Solução do Trabalho II de Cálculo II

1) $f = t^3 \qquad -2 < t < 2$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f\,dt = \frac{1}{2}\int_{-2}^{2} t^3\,dt = 0$$

Pois f é ímpar.

$$a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f \cos\frac{n\pi t}{\ell}\,dt = \frac{1}{\ell}\int_{-\ell}^{\ell} t^3 \cos\frac{n\pi t}{\ell}\,dt$$

$t^3 \cos\frac{n\pi t}{\ell}$ é ímpar, logo $\underline{a_n = 0}$

$$b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} t^3 \operatorname{sen}\frac{n\pi t}{\ell}\,dt = \frac{2}{\ell}\int_{0}^{\ell} t^3 \operatorname{sen}\frac{n\pi t}{\ell}\,dt$$

$u = t^3 \quad du = 3t^2\,dt$

$dv = \operatorname{sen}\frac{n\pi t}{\ell}\,dt \quad v = -\frac{\ell}{n\pi}\cos\frac{n\pi t}{\ell}$

$$b_n = \frac{2}{a}\int_0^a t^3 \sin\frac{n\pi t}{a}\,dt = \frac{2}{a}\left[-\frac{t^3 a}{n\pi}\cos\frac{n\pi t}{a}\Big|_0^a + \frac{3a}{n\pi}\int_0^a t^2\cos\frac{n\pi t}{a}\,dt\right]$$

$$= \frac{2}{a}\left[-\frac{a^4}{n\pi}\cos n\pi + \frac{3a}{n\pi}\underbrace{\int_0^a t^2\cos\frac{n\pi t}{a}\,dt}_{I_1}\right]$$

$$I_1 = \int_0^a t^2\cos\frac{n\pi t}{a}\,dt = \cancelto{0}{\frac{a t^2}{n\pi}\sin\frac{n\pi t}{a}\Big|_0^a} - \frac{2a}{n\pi}\underbrace{\int_0^a t\sin\frac{n\pi}{a}t\,dt}_{I_2}$$

$u = t^2 \quad du = 2t\,dt \qquad v = \frac{a}{n\pi}\sin\frac{n\pi t}{a}$

$dv = \cos\frac{n\pi t}{a}\,dt$

$$I_2 = \int_0^a t \sin\frac{n\pi}{a} t \, dt = \frac{-ta}{n\pi} \cos\frac{n\pi t}{a} \Big|_0^a + \frac{a}{n\pi} \int_0^a \cos\frac{n\pi t}{a} \, dt$$

$$u = t \qquad du = dt$$

$$dv = \sin\frac{n\pi t}{a} \, dt$$

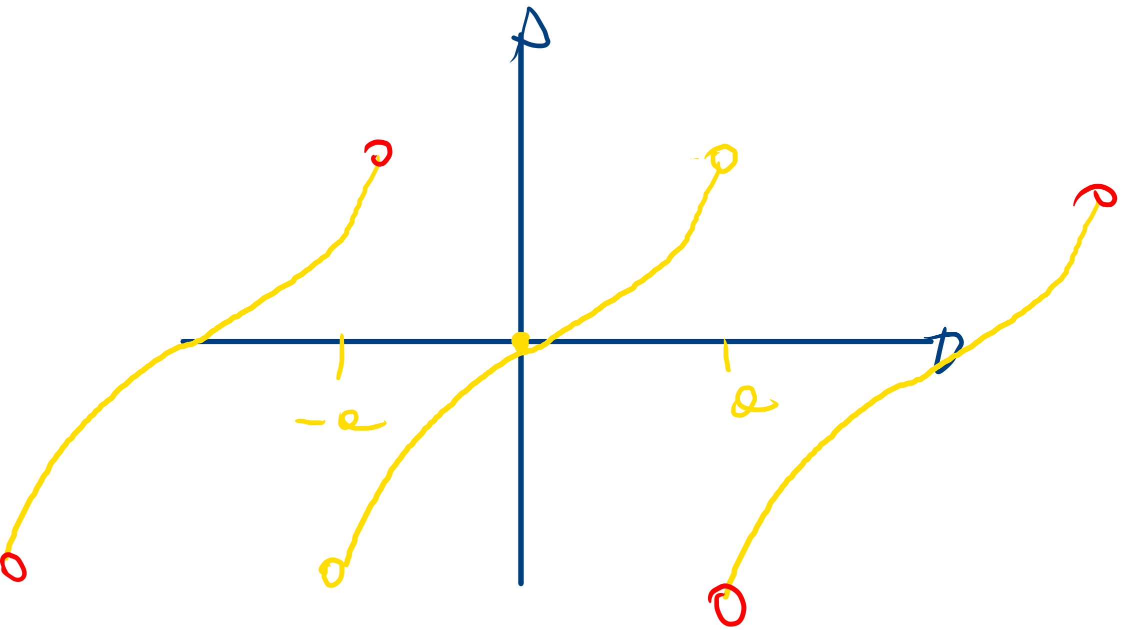
$$v = -\frac{a}{n\pi} \cos\frac{n\pi t}{a}$$

$$I_2 = \frac{-a^2}{n\pi} \cos n\pi + \frac{a^2}{n^2\pi^2} \cancel{\sin\frac{n\pi t}{a}} \Big|_0^a$$

$$0$$

$$b_n = \frac{2}{a} \left[-\frac{a^4}{n\pi} \cos n\pi + \frac{3a}{n\pi} \left\{ \frac{-2a}{n\pi} \left(-\frac{a^2}{n\pi} \cos n\pi \right) \right\} \right]$$

$$= -\frac{2a^3}{n\pi} \cos n\pi + \frac{12a^3}{n^3\pi^3} \cos n\pi = \frac{(-1)^n a^3 2}{n\pi} \left(\frac{6}{n^2\pi^2} - 1 \right)$$



$$f(t) = \begin{cases} 0 & t = \pm a \\ t^3 & -a < t < a \end{cases}$$