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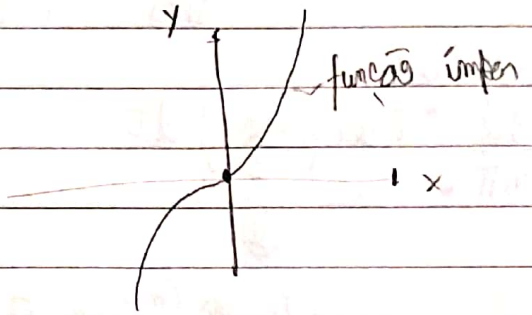
Trabalho 11 - CIV

$$f(t) = t^3, \quad -6 < t \leq 6$$

Pelo esboço:



Como $f(t)$ é Impar, então:

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt = \frac{1}{6}\int_{-6}^{6} t^3\,dt \Rightarrow \frac{1}{6}\left[\frac{t^4}{4}\right]\Bigg|_{-6}^{6} \Rightarrow \frac{1}{6}\left[\frac{(6)^4}{4} - \frac{(-6)^4}{4}\right]$$

$$a_0 = 0$$

Para a_n:

$$a_n = \frac{1}{6}\int_{-6}^{6} f(t)\cos\left(\frac{n\pi t}{6}\right)dt \Rightarrow \frac{1}{6}\int_{-6}^{6} t^3\cos\left(\frac{n\pi t}{6}\right)dt$$

Como $t^3\cos\frac{n\pi t}{6}$ é uma função impar, $a_n = 0$

Assim, em b_n:

$$b_n = \frac{1}{6}\int_{-6}^{6} t^3 \operatorname{sen}\left(\frac{n\pi t}{6}\right)dt = \frac{2}{6}\int_{0}^{6} t^3 \operatorname{sen}\left(\frac{n\pi t}{6}\right)dt$$

Pela integral por partes:

$u = t^3$ $\quad dv = \operatorname{sen}\left(\frac{n\pi t}{6}\right)dt \Rightarrow v = \int \operatorname{sen}\left(n\pi t/6\right)dt$

$du = 3t^2\,dt$ $\quad v = \frac{-6}{n\pi}\cos\left(n\pi t/6\right)$

$$\underbrace{\left[\frac{-t^3 6}{n\pi}\cos\left(n\pi t/6\right)\right]\Big|_0^6 + \frac{18}{n\pi}\int_0^6 t^2\cos\left(n\pi t/6\right)dt}_{uv - \int v\,du}\cdot\frac{2}{6}$$

$$b_n \Rightarrow \frac{2}{6}\left[-(6)^3(6)\cos\left(\frac{n\pi 6}{6}\right) - 0\right] + \frac{18}{n\pi}\int_0^6 t^2\cos\left(\frac{n\pi t}{6}\right)dt$$

$$\Rightarrow \frac{2}{6}\left[-(6)^4\cos(n\pi) + \frac{18}{n\pi}\int_0^6 t^2\cos\left(\frac{n\pi t}{6}\right)dt\right]$$

(I)

(I): $\int_0^6 t^2\cos\left(n\pi t/6\right)dt$ $\quad u = t^2 \;\vert\; dv = \cos(n\pi t/6)\,dt$

$du = 2t\,dt \;\vert\; v = \frac{6}{\pi n}\operatorname{sen}\left(\frac{n\pi t}{6}\right)$

$uv - \int v\,du$

$$\left[\frac{6t^2}{\pi n}\operatorname{sen}\left(\frac{\pi n t}{6}\right)\right]\Big|_0^6 - \frac{12}{n\pi}\int_0^6 t\operatorname{sen}\left(\frac{n\pi t}{6}\right)dt$$

(II)

(II): $\int_0^6 t\operatorname{sen}\left(\frac{n\pi t}{6}\right)dt$ $\quad u = t \;\vert\; dv = \operatorname{sen}(n\pi t/6)\,dt$

$du = dt \;\vert\; v = \frac{-6}{n\pi}\cos(n\pi t/6)$

$uv - \int v\,du$

$$\left[\frac{-6t}{n\pi}\cos\left(\frac{n\pi t}{6}\right)\right]\Big|_0^6 + \frac{6}{n\pi}\int_0^6 \cos\left(\frac{n\pi t}{6}\right)dt$$

$$\Rightarrow \frac{-36}{n\pi}\cos n\pi + \frac{36}{n^2\pi^2}\left[\operatorname{sen}\left(n\pi t/6\right)\right]\Big|_0^6$$

$$\frac{36}{n^2\pi^2}\left[\operatorname{sen}(n\pi) - \operatorname{sen}(0)\right] \quad \textcircled{0}$$

Assim:

$$b_n = \frac{2}{6}\left[\frac{-(6)^4}{n\pi}\cos(n\pi) + \frac{18}{n\pi}\left(-\frac{12}{n\pi}\left\{\frac{-36}{n\pi}\cos n\pi\right\}\right)\right]$$

$$b_n = \frac{-432}{n\pi}\cos(n\pi) + \frac{2.592}{n^3\pi^3}\cos(n\pi)$$

$$\therefore b_n = (-1)^n\frac{432}{n\pi}\left(\frac{6}{n^2\pi^2} - 1\right)$$

$$f(t) = \begin{cases} 0, & t = -6 \\ t^3, & -6 < t \leq 6 \end{cases}$$

credeal