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(20150465)

1.

$$f(x,y) = \begin{cases} c(2x+y) ; & (0 \le x \le 2) \text{ e } (0 \le y \le 3) \\ 0, & \text{caso contrário} \end{cases}$$

a)

$f(x,y) \ge 0$, para $(0 \le x \le 2)$ e $(0 \le y \le 3)$

$$\int_a^b \int_c^d f(x,y)\, dy dx = 1$$

$$\int_0^2 \int_0^3 c(2x+y)\, dy dx \Rightarrow c \int_0^2 \left[2xy + y^2/2\right]_0^3 dx \Rightarrow c \int_0^2 \left[2x(3) + 3^2/2 - (2x(0) + 0^2/2)\right] dx$$

$$\Rightarrow c \int_0^2 [6x + 9/2]\, dx \Rightarrow c\left[3x^2 + \frac{9x}{2}\right]_0^2 \Rightarrow c\left[3(2)^2 + \frac{9(2)}{2} - \left(3(0)^2 + \frac{9(0)}{2}\right)\right] = c[12 + 9]$$

$$c[21] \Rightarrow 21c$$

$$21c = 1$$

$$\boxed{c = 1/21}$$

$$f(x,y) = \begin{cases} \frac{1}{21}(2x+y) ; & (0 \le x \le 2) \text{ e } (0 \le y \le 3) \\ 0, & \text{caso contrário} \end{cases}$$

b) Independentemente

$$f(x,y) = f_1(x) \cdot f_2(y)$$

$$f_1(x) = \int_0^3 f(x,y)\, dy \Rightarrow \frac{1}{21}\int_0^3 (2x+y)\, dy$$

$$f_1(x) = \frac{1}{21}\left[2xy + y^2/2\right]_0^3 \Rightarrow \frac{1}{21}\left[2x(3) + (3)^2/2 - 0\right]$$

$$\Rightarrow \frac{1}{21}[6x + 9/2] = \frac{1}{21}(6x + 9/2)$$

$f_2(y) = \int_0^2 \frac{1}{21}(2x+y)\,dx \Rightarrow \frac{1}{21}\left[x^2 + xy\right]\Big|_0^2 = \frac{1}{21}\left[(2)^2 + 2y - 0\right]$

$\frac{1}{21}\left[4 + 2y\right]$

$f_1(x) \cdot f_2(y) = \frac{1}{21}\left[(4+2y)\cdot\left(6x + 9/2\right)\right]$

$= \frac{1}{21}\left[24x + 18 + 12xy + 9y\right]$

$f_1(x) \cdot f_2(y) = \frac{1}{21}\left[24x + 9y + 18 + 12xy\right] \neq f(x,y) = \frac{1}{21}\left[2x + y\right]$

e) $P(1 \leq Y \leq 2 \mid X = 1)$

$f(y|x) = \dfrac{f(x,y)}{f_1(x)}$

$f(y|x) = \dfrac{\frac{1}{21}\left[2x+y\right]}{\frac{1}{21}\left(6x + 9/2\right)} = \dfrac{2x+y}{6x + 9/2}$

$6 + \frac{9}{2} = \frac{12+9}{2} = \frac{21}{2}$

$\int_1^2 f(y|x)\,dy = \int_1^2 \dfrac{y+2}{6+9/2}\,dy = \dfrac{2}{21}\int_1^2 y+2\,dy$

$\frac{1}{2} + 2 = \frac{1+4}{2} = \frac{5}{2}$

$\frac{2}{21}\left[y^2/2 + 2y\right]\Big|_1^2 \Rightarrow \frac{2}{21}\left[(2)^2/2 + 2(2) - \left((1)^2/2 + 2(1)\right)\right] \Rightarrow \frac{2}{21}\left[2 + 4 - \left(1/2 + 2\right)\right]$

$\frac{2}{21}\left[6 - 5/2\right] = \frac{2}{21}\left[\frac{12-5}{2}\right] = \frac{2}{21}\left[7/2\right] = 1/3$

2.

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4. Distribuição de Binomial

$p = 0,03$ (sucesso)

total de 40.000 (n)

X: par de sapatos

$1 - p = 0,97$ (fracasso)

$X = \{0, 1, \dots, 40.000\}$

$$f(x;p;n) = \begin{cases} \binom{n}{x} p^x (1-p)^{(n-x)}, & x = 0, 1, \dots, n \\ 0, & \text{caso contrário} \end{cases}$$

$$E[X] = E[I_1 + \dots + I_n] = np$$

$$E[X] = (40.000)(0,03)$$

$$\boxed{E[X] = 1.200}$$

variância:

$$Var = np(1-p) = (40.000)(0,03)(0,97) = 1.164$$

$$\sigma = \sqrt{Var} = \sqrt{1.164}$$

$$\boxed{\sigma \approx 34,11}$$

O parâmetro foi os valores que X (V.A.) pode assumir

5.

$\mu = 58$ min

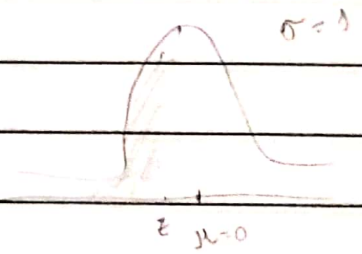
$\sigma = 12$ min

$$Z = \frac{x - \mu}{\sigma}$$

a) $x = 50$ min

$$Z = \frac{50 - 58}{12} \approx -0,666$$

$$\boxed{\Phi(Z) \approx 0,25463}$$



b) $x = 85$ min

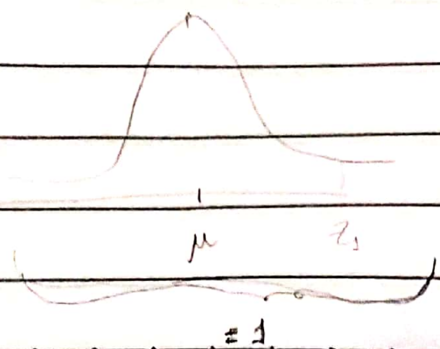
$$Z = \frac{85 - 58}{12} \approx 2,25$$

$$\Phi(Z) \approx 0,98778$$

$$\Phi(-x) = 1 - \Phi(x)$$

$$\Phi(-Z) = 1 - 0,98778$$

$$\boxed{\Phi(-Z) \approx 0,01222}$$



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c) $x = 40$ min

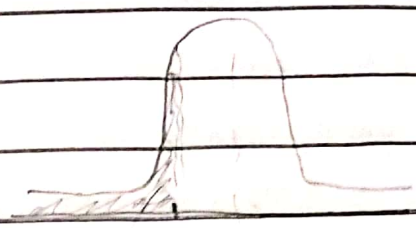
$\mu = 58$ min

$\sigma = 12$ min

$n = 50$ candidatos

$$Z = \frac{40 - 58}{12}$$

$$Z = -1,5$$



$\Phi(-1,5) \approx 0,06681$

$(0,06681)(50) \approx 3,3405 \approx 4$ candidatos