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Lista 3 - C IV

I) $x^3y' + x^2y - y^2 = 2x^4 \quad \div x^3$

$$y' + \frac{y}{x} - \frac{y^2}{x^3} = 2x$$

$$y' = \frac{y^2}{x^3} - \frac{y}{x} + 2x \Rightarrow \text{Eq. de Riccati}$$

. Solução particular

$y_1 = x^2$

$y_1' = 2x$

$$\rightarrow 2x = \frac{x^4}{x^3} - \frac{x^2}{x} + 2x$$

$$2x = \cancel{x} - \cancel{x} + 2x$$

$$2x = 2x \checkmark$$

$$y_1 = x^2 \text{ é solução particular}$$

Assim

$z = y - x^2 \rightarrow y = z + x^2$

$y' = z' + 2x$

$$(z' + 2x) = \frac{(z+x^2)^2}{x^3} - \frac{(z+x^2)}{x} + 2x$$

$$z' + \cancel{2x} = \frac{z^2 + 2zx^2 + x^4}{x^3} - \frac{z}{x} - x + \cancel{2x}$$

$$z' = \frac{z^2}{x^3} + \frac{2z}{x} + \cancel{x} - \cancel{\frac{z}{x}} - \cancel{x}$$

$$z' = \frac{z^2}{x^3} + \frac{z}{x} \Rightarrow \text{Eq de Bernoulli}$$

$$z' = P(x)z + Q(x)z^{\alpha}$$

$$\alpha = 2 \neq 0 \neq 1$$

$$z' = \frac{z}{x} + \frac{z^2}{x^3} \quad \div z^2$$

$$\frac{z'}{z^2} = \frac{1}{xz} + \frac{1}{x^3}$$

$$z' \cdot z^{-2} = \frac{1}{x} \cdot z^{-1} + \frac{1}{x^3}$$

$$u = z^{1-\alpha} = z^{-1}$$

$$u' = z^{-2} \cdot z'$$

$$u' = \frac{1}{x}u + \frac{1}{x^3} \Rightarrow \text{Linear}$$

$$u' + A(x)\,u = B(x)$$

$$A(x) = -1/x \ ; \ B(x) = 1/x^3$$

$$u(x) = \frac{1}{I(x)}\left(\int I(x)\,B(x)\,dx + C\right)$$

$$I(x) = e^{\int A(x)dx}$$

$$I(x) = e^{\int -1/x \, dx} \Rightarrow e^{-\ln|x|} \Rightarrow e^{\ln|x^{-1}|} \Rightarrow x^{-1}$$

$$\int x^{-1} \cdot \frac{1}{x^3}\,dx \Rightarrow \int \frac{1}{x^2}\,dx \Rightarrow -1/x + C$$

$$u(x) = \frac{1}{x^{-1}}\left(\frac{-1}{x} + C\right)$$

$$u(x) = x\left(-1/x + C\right)$$

$$u = z^{-1} \Rightarrow \frac{1}{z} = x\left(-1/x + C\right)$$

$$z = \frac{1}{x(-1/x + C)} \Rightarrow z = y - x^2 \Rightarrow y = \frac{1}{x(-1/x + C)} + x^2$$

$$y = xy' + \sqrt{(y')^2+1} \Rightarrow \text{Eq de Clairaut}$$

derivando em relação a x

$$\cancel{y'} = xy'' + \cancel{y'} + \frac{2y'}{2\sqrt{1+(y')^2}}\, y''$$

$$0 = xy'' + \frac{2y'}{2\sqrt{1+(y')^2}} \cdot y''$$

$$0 = \left(x + \frac{y'}{\sqrt{1+(y')^2}}\right) y''$$

Assim:

$$y'' = 0 \Rightarrow y' = C$$

ou

$$x = \frac{-y'}{\sqrt{1+(y')^2}}$$

$$x = \frac{-t}{\sqrt{1+t^2}}$$

Para $y' = C$

$$\Rightarrow \quad y = xC + \sqrt{C^2+1} \Rightarrow \text{famílias de reta}$$

Para $x = -\dfrac{t}{\sqrt{1+t^2}}$; $y' = t$

$$y = -\frac{t^2}{\sqrt{1+t^2}} + \sqrt{1+t^2} \Rightarrow \text{famílias envoltória}$$

Duas possíveis soluções:

$$\begin{cases} y = xC + \sqrt{1+C^2} & \text{[reta]} \\ y = \dfrac{-t^2}{\sqrt{1+t^2}} + \sqrt{1+t^2} \;;\; x = \dfrac{-t}{\sqrt{1+t^2}} & \text{[Envoltória]} \end{cases}$$

II) $y = xy' + \sqrt{(y')^2+1} \Rightarrow$ Eq. de Clairaut

$$\begin{cases} y = xC + \sqrt{1+C^2} \\ y' = C \end{cases}$$

$$(xC + \sqrt{1+C^2}) = xC + \sqrt{(C)^2+1} \quad \checkmark$$

III) $xy'' = y' + x(y')^2$

$$y'' = \frac{y' + x(y')^2}{x}$$

$$y' = v$$

$$v' = \frac{v + x(v)^2}{x}$$

$$v' - \frac{1}{x}v = v^2 \Rightarrow \text{Eq. de Bernoulli}$$

$$P(x) = -1/x \; ; \; g(x) = 1 \; , \; n = 2$$

$$v' - \frac{1}{x}v = v^2 \quad \div v^2$$

$$\frac{v'}{v^2} - \frac{1}{vx} = 1$$

$$v = y^{1-n} \Rightarrow v' = (1-n)\frac{y'}{y^n} \Rightarrow v = y^{-1} \Rightarrow v' = -1y^{-2} \Rightarrow -1/y^2$$

$$v' = \frac{-v'}{v^2} \Rightarrow -v' - \frac{v}{x} = 1$$

$$-v' - \frac{v}{x} = 1 \Rightarrow \text{Eq. Linear}$$

$$v' + \frac{1}{x} v = -1$$

$$I(x) = e^{\int \frac{1}{x} dx} \Rightarrow e^{\ln|x|} \Rightarrow x$$

$$v(x) = \frac{1}{x} \left(\int I(x) \cdot B(x)\, dx + C \right)$$

$$\int x \cdot (-1)\, dx = -\frac{x^2}{2}$$

$$v(x) = \frac{1}{x} \left(-\frac{x^2}{2} + C \right)$$

$$v(x) = -\frac{x}{2} + \frac{C}{x} = \frac{-x^2 + 2C}{2x}$$

$$v = v^{-1}$$

$$v = \frac{2x}{-x^2 + 2C}$$

$$v = y'$$

$$u = -x^2 + C_1$$

$$y' = \frac{2x}{-x^2 + C_1} \Rightarrow \int y' dx \Rightarrow \int \frac{2x}{-x^2 + C_1} dx \qquad du = -2x\, dx$$

$$-\int \frac{1}{u} du \Rightarrow -\ln|u| + C \Rightarrow -\ln|x^2 + C_1| + K$$

$$\therefore y = -\ln(-x^2 + C_1) + K$$

$$y'' = \frac{2(x^2 + C)}{(x^2 - C)^2}$$

$$x \left(\frac{2(x^2 + C)}{(x^2 - C)^2} \right) = \frac{2x}{-x^2 + C} + x \left(\frac{2x}{-x^2 + C_1} \right)^2$$

$$x \left(\frac{2(x^2 + C)}{(x^4 - 2x^2 C + C^2)} \right) = \frac{2x}{-x^2 + C} + x \frac{4x^2}{-x^4 + 2x^2 C + C^2} \quad ✓$$

$$\therefore y = -\ln(-x^2 + C_1) + K \rightarrow \text{uma das soluções}$$