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Lista 12 - C IV

b)

$$f(t) = \cos(2\pi a t)$$

A transformada pode ser obtida pela definição:

$$\mathcal{F}f = \int_{-\infty}^{\infty} e^{-2\pi i s t} \cos(2\pi a t) dt$$

$$\mathcal{F}f = \int_{-\infty}^{\infty} e^{-2\pi i s t} \left(\frac{e^{2\pi i a t} + e^{-2\pi i a t}}{2} \right) dt$$

$$\mathcal{F}f = \frac{1}{2} \int_{-\infty}^{\infty} e^{-2\pi i t(s-a)} + e^{-2\pi i t(s+a)} dt$$

com uma tabela;

$$\mathcal{F}f = \frac{1}{2} \left[\delta(s-a) + \delta(s+a) \right] //$$

a)

$$y'' + 3y = t \quad , \quad 0 \leq t \leq 1$$

$$y'(0) = y'(1) = 0$$

↳ Condição de Neumann

Assim, propondo:

$$y(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n \cos(n\pi t/L)$$

$$y'(t) = \sum_{n=1}^{\infty} -\frac{n\pi}{L} C_n \operatorname{sen}(n\pi t/L)$$

$$y''(t) = \sum_{n=1}^{\infty} -\frac{n^2\pi^2}{L^2} C_n \cos(n\pi t/L)$$

Credeal

Como $f(t) = t$ e $L=1$, então:

$$C_0 = \frac{2}{L}\int_0^L f(t)\,dt \qquad C_n = \frac{2}{L}\int_0^L f(t)\cos(n\pi t/L)\,dt$$

$$C_0 = 2\int_0^1 t\,dt \Rightarrow 2\left[\frac{t^2}{2}\right]\Big|_0^1 = 2[1/2 - 0] = 1$$

Assim, o C_n será:

$$C_n = 2\int_0^1 t\cos(n\pi t)\,dt \Rightarrow u = t;\ du = dt;\ dv = \cos(n\pi t)dt;$$

$$\Rightarrow v = \operatorname{sen}(n\pi t)/n\pi \Rightarrow uv - \int v\,du \Rightarrow 2\left[\frac{t}{n\pi}\operatorname{sen}(n\pi t)\Big|_0^1 - \frac{1}{n\pi}\int_0^1 \operatorname{sen}(n\pi t)dt\right]$$

$$\Rightarrow 2\left[\frac{1}{n^2\pi^2}\cos(n\pi t)\right]\Big|_0^1 - \frac{2}{n^2\pi^2}((-1)^n - 1) \Rightarrow \sum_{n=1}^{\infty} -C_n n^2\pi^{2}\cos(n\pi t) + \dots$$

$$\dots + 3\left(\frac{C_0}{2} + \sum_{n=1}^{\infty} C_n\cos(n\pi t)\right) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{2}{n^2\pi^2}((-1)^n - 1)\cos(n\pi t)$$

$$\Rightarrow \sum_{n=1}^{\infty}\left(\cos(n\pi t)\left[C_n\{3 - n^2\pi^2\} - \frac{2}{n^2\pi^2}((-1)^n - 1)\right]\right) + \left(\frac{3C_0}{2} - \frac{1}{2}\right) = 0$$

$$\Rightarrow C_0 = 1/3;\quad C_n = \frac{2((-1)^n - 1)}{n^2\pi^2(3 - n^2\pi^2)}$$

Assim, substituindo:

$$\therefore\ Y(t) = \frac{1}{6} + \sum_{n=1}^{\infty}\frac{2((-1)^n - 1)}{n^2\pi^2(3 - n^2\pi^2)}\cos(n\pi t)$$

$$y'(t) = \sum_{n=1}^{\infty}\frac{-2((-1)^n - 1)}{n\pi(3 - n^2\pi^2)}\operatorname{sen}(n\pi t)$$

. $y'(0) = 0 \rightarrow$ verificando:

$$y'(0) = \sum_{n=1}^{\infty}\frac{-2}{n\pi}\frac{((-1)^n - 1)}{(3 - n^2\pi^2)}\underbrace{\operatorname{sen}(n\pi 0)}_{0} = 0$$

. $y'(1) = 0 \rightarrow$ verificando:

$$y'(1) = \sum_{n=1}^{\infty}\frac{-2}{n\pi}\frac{((-1)^n - 1)}{(3 - n^2\pi^2)}\underbrace{\operatorname{sen}(n\pi)}_{0} = 0$$