Soluções da Lista 14 de Cálculo IV

$$\frac{\partial^2 u}{\partial t^2} = k^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(0,t) = 0 = u(L,t)$$

$$u(x,0) = 0$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = x(L-x)$$

Para $u = XT$

$$\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{Tk^2}\frac{d^2T}{dt^2} = -l^2 < 0$$

(se $l^2 = 0$ e $l^2 > 0$ temos a sol. trivial)

$\rightarrow$ $X = A \operatorname{sen} lx + B \cos lx$

$T = C \operatorname{sen} lkt + D \cos ltk$

$X(0) = 0 = B$

$X(L) = 0 = A \operatorname{sen} lL \rightarrow lL = n\pi$

$l = \frac{n\pi}{L}$ $\quad n = 1, 2, 3 \ldots$

$$u(x,t) = \sum_{n=1}^{\infty} \left(C_n \operatorname{sen} \frac{n\pi}{L} tk + D_n \cos \frac{n\pi}{L} tk \right) \operatorname{sen} \frac{n\pi x}{L}$$

$$u(x,0) = 0 = \sum_{n=1}^{\infty} D_n \operatorname{sen} \frac{n\pi x}{L} \rightarrow D_n = 0$$

$$u(x,t) = \sum_{n=1}^{\infty} C_n \operatorname{sen} \frac{n\pi}{L} tk \operatorname{sen} \frac{n\pi x}{L}$$

$$\frac{\partial u}{\partial t} = \sum_{n=1}^{\infty} C_n \frac{n\pi k}{L} \cos \frac{n\pi}{L} tk \operatorname{sen} \frac{n\pi x}{L}$$

$$\sum_{n=1}^{\infty} C_n \frac{n\pi k}{L} \operatorname{sen} \frac{n\pi x}{L} = x(L-x)$$

$$k\frac{n\pi}{l} C_n = \frac{2}{l}\int_0^l x(l-x)\,\text{sen}\,\frac{n\pi x}{l}\,dx$$

$$\int_0^l x\,\text{sen}\,\frac{n\pi x}{l}\,dx = -\frac{xl}{n\pi}\cos\frac{n\pi x}{l}\Big|_0^l + \frac{l}{n\pi}\int_0^l \cos\frac{n\pi x}{l}\,dx$$

$u = x$

$du = dx$

$dv = \text{sen}\,\frac{n\pi x}{l}\,dx$

$v = -\frac{l}{n\pi}\cos n\pi x$

$$= -\frac{l^2}{n\pi}\cos n\pi + \cancel{\frac{l^2}{n^2\pi^2}\,\text{sen}\,\frac{n\pi x}{l}\Big|_0^l}$$

$$= -\frac{l^2}{n\pi}(-1)^n$$

$$\int_0^l x^2 \,\text{sen}\frac{n\pi x}{l}\,dx = -\frac{x^2 l}{n\pi}\cos\frac{n\pi x}{l}\Big|_0^l + \frac{2l}{n\pi}\int_0^l x\cos\frac{n\pi x}{l}\,dx$$

$u = x^2 \quad du = 2x\,dx$

$dv = \text{sen}\frac{n\pi x}{l}\,dx$

$v = -\frac{l}{n\pi}\cos\frac{n\pi x}{l}$

$$= -\frac{l^3}{n\pi}(-1)^n + \frac{2l}{n\pi}\int_0^l x\cos\frac{n\pi x}{l}\,dx$$

$$\int_0^l x\cos\frac{n\pi x}{l}\,dx = \underset{0}{\cancel{\frac{xl}{n\pi}\,\text{sen}\frac{n\pi x}{l}\Big|_0^l}} - \frac{l}{n\pi}\int_0^l \text{sen}\frac{n\pi x}{l}\,dx$$

$u = x \quad du = dx$

$dv = \cos\frac{n\pi x}{l}\,dx$

$v = \frac{l}{n\pi}\,\text{sen}\frac{n\pi x}{l}$

$$= +\frac{l^2}{n^2\pi^2}\cos\frac{n\pi x}{l}\Big|_0^l = \frac{l^2}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$\frac{k n\pi c_n}{L} = \frac{2}{L}\left[L\left\{-\frac{L^2}{n\pi}(-1)^n\right\} - \left\{-\frac{L^3}{n\pi}(-1)^n + \frac{2L}{n\pi}\left[\frac{L^2}{n^2\pi^2}\left((-1)^n - 1\right)\right]\right\}\right]$$

$$\frac{n\pi c_k}{L} = \frac{2}{L}\left[\cancel{-\frac{L^3}{n\pi}(-1)^n + \frac{L^3}{n\pi}(-1)^n} - \frac{2L^3}{n^3\pi^3}\left((-1)^n - 1\right)\right]$$

$$= -\frac{4L^2}{n^3\pi^3}\left((-1)^n - 1\right)$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{-4L^2}{n^3\pi^3}\left((-1)^n - 1\right) \operatorname{sen} \frac{n\pi t}{L k} \operatorname{sen} \frac{n\pi x}{L}$$