Carlos Luilguer Almeida Santos
20150465

Trabalho 4 - C IV

a)

$$(b-x^2)y''-2xy'=0$$
$$y_1=a$$

$y_1=6$
$b=7$

$$(7-x^2)y''-2xy'=0$$
$$y_1=6$$

$$y''-\frac{2x}{(7-x^2)}y'=0$$

$$y''=\frac{2x}{(7-x^2)}y'$$

Substituição $y'=v$

$$v'=\frac{2x}{(7-x^2)}v \Rightarrow \text{Isto vira redução de ordem EDO de 1º ordem}$$

$$\frac{v'}{v}=\frac{2x}{(7-x^2)}$$

$u=7-x^2$
$du=-2x\,dx$

$$\frac{1}{v}\frac{dv}{dx}=\frac{2x}{(7-x^2)}\Rightarrow\int\frac{dv}{v}=\int\frac{2x}{(7-x^2)}dx$$

$$\Rightarrow \ln|v|=-\ln|7-x^2|+C\Rightarrow e^{\ln|v|}=e^{\ln|(7-x^2)^{-1}|+C}\Rightarrow v=(7-x^2)^{-1}\cdot e^C$$

$$\Rightarrow v=\frac{K}{(7-x^2)}\Rightarrow v=y'\Rightarrow y'=\frac{K}{(7-x^2)}\Rightarrow\int y'\,dx\Rightarrow\int\frac{K}{(7-x^2)}dx$$

$$\Rightarrow -K\int\frac{1}{7-x^2}dx\Rightarrow -K\int\frac{1}{(x-\sqrt{7})(x+\sqrt{7})}dx$$

$$\frac{1}{(x-\sqrt{7})(x+\sqrt{7})}=\frac{A}{(x-\sqrt{7})}+\frac{B}{(x+\sqrt{7})}\Rightarrow\frac{A(x+\sqrt{7})+B(x-\sqrt{7})}{(x-\sqrt{7})(x+\sqrt{7})}$$

$\Rightarrow 1 = A(x+\sqrt{7}) + B(x-\sqrt{7})$

$x = -\sqrt{7}$, $x = +\sqrt{7}$

$1 = A(\sqrt{7}+\sqrt{7}) + 0$

$1 = 2\sqrt{7}A$

$A = 1/2\sqrt{7}$

$1 = 0 + B(-\sqrt{7}-\sqrt{7})$

$B = -1/2\sqrt{7}$

$$\frac{1}{(x-\sqrt{7})(x+\sqrt{7})} \quad \frac{1}{2\sqrt{7}(x-\sqrt{7})} - \frac{1}{2\sqrt{7}(x+\sqrt{7})}$$

$$K\int\left(\frac{1}{2\sqrt{7}(x-\sqrt{7})} - \frac{1}{2\sqrt{7}(x+\sqrt{7})}\right)dx \Rightarrow K\left[\frac{1}{2\sqrt{7}}\int\frac{1}{(x-\sqrt{7})}dx - \frac{1}{2\sqrt{7}}\int\frac{1}{(x+\sqrt{7})}dx\right]$$

$u = x-\sqrt{7}$ | $u = x+\sqrt{7}$

$du = dx$ | $du = dx$

$$K\left[\frac{1}{2\sqrt{7}}\ln|x-\sqrt{7}| - \frac{1}{2\sqrt{7}}\ln|x+\sqrt{7}|\right] + C$$

$$y = C_1\left[\frac{\ln|x+\sqrt{7}|}{2\sqrt{7}} - \frac{\ln|x-\sqrt{7}|}{2\sqrt{7}}\right] + C_2$$

$$\therefore y = C_1\frac{(\ln|x+\sqrt{7}|) - \ln(|x-\sqrt{7}|)}{2\sqrt{7}}$$

b)

$$y' = C_1\left[\frac{1}{2\sqrt{7}(x+\sqrt{7})} - \frac{1}{2\sqrt{7}(x-\sqrt{7})}\right] + 0 \Rightarrow y' = \frac{-C_1}{x^2-7}$$

$$y'' = C_1[x^2-7]^{-1} \Rightarrow -1C_1[x^2-7]^{-2}(2x) \Rightarrow \frac{2xC_1}{(x^2-7)^2}$$

$$y'' = \frac{2x}{(7-x^2)}y'$$

$$+\frac{2xC_1}{(x^2-7)^2} = \frac{2x}{(7-x^2)}\left(\frac{+C_1}{x^2-7}\right) \Rightarrow 0 ✓$$

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c) $y(8) = 6 \;;\; y'(8) = 8$

$$y = C_1 \left[\frac{\ln|x+\sqrt{7}|}{2\sqrt{7}} - \frac{\ln|x-\sqrt{7}|}{2\sqrt{7}} \right] + C_2$$

$$y' = \frac{+C_1}{x^2-7}$$

$$\begin{cases} \text{I} \quad 6 = \dfrac{C_1}{2\sqrt{7}} \left[\ln|8+\sqrt{7}| - \ln|8-\sqrt{7}| \right] + C_2 \\ \text{II} \quad 8 = \dfrac{C_1}{8^2-7} \rightarrow 57(8) = C_1 \Rightarrow \boxed{C_1 = 456} \end{cases}$$

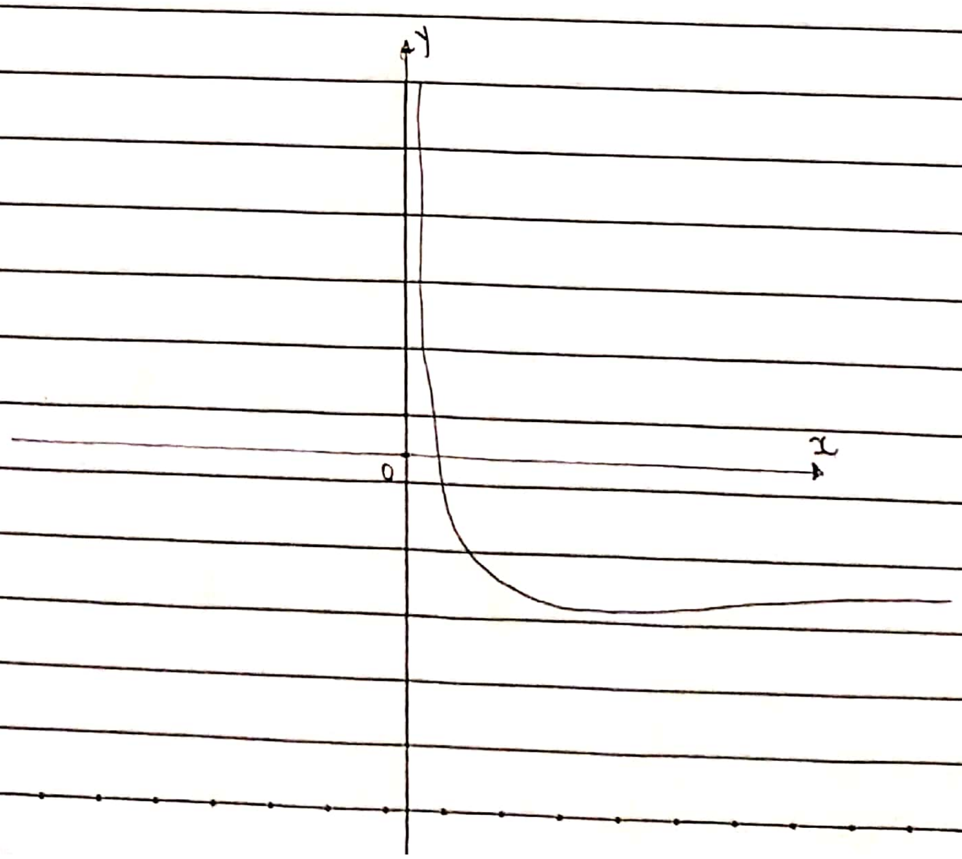
I

$$6 = \frac{[456]}{2\sqrt{7}} \left[0{,}687 \right] + C_2 \Rightarrow C_2 = 6 - \frac{[456]}{2\sqrt{7}} \left[0{,}687 \right]$$

$$\boxed{C_2 \cong -53{,}20}$$

$$\therefore y = [456] \left[\frac{\ln|x+\sqrt{7}|}{2\sqrt{7}} - \frac{\ln|x-\sqrt{7}|}{2\sqrt{7}} \right] - 53{,}20$$

d)



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