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Lista 8 - C IV

a)

$$x^2(1+2x)y''+x(3+5x)y'+(1-2x)y=0$$

$A(x) = (1+2x)$; $\alpha_0 = 1$; $\alpha_1 = 2$; $\alpha_2 = 0$

$B(x) = (3+5x)$; $\beta_0 = 3$; $\beta_1 = 5$; $\beta_2 = 0$

$C(x) = (1-2x)$; $\gamma_0 = 1$; $\gamma_1 = -2$; $\gamma_2 = 0$

$$P_0(r) = r(r-1)+3r+1 = r^2+2r+1$$
$$P_0(r) = (r+1)^2$$

$$r = \frac{-2 \pm \sqrt{2^2-4(1)(1)}}{2}$$

$$r = \frac{-2 \pm 0}{2} \quad r_{1,2} = -1 \Rightarrow \text{Caso } 2,$$

$$P_1(r) = 2r(r-1)+5r-2 = 2r^2+3r-2$$
$$P_1(r) = (r-1/2)(r+2).$$

$$r = \frac{-3 \pm \sqrt{3^2-4(2)(-2)}}{2(2)}$$

$$r = \frac{-3 \pm \sqrt{25}}{4}$$

$$r_1 = \frac{-3+5}{4} = 1/2$$

$$r_2 = \frac{-3-5}{4} = -2$$

Assim, para o Caso 2:

$$y_1(x) = x^{r_1}\sum_{n=0}^{\infty} a_n(r_1)x^n$$

$$y_2(x) = y_1 \ln x + x^{r_1}\sum_{n=1}^{\infty} a_n'(r_1)x^n$$

Para $r = -1$:

$$a_0(-1) = 1 \; ; \; a_1(-1) = -\frac{P_1(r)}{P_0(r+1)} = -\frac{(-3/2)}{1} = 3/2$$

$P_0(r+1) = (0+1)^2 = 1$

$P_1(-1) = (-1-1/2)(-1+2)$

$= (-3/2)(1)$

$$a_n(r) = -\frac{(P_1(m+r-1)a_{m-1}(r) + P_0(r \;\;))}{P_0(m+r)}$$

$$a_n(r) = -\frac{(P_1(n-2)a_{n-1})}{P_0(n-1)}$$

$P_1(n) = (n-1/2)(n+2)$ $\quad$ $P_0(n) = (n+1)^2$

$P_1(n-2) = (n-2-1/2)(n-2+2)$ $\quad$ $P_0(n-1) = (n-1+1)^2$

$= n(n-5/2)$ $\quad$ $= n^2$

Assim,

$$a_n = -\frac{(n(n-5/2))a_{n-1}}{n^2} \Rightarrow -\frac{(n-5/2)a_{n-1}}{n} \quad \text{para } n \geq 1$$

Logo, $Y_1(x)$ será:

$$Y_1(x) = x^{r_1} \sum_{n=0}^{\infty} a_n(r_1) x^n$$

Para $n = 1$

$$a_1 = -\frac{(1-5/2)a_0}{1} = +3/2$$

para $n = 2$

$$a_2 = -\frac{(2-5/2)a_1}{2} \Rightarrow a_2 = -\frac{(-1/2)a_1}{2} \Rightarrow \frac{(1/2)(3/2)}{2} \Rightarrow 3/8$$

Para $n = 3$

$$a_3 = -\frac{(3-5/2)a_2}{3} = -\frac{1/2\, a_2}{3} = -1/6(3/8) = -1/16$$

Logo:

$$Y_1 = x^{-1}\left(1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3 + \dots\right)$$

Por outro lado, $Y_2(x)$ será:

$$\frac{\partial Y_1}{\partial r} = Y_1 \ln x + x^{r_1} \sum_{n=1}^{\infty} a_n'(r_1) x^n$$

$$a_n(r) = -\frac{\left((n+r-3/2)a_{n-1}\right)}{(n+r+1)}$$

Derivando...

$$\frac{\partial a_n(r)}{\partial r} = -\left[\frac{\left(a_{n-1} + (n+r-3/2)a'_{n-1}\right)}{(n+r+1)} - \frac{(n+r-3/2)a_{n-1}}{(n+r+1)^2}\right] = a'_n(r)$$

$$a'_0 = 0 \quad ; \quad a'_1(r) = \left(\frac{1}{r+2} - \frac{r-1/2}{(r+2)^2}\right)$$

$$a'_1(-1) = \left(\frac{1}{-1+2} - \frac{(-1)-1/2}{(-1+2)^2}\right) = 1 - (-3/2) = 1+\frac{3}{2} = 5/2$$

Para $n=2$

$$a'_2(-1) = -\left[\frac{\left(a_1 + (2-1-3/2)a'_1\right)}{(2-1+1)} - \frac{(2-1-3/2)a_1}{(2-1+1)^2}\right]$$

$$= -\left[\frac{a_1 + (-1/2)a'_1}{2} - \frac{(-1/2)a_1}{4}\right] \Rightarrow -\left[\frac{(3/2)+(-1/2)(5/2)}{2} + \frac{(1/2)(3/2)}{4}\right]$$

$$a'_2(-1) = -5/16$$

$n=3$

$$a'_3(-1) = -\left[\frac{a_2 + (3-1-3/2)a'_2}{(3-1+1)} - \frac{(3-1-3/2)a_2}{(3-1+1)^2}\right]$$

$$a'_3(-1) = -\left[\frac{(3/8)+(1/2)(-5/16)}{3} - \frac{(1/2)(3/8)}{9}\right] = -5/96$$

para $n=4$

$$a'_4(-1) = -\left[\frac{a_3 + (4-1-3/2)a'_3}{(4-1+1)} - \frac{(4-1-3/2)a_3}{(4-1+1)^2}\right]$$

$$= -\left[\frac{-1/16 + (3/2)(-5/96)}{4} - \frac{(3/2)(-1/16)}{16}\right] = \frac{15}{512}$$

Assim:

$$y_2 = y_1 \ln x + x^{-1}\left[\frac{5}{2}x - \frac{5}{16}x^2 - \frac{5}{96}x^3 + \frac{15}{512}x^4 + \dots\right]$$

$$6x^2y'' + x(10-x)y' - (2+x)y = 0$$

freedeal

$A(x) = 6$ $\qquad \alpha_0 = 6 \; ; \; \alpha_1 = 0 \; ; \; \alpha_2 = 0$

$B(x) = (10 - x)$ $\qquad \beta_0 = 10 \; ; \; \beta_1 = -1 \; ; \; \beta_2 = 0$

$C(x) = (-2 - x)$ $\qquad \gamma_0 = -2 \; ; \; \gamma_1 = -1 \; ; \; \gamma_0 = 0$

Assim, pelo polinômio indicial:

$$P_0(n) = 6n(n-1) + 10n - 2$$
$$= 6n^2 - 6n + 10 - 2$$
$$P_0(n) = 6n^2 + 4n - 2$$

$$n = \frac{-4 \pm \sqrt{4^2 - 4(6)(-2)}}{2(6)}$$

$$n = \frac{-4 \pm \sqrt{16+48}}{12} \Rightarrow n = \frac{-4 \pm 8}{12} \Rightarrow n_1 = \frac{-4+8}{12} = \frac{4}{12} = \frac{1}{3} \; ; \; n_2 = \frac{-4-8}{12} = -1$$

Para $n_1 > n_2$

$$\frac{1}{3} - (-1) = \frac{4}{3} \Rightarrow \text{Caso } 1$$

Assim:

$$P_0(n) = (n - 1/3)(n+1)$$
$$P_1(n) = -(n+1)$$
$$P_2(n) = 0$$

Para $n_2 = -1$

$$a_0(-1) = 1 \; ; \; a_1(-1) = -\frac{P_1(n)}{P_0(n+1)}$$

$P_1(-1) = -\cancel{(-1+1)}^{\,0}$ | $P_0(n+1) = (n+1-1/3)(n+1+1)$ | $a_1(-1) = -\frac{0}{\cancel{-1/3}} = 0$

$P_1(-1) = 0$ | $= (-1+1-1/3)(-1+1+1)$

$= (-1/3)(1)$

$$a_n(n) = -\frac{(P_1(m+n-1)a_{n-1} + \cancel{P_2(\dots)}^{\,0})}{P_0(m+n)}$$

$P_1(m+n-1) = -(m+n-1+1)$ | $P_0(m+n) = (m+n-1/3)(m+n+1)$

$P_1(m+n-1) = -(n-1)$ | $P_0(m+n) = n(n-4/3)$

Assim:

$$a_n(n) = -\frac{(-(n-1))a_{n-1}}{n(n-4/3)} \Rightarrow a_n(n) = +\frac{(n-1)a_{n-1}}{n(n-4/3)} \quad \text{para } n \geq 1$$

$$n=2 \Rightarrow a_2 = \frac{(2-1)a_1}{2(2-4/3)} = 0 \Rightarrow p/\; n = 1, 2, 3, \dots \; a_n(n) = 0.$$

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Portanto, a solução y_1 será:

$$y_1 = x^{-1}(1 + 0x + 0x^2 + 0x^3 + \dots)$$
$$y_1 = x^{-1}$$

para $r_2 = 1/3$

$$a_0(1/3) = 1 \quad ; \quad a_1(1/3) = -\frac{P_1(r)}{P_0(r+1)}$$

$P_1(r) = -(r+1)$
$= -(1/3 + 1)$
$= -4/3$

$P_0(r+1) = (r+1-1/3)(r+1+1)$
$= (1/3+1-1/3)(1/3+2)$
$P_0(r+1) = (7/3)$

Assim

$$a_1(1/3) = -\frac{(-4/3)}{7/3} = +4/7$$

$P_1(n+r-1) = -(n+r-1+1)$
$= -(n+1/3)$

$$a_n(r) = -\frac{(P_1(n+r-1)\,a_{n-1})}{P_0(n+r)}$$

$P_0(n+r) = (n+r-1/3)(n+r+1)$
$= (n+1/3-1/3)(n+1/3+1)$
$P_0(n+r) = (n)(n+4/3)$

$$a_n(r) = -\frac{(-(n+1/3))\,a_{n-1}}{n(n+4/3)}$$

$$a_n(r) = +\frac{(n+1/3)\,a_{n-1}}{n(n+4/3)} \quad p/\, n \geq 1$$

$$a_1 = \frac{(1+1/3)\,a_0}{1(1+4/3)} = \frac{4/3}{7/3} = 4/7$$

$$n=2 \Rightarrow a_2 = \frac{(2+1/3)\,a_1}{2(2+4/3)} = \frac{(7/3)\,a_1}{2(10/3)} = 1/5$$

$$n=3 \Rightarrow a_3 = \frac{(3+1/3)\,a_2}{3(3+4/3)} = 2/39$$

$$\therefore y_2 = x^{1/3}(1 + 4/7\,x + 1/5\,x^2 + 2/39\,x^3 + \dots)$$

Logo, a solução geral será:

$$y = C_1 y_1 + C_2 y_2$$

$$\therefore y(x) = C_1 x^{-1} + C_2 x^{1/3}(1 + 4/7\,x + 1/5\,x^2 + 2/39\,x^3 + \dots)$$