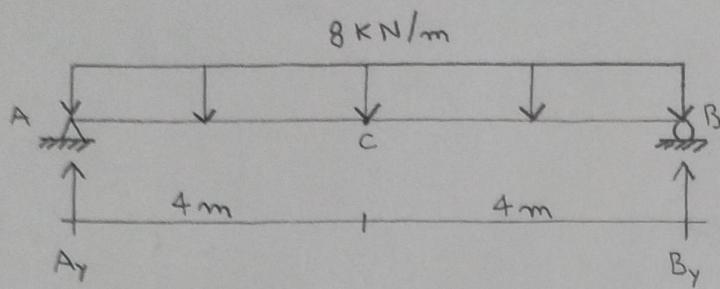


CASTIGLIANO

DETERMINE θ_A y Δ_c



Δ_c AB \rightarrow

$$\sum M_A = 0$$

$$-8(8)(4) + 8B_y - 4P = 0$$

$$B_y = 32 + 0,5P$$

$$A_y = 32 + 0,5P$$

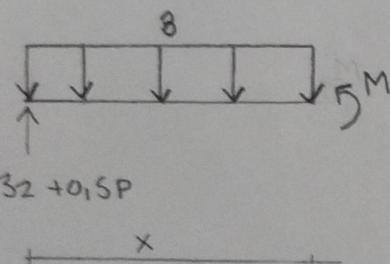
θ_A

$$-8(8)(4) + 8B_y + \bar{M} = 0$$

$$B_y = 32 + \frac{\bar{M}}{8}$$

$$A_y = 32 + \frac{\bar{M}}{8}$$

Δ_c AB $\rightarrow 0 \leq x \leq 8$



$$\sum M_v = 0$$

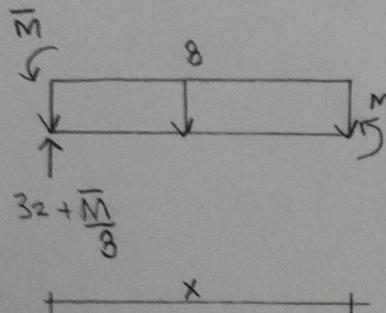
$$M + 8x\left(\frac{x}{2}\right) - x(32 + 0,5P) = 0$$

$$\frac{\partial M}{\partial P} = 0,5x$$

$$M = x(32 + 0,5P) - 4x^2$$

$$M = 32x + 0,5Px - 4x^2$$

θ_A AB $\rightarrow 0 \leq x \leq 8$



$$\sum M_v = 0$$

$$M + 8(x)\left(\frac{x}{2}\right) - x(32 + \frac{\bar{M}}{8}) + \bar{M} = 0 \quad \frac{\partial M}{\partial P} = \frac{x}{8} - 1$$

$$M = x(32 + \frac{\bar{M}}{8}) - 4x^2 - \bar{M}$$

$$M = 32x + \frac{\bar{M}x}{8} - 4x^2 - \bar{M}$$

$$\Delta_c = \int_0^4 (0,5x)(32x - 4x^2) dx$$

$$= \int_0^4 32x^2 - 4x^3$$

$$= \left[\frac{32x^3}{3} - \frac{4x^4}{4} \right]_0^4$$

$$\boxed{\Delta_c = \frac{426,67}{EI} \text{ KN} \cdot \text{m}^3 \downarrow}$$

$$\Theta_A = \int_0^8 \left(\frac{x}{8}(-1) \right) (32x - 4x^2) dx$$

$$= \int_0^8 \frac{x}{8}(32x) + \frac{x}{8}(-4x^2) - 1(32x) - 1(-4x^2) dx$$

$$= \int_0^8 4x^2 - \frac{x^3}{2} - 32x + 4x^2 dx$$

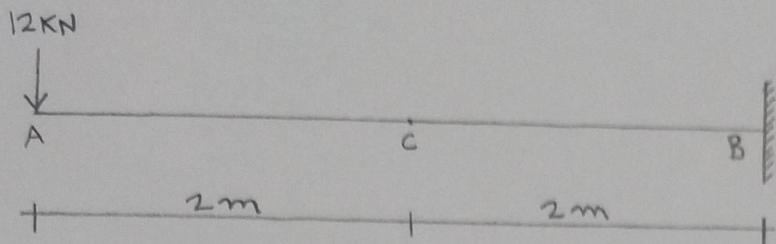
$$= \int_0^8 -\frac{x^3}{2} + 8x^2 - 32x$$

$$= \left[-\frac{x^4}{8} + \frac{8x^3}{3} - 16x^2 \right]_0^8$$

$$\boxed{\Theta_A = \frac{170,6}{EI} \text{ KN} \cdot \text{m}^2 \cancel{\downarrow}}$$

CASTIGLIANO

DETERMINE θ_c y Δ_c



$\Delta_C \rightarrow 0 \leq x \leq 2$

$$\begin{array}{c}
 12 \\
 \downarrow \\
 \text{---} \xrightarrow{\text{M}} \\
 + \quad x \quad +
 \end{array}
 \quad
 \begin{array}{l}
 \sum M = 0 \\
 M + 12x = 0 \\
 M = -12x
 \end{array}
 \quad
 \frac{\partial M}{\partial P} = 0$$

$\Delta_c \text{ AB} \rightarrow 0 \leq x \leq 2$

$$\begin{array}{c}
 12 \\
 \downarrow \\
 \text{---} \xrightarrow{\text{P}} \xrightarrow{\text{M}} \\
 + \quad 2 \quad | \quad x-2 \quad + \\
 + \quad x \quad +
 \end{array}
 \quad
 \begin{array}{l}
 \sum M_V = 0 \\
 M + P(x-2) + 12x = 0 \\
 M = -12x - Px - 2P
 \end{array}
 \quad
 \frac{\partial M}{\partial P} = -x - 2$$

$\theta_c \text{ AB} \rightarrow 0 \leq x \leq 2$

$$\begin{array}{c}
 12 \\
 \downarrow \\
 \text{---} \xrightarrow{\bar{M}} \xrightarrow{M} \\
 + \quad x+2 \quad +
 \end{array}
 \quad
 \begin{array}{l}
 \sum M = 0 \\
 M + \bar{M} + 12(x+2) = 0 \\
 M = -\bar{M} - 12(x+2)
 \end{array}
 \quad
 \frac{\partial M}{\partial P} = -1$$

$$\Delta_c = \int_0^2 (-12x)(-x - 2) dx$$

$$= \int_0^2 12x^2 + 24x dx$$

$$= 4x^3 + 12x^2 \Big|_0^2$$

$$\boxed{\Delta_c = \frac{80}{EI} \text{ KN} \cdot \text{m}^3 \downarrow}$$

$$\Theta_c = \int_0^2 -12(x+2)(-1) dx$$

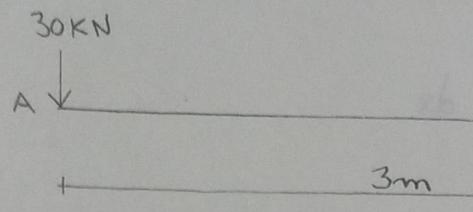
$$= \int_0^2 12x + 24 dx$$

$$= 6x^2 + 24x \Big|_0^2$$

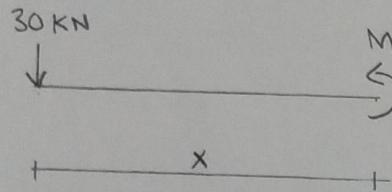
$$\boxed{\Theta_c = \frac{72}{EI} \text{ KN} \cdot \text{m}^2 \nearrow}$$

TRABAJO VIRTUAL

DETERMINE Δ_A y θ_A



$\Delta_A \rightarrow 0 \leq x \leq 3$



$$\sum M = 0$$

$$M + 30x = 0$$

$$M = -30x$$

$$\Delta_A = \int_0^3 -30x(-x) dx$$

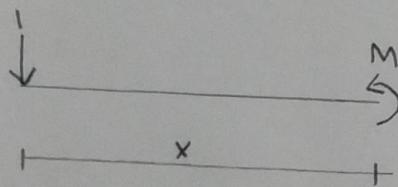
$$= \int_0^3 30x^2 dx$$

$$= \frac{30x^3}{3} \Big|_0^3$$

$$= 10x^3 \Big|_0^3$$

$$\Delta_A = \frac{270}{EI} \text{ KN} \cdot \text{m}^3 \downarrow$$

Δ_A VIRTUAL $AB \rightarrow 0 \leq x \leq 3$



$$\sum M = 0$$

$$M + x = 0$$

$$M = -x$$

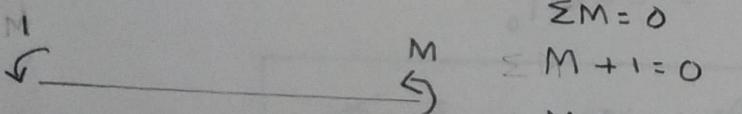
$$\theta_A = \int_0^3 -30x(-1) dx$$

$$= \int_0^3 30x dx$$

$$= \frac{30x^2}{2}$$

$$= \frac{135}{EI} \text{ KN} \cdot \text{m}^2 \swarrow$$

θ_A VIRTUAL $AB \rightarrow 0 \leq x \leq 3$



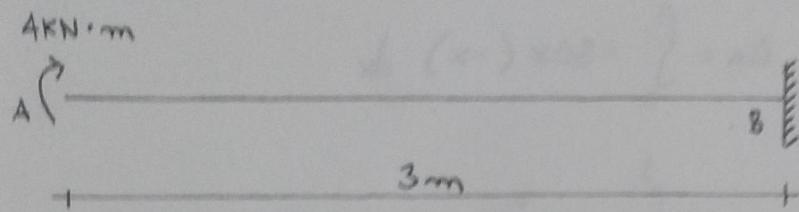
$$\sum M = 0$$

$$\sum M + 1 = 0$$

$$M = -1$$

TRABAJO VIRTUAL

DETERMINE Δ_A y θ_A



$$\Delta_A = \int_0^3 4(-x) \, dx$$

$$= \int_0^3 -4x \, dx$$

$$= -\frac{4x^2}{2} \Big|_0^3$$

$$= -2x^2 \Big|_0^3$$

$$\Delta_A \rightarrow 0 \leq x \leq 3$$

$$\begin{array}{l} 4 \\ \curvearrowleft \\ \hline M \\ \curvearrowright \\ M - 4 = 0 \\ M = 4 \end{array}$$

$$\begin{aligned} \sum M &= 0 \\ M - 4 &= 0 \\ M &= 4 \end{aligned}$$

$$\Delta_A \text{ VIRTUAL } AB \rightarrow 0 \leq x \leq 3$$

$$\begin{array}{l} \downarrow \\ \hline M \\ \curvearrowright \\ M + 1(x) = 0 \\ M = -x \end{array}$$

$$\begin{aligned} \sum M &= 0 \\ M + 1(x) &= 0 \\ M &= -x \end{aligned}$$

$$\boxed{\Delta_A = \frac{18}{EI} \text{ KN} \cdot \text{m}^3 \uparrow}$$

$$\theta_A = \int_0^3 4(-1) \, dx$$

$$= \int_0^3 -4 \, dx$$

$$= -4x \Big|_0^3$$

$$\boxed{\theta_A = \frac{12}{EI} \text{ KN} \cdot \text{m}^2 \rightarrow}$$

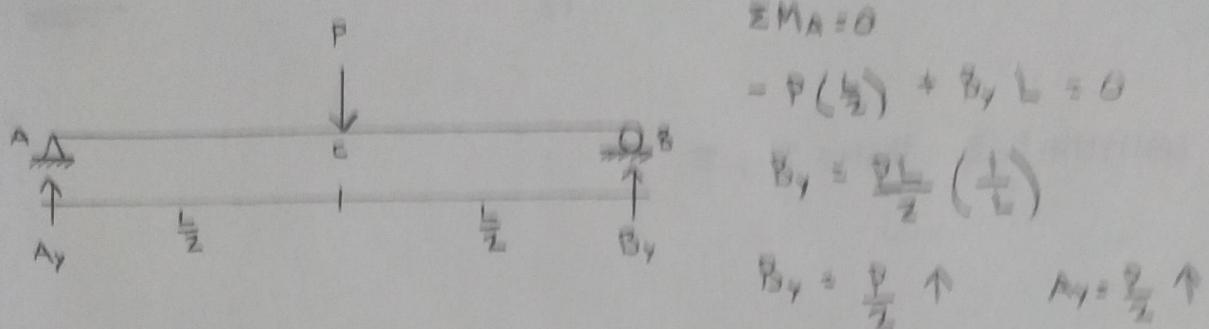
$$\theta_A \text{ VIRTUAL } AB \rightarrow 0 \leq x \leq 3$$

$$\begin{array}{l} \downarrow \\ \hline M \\ \curvearrowright \\ M + 1 = 0 \\ M = -1 \end{array}$$

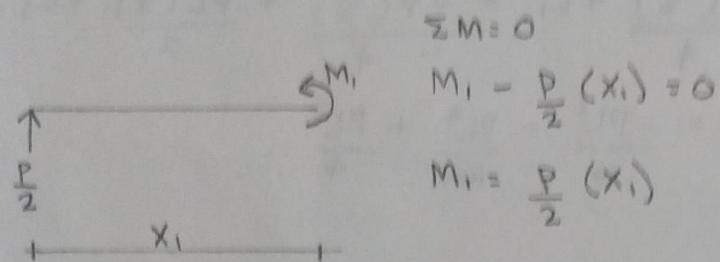
$$\begin{aligned} \sum M &= 0 \\ M + 1 &= 0 \\ M &= -1 \end{aligned}$$

DOBLE INTEGRACIÓN

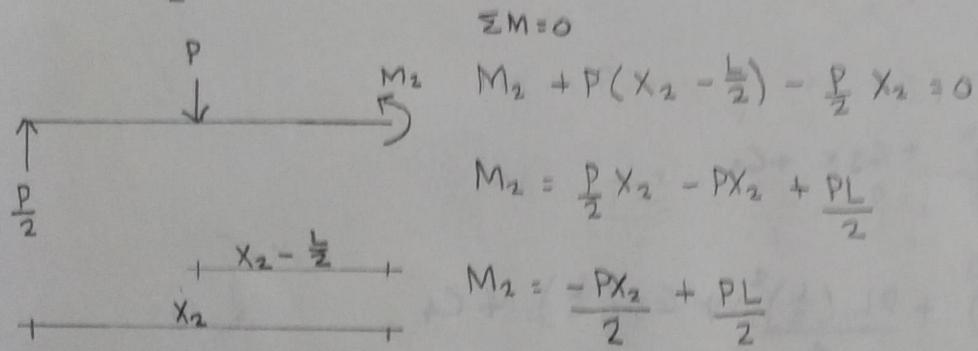
DETERMINE LA ECUACIÓN DE LA CURVA ELÁSTICA



TRAMO I $\rightarrow 0 \leq x_1 \leq \frac{L}{2}$



TRAMO II $\rightarrow \frac{L}{2} \leq x_2 \leq L$



$$EI \theta^I = \int \frac{P}{2}(x_1) dx \Rightarrow EI \theta^I = \frac{Px_1^2}{4} + C_1 \quad (1)$$

$$EI \Delta_{AC}^I = \int \frac{Px_1^2}{4} + C_1 dx \Rightarrow EI \Delta_{AC}^I = \frac{Px_1^3}{12} + C_1 x + C_2 \quad (2)$$

$$EI \theta_{AB}^I = \int -\frac{Px_2}{2} + \frac{PL}{2} dx \Rightarrow EI \theta_{AB}^I = -\frac{Px_2^2}{4} + \frac{PLx_2}{2} + C_3 \quad (3)$$

$$EI \Delta_{AB}^I = \int -\frac{Px_2^2}{4} + \frac{PLx_2}{2} + C_3 dx \Rightarrow EI \Delta_{AB}^I = -\frac{Px_2^3}{12} + \frac{PLx_2^2}{4} + C_3 x_2 + C_4 \quad (4)$$

$$X = 0 \quad \Delta_x^I = 0$$

$$EI \Delta_x^I = \frac{PX_1^3}{12} + C_1 X + C_2 \Rightarrow EI(0) = \frac{P(0)^3}{12} + C_1(0) + C_2 \Rightarrow \underline{\underline{C_2 = 0}}$$

PRINCIPIO DE CONTINUIDAD TRAMO I y II

$$\Theta_{x=\frac{L}{2}}^I = \Theta_{x=\frac{L}{2}}^{II}$$

$$\frac{PX^2}{4} + C_1 = -\frac{PX^2}{4} + \frac{PLX}{2} + C_3 \Rightarrow \frac{P(\frac{L}{2})^2}{4} + C_1 = -\frac{P(\frac{L}{2})^2}{4} + \frac{PL(\frac{L}{2})}{2} + C_3$$

$$\frac{PL^2}{16} + C_1 = -\frac{PL^2}{16} + \frac{PL^2}{4} + C_3 \Rightarrow C_1 - C_3 = \frac{-PL^2}{16} - \frac{PL^2}{16} + \frac{PL^2}{4}$$

$$C_1 - C_3 = \frac{PL^2}{8} \quad (5)$$

$$\Delta_{x=0}^I = \Delta_{x=0}^{II}$$

$$\frac{PX^3}{12} + C_1 X = -\frac{PX^3}{12} + \frac{PLX^2}{4} + C_3 X + C_4$$

$$\frac{P(\frac{L}{2})^3}{12} + C_1(\frac{L}{2}) = -\frac{P(\frac{L}{2})^3}{12} + \frac{PL(\frac{L}{2})^2}{4} + C_3(\frac{L}{2}) + C_4$$

$$\frac{PL^3}{96} + C_1(\frac{L}{2}) = -\frac{PL^3}{96} + \frac{PL^3}{16} + C_3(\frac{L}{2}) + C_4$$

$$\frac{PL^3}{96} + C_1(\frac{L}{2}) - C_3(\frac{L}{2}) + \frac{PL^3}{96} - \frac{PL^3}{16} = C_4 \Rightarrow C_4 = \frac{-PL^3}{24} + \frac{L}{2}(C_1 - C_3)$$

$$C_4 = \frac{-PL^3}{24} + \frac{L}{2} \left(\frac{PL^2}{8} \right)$$

$$\underline{\underline{C_4 = \frac{PL^3}{48}}}$$

$$X = L \quad \Delta^{\text{II}} = 0$$

$$EI \Delta^{\text{II}} = \frac{-PX^3}{12} + \frac{PLX^2}{4} + C_3 X + C_4$$

$$0 = \frac{-PL^3}{12} + \frac{PL(L)^2}{4} + C_3(L) + \left(\frac{PL^3}{48}\right)$$

$$0 = \frac{-PL^3}{12} + \frac{PL^3}{4} + C_3(L) + \frac{PL^3}{48}$$

$$\underline{C_3 = -\frac{3PL^2}{16}}$$

$$C_1 - C_3 = \frac{PL^2}{8}$$

$$C_1 = \frac{PL^2}{8} - \frac{3PL^2}{16}$$

$$\underline{C_1 = \frac{-PL^2}{16}}$$

$$EI \Delta^{\text{I}} = \frac{PX^3}{12} + C_1 X + C_2 \Rightarrow EI \Delta^{\text{I}} = \left(\frac{PX^3}{12} + \left(\frac{-PL^2}{16} \right) X \right) \frac{1}{EI} \quad ②$$

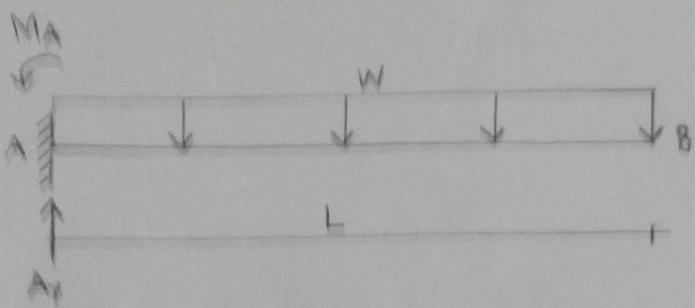
$$EI \Theta^{\text{I}} = \frac{PX^2}{4} + C_1 \Rightarrow EI \Theta^{\text{I}} = \left(\frac{PX^2}{4} + \left(\frac{-PL^2}{16} \right) \right) \frac{1}{EI} \quad ①$$

$$EI \Delta^{\text{II}} = \frac{-PX^3}{12} + \frac{PLX^2}{4} + C_3 X + C_4 \Rightarrow \underline{EI \Delta^{\text{II}} = \frac{-PX^3}{12} + \frac{PLX^2}{4} - \left(\frac{3PL^2}{16} \right) + \left(\frac{PL^3}{48} \right)}$$

$$EI \Theta^{\text{II}} = \frac{-PX^2}{4} + \frac{PLX}{2} + C_3 \Rightarrow \underline{EI \Theta^{\text{II}} = \frac{-PX^2}{4} + \frac{PLX}{2} - \frac{3PL^2}{16}}$$

PROBLEMA DE INTEGRACIÓN

DETERMINE LA ECUACIÓN DE LA CURVA ELÁSTICA.



$$\sum M_A = 0$$

$$M_A - WL \left(\frac{L}{2}\right) = 0$$

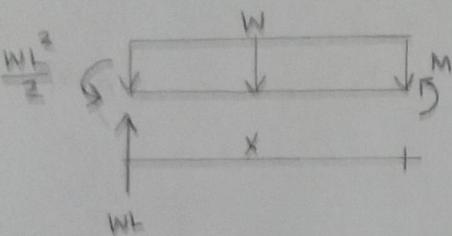
$$M_A = \frac{WL^2}{2}$$

$$\sum F_y = 0$$

$$A_y - WL = 0$$

$$A_y = WL$$

TRAMO I $\leftarrow 0 \leq x \leq L$



$$\sum M = 0$$

$$\frac{WL^2}{2} - WLx + Wx \left(\frac{x}{2}\right) + M = 0$$

$$M = -\frac{WL^2}{2} - \frac{Wx^2}{2} + WLx$$

$$EI\theta = \int -\frac{WL^2}{2} - \frac{Wx^2}{2} + WLx = -\frac{WL^2x}{2} - \frac{Wx^3}{6} + \frac{WLx^2}{2} + C_1$$

$$EI\Delta = \int -\frac{WL^2x}{2} - \frac{Wx^3}{6} + \frac{WLx^2}{2} + C_1 = -\frac{WL^2x^2}{4} - \frac{Wx^4}{24} + \frac{WLx^3}{6} + C_1x + C_2$$

$$x=0 \quad \Delta=0$$

$$EI\Delta = -\frac{WL^2x^2}{4} - \frac{Wx^4}{24} - \frac{WLx^3}{6} + C_1x + C_2 \Rightarrow \underline{\underline{C_2=0}}$$

$$x=0 \quad \theta=0$$

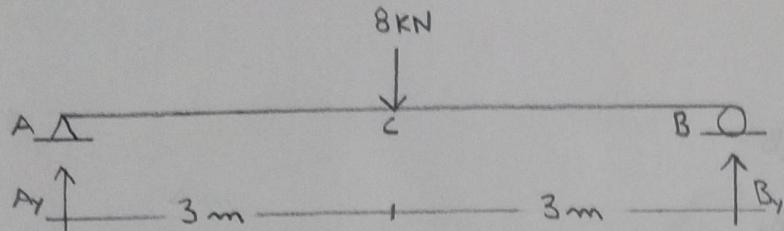
$$EI\theta = -\frac{WL^2x}{2} - \frac{Wx^3}{6} + \frac{WLx^2}{2} + C_1 \Rightarrow \underline{\underline{C_1=0}}$$

$$\theta = \left(-\frac{WL^2x}{2} - \frac{Wx^3}{6} + \frac{WLx^2}{2} \right) \left(\frac{1}{EI} \right)$$

$$\Delta = \left(-\frac{WL^2x^2}{4} - \frac{Wx^4}{24} + \frac{WLx^3}{6} \right) \left(\frac{1}{EI} \right)$$

VIGA CONJUGADA

DETERMINE θ_A y Δ_C



$$\sum M_A = 0$$

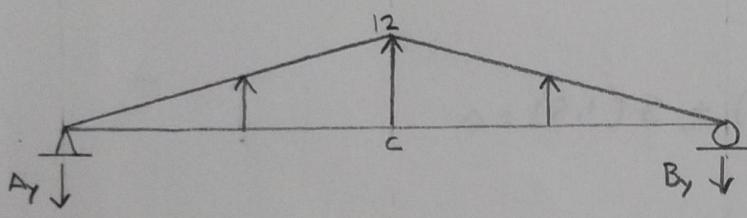
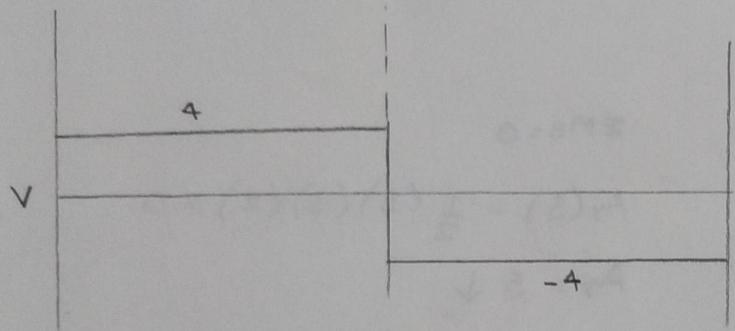
$$-3(8) + 6B_y = 0$$

$$B_y = 4 \text{ kN } \uparrow$$

$$\sum F_y = 0$$

$$A_y - 8 + 4 = 0$$

$$A_y = 4 \text{ kN } \uparrow$$



$$\sum M_B = 0$$

$$-\frac{1}{2}(12)(3)(2) - \frac{1}{2}(12)(3)(4) + 6A_y = 0$$

$$-36 - 72 + 6A_y = 0$$

$$A_y = 18 \downarrow$$

$$\sum M_c = 0$$

$$-\frac{1}{2}(12)(3)(1) + 3(18) + M_c = 0$$

$$M_c - 18 + 54 = 0$$

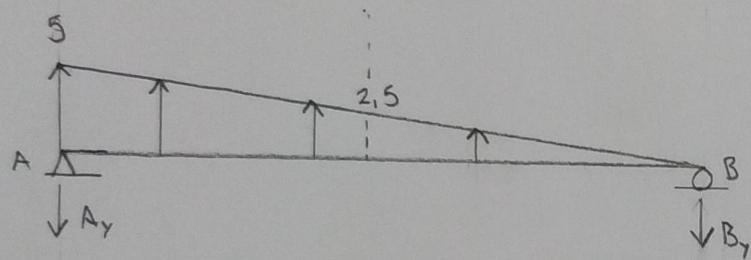
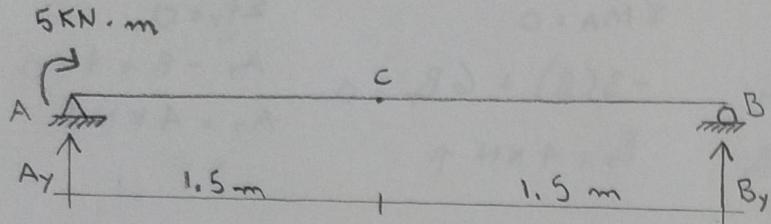
$$M_c = -36 \leftarrow$$

$$\theta_A = V_A = \frac{18}{EI} \text{ KN} \cdot \text{m}^2 \quad \triangleleft$$

$$\Delta_C = M_c = \frac{36}{EI} \text{ KN} \cdot \text{m}^3 \quad \downarrow$$

VIGA CONJUGADA

DETERMINE θ_A y Δ_c



$$\sum M_B = 0$$

$$A_y(3) - \frac{1}{2}(5)(3)(2) = 0$$

$$A_y = 5 \downarrow$$

$$\sum M_C = 0$$

$$M_C - 2,5(1,5)(0,75) - \frac{1}{2}(2,5)(1,5)(1) + 5(1,5) = 0$$

$$M_C - 2,8125 - 1,875 + 7,5 = 0$$

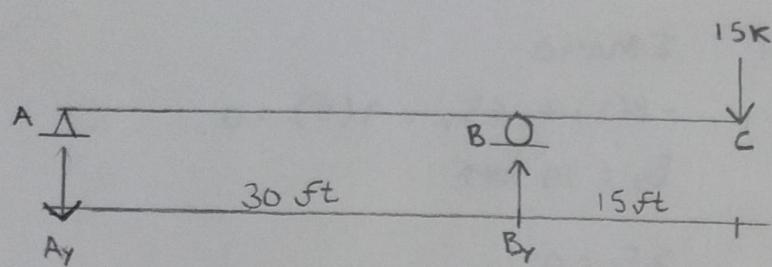
$$M_C = 2,8125 \downarrow$$

$$\Theta_A = V_A = \frac{5}{EI} \text{ KN} \cdot \text{m}^2 \quad \text{A}$$

$$\Delta_c = M_c = \frac{2,8125}{EI} \text{ KN} \cdot \text{m}^3 \quad \downarrow$$

VIGA CONJUGADA

DETERMINE θ_c y Δ_c



$$\sum M_A = 0$$

$$30 B_y - 45 (15) = 0$$

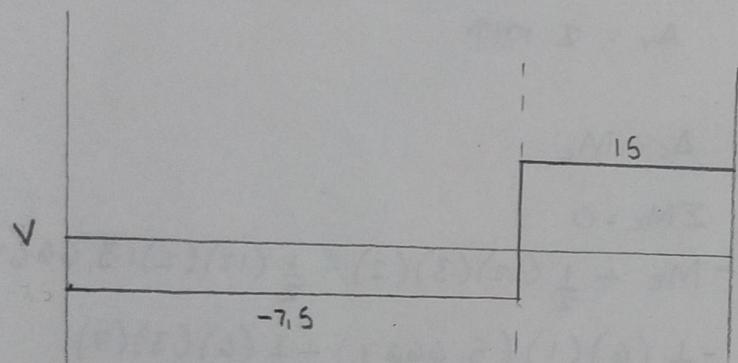
$$B_y = 22,5 \text{ k} \uparrow$$

$$\sum F_y = 0$$

$$-A_y + 22,5 - 15 = 0$$

$$A_y = 7,5 \text{ k} \downarrow$$

$$\Delta_c = M_c$$



$$\sum M_c = 0$$

$$-M_c + \frac{1}{2} (225)(15)(10) +$$

$$\frac{1}{2} (225)(30)(25) - 1125(45) = 0$$

$$-M_c + 16875 + 84375 - 50625 = 0$$

$$M_c = -\frac{50625}{EI} \text{ k} \cdot \text{ft}^3 \downarrow$$

$$\theta_c = V_c$$

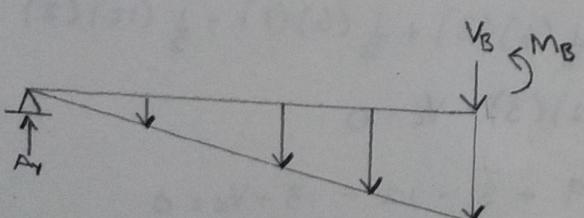
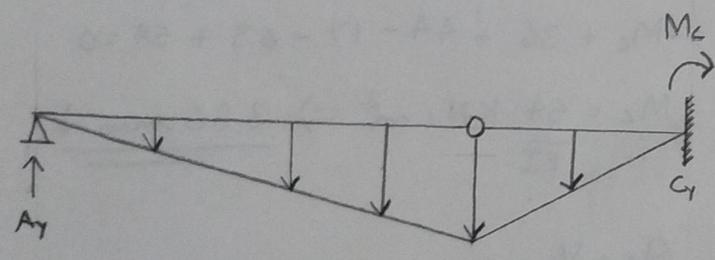
$$\sum F_y = 0$$

$$-V_c - \frac{1}{2} (225)(15) - \frac{1}{2} (225)(30)$$

$$+ 1125 = 0$$

$$-V_c - 1687,5 - 3375 + 1125 = 0$$

$$V_c = -\frac{3937,5}{EI} \text{ k} \cdot \text{ft}^2 \quad \Delta$$



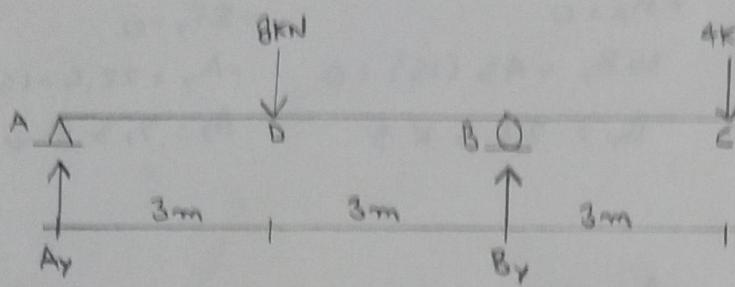
$$\sum M_B = 0$$

$$\frac{1}{2} (225)(30)(10) - 30 A_y = 0$$

$$A_y = 1125 \uparrow$$

VIGA CONJUGADA

DETERMINE θ_c y v_c , ($E = 200 \text{ GPa}$, $I = 76 \times 10^6 \text{ mm}^4$)



4kN
↓
C

$$\sum M_A = 0$$

$$-8(3) + 6B_y - 4(4) = 0$$

$$B_y = 10 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$A_y - 8 + 10 - 4 = 0$$

$$A_y = 2 \text{ kN} \uparrow$$

$$\Delta_c = M_c$$

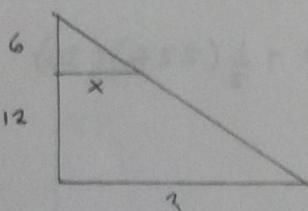
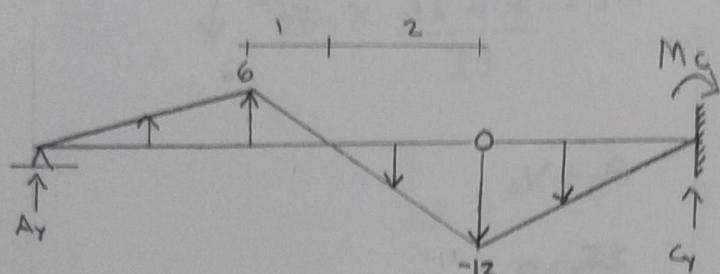
$$\sum M_C = 0$$

$$-M_c + \frac{1}{2}(12)(3)(2) + \frac{1}{2}(12)(2)(3,6667) \\ - \frac{1}{2}(6)(1)(5,6667) - \frac{1}{2}(6)(3)(7)$$

$$-9(-6) = 0$$

$$-M_c + 36 + 44 - 17 - 63 + 54 = 0$$

$$M_c = \frac{54}{EI} \text{ KN} \cdot \text{m}^3 \Rightarrow \underline{\underline{3,86 \text{ mm}}} \downarrow$$



$$\frac{x}{6} = \frac{3}{18}$$

$$x = 1$$

$$\theta_c = v_c$$

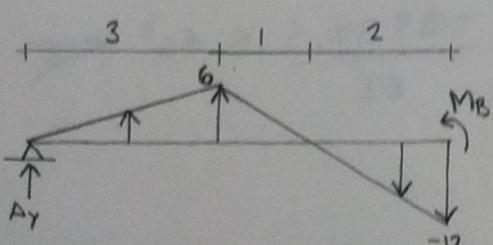
$$\sum F_y = 0$$

$$-6 + \frac{1}{2}(6)(3) + \frac{1}{2}(6)(1) - \frac{1}{2}(12)(2)$$

$$-\frac{1}{2}(12)(3) - v_c = 0$$

$$-6 + 9 + 3 - 12 - 18 - v_c = 0$$

$$v_c = \frac{-24}{EI} \text{ KN} \cdot \text{m}^2 \Rightarrow \underline{\underline{0,001714 \text{ RAD}}} \triangleleft$$



$$\sum M_B = 0$$

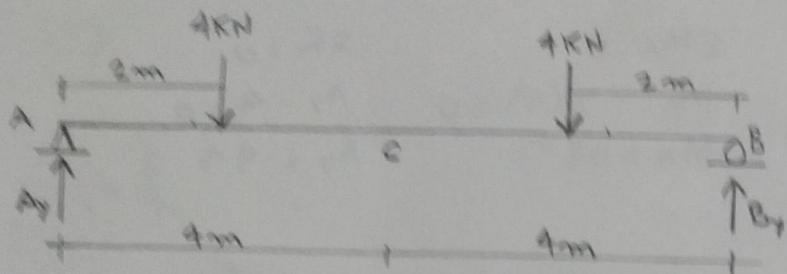
$$\frac{1}{2}(12)(2)(0,6667) - \frac{1}{2}(6)(1)(2,6667) - \frac{1}{2}(6)(3)(4) - 6A_y = 0$$

$$8 - 8 - 36 - 6A_y = 0$$

$$A_y = -6 \downarrow$$

VIGA CONJUGADA

DETERMINE θ_A y Δ_c



$$\sum M_A = 0$$

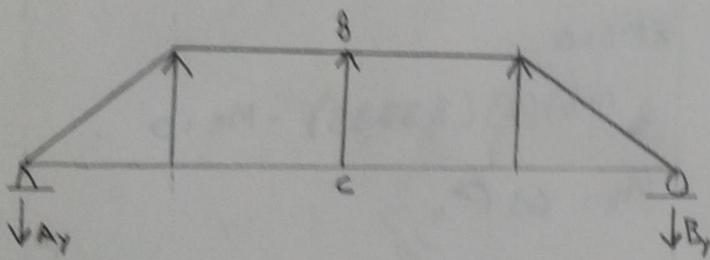
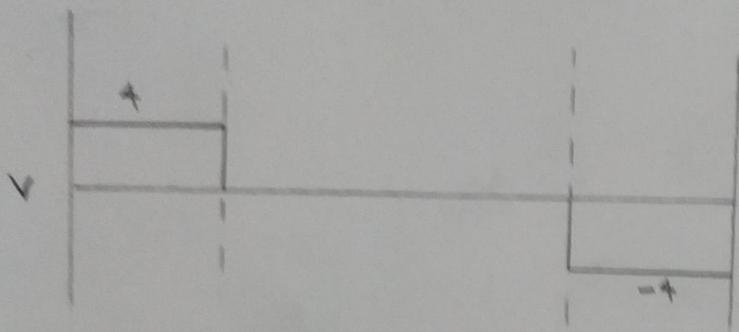
$$-2(4) - 6(4) + 8B_{cy} = 0$$

$$B_{cy} = 4 \uparrow$$

$$\sum F_y = 0$$

$$Ay - 4 - 4 + 4 = 0$$

$$Ay = 4 \uparrow$$



$$\sum M_B = 0$$

$$-\frac{1}{2}(8)(2)(1,3333) - 8(4)(4)$$

$$-\frac{1}{2}(8)(2)(6,6667) + 8A_y = 0$$

$$-10,6664 - 128 - 53,3334 + 8A_y = 0$$

$$A_y = 24 \downarrow$$

$$\sum M_C = 0$$

$$-8(2)(1) - \frac{1}{2}(8)(2)(2,6667) + 4(-24) + M_c = 0$$

$$M_c = 16 - 21,3336 + 96 = 0$$

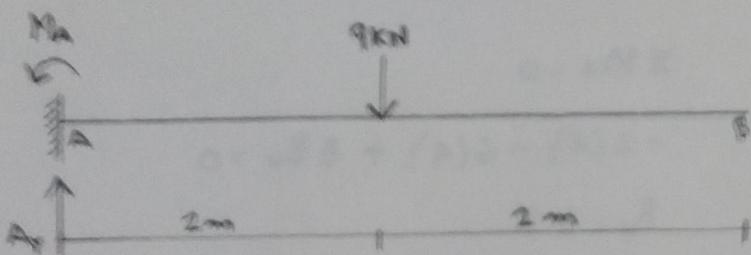
$$M_c = -58,7 \rightarrow$$

$$\theta_A = V_A = \frac{24}{EI} \text{ KN} \cdot m^2 \quad \Delta$$

$$\Delta_c = M_c = \frac{58,7}{EI} \text{ KN} \cdot m^3 \quad \downarrow$$

VIGA CONJUGADA

DETERMINE θ_B y A_B



$$\sum M_A = 0$$

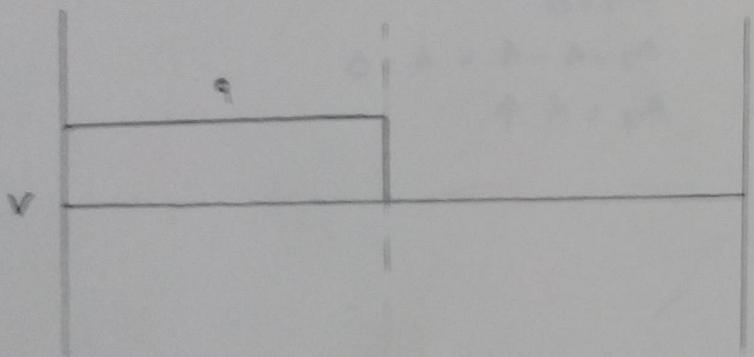
$$M_A - 2(9) = 0$$

$$M_A = 18 \text{ ↗}$$

$$\sum F_y = 0$$

$$A_y - 9 = 0$$

$$A_y = 9 \uparrow$$



$$\sum M_B = 0$$

$$\frac{1}{2}(18)(2)(3,3333) - M_B = 0$$

$$M_B = 60 \text{ ↗}$$

$$\sum F_y = 0$$

$$-\frac{1}{2}(18)(2) + B_y = 0$$

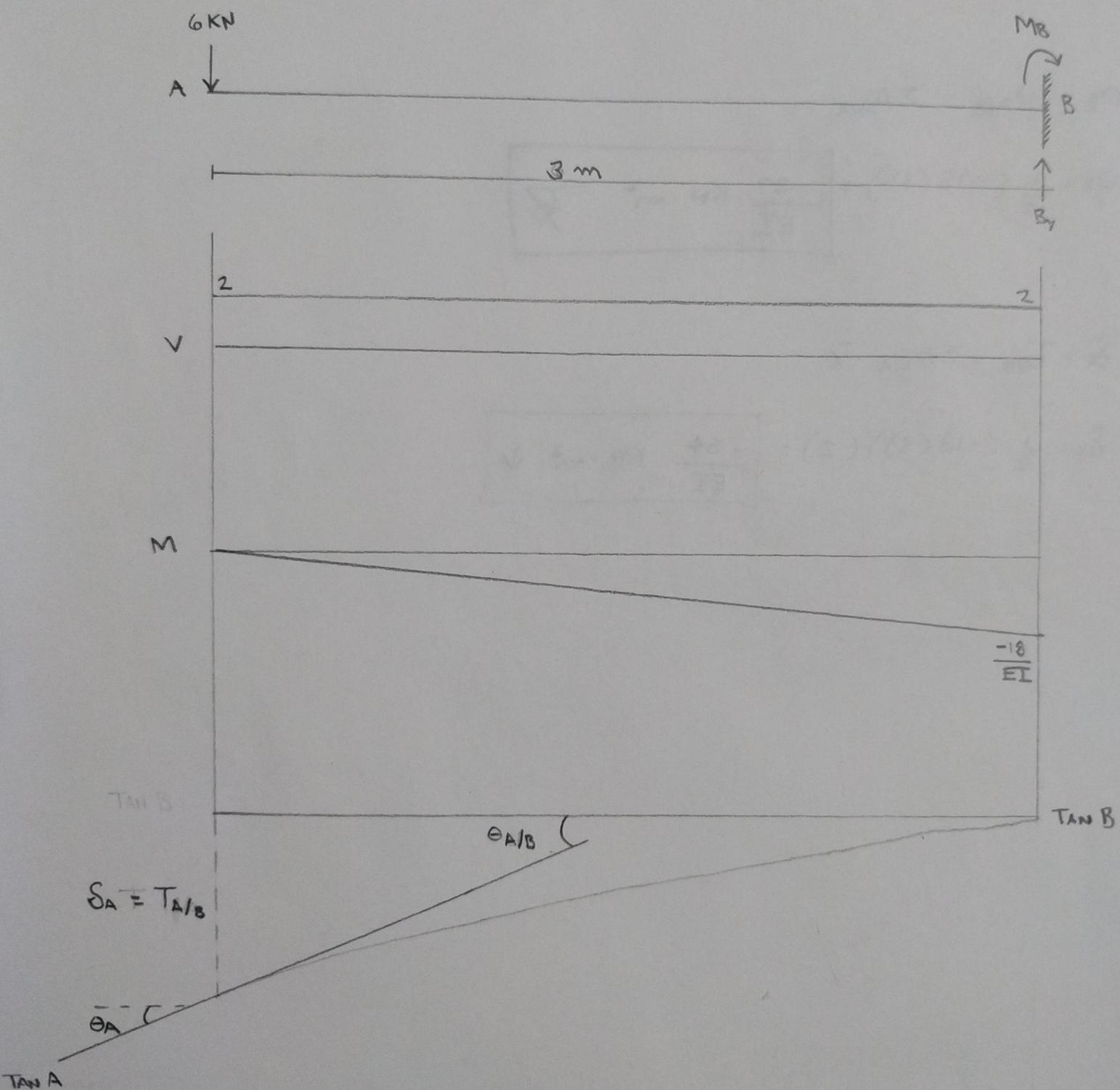
$$B_y = 18 \uparrow$$

$$\theta_B = V_B = \frac{18}{EI} \text{ KN} \cdot \text{m}^2$$

$$\Delta R = M_B = \frac{60}{EI} \text{ KN} \cdot \text{m}^3$$

AREA MOMENTO

DETERMINE LA PENDIENTE Y LA DEFLEXION EN A . $EI = \text{CONSTANTE}$



REACIONES

$$\sum F_y = 0$$

$$-6 + B_y = 0$$

$$B_y = 6 \text{ KN} \uparrow$$

$$\sum M_B = 0$$

$$-M_B + 6(3) = 0$$

$$M_B = 18 \text{ KN} \leftarrow$$

$$\Theta_B = \Theta_{A/B} = \sum A_{B/A}$$

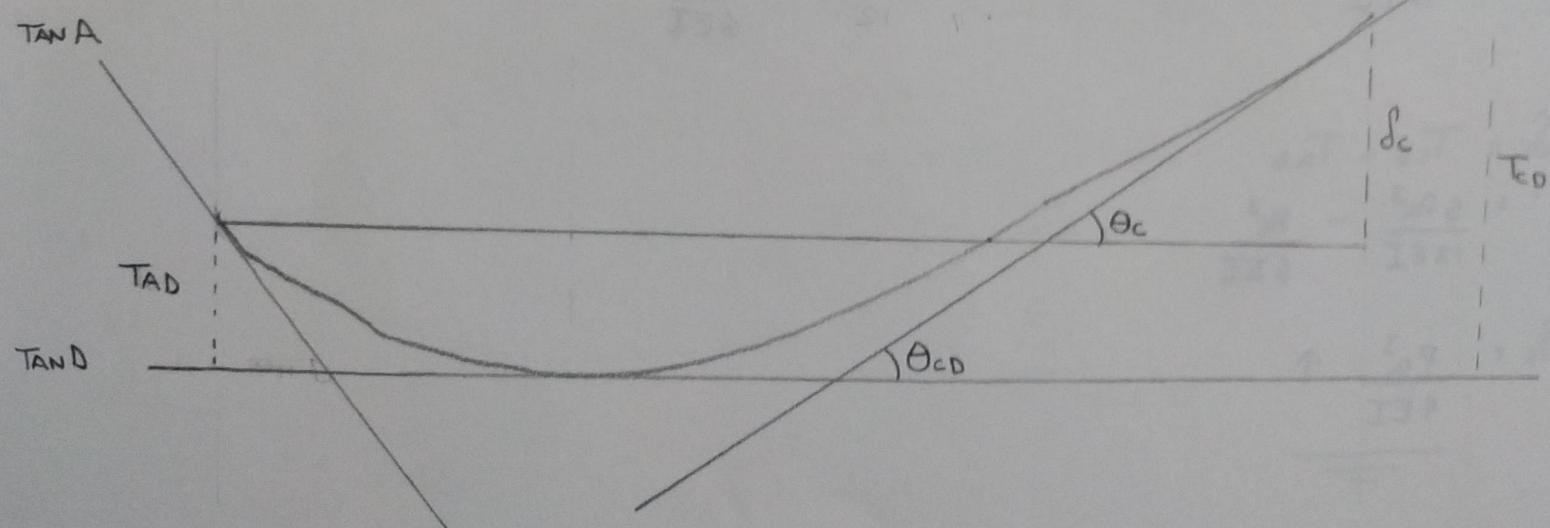
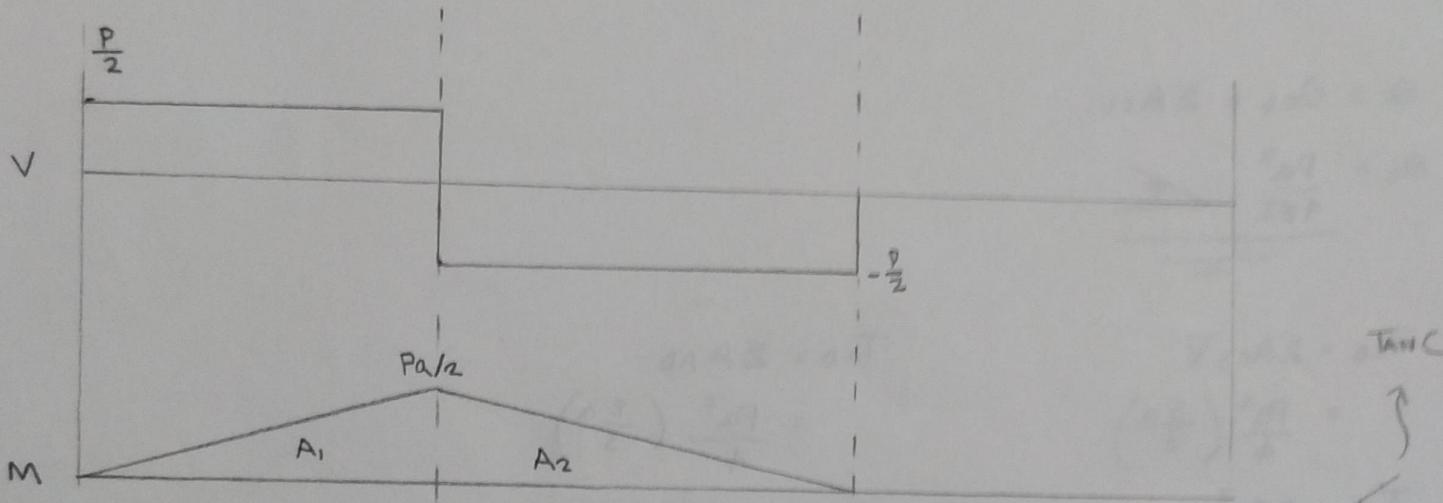
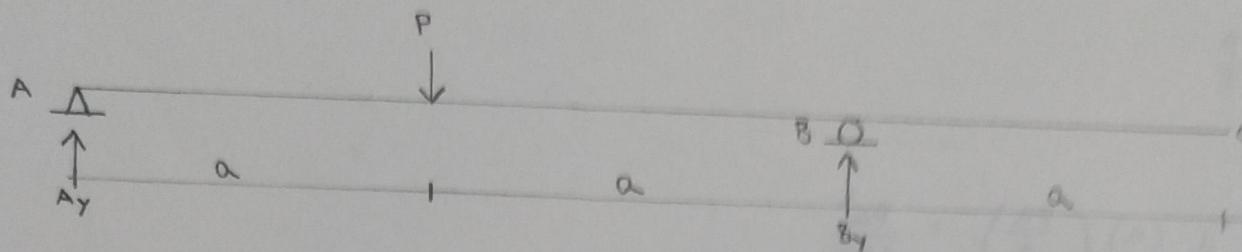
$$\Theta_B = \frac{1}{2} (-18(3)) = \boxed{\frac{-27}{EI} \text{ KN} \cdot m^2 \downarrow}$$

$$\delta_A = T_{A/B} = \sum A_{B/A} \cdot \tilde{x}$$

$$\delta_A = \frac{1}{2} (-18(3))(2) = \boxed{\frac{-54}{EI} \text{ KN} \cdot m^3 \downarrow}$$

AREA MOMENTO

DETERMINE LA PENDIENTE Y EL DESPLAZAMIENTO EN C,
 $EI = \text{CONSTANTE}$.



REACCIONES

$$\sum M_A = 0$$

$$-aP + 2aB_y = 0$$

$$B_y = \frac{aP}{2a}$$

$$B_y = \frac{P}{2}$$

$$\sum F_y = 0$$

$$A_y - P + \frac{P}{2}$$

$$A_y = \frac{P}{2}$$

$$A_1 = A_2 = \frac{1}{2} (a) \left(\frac{Pa}{2} \right) = \frac{Pa^2}{4EI}$$

$$\Theta_c = \Theta_{c0} = \sum A_{cd}$$

$$\Theta_c = \frac{\frac{Pa^2}{4EI}}{=}$$

$$T_{CD} = \sum A_{cd} \bar{x}$$

$$= \frac{Pa^2}{4} \left(\frac{5}{3}a \right)$$

$$T_{CD} = \frac{5Pa^3}{12EI}$$

$$T_{AD} = \sum A_{AD} -$$

$$= \frac{Pa^2}{4} \left(\frac{2}{3}a \right)$$

$$T_{AD} = \frac{2Pa^2}{12} = \frac{Pa^3}{6EI}$$

$$\delta_c = T_{CD} - T_{AD}$$

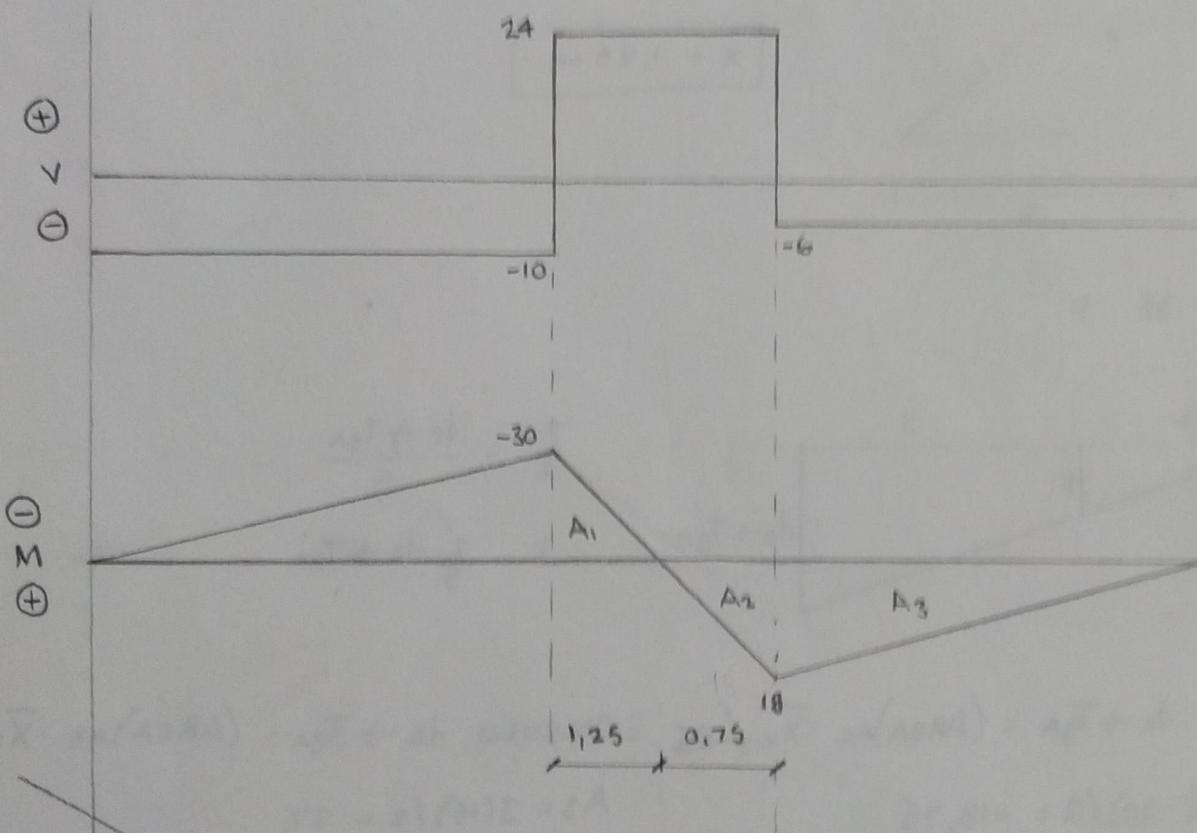
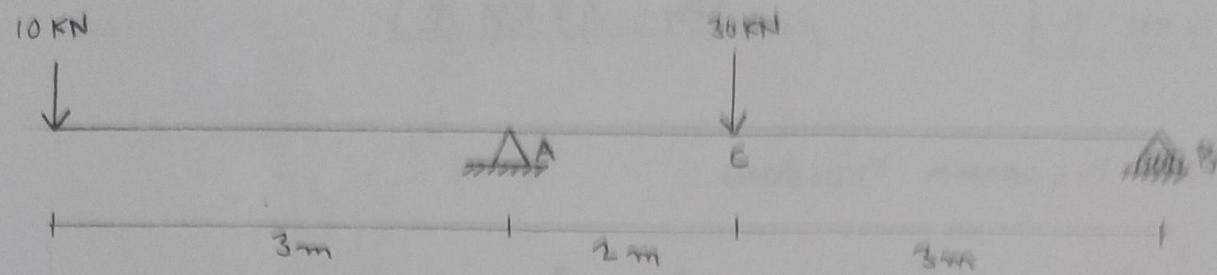
$$= \frac{5Pa^3}{12EI} - \frac{Pa^3}{6EI}$$

$$\delta_c = \frac{\frac{Pa^3}{4EI}}{=}$$

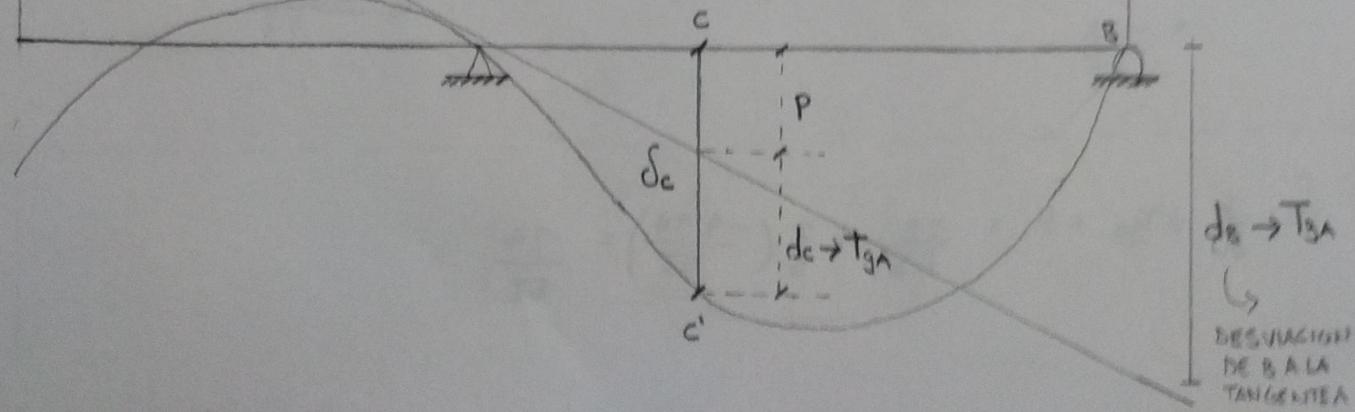
AREA MOMENTO

CALENDAR LA DEFLEXION EN EL PUNTO C

EI = CONSTANTE



CURVA
ELASTICA



EQUILIBRIO

$$\sum M_A = 0$$

$$10(3) - 30(2) + 5B_y = 0$$

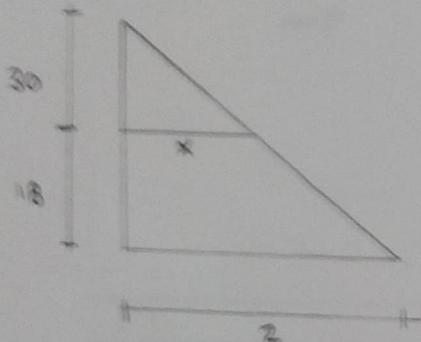
$$B_y = 6 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$-10 - 30 + 6 + A_y = 0$$

$$A_y = 34 \text{ kN} \uparrow$$

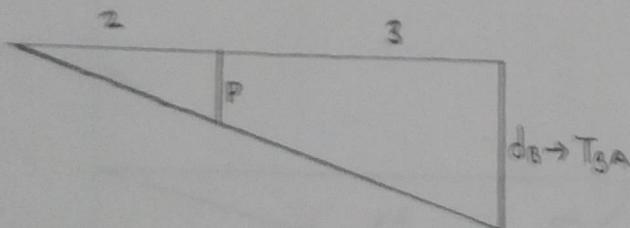
DISTANCIAS. DIAGRAMA MOMENTO



$$\frac{x}{30} = \frac{2}{48}$$

$$x = 1,25 \text{ m}$$

CALCULO DE P



$$\frac{P}{2} = \frac{d_B \rightarrow T_{BA}}{5}$$

$$P = \frac{2}{5} d_B \rightarrow T_{BA}$$

$$\text{CALCULO } d_c \rightarrow T_{BA} = (\text{AREA})_{AC} \cdot \bar{x}_c / EI$$

$$A_1 = 1,25(-30)/2 = -18,75$$

$$A_2 = 0,75(18)/2 = 6,75$$

$$d_c \rightarrow T_{BA} = (-18,75)(1,58) + (6,75)(0,25)$$

$$= \frac{-28}{EI}$$

$$\text{CALCULO } d_B \rightarrow T_{BA} = (\text{AREA})_{AB} \cdot \bar{x}_B / EI$$

$$A_3 = 3(18)/2 = 27$$

$$d_B \rightarrow T_{BA} = (-18,75)(4,58) + (6,75)(3,25)$$

$$+ 27(2)$$

$$= \frac{-9,94}{EI}$$

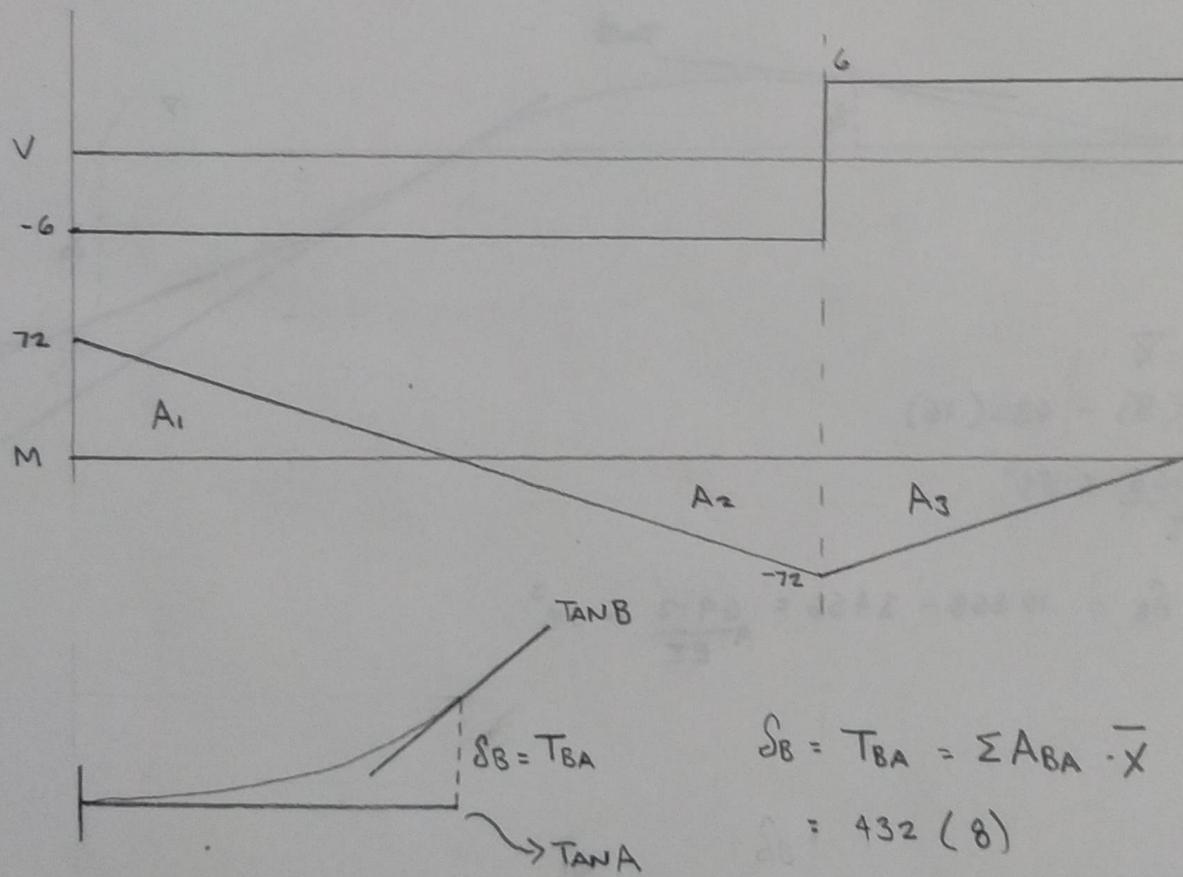
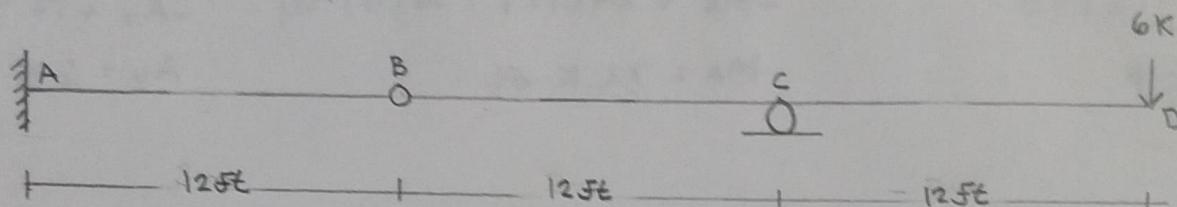
$$d_c = d_c \rightarrow T_{BA} + P = \frac{28}{EI} + \frac{1}{5} \left(\frac{-9,94}{EI} \right) = \frac{24}{EI}$$

AREA MOMENTO

X

DETERMINE LA PENDIENTE Y EL DESPLAZAMIENTO EN D.

SUPONGA QUE A ES UN SOPORTE FIJO, B ES ARTICULACIÓN Y D UN RODILLO. EI ES CONSTANTE.



$$\delta_B = T_{BA} = \sum A_{BA} \cdot \bar{x}$$

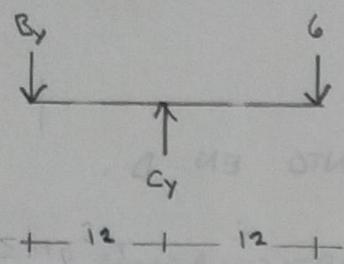
$$= 432 (8)$$

$$\delta_B = \frac{3456}{EI} K \cdot ft^3 \uparrow$$

$$A_1 = \frac{1}{2} (12)(72) = \frac{432}{EI} K \cdot ft^2$$

$$A_3 = A_2 = -\frac{432}{EI} K \cdot ft^2$$

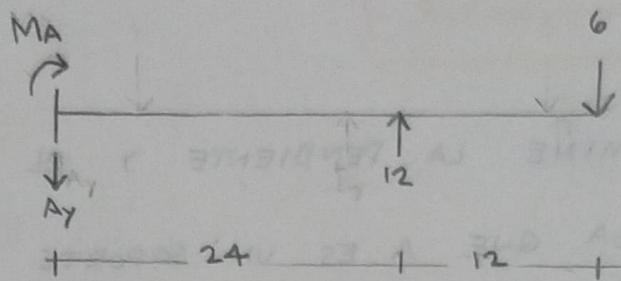
REACCIONES



$$\sum M_B = 0$$

$$12C_y - 24(6) = 0$$

$$C_y = 12 \text{ K} \uparrow$$



$$\sum M_A = 0$$

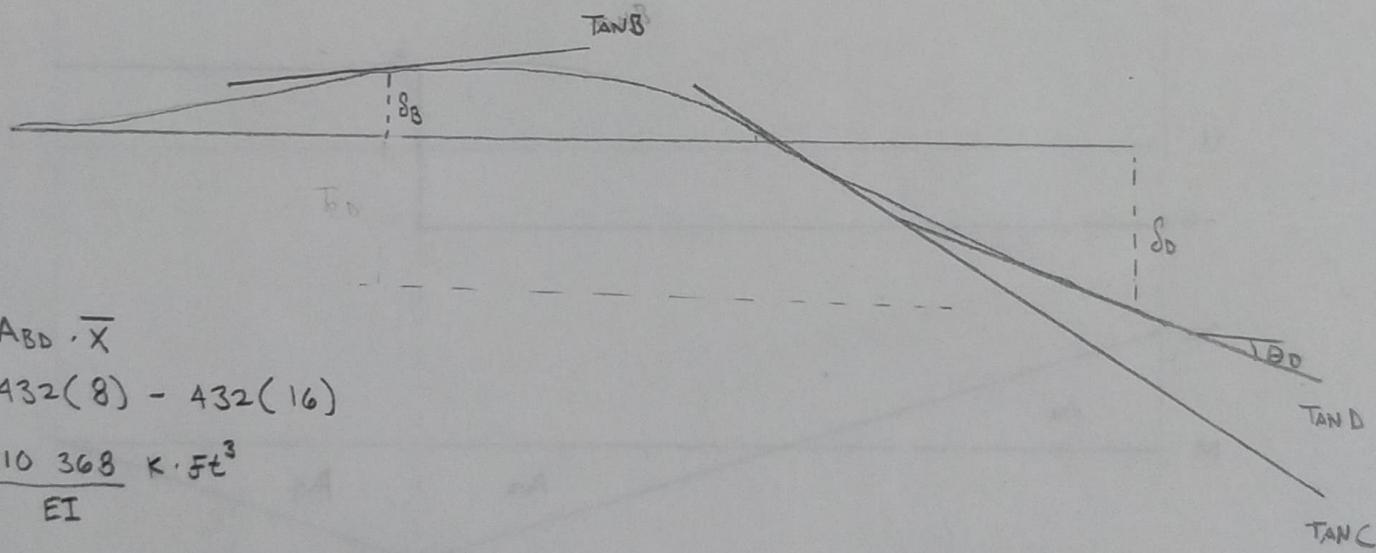
$$-M_A + 24(12) - 36(6) = 0$$

$$M_A = 72 \text{ K} \cdot \text{ft}$$

$$\sum F_y = 0$$

$$-A_y + 12 - 6 = 0$$

$$A_y = 6 \text{ K} \downarrow$$



$$T_{BD} = \sum A_{BD} \cdot \bar{x}$$

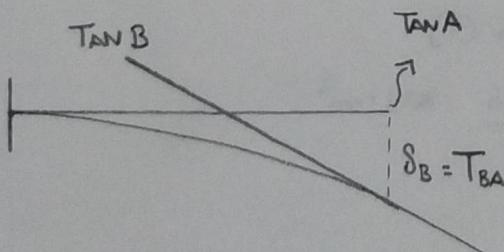
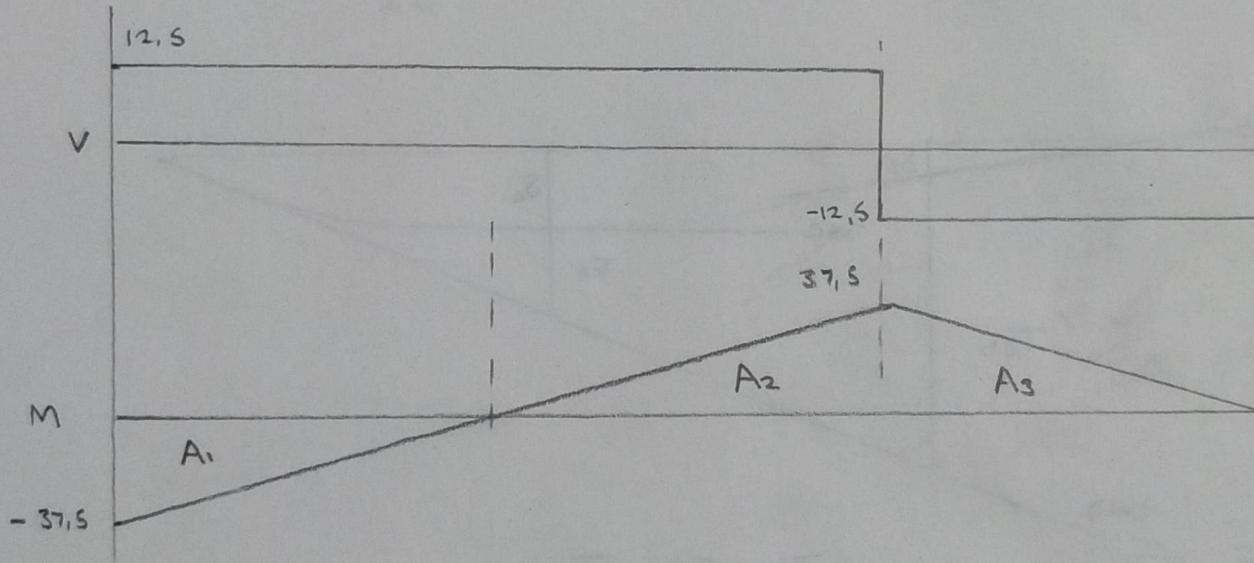
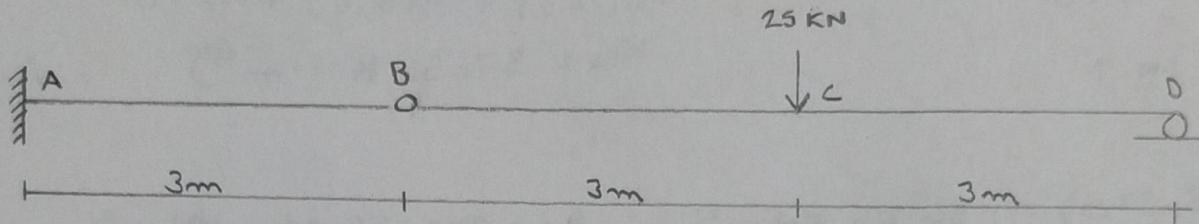
$$= -432(8) - 432(16)$$

$$T_{BD} = \frac{-10368}{EI} \text{ K} \cdot \text{ft}^3$$

$$S_D = T_{BD} - S_B = -10368 - 3456 = \frac{6912}{EI} \text{ K} \cdot \text{ft}^3$$

AREA MOMENTO

DETERMINE EL DESPLAZAMIENTO EN C. A ES UN SOPORTE FIJO Y B UNA ARTICULACIÓN Y D UN RODILLO. EI ES CONSTANTE.



$$T_{BA} = \sum A_{BA} \cdot \bar{x}$$

$$= 56,25(2) - 56,25(1)$$

$$T_{BA} = \frac{112,5}{EI} \text{ KN} \cdot \text{m}^3$$