Solução do Trabalho 2 de Cálculo IV

a) $\left(\frac{2xy}{x^2+1} - x^2\right)dx - \left(2 - \ln(x^2+1)\right)dy = 0$

$M = \frac{2xy}{x^2+1} - x^2 \qquad N = -2 + \ln(x^2+1)$

$M_y = \frac{2x}{x^2+1} \qquad N_x = \frac{2x}{x^2+1} \qquad N_x = M_y$

Exato pois M e N contínuos

$$\frac{\partial f}{\partial x} = M = \frac{2xy}{x^2+1} - x^e$$

$$f = \int \frac{2xy}{x^2+1} - x^e \, dx = \int \frac{2xy}{x^2+1} \, dx - \int x^e \, dx$$

$e \geq 1$

$$u = x^2+1 \quad du = 2x \, dx$$

$$\int \frac{2xy}{x^2+1} \, dx = y \int \frac{du}{u} = y \ln|x^2+1| + c$$

$$f = y \ln|x^2+1| - \frac{1}{e+1} x^{e+1} + h(y)$$

$$\frac{\partial f}{\partial y} = N = \cancel{\ln|x^2+1|} + \frac{\partial h}{\partial y} = -2 + \cancel{\ln(x^2+1)}$$

$$h = \int -2\,dy = -2y + c$$

$$f = y\ln|x^2+1| - \frac{1}{e+1}x^{e+1} - 2y + c = cte$$

$$y\ln|x^2+1| - \frac{x^{e+1}}{e+1} - 2y = d$$

$$y = \frac{x^{e+1}}{(e+1)(\ln|x^2+1|-2)} + \frac{d}{\ln|x^2+1|-2}$$

b) Verificação:

$$y \ln|x^2+1| - \frac{x^{e+1}}{e+1} - 2y = d$$

$$y' \ln|x^2+1| + \frac{y\,2x}{x^2+1} - x^e - 2y' = 0$$

$$(\ln|x^2+1| - 2)\,y' + \left(\frac{2xy}{x^2+1} - x^e\right) = 0$$

$$\left(\frac{2xy}{x^2+1} - x^e\right) dx - \left(2 - \ln|x^2+1|\right) dy = 0$$

Ok

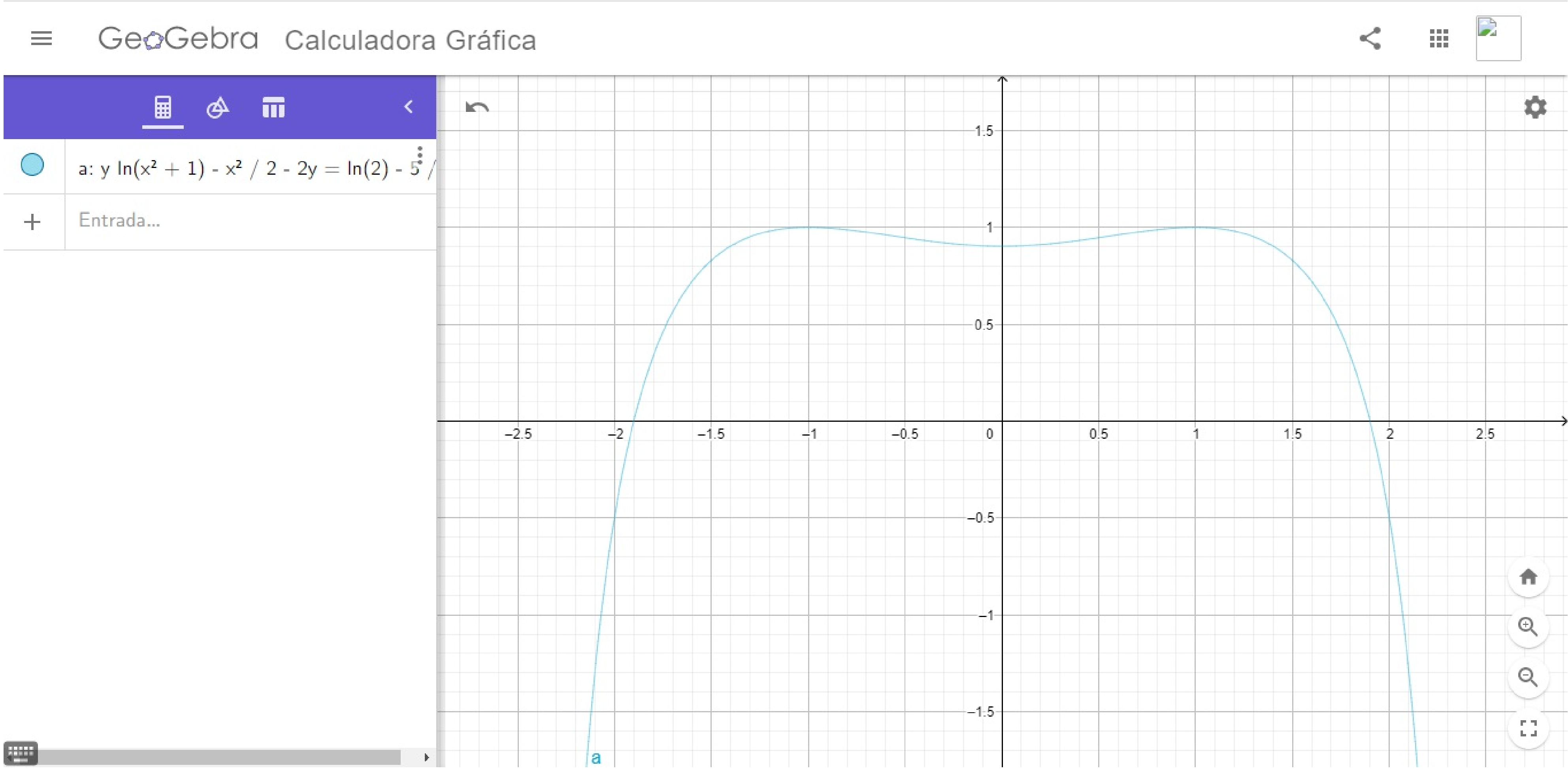
c) $y(x_0) = y_0$

$$d = y_0 \ln|x_0^2+1| - \frac{x_0^{a+1}}{a+1} - 2y_0$$

Para $x_0 = 1$ $\quad y_0 = 1$ $\quad a = 1$

$$d = \ln 2 - \frac{1}{2} - 2 = \ln 2 - \frac{5}{2}$$

d)


GeoGebra Calculadora Gráfica
a: y ln(x² + 1) - x² / 2 - 2y = ln(2) - 5 /
Entrada...
a