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Trabalho 9 - CIV

a)

$$y'' + 4y = 4 \; ; \; y(0) = 5 \; ; \; y'(0) = 6$$

. Aplicando a transformada:

$$\mathcal{L}\{y'' + 4y\} = \mathcal{L}\{4\}$$

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y\} = 4\mathcal{L}\{1\}$$

Pela Transformada da derivada:

$$s^2\mathcal{L}\{y\} - s\,y(0) - y'(0) + 4\mathcal{L}\{y\} = 4\mathcal{L}\{1\}$$

$$s^2 Y(s) - 5s - 6 + 4Y(s) = 4 \cdot \frac{1}{s}$$

$$s^2 Y(s) + 4Y(s) = \frac{4}{s} + 5s + 6$$

$$Y(s)(s^2+4) = \frac{4}{s} + 5s + 6$$

$$Y(s) = \frac{4}{s(s^2+4)} + \frac{5s}{(s^2+4)} + \frac{6}{(s^2+4)}$$

Assim, pela Transformada inversa:

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{4}{s(s^2+2^2)} + \frac{5s}{(s^2+2^2)} + \frac{6}{(s^2+2^2)}\right\}$$

Assim, com a tabela:

$$\mathcal{L}^{-1}\left\{\frac{4}{s(s^2+2^2)}\right\} = \frac{1}{s(s^2+w^2)} = \frac{1}{w^2}(1-\cos wt) \Rightarrow \frac{4}{2^2}(1-\cos 2t)$$

$$\mathcal{L}^{-1}\left\{\frac{5s}{(s^2+2^2)}\right\} = \frac{s}{s^2+w^2} = \cos wt \Rightarrow 5\cos 2t$$

$$\mathcal{L}^{-1}\left\{\frac{6}{(s^2+2^2)}\right\} = \frac{w}{s^2+w^2} = \frac{1}{w}\operatorname{sen} wt \Rightarrow 3 \cdot \frac{1}{2}\operatorname{sen} 2t$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

$$y(t) = (1-\cos 2t) + 5\cos 2t + 3\operatorname{sen} 2t$$

$$\therefore y(t) = 4\cos 2t + 3\operatorname{sen} 2t + 1$$

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Verificando:

$$y(t) = 4\cos 2t + 3\,\text{sen}\,2t + 1$$
$$y''(t) = -12\,\text{sen}\,2t - 16\cos 2t$$

Substituindo na EDO:

$$y'' + 4y = 4$$
$$[-12\,\text{sen}\,2t - 16\cos 2t] + 4[4\cos 2t + 3\,\text{sen}\,2t + 1] = 4$$
$$-\cancel{12\,\text{sen}\,2t} - \cancel{16\cos 2t} + \cancel{16\cos 2t} + \cancel{12\,\text{sen}\,2t} + 4 = 4$$
$$4 = 4$$

$\therefore$ $y(t) = 4\cos 2t + 3\,\text{sen}\,2t + 1$ é sol. do P.V.I.