

Lista 1 CIV

a) $x^2 \frac{dy}{dx} = y - xy$; $y(-1) = -1$

P.V.I: $y(-1) = -1$

$$x^2 \frac{dy}{dx} = y(1-x)$$

$$\int \frac{dy}{y} = \int \frac{(1-x) dx}{x^2}$$

$$\ln|y| = \int (1-x)(x^{-2}) dx$$

$$\ln|y| = \int x^{-2} - x^{-1} dx$$

$$\ln|y| = -\int \frac{1}{x} dx + \int x^{-2} dx$$

$$\ln|y| = -\ln|x| + \frac{x^{-1}}{-1} + C$$

$$\ln|y| = -\ln|x| - \frac{1}{x} + C$$

$$e^{\ln|y|} = e^{-\ln|x| - \frac{1}{x} + C}$$

$$|y| = + \frac{1}{x} \cdot e^{-\frac{1}{x}} \cdot e^C$$

$$y = + K/x \cdot e^{-1/x}$$

$$y = + K/x \cdot e^{-1/x}$$

$$-1 = K/(-1) \cdot e^{-1/(-1)}$$

$$-1 = K/(-1) \cdot e$$

$$K = 1/e$$

$$\therefore y = -1/e x \cdot e^{-1/x}$$

b) $x \ln(y) \frac{dy}{dx} = \left(\frac{x+1}{y} \right)^2$

$$x \ln(y) \frac{dy}{dx} = \frac{(x+1)^2}{y^2}$$

$$\ln(y) \cdot y^2 \frac{dy}{dx} = \frac{(x+1)^2}{x}$$

$$\int y^{\frac{1}{3}} \ln(y) dy = \int \frac{(x+1)^2}{x} dx$$

$$\textcircled{I} \int y^{\frac{1}{3}} \ln(y) dy$$

$$u = \ln(y) \quad \frac{1}{y}$$

$$dv = y^{\frac{1}{3}} \quad \frac{1}{3} y^{\frac{4}{3}}$$

$$u \cdot v - \int v du$$

$$\ln(y) \cdot y^{\frac{4}{3}} - \int y^{\frac{4}{3}} \cdot \frac{1}{y} dy$$

$$\ln(y) \cdot y^{\frac{4}{3}} - \frac{1}{3} \int y^{\frac{1}{3}} dy$$

$$\ln(y) \cdot y^{\frac{4}{3}} - \frac{1}{3} \cdot y^{\frac{4}{3}} + C$$

$$\ln(y) \cdot y^{\frac{4}{3}} - \frac{y^{\frac{4}{3}}}{9} + C$$

$$\frac{1}{3} \ln(y) \cdot y^{\frac{4}{3}} - \frac{y^{\frac{4}{3}}}{9}$$

Assim:

$$\frac{1}{3} \ln(y) \cdot y^{\frac{4}{3}} - \frac{y^{\frac{4}{3}}}{9} = \frac{x^2}{2} + 2x + \ln|x| + C$$

$$\therefore \frac{1}{3} \ln|y| \cdot y^{\frac{4}{3}} - \frac{y^{\frac{4}{3}}}{9} = \frac{x^2}{2} + 2x + \ln|x| + C$$

$$\textcircled{II} \int \frac{(x+1)^2}{x} dx$$

$$u = x+1$$

$$\frac{du}{dx} = 1$$

$$\int \frac{x^2 + 2x + 1}{x} dx$$

$$\int x dx + \int 2 dx + \int \frac{1}{x} dx$$

$$\frac{x^2}{2} + 2x + \ln|x| + C$$