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Trabalho 6 - C.IV

a)

$$y'' + 36y = \operatorname{sen}(6x)(\sec(6x))^2$$

Solução homogênea:

$$y'' + 36y = 0$$
$$r^2 + 36 = 0$$
$$r = \pm 6i$$
$$y_h = C_1 e^0 \cos(6x) + C_2 e^0 \operatorname{sen}(6x)$$
$$\therefore y_h = C_1 \cos(6x) + C_2 \operatorname{sen}(6x)$$

Assim, calculando o Wronskiano:

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos(6x) & \operatorname{sen}(6x) \\ -6\operatorname{sen}(6x) & 6\cos(6x) \end{vmatrix} \begin{array}{l} 6\cos^2(6x) - [-6\operatorname{sen}^2(6x)] \\ = \underline{\underline{6}} \neq 0 \end{array}$$

Pela variação de parâmetros:

$$P_0(x)y'' + P_1(x)y' + P_2(x)y = F(x)$$

Propor:

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p' = u_1' y_1 + u_1 y_1' + u_2' y_2 + u_2 y_2'$$

$$u_1' y_1 + u_2' y_2 = 0 \Rightarrow \text{Supondo a segunda relação}$$

Assim:

$$y_p' = u_1 y_1' + u_2 y_2'$$

Derivando novamente:

$$y_p'' = u_1' y_1' + u_1 y_1'' + u_2' y_2' + u_2 y_2''$$

Substituindo na EDO:

$$y'' + 36y = \operatorname{sen}(6x)(\sec(6x))^2$$

$$[u_1'y_1' + u_1y_1'' + u_2'y_2' + u_2y_2''] + 6(u_1y_1 + u_2y_2) = \operatorname{sen}(6x)\sec^2(6x)$$

$$u_1(\underbrace{y_1'' + 6y_1}_{=0}) + u_2(\underbrace{y_2'' + 6y_2}_{=0}) + (u_1'y_1' + u_2'y_2') = \operatorname{sen}(6x)\sec^2(6x)$$

Assim:

$$(u_1'y_1' + u_2'y_2') = \operatorname{sen}(6x)\sec^2(6x)$$

Logo:

$$\begin{cases} \text{I} \quad u_1'y_1' + u_2'y_2' = \operatorname{sen}(6x)\sec^2(6x) & \times(y_2) \\ \text{I} \quad u_1'y_1 + u_2'y_2 = 0 & \times(y_2') \end{cases}$$

$$u_1'y_1y_2' + u_2'y_2y_2' = 0$$

$$- \quad u_1'y_1'y_2 + u_2'y_2'y_2 = y_2\operatorname{sen}(6x)\sec^2(6x)$$

$$u_1'(y_1y_2' - y_1'y_2) = -y_2\operatorname{sen}(6x)\sec^2(6x)$$

$$y_1y_2' - y_1'y_2 = W(y_1,y_2) = 6$$

Assim:

$$u_1' = -\frac{y_2\operatorname{sen}(6x)\sec^2(6x)}{6}; \quad u_2' = \frac{y_1\operatorname{sen}(6x)\sec^2(6x)}{6}$$

Como:

$$y_1 = \cos(6x); \quad y_2 = \operatorname{sen}(6x)$$

Temos

$$u_1' = \frac{-[\operatorname{sen}^2(6x)\sec^2(6x)]}{6} \Rightarrow u_1 = \int u_1'\,dx \Rightarrow u_1 = -\int \frac{\operatorname{sen}^2(6x)\sec^2(6x)}{6}dx$$

$$\Rightarrow u_1 = -\tfrac{1}{6}\int \operatorname{sen}^2(6x)\sec^2(6x)\,dx \Rightarrow \begin{matrix} u=6x \\ du/6 = dx \end{matrix} \Rightarrow -\tfrac{1}{36}\int \operatorname{sen}^2(u)\cdot\sec^2(u)\,du$$

$$\Rightarrow \operatorname{sen}^2(u)\cdot\sec^2(u) = tg^2(u) \Rightarrow -\tfrac{1}{36}\int \tan^2(u)\,du \Rightarrow Tg^2(u) = \sec^2(u) - 1 \Rightarrow -\tfrac{1}{36}\int \ldots$$

$$\ldots(\sec^2(u)-1)\,du \Rightarrow -\tfrac{1}{36}\left[\int \sec^2(u)\,du - \int 1\,du\right] \Rightarrow -\tfrac{1}{36}[Tg(u) - u] \Rightarrow u = 6x$$

$$\Rightarrow -\tfrac{1}{36}[tg(6x) - 6x] = -\frac{Tan(6x)}{36} + \frac{x}{6} + \cancel{C}_{\,0}$$

$$\therefore u_1 = -\frac{Tan(6x)}{36} + \frac{x}{6}$$

Por outro lado:

$$u_2' = \frac{\cos(6x)\operatorname{sen}(6x)\sec^2(6x)}{6} \Rightarrow u_2 = \int u_2'\,dx \Rightarrow 1/6 \int \cos(6x)\operatorname{sen}(6x)\sec^2(6x)\,dx$$

$\Rightarrow \sec(6x) = 1/\cos(6x) \Rightarrow 1/6 \int \operatorname{sen}(6x)\cos(6x) \cdot \frac{1}{\cos^2(6x)}\,dx \Rightarrow 1/6 \int \operatorname{sen}(6x) \cdot \sec(6x)\,dx$

$\Rightarrow \operatorname{sen}(6x) \cdot \sec(6x) = tg(6x) \Rightarrow 1/6 \int tg(6x)\,dx \Rightarrow u = 6x ; du/6 = dx \Rightarrow 1/6 \int tg(u)\,du/6$

$\Rightarrow tg(u) = \operatorname{sen}(u)/\cos(u) \Rightarrow 1/36 \int \operatorname{sen}(u)/\cos(u)\,du \Rightarrow a = \cos u ; da = -\operatorname{sen} u\,du$

$\Rightarrow -1/36 \int 1/a\,da \Rightarrow -1/36 [\ln|a|] \Rightarrow -1/36 [\ln|\cos(u)|]$

$$\therefore u_2 = -\frac{\ln|\cos(6x)|}{36}$$

Assim, a solução particular será:

$$\therefore y_p = \cos(6x) \cdot \left[-\frac{\tan(6x)}{36} + \frac{x}{6}\right] + \operatorname{sen}(6x)\left[-\frac{\ln|\cos(6x)|}{36}\right]$$

b) Verificando:

$$y_p' = -\frac{\cos(6x)\ln(\cos(6x))}{6} - 6\operatorname{sen}(6x)\left(\frac{x}{6} - \frac{\tan(6x)}{36}\right) + \frac{\operatorname{sen}^2(6x)}{6\cos(6x)} + \cos(6x)\ldots$$

$$\ldots\left(\frac{1}{6} - \frac{\sec^2(6x)}{6}\right)$$

$$y_p'' = \operatorname{sen}(6x)\ln|\cos(6x)| - 2\cos(6x)\sec^2(6x)\tan(6x) + \cos(6x)\tan(6x) + \ldots$$

$$\ldots + \frac{\operatorname{sen}^3(6x)}{\cos^2(6x)} + 2\sec^2(6x) + \operatorname{sen}(6x) - 6x\cos(6x)$$

Substituindo na EDO:

$$y'' + 36y = \operatorname{sen}(6x)\sec^2(6x)$$

$$[\operatorname{sen}(6x)\ln|\cos(6x)| - 2\cos(6x)\sec^2(6x)\tan(6x) + \cancel{\cos(6x)\tan(6x)} + \operatorname{sen}^3(6x)/\cos^2(6x) + \ldots$$

$$\ldots + 2\sec^2(6x)\operatorname{sen}(6x) + \operatorname{sen}(6x) - 6x\cancel{\cos(6x)}] + \cancel{36}\left([-\frac{\cancel{\cos(6x)\tan(6x)}}{\cancel{36}} + \frac{x\cos(6x)}{\cancel{6}} - \ldots\right.$$

$$\left.\ldots - \frac{\operatorname{sen}(6x)\ln|\cos(6x)|}{\cancel{36}}\right) =$$

$$\frac{-2\cos(6x)\sec^2(6x)\tan(6x)+\sin^2(6x)+2\sec^2(6x)\sin(6x)+}{\cos^2(6x)}$$

$\sin(6x) =$

$$-2\cancel{\cos(6x)}\frac{\sin(6x)}{\cancel{\cos(6x)}}\sec^2(6x)+2\sin(6x)\sec^2(6x)+\frac{\sin^3(6x)}{\cos^2(6x)}+\sin(6x)$$

$$\Rightarrow \frac{\sin^2(6x)}{\cos^2(6x)}\cdot\sin(6x)+\sin(6x)=\tan(6x)\cdot\sin(6x)+\sin(6x)$$

Como: $\tan^2(6x)+1=\sec^2(6x)$

$$(\sec^2(6x)-1)\sin(6x)+\sin(6x)=\sin(6x)\sec^2(6x)-\cancel{\sin(6x)}+\cancel{\sin(6x)}$$

$$=\sin(6x)\sec^2(6x)$$

$\therefore$ sol. geral $y=y_p+C_1y_1+C_2y_2$

c)

PVI:

$$\begin{cases} y(6)=5 \\ y'(6)=6 \end{cases}$$

(I) $y'=-6C_1\sin(6x)+6C_2\cos(6x)+\dfrac{\sin(6x)\tan(6x)}{6}+\dfrac{\sin^3(6x)}{6\cos(6x)}-x\sin(6x)-\dots$

$\dots-\dfrac{\cos(6x)\sec^2(6x)}{6}+\dfrac{\cos(6x)}{6}-\dfrac{\cos(6x)\ln|\cos(6x)|}{6}$

(I) $y=C_1\cos(6x)+C_2\sin(6x)-\dfrac{\cos(6x)\tan(6x)}{36}+\dfrac{x\cos(6x)}{6}-\dfrac{\sin(6x)\ln|\cos(6x)|}{36}$

Substituindo os valores:

(I): $5=C_1\cos(36)+C_2\sin(36)-\underbrace{\dfrac{\cos(36)\tan(36)}{36}}_{+0,027}+\underbrace{\dfrac{6\cos(36)}{6}}_{-0,127}-\underbrace{\dfrac{\sin(36)\ln|\cos(36)|}{36}}_{-0,0366}$

$5=C_1\cos(36)+C_2\sin(36)-1,1476$

(II): $6=-6C_1\sin(36)+6C_2\cos(36)+\underbrace{\dfrac{\sin(36)\tan(36)}{6}}_{-1,281}+\underbrace{\dfrac{\sin^3(36)}{6\cos(36)}}_{-1,381}-\underbrace{6\sin(36)}_{5,960}$

$-\underbrace{\dfrac{\cos(36)\sec^2(36)}{6}}_{1,303}+\underbrace{\dfrac{\cos(36)}{6}}_{-0,021}-\underbrace{\dfrac{\cos(36)\ln|\cos(36)|}{6}}_{0,043}\simeq 4,712$

Assim:

$$\begin{cases} \text{I} \quad 5 = C_1 \cos(36) + C_2 \operatorname{sen}(36) - 1{,}1476 \\ \text{II} \quad 6 = -6C_1 \operatorname{sen}(36) + 6C_2 \cos(36) + 4{,}712 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} 5 = -0{,}127\,C_1 - 0{,}991\,C_2 - 1{,}1476 \\ 6 = 6C_1\,0{,}991 + 6C_2\,0{,}127 + 4{,}712 \end{cases} \Rightarrow \begin{cases} 6{,}1476 = C_1\,0{,}127 - C_2\,0{,}991 \times(-5{,}946) \\ 1{,}288 = 5{,}946\,C_1 - 0{,}762\,C_2 \times(0{,}127) \end{cases}$$

$$\begin{cases} \text{I} \quad -36{,}553 = -0{,}755\,C_1 + 5{,}892\,C_2 \\ \text{II} \quad 0{,}163 = 0{,}755\,C_1 - 0{,}096\,C_2 \end{cases} +$$

$$-36{,}39 = 5{,}796\,C_2$$

$$C_2 = -36{,}39/5{,}796$$

$$\therefore C_2 \approx -6{,}278$$

II: $0{,}163 = 0{,}755\,C_1 - 0{,}096[-6{,}278]$

$$C_1 = -0{,}434/0{,}755$$

$$C_1 \approx -0{,}582$$

Logo:

$$y = -0{,}582\cos(6x) - 6{,}278\operatorname{sen}(6x) - \frac{\cos(6x)\tan(6x)}{36} + \frac{x\cos(6x)}{6} - \frac{\operatorname{sen}(6x)\ln|\cos(6x)|}{36}$$

GeoGebra Calculadora Gráfica ENTRAR...

$$f : y = -0.582 \cos(6x) - 6.278 \operatorname{sen}(6x) - \frac{\cos(6x) \operatorname{tg}(6x)}{36} + \frac{x \cos(6x)}{6} - \frac{\operatorname{sen}(6x) \ln(\cos(6x))}{36}$$

Entrada...