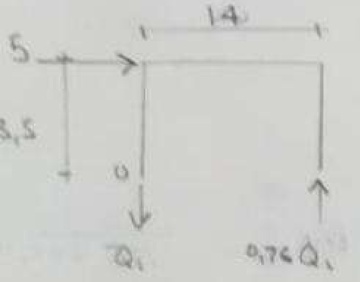
NIVEL #1 = AE = 0,286 BF = 0,214 CG = 1,071 DH

NIVEL #2 = EI = 0,286 FJ = 0,214 GK = 1,071 HJ

NIVEL #3 = JM = 0,75 KN



$\circlearrowleft \Sigma M_0 = 0$

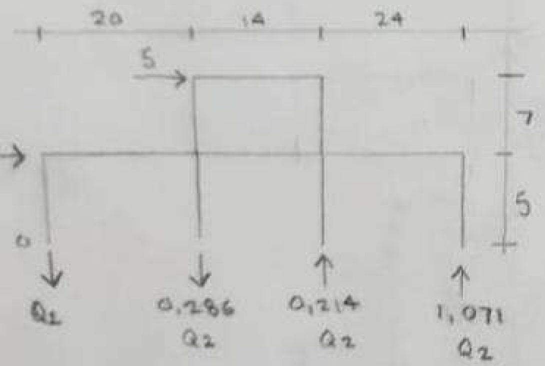
$-5(3,5) + 0,75 Q_1 (14) = 0$

$-17,5 + 10,5 Q_1 = 0$

$Q_1 = 1,67$ KN

$X_c = \dfrac{A(0) + A(20) + A(34) + A(58)}{4}$

$\underline{X_c = 28 \text{ ft}}$



$\circlearrowleft \Sigma M_0 = 0$

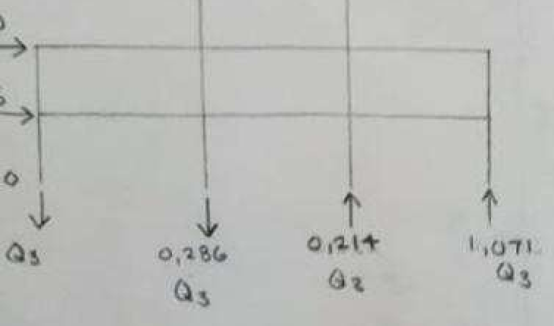
$-10(5) - 5(12) - 20(0,286 Q_2) + 34(0,214 Q_2)$

$+ 58(1,071 Q_2) = 0$

$-50 - 60 - 5,72 Q_2 + 7,276 Q_2 + 62,118 Q_2 = 0$

$-110 + 63,674 Q_2 = 0$

$Q_2 = 1,727 Q_2$



$\circlearrowleft \Sigma M_0 = 0$

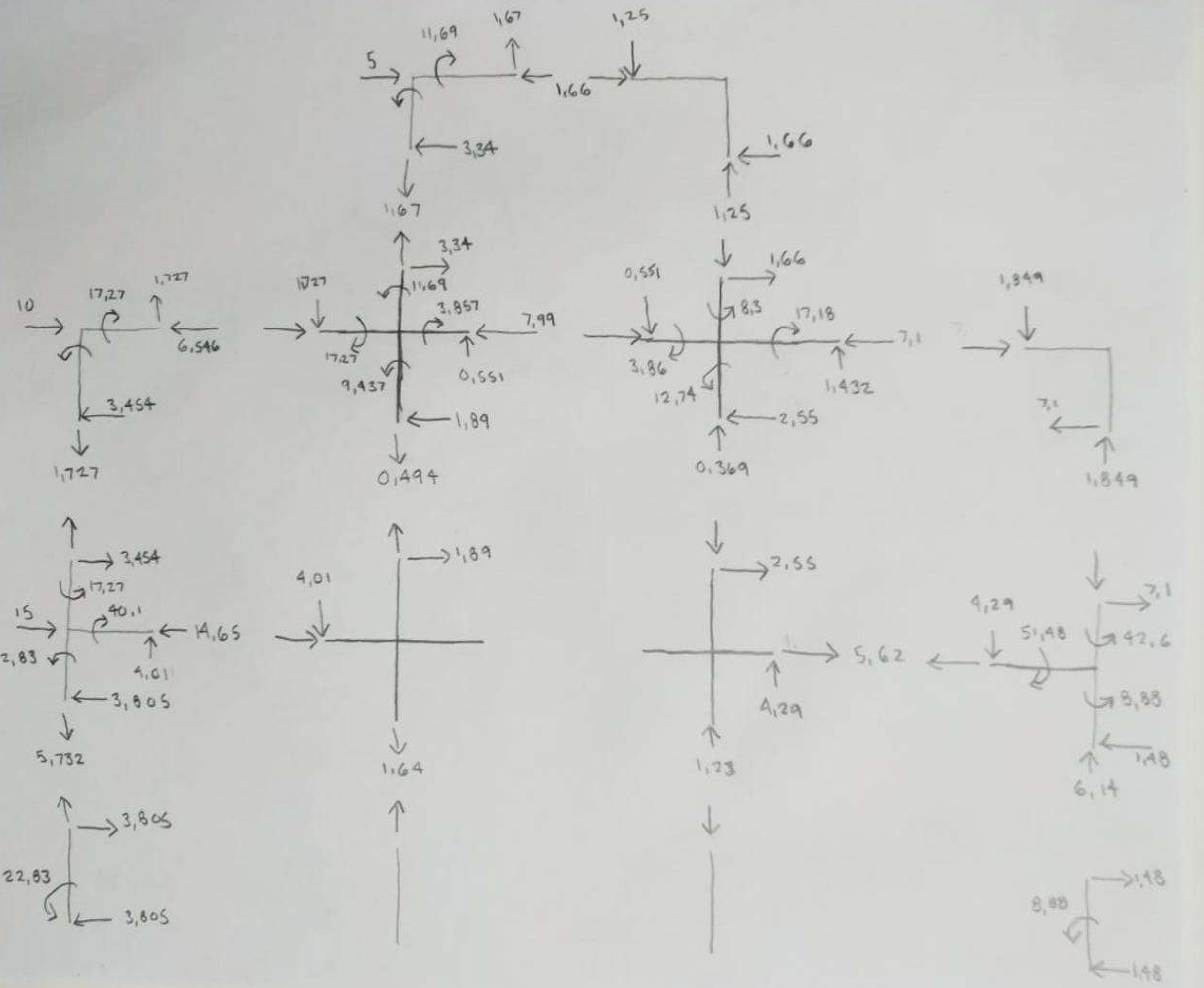
$-6(15) - 16(10) - 23(5) - 20(0,286 Q_3)$

$+ 34(0,214 Q_3) + 58(1,071 Q_3) = 0$

$-90 - 160 - 115 - 5,72 Q_3 + 7,276 Q_3 + 62,118 Q_3 = 0$

$-365 + 63,674 Q_3 = 0$

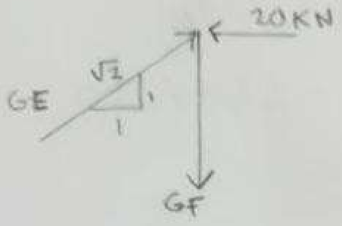
$Q_3 = 5,732 Q_3$



5
11,69
1,67
1,25
1,66
3,34
1,66
1,67
1,25
10
17,27
1,727
6,546
3,454
1,727
17,27
3,34
11,69
3,857
7,99
17,27
9,437
0,551
1,89
0,494
0,551
1,66
8,3
17,18
7,1
3,86
12,74
1,432
2,55
0,369
1,849
7,1
1,849
3,454
17,27
40,1
15
14,65
22,83
4,01
3,805
5,732
1,89
4,01
1,64
2,55
5,62
4,29
1,23
4,29
51,48
7,1
42,6
8,88
1,48
6,14
3,805
22,83
3,805
1,48
8,88
1,48

PARCIAL #1

①



$\Sigma F_x = 0$

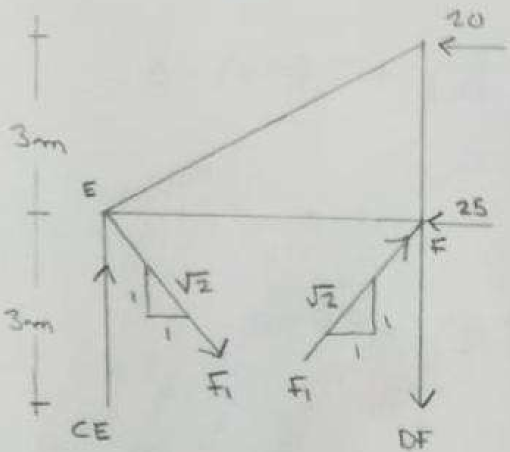
$-20 + GE\left(\frac{1}{\sqrt{2}}\right) = 0$

$\boxed{GE = 28{,}284\ KN}$ C

$\Sigma F_y = 0$

$-GF + 28{,}284\left(\frac{1}{\sqrt{2}}\right) = 0$

$\boxed{GF = 20\ KN}$ T



$\Sigma F_x = 0$

$2F_1\left(\frac{1}{\sqrt{2}}\right) - 20 - 25 = 0$

$F_1 = 31{,}819\ KN$

$\boxed{ED = 31{,}819\ KN}$ T

$\boxed{CF = 31{,}819\ KN}$ C

$\Sigma M_F = 0$

$3(20) - 3CE + 3(31{,}82)\left(\frac{1}{\sqrt{2}}\right) = 0$

$\boxed{CE = 42{,}499\ KN}$ C

$\Sigma M_E = 0$

$3(20) + 3(31{,}82)\left(\frac{1}{\sqrt{2}}\right) - 3DF = 0$

$\boxed{DF = 42{,}499\ KN}$ T

REACCIONES

$\Sigma M_B = 0$

$-3A_y + 6(25) + 9(20) = 0$

$\boxed{A_y = 110\ KN \uparrow}$

$\Sigma F_y = 0$

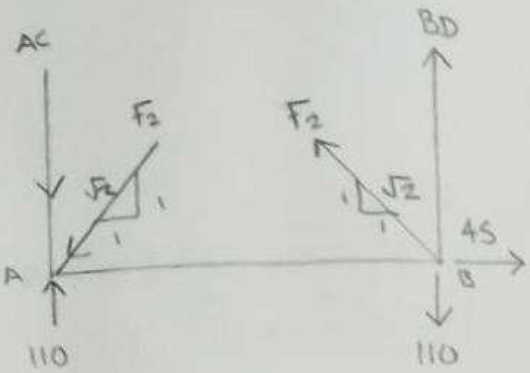
$110 - B_y = 0$

$\boxed{B_y = 110\ KN \downarrow}$

$\Sigma F_x = 0$

$B_x - 20 - 25 = 0$

$\boxed{B_x = 45\ KN \rightarrow}$



$\Sigma F_x = 0$

$45 - 2F_2\left(\frac{1}{\sqrt{2}}\right) = 0$

$\boxed{F_2 = 31{,}819\ \text{KN}}$

$\boxed{AD = 31{,}819\ \text{KN}}$ C

$\boxed{BC = 31{,}819\ \text{KN}}$ T

$\circlearrowleft\!+\ \Sigma M_A = 0$

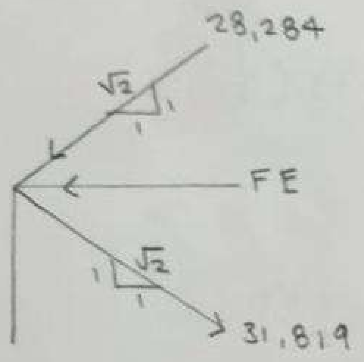
$3BD + 3(31{,}82)\left(\frac{1}{\sqrt{2}}\right) - 3(110) = 0$

$\boxed{BD = 87{,}499\ \text{KN}}$ T

$\circlearrowleft\!+\ \Sigma M_B = 0$

$-3AC - 3(31{,}82)\left(\frac{1}{\sqrt{2}}\right) + 3(110) = 0$

$\boxed{AC = 87{,}499\ \text{KN}}$ C

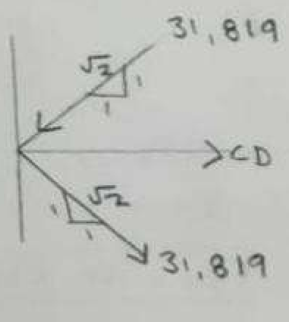


$\Sigma F_x = 0$

$-28{,}284\left(\frac{1}{\sqrt{2}}\right) + 31{,}82\left(\frac{1}{\sqrt{2}}\right) - FE = 0$

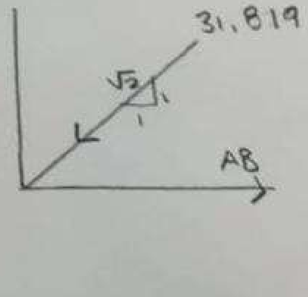
$-20 + 22{,}5 - FE = 0$

$\boxed{FE = 2{,}499\ \text{KN}}$ C



$\Sigma F_x = 0$

$\boxed{CD = 0}$



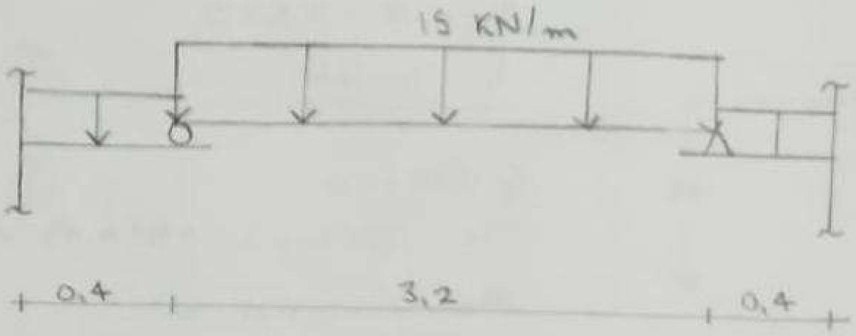
$\Sigma F_x = 0$

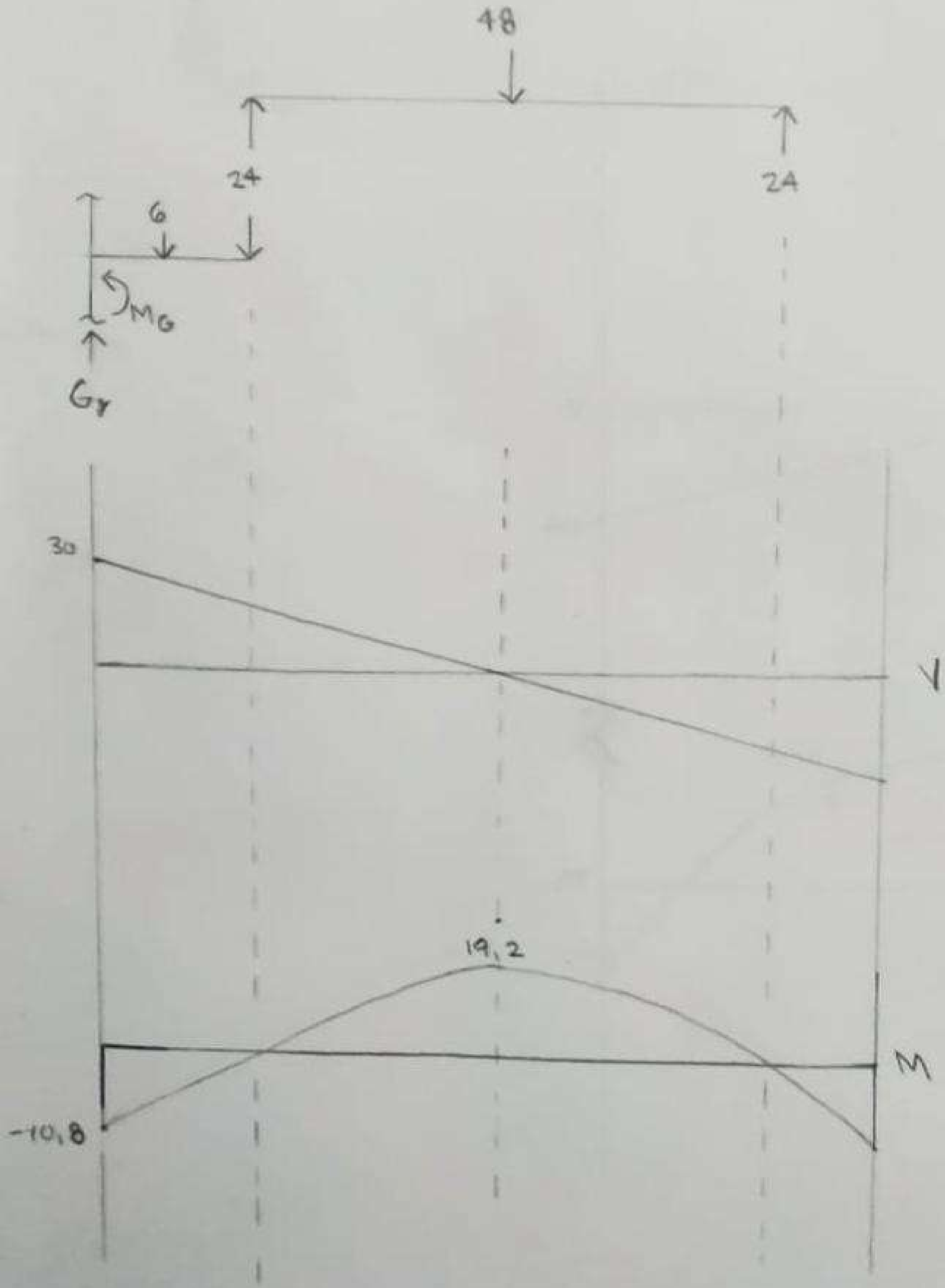
$AB - 31{,}82\left(\frac{1}{\sqrt{2}}\right) = 0$

$\boxed{AB = 22{,}499\ \text{KN}}$ T

(2)

VIGA DE 4 m y 15 KN/m





$\Sigma F_y = 0$

$G_y - 6 - 24 = 0$

$\underline{G_y = 30 \text{ KN}}$

$\Sigma M_G = 0$

$M_G - 6(0,2) - 24(0,4) = 0$

$\underline{M_G = 10,8 \text{ KN} \cdot m^2}$

VIGA DE 4 m Y 30 KN/m

$$\Sigma F_y = 0$$

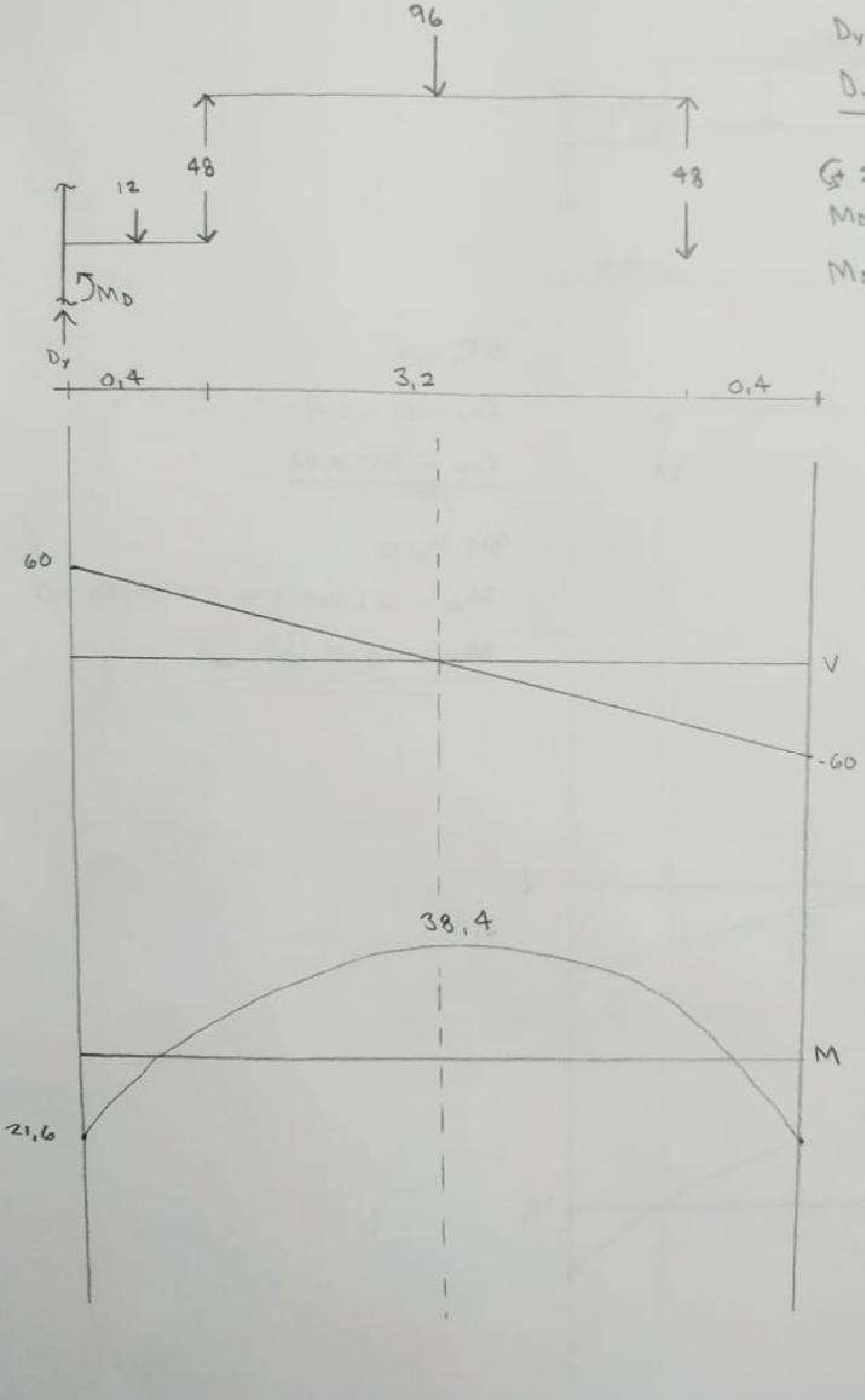
$$D_y - 12 - 48 = 0$$

$$\underline{D_y = 60\ KN}$$

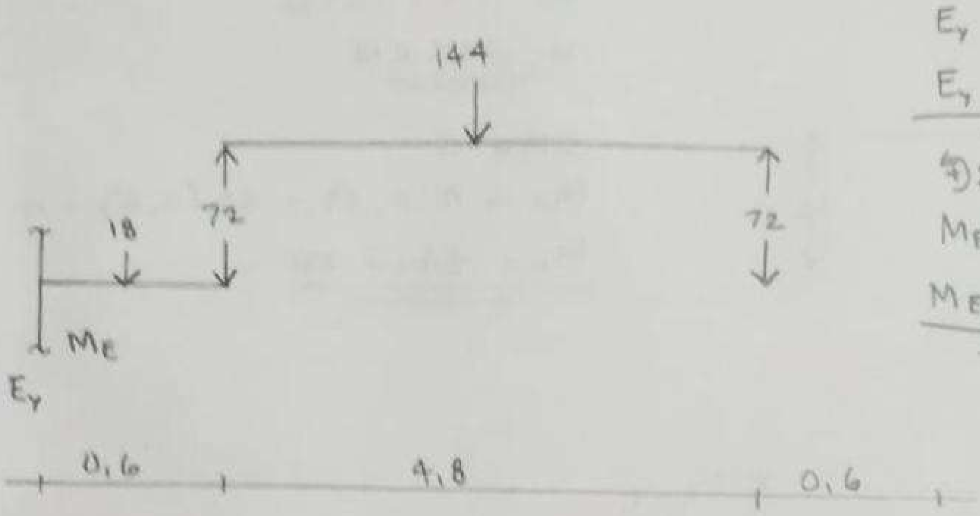
$$\circlearrowleft + \Sigma M_D = 0$$

$$M_D - 12(0{,}2) - 48(0{,}4) = 0$$

$$M_D = 21{,}6\ KN \cdot m^2$$



VIGA DE 6m Y 30 KN/m



$\Sigma F_y = 0$

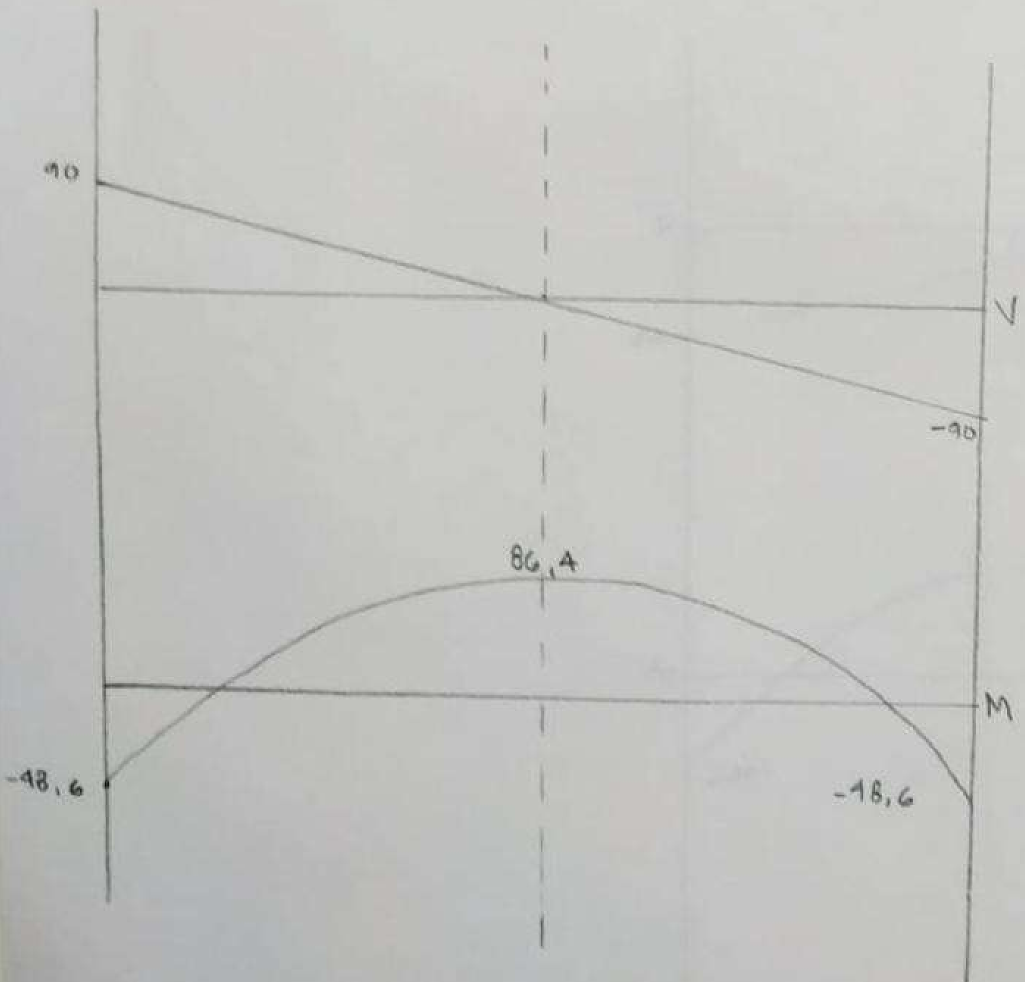
$E_y - 18 - 72 = 0$

$E_y = 90 \text{ KN}$

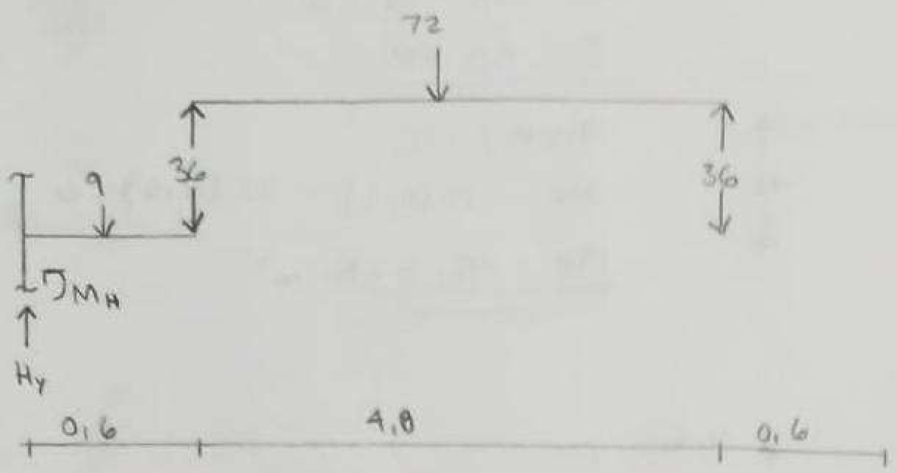
$\circlearrowleft \Sigma M_E = 0$

$M_E - 18(0,3) - 72(0,6) = 0$

$M_E = 48,6 \text{ KN} \cdot m^2$



VIGA DE 6m Y 15 KN/m



$\Sigma F_y = 0$

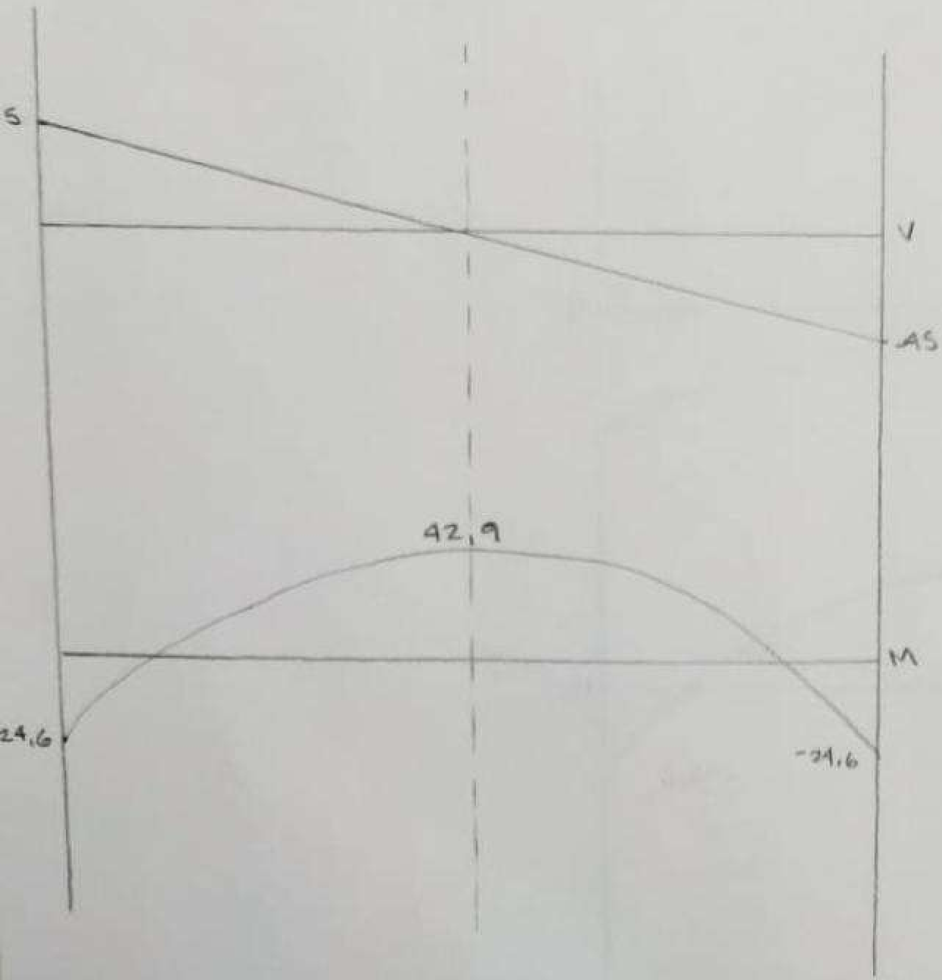
$H_Y - 9 - 36 = 0$

$H_Y = 45\ KN$

$\Sigma M_H = 0$

$M_H - 9(0,3) - 36(0,6) = 0$

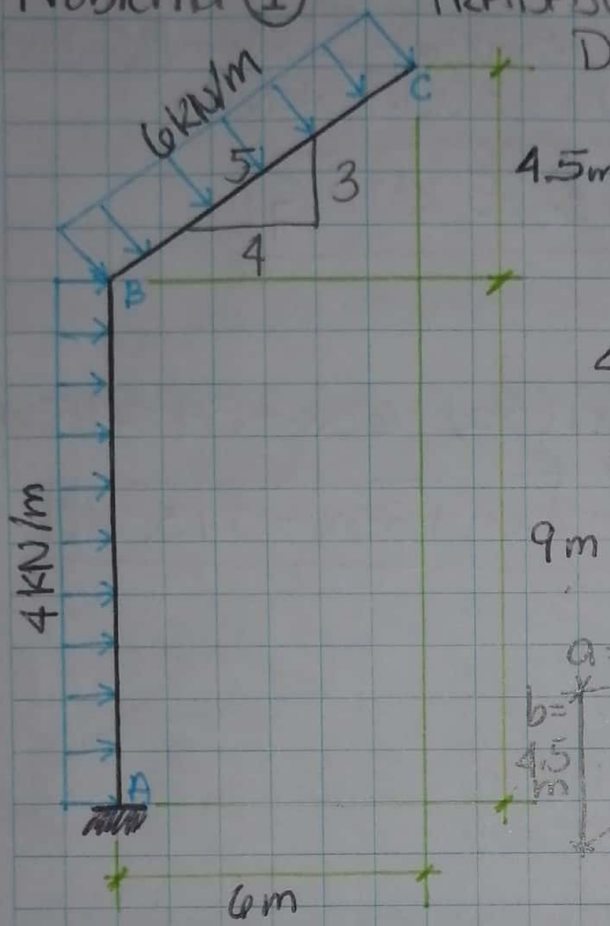
$M_H = 24,3\ KN$



Parcial n°2

Problema (1) TRABAJO VIRTUAL

Determinar la deflexión en C.



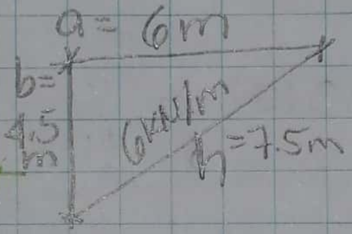
$E = 50 GPa$

$I = 3000 \times 10^6 mm^4$

$4 KN/m\,(9m) = 36 KN$

$6KN/m\,(4/5) = 4.8\ KN/m$ "Y"

$6 KN/m\,(3/5) = 3.6 KN/m$ "X"



$h = \sqrt{a^2 + b^2}$

$= \sqrt{6^2 + 4.5^2}$

$\boxed{h = 7.5m}$

$+\uparrow \Sigma F_y = 0$

$A_y - 4.8 KN/m\,(7.5m) = 0$

$A_y = 4.8 KN/m\,(7.5m)$

$\boxed{A_y = 36 KN \uparrow}$

$\xrightarrow{+} \Sigma F_x = 0$

$A_x + 36 KN + 3.6 KN/m\,(7.5m) = 0$

$A_x = -36 KN - 27 KN$

$A_x = -63 KN$

$\boxed{A_x = 63 KN \leftarrow}$

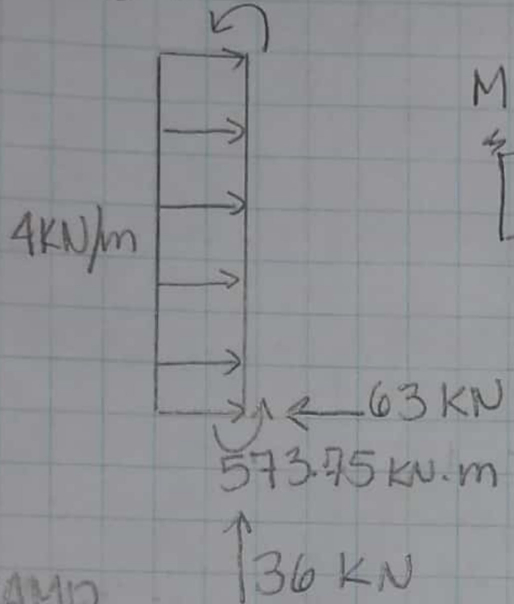
$\circlearrowleft + \sum M_A = 0$

$M_A - 36\,KN(4.5m) - 4.8\,KN/m(7.5m)(3m) - 3.6\,KN/m(7.5m)(11.25m) = 0$

$M_A = 162\,KN\cdot m + 108\,KN\cdot m + 303.75\,KN\cdot m$

$$\boxed{M_A = 573.75\ KN\cdot m \circlearrowleft}$$

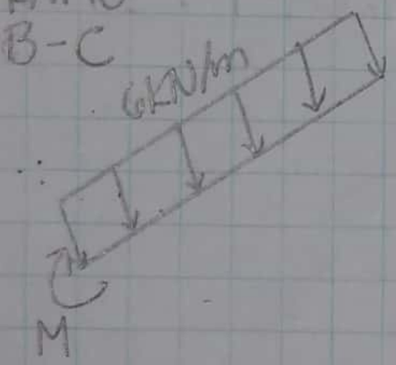
TRAMO A-B



$M + 4(x)(x/2) - 63x + 573.75 = 0$

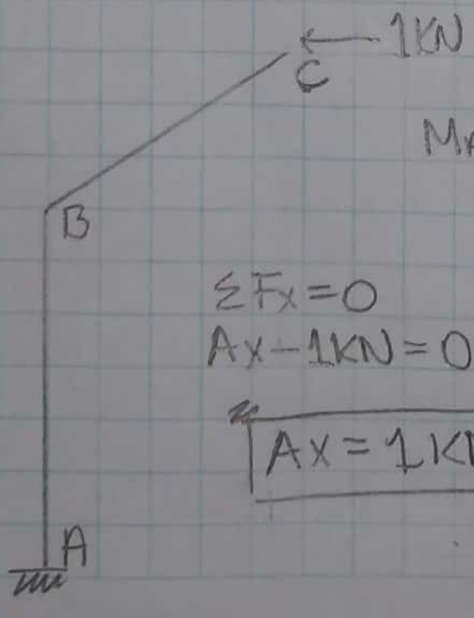
$$\boxed{M = -2x^2 + 63x - 573.75}$$

TRAMO B-C



$-M - \overset{3}{\cancel{6}}(x)(x/\cancel{2}) = 0$

$$\boxed{-3x^2 = M}$$



$\circlearrowleft \sum M_A = 0$

$M_A - 1KN(13.5m) = 0$

$$\boxed{M_A = 13.5\ KN\cdot m \circlearrowleft}$$

$\sum F_x = 0$

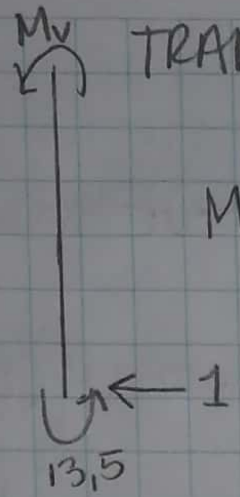
$A_x - 1KN = 0$

$$\boxed{A_x = 1KN \leftarrow}$$

$\sum F_y = 0$

$$\boxed{A_y = 0}$$

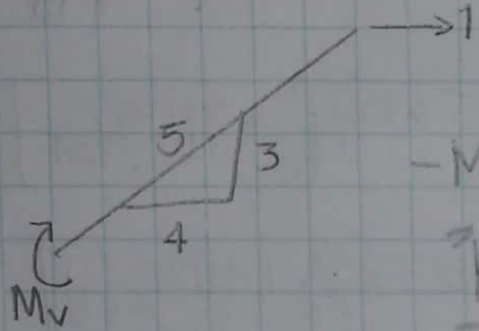
Scanned with CamScanner



TRAMO A-B

$$Mv - x + 13{,}5\,KN \cdot m = 0$$

$$\boxed{Mv = x - 13{,}5\,KN \cdot m}$$



TRAMO B-C

$$-Mv - 3/5\,x = 0$$

$$\boxed{-3/5\,x = Mv}$$

$$E = 50\,GPa \approx 50 \times 10^{6}\,kPa \qquad I = 3000 \times 10^{6}\,mm^{4} \approx 3 \times 10^{-3}\,m^{4}$$

$$EI = 150\,000\,KN \cdot m^{2}$$

SECCIÓN	ORIGEN	LIMITE	M (k·m)	Mv (KN·m)
A – B	A	0 – 9	$-2x^2+63x-573{,}75$	$x-13{,}5$
B – C	C	0 – 7,5	$-3x^2$	$-3/5\,x$

Scanned with CamScanner

$$1\,\Delta C = \sum \int_0^x \frac{M_v\, M}{EI}\, dx$$

$$= \frac{1}{EI}\left[\int_0^9 (-2x^2+63x-573.75)(x-13.5)\,dx + \int_0^{7.5} (-3x^2)(-3/5\,x)\,dx\right.$$

$$= \frac{1}{EI}\left[\int_0^9 (-2x^3+63x^2-573.75x+27x^2-850.5x+7745.62)\,dx + \int_0^{7.5} \left(9x^3/5\right) dx\right]$$

$$= \frac{1}{EI}\left[\int_0^9 (-2x^3+90x^2-1424.25x+7745.62)\,dx + \left(9x^4/20\right)\Big|_0^{7.5}\right]$$

$$= \frac{1}{EI}\left[\left(-x^4/2+30x^3-5697x^2/8+7745.62x\right)\Big|_0^9 + 91125/64\right]$$

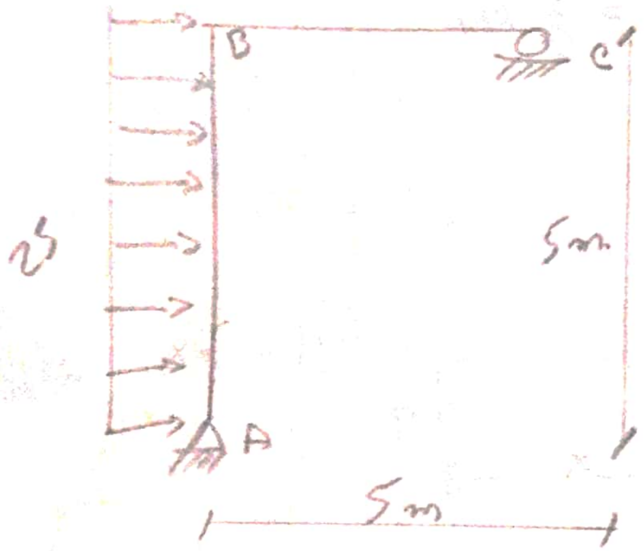
$$= \frac{1}{EI}\left[\left(-6561/2+21870-461457/8+557685/8+91125/64\right)\right]$$

$$\Delta C = \frac{1}{EI}\left(32041.828\ kN\cdot m^3\right)$$

$$\Delta C = \frac{32041.828\ kN\cdot m^3}{150\,000\ kN\cdot m^2}$$

$$\Delta C = 0.214\,m \Rightarrow \boxed{\Delta C = 214\,mm}$$

Problema 2.



$\Sigma M_A = 0$

$-25\left(\frac{5}{2}\right)^2 + R_C(5) = 0$

$R_C = 62{,}5\, kN \uparrow$

$\Sigma F_x = 0$

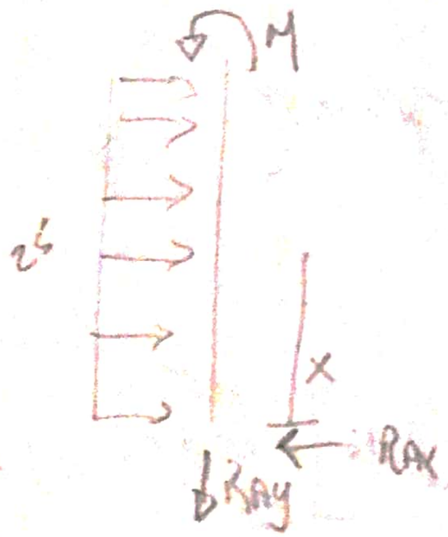
$R_{Ax} = -25(5)$

$R_A = 125\, kN$

$\Sigma F_y = 0$

$R_{Ay} = -62{,}5\, kN \downarrow$

AB $0 \leq x \leq 5$

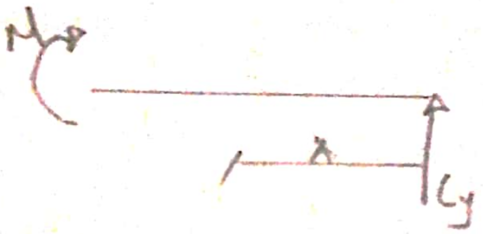


$\Sigma M = 0$

$M = -125x - 25\frac{x^2}{2}$

$\frac{\partial M}{\partial P} = \frac{1}{5}x$

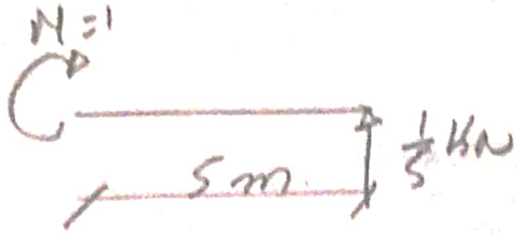
BC



$\Sigma M = 0$

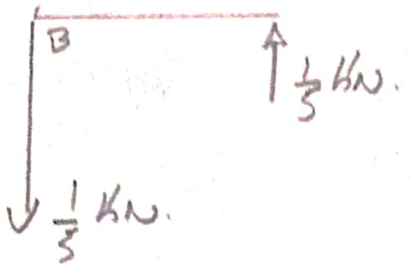
$M = 62{,}5x$

$\frac{\partial M}{\partial P} = 0$



$R(5) = 1\,kN.m$

$R = \frac{1}{5}\,kN.$



$M = \frac{1}{5}x$

Ses.	Direccio	limites	M	
AB	A	0-5	$-125x-25\frac{x^2}{2}$	$\frac{x}{5}$
CB	C	0-5	$62,5x$	0

$$\Delta_C = \frac{1}{EI}\left[\int_0^5 \left(\frac{1}{5}x\right)\left(125x - \frac{25x^2}{2}\right)dx\right] + \int_0^5 2,5x\,(0)\,dx$$

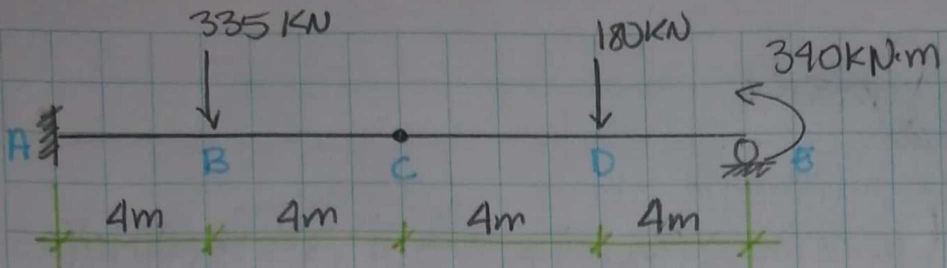
$$\Delta_C = \frac{1}{50000}\left[651,05\right]$$

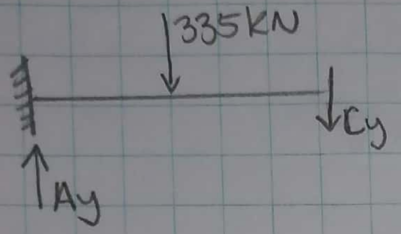
$$\Delta_C = 13\,mm \Rightarrow 0,0125\,m.$$

$E = 50 \times 10^6\,kPa.$

$I = 1 \times 10^6\,mm^4 = 1 \times 10^{-3}\,m^4$

Problema 3





$\circlearrowleft^{+} \sum M_A = 0$

$M_A - 335\,kN(4m) - 132{,}5\,kN(8m) = 0$

$M_A = 1340\,kN\cdot m + 1060\,kN\cdot m$

$\boxed{M_A = 2400\,kN\cdot m\ \circlearrowleft}$

$+\uparrow \sum F_y = 0$

$C_y + 47{,}5 - 180\,kN = 0$

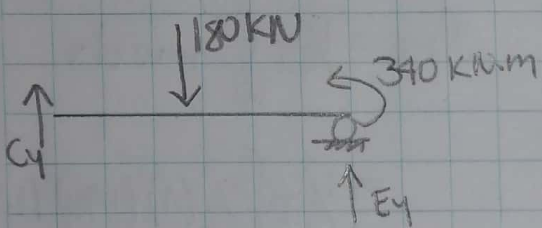
$\boxed{C_y = 132{,}5\,kN\uparrow}$

$+\uparrow \sum F_y = 0$

$A_y - 335\,kN - 132{,}5\,kN = 0$

$A_y = 335\,kN + 132{,}5\,kN$

$\boxed{A_y = 467{,}5\,kN\uparrow}$

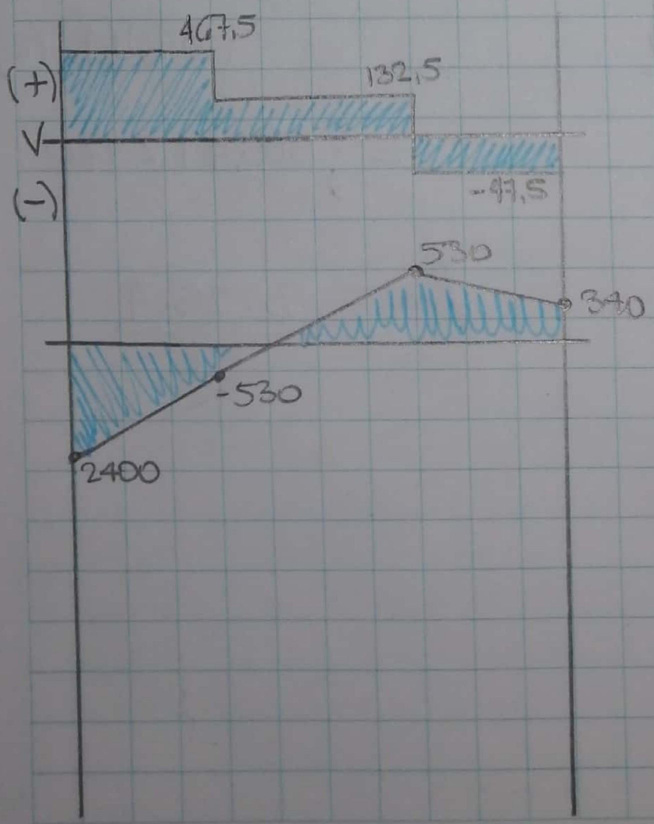


$\circlearrowleft^{+} \sum M_c = 0$

$E_y(8m) - 180(4m) + 340$

$E_y = \dfrac{180\,kN(4m) + 340}{8m}$

$\boxed{E_y = 47{,}5\,kN\uparrow}$



$$t_{CE} = \frac{1}{EI}\int_0^x m x\,dx$$

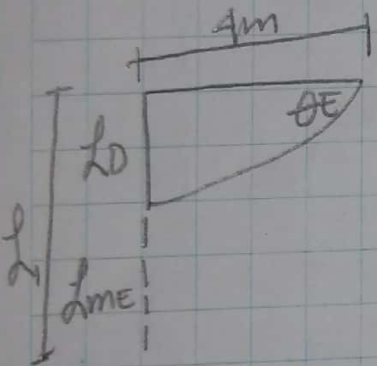
$$= \frac{1}{EI}\int \frac{1}{2}(4)(530)\left(\frac{2}{3}(4)\right) + 340(4)(6) + \frac{1}{2}(4)(190)(5.33)$$

$$= \frac{1}{EI}\int 2.8\times10^3 + 8160 + 2.02\times10^3$$

$$= \frac{13040}{EI}$$

$$\tan\theta_E = \frac{t_{CE}}{8}$$

$$\theta_E = \frac{13040}{8EI}$$



$$t_1 = 4\theta_E$$

$$= 4\left(\frac{13040}{8EI}\right)$$

$$= \frac{13040}{2EI}$$

$$t_{mE} = \frac{1}{EI}\int_d^x m x\,dx$$

$$= \frac{1}{EI}\int \frac{1}{2}(4)(190)(1.33) + 340(4)(2)$$

$$= \int 505.4 + 2720$$

$$t_{mE} = \frac{3225.4}{EI}$$

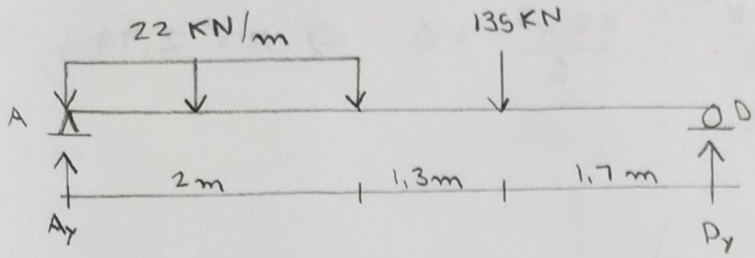
$$t_D = t_1 - t_{mE}$$

$$= \frac{13040}{2EI} - \frac{3225.4}{EI}$$

$$t_D = \frac{3294.6}{EI}$$

PARCIAL #2

PROBLEMA #4



$\Sigma M_A = 0$

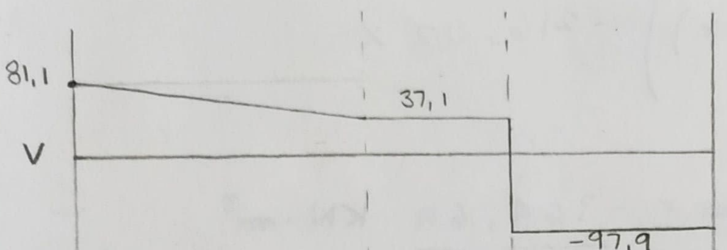
$-22(2)(1) - 135(3,3) + 5D_y = 0$

$D_y = 97,9\ KN \uparrow$

$\Sigma F_y = 0$

$-22(2) - 135 + 97,9 + A_y = 0$

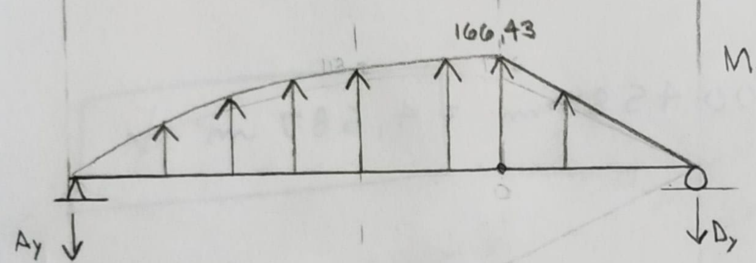
$A_y = 81,1\ KN \uparrow$



$M_1 = 81,1x - \frac{22x^2}{2}$

$M_2 = 81,1x - 44(x-1)$

$M_3 = 81,1x - 44(x-1) - 135(x-3,3)$



$H = \int_0^2 \left(-\frac{22x^2}{2} + 81,1x\right)x = 172,26$

$\Sigma M_A = 0$

$-D_y(5) + \frac{1}{2}(166,43)(1,7)\left(\frac{1}{3}(1,7) + 3,3\right) + 118,2(1,3)(2,65)$

$+ \frac{48,23}{2}(1,3)\left(\left(\frac{2}{3}\right)(1,3) + 2\right) + 172,26 = 0$

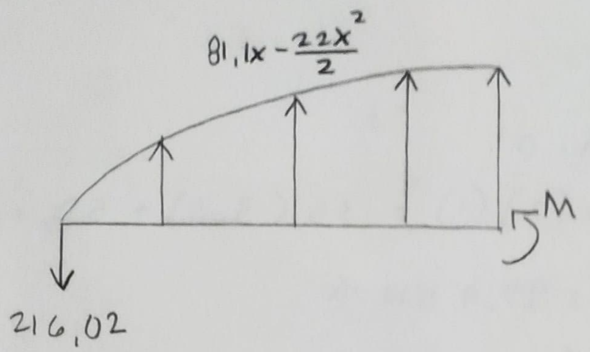
$-5D_y + 547 + 407,2 + 89,87 + 172,26 = 0$

$D_y = 243,27\ KN \downarrow$

$\Sigma F_y = 0$

$-A_y - \frac{48,23}{2}(1,3) - 118,2(1,3) - \frac{1}{2}(166,43)(1,7) - 132,87 + 243,27 = 0$

$A_y = -216,02\ KN \downarrow$



$\Sigma F = 0$

$$\left(\int_0^x 81{,}1x - \frac{22x^2}{2}\right) - 216{,}02 = 0$$

$$\frac{81{,}1x^2}{2} - \frac{22x^3}{6} = 0 \Rightarrow x = 2{,}79\ m$$

$$\Delta_{max} = \left(\int_0^{2{,}79} \left(81{,}x - \frac{22}{2}x^2\right)(2{,}79 - x)\right) - 216{,}02\,x$$

$$\Delta_{max} = 238 - 602{,}69 \Rightarrow \Delta_{max} = \frac{-364{,}69\ KN \cdot m^3}{EI}$$

$$\Delta_{max} = \frac{-364{,}69 \times 10^3}{15 \times 10^9 (5300 \times 10^{-6})} = 0{,}004587\ m = \boxed{4{,}587\ mm \downarrow}$$

ANÁLISIS DE ESTRUCTURAS ESTÁTICAMENTE INDETERMINADAS (HIPERESTÁTICAS) POR EL MÉTODO DE FUERZAS (FLEXIBILIDADES).

- ALEXIS D. CUBILLA M. / 4-801-1260
- ZAHARANI P. GOZAINE J. / 4-794-118
- LUIS A. HERRERA / 4-796-1859
- JOEL A. PÉREZ V. / 4-800-858

MÉTODO DE LAS FLEXIBILIDADES

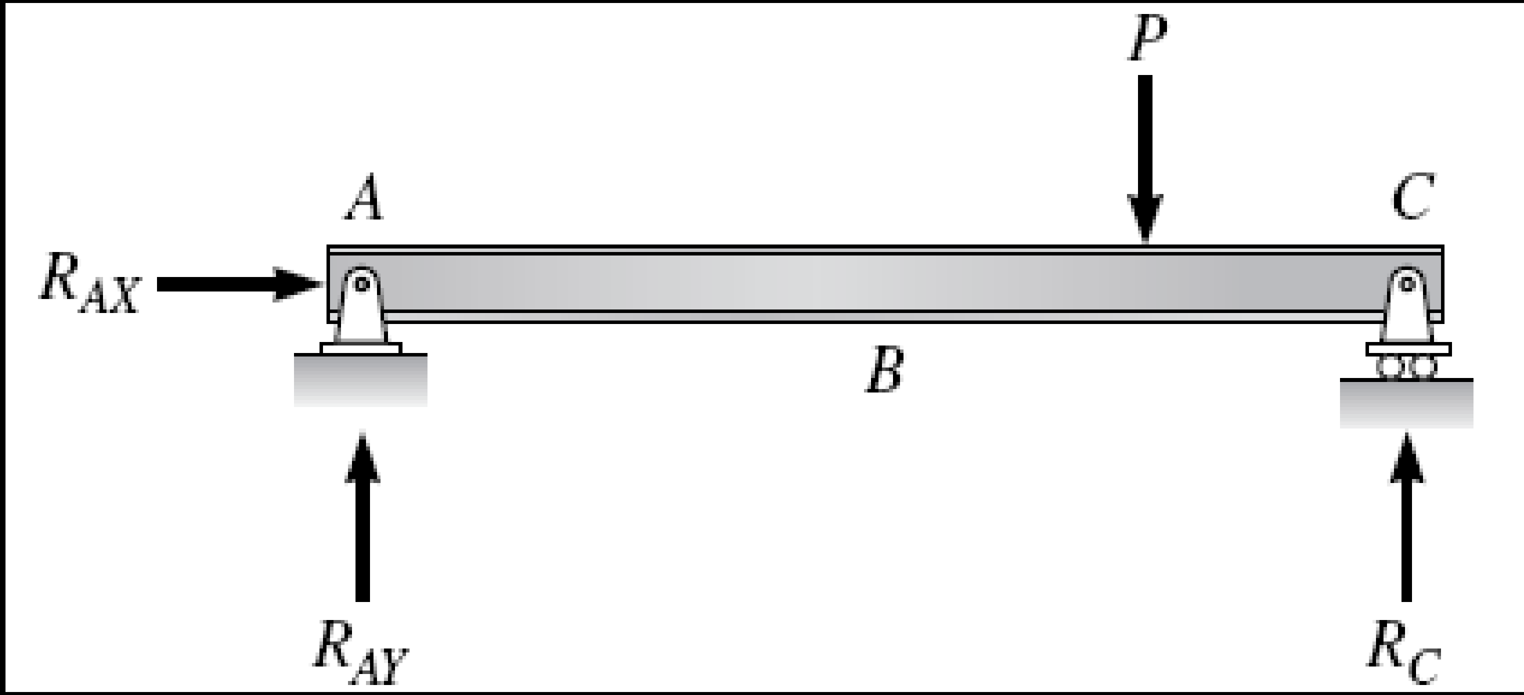
INTRODUCCIÓN

ACERCA DE ESTE MÉTODO

- Este método, también llamado método de la superposición, es utilizado para analizar estructuras indeterminadas lineales.
- Todos los métodos deben satisfacer los requisitos de equilibrio y compatibilidad.

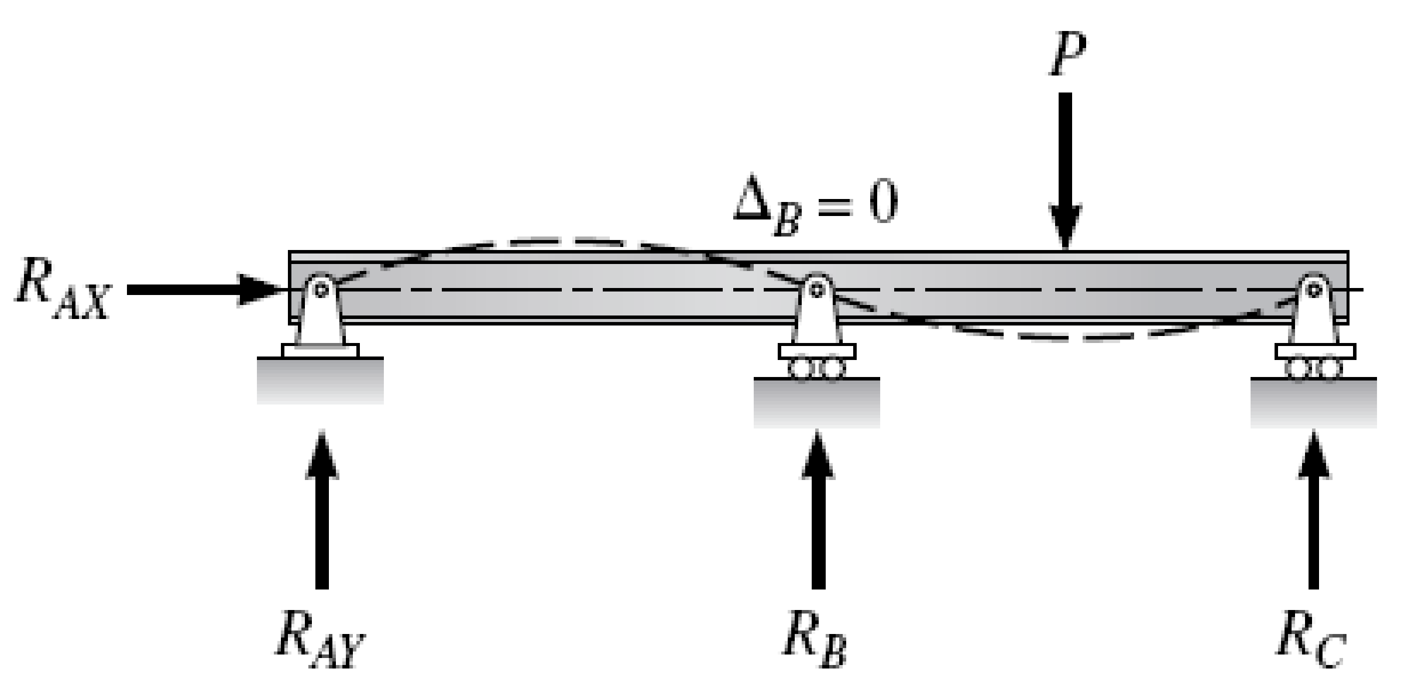
PASO CLAVE

- Una estructura indeterminada se reemplazará por una estructura determinada estable.
- Esta estructura es llamada “estructura base o liberada”
- Se establece a partir de la estructura original imaginando que se remueven temporalmente restricciones tales como los apoyos.



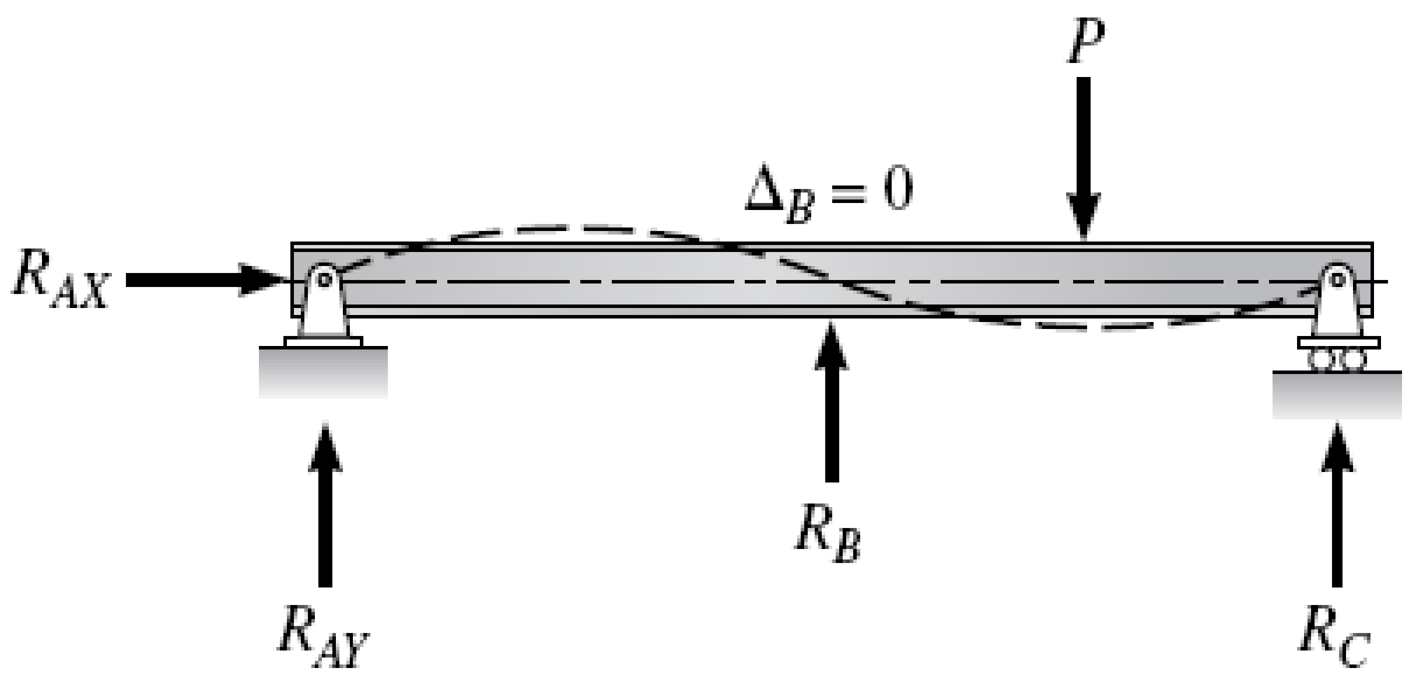
CONCEPTO DE REDUNDANTE

- Para conformar a una estructura estable, se precisan de al menos tres restricciones, que no sean equivalentes a un sistema de fuerzas paralelas o concurrentes.
- Así, se evita el desplazamiento como cuerpo rígido bajo cualquier condición de carga.



CONCEPTO DE REDUNDANTE

- En muchas estructuras la designación de una reacción particular como redundante, es arbitraria.



CONCEPTO DE REDUNDANTE

- En esta imagen se muestra la estructura liberada de la viga anterior (cambia el rodillo en *B* por una reacción).

FUNDAMENTOS DEL MÉTODO DE LAS FLEXIBILIDADES.

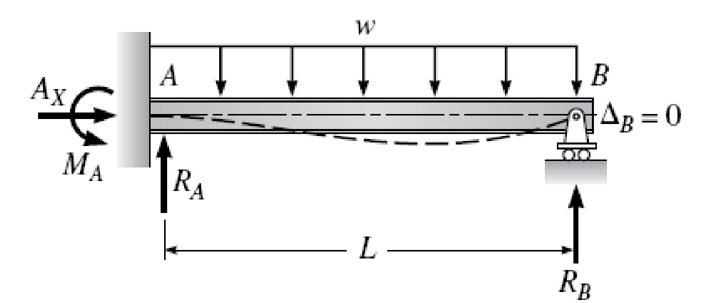
- En este método, uno se imagina que el número suficiente de redundantes (apoyos, por ejemplo) se remueven de la estructura indeterminada para producir una estructura determinada estable liberada.
- Después, se aplican las cargas de diseño (las cargas originales), y las redundantes, de las cuales aún no se conoce su magnitud.

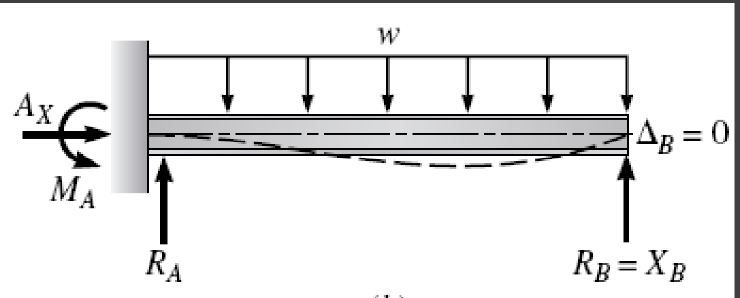
FUNDAMENTOS DEL MÉTODO DE LAS FLEXIBILIDADES

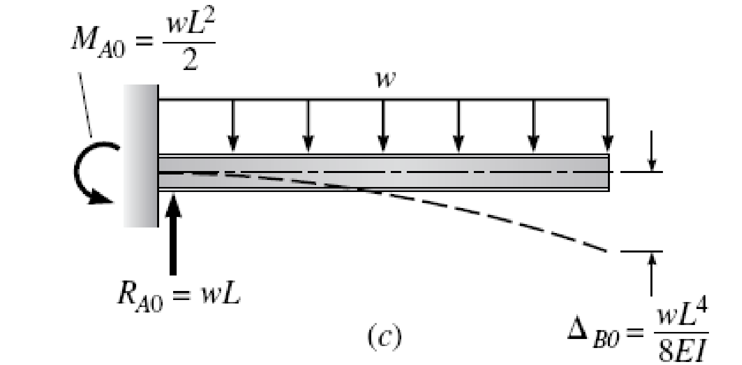
- Después analizamos la estructura determinada liberada para las cargas aplicadas y las redundantes. En este paso, el análisis se divide en dos casos separados: (1) para las cargas reales aplicadas y (2) para cada redundante desconocida. Para cada caso, las deflexiones se calculan en cada punto donde actúe una redundante.

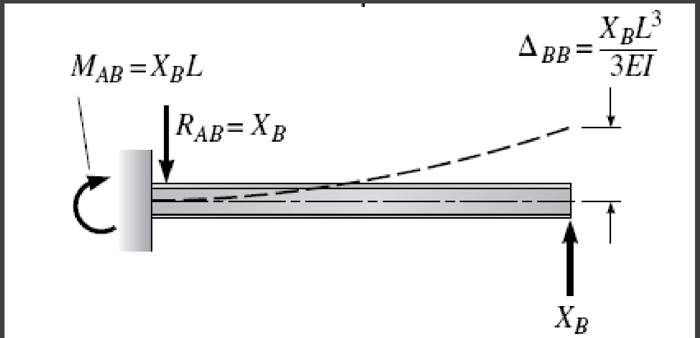
FUNDAMENTOS DEL MÉTODO DE LAS FLEXIBILIDADES

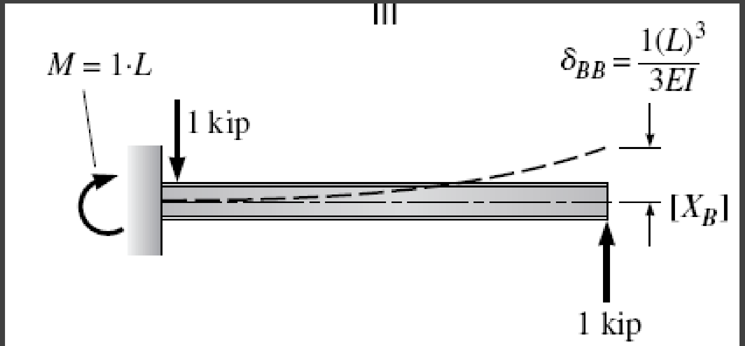
- Para resolver para las redundantes, las deflexiones se suman en cada punto donde actúa una redundante y se igualan al valor conocido de la deflexión.
- Una vez que se determinan los valores de las redundantes, el balance de la estructura se puede analizar con las ecuaciones de la estática.


w
A
B
A_X
M_A
R_A
$\Delta_B = 0$
L
R_B
(a)


w
A_X
M_A
$\Delta_B = 0$
R_A
$R_B = X_B$
(b)


$M_{A0} = \frac{wL^2}{2}$
w
$R_{A0} = wL$
$\Delta_{B0} = \frac{wL^4}{8EI}$
(c)


$M_{AB} = X_B L$
$R_{AB} = X_B$
$\Delta_{BB} = \frac{X_B L^3}{3EI}$
X_B
(d)


$M = 1 \cdot L$
1 kip
$\delta_{BB} = \frac{1(L)^3}{3EI}$
$[X_B]$
1 kip
(e)

FUNDAMENTOS DEL MÉTODO DE LAS FLEXIBILIDADES

- Esta condición de geometría o compatibilidad se puede expresar como:

$$\Delta_B = 0$$

- Podemos escribir la ecuación anterior como:

$$\Delta_{B0} + \Delta_{BB} = 0$$

- De tal forma, obtenemos:

$$-\frac{wl^4}{8EI} + \frac{X_B L^3}{3EI} = 0$$

- Despejando para XB se obtiene:

$$X_B = \frac{3wL}{8}$$

- Por ejemplo, la reacción vertical en el apoyo A vale:

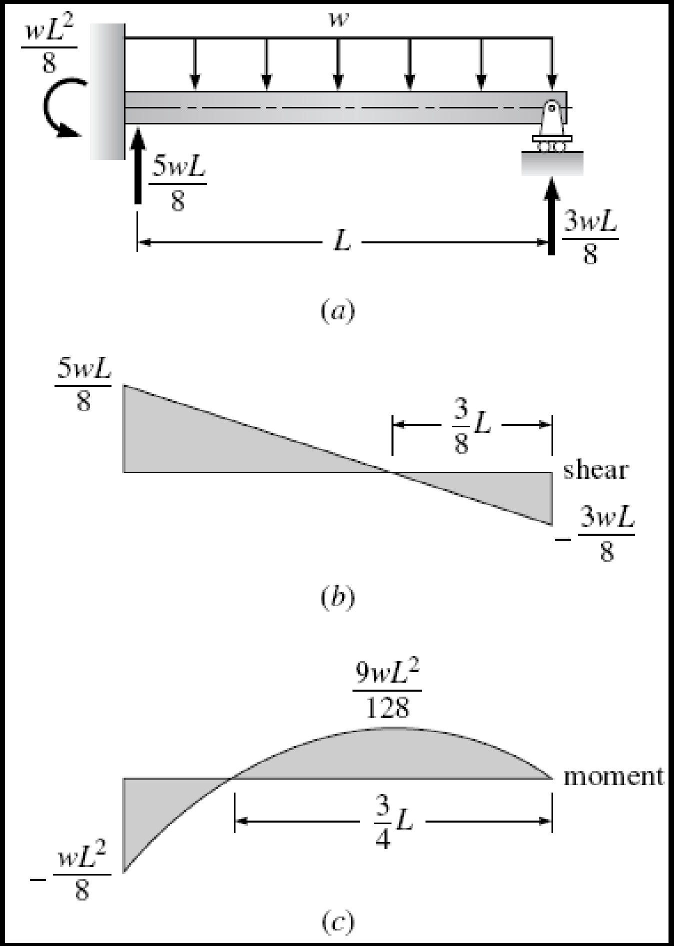
$$R_A = wL - X_B = wL - \frac{3wL}{8} = \frac{5wL}{8}$$

- Similarmente, el momento en A es igual a:

$$M_A = \frac{wL^2}{2} - X_B L = \frac{wL^2}{2} - \frac{3wL(L)}{8} = \frac{wL^2}{8}$$

FUNDAMENTOS DEL MÉTODO DE LAS FLEXIBILIDADES

- Ya que se conocen las reacciones, se pueden dibujar los diagramas de corte y de momento.



FUNDAMENTOS DEL MÉTODO DE LAS FLEXIBILIDADES

- Ya que las vigas en los incisos (d) y (e) son equivalentes se puede decir que:

$$\Delta_{BB} = X_B \delta_{BB}$$

Pero $\Delta_{B0} + \Delta_{BB} = 0$:

$$\Delta_{B0} + X_B \delta_{BB} = 0$$

Y:

$$X_B = -\frac{\Delta_{B0}}{\delta_{BB}}$$

- Aplicando la ecuación anterior, calculamos X_B como:

$$X_B = -\frac{\Delta_{B0}}{\delta_{BB}} = -\frac{-\frac{wL^4}{8EI}}{\frac{L^3}{3EI}} = \frac{3wL}{8}$$

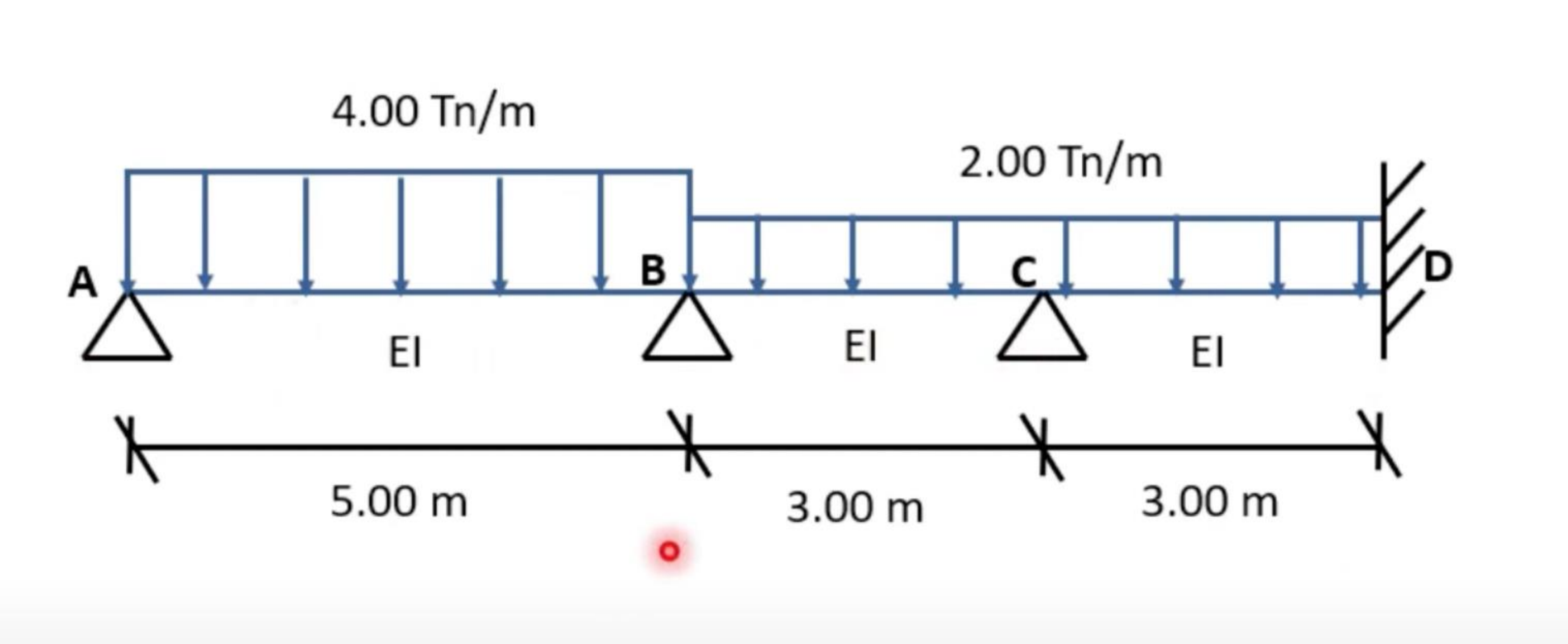
FUNDAMENTOS DEL MÉTODO DE LAS FLEXIBILIDADES

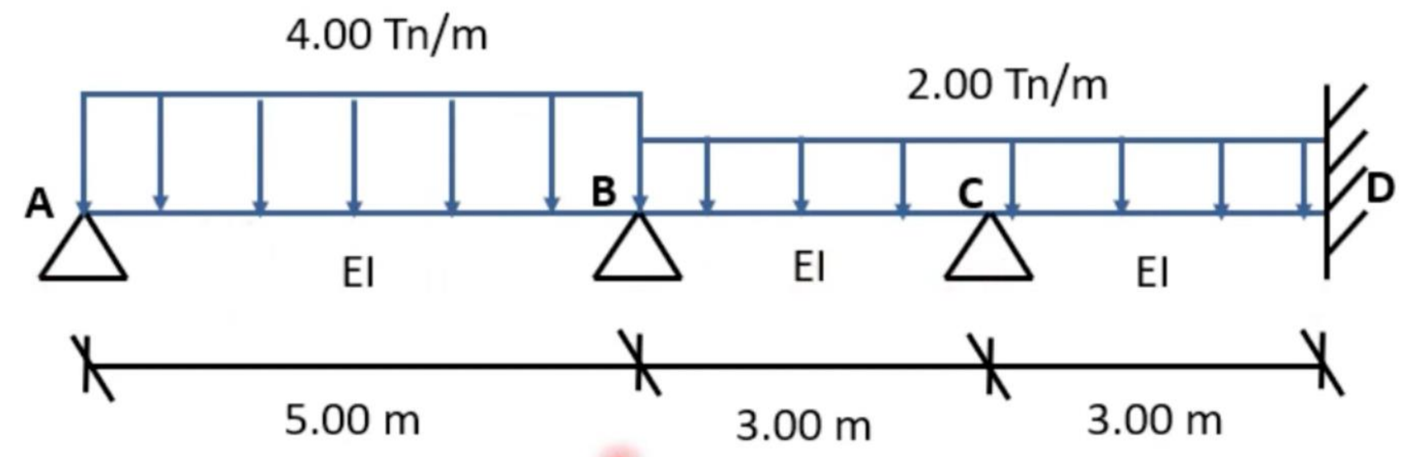
- El momento en el apoyo empotrado es igual a:

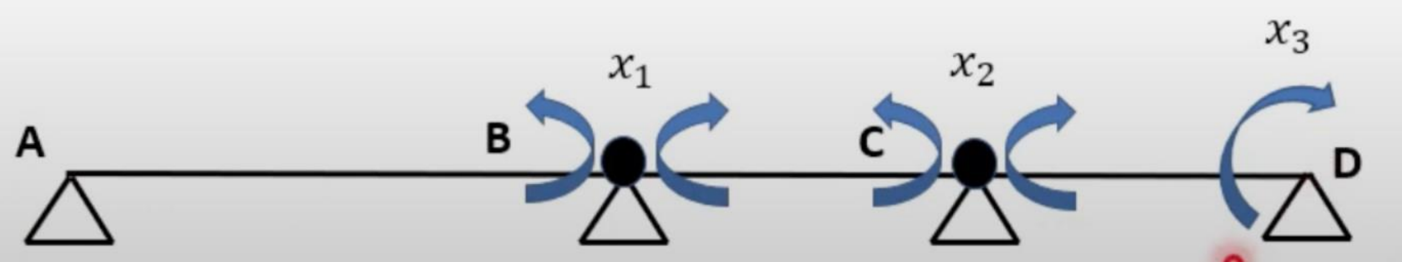
$$M_A = \frac{wL^2}{2} - (1L)X_B = \frac{wL^2}{2} - L\left(\frac{3wL}{8}\right) = \frac{wL^2}{8}$$

EJEMPLO #1

VIGA HIPERESTÁTICA POR EL METODO DE FUERZAS (FLEXIBILIDAD)

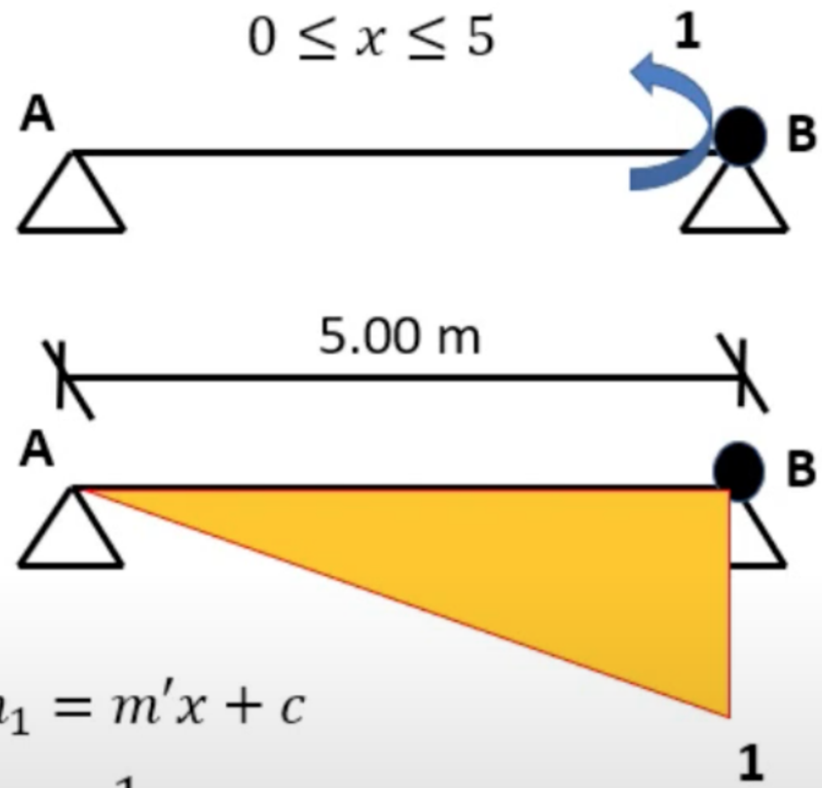






Sistema X-D

Cálculo de "m_1" ($x_1 = 1$; $x_2 = x_3 = 0$)

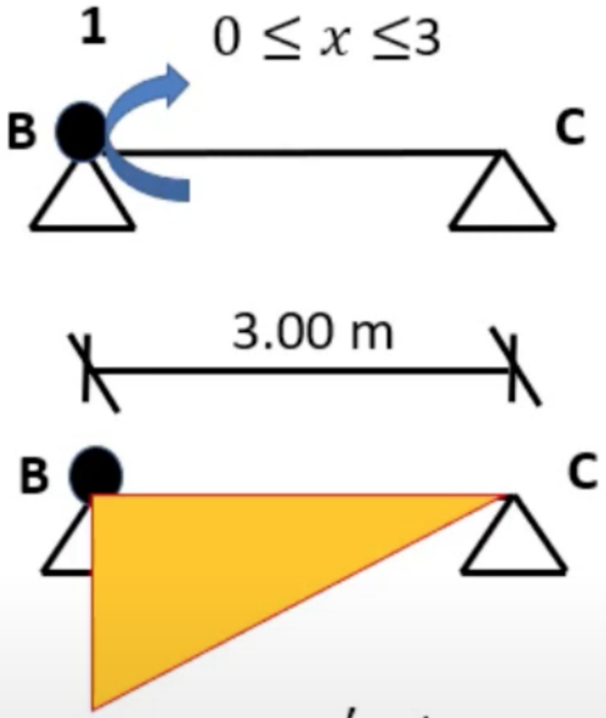


$$m_1 = m'x + c$$

$$m_1 = \frac{1}{5}x + c$$

$$(0) = \frac{1}{5}(0) + c \Rightarrow c = 0$$

$$m_1 = \frac{1}{5}x$$



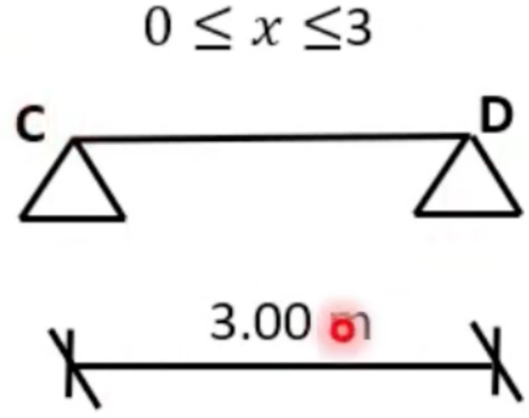
1

$$m_1 = m'x + c$$

$$m_1 = \frac{-1}{3}x + c$$

$$(1) = \frac{-1}{3}(0) + c \Rightarrow c = 1$$

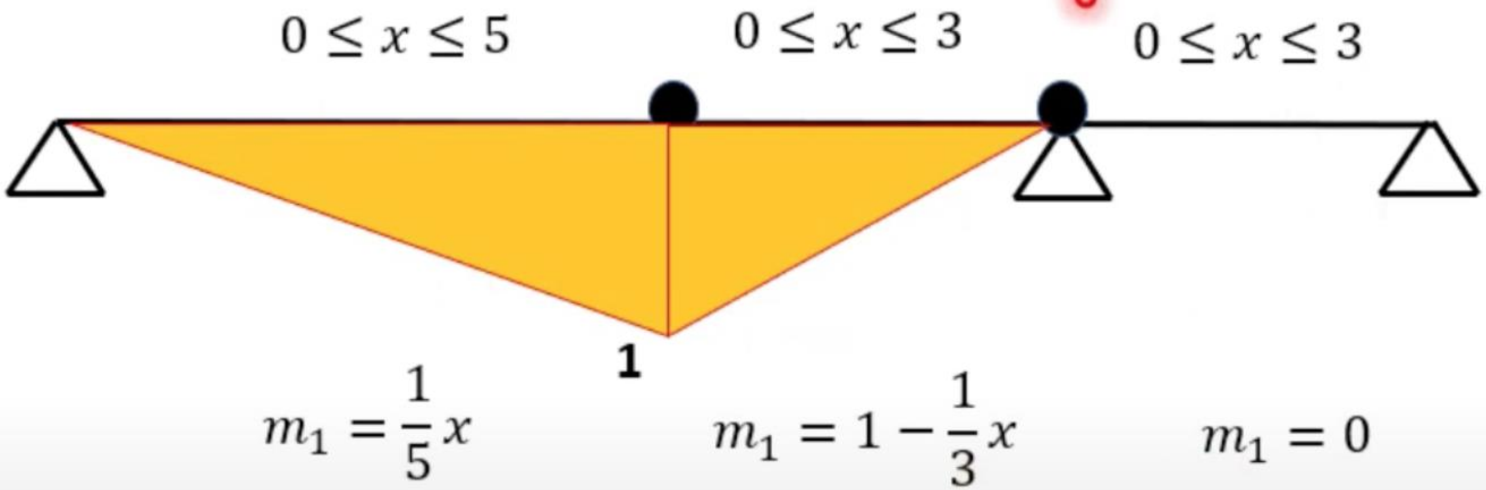
$$m_1 = 1 - \frac{1}{3}x$$



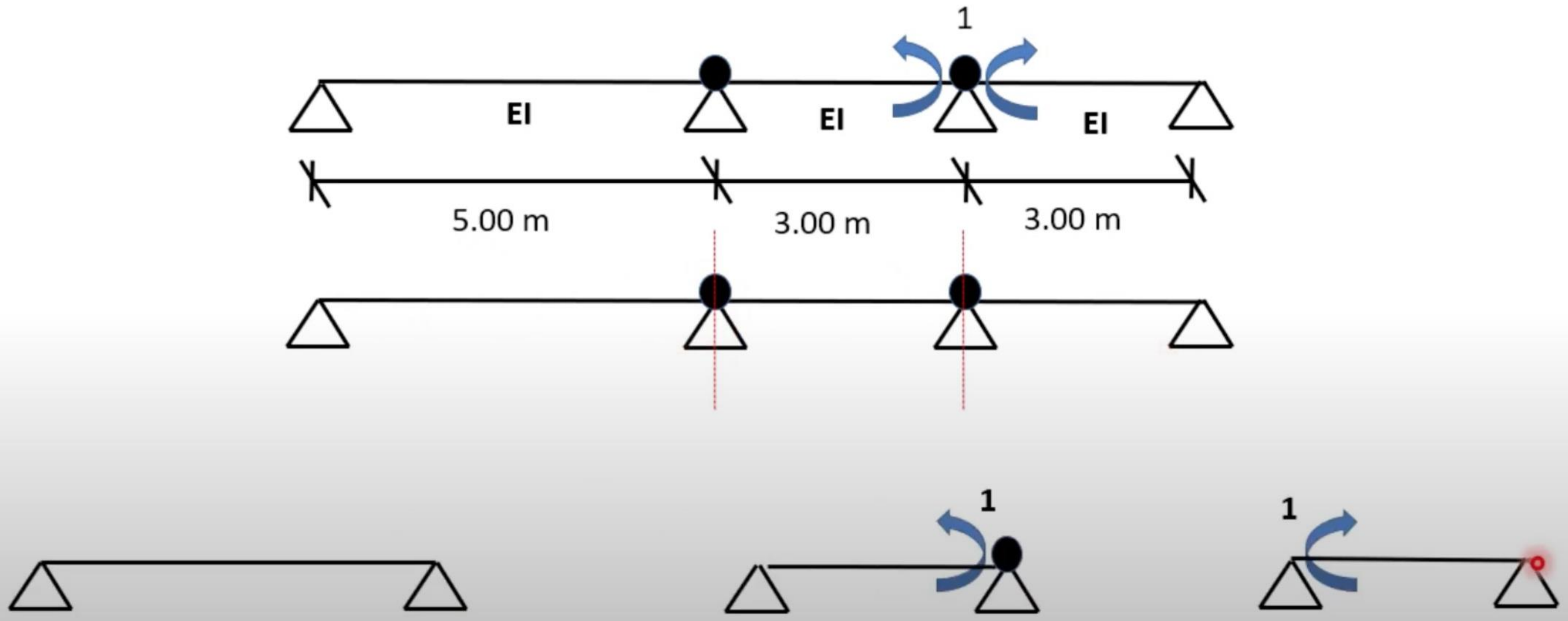
$$m_1 = m'x + c$$

$$m_1 = 0$$

Cálculo de "m_1" ($x_1 = 1$; $x_2 = x_3 = 0$)

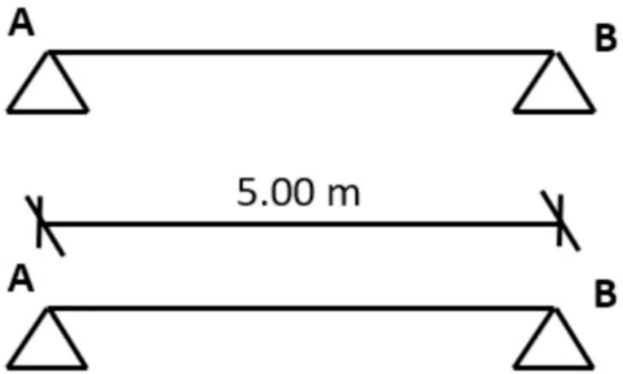


Cálculo de "m_2" ($x_2 = 1$; $x_1 = x_3 = 0$)



Cálculo de "m_2" ($x_2 = 1$; $x_1 = x_3 = 0$)

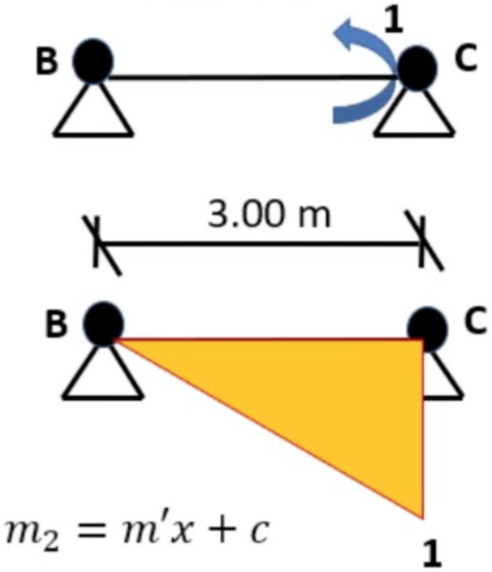
$0 \leq x \leq 5$



$$m_2 = m'x + c$$

$$m_2 = 0$$

$0 \leq x \leq 3$



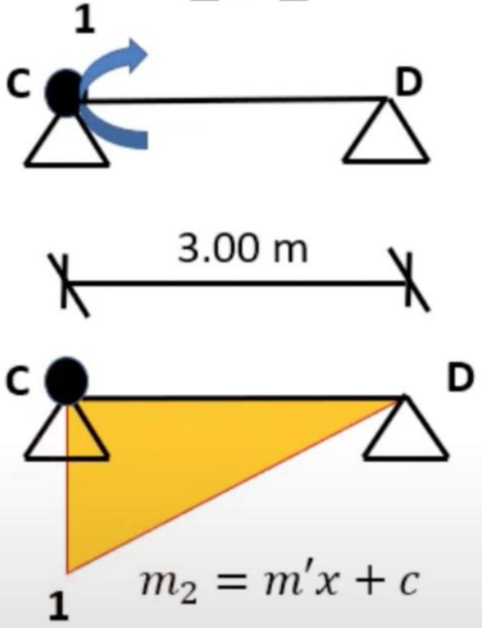
$$m_2 = m'x + c$$

$$m_2 = \frac{1}{3}x + c$$

$$(0) = \frac{1}{3}(0) + c \Rightarrow c = 0$$

$$m_2 = \frac{1}{3}x$$

$0 \leq x \leq 3$



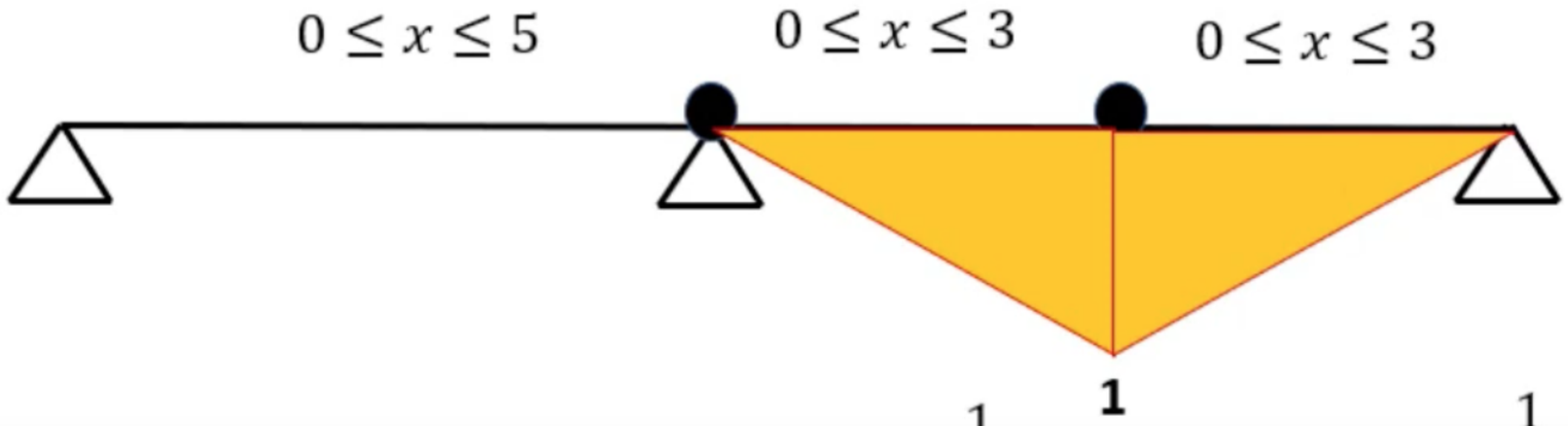
$$m_2 = m'x + c$$

$$m_2 = \frac{-1}{3}x + c$$

$$(1) = \frac{1}{3}(0) + c \Rightarrow c = 1$$

$$m_2 = 1 - \frac{1}{3}x$$

Cálculo de "m_2" ($x_2 = 1$; $x_1 = x_3 = 0$)

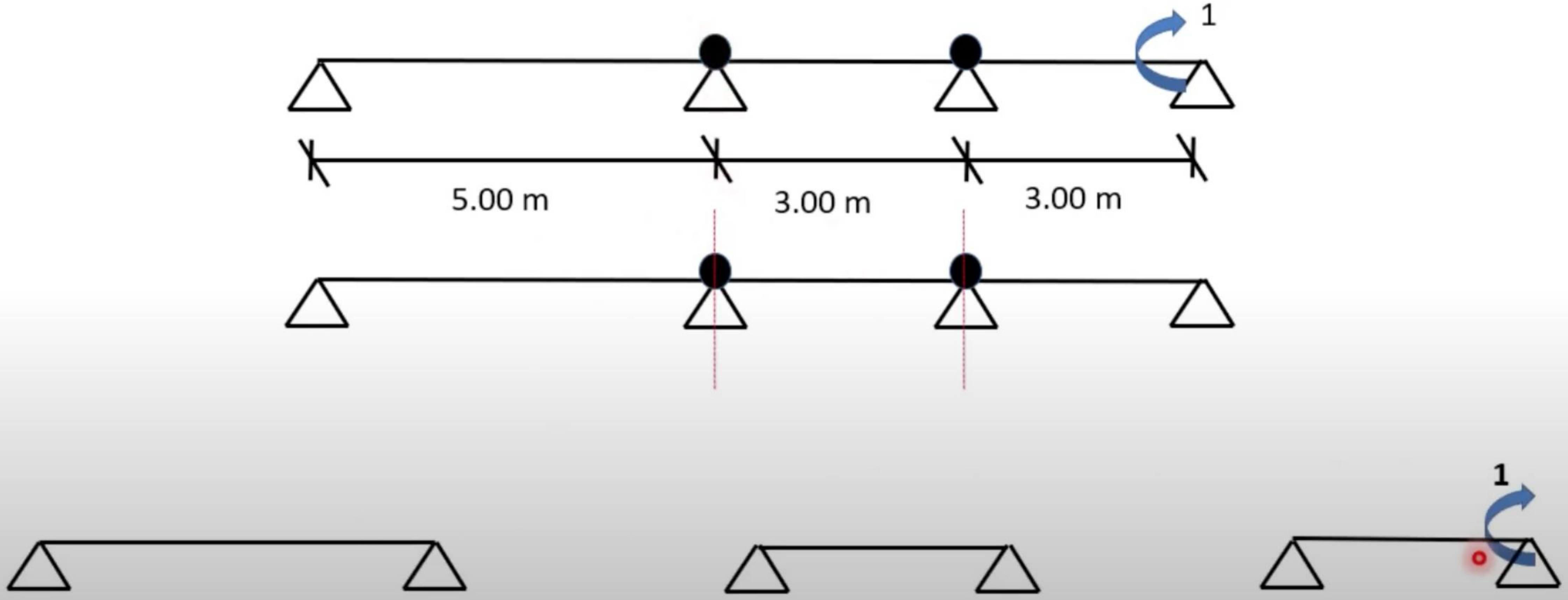


$m_2 = 0$

$m_2 = \frac{1}{3}x$

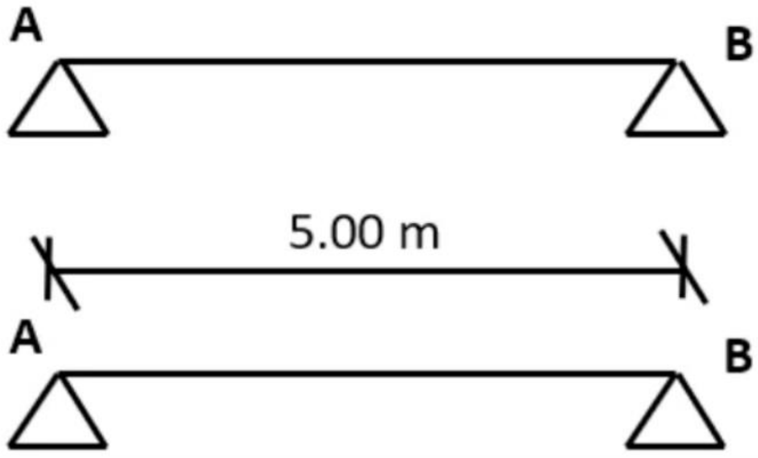
$m_2 = 1 - \frac{1}{3}x$

Cálculo de "m_3" ($x_3 = 1$; $x_1 = x_2 = 0$)



Cálculo de "m_3" ($x_3 = 1$; $x_1 = x_2 = 0$)

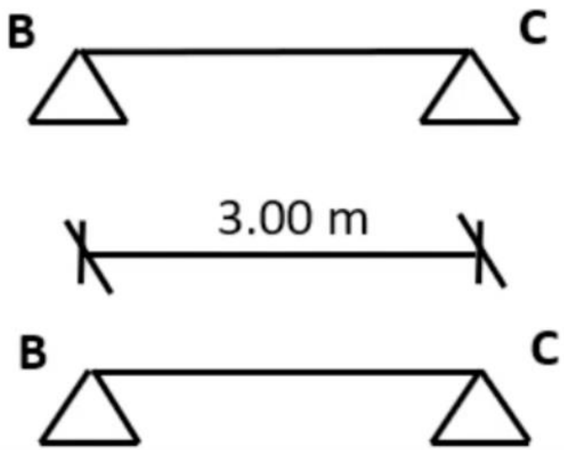
$0 \leq x \leq 5$



$$m_3 = m'x + c$$

$$m_3 = 0$$

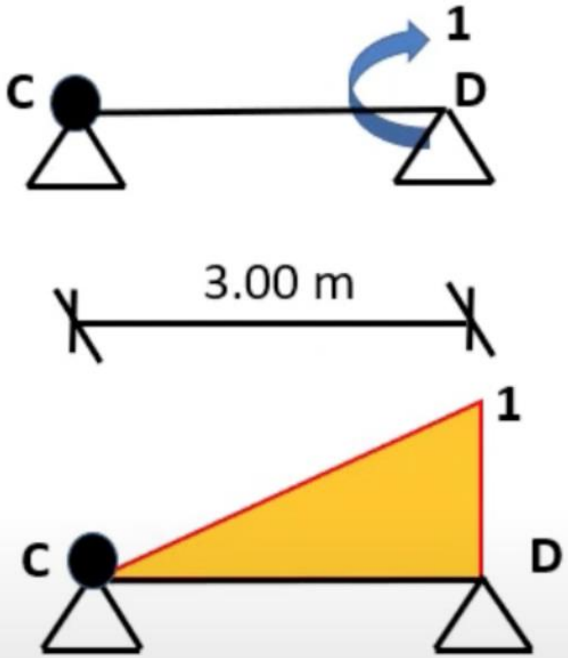
$0 \leq x \leq 3$



$$m_3 = m'x + c$$

$$m_3 = 0$$

$0 \leq x \leq 3$



$$m_3 = m'x + c$$

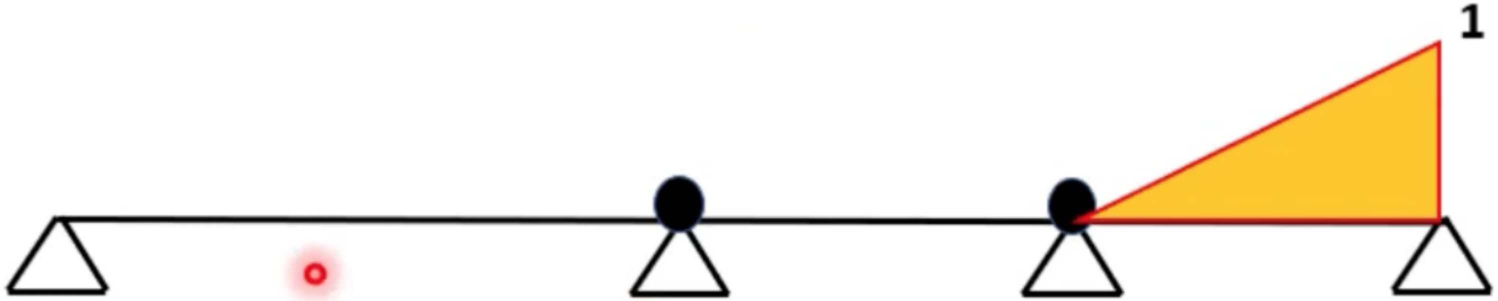
$$m_3 = \frac{-1}{3}x + c$$

$$(0) = \frac{-1}{3}(0) + c \Rightarrow c = 0$$

$$m_3 = -\frac{1}{3}x$$

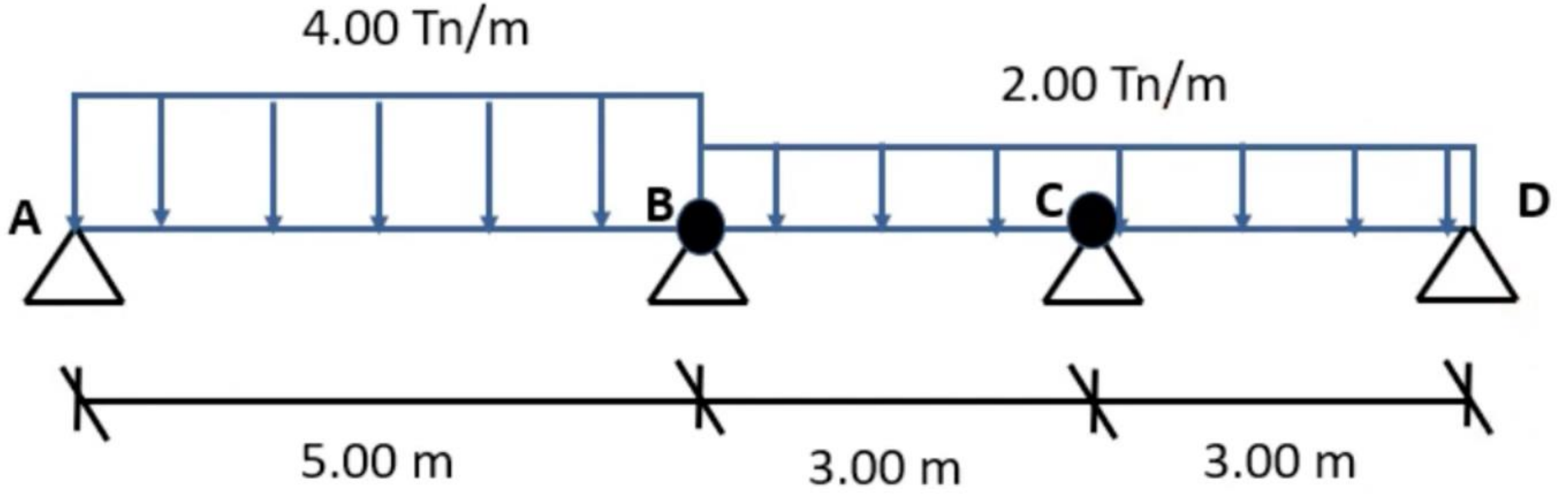
Cálculo de "m_3" ($x_3 = 1$; $x_1 = x_2 = 0$)

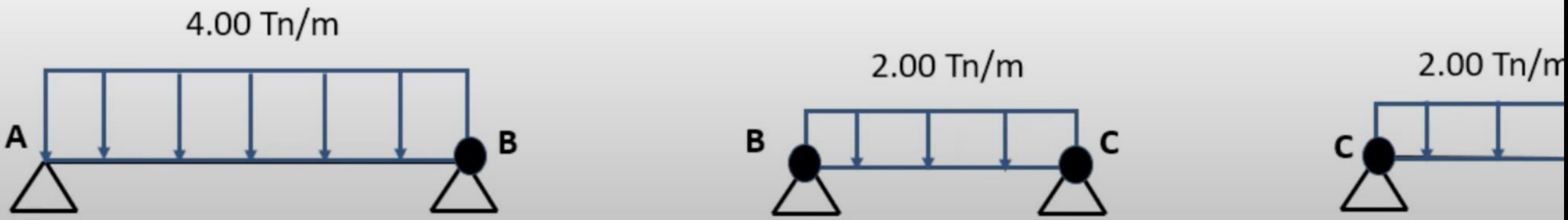
$0 \leq x \leq 5$ $\quad$ $0 \leq x \leq 3$ $\quad$ $0 \leq x \leq 3$



$m_3 = 0$ $\quad$ $m_3 = 0$ $\quad$ $m_3 = -\frac{1}{3}x$

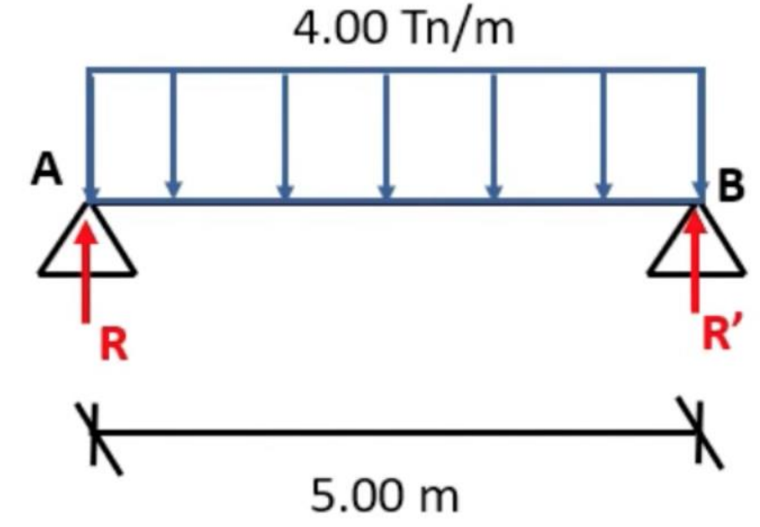
Cálculo de "M" $(Estado\ primario)$





Cálculo de "M" ($Estado\ primario$)

$\boldsymbol{0 \leq x \leq 5}$



$$\sum M_B = 0$$

$$4 \times 5 \times \frac{5}{2} - 5R = 0$$

$$5R = 50$$

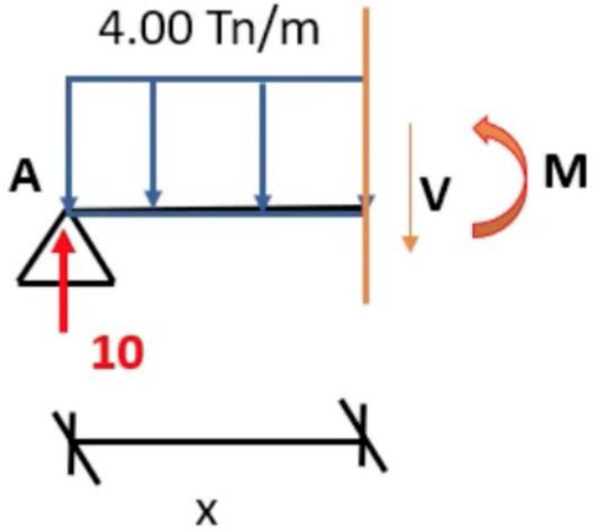
$$R = 10$$

$$\sum F_y = 0$$

$$R + R' - 4 \times 5 = 0$$

$$10 + R' - 20 = 0$$

$$R' = 10$$



$$\sum F = 0$$

$$10 - V - 4x = 0$$

$$\boldsymbol{V = 10 - 4x}$$

$$\sum M = 0$$

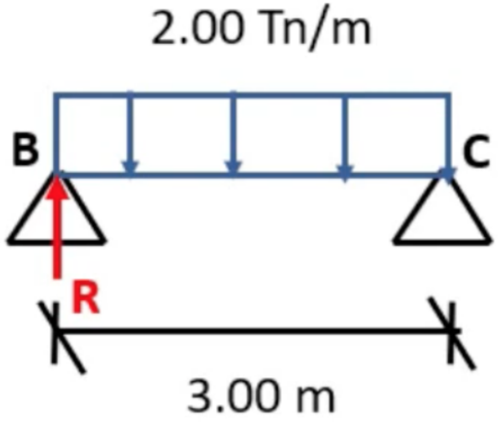
$$-10x + 4x\frac{x}{2} + M = 0$$

$$-10x + 2x^2 + M = 0$$

$$\boldsymbol{M = 10x - 2x^2}$$

Cálculo de "M" ($Estado\ primario$)

$\mathbf{0 \leq x \leq 3}$



$$\sum M_C = 0$$

$$2 \times 3 \times \frac{3}{2} - 3R = 0$$

$$3R = 9$$

$$R = 3$$

$$w = -2$$

$$V = \int w\, dx$$

$$M = \int V\, dx$$

$$V = \int -2\, dx$$

$$V = -2x + c$$

$$(3) = -2(0) + c$$

$$c = 3$$

$$\boldsymbol{V = -2x + 3}$$

$$M = \int -2x + 3\, dx$$

$$M = -x^2 + 3x + d$$

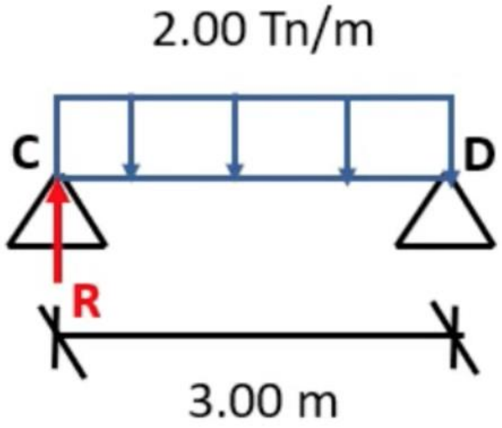
$$(0) = -(0)^2 + 3(0) + d$$

$$d = 0$$

$$\boldsymbol{M = -x^2 + 3x}$$

Cálculo de "M" ($Estado\ primario$)

$\mathbf{0 \leq x \leq 3}$



$$\sum M_D = 0$$

$$2 \times 3 \times \frac{3}{2} - 3R = 0$$

$$3R = 9$$

$$R = 3$$

$$w = -2$$

$$V = \int w\,dx$$

$$M = \int V\,dx$$

$$V = \int -2\,dx$$

$$V = -2x + c$$

$$(3) = -2(0) + c$$

$$c = 3$$

$$\boldsymbol{V = -2x + 3}$$

$$M = \int -2x + 3\,dx$$

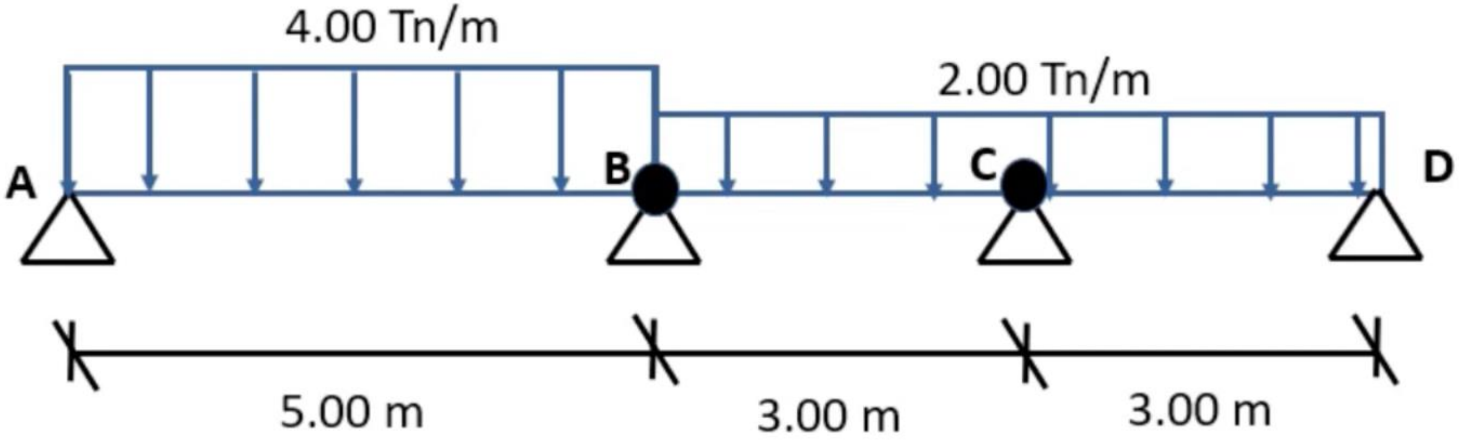
$$M = -x^2 + 3x + d$$

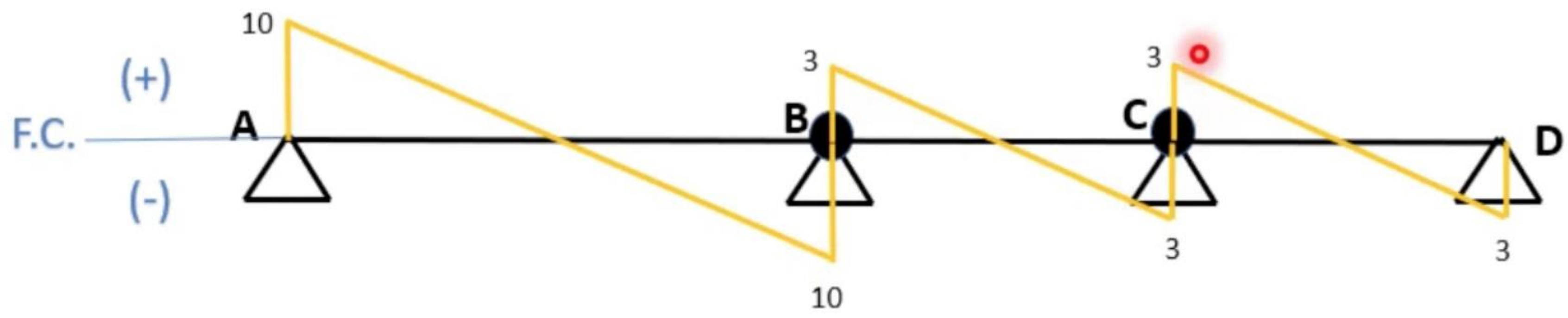
$$(0) = -(0)^2 + 3(0) + d$$

$$d = 0$$

$$\boldsymbol{M = -x^2 + 3x}$$

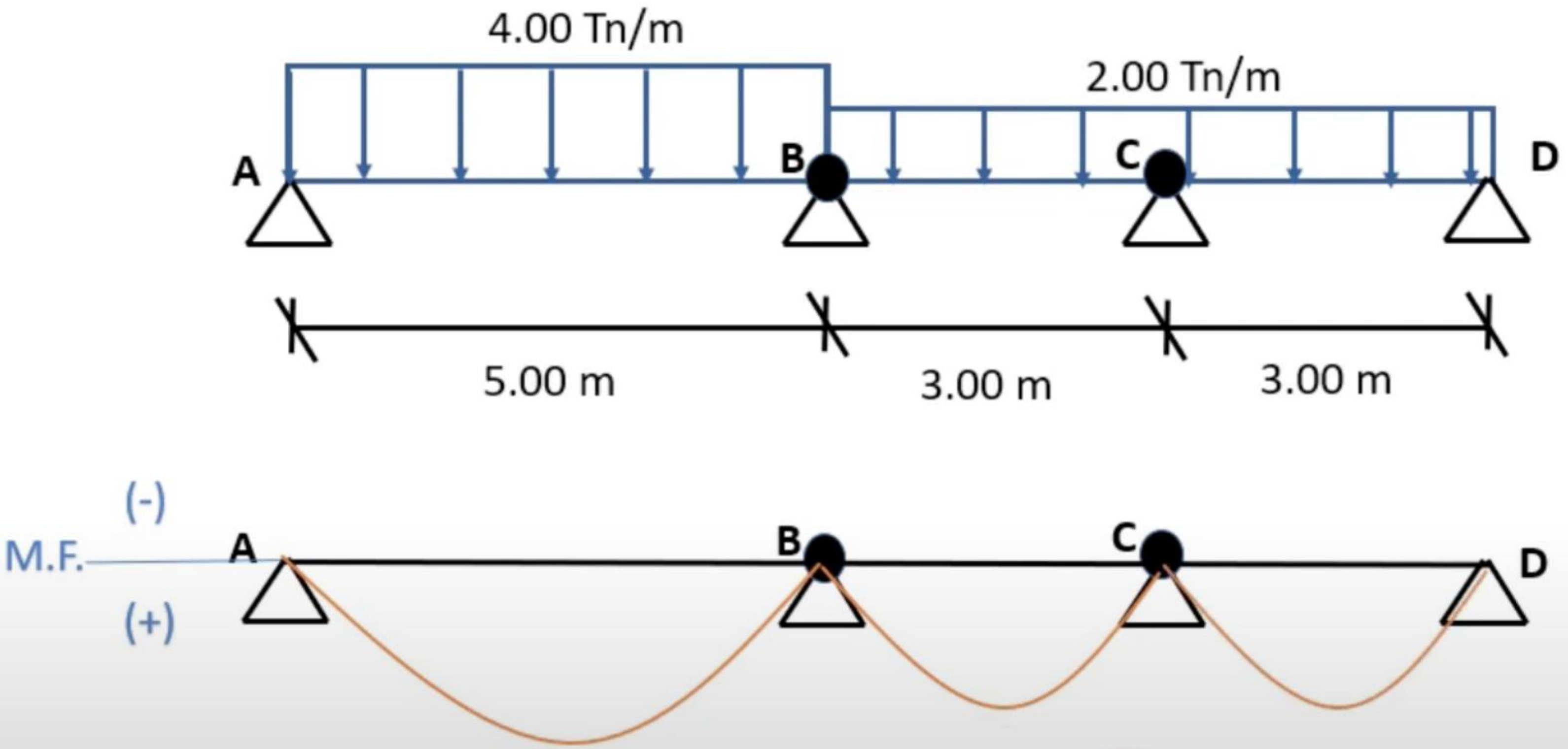
Resumen de "M" $(Estado\ primario)$





$$V = 10 - 4x \qquad V = -2x + 3 \qquad V = -2x + 3$$

Resumen de "M" $(Estado\ primario)$



$M = 10x - 2x^2$ $\quad M = -x^2 + 3x$ $\quad M = -x^2 + 3x$

Cálculo de los coeficientes de flexibilidad

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$f_{11} = \int \frac{m_1 m_1}{EI}$$

$$f_{11} = \frac{1}{EI} \int m_1 m_1$$

$$f_{11} = \frac{1}{EI} [\int_0^5 \left(\frac{x}{5}\right)^2 dx + \int_0^3 \left(1 - \frac{x}{3}\right)^2 dx + \cancel{\int_0^3 (0)^2 dx}]$$

$$f_{11} = \frac{1}{EI} [\int_0^5 \frac{x^2}{25} dx + \int_0^3 1 - \frac{2x}{3} + \frac{x^2}{9} dx]$$

Tabla resumen			
Tramo	$\mathbf{0 \leq x \leq 5}$	$\mathbf{0 \leq x \leq 3}$	$\mathbf{0 \leq x \leq 3}$
m_1	$\frac{x}{5}$	$1 - \frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1 - \frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x - 2x^2$	$-x^2 + 3x$	$-x^2 + 3x$

$$f_{11} = \frac{1}{EI} [\frac{x^3}{3 \times 25}\Big| + x - \frac{x^2}{3} + \frac{x^3}{3 \times 9}\Big|]$$

$$f_{11} = \frac{1}{EI} [\frac{(5)^3}{3 \times 25} + (3) - \frac{(3)^2}{3} + \frac{(3)^3}{3 \times 9}]$$

$$f_{11} = \frac{1}{EI} [\frac{5}{3} + 1]$$

$$\boldsymbol{f_{11} = \frac{8}{3EI}}$$

Cálculo de los coeficientes de flexibilidad

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$f_{22} = \int \frac{m_2 m_2}{EI}$$

$$f_{22} = \frac{1}{EI} \int m_2 m_2$$

$$f_{22} = \frac{1}{EI} [\cancel{\int_0^5 (0)^2 dx} + \int_0^3 \left(\frac{x}{3}\right)^2 dx + \int_0^3 \left(1 - \frac{x}{3}\right)^2 dx]$$

$$f_{22} = \frac{1}{EI} [\int_0^3 \frac{x^2}{9} dx + \int_0^3 1 - \frac{2x}{3} + \frac{x^2}{9} dx]$$

Tabla resumen			
Tramo	$\mathbf{0 \le x \le 5}$	$\mathbf{0 \le x \le 3}$	$\mathbf{0 \le x \le 3}$
m_1	$\frac{x}{5}$	$1 - \frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1 - \frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x - 2x^2$	$-x^2 + 3x$	$-x^2 + 3x$

$$f_{22} = \frac{1}{EI} [\frac{x^3}{3 \times 9}\Big| + x - \frac{x^2}{3} + \frac{x^3}{3 \times 9}\Big|]$$

$$f_{22} = \frac{1}{EI} [\frac{(3)^3}{3 \times 9} + (3) - \frac{(3)^2}{3} + \frac{(3)^3}{3 \times 9}]$$

$$f_{22} = \frac{1}{EI} [1 + 1]$$

$$\boldsymbol{f_{22} = \frac{2}{EI}}$$

Cálculo de los coeficientes de flexibilidad

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$f_{33} = \int \frac{m_3 m_3}{EI}$$

$$f_{33} = \frac{1}{EI}\int m_3 m_3$$

$$f_{33} = \frac{1}{EI}[\cancel{\int_0^5 (0)^2 dx} + \cancel{\int_0^3 (0)^2 dx} + \int_0^3 \left(-\frac{x}{3}\right)^2 dx]$$

$$f_{33} = \frac{1}{EI}[\int_0^3 \frac{x^2}{9} dx]$$

Tabla resumen			
Tramo	$\mathbf{0 \leq x \leq 5}$	$\mathbf{0 \leq x \leq 3}$	$\mathbf{0 \leq x \leq 3}$
m_1	$\frac{x}{5}$	$1 - \frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1 - \frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x - 2x^2$	$-x^2 + 3x$	$-x^2 + 3x$

$$f_{33} = \frac{1}{EI}[\frac{x^3}{3 \times 9}\Big|] \qquad f_{33} = \frac{1}{EI}[\frac{(3)^3}{3 \times 9}] \qquad f_{33} = \frac{1}{EI}[1] \qquad \boldsymbol{f_{33} = \frac{1}{EI}}$$

Cálculo de los coeficientes de flexibilidad

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$f_{12} = \int \frac{m_1 m_2}{EI}$$

$$f_{12} = \frac{1}{EI}\int m_1 m_2$$

$$f_{12} = \frac{1}{EI}[\cancel{\int_0^5 (\frac{x}{5})(0)dx} + \int_0^3 (1-\frac{x}{3})(\frac{x}{3})dx + \cancel{\int_0^3 (0)(1-\frac{x}{3})dx}]$$

$$f_{12} = \frac{1}{EI}[\int_0^3 \frac{x}{3} - \frac{x^2}{9}dx]$$

Tabla resumen			
Tramo	$\mathbf{0 \le x \le 5}$	$\mathbf{0 \le x \le 3}$	$\mathbf{0 \le x \le 3}$
m_1	$\frac{x}{5}$	$1-\frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1-\frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x - 2x^2$	$-x^2 + 3x$	$-x^2 + 3x$

$$f_{12} = \frac{1}{EI}[\frac{x^2}{2\times 3} - \frac{x^3}{3\times 9}\Big|]$$

$$f_{12} = \frac{1}{EI}[\frac{(3)^3}{2\times 3} - \frac{(3)^3}{3\times 9}]$$

$$f_{12} = \frac{1}{EI}[\frac{3}{2} - 1]$$

$$\boldsymbol{f_{12} = \frac{1}{2EI}}$$

Cálculo de los coeficientes de flexibilidad

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$f_{13} = \int \frac{m_1 m_3}{EI}$$

$$f_{13} = \frac{1}{EI} \int m_1 m_3$$

$$f_{13} = \frac{1}{EI} [\int_0^5 \cancel{(\frac{x}{5})(0)} dx + \int_0^3 \cancel{(1 - \frac{x}{3})(0)} dx + \int_0^3 \cancel{(0)(-\frac{x}{3})} dx]$$

$$f_{13} = \frac{1}{EI} [0 + 0 + 0]$$

$$f_{13} = \frac{0}{EI} \qquad \boldsymbol{f_{13} = 0}$$

Tabla resumen			
Tramo	$\mathbf{0 \leq x \leq 5}$	$\mathbf{0 \leq x \leq 3}$	$\mathbf{0 \leq x \leq 3}$
m_1	$\frac{x}{5}$	$1 - \frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1 - \frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x - 2x^2$	$-x^2 + 3x$	$-x^2 + 3x$

Cálculo de los coeficientes de flexibilidad

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$f_{23} = \int \frac{m_2 m_3}{EI}$$

$$f_{23} = \frac{1}{EI}\int m_2 m_3$$

$$f_{23} = \frac{1}{EI}[\cancel{\int_0^5 (0)(0)dx} + \cancel{\int_0^3 (\frac{x}{3})(0)dx} + \int_0^3 (1-\frac{x}{3})(-\frac{x}{3})dx]$$

$$f_{23} = \frac{1}{EI}[\int_0^3 (-\frac{x}{3}+\frac{x^2}{9})dx]$$

$$f_{23} = \frac{1}{EI}[\frac{-x^2}{2\times 3}+\frac{x^3}{3\times 9}\Big|]$$

$$f_{23} = \frac{1}{EI}[\frac{-(3)^3}{2\times 3}+\frac{(3)^3}{3\times 9}]$$

$$f_{23} = \frac{1}{EI}[\frac{-3}{2}+1]$$

$$\boldsymbol{f_{23} = \frac{-1}{2EI}}$$

Tabla resumen			
Tramo	$\mathbf{0 \le x \le 5}$	$\mathbf{0 \le x \le 3}$	$\mathbf{0 \le x \le 3}$
m_1	$\frac{x}{5}$	$1-\frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1-\frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x-2x^2$	$-x^2+3x$	$-x^2+3x$

Cálculo del vector {D}

Tabla resumen			
Tramo	$0 \le x \le 5$	$0 \le x \le 3$	$0 \le x \le 3$
m_1	$\frac{x}{5}$	$1-\frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1-\frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x-2x^2$	$-x^2+3x$	$-x^2+3x$

$$D_1 = \int \frac{m_1 M}{EI}$$

$$\{D\} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}$$

$$D_1 = \frac{1}{EI}\int m_1 M$$

$$D_1 = \frac{1}{EI}[\int_0^5 (\frac{x}{5})(10x-2x^2)dx + \int_0^3 (1-\frac{x}{3})(-x^2+3x)dx + \cancel{\int_0^3 (0)(-x^2+3x)dx}]$$

$$D_1 = \frac{1}{EI}[\int_0^5 2x^2 - \frac{2x^3}{5}dx + \int_0^3 -2x^2 + \frac{x^3}{3} + 3xdx]$$

$$D_1 = \frac{1}{EI}[\frac{2x^3}{3} - \frac{2x^4}{5\times 4}\Big| + \frac{-2x^3}{3} + \frac{x^4}{3\times 4} + \frac{3x^2}{2}\Big|]$$

$$D_1 = \frac{1}{EI}[\frac{2(5)^3}{3} - \frac{2(5)^4}{5\times 4} + \frac{-2(3)^3}{3} + \frac{(3)^4}{3\times 4} + \frac{3(3)^2}{2}]$$

$$D_1 = \frac{1}{EI}[\frac{250}{3} - \frac{125}{2} - 18 + \frac{27}{4} + \frac{27}{2}]$$

$$\boldsymbol{D_1 = \frac{23.0833}{EI}}$$

Cálculo del vector {D}

$$D_2 = \int \frac{m_2 M}{EI}$$

$$\{D\} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}$$

$$D_2 = \frac{1}{EI}\int m_2 M$$

Tabla resumen			
Tramo	$0 \le x \le 5$	$0 \le x \le 3$	$0 \le x \le 3$
m_1	$\frac{x}{5}$	$1-\frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1-\frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x-2x^2$	$-x^2+3x$	$-x^2+3x$

$$D_2 = \frac{1}{EI}\left[\cancel{\int_0^5 (0)(10x-2x^2)dx} + \int_0^3 \left(\frac{x}{3}\right)(-x^2+3x)dx + \int_0^3 \left(1-\frac{x}{3}\right)(-x^2+3x)dx\right]$$

$$D_2 = \frac{1}{EI}\left[\int_0^3 x^2 - \frac{x^3}{3}dx + \int_0^3 -2x^2 + \frac{x^3}{3} + 3x dx\right]$$

$$D_2 = \frac{1}{EI}\left[\frac{(3)^3}{3} - \frac{(3)^4}{3\times 4} + \frac{-2(3)^3}{3} + \frac{(3)^4}{3\times 4} + \frac{3(3)^2}{2}\right]$$

$$D_2 = \frac{1}{EI}\left[\frac{x^3}{3} - \frac{x^4}{3\times 4}\Big| + \frac{-2x^3}{3} + \frac{x^4}{3\times 4} + \frac{3x^2}{2}\Big|\right]$$

$$D_2 = \frac{1}{EI}\left[9 - \frac{27}{4} - 18 + \frac{27}{4} + \frac{27}{2}\right]$$

$$\boldsymbol{D_2 = \frac{4.5}{EI}}$$

Cálculo del vector {D}

Tabla resumen			
Tramo	$\mathbf{0 \le x \le 5}$	$\mathbf{0 \le x \le 3}$	$\mathbf{0 \le x \le 3}$
m_1	$\frac{x}{5}$	$1-\frac{x}{3}$	0
m_2	0	$\frac{x}{3}$	$1-\frac{x}{3}$
m_3	0	0	$-\frac{x}{3}$
M	$10x - 2x^2$	$-x^2 + 3x$	$-x^2 + 3x$

$$D_3 = \int \frac{m_3 M}{EI}$$

$$\{D\} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}$$

$$D_3 = \frac{1}{EI}\int m_3 M$$

$$D_3 = \frac{1}{EI}\left[\cancel{\int_0^5 (0)(10x - 2x^2)}dx + \cancel{\int_0^3 (0)(-x^2 + 3x\,)}dx + \int_0^3 \left(-\frac{x}{3}\,\right)(-x^2 + 3x\,)dx\right]$$

$$D_3 = \frac{1}{EI}\left[\int_0^3 -x^2 + \frac{x^3}{3}\,dx\right]$$

$$D_3 = \frac{1}{EI}\left[\frac{-x^3}{3} + \frac{x^4}{3\times 4}\Big|\right]$$

$$D_3 = \frac{1}{EI}\left[\frac{-(3)^3}{3} + \frac{(3)^4}{3\times 4}\right]$$

$$D_3 = \frac{1}{EI}\left[-9 + \frac{27}{4}\right]$$

$$\boldsymbol{D_3 = \frac{-2.25}{EI}}$$

Cálculo de las redundantes {X}

$$\{\boldsymbol{D}\} + [\boldsymbol{F}]\{\boldsymbol{X}\} = \{\boldsymbol{0}\}$$

$$\begin{Bmatrix} \dfrac{23.0833}{EI} \\ \dfrac{4.5}{EI} \\ \dfrac{-2.25}{EI} \end{Bmatrix} + \begin{bmatrix} \dfrac{8}{3EI} & \dfrac{1}{2EI} & 0 \\ \dfrac{1}{2EI} & \dfrac{2}{EI} & \dfrac{-1}{2EI} \\ 0 & \dfrac{-1}{2EI} & \dfrac{1}{EI} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\cancel{\frac{1}{EI}} \begin{Bmatrix} 23.0833 \\ 4.5 \\ -2.25 \end{Bmatrix} + \cancel{\frac{1}{EI}} \begin{bmatrix} \dfrac{8}{3} & \dfrac{1}{2} & 0 \\ \dfrac{1}{2} & 2 & \dfrac{-1}{2} \\ 0 & \dfrac{-1}{2} & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

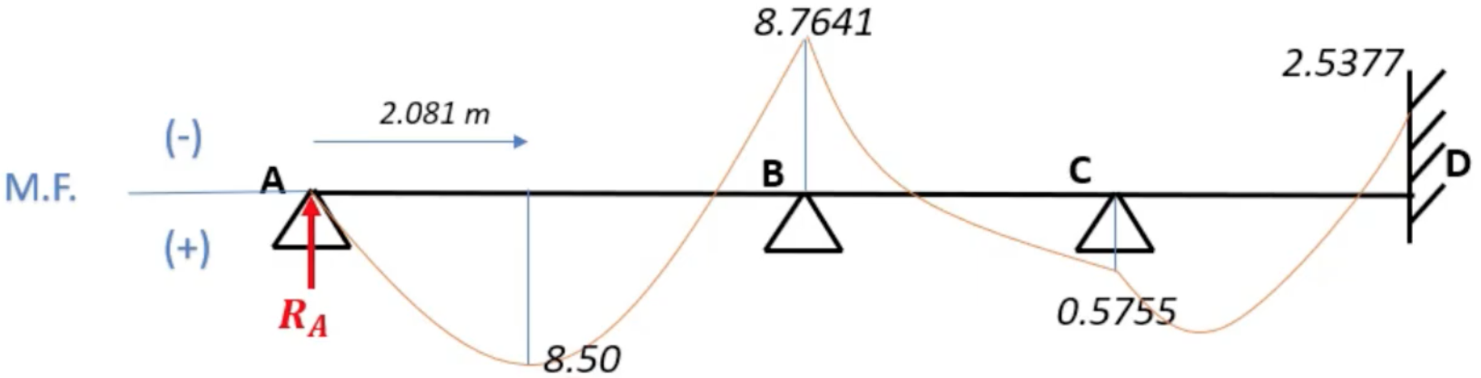
$$\begin{Bmatrix} 23.0833 \\ 4.5 \\ -2.25 \end{Bmatrix} + \begin{bmatrix} \dfrac{8}{3} & \dfrac{1}{2} & 0 \\ \dfrac{1}{2} & 2 & \dfrac{-1}{2} \\ 0 & \dfrac{-1}{2} & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = -\begin{Bmatrix} 23.0833 \\ 4.5 \\ -2.25 \end{Bmatrix} \begin{bmatrix} \dfrac{8}{3} & \dfrac{1}{2} & 0 \\ \dfrac{1}{2} & 2 & \dfrac{-1}{2} \\ 0 & \dfrac{-1}{2} & 1 \end{bmatrix}^{-1}$$

Cálculo de las redundantes {X}

$$\{\boldsymbol{D}\} + [\boldsymbol{F}]\{\boldsymbol{X}\} = \{\boldsymbol{0}\}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -8.7641 \\ 0.5755 \\ 2.5377 \end{Bmatrix}$$



$$M = R_A x - 2x^2$$

$$(-8.7641) = R_A(5) - 2(5)^2$$

$$R_A = 8.247 \cong 8.25\ Tn$$

$$V = R_A - 4x$$

$$V = 8.25 - 4x$$

$$0 = 8.25 - 4x$$

$$x = 2.061\ m$$

$$M = 8.25x - 2x^2$$

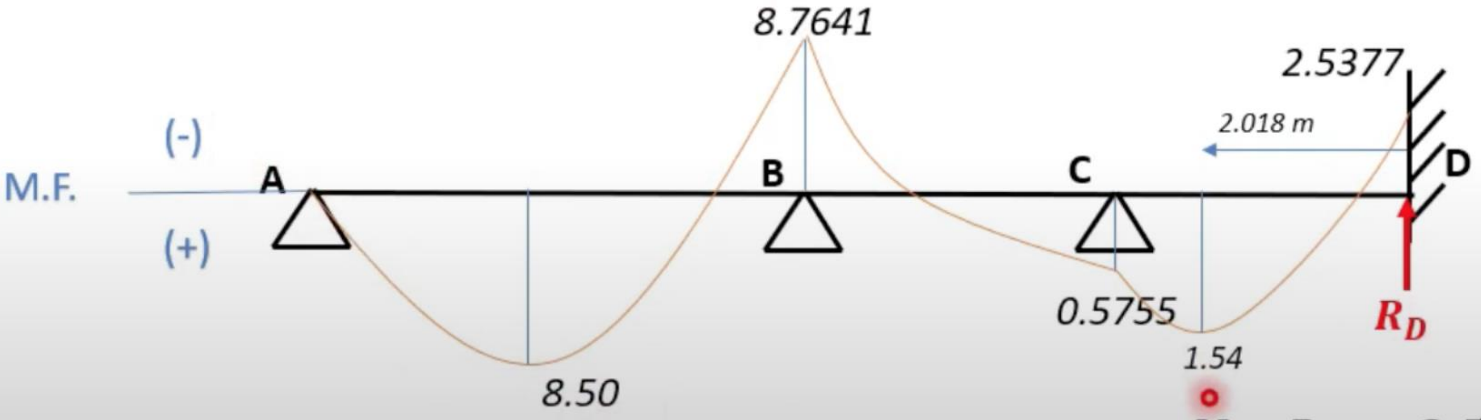
$$M = 8.25(2.061) - 2(2.061)^2$$

$$M = 8.50\ Tn.m$$

Cálculo de las redundantes {X}

$$\{\boldsymbol{D}\} + [\boldsymbol{F}]\{\boldsymbol{X}\} = \{\boldsymbol{0}\}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -8.7641 \\ 0.5755 \\ 2.5377 \end{Bmatrix}$$



$$M = R_D x - 2.5377 - x^2$$

$$0.5755 = R_D(3) - 2.5377 - (3)^2$$

$$R_D = 4.037 \cong 4.04\,Tn$$

$$V = 4.04 - 2x$$

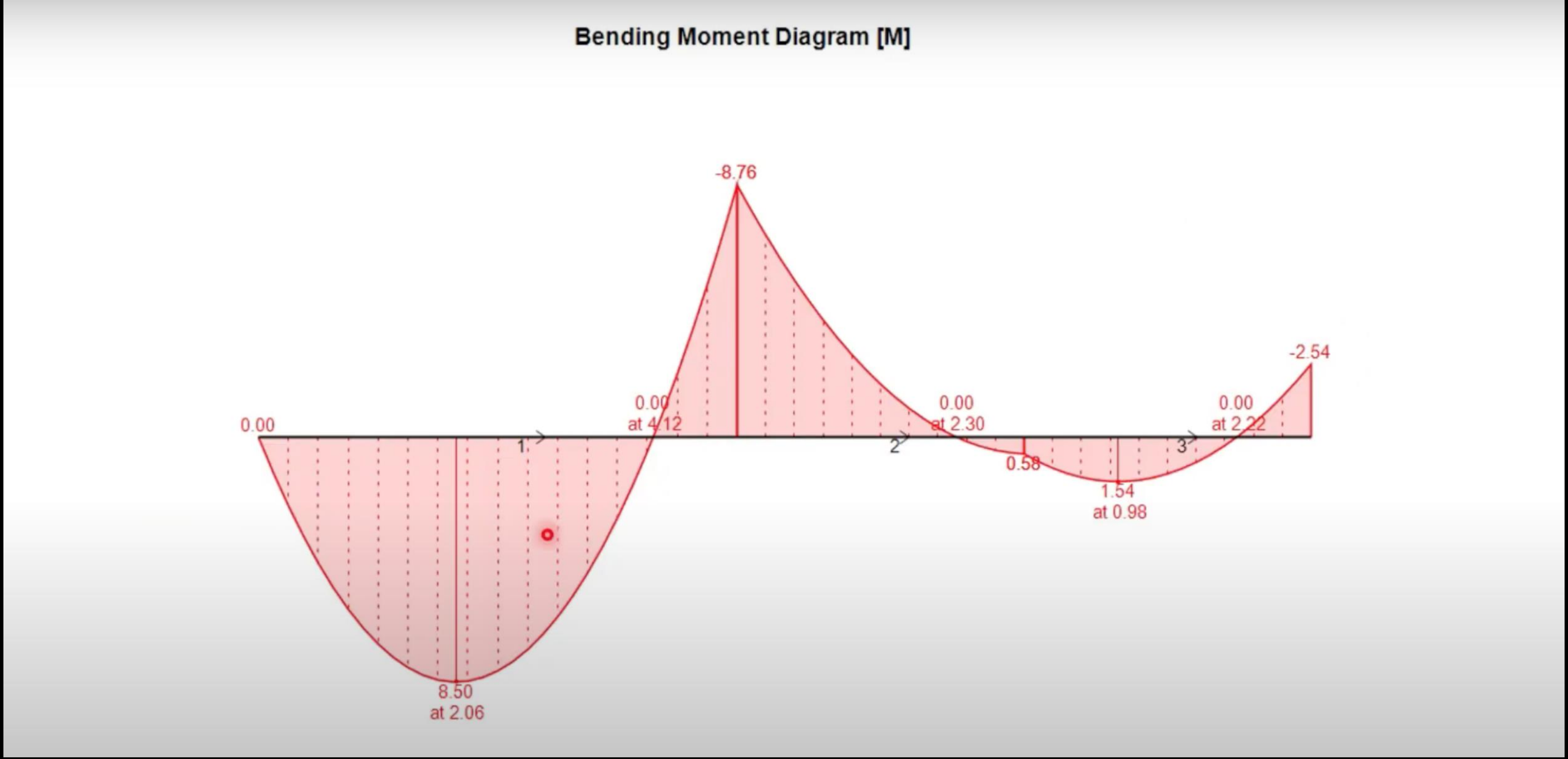
$$0 = 4.04 - 2x$$

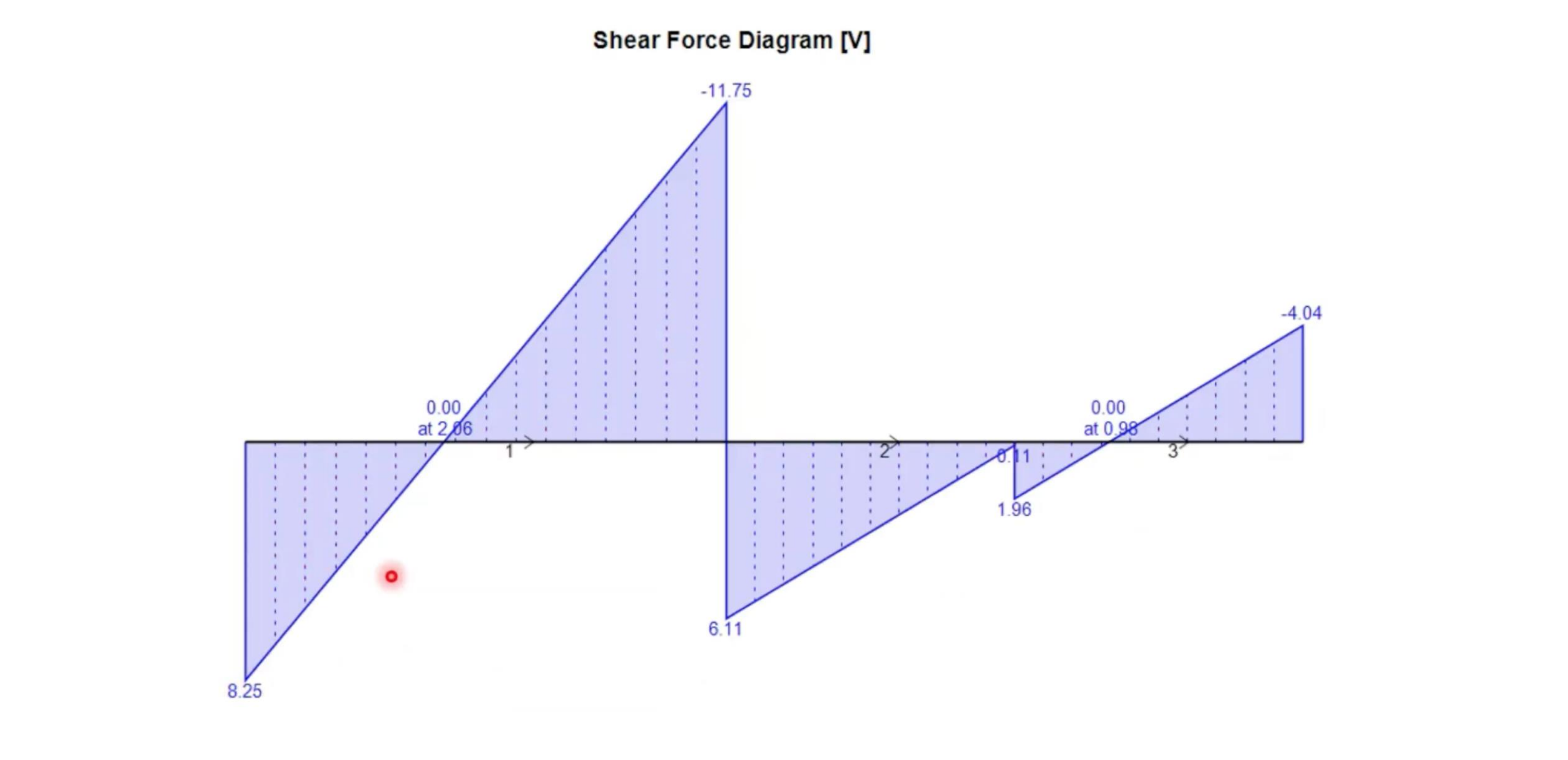
$$x = 2.018\,m$$

$$M = R_D x - 2.5377 - x^2$$

$$M = 4.04(2.081) - 2.5377 - (2.081)^2$$

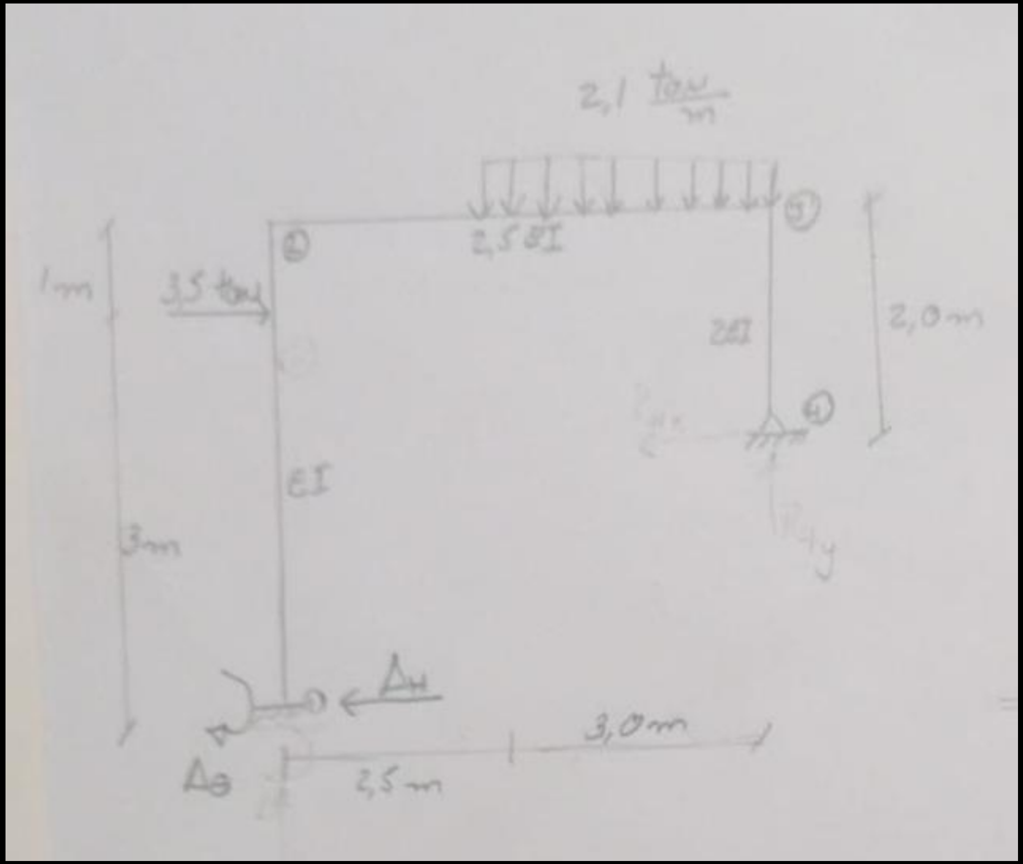
$$M = 1.54\,Tn.m$$


Bending Moment Diagram [M]
-8.76
-2.54
0.00
0.00
at 4.12
0.00
at 2.30
0.00
at 2.22
1
2
3
0.58
1.54
at 0.98
8.50
at 2.06

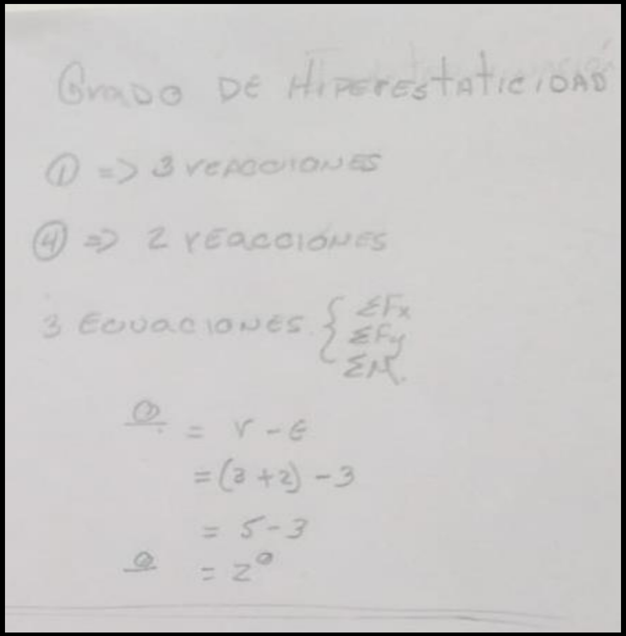

Shear Force Diagram [V]
-11.75
-4.04
0.00
at 2.06
0.00
at 0.98
1
2
3
0.11
1.96
6.11
8.25

Método de fuerza (Flexibilidades) en marcos

Ejemplo: Estructuras estáticamente indeterminadas



CALCULO DE GRADO DE HIPERESTATICIDAD



Grado de Hiperestaticidad

① => 3 reacciones

④ => 2 reacciones

3 Ecuaciones $\begin{cases} \Sigma F_x \\ \Sigma F_y \\ \Sigma M \end{cases}$

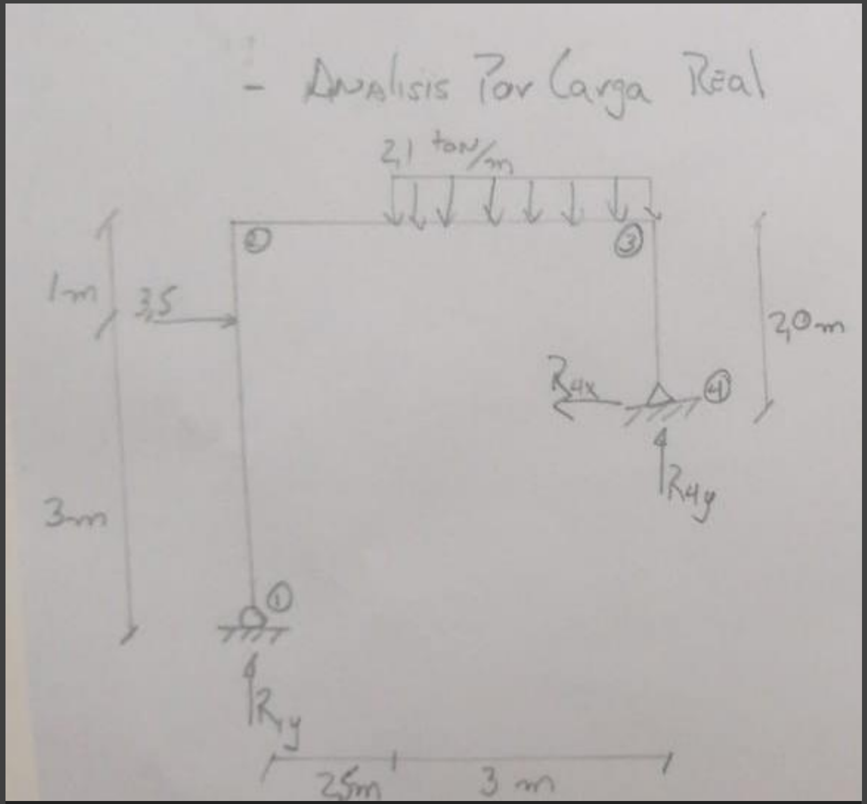
$G = r - e$

$= (3+2) - 3$

$= 5 - 3$

$G = 2°$

ANALISIS POR CARGA REAL



MARCO ISOSTATICO

Reacciones= 3

Ecuaciones = 3

Reacciones.

$\overset{+}{\rightarrow} \sum F_x = 0$

$3,5 - R_{4x} = 0$

$\underline{\underline{R_{4x} = 3,5 \text{ ton} \leftarrow}}$

$\overset{+}{\circlearrowleft} \sum M_1 = 0$

$-(3,5)(3) + 3,5(2) - (2,1)(3,0)(4,0) + 5,5 R_{4y} = 0$

$\underline{\underline{R_{4y} = 5,218 \text{ ton} \uparrow}}$

$+\uparrow \sum F_y = 0$

$R_{1y} - (2,1)(3) + 5,218 = 0$

$\underline{R_{1y} = 1,082 \text{ ton} \uparrow}$

CALCULO DE REACCIONES

CALCULO DE MOMENTOS INTERNOS

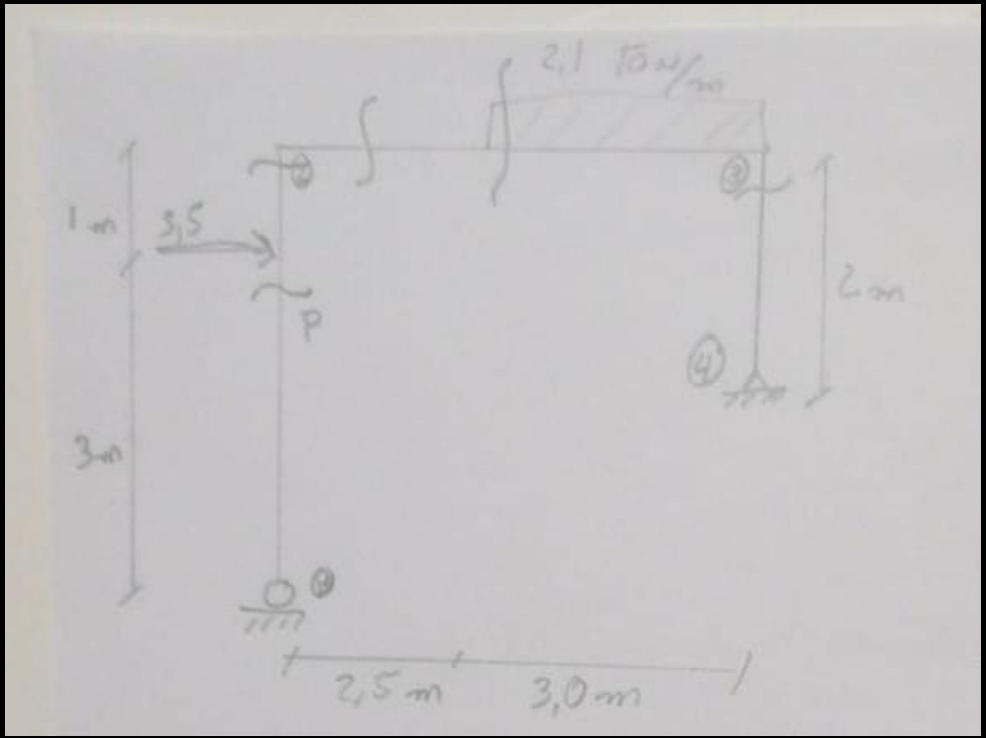
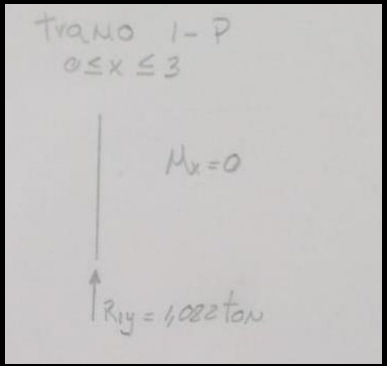
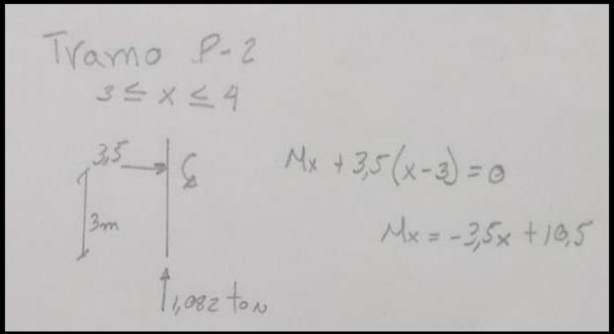
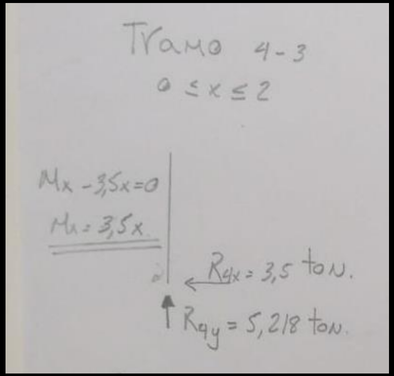
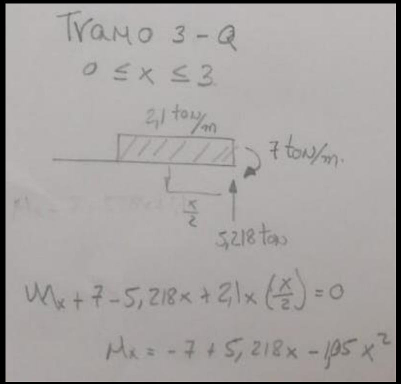


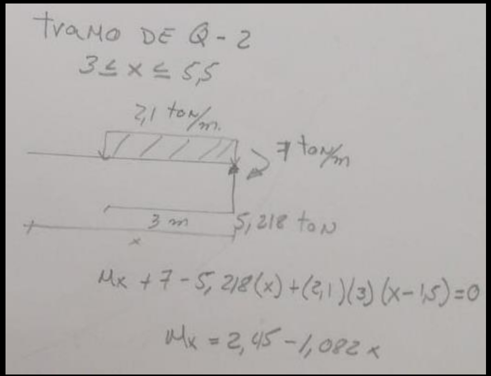
DIAGRAMA DE CORTES EN MARCO ISOSTATICO



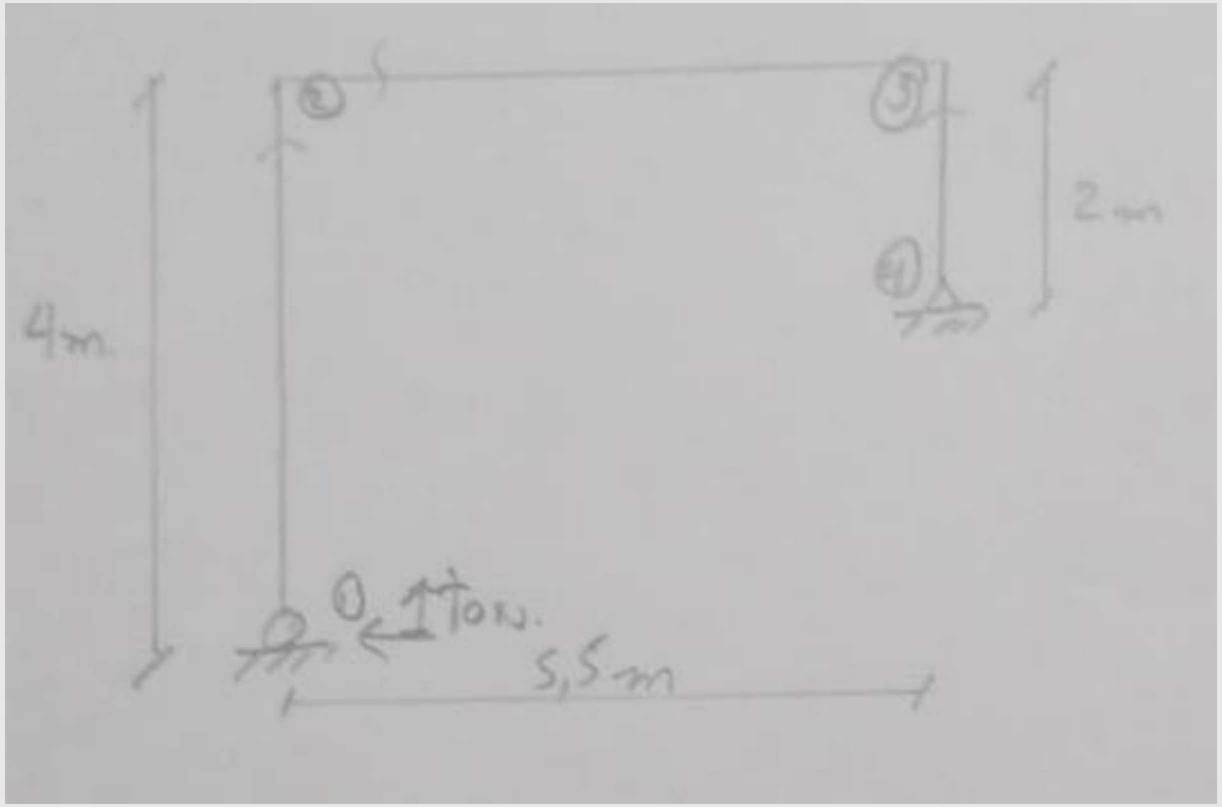








ANALISIS DE CARGA VIRTUAL HORIZONTALES.



REACCIONES.

$\xrightarrow{+} \Sigma F_x = 0$

$-1 + R_{4x} = 0$

$\underline{R_{4x} = 1 \text{ ton} \rightarrow}$

$\circlearrowright^{+} \Sigma M_1 = 0$

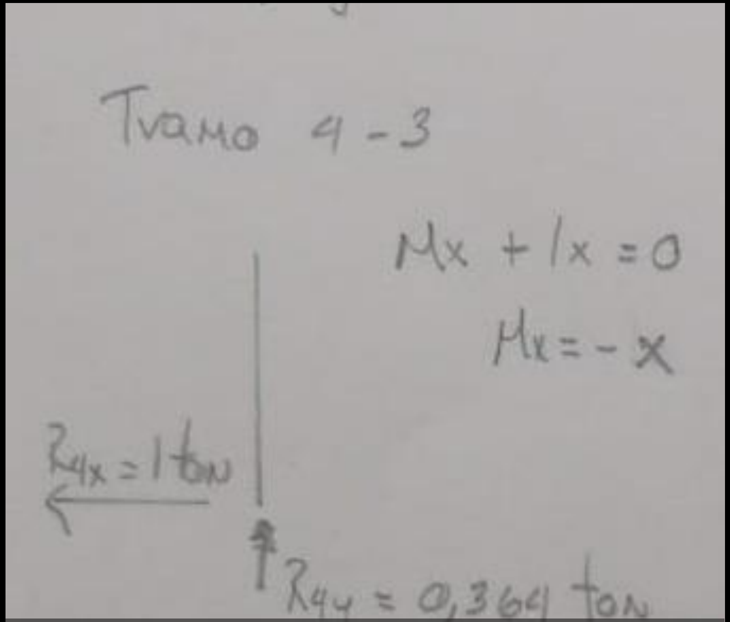
$-1(2) + 5,5(R_{4y}) = 0$

$\underline{R_{4y} = 0,364 \text{ ton} \uparrow}$

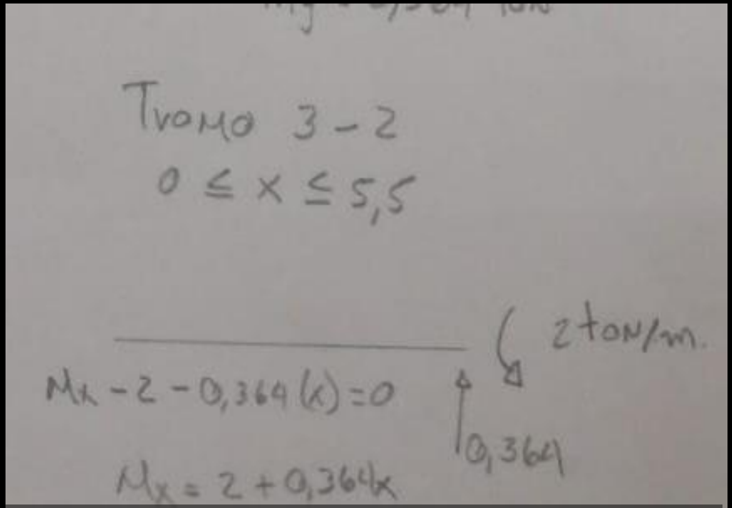
$\underline{R_{1y} = 0,364 \text{ ton} \downarrow}$

DIAGRAMA DE CARGA VIRTUALES HORIZONTAL EN MARCO ISOSTATICO

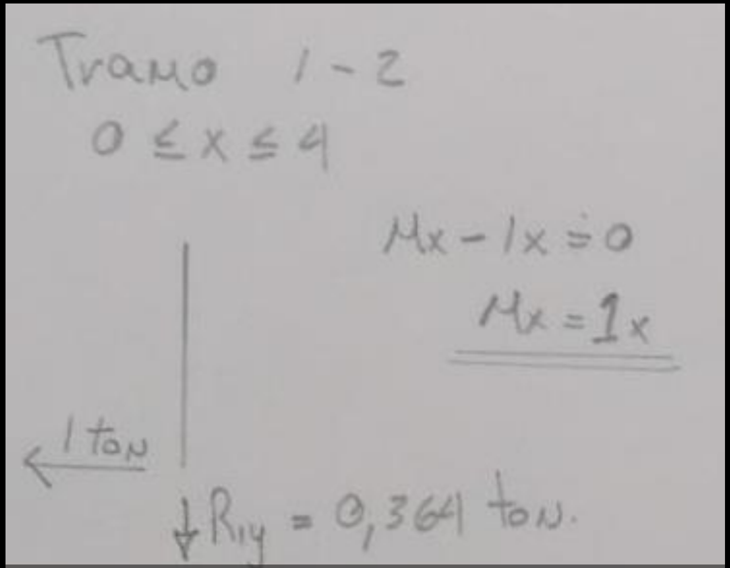
CALCULO DE REACCIONES



CORTE 2



CORTE 3

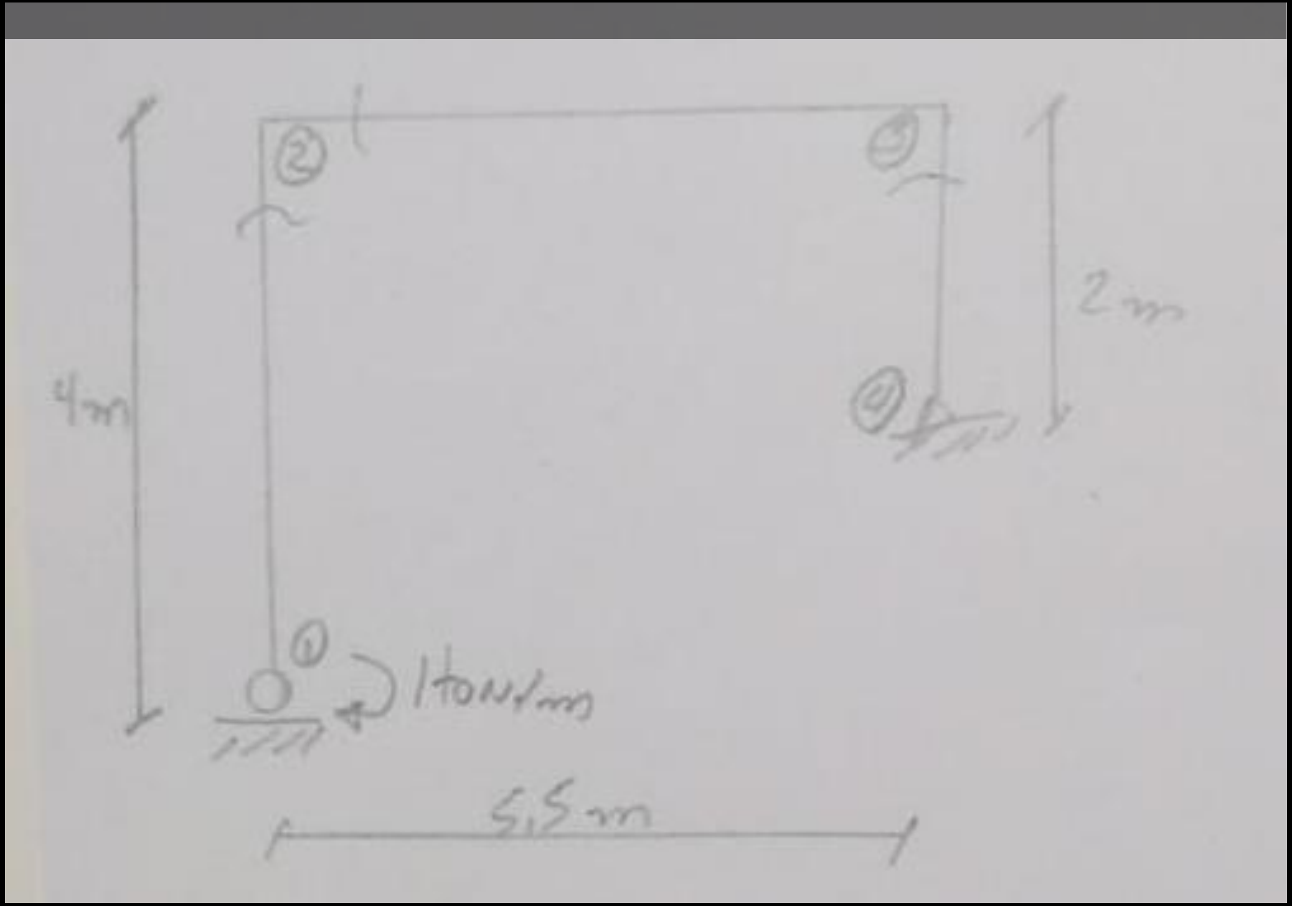


CORTE 1

CALCULO DE MOMENTOS INTERNOS EN MARCO CON CARGA VIRTUAL

ANALISIS DE CARGA VIRTUAL ROTACIONALES.

DIAGRAMA DE CARGA VIRTUALES ROTACIONALES EN MARCO ISOSTATICO



CALCULO DE REACCIONES

REACCIONES.

$\xrightarrow{+} \sum F_x = 0$

$-1 + R_{4x} = 0$

$\underline{\underline{R_{4x} = 1 \text{ ton} \rightarrow}}$

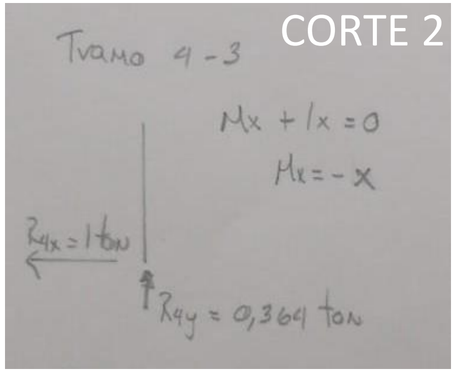
$\circlearrowleft^{+} \sum M_1 = 0$

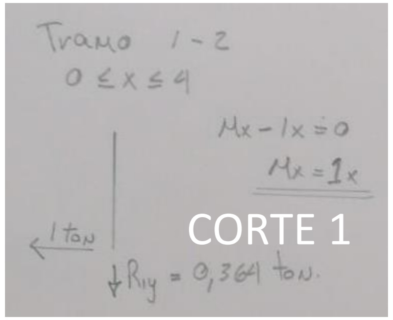
$-1(2) + 5{,}5(R_{4y}) = 0$

$\underline{\underline{R_{4y} = 0{,}364 \text{ ton} \uparrow}}$

$\underline{\underline{R_{1y} = 0{,}364 \text{ ton} \downarrow}}$

CALCULO DE MOMENTOS INTERNOS
EN MARCO CON CARGA VIRTUAL





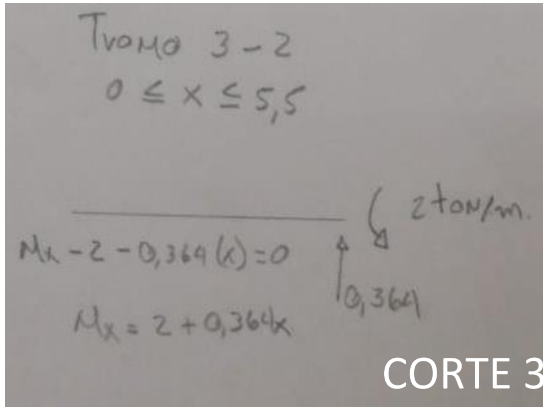


TABLA DE VALORES

tramo	Rango	M(x) carga Real	M_{xH}	$M_{x\theta}$	EI
1 - P	$0 \le x \le 3$	0	x	1	EI
P - 2	$3 \le x \le 4$	$-3,5x + 10,5$	x	1	EI
4 - 3	$0 \le x \le 2$	$3,5x$	$-x$	0	2EI
3 - Q	$0 \le x \le 3$	$-7 + 5,218x - 1,05x^2$	$2 + 0,364x$	$0,182x$	2,5 EI
Q - 2	$3 \le x \le 5,5$	$2,45 - 1,082x$	$2 + 0,364x$	$0,182x$	2,5 EI

FORMULAS DE LOS DESPLAZAMIENTOS

Formula.

$$\Delta_H = \int_0^L \frac{M_x M_{xH}}{EI} dx$$

$$\delta_{HH} = \int_0^L \frac{M_{xH} M_{xH}}{EI} dx$$

$$\delta_{\theta H} = \int_0^L \frac{M_{xH} M_{x\theta}}{EI}$$

$$\Delta_\theta = \int_0^L \frac{M_x M_{x\theta}}{EI} dx$$

$$\delta_{\theta\theta} = \int_0^L \frac{M_{x\theta} M_{x\theta}}{EI} dx$$

DELTA H

Formula.

$$\Delta_H = \int_0^L \frac{M_x M_{xH}}{EI} dx$$

tramo	Rango	M(x) Carga Real	M_{xH}	$M_{x\theta}$	EI
1 - P	$0 \leq x \leq 3$	0	x	1	EI
P - 2	$3 \leq x \leq 4$	$-3{,}5x + 10{,}5$	x	1	EI
4 - 3	$0 \leq x \leq 2$	$3{,}5x$	$-x$	0	2EI
3 - Q	$0 \leq x \leq 3$	$-7 + 5{,}218x - 1{,}05x^2$	$2 + 0{,}364x$	$0{,}182x$	2,5 EI
Q - 2	$3 \leq x \leq 5{,}5$	$2{,}45 - 1{,}082x$	$2 + 0{,}364x$	$0{,}182x$	2,5 EI

$$= \int_0^3 0\, dx + \int_3^4 \frac{(-3{,}5x + 10{,}5)(x)}{EI} dx + \int_0^2 \frac{3{,}5x(-x)}{2EI} dx + \int_0^3 \frac{(-7 + 5{,}218x - 1{,}05x^2(2 + 0{,}364x)}{2{,}5\,EI}$$

$$+ \int_3^{5{,}5} \frac{(2{,}45 - 1{,}082x)(2 + 0{,}364x)}{2{,}5\,EI} dx$$

$$\Delta_H = -\frac{6{,}4167}{EI} - \frac{4{,}6667}{EI} - \frac{6{,}4198}{EI} - \frac{7{,}8259}{EI}$$

$$\Delta_H = -\frac{25{,}3291}{EI}$$

DELTA θ

tramo	Rango	$M_{(x)}$ Carga Real	M_{xH}	$M_{x\theta}$	EI
1 - P	$0 \leq x \leq 3$	0	x	1	EI
P - 2	$3 \leq x \leq 4$	$-3,5x + 10,5$	x	1	EI
4 - 3	$0 \leq x \leq 2$	$3,5x$	$-x$	0	2EI
3 - Q	$0 \leq x \leq 3$	$-7 + 5,218x - 1,05x^2$	$2 + 0,364x$	$0,182x$	2,5 EI
Q - 2	$3 \leq x \leq 5,5$	$2,45 - 1,082x$	$2 + 0,364x$	$0,182x$	2,5 EI

$$\Delta_\theta = \int_0^L \frac{M_x \, M_{x\theta}}{EI} \, dx$$

$$= \int_0^3 0 \, dx + \int_3^4 \frac{(-3,5x + 10,5)(1)}{EI} \, dx + \int_0^2 0 \, dx + \int_0^3 \frac{(-7 + 5,218x - 1,05x^2}{2,5EI}$$

$$+ \int_3^{5,5} \frac{2,45 - 1,082x\,(0,182x)}{2,5EI} \, dx$$

$$\Delta_\theta = -\frac{1,75}{EI} - \frac{0,5073}{EI} - \frac{1,7644}{EI}$$

$$\Delta_\theta = -\frac{4,0217}{EI}$$

DELTA HH

$$\delta_{HH} = \int_0^L \frac{M_{xH} M_{xH}}{EI} dx$$

tramo	Rango	$M_{(x)}$ Carga Real	M_{xH}	$M_{x\theta}$	EI
1 - P	$0 \leq x \leq 3$	0	x	1	EI
P - 2	$3 \leq x \leq 4$	$-3,5x + 10,5$	x	1	EI
4 - 3	$0 \leq x \leq 2$	$3,5x$	$-x$	0	2EI
3 - Q	$0 \leq x \leq 3$	$-7 + 5,218x - 1,05x^2$	$2 + 0,364x$	$0,182x$	2,5 EI
Q - 2	$3 \leq x \leq 5,5$	$2,45 - 1,082x$	$2 + 0,364x$	$0,182x$	2,5 EI.

$$= \int_0^4 \frac{x^2}{EI} dx + \int_0^2 \frac{(-x)^2}{2EI} dx + \int_0^{5,5} \frac{(2 + 0,364x)^2}{2,5\,EI} dx$$

$$= \frac{21,3333}{EI} + \frac{1,3333}{EI} + \frac{20,548}{EI}$$

$$\delta_{HH} = \frac{43,2146}{EI}$$

DELTA θθ

$$\delta_{\theta\theta} = \int_0^L \frac{M_{x\theta} M_{x\theta}}{EI} dx$$

$$= \int_0^4 \frac{1}{EI} dx + \int_0^2 0 \, dx + \int_0^{5,5} \frac{(0,182x)^2}{EI} dx$$

$$= \frac{4}{EI} + \frac{0,7348}{EI}$$

$$\delta_{\theta\theta} = \frac{4,7348}{EI}$$

tramo	Rango	$M_{(x)}$ Carga Real	M_{xH}	$M_{x\theta}$	EI
1 - P	$0 \leq x \leq 3$	0	x	1	EI
P - 2	$3 \leq x \leq 4$	$-3,5x + 10,5$	x	1	EI
4 - 3	$0 \leq x \leq 2$	$3,5x$	$-x$	0	2EI
3 - Q	$0 \leq x \leq 3$	$-7 + 5,218x - 1,05x^2$	$2 + 0,364x$	$0,182x$	2,5 EI
Q - 2	$3 \leq x \leq 5,5$	$2,45 - 1,082x$	$2 + 0,364x$	$0,182x$	2,5 EI

DELTA Hθ

$$\delta_{\theta H} = \int_0^L \frac{M_{xH}\, M_{x\theta}}{EI}$$

$$= \int_0^4 \frac{x\,(1)}{EI}\,dx + \int_0^2 0\,dx + \int_0^{5,5} \frac{2 + 0,364x\,(0,182x)}{2,5\,EI}\,dx$$

$$= \frac{8}{EI} + \frac{3,6718}{EI}$$

$$\delta_{H\theta} = \frac{11,6718}{EI}.$$

tramo	Rango	$M_{(x)}$ Carga Real	M_{xH}	$M_{x\theta}$	EI
1 - P	$0 \le x \le 3$	0	x	1	EI
P - 2	$3 \le x \le 4$	$-3,5x + 10,5$	x	1	EI
4 - 3	$0 \le x \le 2$	$3,5x$	$-x$	0	2EI
3 - Q	$0 \le x \le 3$	$-7 + 5,218x - 1,05x^2$	$2 + 0,364x$	$0,182x$	2,5 EI
Q - 2	$3 \le x \le 5,5$	$2,45 - 1,082x$	$2 + 0,364x$	$0,182x$	2,5 EI.

SUSTITUIMOS DE TERMINOS EN LA ECUACION.

$\Delta_H = -\frac{25,3291}{EI}$ $\Delta_\Theta = -\frac{4,0217}{EI}$ $\delta_{HH} = \frac{43,2146}{EI}$ $\delta_{\Theta\Theta} = \frac{4,7348}{EI}$ $\delta_{H\Theta} = \frac{11,6718}{EI}$

FORMULA DE SUSTITUCION

$$\Delta_H + \delta_{HH} R_{1x} + \delta_{H\Theta} M_1 = 0$$
$$\Delta_\Theta + \delta_{H\Theta} R_{1x} + \delta_{\Theta\Theta} M_1 = 0$$

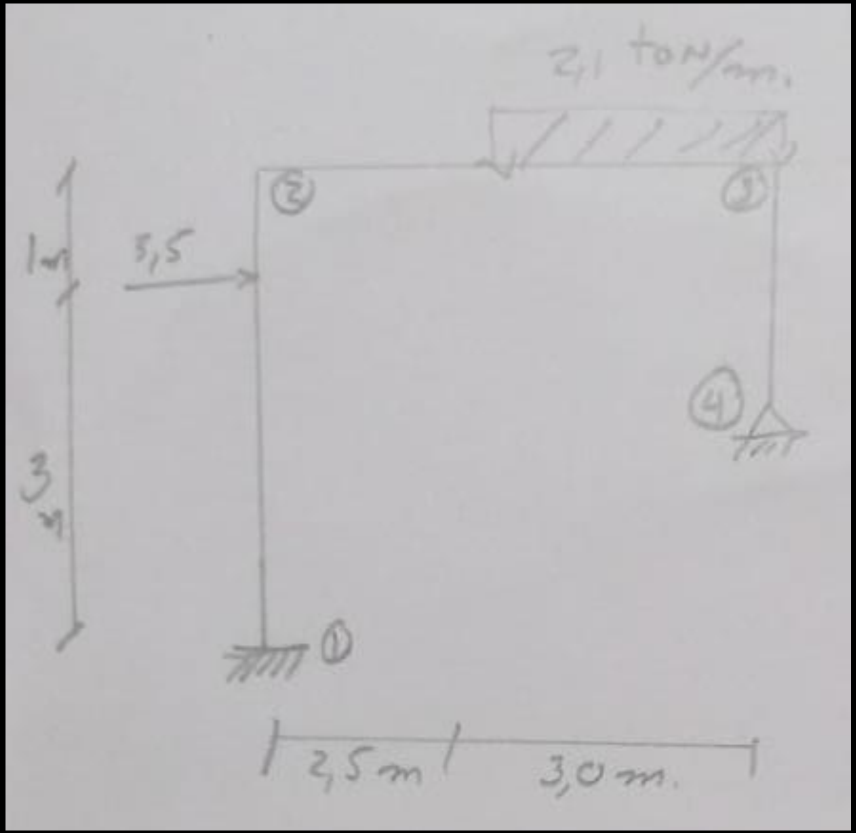
SUSTITUCION

$$-25,329 + 43,2148 R_{1x} + 11,6718 M_1 = 0$$
$$-4,0217 + 11,6718 R_{1x} + 4,7348 M_1 = 0$$

RESPUESTAS DE LAS INCOGNITAS

$$R_{1x} = 1,06$$
$$M_1 = -1,76$$

Marco estáticamente indeterminado grado 2



Valores encontrados en punto 1 por método de fuerza.

$R_{1x} = 1,06$ ton

$M_1 = 1,76$ ton.m ↺

CALCULO DE REACCIONES

$\Sigma F_x = 0$

$3,5 + R_{4x} - 1,06 = 0$

$\underline{R_{4x} = 2,44 \text{ ton.} \leftarrow}$

$+↺ \; \Sigma M_1 = 0$

$1,76 - 3,5(3) + 244(2) - 2,1(3,0)(4,0) + 5,5 R_{4y} = 0$

$\underline{\uparrow \; R_{4y} = 5,284 \text{ ton}}$

$+\uparrow \; \Sigma F_y = 0$

$R_{1y} - (2,1)(3) + 5,284 = 0$

$\underline{+\uparrow \; R_{1y} = 1,016 \text{ Ton.}}$

💪 MÉTODO DE 💪

FUERZAS

ARMADURAS

METODO DE FUERZAS

EN 3 PASOS

GRADO DE
INDETERMINACIÓN

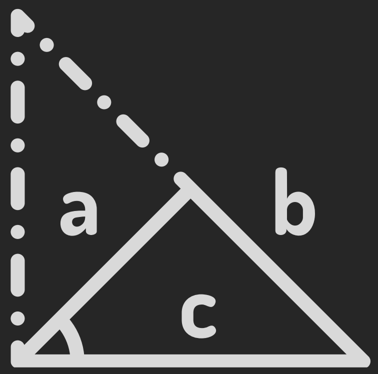
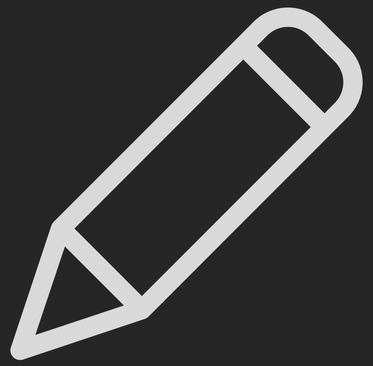


DIAGRAMA DE
ARMADURA REAL E IMAGINARIA

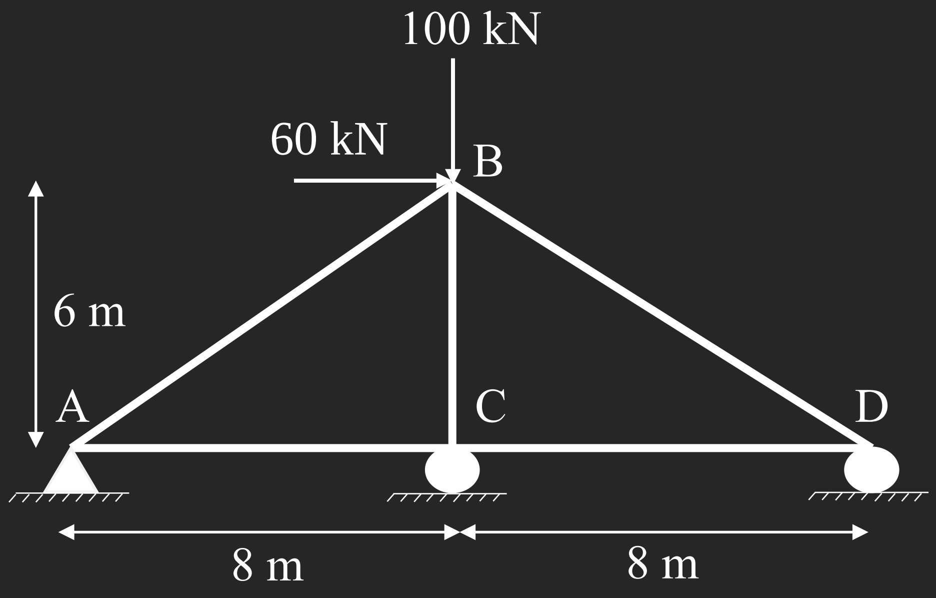


REMPLAZAR
FÓRMULA

GRADO DE

INDETERMINACIÓN

ENUNCIADO: DETERMINAR POR EL METODO DE LAS FUERZAS LA REACCIÓN EN EL APOYO C.



$$A_{BC} = 2000\ mm^2$$

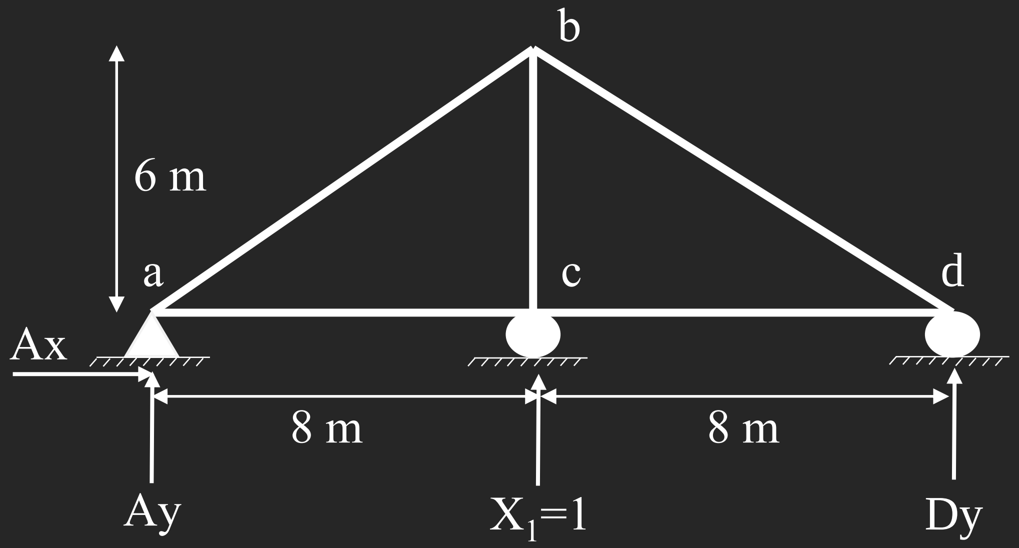
$$A_{AB} = A_{BD} = A_{AC} = A_{CD} = 3000\ mm^2$$

$$Grado\ de\ Indeterminación$$

$$GI = 9 - 2(4)$$

$$GI = 1 \rightarrow Hiperstática\ de\ 1\,°$$

REACCIONES

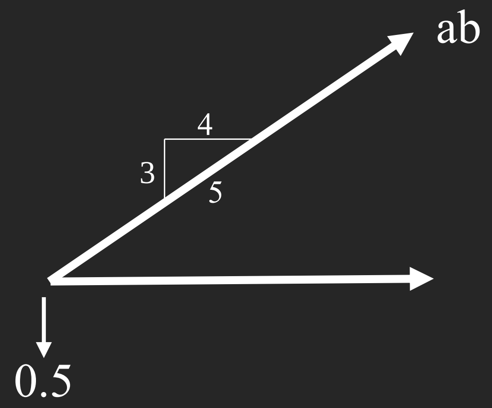


$$\sum M_A = 0$$

$$8(1) + 16(D_y) = 0$$

$$D_y = -0.5kN \downarrow = A_y$$

IMAGINARIA



$$\boldsymbol{Nodo\ \ a}$$

$$\sum F_y = 0$$

$$\frac{3}{5}ab - 0.5 = 0$$

$$ab = 0.83\ kN\ \ \boldsymbol{T}$$

$$\sum F_x = 0$$

$$\frac{4}{5}(0.83) + ac = 0$$

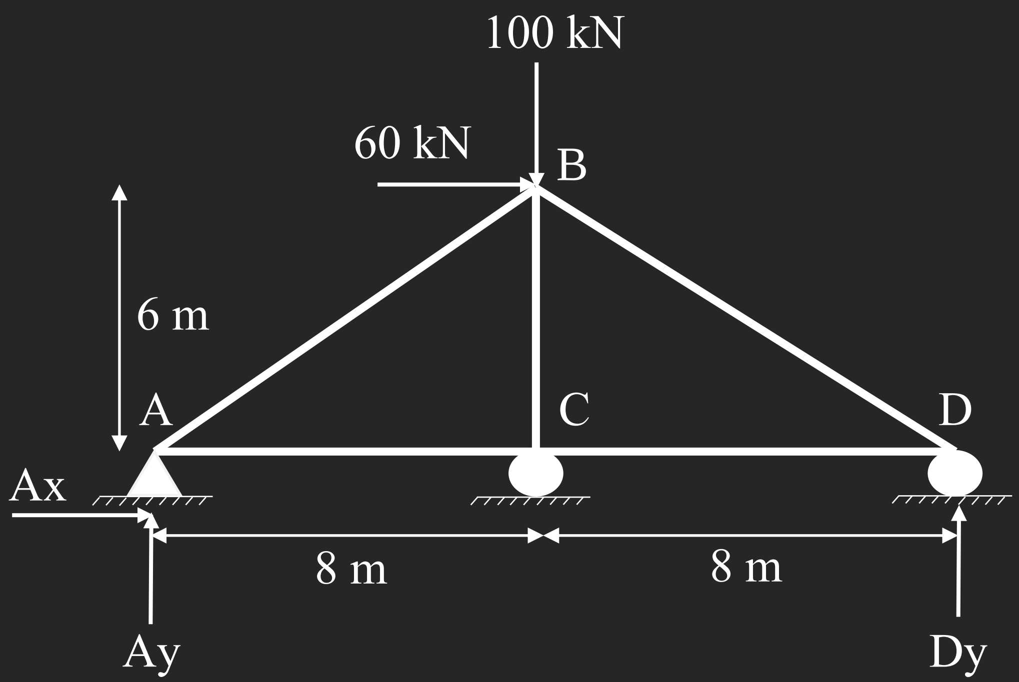
$$ac = -0.664\ kN\ \ \boldsymbol{C}$$

$$\boldsymbol{Nodo\ \ c}$$

$$bc = -1\ kN\ \boldsymbol{C}$$

ARMADURA REAL

REACCIONES



$$\sum M_A = 0$$

$$16(D_y) - 6(60) - 8(100) = 0$$

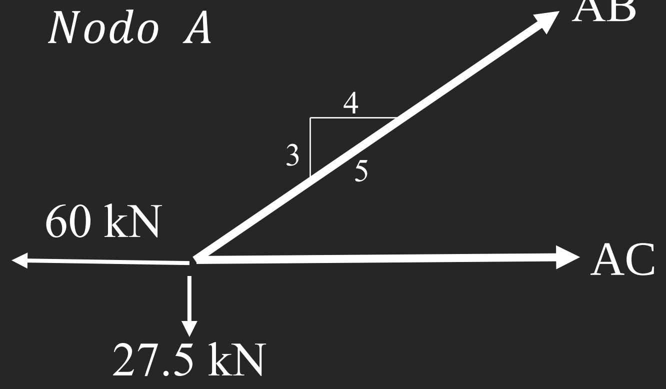
$$D_y = 72.5\ kN \uparrow$$

$$\sum F_y = 0$$

$$-100 + A_y + 72.5 = 0$$

$$A_y = 27.5\ kN\ \uparrow$$

REAL



$$\sum F_y = 0$$

$$\frac{3}{5}AB\ + 27.5 = 0$$

$$AB = -45.83\ \ \boldsymbol{C}$$

$$\sum F_x = 0$$

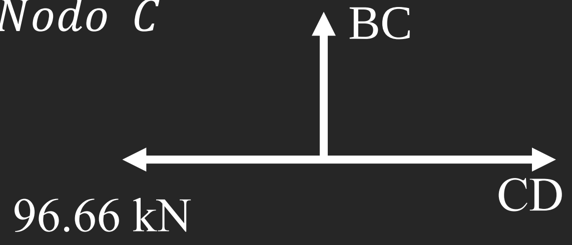
$$\frac{4}{5}(-45.83) + AC\ - 60 = 0$$

$$AC = 96.66\ \ \boldsymbol{T}$$

NODOS EN ARMADURA

REAL

$Nodo\ \ C$



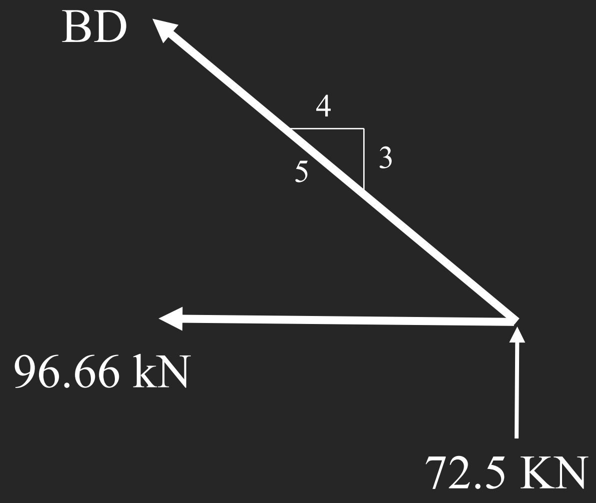
$$\sum F_y = 0$$

$$BC = 0$$

$$\sum F_x = 0$$

$$-96.66 + CD = 0$$

$$CD = 96.66\ \ \boldsymbol{T}$$



$Nodo\ \ D$

$$\sum F_y = 0$$

$$72.5\ + \frac{3}{5} BD = 0$$

$$BD = -120.83\ kN\ \ \boldsymbol{C}$$

REMPLAZAR FUERAS INTERNAS EN LA

ECUACIÓN CANÓNICA

$$\delta_M = \frac{(-1)^2(6)}{E(0.002)} + \left[\frac{(0.83)^2(10) + (-0.66)^2(8)}{E(0.003)}\right](2) = \frac{9916}{E}$$

$$\Delta P = \frac{0}{E(0.002)} + \frac{[(-0.66)(96.66)(8)](2) + (0.83)(-45.83)(10) + (0.83)(-120.83)(10)}{E(0.003)} = \frac{-801335}{E}$$

Remplazamos y Obtenemos

$$\delta_m X_1 + \Delta P = 0$$

$$\frac{9916}{E} X_1 - \frac{801335}{E} = 0$$

$$X_1 = C_y = 80.81\ kN \uparrow$$

GRACIAS

POR SU ATENCIÓN