# Soluções da Lista 10 de Cálculo IV

1) $y'' + 2y' + y = \begin{cases} 4e^{t} & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases}$ $\quad y(0)=0$, $\; y'(0)=0$

$$f(t) = 4e^{t} + u(t-1)\left(-4e^{t}\right)$$

$$= 4e^{t} - 4u(t-1)\, e^{t} \cdot e \cdot e^{-1} = 4e^{t} - 4u(t-1)\, e^{t-1} e$$

$$s^2 \mathcal{L}y + 2s\mathcal{L}y + \mathcal{L}y = \frac{4}{s-1} + e^{-s}\left(\frac{-4e}{s-1}\right)$$

$$\mathcal{L}(y)(s^2+2s+1) = \frac{4}{s-1} - \frac{4e^{-s+1}}{s-1}$$

$$\mathcal{L}y = \frac{4}{(s-1)(s^2+2s+1)} - \frac{4e^{-s+1}}{(s-1)(s^2+2s+1)}$$

$$s^2+2s+1=0$$

$$s_\pm = \frac{-2 \pm \sqrt{4-4\cdot 1 \cdot 1}}{2} = -1$$

$$\mathcal{L}y = \frac{4}{(s+1)^2(s-1)} - \frac{4e^{-s+1}}{(s-1)(s+1)^2}$$

$$-(s+1)(s-1) - 2(s-1) + (s+1)^2$$

$$\cancel{-s^2}+1 \cancel{-2s} + 2 \cancel{+s^2} \cancel{+2s} + 1$$

$$4$$

$$\frac{1}{(s+1)^2(s-1)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s-1} = \frac{1}{4}\left(\frac{-1}{s+1} - \frac{2}{(s+1)^2} + \frac{1}{s-1}\right)$$

$$A(s+1)(s-1) + B(s-1) + C(s+1)^2 = \cancel{A}s^2 \cancel{-A} + \cancel{Bs} \cancel{-B} + C\cancel{s^2} \cancel{+2Cs} + C = 1$$

$$A + C = 0 \qquad -A - B + C = 1 \qquad A = -C$$

$$B + 2C = 0 \qquad C + 2C + C = 1 \qquad B = -2C$$

$$C = 1/4 \qquad A = -1/4 \qquad B = -1/2$$

$$\mathcal{L}y = \frac{4}{(s+1)^2(s-1)} - \frac{4e^{-s+1}}{(s-1)(s+1)^2}$$

$$y = \mathcal{L}^{-1}\left(4 \cdot \frac{1}{4}\left(\frac{-1}{s+1} - \frac{2}{(s+1)^2} + \frac{1}{s-1}\right)\right)$$

$$- \mathcal{L}^{-1}\left(4 \cdot \frac{1}{4}\left(\frac{-1}{s+1} - \frac{2}{(s+1)^2} + \frac{1}{s-1}\right)e^{-s+1}\right)$$

$$= -e^{-t} - 2e^{-t}t + e^{t} + e\,u(t-1)\left(e^{-(t-1)} + 2e^{-(t-1)}(t-1) - e^{t-1}\right)$$

$$= e^{t} - e^{-t} - 2e^{-t} + u(t-1)\left(-e^{-t+2} + 2e^{-t+2}t - e^{t-1}\right)$$

2) $y'' + y = \sin(3t) + 2\delta(t - \pi/2)$

$y(0) = 1$

$y'(0) = -1$

$$s^2 \mathcal{L}(y) + 1 - s + \mathcal{L}(y) = \frac{3}{s^2+9} + 2e^{-\pi/2\, s}$$

$$(s^2+1)\mathcal{L}_y = s - 1 + \frac{3}{s^2+9} + 2e^{-\frac{\pi}{2} s}$$

$$\mathcal{L}y = \frac{s}{s^2+1} - \frac{1}{s^2+1} + \frac{3}{(s^2+1)(s^2+9)} + \frac{2}{s^2+1} e^{-\frac{\pi}{2} s}$$

$$\frac{1}{(s^2+1)(s^2+9)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9} = \frac{1}{8}\left(\frac{1}{s^2+1} - \frac{1}{s^2+9}\right)$$

$$(As+B)(s^2+9) + (Cs+D)(s^2+1) = 1$$

$$\cancel{As^3} + \cancel{9As} + \cancel{Bs^2} + 9B + \cancel{Cs^3} + \cancel{Cs} + \cancel{Ds^2} + D = 1$$

$$A + C = 0 \qquad 9A + C = 0$$

$$B + D = 0 \qquad 9B + D = 1$$

$$8B = 1 \qquad B = \frac{1}{8} \qquad D = -1/8$$

$$A = 0$$

$$C = 0$$

$$y = \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) - \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) + \mathcal{L}^{-1}\left(\frac{3}{(s^2+1)(s^2+9)}\right) + \mathcal{L}^{-1}\left(\frac{2}{s^2+1} e^{-\frac{\pi}{2}s}\right)$$

$$= \cos t - \operatorname{sen} t + \frac{3}{8}\mathcal{L}^{-1}\left(\frac{1}{s^2+1} - \frac{1}{s^2+9}\right) + 2u(t-\pi/2)\operatorname{sen}(t-\pi/2)$$

$$= \cos t - \operatorname{sen} t + \frac{3}{8}\left(\operatorname{sen} t - \frac{\operatorname{sen} 3t}{3}\right) + 2u(t-\pi/2)\operatorname{sen}(t-\pi/2)$$

$$= \cos t - \frac{5}{8}\operatorname{sen} t - \frac{1}{8}\operatorname{sen} 3t + 2u(t-\pi/2)\operatorname{sen}(t-\pi/2)$$