Solução da Lista 11 de Cálculo IV

1) $f(t) = \begin{cases} 0, & -\pi < t < 0 \\ \pi & 0 < t < \pi \end{cases}$

Se $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + b_n \operatorname{sen} nt$

$\rightarrow a_0 = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(t)\, dt \quad a_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(t) \cos nt\, dt \quad b_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(t) \operatorname{sen} nt\, dt$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\cancel{\pi}}\int_0^{\pi} \cancel{\pi}\,dt = \pi$$

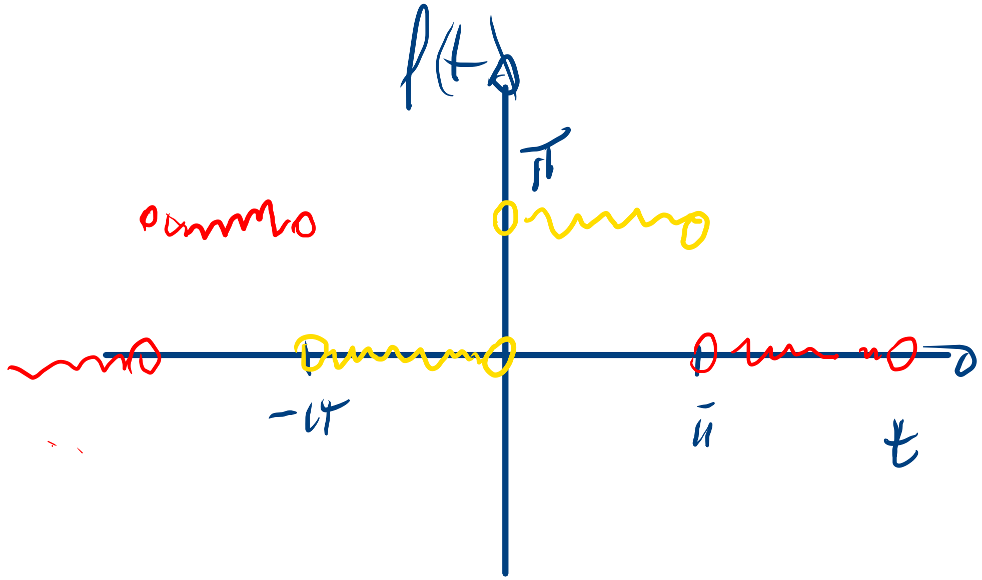
$n \geq 1$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt = \frac{1}{\cancel{\pi}}\int_0^{\pi} \cancel{\pi}\cos nt\,dt = \frac{\sin nt}{n}\Big|_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt = \frac{1}{\cancel{\pi}}\int_0^{\pi} \cancel{\pi}\sin nt\,dt = -\frac{\cos nt}{n}\Big|_0^{\pi} = \frac{-1}{n}\left((-1)^n - 1\right)$$

$$b_n = \frac{1}{n}\left(1-(-1)^n\right)$$

$$f(t) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{1}{n}\left(1-(-1)^n\right) \operatorname{sen} nt$$



$$f(t) = \begin{cases} \pi/2 \ , & t = -\pi, 0, \pi \\ 0 \ , & -\pi < t < 0 \\ \pi \ , & 0 < t < \pi \end{cases}$$

b) $f(t) = e^{t} \qquad -1 < t \leq 1$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt = \int_{-1}^{1} e^{t}\,dt = e^{t}\Big|_{-1}^{1} = e - e^{-1}$$

$$a_n = \int_{-1}^{1} e^{t}\cos n\pi t\,dt = \frac{e^{t}}{n\pi}\cancelto{0}{\operatorname{sen} n\pi t}\Big|_{-1}^{1} - \frac{1}{n\pi}\int_{-1}^{1} e^{t}\operatorname{sen} n\pi t\,dt$$

$u = e^{t} \quad du = e^{t}\,dt$

$dv = \cos n\pi t\,dt$

$v = \dfrac{\operatorname{sen} n\pi t}{n\pi}$

$u = e^{t} \quad du = e^{t}\,dt$

$dv = \operatorname{sen} n\pi t\,dt$

$v = -\dfrac{\cos n\pi t}{n\pi}$

$$a_n = -\frac{1}{n\pi}\left[-e^t\frac{\cos n\pi t}{n\pi} + \frac{1}{n\pi}\int_{-1}^{1} e^t \cos n\pi t \, dt\right]$$

$$= \frac{e^t}{n^2\pi^2}\cos n\pi t\Big|_{-1}^{1} - \frac{1}{n^2\pi^2}\, a_n$$

$$a_n\left(1+\frac{1}{n^2\pi^2}\right) = \frac{e^t \cos n\pi t}{n^2\pi^2}\Big|_{-1}^{1}$$

$$a_n = \frac{e^t \cos n\pi t}{1+n^2\pi^2}\Big|_{-1}^{1} = \frac{1}{1+n^2\pi^2}\left(e\cos n\pi - e^{-1}\cos n\pi\right)$$

$$= \frac{\cos n\pi}{1+n^2\pi^2}\left(e-e^{-1}\right) = \frac{(-1)^n\left(e-e^{-1}\right)}{1+n^2\pi^2}$$

$$b_n = \int_{-1}^{1} e^t \operatorname{sen} n\pi t \, dt = -\frac{e^t \cos n\pi t}{n\pi}\Big|_{-1}^{1} + \frac{1}{n\pi}\int_{-1}^{1} e^t \cos n\pi t \, dt$$

$u = e^t \qquad du = e^t \, dt$

$dv = \operatorname{sen} n\pi t \, dt$

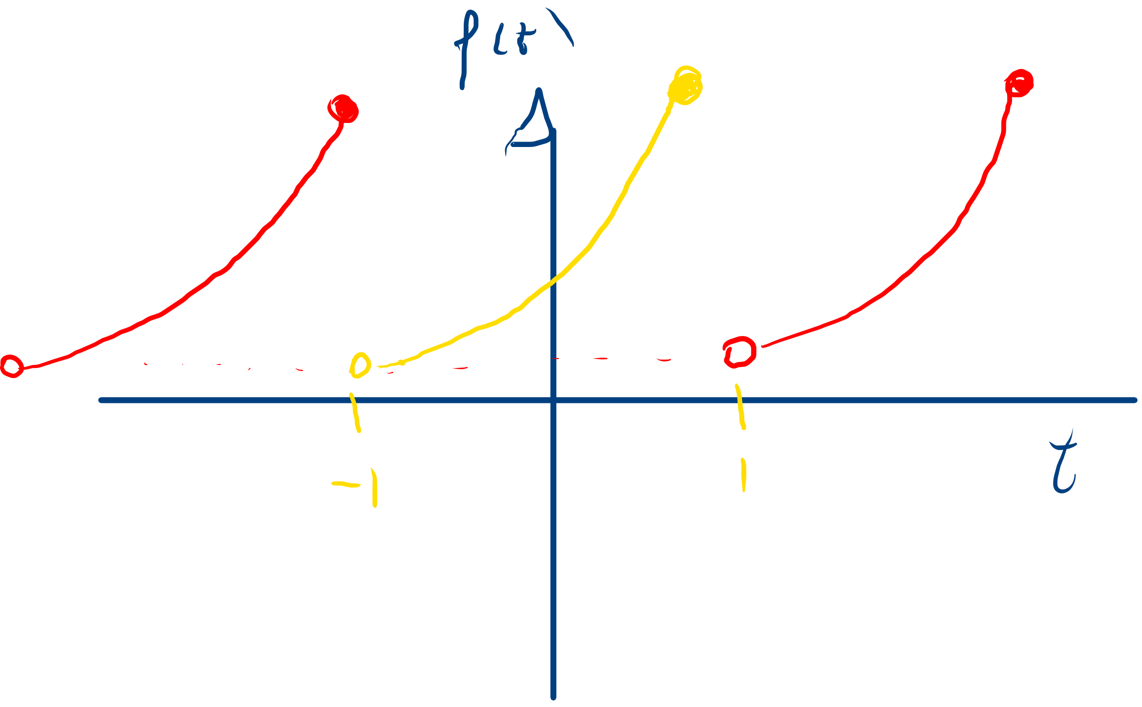
$v = -\dfrac{\cos n\pi t}{n\pi}$

$$b_n = \frac{-1}{n\pi}\left(e \cos n\pi - e^{-1}\cos n\pi\right) + \frac{1}{n\pi} a_n$$

$$b_n = -\frac{(-1)^n}{n\pi}\left(e - e^{-1}\right) + \frac{1}{n\pi}\,\frac{(-1)^n\left(e - e^{-1}\right)}{1+n^2\pi^2}$$

$$b_n = \frac{\left(e - e^{-1}\right)}{n\pi}(-1)^n\left(\frac{1}{1+n^2\pi^2} - 1\right)$$

$$= \frac{(-1)^n\left(e - e^{-1}\right)}{\cancel{n\pi}}\,\frac{-n^2\pi^2}{1+n^2\pi^2} = \frac{(-1)^{n+1}\left(e - e^{-1}\right) n\pi}{1+n^2\pi^2}$$



$$f(t) = \begin{cases} \dfrac{e + e^{-1}}{2}, & t = \pm 1 \\ e^{t} & -1 < t < 1 \end{cases}$$