Solução do Trabalho de Calculo IV

1) $y'' - y = \begin{cases} t, & 0 \leq t < a \\ -t, & t \geq a \end{cases}$ $\qquad y(0) = 0$, $\quad y'(0) = 0$

$$y'' - y = f(t) = t + u(t-a)(-t-t)$$

$$= t - 2u(t-a)(t-a+a)$$

$$f(t) = t - 2u(t-a)(t-a) - 2a\,u(t-a)$$

$$s^2 \mathcal{L}_y - \mathcal{L}_y = \frac{1}{s^2} - 2\frac{e^{-es}}{s^2} - 2e\frac{e^{-es}}{s}$$

$$\mathcal{L}_y = \frac{1}{s^2(s^2-1)} - \frac{2e^{-es}}{s^2(s^2-1)} - \frac{2e\,e^{-es}}{s(s^2-1)}$$

$$\frac{1}{s^2(s^2-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s-1}$$

$$As(s^2-1) + B(s^2-1) + Cs^2(s-1) + Ds^2(s+1) = 1$$

$$\cancel{As^3} - \cancel{As} + \cancel{Bs^2} - B + \cancel{Cs^3} - \cancel{Cs^2} + \cancel{Ds^3} + \cancel{Ds^2} = 1$$

$A + C + D = 0$ $\qquad$ $A = 0$ $\qquad$ $B = -1$

$B - C + D = 0$ $\qquad$ $C + D = 0$ $\qquad$ $2D = 1$

$-A = 0$ $\qquad$ $-C + D = 1$ $\qquad$ $D = 1/2$

$-B = 1$ $\qquad$ $C = -1/2$

$$\frac{1}{s^2(s^2-1)} = \frac{-1}{s^2} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{2}\frac{1}{s-1}$$

$$\frac{1}{s(s^2-1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} = \frac{-1}{s} + \frac{1}{2}\frac{1}{(s+1)} + \frac{1}{2(s-1)}$$

$$A(s^2-1) + Bs(s-1) + Cs(s+1) = \cancel{As^2} - A + \cancel{Bs^2} - \cancel{Bs} + \cancel{Cs^2} + \cancel{Cs} = 1$$

$A + B + C = 0$ $\qquad$ $C - B = 0$ $\qquad$ $A = -1$ $\qquad$ $C = 1/2 = B$

$$\mathcal{L}y = \frac{1}{s^2(s^2-1)} - \frac{2e^{-es}}{s^2(s^2-1)} - \frac{2e\,e^{-es}}{s(s^2-1)}$$

$$y = \mathcal{L}^{-1}\left(\frac{1}{s^2(s^2-1)}\right) - 2\mathcal{L}^{-1}\left(\frac{e^{-es}}{s^2(s^2-1)}\right) - 2e\,\mathcal{L}^{-1}\left(\frac{e^{-es}}{s(s^2-1)}\right)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2(s^2-1)}\right) = \mathcal{L}^{-1}\left(-\frac{1}{s^2} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{2}\frac{1}{s-1}\right)$$

$$= -t - \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}$$

$$\mathcal{L}^{-1}\left(\frac{e^{-as}}{s^2(s^2-1)}\right) = \mathcal{L}\left(\frac{-e^{-as}}{s^2} - \frac{1}{2}\frac{e^{-as}}{s+1} + \frac{1}{2}\frac{e^{-as}}{s-1}\right)$$

$$= \underline{-u(t-a)(t-a)} - \frac{1}{2}u(t-a)e^{-(t-a)} + \frac{1}{2}u(t-a)e^{t-a}$$

$$\mathcal{L}^{-1}\left(\frac{e^{-as}}{s(s^2-1)}\right) = \mathcal{L}^{-1}\left(\frac{-e^{-as}}{s} + \frac{1}{2}\frac{e^{-as}}{s+1} + \frac{1}{2}\frac{e^{-as}}{s-1}\right)$$

$$= -u(t-a) + \frac{1}{2}u(t-a)e^{-(t-a)} + \frac{1}{2}u(t-a)e^{t-a}$$

$$y = -t - \frac{1}{2} e^{-t} + \frac{1}{2} e^{t}$$

$$-2\left(-u(t-e)(t-e) - \frac{1}{2} u(t-e) e^{-(t-e)} + \frac{1}{2} u(t-e) e^{t-e} \right)$$

$$-2e\left(-u(t-e) + \frac{1}{2} u(t-e) e^{-(t-e)} + \frac{1}{2} u(t-e) e^{t-e} \right)$$

$$y = -t + \operatorname{senh} t - 2u(t-e)\left(-t + e + \operatorname{senh}(t-e) - e + e \cosh(t-e) \right)$$

$$y = -t + \operatorname{senh} t - 2u(t-e)\left(-t + \operatorname{senh}(t-e) + e \cosh(t-e) \right)$$