Solução da Lista 1 de Cálculo IV

1) $$x^2 \frac{dy}{dx} = y - xy$$

$$x^2 \frac{dy}{dx} = y(1-x)$$

$$\frac{dy}{y} = \frac{(1-x)}{x^2}\,dx$$

$$\int \frac{dy}{y} = \int \frac{(1-x)}{x^2}\,dx$$

$$\ln|y| = \int \frac{dx}{x^2} - \int \frac{dx}{x}$$

$$\ln|y| = -\frac{1}{x} - \ln|x| + C^*$$

$$|y| = e^{-1/x}\, e^{-\ln|x|}\, e^{C^*}$$

$$\boxed{y = \frac{c}{x} e^{-1/x}}$$ Sol. geral

$c = \pm e^{c^*}$

Note que $y=0$ também é solução.

Verificação:

$$\frac{dy}{dx} = -\frac{c}{x^2} e^{-1/x} + \frac{c}{x^3} e^{-1/x}$$

$$=$$

Na EDO

$$x^2 e^{-1/x}\left(\frac{1}{x^3} - \frac{1}{x^2}\right) = \frac{c}{x} e^{-1/x} - x\frac{c}{x} e^{-1/x}$$

$$c e^{-1/x}\left(\frac{1}{x} - 1\right) = c e^{-1/x}\left(\frac{1}{x} - 1\right)$$

ok

O PVI : $y(-1) = -1$

$$y = \frac{c}{x} e^{-1/x} \quad \left\{ \quad -1 = \frac{c}{-1} e^{-1/-1} \right.$$

$$c = e^{-1}$$

2) $x\ln(y)\frac{dy}{dx} = \left(\frac{x+1}{y}\right)^2$

$$y^2\ln(y)\,dy = \frac{(x+1)^2}{x}\,dx$$

$$\int y^2\ln(y)\,dy = \int\frac{(x+1)^2}{x}\,dx$$

$$\int\frac{(x+1)^2}{x}\,dx = \int\left(\frac{x^2+2x+1}{x}\right)dx = \frac{x^2}{2} + 2x + \ln|x| + C$$

$$\int y^2\ln(y)\,dy = I \qquad u = \ln(y) \quad du = \frac{1}{y}\,dy$$

$$dv = y^2\,dy \qquad v = \frac{y^3}{3}$$

$$I = \frac{y^3}{3}\ln(y) - \int \frac{y^3}{3y}\,dy$$

$$= \frac{y^3}{3}\ln(y) - \frac{1}{3}\frac{y^3}{3} + d$$

$$\boxed{\frac{y^3}{3}\ln(y) - \frac{y^3}{9} = \frac{x^2}{2} + 2x + \ln|x| + c}$$

Verificação: Derivando ambos os lados:

$$\frac{\cancel{3}y^2}{\cancel{3}}\ln(y)y' + \frac{y^3}{3y}y' - \frac{3y^2}{9}y' = \frac{2x}{2} + 2 + \frac{1}{x}$$

$$\left(y^2\ln(y) + \cancel{\frac{y^2}{3} - \frac{y^2}{3}}\right)y' = x + 2 + \frac{1}{x}$$

$$y^2\ln(y)\,y' = \frac{x^2+2x+1}{x}$$

$$x\ln(y)\frac{dy}{dx} = \left(\frac{x+1}{y}\right)^2$$

OK