = A\cos\lambda x + B\,\mathrm{sen}\,\lambda x$

$X(0) = 0 = A$

$X(L) = 0 = B\,\mathrm{sen}\,\lambda L$

$\lambda L = n\pi$

$\lambda = \frac{n\pi}{L} \quad n = 1, 2, 3 \ldots$

$T = T_0 e^{-\lambda^2 t k}$

$\longrightarrow$

$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n\pi}{L}\right)^2 t k} \,\mathrm{sen}\, \frac{n\pi x}{L}$

Como

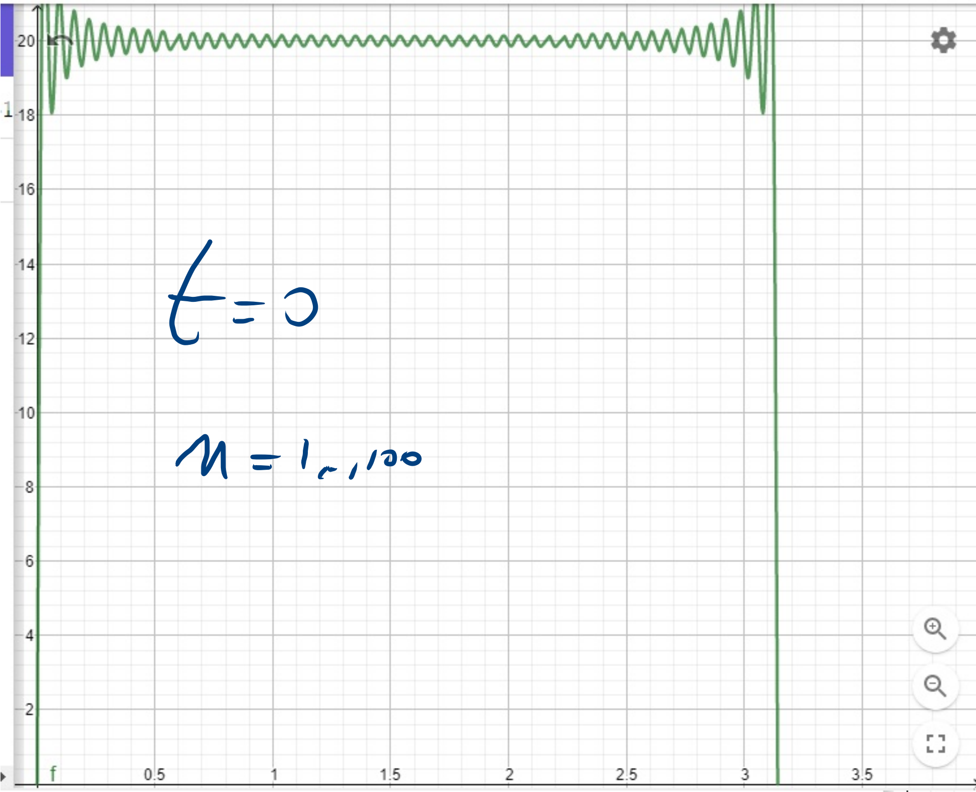
$u(x,0) = 20 = \sum_{n=1}^{\infty} B_n \,\mathrm{sen}\, \frac{n\pi x}{L}$

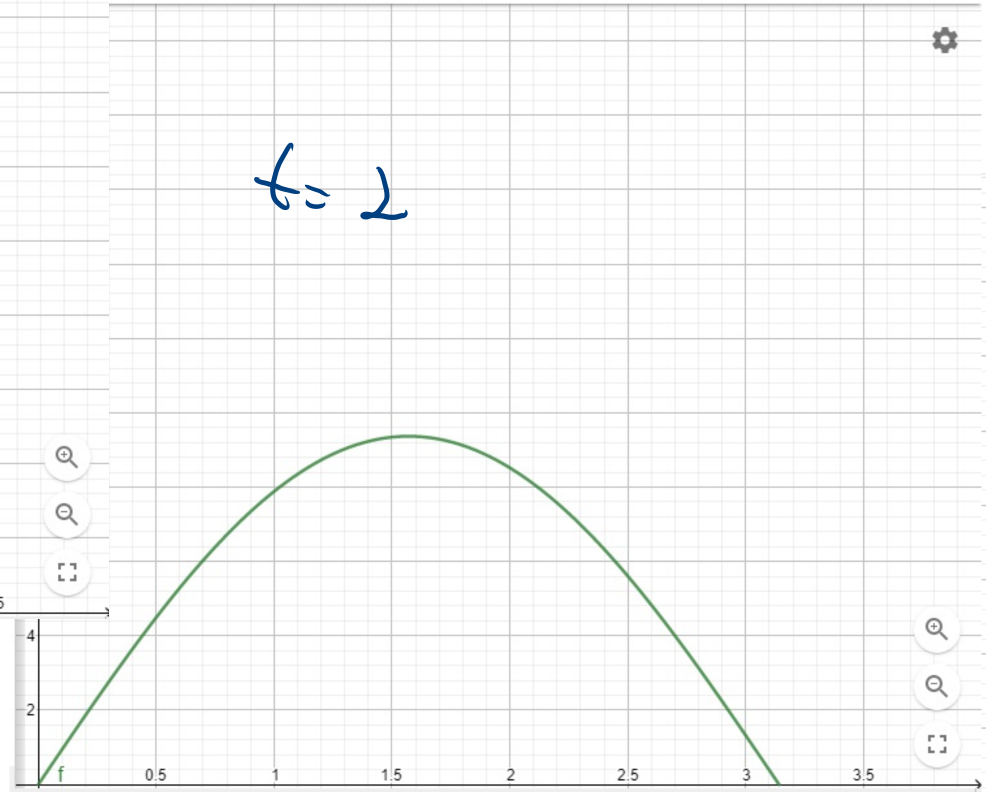
$B_n = \frac{2}{L}\int_0^L 20 \,\mathrm{sen}\, \frac{n\pi x}{L}\, dx$

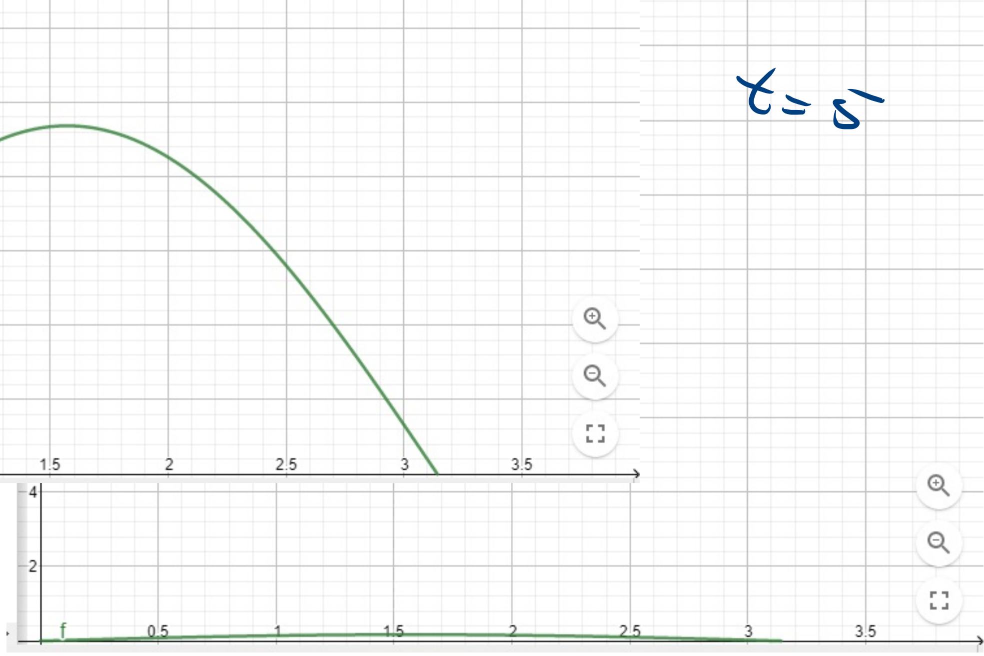
$$B_n = \frac{40}{L}\int_0^L \sin\frac{n\pi x}{L}\,dx = -\frac{40}{L}\frac{L}{n\pi}\cos\frac{n\pi x}{L}\Big|_0^L$$

$$= -\frac{40}{n\pi}\left(\cos n\pi - 1\right) = \frac{40}{n\pi}\left(1 - (-1)^n\right)$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{40}{n\pi}\left(1 - (-1)^n\right)\sin\left(\frac{n\pi x}{L}\right) e^{-\left(\frac{n\pi}{L}\right)^2 k t}$$


t = 0
n = 1,…,100
f


t = 2
f


t = 5
f