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Lista 6 - C IV

a)

$$2y''' - 11y'' + 12y' + 9y = 0$$

$2r^3 - 11r^2 + 12r + 9 = 0$

$r(3) = 0$

3	2	-11	12	9
	2	-5	-3	0

$2r^2 - 5r - 3 = 0$

$r = \frac{+5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$r = \frac{-5 \pm \sqrt{25 + 24}}{4}$

$r = \frac{+5 \pm 7}{4}$

$r_1 = \frac{+5+7}{4} = 3$

$r_2 = \frac{+5-7}{4} = \frac{-2}{4} = \frac{-1}{2}$

$$y(x) = C_1 e^{3x} + C_2 x e^{3x} + C_3 e^{-\frac{1}{2}x}$$

Verificando:

$y' = 3C_1 e^{3x} + C_2 e^{3x} + 3C_2 x e^{3x} - \frac{1}{2} C_3 e^{-\frac{1}{2}x}$

$y'' = 9C_1 e^{3x} + 3C_2 e^{3x} + 3C_2 e^{3x} + 9C_2 x e^{3x} + \frac{1}{4} C_3 e^{-\frac{1}{2}x}$

$= 9C_1 e^{3x} + 6C_2 e^{3x} + 9C_2 x e^{3x} + \frac{1}{4} C_3 e^{-\frac{1}{2}x}$

$y''' = 27C_1 e^{3x} + 18C_2 e^{3x} + 9C_2 e^{3x} + 27C_2 x e^{3x} - \frac{1}{8} C_3 e^{-\frac{1}{2}x}$

$= 27C_1 e^{3x} + 27C_2 e^{3x} + 27C_2 x e^{3x} - \frac{1}{8} C_3 e^{-\frac{1}{2}x}$

Substituindo:

$2[27C_1 e^{3x} + 27C_2 e^{3x} + 27C_2 x e^{3x} - \frac{1}{8} C_3 e^{-\frac{1}{2}x}] - 11[9C_1 e^{3x} + 6C_2 e^{3x} + 9C_2 x e^{3x} + \frac{1}{4} C_3 e^{-\frac{1}{2}x}] \ldots$

$\ldots + 12[3C_1 e^{3x} + C_2 e^{3x} + 3C_2 x e^{3x} - \frac{1}{2} C_3 e^{-\frac{1}{2}x}] + 9[C_1 e^{3x} + C_2 x e^{3x} + C_3 e^{-\frac{1}{2}x}] = 0$

$\cancel{54C_1 e^{3x}} + \cancel{54C_2 e^{3x}} + \cancel{54C_2 x e^{3x}} - \frac{1}{4} C_3 e^{-x/2} - \cancel{99C_1 e^{3x}} - \cancel{66C_2 e^{3x}} - \cancel{99C_2 x e^{3x}} - \frac{11}{4} C_3 e^{-x/2}$

$+ \cancel{36C_1 e^{3x}} + \cancel{12C_2 e^{3x}} + \cancel{36C_2 x e^{3x}} - 6C_3 e^{-x/2} + \cancel{9C_1 e^{3x}} + \cancel{9C_2 x e^{3x}} + \cancel{9C_3 e^{-x/2}} = 0$ ✓

$\therefore y = C_1 e^{3x} + C_2 x e^{3x} + C_3 e^{-x/2}$ é solução

$$4y'''' - 13y'' + 9y = 0$$

$4r^4 - 13r^2 + 9 = 0$

$r(1) = 0$

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Digitalizado com CamScanner

$$4r^4 - 13r^2 + 9 = 0$$
$$r(1) = 0$$

1	4	0	-13	0	9
-1	4	4	-9	-9	0
	4	0	-9	0	

$4r^3 + 4r^2 - 9r - 9 = 0$

$r(-1) = 0$

$4r^2 + 0r - 9 = 0$

$4r^2 = 9$

$r^2 = 9/4$

$r = \pm\sqrt{9/4}$

$r = \pm 3/2$

$$\therefore y(x) = C_1 e^{x} + C_2 e^{-x} + C_3 e^{3/2 x} + C_4 e^{-3/2 x}$$

$$y' = C_1 e^{x} - C_2 e^{-x} + 3/2\, C_3 e^{3/2 x} - 3/2\, C_4 e^{-3/2 x}$$
$$y'' = C_1 e^{x} + C_2 e^{-x} + 9/4\, C_3 e^{3/2 x} + 9/4\, C_4 e^{-3/2 x}$$
$$y''' = C_1 e^{x} - C_2 e^{-x} + 27/8\, C_3 e^{3/2 x} - 27/8\, C_4 e^{-3/2 x}$$
$$y'''' = C_1 e^{x} + C_2 e^{-x} + 81/16\, C_3 e^{3/2 x} + 81/16\, C_4 e^{-3/2 x}$$

Verificando:

$$4[C_1 e^{x} + C_2 e^{-x} + 81/16\, C_3 e^{3/2 x} + 81/16\, C_4 e^{-3/2 x}] - 13[C_1 e^{x} + C_2 e^{-x} + 9/4\, C_3 e^{3/2 x} + 9/4\, C_4 e^{-3/2 x}] \ldots$$
$$\ldots + 9[C_1 e^{x} + C_2 e^{-x} + C_3 e^{3/2 x} + C_4 e^{-3/2 x}] = 0$$
$$4\cancel{C_1} e^{x} + 4C_2 e^{-x} + 324/16\, C_3 e^{3/2 x} + \cancel{324/16\, C_4} e^{-3/2 x} - 13\cancel{C_1} e^{x} - 13\cancel{C_2} e^{-x} - \cancel{117/4}\, C_3 e^{3/2 x} - \cancel{117/4\, C_4} e^{-3/2 x}$$
$$+ 9\cancel{C_1} e^{x} + 9\cancel{C_2} e^{-x} + 9\cancel{C_3} e^{3/2 x} + 9\cancel{C_4} e^{-3/2 x} = 0 \quad \text{é solução}$$

$$\therefore y = C_1 e^{x} + C_2 e^{-x} + C_3 e^{3/2 x} + C_4 e^{-3/2 x}$$

$$x^2 y'' - 4xy' + (x^2 + 6) y = x^4$$
$$y_1 = x^2 \cos(x) \quad , \quad y_2 = x^2 \operatorname{sen}(x)$$

Assim, calculando o Wronskiano

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x^2 \cos x & x^2 \operatorname{sen}(x) \\ 2x\cos x - x^2 \operatorname{sen} x & 2x \operatorname{sen} x + x^2 \cos(x) \end{vmatrix}$$

$$= x^2 \cos x\,(2x \operatorname{sen} x + x^2 \cos(x)) - [x^2 \operatorname{sen}(x)(2x\cos x - x^2 \operatorname{sen} x)]$$
$$= \cancel{2x^3 \cos x \operatorname{sen} x} + x^4 \cos^2 x - \cancel{2x^3 \operatorname{sen} x \cos x} + x^4 \operatorname{sen}^2 x$$
$$x^4(\cos^2 x + \operatorname{sen}^2 x) = x^4 \neq 0$$

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Daí:

$$u_1' = \frac{-y_2 x^4}{W} \quad ; \quad u_2' = \frac{y_1 x^4}{W}$$

$u_1 = \int \frac{-x^2 \operatorname{sen} x \, x^4}{x^4} dx \Rightarrow -\int x^2 \operatorname{sen} x \, dx \Rightarrow u = x^2$, $dv = \operatorname{sen} x$

$du = 2x\,dx$, $v = -\cos x$

$u \cdot v - \int v\,du \Rightarrow x^2(-\cos x) - \int -\cos x \, 2x \, dx \Rightarrow -x^2\cos x + 2\int \cos x \, x \, dx$

$u = x$; $dv = \cos x$ $\Rightarrow$

$du = dx$; $v = \operatorname{sen} x$ $\quad 2[x \operatorname{sen} x - \int \operatorname{sen} x \, dx] \Rightarrow 2[x \operatorname{sen} x + \cos x] \Rightarrow 2x \operatorname{sen} x + 2\cos x$

$\Rightarrow u_1 = -x^2\cos x + 2x \operatorname{sen} x + 2\cos x + \cancel{C}^0$ //

Por outro lado:

$u_2 = \int \frac{x^2 \cos x \, x^4}{x^4} dx \Rightarrow \int x^2 \cos x \, dx \Rightarrow u = x^2$; $dv = \cos x$

$du = 2x\,dx$; $v = \operatorname{sen} x$

$u \cdot v - \int v\,du \Rightarrow x^2 \operatorname{sen} x - \int \operatorname{sen} x \, 2x \, dx \Rightarrow x^2 \operatorname{sen} x - 2\int x \operatorname{sen} x \, dx$ $\quad u = x$; $dv = \operatorname{sen} x$

$du = dx$; $v = -\cos x$

$u \cdot v - \int v\,du \Rightarrow x(-\cos x) - \int -\cos x \, dx \Rightarrow [x\cos x + \operatorname{sen} x]2$

$$u_2 = x^2 \operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x + \cancel{C}_0$$

Assim

$$y_p = y_1 u_1 + y_2 u_2$$

$$y_p = x^2\cos x(-x^2\cos x + 2x \operatorname{sen} x + 2\cos x) + x^2 \operatorname{sen} x(x^2 \operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x)$$

$$y_p' = -4x(\operatorname{sen}^2 x + (-x^3 - x)\cos x \operatorname{sen} x - \cos^2 x)$$

$$y_p'' = -4((x^4 + x^2 + 1)\operatorname{sen}^2 x + (2x - 4x^3)\cos x \operatorname{sen} x + (-x^4 - x^2 - 1)\cos^2 x)$$

Substituindo na EDO:

$x^2[-4((x^4+x^2+1)\operatorname{sen}^2 x + (2x - 4x^3)\cos x \operatorname{sen} x + (-x^4 - x^2 - 1)\cos^2 x)] - 4x[-4x(\operatorname{sen}^2 x + \ldots$
$+ (-x^3 - x)\cos x \operatorname{sen} x - \cos^2 x)] + (x^2 + 6)[x^2\cos x(-x^2\cos x + 2x \operatorname{sen} x + 2\cos x) + x^2 \operatorname{sen} x(x^2$
$\operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x)] = x^4$

$$\operatorname{sen} x\, y'' + (2\operatorname{sen} x - \cos x)\, y' + (\operatorname{sen} x - \cos x)\, y = e^{-x}$$

$$y_1 = e^{-x} \quad , \quad y_2 = e^{-x}\cos x$$

$$W(y_1,y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x} & e^{-x}\cos x \\ -e^{-x} & -e^{-x}(\operatorname{sen} x + \cos x) \end{vmatrix} = -e^{-2x}(\operatorname{sen} x + \cancel{\cos x}) - \left[\cancel{-e^{-2x}\cos x}\right]$$

$$W(y_1,y_2) = -e^{-2x}\operatorname{sen} x$$

$$u_1' = \frac{-y_2\, e^{-x}}{-e^{-2x}\operatorname{sen} x} \quad ; \quad u_2' = \frac{y_1\, e^{-x}}{-e^{-2x}\operatorname{sen} x}$$

$$u_1 = -\int \frac{\cancel{e^{-2x}}\cos x}{-\cancel{e^{-2x}}\operatorname{sen} x}\,dx \Rightarrow u_1 = \int \frac{\cos x}{\operatorname{sen}}\,dx \;\; u = \operatorname{sen} x \;\; du = \cos x\,dx \Rightarrow \int \frac{1}{u}\,du \Rightarrow \ln|u|$$

$$\Rightarrow \ln|\operatorname{sen} x| + \cancel{C_0} \Rightarrow u_1 = \ln|\operatorname{sen} x|$$

$$u_2 = \int \frac{\cancel{e^{-2x}}}{-\cancel{e^{-2x}}\operatorname{sen} x}\,dx = \int \frac{1}{\operatorname{sen} x}\,dx \Rightarrow \frac{\ln|1-\cos(x)|}{2} - \frac{\ln|\cos x + 1|}{2} + \cancel{C_0}$$

$$y_p = e^{-x}\ln|\operatorname{sen} x| + e^{-x}\cos x\left[\frac{\ln|1-\cos x|}{2} - \frac{\ln|\cos x + 1|}{2}\right]$$

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