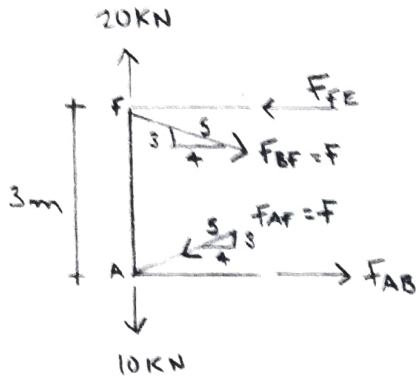
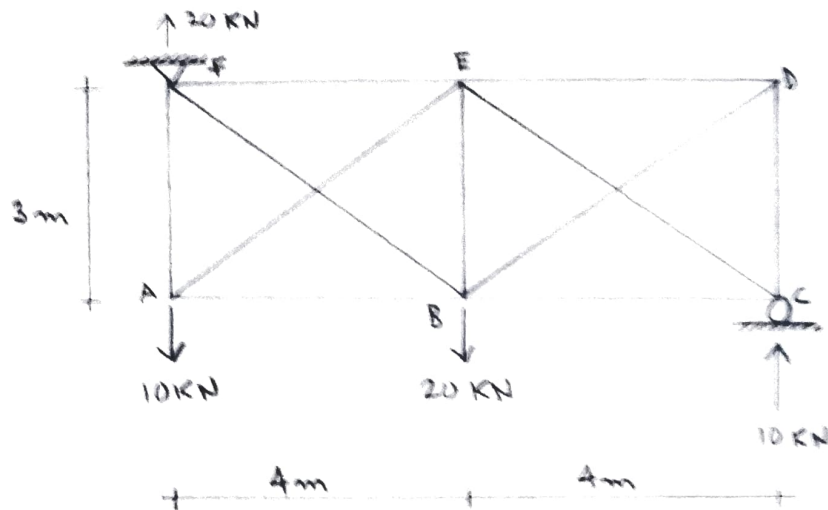


ANÁLISIS APROXIMADO DE ARMADURAS



$$\sum M_A = 0$$

$$3F_{FE} - 3\left(\frac{4}{5}\right)F = 0$$

$$F_{FE} = 0,8F$$

$$F_{FE} = 6,66 \text{ kN } C$$

$$\sum F_y = 0$$

$$20 - 10 - 2\left(\frac{3}{5}\right)F = 0$$

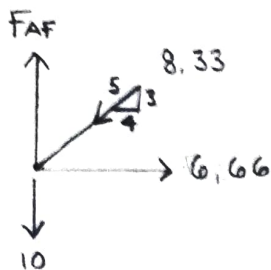
$$10 - 1,2F = 0$$

$$F = 8,33 \text{ kN}$$

$$\sum M_F = 0$$

$$-\frac{4}{5}(8,33) + 3F_{AB} = 0$$

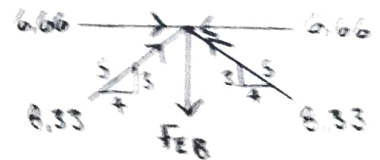
$$F_{AB} = 2,22 \text{ kN } T$$



$$\sum F_y = 0$$

$$F_{AF} - \frac{3}{5}(8,33) - 10 = 0$$

$$F_{AF} = 15 \text{ kN } T$$



$$\sum F_y = 0$$

$$2\left(\frac{3}{5}\right)8,33 - F_{EB} = 0$$

$$F_{EB} = 10 \text{ kN } T$$

$$\sum M_C = 0$$

$$-3F_{ED} + \frac{4}{5}F_1 = 0$$

$$-3F_{ED} + 6,66 = 0$$

$$F_{ED} = 2,22 \text{ kN } C$$

$$\sum F_y = 0$$

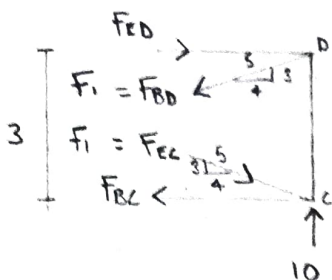
$$-2\left(\frac{3}{5}\right)F_1 + 10 = 0$$

$$F_1 = 8,33 \text{ kN}$$

$$\sum M_D = 0$$

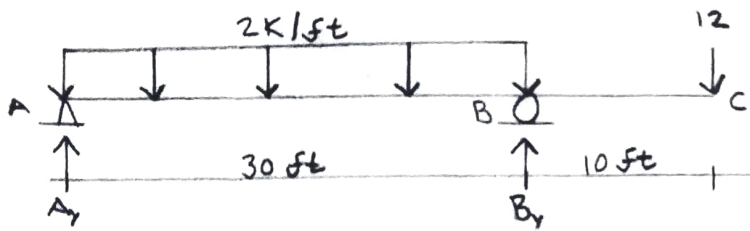
$$-3F_{BC} + 3\left(\frac{4}{5}\right)(8,33) = 0$$

$$F_{BC} = 6,66 \text{ kN } T$$



METODO CASTIGLIANO

DETERMINE LA DEFLEXIÓN EN EL PUNTO C DE LA VIGA POR EL SEGUNDO TEOREMA DE CASTIGLIANO. $E = 29\,000\text{ KSI}$, $I = 2000\text{ m}^4$



$$\Sigma M_A = 0$$

$$-2(30)(15) + 30B_y - 40P = 0$$

$$-900 + 30B_y - 40P = 0$$

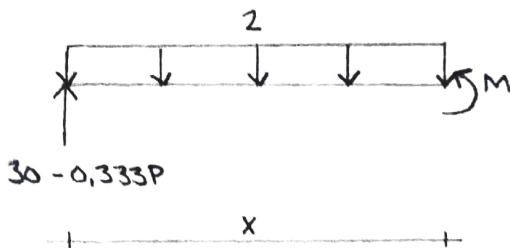
$$B_y = 30 + 1,3333P$$

$$\Sigma F_y = 0$$

$$A_y - 2(30) + 30 + 1,3333P - P = 0$$

$$A_y = 30 - 0,3333P$$

TRAMO AB $\rightarrow 0 \leq x \leq 30$



$$\Sigma M = 0$$

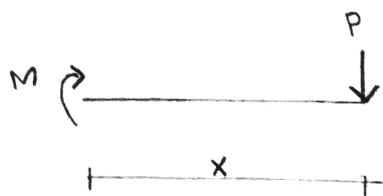
$$2x\left(\frac{x}{2}\right) - x(30 - 0,3333P) + M = 0$$

$$x^2 - 30x + 0,3333Px + M = 0$$

$$M = -0,3333Px + 30x - x^2$$

$$\frac{\partial M}{\partial P} = -0,3333x$$

TRAMO CB $\leftarrow 0 \leq x \leq 10$



$$\Sigma M = 0$$

$$-M - Px = 0$$

$$M = -Px$$

$$\frac{\partial M}{\partial P} = -x$$

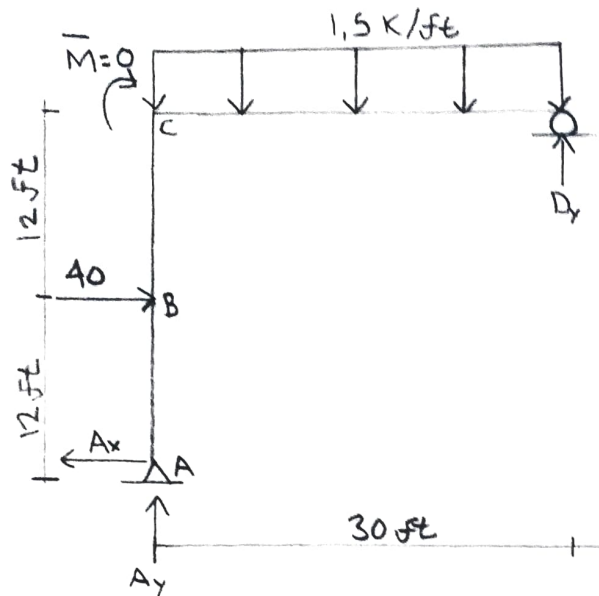
$$\Delta_c = \int_0^{30} (-0,3333x)(-0,3333x(12)) + 30x - x^2 + \int_0^{10} (-x)(-12x)$$

$$\Delta_c = \frac{-6501,5(12)^3}{29000(2000)} = -0,194\text{ m} \downarrow$$

METODO CASTIGLIANO

DETERMINAR ROTACIÓN EN C.

$$E = 29\,000, I = 2\,500 \text{ m}^4$$



$$\Sigma M_A = 0$$

$$-40(12) - 1.5(30)(15) - \bar{M} + 30D_y = 0$$

$$-480 - 675 - \bar{M} + 30D_y = 0$$

$$D_y = 38.5 + \frac{\bar{M}}{30} \uparrow$$

$$\Sigma F_y = 0$$

$$A_y - 1.5(30) + 38.5 + \frac{\bar{M}}{30} = 0$$

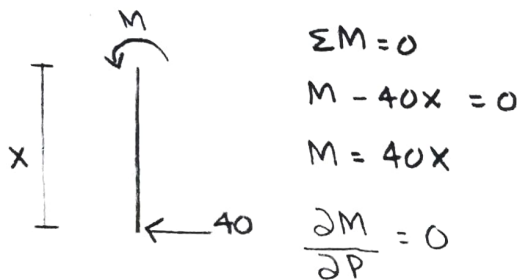
$$A_y = 6.5 - \frac{\bar{M}}{30} \uparrow$$

$$\Sigma F_x = 0$$

$$-Ax + 40 = 0$$

$$Ax = 40$$

TRAMO AB $\rightarrow 0 \leq x \leq 12$



$$\Sigma M = 0$$

$$M - 40x = 0$$

$$M = 40x$$

$$\frac{\partial M}{\partial P} = 0$$

TRAMO CB $\rightarrow 0 \leq x \leq 12$



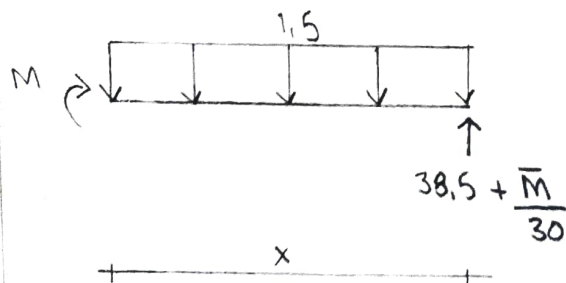
$$\Sigma M = 0$$

$$-M + 480 = 0$$

$$M = 480$$

$$\frac{\partial M}{\partial P} = 0$$

TRAMO DC $\rightarrow 0 \leq x \leq 30$



$$\Sigma M = 0$$

$$-M - 1.5x\left(\frac{x}{2}\right) + x\left(38.5 + \frac{\bar{M}}{30}\right) = 0$$

$$\frac{\partial M}{\partial P} = \frac{x}{30}$$

$$-M - 0.75x^2 + 38.5x + \frac{\bar{M}}{30}x = 0$$

$$M = -0.75x^2 + 38.5x + \frac{\bar{M}}{30}x$$

$$\theta_c = \int_0^{30} \left(\frac{x}{30}\right)(-0.75x^2 + 38.5x) dx = \frac{6487.5}{EI} \text{ K} \cdot \text{ft}^2$$

$$\theta_c = \frac{6487.5(12)^2}{29000(2500)} = 0.01289 \text{ RAD}$$

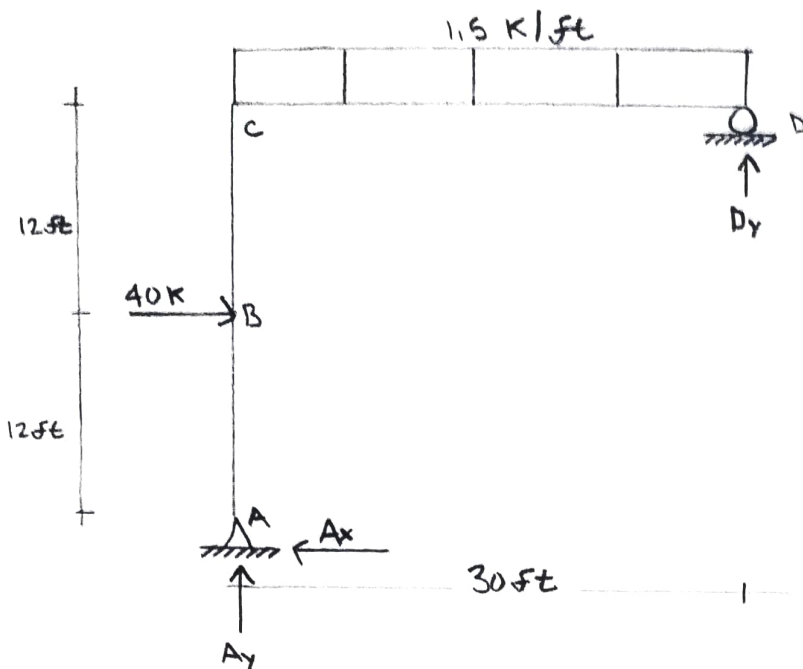
METODO DE TRABAJO VIRTUAL

DETERMINE LA ROTACIÓN EN EL NUDO C DEL MARCO QUE SE MUESTRA EN LA FIGURA.

$EI = \text{CONSTANTE}$

$E = 29\,000 \text{ KSI}$

$I = 2\,500 \text{ in}^4$



$$\sum M_A = 0$$

$$-40(12) - 1.5(30)(15) + 30D_y = 0$$

$$D_y = 38.5 \text{ K} \cdot \text{ft} \uparrow$$

$$\sum F_y = 0$$

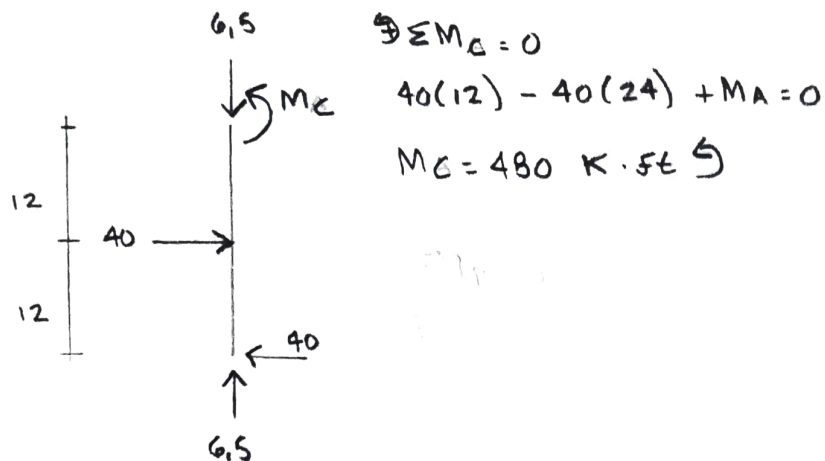
$$A_y - 1.5(30) + 38.5 = 0$$

$$A_y = 6.5 \text{ K} \uparrow$$

$$\sum F_x = 0$$

$$40 - A_x = 0$$

$$A_x = 40 \text{ K} \leftarrow$$



$$\sum M_C = 0$$

$$40(12) - 40(24) + M_A = 0$$

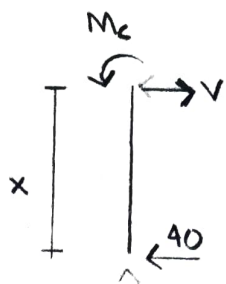
$$M_C = 480 \text{ K} \cdot \text{ft} \curvearrowright$$



$$45(15) + 6.5(30) - 480 + M_D = 0$$

$$M_D = 0$$

$$AB \rightarrow 0 \leq x \leq 12$$



$$\sum M_C = 0$$

$$M_C - 40(x) = 0$$

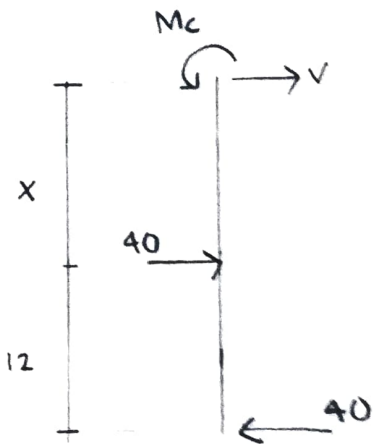
$$M_C = 40x$$

$$\sum F_x = 0$$

$$V - 40 = 0$$

$$V = 40$$

$$BC \rightarrow 12 \leq x \leq 24$$



$$\sum M_c = 0$$

$$M_c + 40x - 40(12+x) = 0$$

$$M_c + 40x - 480 + 40x = 0$$

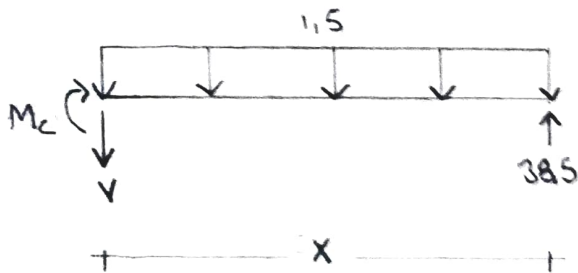
$$M_c = 480$$

$$\sum F_x = 0$$

$$V - 40 + 40 = 0$$

$$V = 0$$

$$DC \rightarrow 0 \leq x \leq 30$$



$$\sum M_c = 0$$

$$-M_c - 1.5(x)\left(\frac{x}{2}\right) + 38.5(x) = 0$$

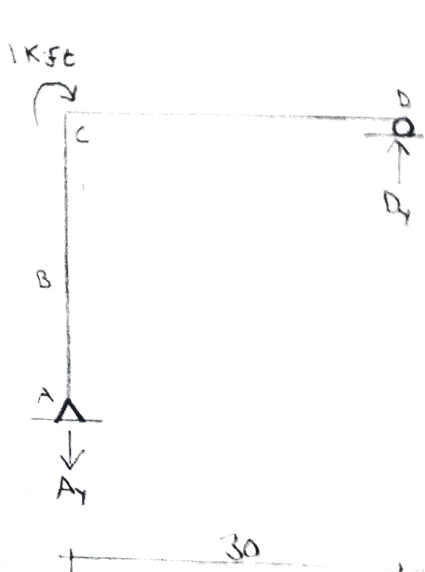
$$M_c = 38.5x - 0.75x^2$$

$$\sum F_y = 0$$

$$-V - 1.5(x) + 38.5 = 0$$

$$V = 38.5 - 1.5x$$

MARCO VIRTUAL



$$\sum M_A = 0$$

$$-1 + 30D_y = 0$$

$$D_y = \frac{1}{30} \uparrow$$

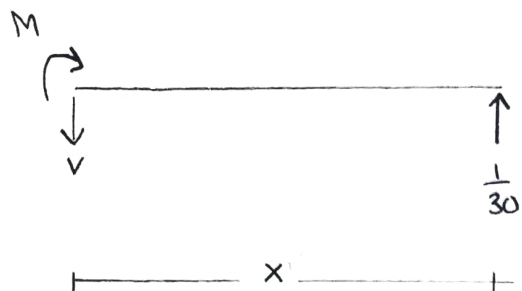
$$\sum F_x = 0$$

$$-A_y + \frac{1}{30} = 0$$

$$A_y = \frac{1}{30} \downarrow$$

$$CD \rightarrow 0 \leq x \leq 30$$

2



$$\sum M = 0$$

$$-M + \frac{1}{30}x = 0$$

$$M = \frac{1}{30}x$$

$$\sum F_y = 0$$

$$-V + \frac{1}{30} = 0$$

$$V = \frac{1}{30}$$

SEGMENTO	ORIGEN	LIMITES	M (K.ft)	M _v (K.ft)
AB	A	0-12	40x	0
BC	B	12-24	480	0
DC	D	0-30	38,5x - 0,75x ²	$\frac{1}{30}x$

$$I(\theta_c) = \int \frac{M_v \cdot M}{EI} dx$$

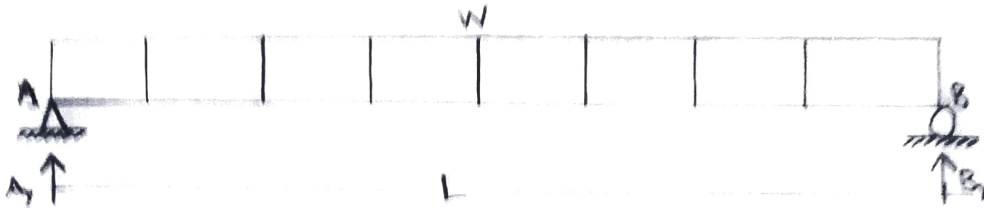
$$= \int_0^{30} \frac{x}{30} (38,5x - 0,75x^2)$$

$$\theta_c = \frac{6487,5 (12)^2}{29000 (2500)} = 0,01289 \text{ RAD}$$



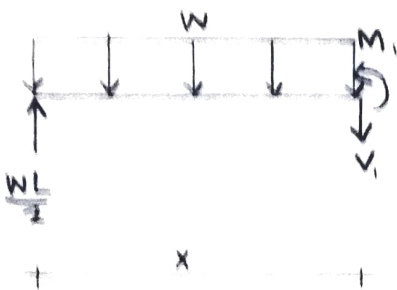
METODO DE DUPLA INTEGRACIÓN

DETERMINE LA ECUACIÓN DE LA PENDIENTE Y LA DEFLEXIÓN DE LA VIGA MOSTRADA. TAMBIEN CALCULE LA PENDIENTE EN CADA EXTREMO Y LA DEFLEXIÓN EN LA MITAD.



$$A_y = \frac{WL}{2} \uparrow$$

$$B_y = \frac{WL}{2} \uparrow$$



$$\sum M_A = 0$$

$$-\frac{WL}{2}(x) + W(x)\left(\frac{x}{2}\right) + M_1 = 0$$

$$-\frac{WLx}{2} + \frac{Wx^2}{2} + M_1 = 0$$

$$M_1 = \frac{WLx}{2} - \frac{Wx^2}{2} \Rightarrow M_1 = \frac{W}{2}(Lx - x^2)$$

$$\frac{M}{EI} = \frac{W}{2EI}(Lx - x^2)$$

$$\theta = \int \frac{W}{2EI}(Lx - x^2)$$

$$= \frac{W}{2EI} \int Lx - x^2$$

$$\theta = \frac{W}{2EI} \left(\frac{Lx^2}{2} - \frac{x^3}{3} \right) + C_1$$

$$\delta = \int \frac{W}{2EI} \left(\frac{Lx^2}{2} - \frac{x^3}{3} \right) + C_1$$

$$= \frac{W}{2EI} \int \frac{Lx^2}{2} - \frac{x^3}{3} + C_1$$

$$\delta = \frac{W}{2EI} \left(\frac{Lx^3}{6} - \frac{x^4}{12} \right) + C_1x + C_2$$

$$\Delta = \frac{W}{2EI} \left(\frac{Lx^3}{6} - \frac{x^4}{12} \right) + C_1 x + C_2$$

$$0 = \frac{W}{2EI} \left(\frac{L(0)^3}{6} - \frac{0^4}{12} \right) + C_1(0) + C_2$$

→ SE EVALUA EN $x=0$

CUANDO $x=0$ LA DEFORMADA ES IGUAL A 0 PORQUE EN EL APOYO NO HAY DEFORMADA

$$C_2 = 0$$

$$\Delta = \frac{W}{2EI} \left(\frac{Lx^3}{6} - \frac{x^4}{12} \right) + C_1 x + C_2$$

$$0 = \frac{W}{2EI} \left(\frac{L(L)^3}{6} - \frac{L^4}{12} \right) + C_1(L) + 0$$

→ SE EVALUA EN $x=L$

$$0 = \frac{W}{2EI} \left(\frac{12L^4}{12} - \frac{L^4}{12} \right) + C_1 L$$

$$C_1 L = -\frac{W}{2EI} \left(\frac{L^4}{12} \right)$$

$$C_1 = \frac{-WL^4}{24EI} \left(\frac{1}{L} \right) \Rightarrow C_1 = \frac{-WL^3}{24EI}$$

$$\Theta = \frac{W(6x^2L - 4x^3 - L^3)}{24EI}$$

$$x=0 \Rightarrow \Theta_A = \frac{-WL^3}{24EI} \quad \nabla$$

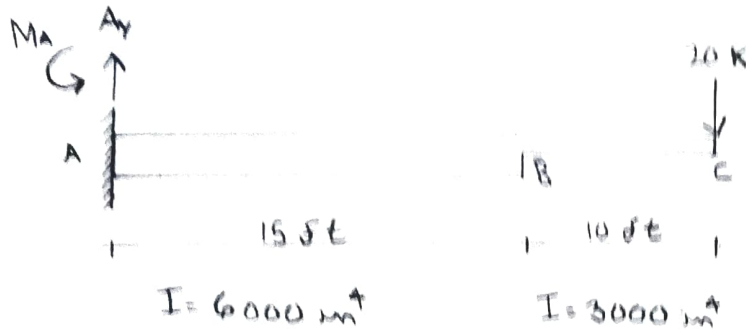
$$x=L \Rightarrow \Theta_B = \frac{WL^3}{24EI} \quad \searrow$$

$$\delta_c = \frac{WL}{12EI} \left(Lx^2 - \frac{x^3}{2} - \frac{L^3}{2} \right)$$

$$x = \frac{L}{2} \Rightarrow \Delta_c = \frac{-5WL^4}{384EI} \quad \downarrow$$

METODO DE AREA MOMENTO

DETERMINE LA PENDIENTE Y LA DEFLEXIÓN EN LOS PUNTOS B, C DE LA VIGA EN CANTILIVER QUE SE MUESTRA POR EL METODO ÁREA MOMENTO.



$$E = 29,000 \text{ KSI}$$

$$\sum F_y = 0$$

$$A_y - 20 = 0$$

$$A_y = 20 \text{ K} \uparrow$$

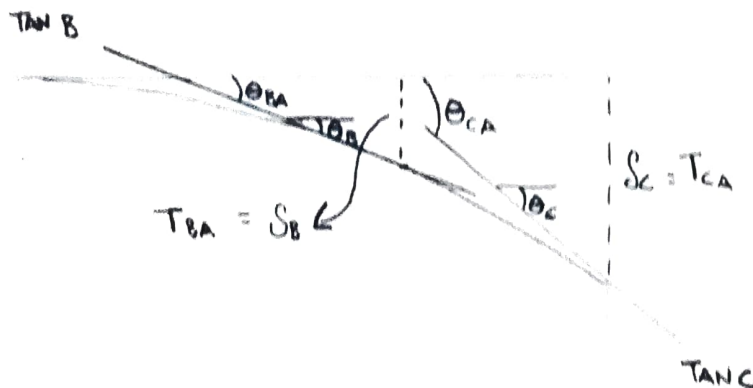
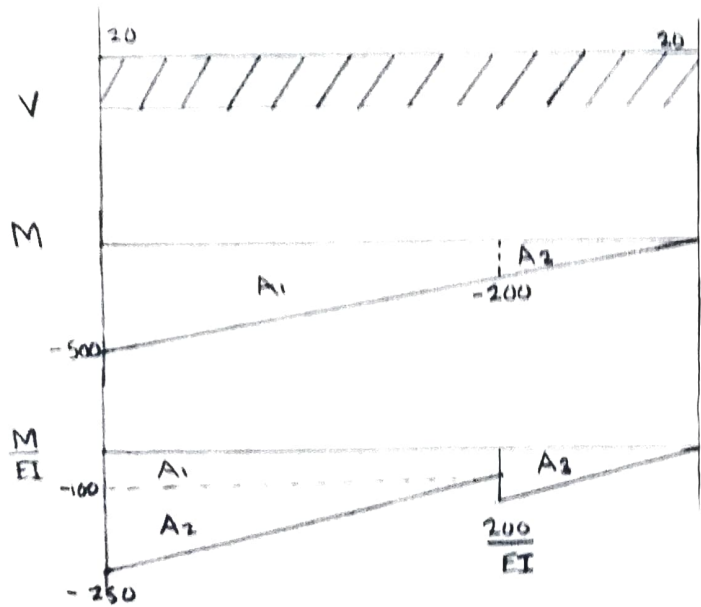
$$\sum M_A = 0$$

$$M_a - 25(20) = 0$$

$$M_a = 500 \text{ K} \cdot \text{ft} \curvearrowright$$

$$I = I_{BC} = 3000$$

$$I_{AB} = 6000 = 2(3000) = 2I$$



$$A_1 = -100(15) = -1500$$

$$A_2 = -\frac{1}{2}(150)(15) = -1125$$

$$A_3 = -\frac{1}{2}(200)(10) = -1000$$

$$\theta_c = \theta_{cA} = \sum A_{cA} = -1500 - 1125 - 1000 = \frac{-3625(12)^2}{(29000)(3000)}$$

$$\theta_c = 0,006 \text{ RAD} \quad \swarrow$$

$$\delta_c = T_{cA} = \sum A_{cA} \cdot \bar{X} = -1000(6,667) - 1125(20) + 1500(17,5)$$

$$\delta_c = \frac{55417(12)^3}{(29000)(3000)} = 1,1 \text{ m} \downarrow$$

$$\theta_B = \theta_{BA} = \sum A_{BA} = -1500 - 1125 = \frac{-2625(12)^2}{29000(3000)} = 0,0043 \text{ RAD} \quad \swarrow$$

$$\delta_B = T_{BA} = \sum A_{BA} \cdot \bar{X} = -1125(10) - 1500(7,5) = \frac{22500(12)^3}{29000(3000)}$$

$$\delta_B = 0,4469 \text{ m} \downarrow$$