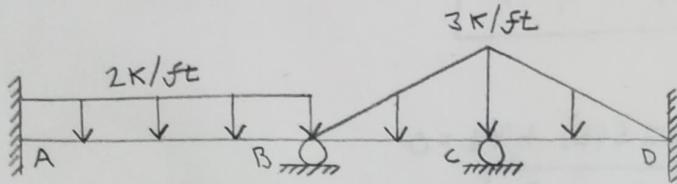


**TAREA #1**

① POR EL METODO DE PENDIENTE - DEFLEXION  
ENCUENTRE LAS ROTACIONES EN B Y C Y  
TAMBIEN LAS REACCIONES DE LA VIGA.



ANGULOS

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{15}(2\theta_B) + 37,5 + \frac{2EI}{10}(2\theta_B + \theta_C) - 10 = 0$$

FEM

$$AB = -\frac{WL^2}{12} = -37,5$$

$$BA = 37,5$$

$$BC = -\frac{WL^2}{30} = -10$$

$$CB = \frac{WL^2}{20} = 15$$

$$CD = -\frac{WL^2}{20} = -15$$

$$DC = \frac{WL^2}{30} = 10$$

MOMENTO

$$M_{AB} = \frac{2EI}{15}(\theta_B) - 37,5 = -43,45 \text{ K.Ft}$$

$$M_{BA} = \frac{2EI}{15}(2\theta_B) + 37,5 = 25,61 \text{ K.Ft}$$

$$M_{BC} = \frac{2EI}{10}(2\theta_B + \theta_C) - 10 = -25,61 \text{ K.Ft}$$

$$M_{CB} = \frac{2EI}{10}(2\theta_C + \theta_B) + 10 = 10,54 \text{ K.Ft}$$

$$M_{CD} = \frac{2EI}{10}(2\theta_C) - 15 = -10,54 \text{ K.Ft}$$

$$M_{DC} = \frac{2EI}{10}(\theta_C) + 10 = 12,23 \text{ K.Ft}$$

ANGULOS

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{15}(2\theta_B) + 37,5 + \frac{2EI}{10}(2\theta_B + \theta_C) - 10 = 0$$

$$\frac{4EI}{15}\theta_B + 37,5 + \frac{2EI}{5}\theta_B + \frac{2EI}{5}\theta_C - 10 = 0$$

$$\frac{2EI}{3}\theta_B + \frac{EI}{5}\theta_C + 27,5 = 0$$

$$\theta_B = -\frac{3}{10}\theta_C - \frac{41,25}{EI}$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2EI}{10}(2\theta_C + \theta_B) + 10 + \frac{2EI}{10}(2\theta_C) - 10 = 0$$

$$\frac{2EI}{5}\theta_C + \frac{EI}{5}\theta_B + \frac{2EI}{5}\theta_C = 0$$

$$\frac{EI}{5}\theta_B + \frac{4EI}{5}\theta_C = 0$$

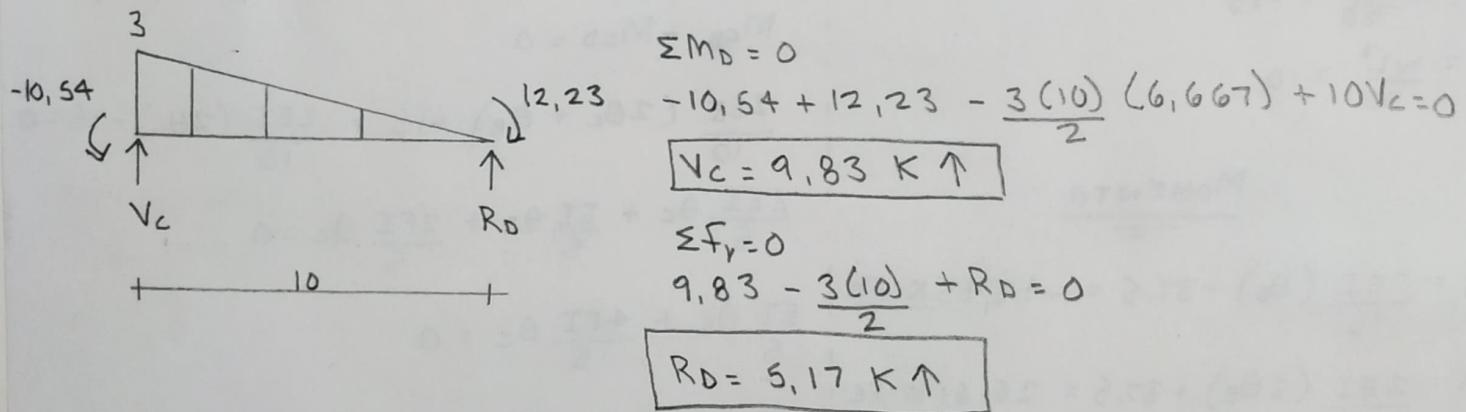
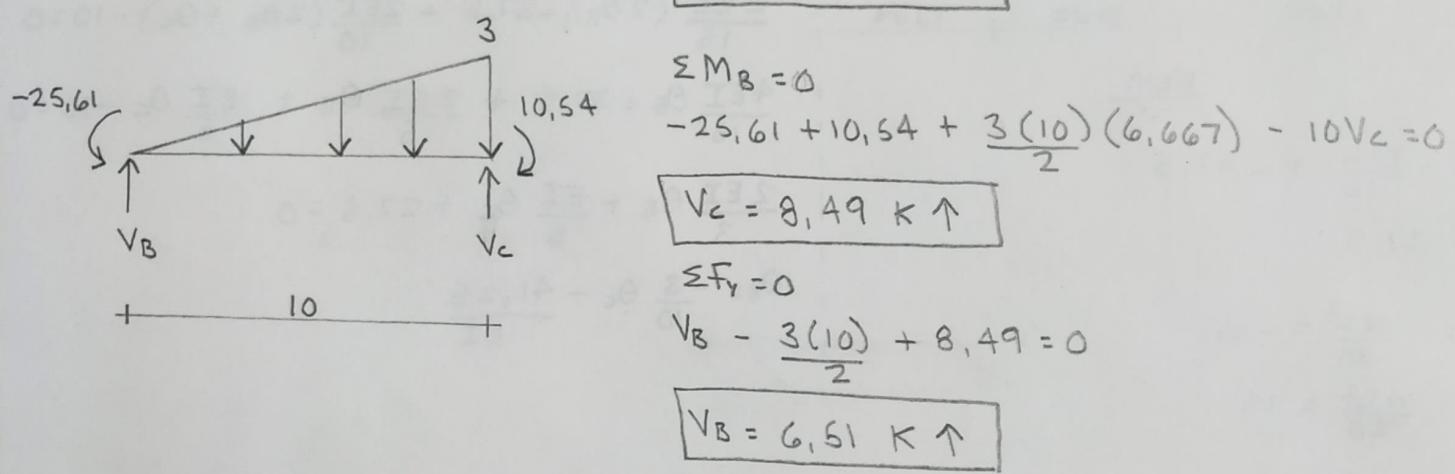
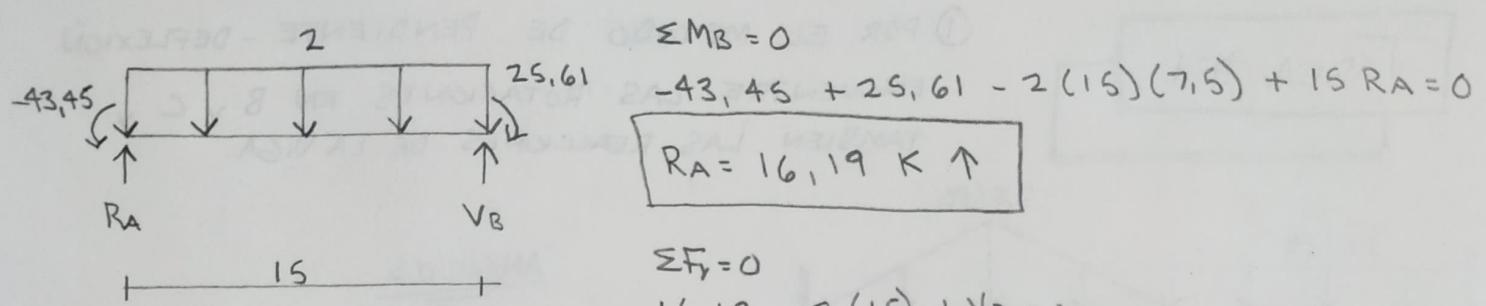
$$\theta_B = -4\theta_C \Rightarrow \theta_B = -\frac{1660}{37EI}$$

$$\theta_B = \theta_B$$

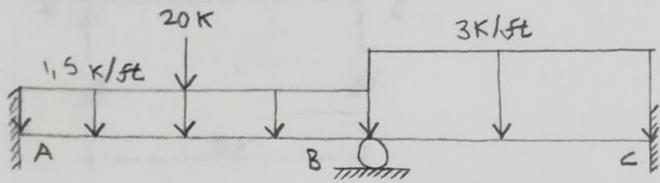
$$-\frac{3}{10}\theta_C - \frac{41,25}{EI} = -4\theta_C$$

$$3,7\theta_C = \frac{41,25}{EI}$$

$$\theta_C = \frac{825}{74EI}$$



(2) POR EL METODO DE PENDIENTE DE DEFLEXION, DIBUJE LOS DIAGRAMAS DE CORTANTE Y MOMENTO.  $E = 29000 \text{ KSI}$ ,  $I = 1650 \text{ in}^4$

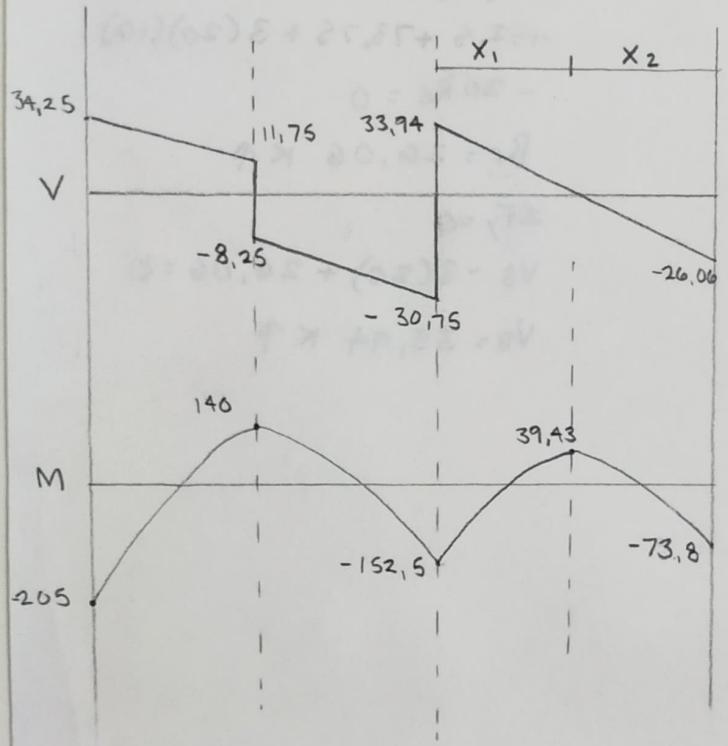


+ 15 ft | 15 ft | 20 ft +

### MOMENTOS

$$M_{AB} = \frac{2EI}{30}(\theta_B) - 187,5 = -205 \text{ k-ft}$$

$$M_{BA} = \frac{2EI}{30}(2\theta_B) + 187,5 = 152,5 \text{ k-ft}$$



$$M_{BC} = \frac{2EI}{20}(2\theta_B) - 100 = -152,5 \text{ k-ft}$$

$$M_{CB} = \frac{2EI}{20}(\theta_B) + 100 = 73,75 \text{ k-ft}$$

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{30}(2\theta_B) + 187,5 + \frac{2EI}{20}(2\theta_B) - 100 = 0$$

$$0,1333 EI \theta_B + 0,2EI \theta_B + 87,5 = 0$$

$$\theta_B = \frac{-262,53}{EI}$$

### FEM

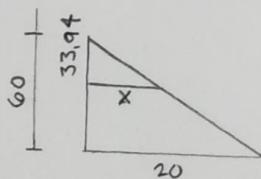
$$AB = -\frac{WL^2}{12} - \frac{PL}{8} = -\frac{1.5(30)^2}{12} - \frac{20(30)}{8}$$

$$AB = -187,5$$

$$BA = 187,5$$

$$BC = -\frac{WL^2}{12} = -\frac{3(20)^2}{12} = -100$$

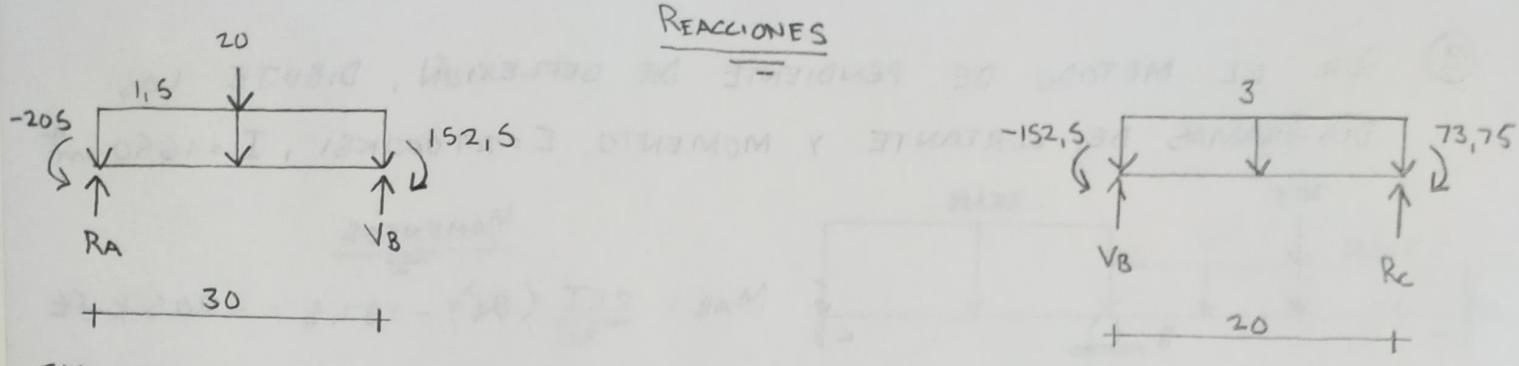
$$CB = 100$$



$$\frac{x}{33,94} = \frac{20}{60}$$

$$x_1 = 11,31$$

$$x_2 = 8,69$$



$$\sum M_B = 0$$

$$-20 \cdot 15 + 152,5 - 1,5(30)(15) - 20(15) + 30 R_A = 0$$

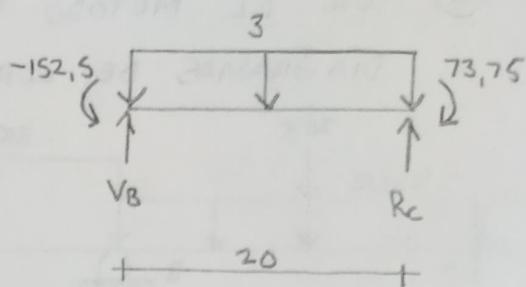
$$R_A = 34,25 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$34,25 - 20 - 1,5(30) + V_B = 0$$

$$V_B = 30,75 \text{ kN} \uparrow$$

### REACCIONES



$$\sum M_B = 0$$

$$-152,5 + 73,75 + 3(20)(10)$$

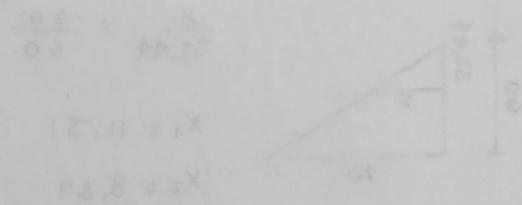
$$-20 R_C = 0$$

$$R_C = 26,06 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$V_B - 3(20) + 26,06 = 0$$

$$V_B = 33,94 \text{ kN} \uparrow$$

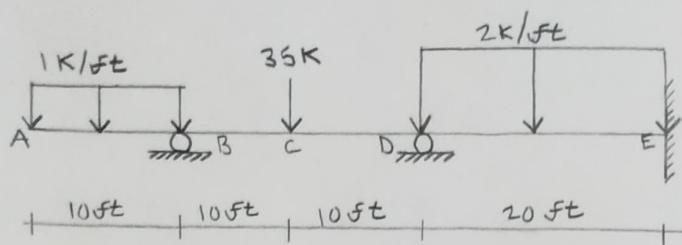


$$\frac{(12,00) \cdot 2}{2} - \frac{(12,00) \cdot 2}{2} = \frac{12,00}{2} - \frac{12,00}{2}$$

$$12,00 - \frac{12,00}{2} \cdot 2 = \frac{12,00}{2} - 12,00$$

$$0 = 0$$

③ POR EL METODO DE PENDIENTE DE DEFLEXION, DIBUJE LOS DIAGRAMAS DE CORTANTE Y MOMENTO. EI = CONSTANTE.



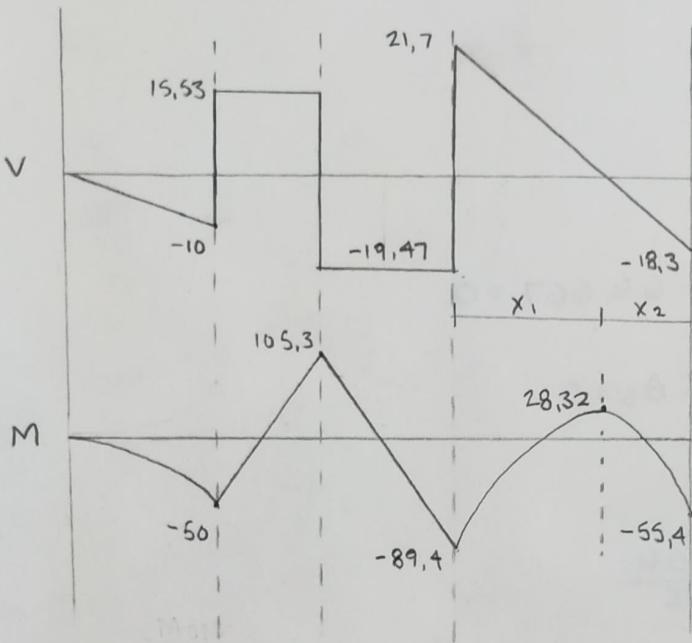
FEM

$$BD = -\frac{PL}{8} = -87,5$$

$$DB = 87,5$$

$$DE = -\frac{WL^2}{12} = -66,667$$

$$ED = 66,667$$



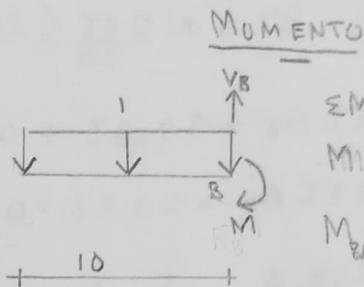
MOMENTO

$$M_{BD} = \frac{2EI}{20} (2\theta_B + \theta_D) - 87,5 = -503 K \cdot ft$$

$$M_{DB} = \frac{2EI}{20} (2\theta_D + \theta_B) + 87,5 = 89,29 K \cdot ft$$

$$M_{DE} = \frac{2EI}{20} (2\theta_D) + 66,667 = -89,28 K \cdot ft$$

$$M_{ED} = \frac{2EI}{20} (\theta_D) + 66,667 = 55,36 K \cdot ft$$



$$\sum M_B = 0$$

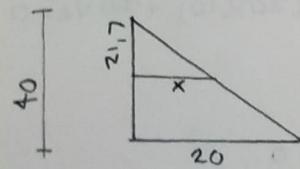
$$M_1 - 1(10)(5) = 0$$

$$M_1 = 50 K \cdot ft$$

$$\sum F_y = 0$$

$$-1(10) + V_B = 0$$

$$V_B = 10 \uparrow$$



$$\frac{X}{21,7} = \frac{20}{40}$$

$$X_1 = 10,85$$

$$X_2 = 9,15$$

### ANGULOS

$$M_{BA} + M_{BD} = 0$$

$$50 + \frac{2EI}{20} (2\theta_B + \theta_D) - 87,5 = 0$$

$$-37,5 + 0,2 EI \theta_B + 0,1 EI \theta_D = 0$$

$$\theta_B = -0,5 \theta_D + \frac{187,5}{EI}$$

$$M_{DB} + M_{DE} = 0$$

$$\frac{2EI}{20} (2\theta_D + \theta_B) + 87,5 + \frac{2EI}{20} (2\theta_D) - 66,667 = 0$$

$$0,2 EI \theta_D + 0,1 EI \theta_B + 20,83 + 0,2 EI \theta_D = 0$$

$$0,1 EI \theta_B + 0,4 EI \theta_D + 20,83 = 0$$

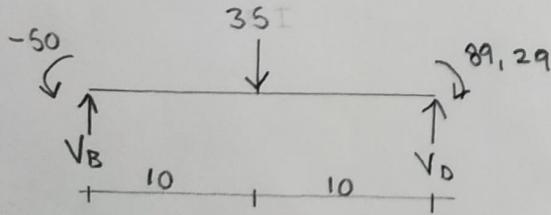
$$\theta_B = -4 \theta_D - \frac{208,3}{EI} \Rightarrow \theta_B = \frac{244,06}{EI}$$

$$\theta_B = \theta_B$$

$$-0,5 \theta_D + \frac{187,5}{EI} + 4 \theta_D + \frac{208,3}{EI} = 0$$

$$\theta_D = -\frac{113,09}{EI}$$

### REACCIONES



$$\sum M_B = 0$$

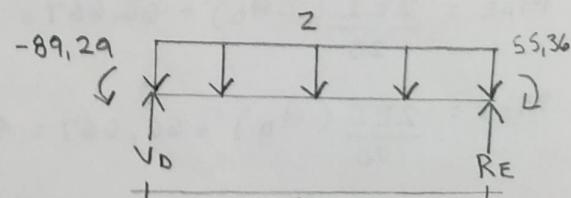
$$-50 + 89,29 - 35(10) + 20V_B = 0$$

$$V_B = 15,53 \uparrow$$

$$\sum F_y = 0$$

$$15,53 - 35 + V_D = 0$$

$$V_D = 19,47 \uparrow$$



$$\sum M_E = 0$$

$$-89,29 + 55,36 - 2(20)(10) + 20V_D = 0$$

$$V_D = 21,7 \uparrow$$

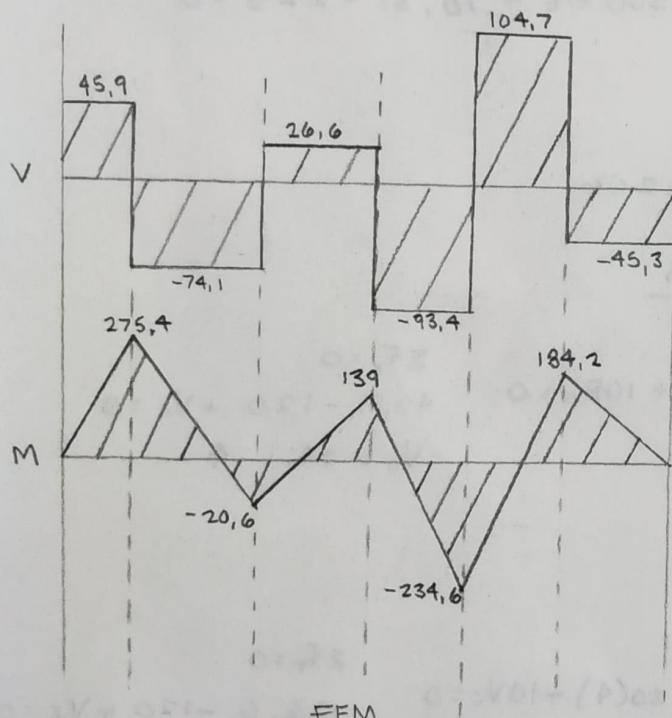
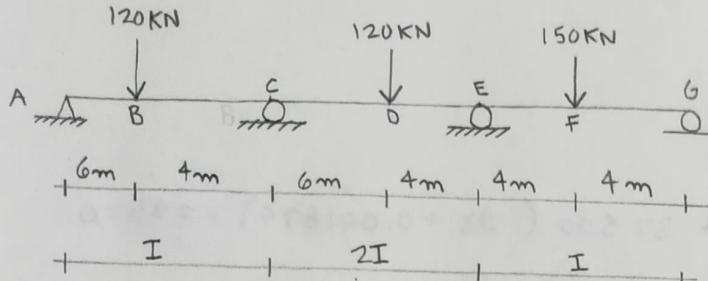
$$\sum F_y = 0$$

$$21,7 - 2(20) + R_E = 0$$

$$R_E = 18,3 \uparrow$$

④ POR EL METODO DE PENDIENTE DEFLEXION DETERMINAR LOS DIAGRAMAS DE CORTANTE Y MOMENTO. EL ASENTAMIENTO EN A = 10 mm , E = 40 mm , C = 65 mm y G = 25 mm .

$$E = 200 \text{ GPa} \text{ y } I = 500 (10^6) \text{ mm}^4$$



$$CA = \frac{P}{L^2} (b^2 a + \frac{a^2 b}{2}) =$$

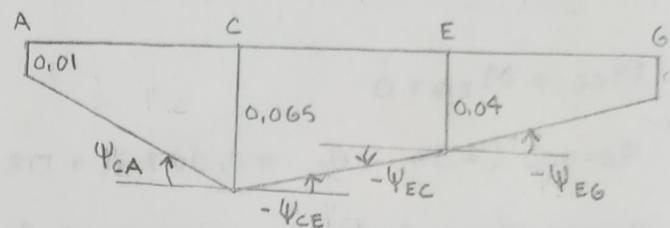
$$CA = \frac{120}{10^2} (6^2(4) + \frac{4^2(6)}{2}) = 230,4$$

$$CE = -\frac{Pb^2 a}{L^2} = -\frac{120(4)^2(6)}{10^2} = -115,2$$

$$EC = \frac{Pa^2 b}{L^2} = \frac{120(6)^2(4)}{10^2} = 172,8$$

$$EG = -\frac{3PL}{16} = -\frac{3(150)(8)}{16} = -225$$

### ROTACIONES



$$\psi_{AC} = 0,065 - 0,01 / 10 = 0,0055$$

$$\psi_{CE} = 0,065 - 0,04 / 10 = -0,0025$$

$$\psi_{EC} = -0,0025$$

$$\psi_{EG} = 0,04 - 0,025 / 8 = -0,001875$$

### MOMENTOS

$$EI = \frac{200 \times 10^9 (500 \times 10^{-6})}{1000} = 100000$$

$$M_{CA} = \frac{3(100000)}{10} ((\theta_C - 0,0055) + 230,4)$$

$$M_{CA} = 30000 ((\theta_C - 0,0055) + 230,4) = 21,23$$

$$M_{CE} = \frac{2(200000)}{10} (2\theta_C + \theta_E - (-0,0075) - 115,2)$$

$$M_{CE} = 40000 (2\theta_C + \theta_E + 0,0075) - 115,2 = -21,23$$

$$M_{EC} = \frac{2(200000)}{10} (2\theta_E + \theta_C - (-0,0075)) + 172,8$$

$$M_{EC} = 40000 (2\theta_E + \theta_C + 0,0075) + 172,8 = 237,42$$

$$M_{EG} = \frac{3(100000)}{8} ((\theta_E - (-0,001875)) - 225)$$

$$M_{EG} = 37500 (\theta_E + 0,001875) - 225 = -237,42$$

### ANGULOS

$$M_{CA} + M_{CE} = 0$$

$$30\,000 (\theta_C - 0,0055) + 230,4 + 40\,000 (2\theta_C + \theta_E + 0,0075) - 115,2 = 0$$

$$30000 \theta_C - 165 + 230,4 + 80\,000 \theta_C + 40\,000 \theta_E + 300 - 115,2 = 0$$

$$110\,000 \theta_C + 40\,000 \theta_E + 250,2 = 0$$

$$M_{EC} + M_{EG} = 0$$

$$40\,000 (2\theta_E + \theta_C + 0,0075) + 172,8 + 37\,500 (\theta_E + 0,001875) - 225 = 0$$

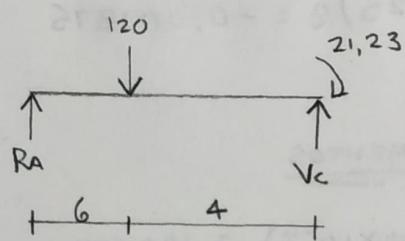
$$80\,000 \theta_E + 40\,000 \theta_C + 300 + 172,8 + 37\,500 \theta_E + 70,31 - 225 = 0$$

$$40\,000 \theta_C + 117\,500 \theta_E + 318,11 = 0$$

$$\theta_C = -0,00147232$$

$$\theta_E = -0,0022061$$

### REACCIONES



$$\sum M_C = 0$$

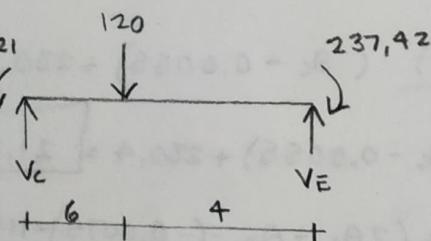
$$21,23 - 120(4) + 10R_A = 0$$

$$R_A = 45,9 \uparrow$$

$$\sum F_y = 0$$

$$45,9 - 120 + V_C = 0$$

$$V_C = 74,1 \uparrow$$



$$\sum M_E = 0$$

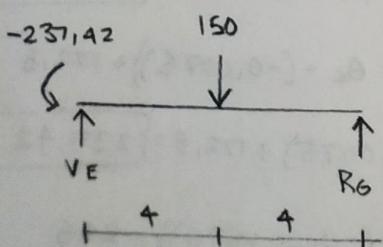
$$-237,42 + 237,42 - 120(4) + 10V_C = 0$$

$$V_C = 26,6 \uparrow$$

$$\sum F_y = 0$$

$$26,6 - 120 + V_E = 0$$

$$V_E = 93,4 \uparrow$$



$$\sum M_E = 0$$

$$-237,42 + 150(4) - 8R_G = 0$$

$$R_G = 45,3 \uparrow$$

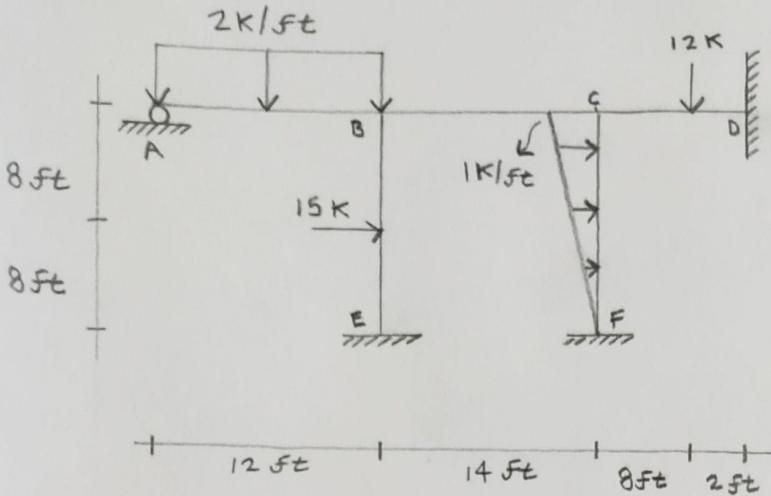
$$\sum F_y = 0$$

$$V_E - 150 + 45,3 = 0$$

$$V_E = 104,7 \uparrow$$

## TALLER # 2

DETERMINAR LOS MOMENTOS. EI = CONSTANTE



### FEM

$$BA = \frac{WL^2}{8} = \frac{2(12)^2}{8} = 36$$

$$BE = \frac{PL}{8} = \frac{15(16)}{8} = 30$$

$$EB = -30$$

$$CF = \frac{WL^2}{20} = \frac{1(16)^2}{20} = 12,8$$

$$FC = -\frac{WL^2}{30} = -\frac{1(16)^2}{30} = -8,53$$

$$CD = -\frac{Pb^2a}{L^2} = -\frac{12(2)^2(8)}{10^2} = -3,84$$

$$DC = \frac{Pa^2b}{L^2} = \frac{12(8)^2(2)}{10^2} = 15,36$$

### MOMENTOS

$$M_{BA} = \frac{3EI}{12}(\theta_B) + 36 = 14,85 \text{ K.ft}$$

$$M_{BE} = \frac{2EI}{16}(2\theta_B) + 30 = 8,85 \text{ K.ft}$$

$$M_{EB} = \frac{2EI}{16}(\theta_B) - 30 = -40,57 \text{ K.ft}$$

$$M_{BC} = \frac{2EI}{14}(2\theta_B + \theta_C) = -23,69 \text{ K.ft}$$

$$M_{CB} = \frac{2EI}{14}(2\theta_C + \theta_B) = -11,13 \text{ K.ft}$$

$$M_{CF} = \frac{2EI}{16}(2\theta_C) + 12,8 = 13,64 \text{ K.ft}$$

$$M_{FC} = \frac{2EI}{16}(\theta_C) - 8,53 = -8,11 \text{ K.ft}$$

$$M_{CD} = \frac{2EI}{10}(2\theta_C) - 3,84 = -2,5 \text{ K.ft}$$

$$M_{DC} = \frac{2EI}{10}(\theta_C) + 15,36 = 16,03 \text{ K.ft}$$

### ANGULOS

$$M_{BA} + M_{BE} + M_{BC} = 0$$

$$0,25\theta_B + \frac{36}{EI} + 0,25\theta_B + \frac{30}{EI} + 0,286\theta_B$$

$$+ 0,143\theta_C = 0$$

$$0,786\theta_B + 0,143\theta_C = -\frac{66}{EI}$$

$$M_{CB} + M_{CF} + M_{CD} = 0$$

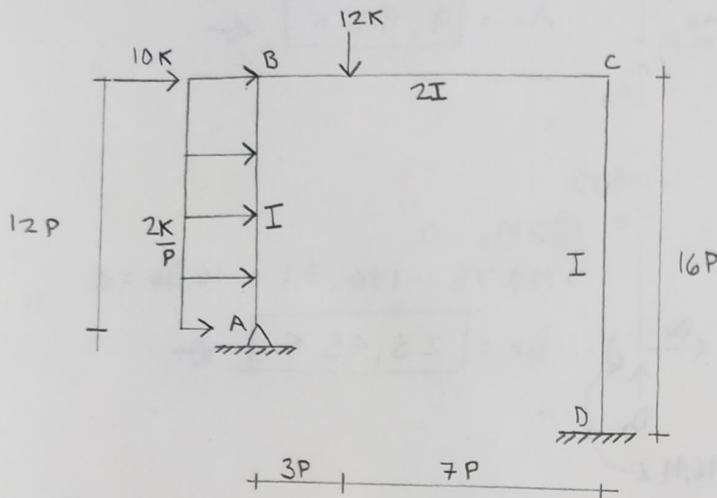
$$0,286\theta_C + 0,143\theta_B + 0,25\theta_C + \frac{12,8}{EI}$$

$$0,4\theta_C - \frac{3,84}{EI} = 0$$

$$0,143\theta_B + 0,936\theta_C = -\frac{8,96}{EI}$$

$$\theta_B = -\frac{84,58}{EI} \quad \theta_C = \frac{3,35}{EI}$$

### TALLER #3



$$\Psi_{AB} = \frac{\Delta}{12}$$

$$\Psi_{DC} = \frac{\Delta}{16}$$

$$12\Psi_{AB} = 16\Psi_{DC}$$

$$\Psi_{DC} = 0,75 \Psi_{AB}$$

$$M_{BA} + M_{BC} = 0$$

$$0,25\theta_B - 0,25\Psi + 0,8\theta_B + 0,4\theta_C$$

$$-17,64/EI + 36/EI = 0$$

$$1,05\theta_B + 0,4\theta_C - 0,25\Psi = \frac{18,36}{EI}$$

FEM

$$BA = \frac{WL^2}{8} = \frac{2(12)^2}{8} = 36$$

$$BC = -\frac{Pb^2a}{L^2} = -\frac{12(7)^2(3)}{10^2} = -17,64$$

$$CB = \frac{Pa^2b}{L^2} = \frac{12(3)^2(7)}{10^2} = 7,56$$

$$MCB + MCD = 0$$

$$0,8\theta_C + 0,4\theta_B + \frac{7,56}{EI} + 0,25\theta_C$$

$$-0,281\Psi = 0$$

$$0,4\theta_B + 1,05\theta_C - 0,281\Psi = \frac{-7,56}{EI}$$

MOMENTOS

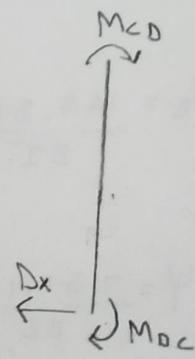
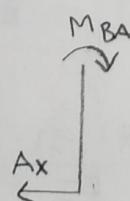
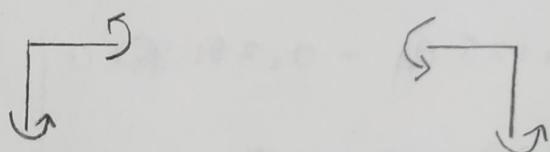
$$M_{BA} = \frac{3EI}{12}(\theta_B - \Psi) + 36 = -118,07$$

$$M_{BC} = \frac{4EI}{10}(2\theta_B + \theta_C) - 17,64 = 154,79$$

$$M_{CB} = \frac{4EI}{10}(2\theta_C + \theta_B) + 7,56 = 178,56$$

$$M_{CD} = \frac{2EI}{16}(2\theta_C - 2,25\Psi) = -178,56$$

$$M_{DC} = \frac{2EI}{16}(\theta_C - 2,25\Psi) = -196,41$$



$$\rightarrow \sum F_x = 0$$

$$24 + 10 - Ax - Dx = 0$$

$$34 - Ax - Dx = 0$$

$$\leftarrow \sum M_B = 0$$

$$M_{BA} + 12Ax = 0$$

$$Ax = -\frac{M_{BA}}{12}$$

$$\leftarrow \sum M_c = 0$$

$$M_{CD} + M_{DC} + 16Dx = 0$$

$$Dx = -\frac{M_{CD} - M_{DC}}{16}$$

$$34 + \frac{M_{BA}}{12} + \frac{M_{CD} + M_{DC}}{16} = 0$$

$$\frac{544}{EI} + 1,333(0,25\theta_B - 0,25\psi)$$

$$+ 0,25\theta_C - 0,281\psi + \frac{36}{EI}$$

$$+ 0,125\theta_C - 0,281\psi = 0$$

$$0,333\theta_B + 0,375\theta_C - 0,895\psi = -\frac{580}{EI}$$

$$\theta_B = \frac{144,88}{EI}$$

$$\theta_C = \frac{141,31}{EI}$$

$$\psi = \frac{761,16}{EI}$$

### REACCIONES

$$-118,07$$

$$\leftarrow \sum M_B = 0$$

$$-118,07 + 12Ax = 0$$

$$Ax = 9,84 \text{ kN} \quad \leftarrow$$

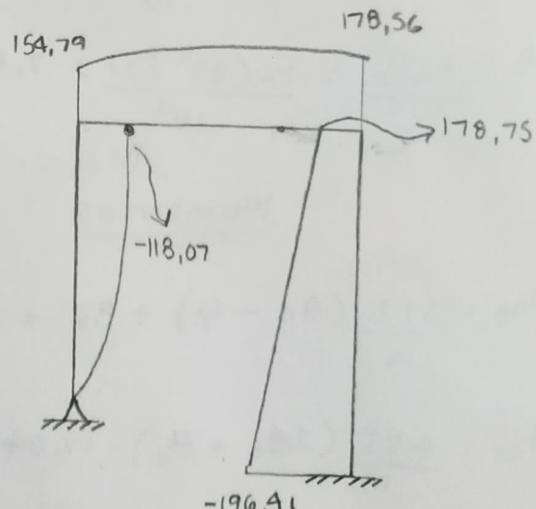
$$-178,75$$

$$\leftarrow \sum M_c = 0$$

$$-178,75 - 196,41 + 16Dx = 0$$

$$Dx = 23,45 \text{ kN} \quad \leftarrow$$

$$-196,41$$



# TALLER # 4

## FACTOR RIGIDEZ

$$R_1 = \frac{3EI}{10} = 0,3 EI$$

$$R_2 = \frac{4EI}{12} = 0,3333 EI$$

$$R_3 = \frac{4EI}{15} = 0,2667 EI$$

## FACTOR DISTRIBUCIÓN

$$D_{BA} = \frac{R_1}{R_1 + R_2} = \frac{0,3}{0,3 + 0,3333} = 0,474$$

$$D_{BC} = \frac{R_2}{R_1 + R_2} = \frac{0,3333}{0,3 + 0,3333} = 0,526$$

$$D_{CB} = \frac{R_2}{R_2 + R_3} = \frac{0,3333}{0,3333 + 0,2667} = 0,555$$

$$D_{CD} = \frac{R_3}{R_2 + R_3} = \frac{0,2667}{0,3333 + 0,2667} = 0,445$$

## FEM

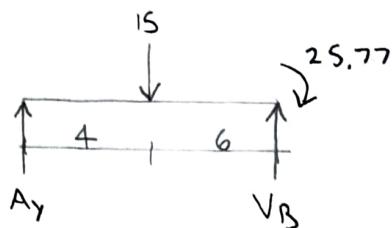
$$BA = \left(\frac{P}{L^2}\right) \left(b^2 a + \frac{a^2 b}{2}\right) = \frac{15}{10^2} (4^2 (6) + \frac{6^2 (4)}{2}) = \underline{\underline{25,2}}$$

$$BC = -\frac{WL^2}{12} = -\frac{2(12)^2}{12} = \underline{\underline{-24}} \quad CB = \underline{\underline{24}}$$

$$CD = -\frac{WL^2}{30} = -\frac{2(15)^2}{30} = \underline{\underline{-15}}$$

$$DC = \frac{WL^2}{20} = \frac{2(15)^2}{20} = \underline{\underline{22,5}}$$

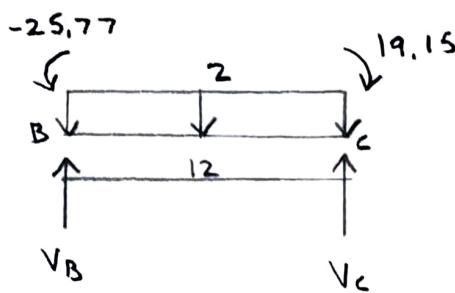
A	B	C	D
	$\leftarrow 0 \quad \frac{1}{2} \rightarrow$ 0,474 0,526	$\leftarrow \frac{1}{2} \quad \frac{1}{2} \rightarrow$ 0,555 0,445	
M	25,2 -24	24 -15	22,5
$1^{\circ} D$	-0,57 -0,63	-5 -4	
T	$\underline{-2,5}$	$\underline{-0,315}$	
$2^{\circ} D$	1,18 1,32	0,17 0,14	
T	$\underline{0,09}$	$\underline{0,66}$	
	$-0,04 -0,05$	$-0,36 -0,3$	
$M_{AB} = 0$	$M_{BC} = -25,77 \text{ K}\cdot\text{p}$	$M_{CD} = -19,16 \text{ K}\cdot\text{p}$	
$M_{BC} = 25,77 \text{ K}\cdot\text{p}$	$M_{CB} = 19,16 \text{ K}\cdot\text{p}$	$M_{DC} = 20,57 \text{ K}\cdot\text{p}$	



REACCIONES

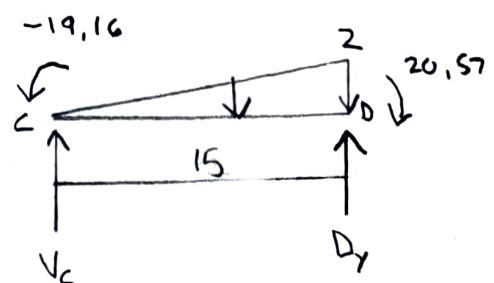
$$\begin{aligned} & \uparrow \sum M_B = 0 \\ & 25,77 - 15(6) + 10 A_y = 0 \\ & A_y = 6,42 \text{ K} \uparrow \end{aligned}$$

$$\begin{aligned} & \uparrow \sum F_y = 0 \\ & 6,42 - 15 + V_B = 0 \\ & V_B = 8,58 \text{ K} \uparrow \end{aligned}$$



$$\begin{aligned} & \uparrow \sum M_c = 0 \\ & -25,77 + 19,15 - 2(12)(6) + 12 V_B = 0 \\ & V_B = 12,55 \text{ K} \uparrow \end{aligned}$$

$$\begin{aligned} & \uparrow \sum F_y = 0 \\ & 12,55 - 2(12) + V_C = 0 \\ & V_C = 11,45 \text{ K} \uparrow \end{aligned}$$



$$\begin{aligned} & \uparrow \sum M_c = 0 \\ & -19,16 + 20,57 + \frac{2(15)}{2}(10) - 15 D_y = 0 \\ & D_y = 10,01 \text{ K} \uparrow \end{aligned}$$

$$\begin{aligned} & \uparrow \sum F_y = 0 \\ & V_C - \frac{2(15)}{2} + 10,01 = 0 \end{aligned}$$

$$V_C = 4,99 \text{ K} \uparrow$$

## TALLER # 5

### FACTOR DE RIGIDEZ

$$R_{BA} = \frac{4EI}{L} = \frac{4EI}{16} = 0,25$$

$$R_{BC} = \frac{3EI}{L} = \frac{3EI}{10} = 0,3 EI$$

$$R_{BD} = \frac{4EI}{L} = \frac{4EI}{12} = 0,3333 EI$$

$$R_{DE} = \frac{3EI}{L} = \frac{3EI}{12} = 0,25 EI$$

$$R_{DF} = \frac{4EI}{L} = \frac{4EI}{14} = 0,2857 EI$$

### FACTOR DE DISTRIBUCIÓN

$$D_{BC} = \frac{R_{BC}}{R_{BC} + R_{BA} + R_{BD}} = \frac{0,3}{0,3 + 0,25 + 0,3333} = 0,34$$

$$D_{BA} = \frac{R_{BA}}{R_{BC} + R_{BA} + R_{BD}} = \frac{0,25}{0,3 + 0,25 + 0,3333} = 0,283$$

$$D_{BD} = \frac{R_{BD}}{R_{BC} + R_{BA} + R_{BD}} = \frac{0,3333}{0,3 + 0,25 + 0,3333} = 0,377$$

$$D_{DB} = \frac{R_{BD}}{R_{BD} + R_{DE} + R_{DF}} = \frac{0,3333}{0,3333 + 0,25 + 0,2857} = 0,383$$

$$D_{DE} = \frac{R_{DE}}{R_{BD} + R_{DE} + R_{DF}} = \frac{0,25}{0,3333 + 0,25 + 0,2857} = 0,288$$

$$D_{DF} = \frac{R_{DF}}{R_{BD} + R_{DE} + R_{DF}} = \frac{0,2857}{0,3333 + 0,25 + 0,2857} = 0,329$$

FEM

$$AB = -\frac{\rho L}{8} = -\frac{15(16)}{8} = -30 \quad BA = 30$$

$$BC = \frac{Pa^2b}{L^2} = \frac{10(4)^2(6)}{10^2} = 9,6$$

$$BD = -\frac{WL^2}{30} = -\frac{3(12)^2}{30} = -14,4$$

$$DB = \frac{WL^2}{20} = \frac{3(12)^2}{20} = 21,6$$

$$DF = -\frac{WL^2}{12} = -\frac{2(14)^2}{12} = -32,67 \quad FD = 32,67$$

$$M_{BC} = 4,67 \text{ Kft}$$

$$M_{BD} = -24,7 \text{ Kft}$$

$$M_{BA} = 20,2 \text{ Kft}$$

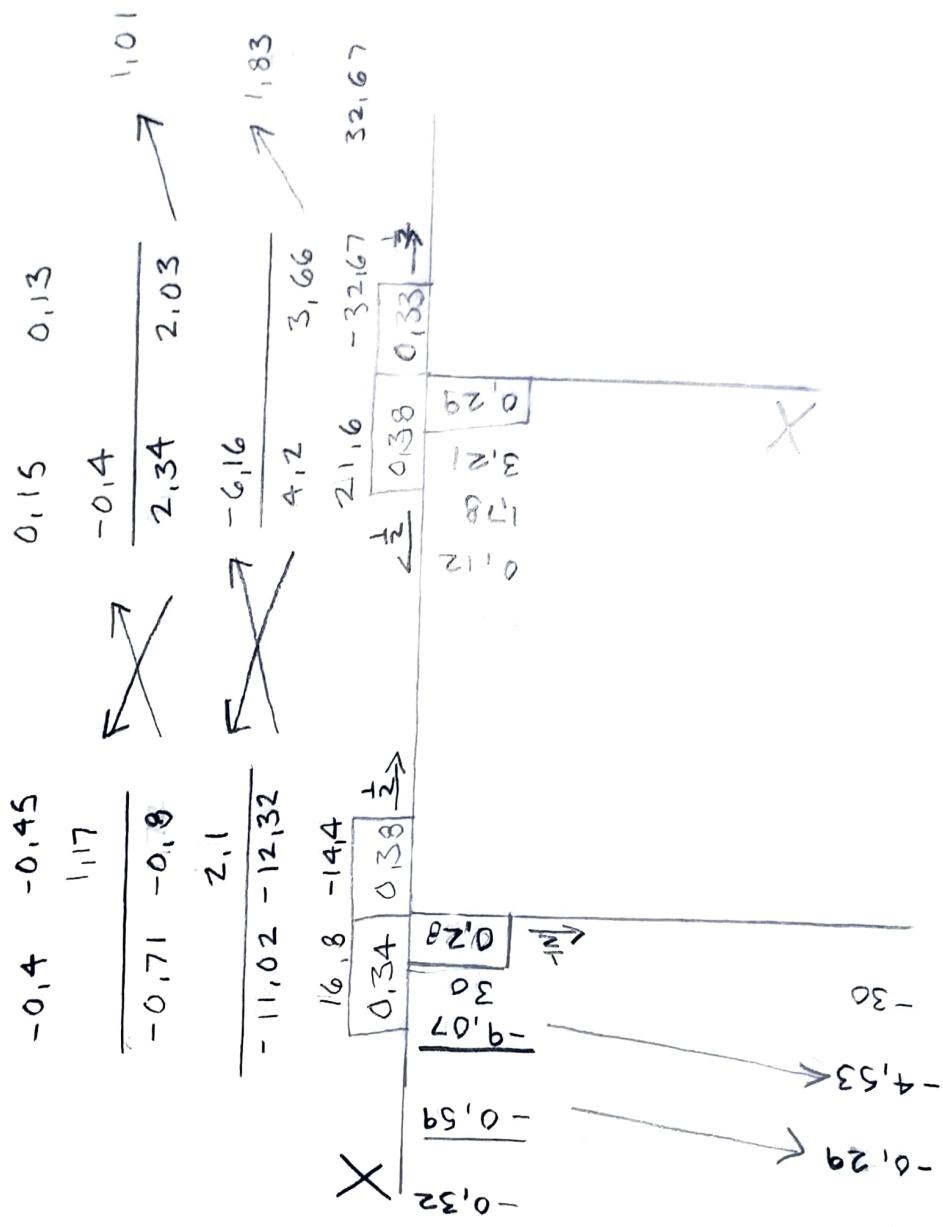
$$M_{AB} = -34,82 \text{ Kft}$$

$$M_{DB} = 21,73 \text{ Kft}$$

$$M_{DE} = 5,11 \text{ Kft}$$

$$M_{DF} = -26,85 \text{ Kft}$$

$$M_{FD} = 35,51 \text{ Kft}$$



# TALLER #6

F.R

$$R_{AB} = \frac{2EI}{L} = \frac{2EI}{10} = 0,2$$

$$R_{BC} = \frac{4EI}{L} = \frac{4EI}{12} = 0,3333$$

$$R_{CD} = \frac{4EI}{L} = \frac{4EI}{16} = 0,25$$

F.D

$$D_{BA} = \frac{R_{AB}}{R_{AB} + R_{BC}} = \frac{0,2}{0,2 + 0,3333} = 0,37$$

$$D_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} = \frac{0,3333}{0,2 + 0,3333} = 0,63$$

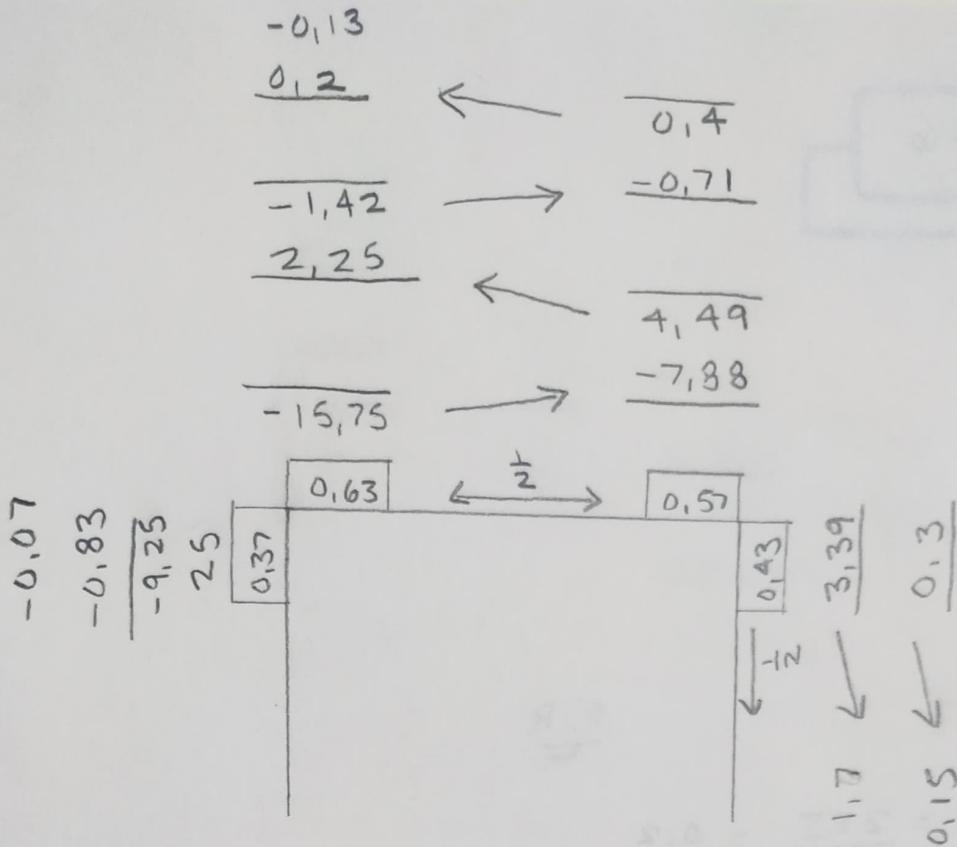
$$D_{CB} = \frac{R_{BC}}{R_{BC} + R_{CD}} = \frac{0,3333}{0,3333 + 0,25} = 0,57$$

$$D_{CD} = \frac{R_{CD}}{R_{BC} + R_{CD}} = \frac{0,25}{0,3333 + 0,25} = 0,43$$

FEM

$$BA = \frac{WL^2}{8} = \frac{2(10)^2}{8} = 25$$

MARCO 1



$$M_{AB} = 0$$

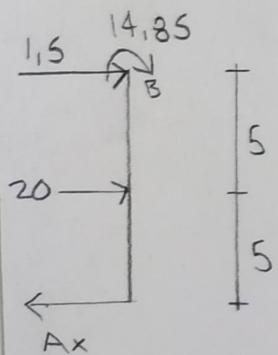
$$M_{BA} = 14.85$$

$$M_{BC} = -14.85$$

$$M_{CB} = -3.7$$

$$M_{CD} = 3.7$$

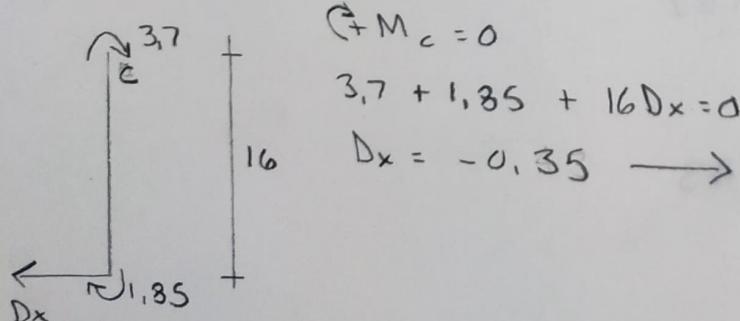
$$M_{DC} = 1.85$$



$$\begin{aligned} &\text{↶ } \sum M_B = 0 \\ &14.85 - 20(5) + 10Ax = 0 \\ &Ax = 8.51 \leftarrow \end{aligned}$$

$$R = 1.5 + 20 + 0.35 - 8.51$$

$$R_1 = 13.34$$

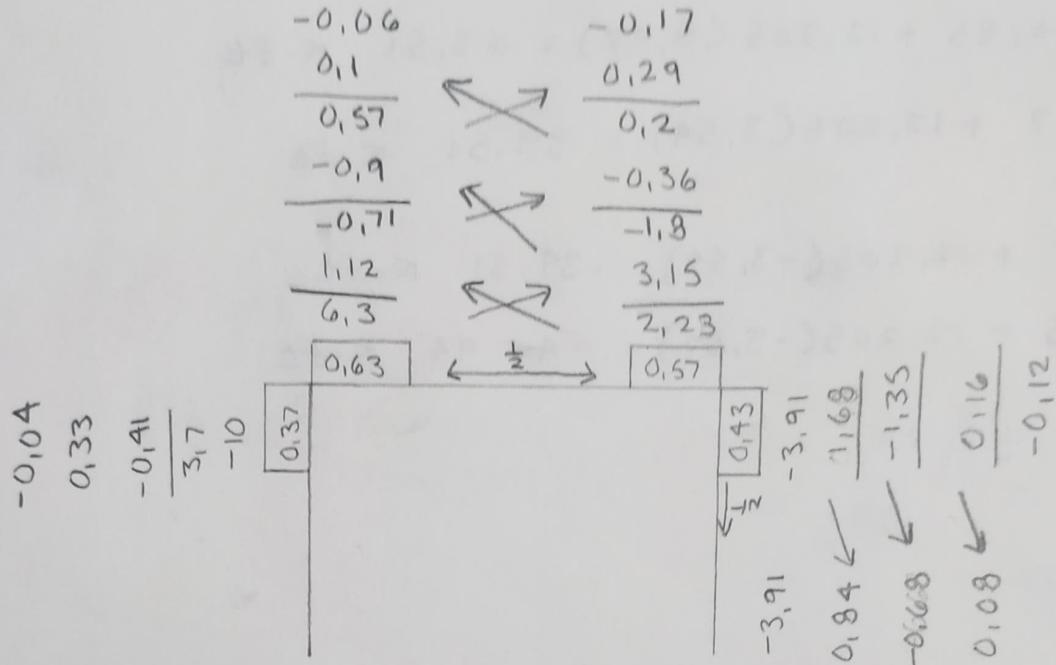


$$\begin{aligned} &\text{↶ } M_c = 0 \\ &3.7 + 1.85 + 16Dx = 0 \\ &Dx = -0.35 \rightarrow \end{aligned}$$

$$M_{BA} = -10$$

$$\Delta = \frac{M L^2}{6EI} = \frac{-10 (10)^2}{6EI} = \frac{-160,67}{EI}$$

$$M_{CD} = M_{DC} = \frac{6EI\Delta}{L^2} = \frac{6(-160,67)}{16^2} = -3,91$$



$$M_{AB} = 0$$

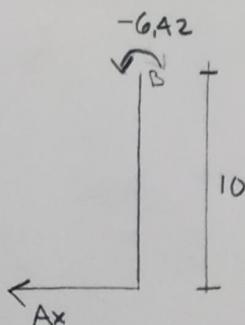
$$M_{BC} = 6,42$$

$$M_{CD} = -3,54$$

$$M_{BA} = -6,42$$

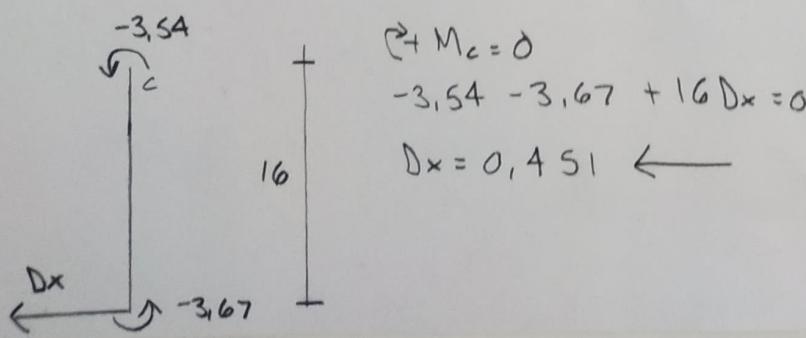
$$M_{CB} = 3,54$$

$$M_{DC} = -3,67$$



$$\begin{aligned} \sum M_B &= 0 \\ -6,42 + 10Ax &= 0 \\ Ax &= 0,642 \leftarrow \end{aligned}$$

$$R_2 = 0,642 + 0,951 = 1,093$$



$$\begin{aligned} \sum M_C &= 0 \\ -3,54 - 3,67 + 16Dx &= 0 \\ Dx &= 0,451 \leftarrow \end{aligned}$$

MOMENTOS FINALES

$$M_{AB} = 0$$

$$M_{BA} = 14,85 + 12,205(-6,42) = -63,51 \text{ K.ft}$$

$$M_{BC} = -14,85 + 12,205(6,42) = 63,51 \text{ K.ft}$$

$$M_{CB} = -3,7 + 12,205(3,54) = 39,51 \text{ K.ft}$$

$$M_{CD} = 3,7 + 12,205(-3,54) = -39,51 \text{ K.ft}$$

$$M_{DC} = 1,85 + 12,205(-3,67) = -42,94 \text{ K.ft}$$

TALLER #7

ELEMENTO 1

$$\lambda_x = \frac{0 - 2}{2} = -1 \quad \lambda_y = \frac{0 - 0}{2} = 0 \quad L = 2 \quad \frac{AE}{L} = 150 \times 10^6$$

$$K_1 = 150 \times 10^6 \begin{bmatrix} 5 & 6 & 7 & 8 \\ 150 \times 10^6 & 0 & -150 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 \\ -150 \times 10^6 & 0 & 150 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} 5 \\ 6 \\ 7 \\ 8 \end{array}$$

ELEMENTO 2

$$\lambda_x = \frac{2 - 4}{2} = -1 \quad \lambda_y = \frac{0 - 0}{2} = 0 \quad L = 2 \quad \frac{AE}{L} = 150 \times 10^6$$

$$K_2 = 150 \times 10^6 \begin{bmatrix} 1 & 2 & 5 & 6 \\ 150 \times 10^6 & 0 & -150 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 \\ -150 \times 10^6 & 0 & 150 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array}$$

### ELEMENTO 3

$$\lambda_x = \frac{2-4}{2,8284} = -0,7071$$

$$\lambda_x = \frac{2-0}{2,8284} = 0,7071 \quad L = 2,8284$$

$$\frac{\Delta E}{L} = 106 \ 067 \ 034,4$$

$$K_3 = 106 \ 067 \ 034,4$$

1	2	3	4
$53,03 \times 10^6$	$-53,03 \times 10^6$	$-53,03 \times 10^6$	$53,03 \times 10^6$
$-53,03 \times 10^6$	$53,03 \times 10^6$	$53,03 \times 10^6$	$-53,03 \times 10^6$
$-53,03 \times 10^6$	$53,03 \times 10^6$	$53,03 \times 10^6$	$-53,03 \times 10^6$
$53,03 \times 10^6$	$-53,03 \times 10^6$	$-53,03 \times 10^6$	$53,03 \times 10^6$

### ELEMENTO 4

$$\lambda_x = \frac{2-2}{2} = 0$$

$$\lambda_y = \frac{2-0}{2} = 1$$

$$L = 2$$

$$\frac{\Delta E}{L} = 150 \times 10^6$$

$$K_4 = 150 \times 10^6$$

5	6	3	4	5
0	0	0	0	0
0	$150 \times 10^6$	0	$-150 \times 10^6$	6
0	0	0	0	3
0	$-150 \times 10^6$	0	$150 \times 10^6$	4

ELEMENTO 5

$$\lambda_x = \frac{0 - 2}{2,8284} = -0,7071 \quad \lambda_y = \frac{2 - 0}{2,8284} = 0,7071 \quad L = 2,8284$$

$$\frac{\Delta E}{L} = 106 \quad 067 \quad 034,4$$

$$K_5 = 106 \quad 067 \quad 034,4$$

$$\begin{bmatrix} 3 & 4 & 7 & 8 \\ 53,03 \times 10^6 & 53,03 \times 10^6 & -53,03 \times 10^6 & -53,03 \times 10^6 \\ -53,03 \times 10^6 & 53,03 \times 10^6 & -53,03 \times 10^6 & -53,03 \times 10^6 \\ -53,03 \times 10^6 & -53,03 \times 10^6 & 53,03 \times 10^6 & 53,03 \times 10^6 \\ 53,03 \times 10^6 & -53,03 \times 10^6 & 53,03 \times 10^6 & 53,03 \times 10^6 \end{bmatrix}$$

ELEMENTO 6

$$\lambda_x = \frac{0 - 2}{2} = -1 \quad \lambda_y = \frac{2 - 2}{2} = 0 \quad L = 2 \quad \frac{\Delta E}{L} = 150 \times 10^6$$

$$K_6 = 150 \times 10^6$$

$$\begin{bmatrix} 3 & 4 & 9 & 10 \\ 150 \times 10^6 & 0 & -150 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 \\ -150 \times 10^6 & 0 & 150 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 9 \\ 10 \end{bmatrix}$$

	1	2	3	4	5	6	7	8	9	10
1	203,03	-53,03	-53,03	53,03	-150	0	0	0	0	0
2	-53,03	53,03	53,03	-53,03	0	0	0	0	0	0
3	-53,03	53,03	256,67	0	0	0	-53,03	-53,03	-150	0
4	53,03	-53,03	0	256,67	0	-150	-53,03	-53,03	0	0
5	-150	0	0	0	300	0	-150	0	0	0
6	0	0	0	-150	0	150	0	0	0	0
7	0	0	-53,03	-53,03	-150	0	203,03	53,03	0	0
8	0	0	-53,03	-53,03	0	0	53,03	53,03	0	0
9	0	0	-150	-150	0	0	0	150	0	0
10	0	0	0	0	0	0	0	0	0	0

K =

TALLER # 8

ELEMENTO 1

$$\delta_x = \frac{48}{67,8822} = 0,7071 \quad \delta_y = \frac{48}{67,8822} = 0,7071 \quad \frac{\Delta E}{L} = 320,409$$

$$K_1 = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 160,204 & 160,204 & -160,204 & -160,204 \\ 160,204 & 160,204 & -160,204 & -160,204 \\ -160,204 & -160,204 & 160,204 & 160,204 \\ -160,204 & -160,204 & 160,204 & 160,204 \end{bmatrix}$$

ELEMENTO 2

$$\lambda_x = 0 \quad \delta_y = \frac{-48}{48} = -1 \quad \frac{\Delta E}{L} = 454,5454$$

$$K_2 = \begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 453,125 & 453,125 & -453,125 \\ 0 & 0 & 0 & 0 \\ 0 & -453,125 & 0 & 453,125 \end{bmatrix}$$

### III ELEMENTO 3

$$\delta_x = \frac{36}{60} = 0,6$$

$$\delta_y = \frac{-43}{60} = -0,7$$

$$\frac{AE}{L} = 362,4502$$

$$K_3 = \begin{bmatrix} 1 & 2 & 7 & 8 \\ 130,5 & -174 & -130,5 & 174 \\ -174 & 232 & 174 & -232 \\ -130,5 & 174 & 130,5 & -174 \\ 174 & -232 & -174 & 232 \end{bmatrix}$$

$$K = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 290,704 & -13,796 & -160,204 & -160,204 & 0 & 0 & -130,5 & 174 \\ -13,796 & 845,329 & -160,204 & -160,204 & 0 & -453,125 & 174 & -232 \\ -160,204 & -160,204 & 160,204 & 160,204 & 0 & 0 & 0 & 0 \\ -160,204 & -160,204 & 160,204 & 160,204 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -453,125 & 0 & 0 & 0 & 453,125 & 0 & 0 \\ -130,5 & 174 & 0 & 0 & 0 & 0 & 130,5 & -174 \\ 174 & -232 & 0 & 0 & 0 & 0 & -174 & 232 \end{bmatrix}$$

$$\begin{bmatrix} -0,5 \\ 0 \end{bmatrix} = \begin{bmatrix} 290,704 & -13,796 \\ -13,796 & 845,329 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$D_1 = -0,001721 \text{ m}$$

$$D_2 = -0,000028 \text{ m}$$

$$\begin{bmatrix} Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} -160,204 & -160,204 \\ -160,204 & -160,204 \\ 0 & 0 \\ 0 & -453,125 \\ -130,5 & 174 \\ 174 & -232 \end{bmatrix} \begin{bmatrix} -0,001721 \\ -0,000028 \end{bmatrix}$$

$$Q_3 = 0,28 \text{ K}$$

$$Q_4 = 0,28 \text{ K}$$

$$Q_5 = 0$$

$$Q_6 = 0,013 \text{ K}$$

$$Q_7 = 0,22 \text{ K}$$

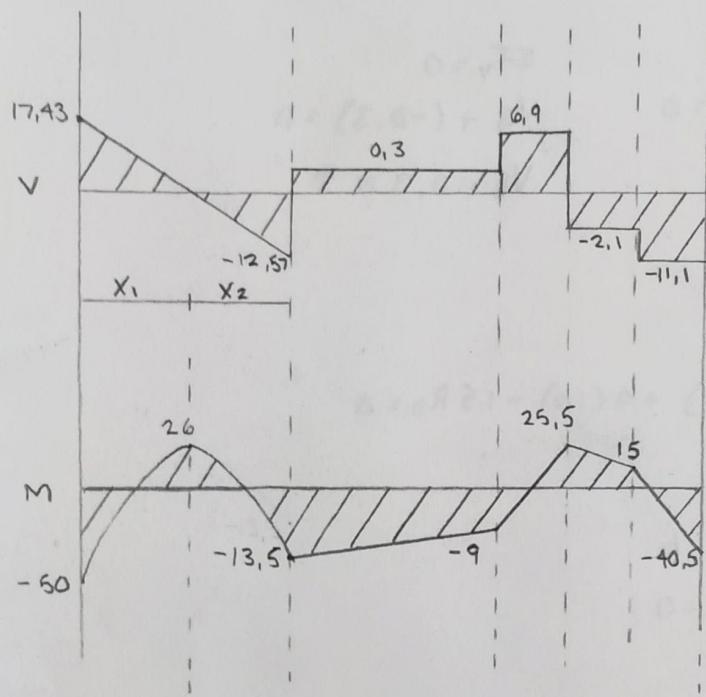
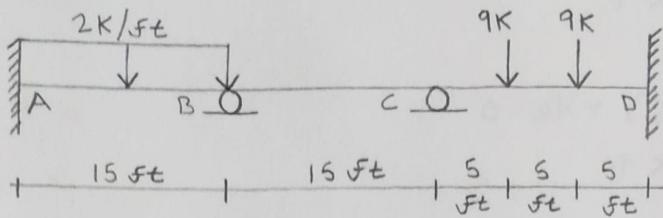
$$Q_8 = -0,293 \text{ K}$$

$$q_F = \begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -0,00172 \\ -0,00003 \\ 0 \\ 0 \end{bmatrix}$$

$$q_F = -0,01273 \text{ K} = -12,73 \text{ K} \quad \textcircled{C}$$

TALLER #1

POR EL METODO PENDIENTE - DEFLEXIÓN DIBUJAR  
LOS DIAGRAMAS DE CORTANTE Y MOMENTO



FEM

$$AB = -\frac{WL^2}{12} = -\frac{2(15)^2}{12} = -37,5$$

$$BA = 37,5$$

$$CD = -\frac{2PL}{9} = -\frac{2(9)(15)}{9} = -30$$

$$DC = 30$$

# MOMENTO

$$M_{AB} = \frac{2EI}{15} (\theta_B) - 37,5 = -50 \text{ k}\cdot\text{ft}$$

$$M_{BA} = \frac{2EI}{15} (2\theta_B) + 37,5 = 13,5 \text{ k}\cdot\text{ft}$$

$$M_{CB} + M_{CD} = 0$$

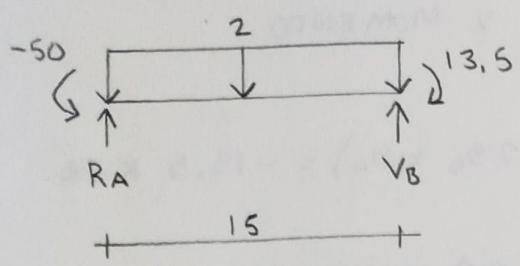
$$\frac{2EI}{15} (2\theta_c + \theta_B) + \frac{2EI}{15} (2\theta_c) - 30 = 0$$

$$0.2667 EI \theta_c + 0.1333 EI \theta_B + 0.2667 EI \theta_c - 30 = 0$$

$$0,1333 EI \theta_B + 0,5334 EI \theta_C - 30 = 0$$

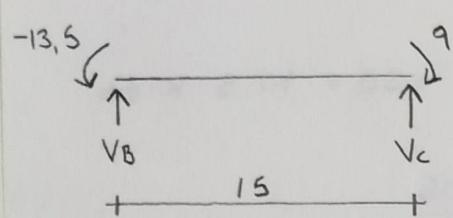
$$\Theta_B = -\frac{89,98}{EI} \quad \text{ET} \quad \Theta_C = \frac{78,73}{EI}$$

REACCIONES



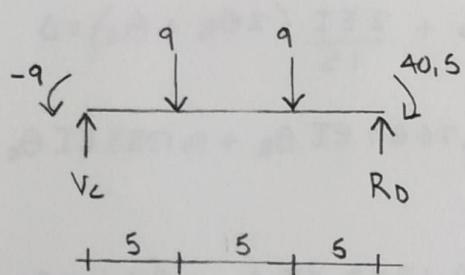
$$\sum M_B = 0 \\ -50 + 13.5 - 2(15)(7.5) + 15 R_A = 0 \\ R_A = 17.43 \text{ kN} \uparrow$$

$$\sum F_y = 0 \\ 17.43 - 2(15) + V_B = 0 \\ V_B = 12.57 \text{ kN} \uparrow$$



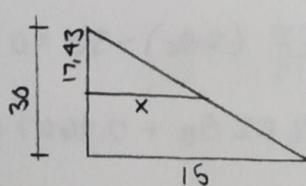
$$\sum M_B = 0 \\ -13.5 + 9 - 15 V_C = 0 \\ V_C = -0.3 \text{ kN} \downarrow$$

$$\sum F_y = 0 \\ V_B + (-0.3) = 0 \\ V_B = 0.3 \text{ kN} \uparrow$$



$$\sum M_C = 0 \\ -9 + 40.5 + 9(5) + 9(10) - 15 R_D = 0 \\ R_D = 11.1 \text{ kN} \uparrow$$

$$\sum F_y = 0 \\ V_C - 9 - 9 + 11.1 = 0 \\ V_C = 6.9 \text{ kN} \uparrow$$



$$\frac{x}{17.43} = \frac{15}{30}$$

$$x_1 = 8.715 \text{ ft}$$

$$x_2 = 6.285 \text{ ft}$$

TALLER #9

EI = CONSTANTE

≡ ELEMENTO 1

$$L = 4$$

$$K_1 = \begin{bmatrix} 6 & 7 & 4 & 3 \\ 0,19 & 0,38 & -0,19 & 0,38 \\ 0,38 & 1 & -0,38 & 0,5 \\ -0,19 & -0,38 & 0,19 & -0,38 \\ 0,38 & 0,5 & -0,38 & 1 \end{bmatrix}$$

≡ ELEMENTO 2

$$L = 3$$

$$K_2 = \begin{bmatrix} 4 & 2 & 5 & 1 \\ 0,44 & 0,67 & -0,44 & 0,67 \\ 0,67 & 1,33 & -0,67 & 0,67 \\ -0,44 & -0,67 & 0,44 & -0,67 \\ 0,67 & 0,67 & -0,67 & 1,33 \end{bmatrix}$$

$$K = \begin{bmatrix} 1,333 & 0,667 & 0 & 0,667 & -0,667 & 0 & 0 \\ 0,667 & 1,333 & 0 & 0,667 & -0,667 & 0 & 0 \\ 0 & 0 & 1 & -0,375 & 0 & 0,375 & 0,5 \\ 0,667 & 0,667 & -0,375 & 0,632 & -0,444 & -0,188 & -0,375 \\ -0,667 & -0,667 & 0 & -0,444 & 0,444 & 0 & 0 \\ 0 & 0 & 0,375 & -0,188 & 0 & 0,188 & 0,375 \\ 0 & 0 & 0,5 & -0,375 & 0 & 0,375 & 1 \end{bmatrix}$$

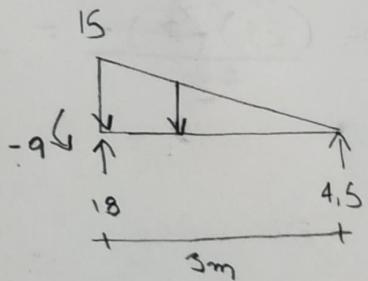
$$\Delta_U = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta_K = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_K = \begin{bmatrix} 0 \\ -9 \\ 0 \\ 0 \\ Q_5 \\ Q_6 \\ Q_7 \end{bmatrix}$$

$$Q_U = \begin{bmatrix} 0 \\ -9 \\ 0 \\ 0 \\ Q_5 \\ Q_6 \\ Q_7 \end{bmatrix}$$

$$F_{EM\ 2} = \frac{-w L^2}{15} = -\frac{15(3)^2}{15} = -9 \downarrow \quad F_2 = -18 \downarrow$$



$$\begin{bmatrix} 0 \\ -9 \\ 0 \\ -18 \end{bmatrix} = \begin{bmatrix} 1,333 & 0,667 & 0 & 0,667 \\ 0,667 & 1,333 & 0 & 0,667 \\ 0 & 0 & 1 & -0,375 \\ 0,667 & 0,667 & -0,375 & 0,632 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$D_1 = \frac{111,1667}{EI} \quad D_2 = \frac{97,6667}{EI} \quad D_3 = -\frac{120}{EI} \quad D_4 = -\frac{320}{EI}$$

$$\begin{bmatrix} Q_5 \\ Q_6 \\ Q_7 \end{bmatrix} = \begin{bmatrix} -0,667 & -0,667 & 0 & -0,444 \\ 0 & 0 & 0,375 & -0,188 \\ 0 & 0 & 0,5 & -0,375 \end{bmatrix} \begin{bmatrix} 111,1667 \\ 97,6667 \\ -120 \\ -320 \end{bmatrix}$$

$$Q_5 = 3 \text{ kN} \quad Q_6 = 15 \text{ kN} \quad Q_7 = 60 \text{ kN}$$

$$R_5 = 3 + 4,5 = 7,5 \text{ kN} \uparrow$$

$$R_6 = 15 \text{ kN} \uparrow$$

$$R_7 = 60 \text{ kN} \cdot \text{m} \curvearrowright$$