Carlos Luilguer Almeida Santos
20150465

Lista 4 - C IV

a)

$4x^2(\operatorname{sen}(x))y'' - 4x(x\cos(x) + \operatorname{sen}(x))y' + (2x\cos(x) + 3\operatorname{sen}(x))y = 0 \; ; y_1 = x^{1/2}$

$\div 4x^2(\operatorname{sen} x)$

$$y'' - \left(\frac{\cos x}{\operatorname{sen} x} + \frac{1}{x}\right)y' + \left(\frac{\cos x}{2x\operatorname{sen} x} + \frac{3}{4x^2}\right)y = 0$$

Pelo teorema de Abel:

$$y_2(x) = y_1(x) \cdot \int \frac{e^{-\int p(x)\,dx}}{(y_1(x))^2}\,dx$$

$p(x) = -\frac{\cos x}{\operatorname{sen} x} - \frac{1}{x} \Rightarrow -\int -\frac{\cos x}{\operatorname{sen} x} - \frac{1}{x}\,dx \Rightarrow \int \frac{\cos x}{\operatorname{sen} x}\,dx + \int \frac{1}{x}\,dx \quad u = \operatorname{sen} x, \; du = \cos x\,dx$

$\Rightarrow \int 1/u\,du + \int 1/x\,dx \Rightarrow \ln|\operatorname{sen} x| + \ln|x| \Rightarrow e^{\ln|\operatorname{sen} x| + \ln|x|} = x\operatorname{sen} x$

$\Rightarrow y_2 = x^{1/2}\int \frac{x \cdot \operatorname{sen} x}{(x^{1/2})^2}\,dx \Rightarrow x^{1/2}\int \operatorname{sen} x\,dx \Rightarrow x^{1/2}[-\cos x] + C$

$$\therefore y(x) = C_1 x^{1/2} - C_2(x^{1/2}\cos x)$$

$y_2 = -x^{1/2}\cos x \; ; \; y_2' = x^{1/2}\operatorname{sen} x - \cos x/2x^{1/2} \; ; \; y_2'' = \frac{3\operatorname{sen} x}{2x^{1/2}} + \frac{2x\cos x}{2x^{1/2}} - \frac{2x\operatorname{sen} x}{4x^{1/2}} + \dots$

$\dots + \cos x/4x^{3/2}$

Substituindo: $y'' - \left(\frac{\cos x}{\operatorname{sen} x} + \frac{1}{x}\right)y' + \left(\frac{\cos x}{2x\operatorname{sen} x} + \frac{3}{4x^2}\right)y = 0$

Foroni

$$\left[\frac{3\operatorname{sen}x}{2x^{1/3}} + \frac{2x\cos x}{2x^{1/3}} - \frac{2x\operatorname{sen}x}{4x^{3/3}} + \frac{\operatorname{sen}x}{4x^{3/3}}\right] - \left(\frac{\cos x}{\operatorname{sen}x} + \frac{1}{x}\right)\left[\frac{x^{1/3}\cos x - \operatorname{sen}x}{2x^{1/3}}\right]$$

$$+ \left(\frac{\cos x}{2x\operatorname{sen}x} + \frac{3}{4x^2}\right)\left[-x^{1/3}\operatorname{sen}x\right] = 0$$

$$\left[\frac{3\operatorname{sen}x}{2x^{1/3}} + x^{1/3}\cos x - \frac{\operatorname{sen}x}{2x^{1/3}} + \frac{\operatorname{sen}x}{4x^{1/3}}\right] - \frac{\cos x\, x^{1/3}\operatorname{sen}x}{\operatorname{sen}x} + \cancel{\frac{\cos^2 x}{\operatorname{sen}x\, 2x^{1/3}}} - \cancel{\frac{\operatorname{sen}x}{x^{1/3}}}$$

$$+ \cancel{\frac{\cos x}{2x^{1/3}}} - \frac{\cos^2 x}{2x^{1/3}\operatorname{sen}x} - \frac{3\operatorname{sen}x}{4x^{1/3}} = 0 \therefore \text{solução}$$

$$y'' + 3y = 0,\; y(\pi/3) = 2,\; y'(\pi/3) = -1$$

$$y'' + 3y = 0$$

$r^2 + 0 + 3 = 0$

$r^2 + 3 = 0$

$r^2 = \pm\sqrt{-3}$

$r = \pm\sqrt{3}\,i$

$$\therefore y(x) = e^0\left(c_1\cos(\sqrt{3}x) + c_2\operatorname{sen}(\sqrt{3}x)\right)$$

$$y(x) = c_1\cos(\sqrt{3}x) + c_2\operatorname{sen}(\sqrt{3}x)$$

$$y'(x) = c_1(-\operatorname{sen}(\sqrt{3}x))\sqrt{3} + c_2\cos(\sqrt{3}x)\sqrt{3}$$

$$\begin{cases} c_1\cos(\sqrt{3}(\pi/3)) + c_2\operatorname{sen}(\sqrt{3}(\pi/3)) = 2 \\ -\sqrt{3}c_1\operatorname{sen}(\sqrt{3}(\pi/3)) + \sqrt{3}c_2\cos(\sqrt{3}(\pi/3)) = -1 \end{cases}$$

$$\begin{cases} c_1(1/2) + c_2(\sqrt{3}/2) = 2 \\ -\sqrt{3}c_1(\sqrt{3}/2) + \sqrt{3}c_2(1/2) = -1 \end{cases} \Rightarrow \begin{cases} \text{I}\quad \frac{1}{2}c_1 + \frac{\sqrt{3}}{2}c_2 = 2 \\ \text{II}\quad -\frac{3}{2}c_1 + \frac{\sqrt{3}}{2}c_2 = -1 \end{cases}$$

I - II:

$+\frac{4}{2}c_1 = 3$

$c_1 = 6/4$

$\therefore c_1 = 3/2$

$1/1$

I: $\frac{1}{2}(3/2) + \frac{\sqrt{3}}{2}c_2 = 2$

$\frac{3}{4} + \frac{\sqrt{3}}{2}c_2 = 2$

$c_2 = \frac{5}{4}\cdot\frac{2}{\sqrt{3}}$

$\therefore c_2 = \frac{5\sqrt{3}}{6}$

$$\therefore y(x) = \left(\tfrac{3}{2}\right)\cos(\sqrt{3}x) + \left(\tfrac{5\sqrt{3}}{6}\right)\operatorname{sen}(\sqrt{3}x)$$

Verificando:

$y''(x) = -3C_1\cos(\sqrt{3}x) - 3C_2\operatorname{sen}(\sqrt{3}x)$

$$(-3C_1\cos(\sqrt{3}x) - 3C_2\operatorname{sen}(\sqrt{3}x)) + 3(C_1\cos(\sqrt{3}x) + C_2\operatorname{sen}(\sqrt{3}x)) = 0 \quad ✓$$

$y'' - 6y' - 7y = 0$

$r^2 - 6r - 7 = 0$

$r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$r = \dfrac{+6 \pm \sqrt{36 - 4(1)(-7)}}{2}$

$r = \dfrac{6 \pm \sqrt{36 + 28}}{2}$

$r = \dfrac{6 \pm 8}{2}$ → $r_1 = 7$, $r_2 = -1$

$$\therefore y(x) = C_1 e^{7x} + C_2 e^{-x}$$

Verificando:

$y'(x) = 7C_1 e^{7x} - C_2 e^{-x}$

$y''(x) = 49C_1 e^{7x} + C_2 e^{-x}$

$$y'' - 6y' - 7y = 0$$

$$(49C_1e^{7x} + C_2e^{-x}) - 6(7C_1e^{7x} - C_2e^{-x}) - 7(C_1e^{7x} + C_2e^{-x}) = 0$$

$$49C_1e^{7x} + C_2e^{-x} - 42C_1e^{7x} + 6C_2e^{-x} - 7C_1e^{7x} - 7C_2e^{-x} = 0$$

$$49C_1e^{7x} - 49C_1e^{7x} + 7C_2e^{-x} - 7C_2e^{-x} = 0$$

$$\therefore y(x) = C_1e^{7x} + C_2e^{-x} \text{ é solução}$$

$y'' + 4y' + 4y = 0$

$r^2 + 4r + 4 = 0$

$r = \dfrac{-4 \pm \sqrt{16 - 4(1)(4)}}{2}$

$r = \dfrac{-4 \pm \sqrt{0}}{2}$

→ $r_1 = r_2 = -2$

$$\therefore y(x) = C_1 e^{-2x} + C_2 x e^{-2x}$$

Verificando:

$y'(x) = e^{-2x}(-2C_1 - 2C_2 x + C_2)$

$y''(x) = 4e^{-2x}(C_2 x - C_2 + C_1)$

$$y'' + 4y' + 4y = 0$$

$[4e^{-2x}(C_2 x - C_2 + C_1)] + 4[e^{-2x}(-2C_1 - 2C_2 x + C_2)] + 4[e^{-2x}(C_1 + C_2 x)]$

é solução ✓