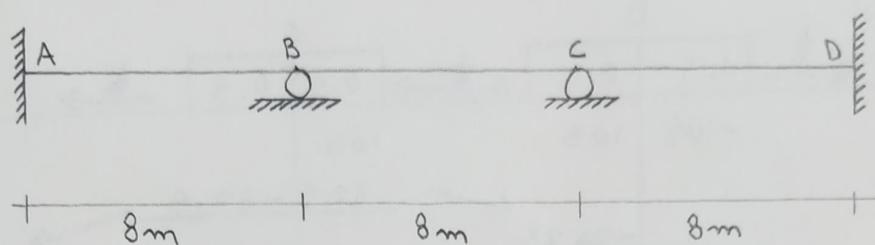


HARDY CROSS

DETERMINE LOS MOMENTOS, DEBIDO A UN ASENTAMIENTO DE 20 MM EN EL APOYO B.



$$E = 70 \text{ GPa} \quad I = 800 (10^6) \text{ mm}^4$$

F.R

$$R_{AB} = R_{BC} = R_{CD} = \frac{4EI}{L} = \frac{4EI}{8} = 0,5$$

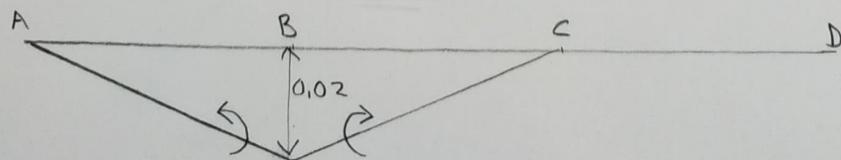
F.D

$$D_{BA} = \frac{R_{AB}}{R_{AB} + R_{BC}} = \frac{0,5}{0,5 + 0,5} = 0,5$$

$$D_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} = \frac{0,5}{0,5 + 0,5} = 0,5$$

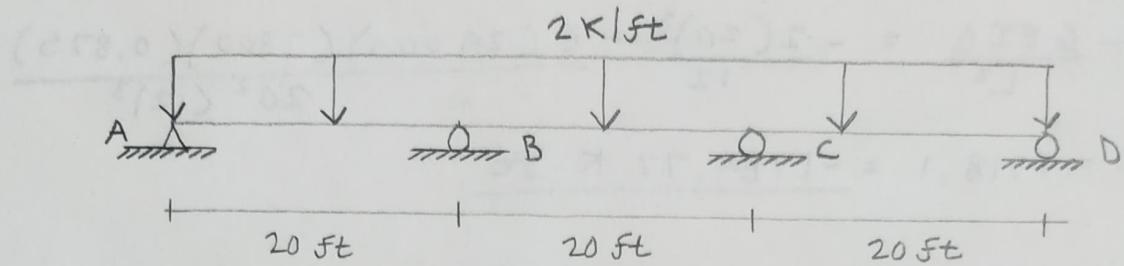
$$D_{CB} = \frac{R_{BC}}{R_{BC} + R_{CD}} = \frac{0,5}{0,5 + 0,5} = 0,5$$

$$D_{CD} = \frac{R_{CD}}{R_{BC} + R_{CD}} = \frac{0,5}{0,5 + 0,5} = 0,5$$



HARDY CROSS

DETERMINE LOS MOMENTOS, DEBIDO A UN
ASENTAMIENTO DE $5/8$ in EN B, DE $1\frac{1}{2}$ in
EN C Y DE $\frac{3}{4}$ in EN D.



$$E = 29000 \text{ ksi}$$

$$I = 7800 \text{ in}^4$$

F.R

$$R_{AB} = R_{CD} = \frac{3EI}{20} = 0,15$$

$$R_{BC} = \frac{4EI}{20} = 0,2$$

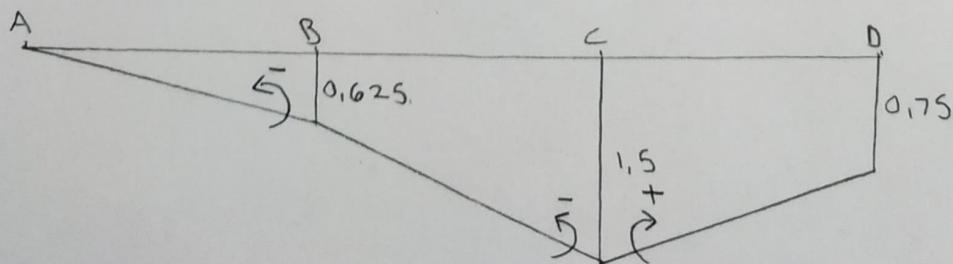
F.D

$$D_{BA} = \frac{R_{AB}}{R_{AB} + R_{BC}} = \frac{0,15}{0,15 + 0,2} = 0,43$$

$$D_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} = \frac{0,2}{0,15 + 0,2} = 0,57$$

$$D_{CB} = \frac{R_{BC}}{R_{BC} + R_{CD}} = \frac{0,2}{0,2 + 0,15} = 0,57$$

$$D_{CD} = \frac{R_{CD}}{R_{CB} + R_{CD}} = \frac{0,15}{0,2 + 0,15} = 0,43$$



FEM

$$BA = \frac{WL^2}{12} - \frac{6EI\Delta}{L^2} = \frac{2(20)^2}{12} - \frac{6(29000)(7800)(0.625)}{20^2 (12)^3}$$

$$BA = 66,67 - 1227,21 = \underline{-1160,54 \text{ K.ft}}$$

$$BC = \frac{-WL^2}{12} - \frac{6EI\Delta}{L^2} = -\frac{2(20)^2}{12} - \frac{6(29000)(7800)(0.875)}{20^2 (12)^3}$$

$$BC = -66,67 - 1718,1 = \underline{-1784,77 \text{ K.ft}}$$

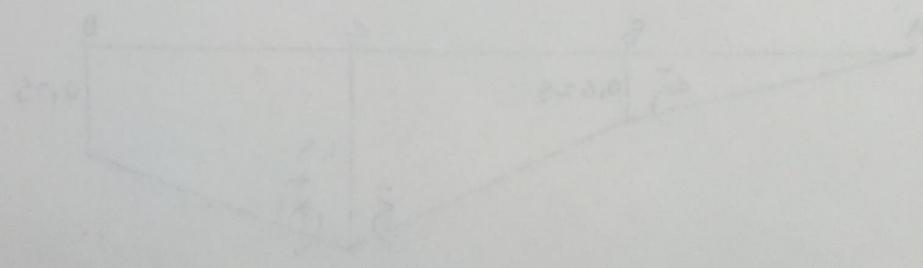
$$CB = 66,67 - 1718,1 = \underline{-1651,43 \text{ K.ft}}$$

$$CD = \frac{-WL^2}{12} + \frac{6EI\Delta}{L^2} = -\frac{2(20)^2}{12} + \frac{6(29000)(7800)(0.75)}{20^2 (12)^2}$$

$$CD = -66,67 + 1472,66 = \underline{1406,9 \text{ K.ft}}$$

$$AB = -66,67 - 1227,21 = \underline{-1293,88 \text{ K.ft}}$$

$$DC = 66,67 + 1472,66 = \underline{1539,33 \text{ K.ft}}$$



A

B

C

D

 $-1293,88$ $-1160,54$ $-1784,77$ $-1651,43$ 1406 $1293,88$ $1266,48$ $1678,82$ $139,9$ $105,54$ $1539,33$ $646,98$ $69,95$ $839,41$ $-769,66$ $-1539,33$ $-308,28$ $-408,65$ $-39,76$ -30 $-19,88$ $-19,88$ $-204,32$ $8,55$ $11,33$ $116,46$ $87,86$ $58,23$ $58,23$ $5,66$ $-25,04$ $-33,19$ $-3,23$ $-2,43$ $-1,61$ $-1,61$ $-16,6$ $0,69$ $0,92$ $9,46$ $7,14$ $4,73$ $4,73$ $0,46$ $-2,03$ $-2,7$ $-0,26$ $-0,2$ $-0,13$ $-0,13$ $-1,35$ $0,06$ $0,07$ $0,77$ $0,58$ $0,39$ $0,39$ $0,04$ $-0,02$ $-0,17$ $-0,22$ $-0,02$ $-0,02$

$$M_{AB} = 0$$

$$M_{BC} = -426,7$$

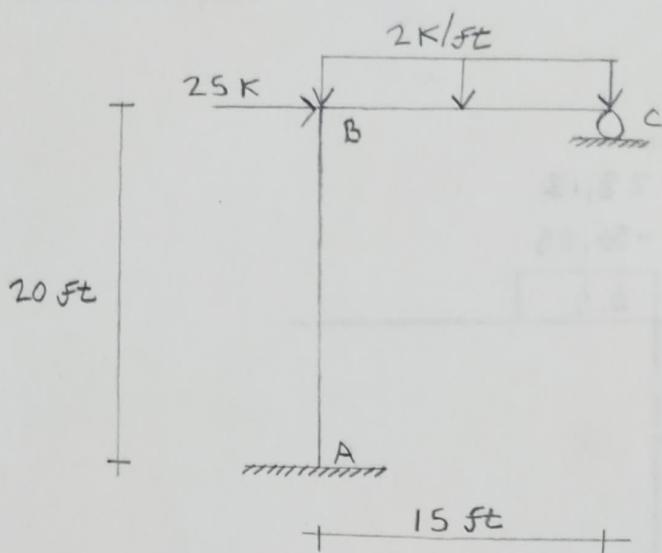
$$M_{CD} = 804,81$$

$$M_{BA} = 426,7$$

$$M_{CB} = -804,81$$

$$M_{DC} = 0$$

HARDY CROSS



F.R

$$R_{AB} = \frac{4EI}{L} = \frac{4EI}{20} = 0,2$$

$$R_{BC} = \frac{3EI}{L} = \frac{3EI}{15} = 0,2$$

F.D

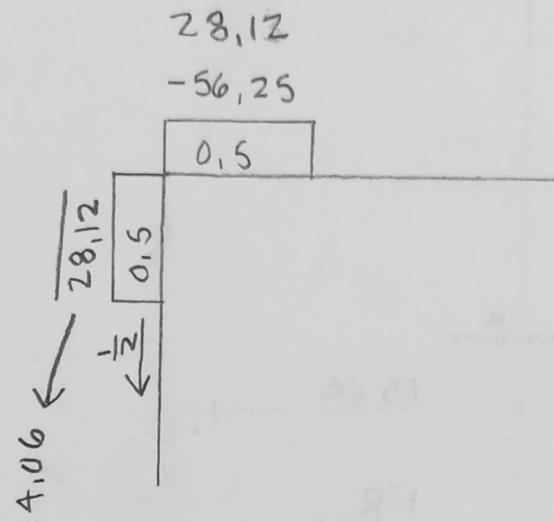
$$D_{BA} = \frac{R_{AB}}{R_{AB} + R_{BC}} = \frac{0,2}{0,2 + 0,2} = 0,5$$

$$D_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} = \frac{0,2}{0,2 + 0,2} = 0,5$$

FEM

$$BC = -\frac{WL^2}{8} = -\frac{2(15^2)}{8} = -56,25$$

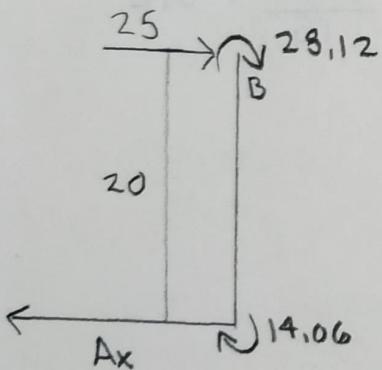
MARCO 1



$$M_{AB} = 14.06$$

$$M_{BA} = 28.12$$

$$M_{BC} = -28.12$$



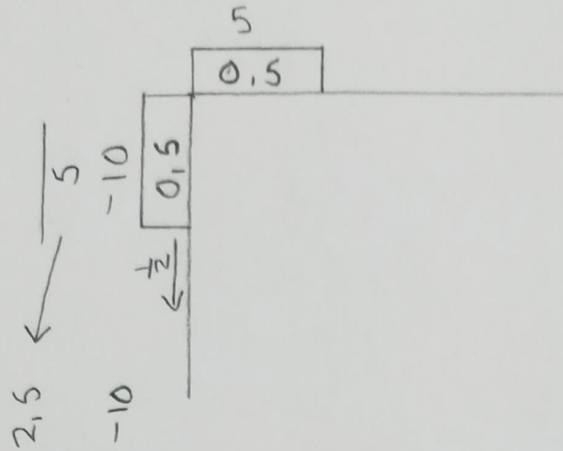
$$\sum M_B = 0$$

$$28.12 + 14.06 + 20 A_x = 0$$

$$A_x = -2.109 \rightarrow$$

$$R_1 = 25 + 2.109 = 27.109$$

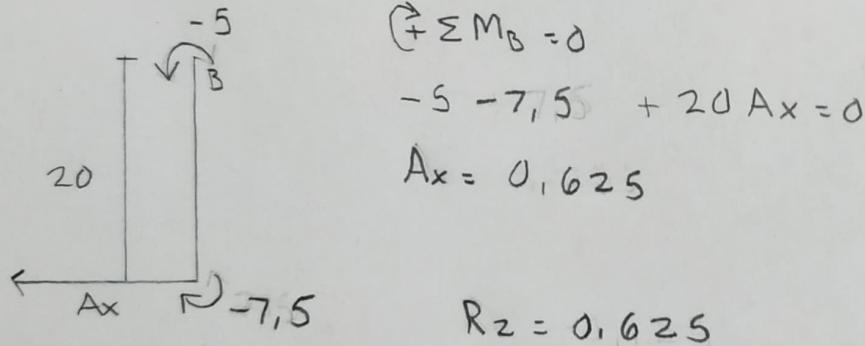
MARCO 2



$$M_{AB} = -7,5$$

$$M_{BA} = -5$$

$$M_{BC} = 5$$



$$\begin{aligned} \sum M_B &= 0 \\ -5 - 7,5 + 20 A_x &= 0 \\ A_x &= 0,625 \end{aligned}$$

$$R_z = 0,625$$

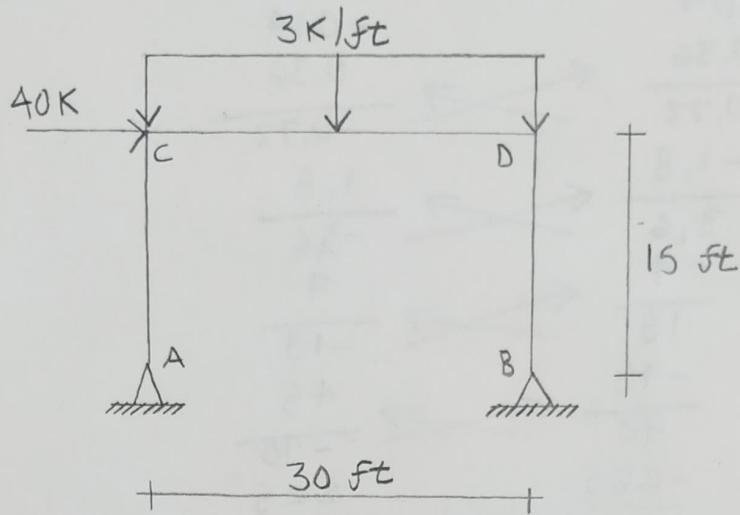
MOMENTOS FINALES

$$M_{AB} = 14,06 + 43,3744 (-7,5) = -311,25 \text{ k.ft}$$

$$M_{BA} = 28,12 + 43,3744 (-5) = -188,75 \text{ k.ft}$$

$$M_{BC} = -28,12 + 43,3744 (5) = 188,75 \text{ k.ft}$$

HARDY CROSS



F.R

$$R_{AC} = R_{BD} = \frac{3EI}{L} = \frac{3EI}{15} = 0,2$$

$$R_{CD} = \frac{4EI}{L} = \frac{4EI}{30} = 0,1333$$

F.D

$$D_{CA} = \frac{R_{AC}}{R_{AC} + R_{CD}} = \frac{0,2}{0,2 + 0,1333} = 0,6$$

$$D_{CD} = \frac{R_{CD}}{R_{AC} + R_{CD}} = \frac{0,1333}{0,2 + 0,1333} = 0,4$$

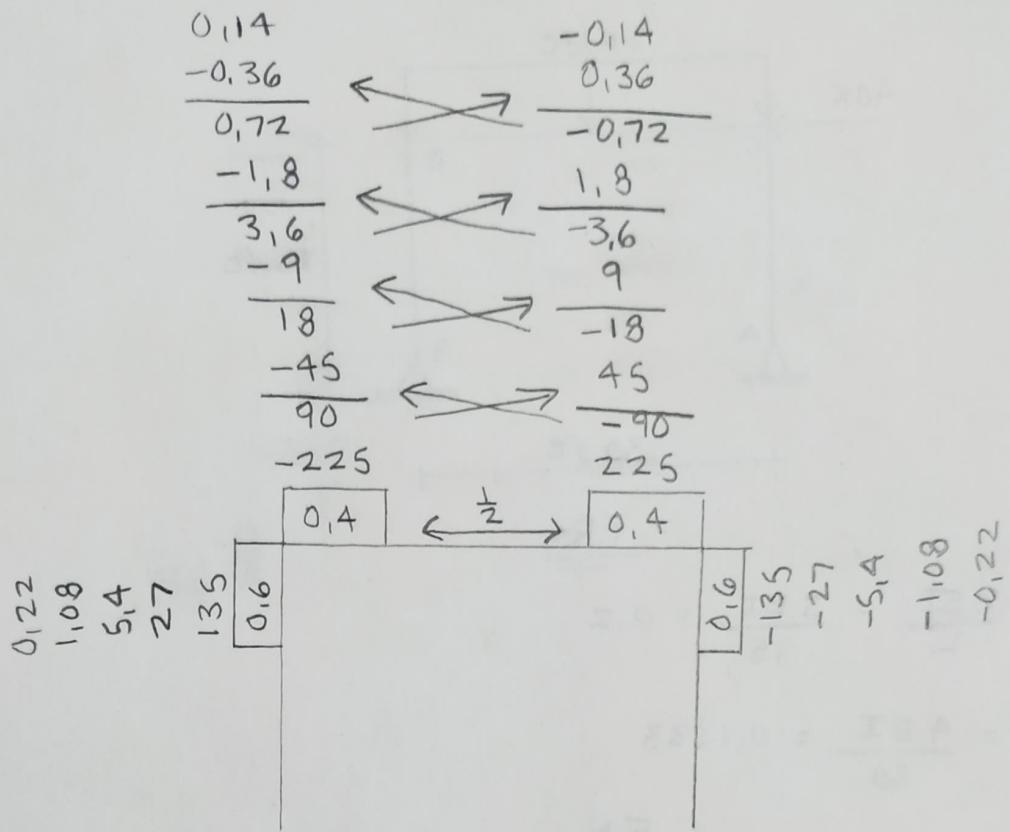
$$D_{DC} = \frac{R_{CD}}{R_{CD} + R_{BD}} = \frac{0,1333}{0,1333 + 0,2} = 0,4$$

$$D_{DB} = \frac{R_{BD}}{R_{CD} + R_{BD}} = \frac{0,2}{0,1333 + 0,2} = 0,6$$

FEM

$$CD = -\frac{WL^2}{12} = -\frac{3(30)^2}{12} = -225$$

$$DC = 225$$



$$M_{AC} = 0$$

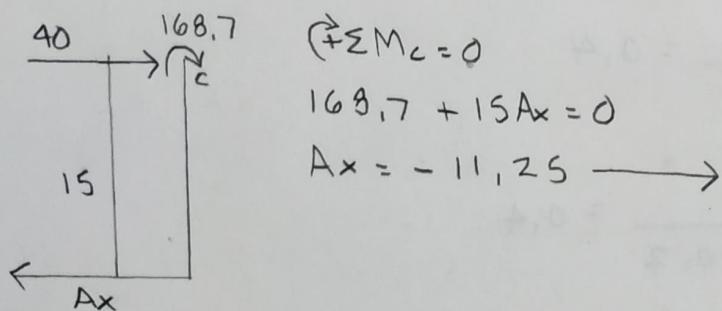
$$M_{CD} = -168.7$$

$$M_{DB} = -168.7$$

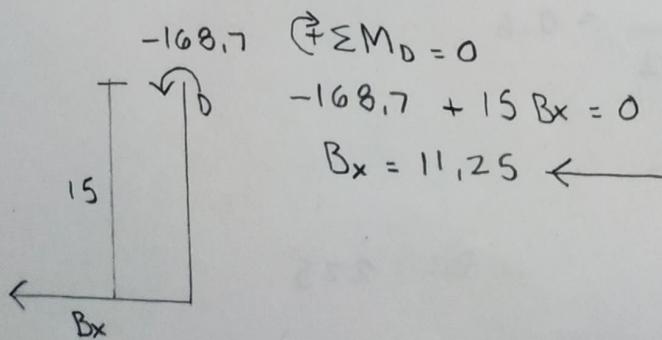
$$M_{CA} = 168.7$$

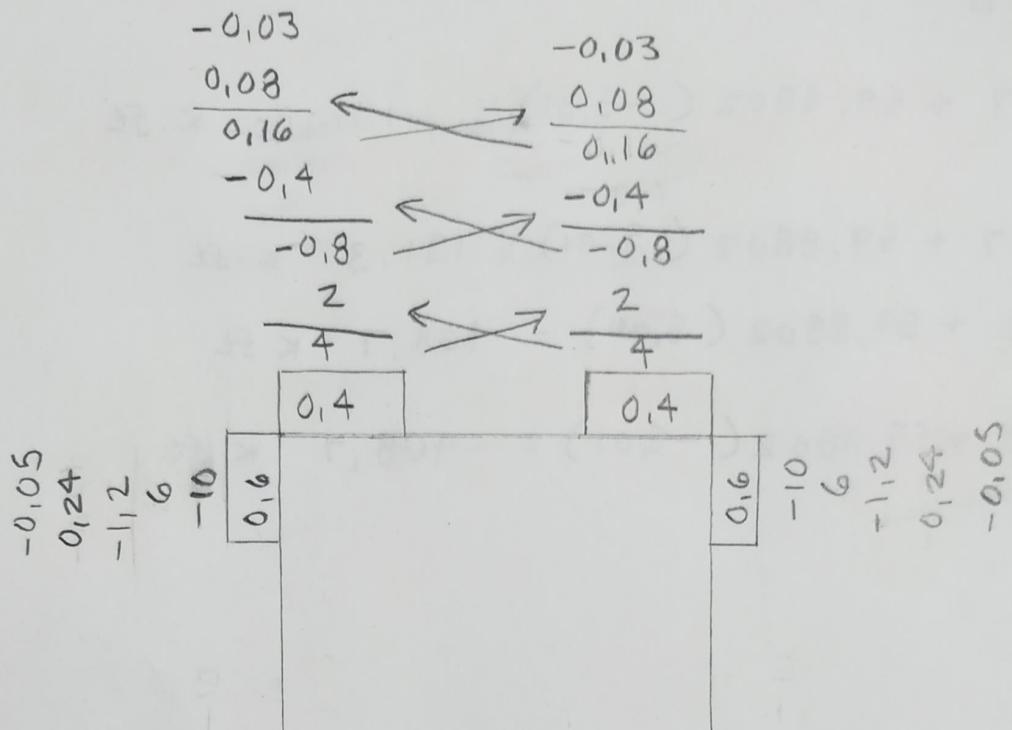
$$M_{DC} = 168.7$$

$$M_{BD} = 0$$



$$R_1 = 40 + 11.25 - 11.25 = 40$$





$$M_{AC} = 0$$

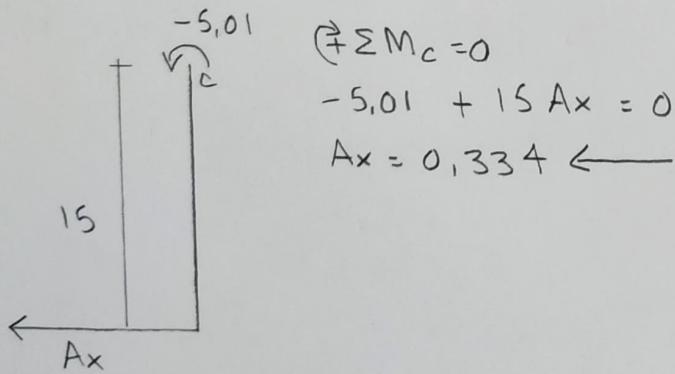
$$M_{CD} = 5,01$$

$$M_{DB} = -5,01$$

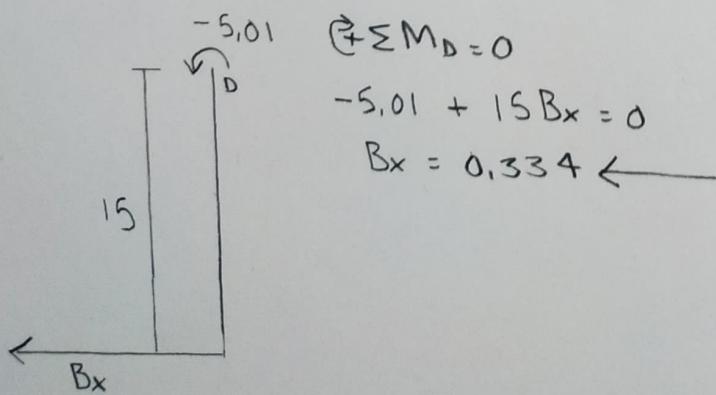
$$M_{CA} = -5,01$$

$$M_{DC} = 5,01$$

$$M_{BD} = 0$$



$$R_2 = 0,334 + 0,334 = 0,668$$



MOMENTOS FINALES

$$M_{Ac} = M_{Bd} = 0$$

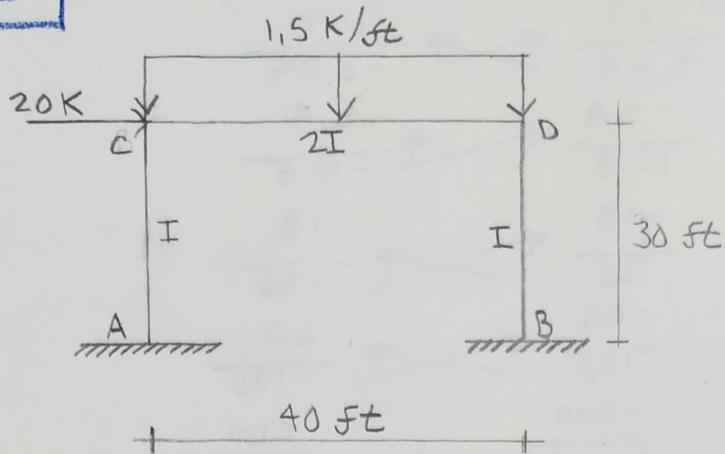
$$M_{cA} = 168,7 + 59,8802 (-5,01) = -131,3 \text{ k.ft}$$

$$M_{cd} = -168,7 + 59,8802 (5,01) = 131,3 \text{ k.ft}$$

$$M_{dc} = 168,7 + 59,8802 (5,01) = 468,7 \text{ k.ft}$$

$$M_{dB} = -168,7 + 59,8802 (-5,01) = -468,7 \text{ k.ft}$$

HARDY CROSS



F.R

$$R_{AC} = R_{BD} = \frac{4EI}{L} = \frac{4E(I)}{30} = 0,1333 EI$$

$$R_{CD} = \frac{4EI}{L} = \frac{4E(2I)}{40} = 0,2$$

F.D

$$D_{CA} = \frac{R_{AC}}{R_{AC} + R_{CD}} = \frac{0,1333}{0,1333 + 0,2} = 0,4$$

$$D_{CD} = \frac{R_{CD}}{R_{AC} + R_{CD}} = \frac{0,2}{0,1333 + 0,2} = 0,6$$

$$D_{DC} = \frac{R_{CD}}{R_{CD} + R_{BD}} = \frac{0,2}{0,2 + 0,1333} = 0,6$$

$$D_{DB} = \frac{R_{BD}}{R_{CD} + R_{BD}} = \frac{0,1333}{0,2 + 0,1333} = 0,4$$

FEM

$$CD = -\frac{WL^2}{12} = -\frac{1,5(40)^2}{12} = -200$$

$$DC = 200$$

$$\begin{array}{r}
 0,29 \\
 -0,48 \\
 \hline
 0,97 \\
 -1,62 \\
 \hline
 3,24 \\
 -5,4 \\
 \hline
 10,8 \\
 -18 \\
 \hline
 36 \\
 -60 \\
 \hline
 120 \\
 -200 \\
 \hline
 0,6 \quad | \quad 0,4 \\
 \hline
 \frac{1}{2} \quad | \quad \frac{1}{2} \\
 \hline
 0,4 \quad | \quad 0,4 \\
 \hline
 -80 \\
 \hline
 200 \\
 \hline
 -0,29 \\
 0,48 \\
 \hline
 -0,97 \\
 1,62 \\
 \hline
 -3,24 \\
 5,4 \\
 \hline
 -10,8 \\
 18 \\
 \hline
 -36 \\
 60 \\
 \hline
 -120 \\
 200 \\
 \hline
 -0,65 \\
 7,2 \\
 \hline
 24 \\
 80 \\
 \hline
 40 \\
 \hline
 0,19
 \end{array}$$

$$M_{AC} = 57$$

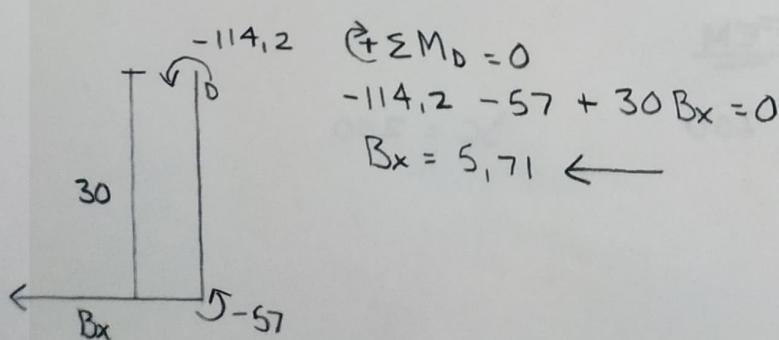
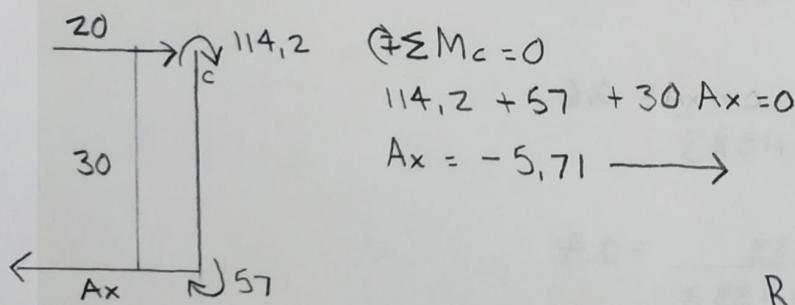
$$M_{CA} = 114,2$$

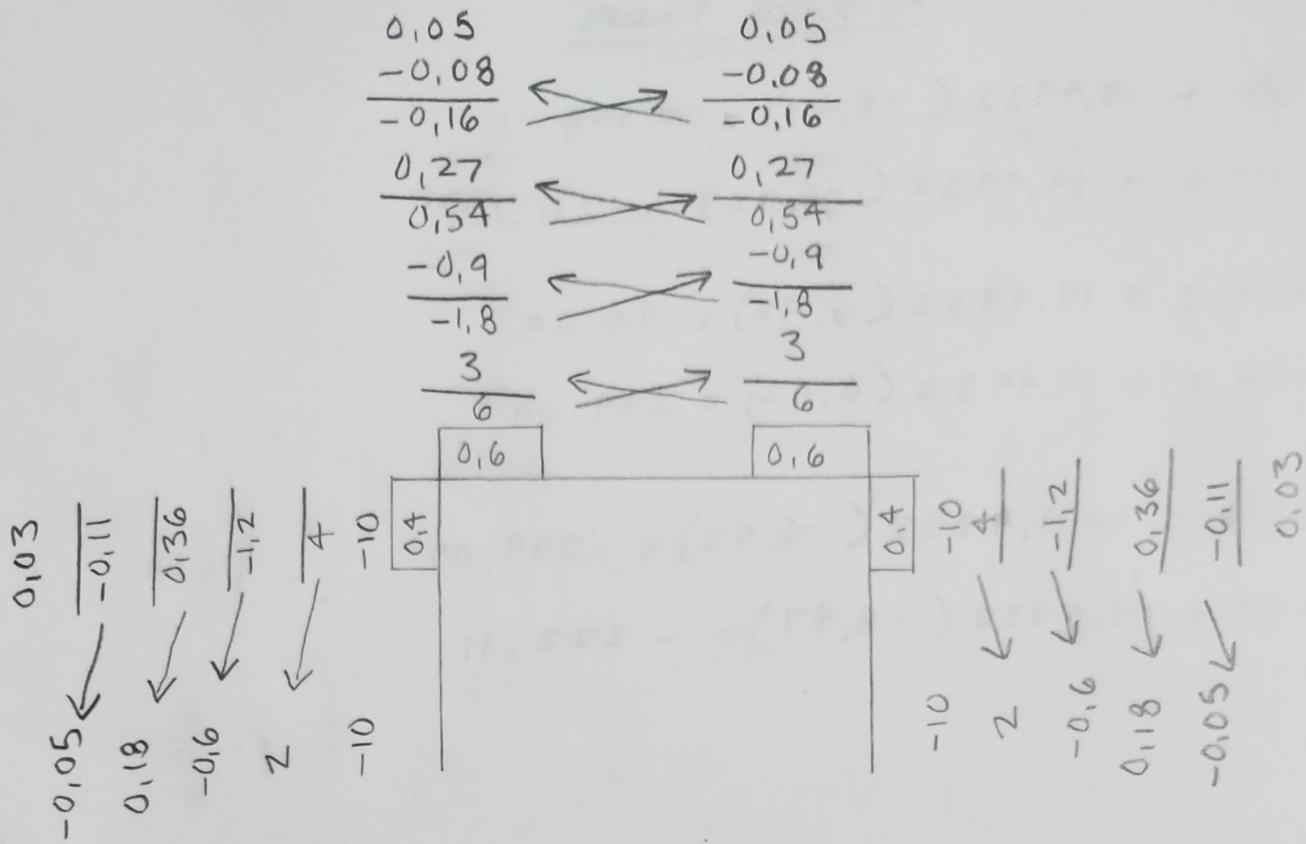
$$M_{CD} = -114,2$$

$$M_{DC} = 114,2$$

$$M_{DB} = -114,2$$

$$M_{BD} = -57$$





$$M_{AC} = -8,47$$

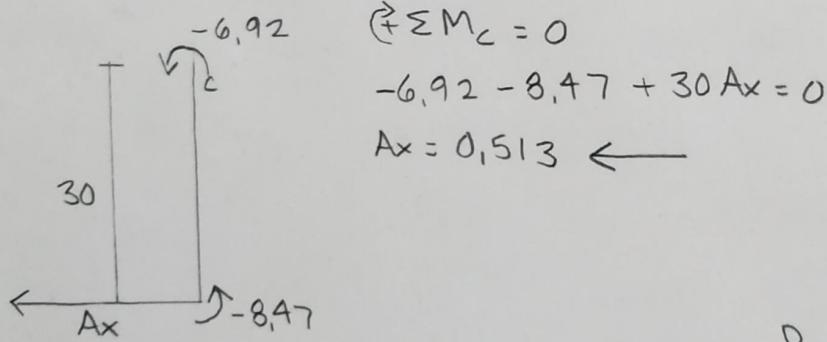
$$M_{CD} = 6,92$$

$$M_{DB} = -6,92$$

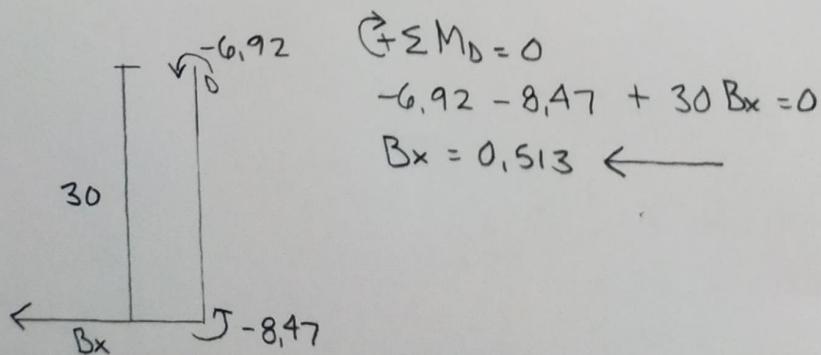
$$M_{CA} = -6,92$$

$$M_{DC} = 6,92$$

$$M_{BD} = -8,47$$



$$R_2 = 0,513 + 0,513 = 1,026$$



MOMENTOS FINALES

$$M_{AC} = 57 + 19,4932 (-8,47) = -108,11$$

$$M_{CA} = 114,2 + 19,4932 (-6,92) = -20,69$$

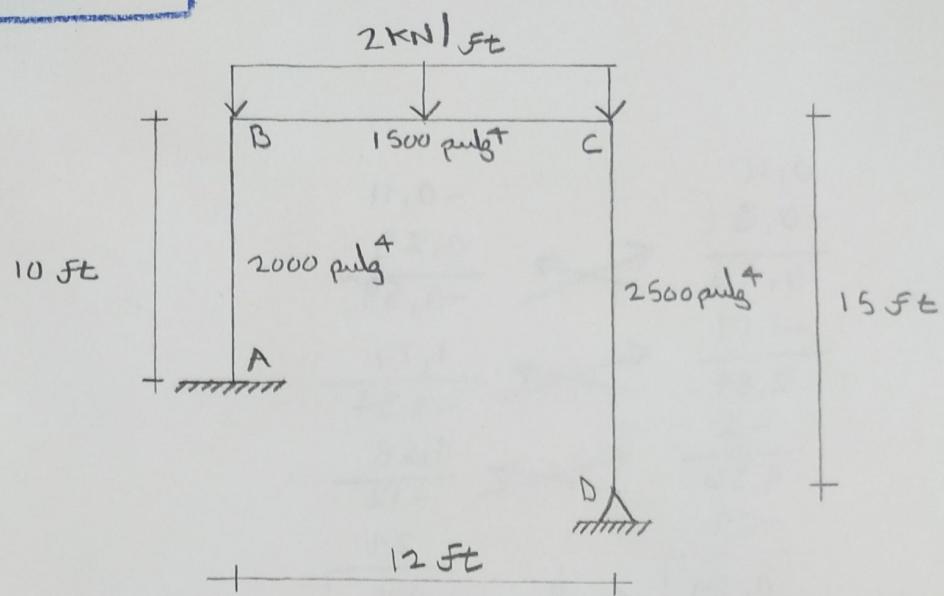
$$M_{CD} = -114,2 + 19,4932 (6,92) = 20,69$$

$$M_{DC} = 114,2 + 19,4932 (6,92) = 249,09$$

$$M_{DB} = -114,2 + 19,4932 (-6,92) = -249,09$$

$$M_{BD} = -57 + 19,4932 (-8,47) = -222,11$$

HARDY CROSS



F.R

$$R_{AB} = \frac{4EI}{L} = \frac{4E(1,3333)}{10} = 0,5333 E$$

$$R_{BC} = \frac{4EI}{L} = \frac{4E(1)}{12} = 0,3333 E$$

$$R_{CD} = \frac{3EI}{L} = \frac{3E(1,6667)}{15} = 0,3333 E$$

F.D

$$D_{BA} = \frac{R_{AB}}{R_{AB} + R_{BC}} = \frac{0,5333}{0,5333 + 0,3333} = 0,61$$

$$D_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} = \frac{0,3333}{0,5333 + 0,3333} = 0,39$$

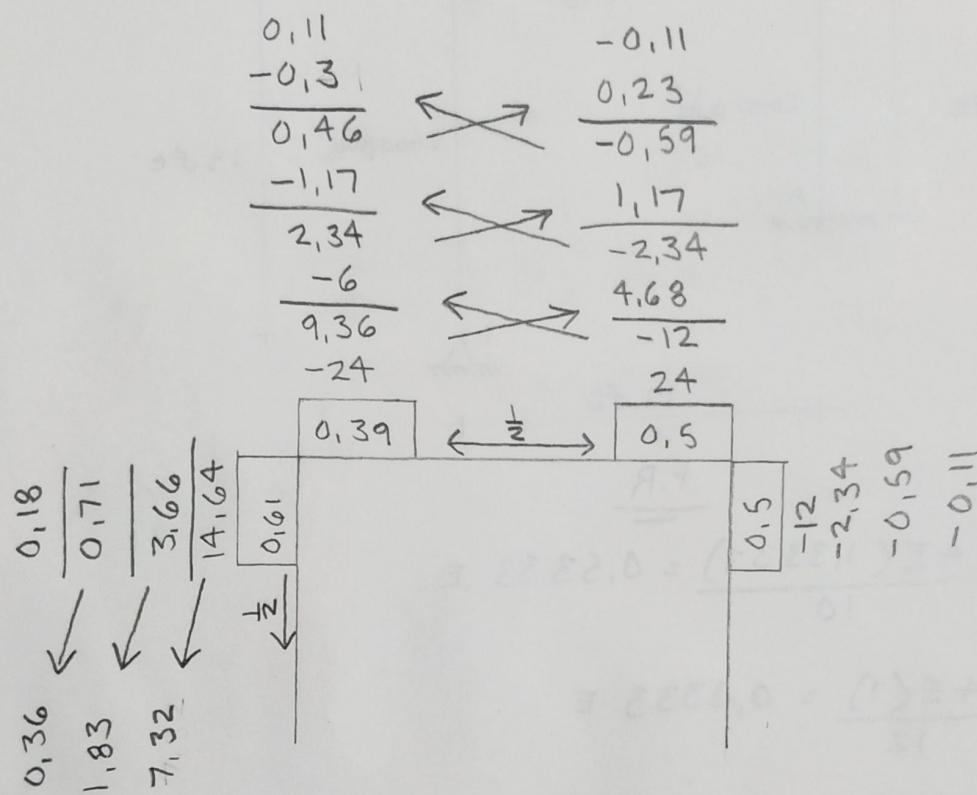
$$D_{CB} = \frac{R_{BC}}{R_{BC} + R_{CD}} = \frac{0,3333}{0,3333 + 0,3333} = 0,5$$

$$D_{CD} = \frac{R_{CD}}{R_{BC} + R_{CD}} = \frac{0,3333}{0,3333 + 0,3333} = 0,5$$

$$BC = \frac{-WL^2}{12} = \frac{-2(12)^2}{12} = -24$$

$$CB = 24$$

FEM



$$M_{AB} = 9.51$$

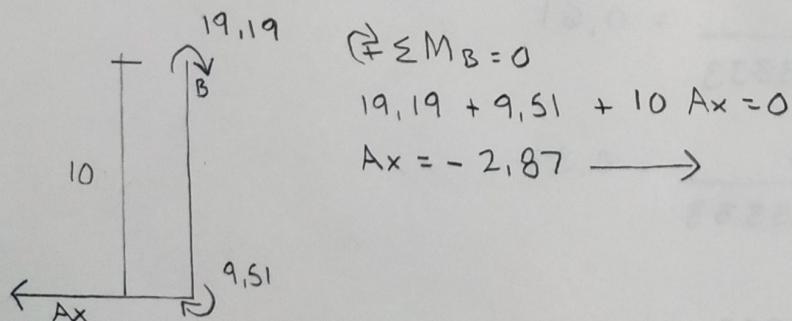
$$M_{BC} = -19.2$$

$$M_{CD} = -15.04$$

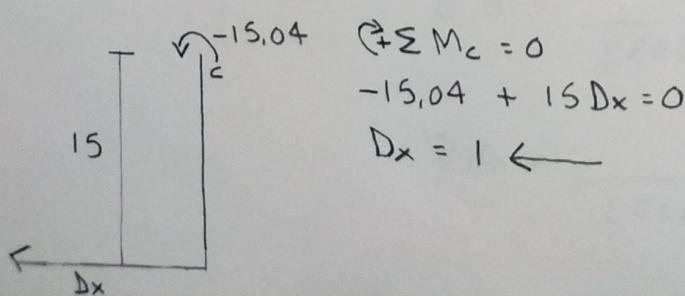
$$M_{BA} = 19.19$$

$$M_{CB} = 15.04$$

$$M_{DC} = 0$$

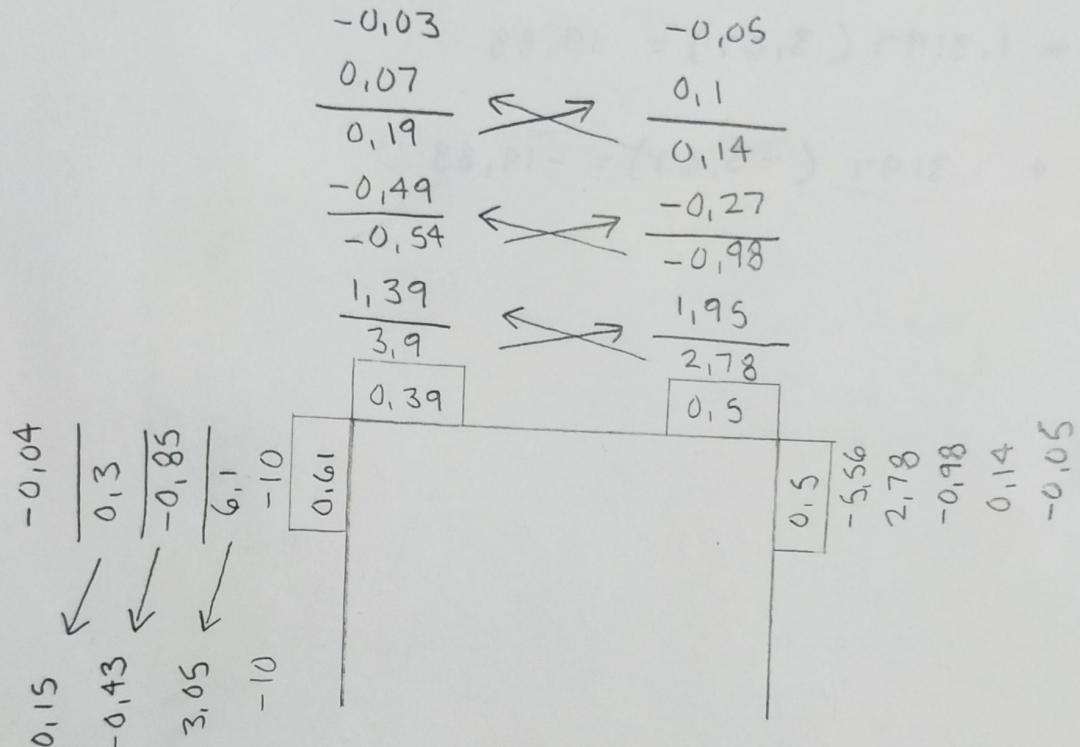


$$R_1 = 2.87 - 1 = 1.87$$



$$M_{AB} = M_{BA} = -10 \quad \Delta = \frac{-10 (10)^2}{6E(1,3333)} = \frac{-125}{E}$$

$$M = \frac{6EI\Delta}{L^2} = \frac{6(1,6667)(-125)}{15^2} = -5,56$$



$$M_{AB} = -7,23$$

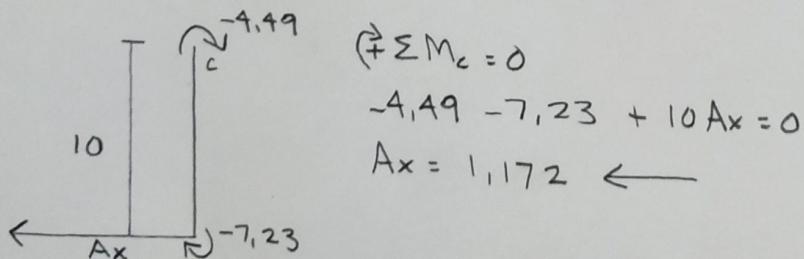
$$M_{BA} = -4,49$$

$$M_{BC} = 4,49$$

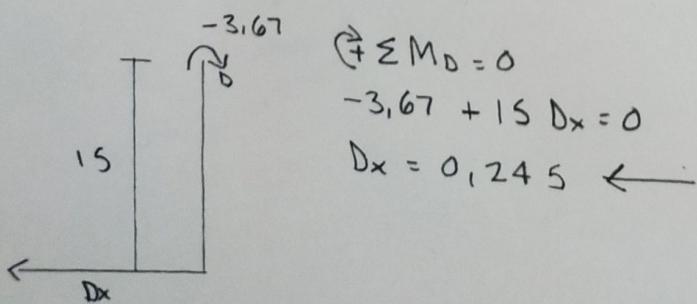
$$M_{CB} = 3,67$$

$$M_{CD} = -3,67$$

$$M_{DC} = 0$$



$$R_2 = 1,172 + 0,245 = 1,417$$



MOMENTOS FINALES

$$M_{AB} = 9,51 + 1,3197 (-7,23) = -0,1$$

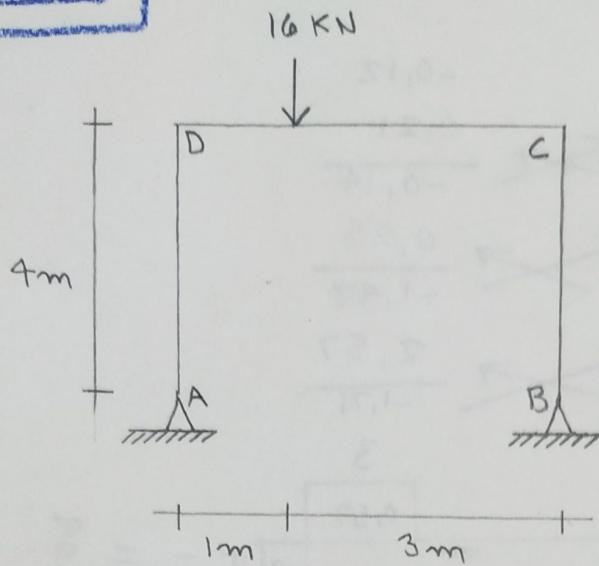
$$M_{BA} = 19,19 + 1,3197 (-4,49) = 13,26$$

$$M_{BC} = -19,2 + 1,3197 (4,49) = -13,26$$

$$M_{CB} = 15,04 + 1,3197 (3,67) = 19,88$$

$$M_{CD} = -15,04 + 1,3197 (-3,67) = -19,88$$

HARDY CROSS



$$R_{AD} = R_{BC} = \frac{3EI}{L} = \frac{3EI}{4} = 0,75$$

$$\underline{\underline{F.R}}$$

$$R_{DC} = \frac{4EI}{L} = \frac{4EI}{4} = 1$$

F.D

$$D_{DA} = \frac{R_{AD}}{R_{AD} + R_{DC}} = \frac{0,75}{0,75 + 1} = 0,43$$

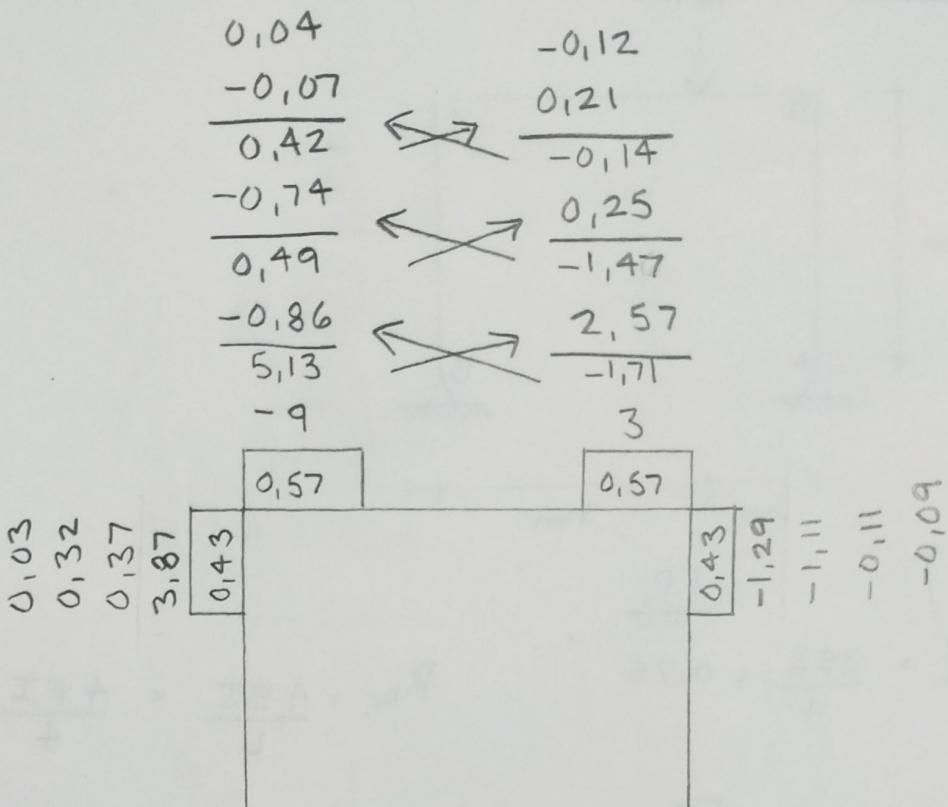
$$D_{DC} = \frac{R_{DC}}{R_{AD} + R_{DC}} = \frac{1}{0,75 + 1} = 0,57$$

$$D_{CD} = \frac{R_{DC}}{R_{DC} + R_{BC}} = \frac{1}{1 + 0,75} = 0,57$$

$$D_{CB} = \frac{R_{CB}}{R_{CD} + R_{CB}} = \frac{0,75}{1 + 0,75} = 0,43$$

$$D_C = -\frac{Pb^2a}{L^2} = -\frac{16(3)^2(1)}{4^2} = -9$$

$$CD = \frac{Pa^2b}{L^2} = \frac{16(1)^2(3)}{4^2} = 3$$



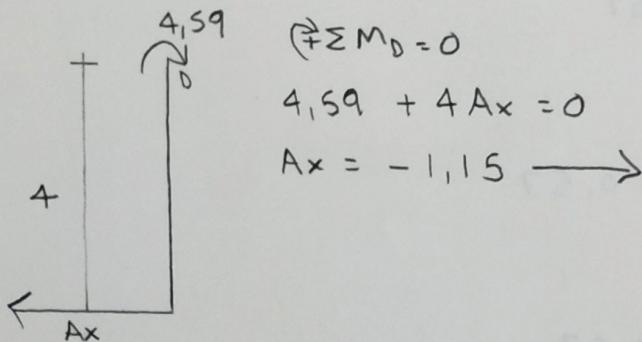
$$M_{AD} = M_{BC} = 0$$

$$M_{DC} = -4,59$$

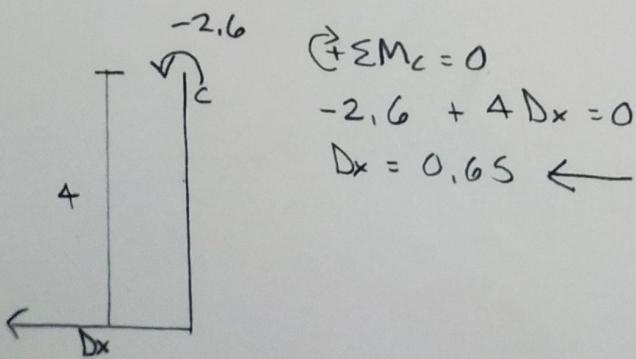
$$M_{CB} = -2,6$$

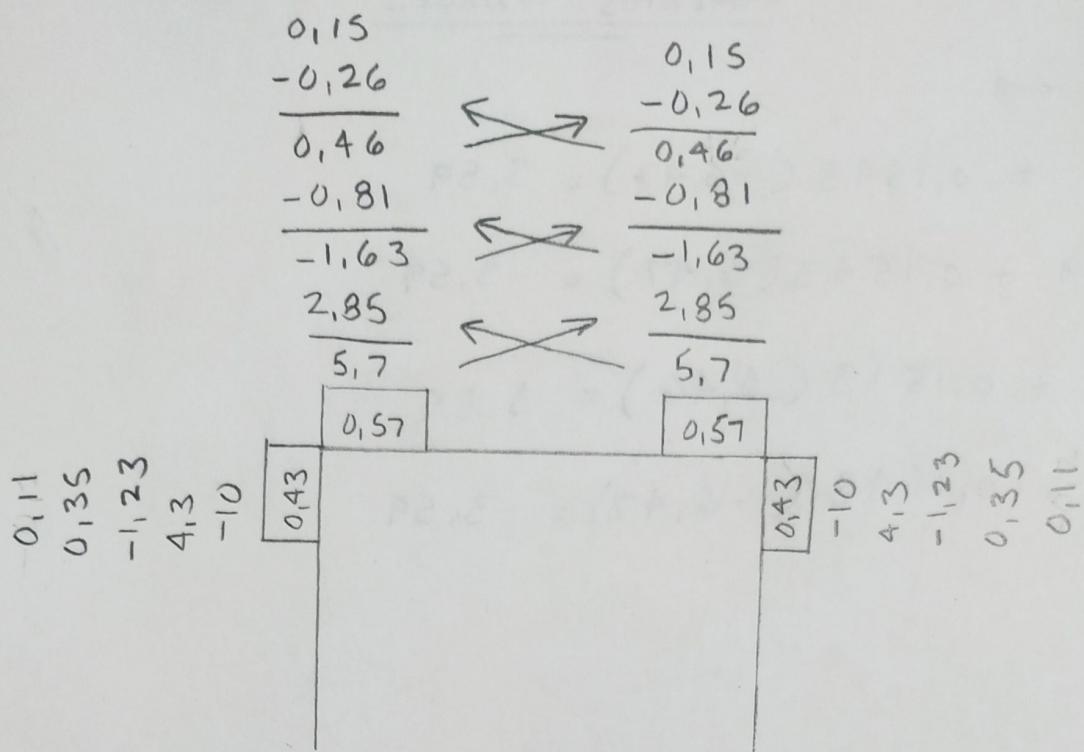
$$M_{DA} = 4,59$$

$$M_{CD} = 2,59$$



$$R_1 = 1,15 - 0,65 = 0,5$$





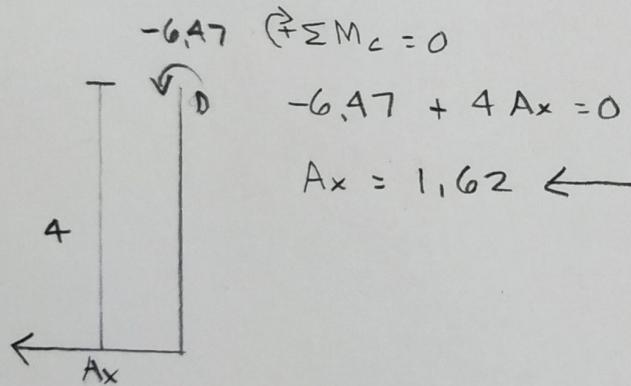
$$M_{AD} = M_{BC} = 0$$

$$M_{DC} = 6.46$$

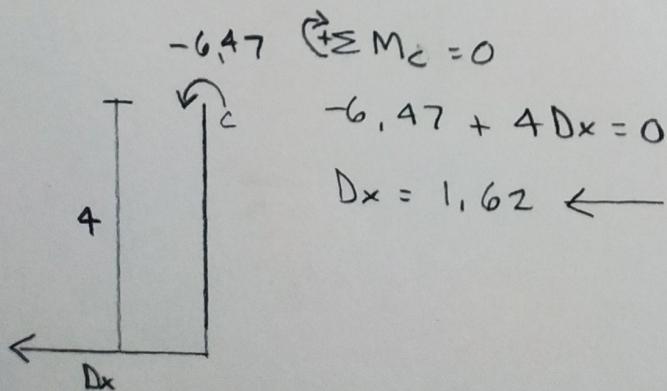
$$M_{CB} = -6.47$$

$$M_{DA} = -6.47$$

$$M_{CD} = 6.46$$



$$R_2 = -1.62 - 1.62 = 3.24$$



MOMENTOS FINALES

$$M_{AD} = M_{BC} = 0$$

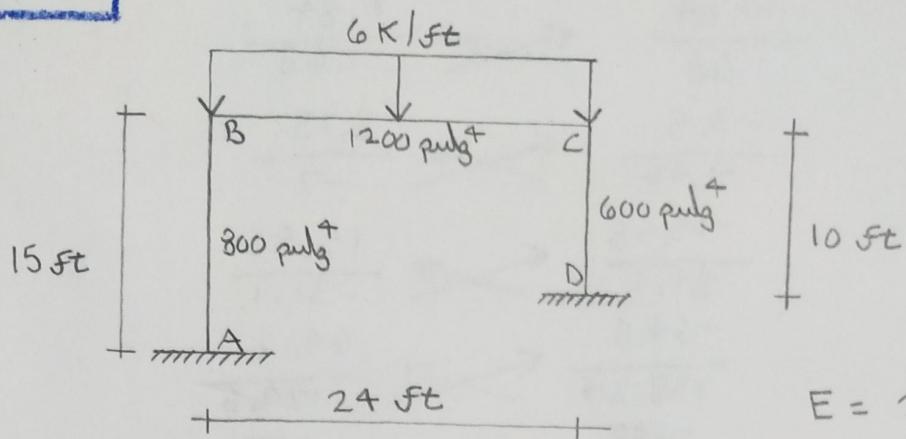
$$M_{DA} = 4,59 + 0,1543(-6,47) = 3,59$$

$$M_{DC} = -4,59 + 0,1543(6,47) = -3,59$$

$$M_{CD} = 2,59 + 0,1543(6,46) = 3,59$$

$$M_{CB} = -2,6 + 0,1543(-6,47) = -3,59$$

HARDY CROSS



$$E = 29(10^3) \text{ ksi}$$

F.R

$$R_{AB} = \frac{4EI}{L} = \frac{4E(1,333)}{15} = 0,3555$$

$$R_{BC} = \frac{4EI}{L} = \frac{4E(2)}{24} = 0,3333$$

$$R_{CD} = \frac{4EI}{L} = \frac{4E(1)}{10} = 0$$

F.D

$$D_{BA} = \frac{R_{AB}}{R_{AB} + R_{BC}} = \frac{0,3555}{0,3555 + 0,3333} = 0,52$$

$$D_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} = \frac{0,3333}{0,3555 + 0,3333} = 0,48$$

$$D_{CB} = \frac{R_{BC}}{R_{BC} + R_{CD}} = \frac{0,3333}{0,3333 + 0,4} = 0,45$$

$$D_{CD} = \frac{R_{CD}}{R_{BC} + R_{CD}} = \frac{0,4}{0,3333 + 0,4} = 0,55$$

FEM

$$BC = -\frac{wL^2}{12} = -\frac{6(24)^2}{12} = -288$$

$$CB = 288$$

MARCO 1

$$\begin{array}{r}
 0,1 \\
 0,22 \\
 0,91 \\
 4,04 \\
 16,95
 \end{array}$$

$$\begin{array}{r}
 0,09 \\
 -0,19 \\
 0,4 \\
 -0,84 \\
 1,68 \\
 -3,5 \\
 7,46 \\
 -15,55 \\
 31,1 \\
 -64,8 \\
 138,24 \\
 -288
 \end{array}
 \begin{array}{c}
 \cancel{\swarrow} \quad \cancel{\searrow} \\
 \cancel{\swarrow} \quad \cancel{\searrow}
 \end{array}
 \begin{array}{r}
 -0,09 \\
 0,2 \\
 -0,38 \\
 0,84 \\
 -1,68 \\
 3,73 \\
 -7 \\
 15,55 \\
 -31,1 \\
 69,12 \\
 -129,6 \\
 288
 \end{array}$$

$$\begin{array}{c}
 0,44 \\
 1,82 \\
 8,09 \\
 33,7 \\
 149,76 \\
 0,48 \\
 0,52 \\
 0,55 \\
 -158,4 \\
 -38,02 \\
 -8,55 \\
 -2,05 \\
 -0,46 \\
 -0,11
 \end{array}$$

$$M_{AB} = 96,9$$

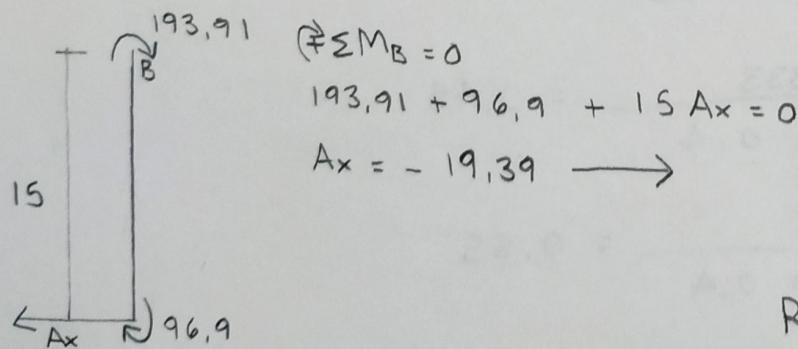
$$M_{BC} = -193,91$$

$$M_{CD} = -207,59$$

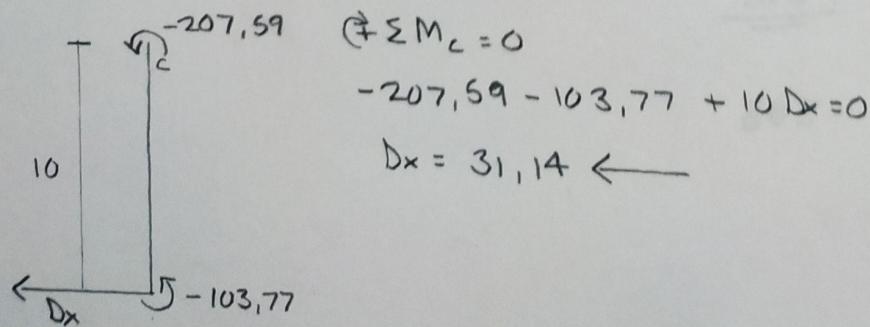
$$M_{BA} = 193,91$$

$$M_{CB} = 207,59$$

$$M_{DC} = -103,77$$



$$R_1 = 19,39 - 31,14 = 11,75$$

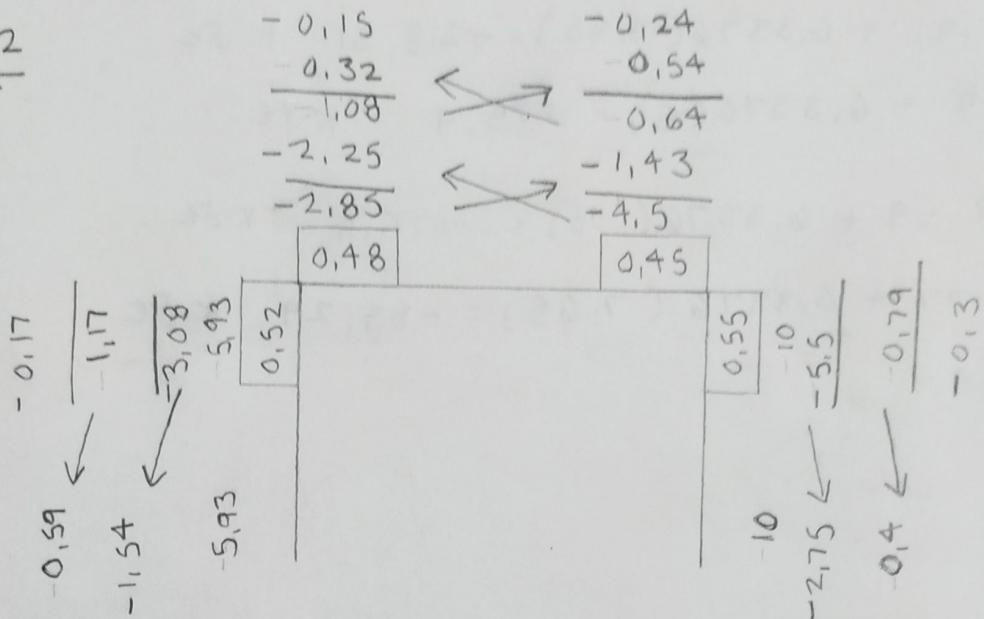


$$M_{CD} = M_{DC} = 10$$

$$\Delta = \frac{M L^2}{6EI} = \frac{10 (10)^2}{6E(1)} = \frac{166,67}{E}$$

$$M_{AB} = M_{BA} = \frac{6EI\Delta}{L^2} = \frac{6(1,3333)(166,67)}{15^2} = 5,93$$

MARCO 2



$$M_{AB} = 4,98$$

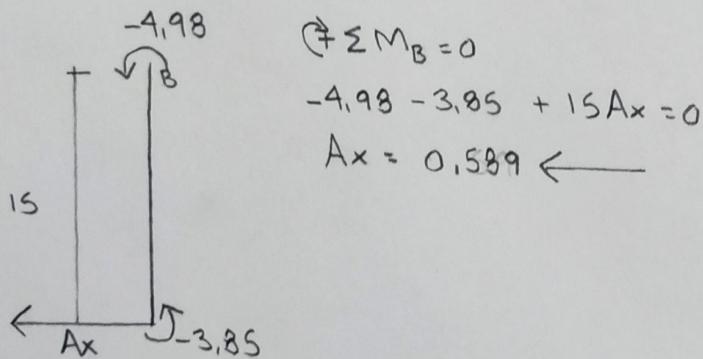
$$M_{BA} = -3,85$$

$$M_{CD} = 5$$

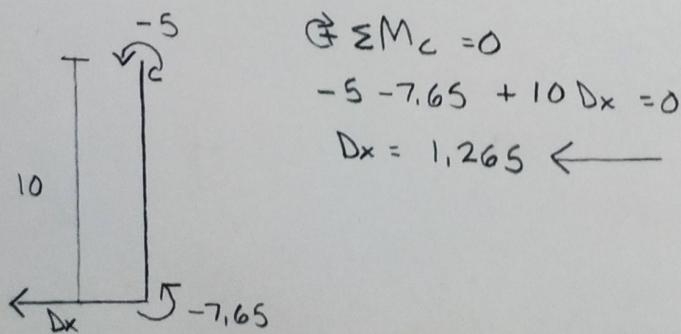
$$M_{BA} = 3,85$$

$$M_{CB} = -5$$

$$M_{DC} = -7,65$$



$$R_2 = -0,589 - 1,265 = 1,854$$



MOMENTOS FINALES

$$M_{AB} = 96,9 + 6,3376 (-4,98) = 128,46 \text{ K.ft}$$

$$M_{BA} = 193,91 + 6,3376 (-3,85) = 218,31 \text{ K.ft}$$

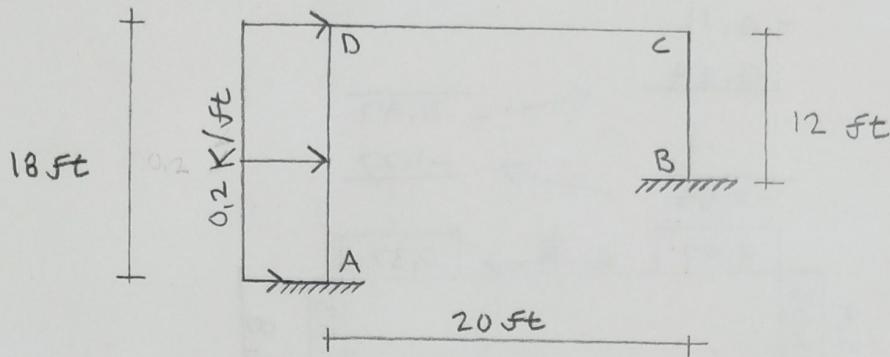
$$M_{BC} = -193,91 + 6,3376 (-3,85) = -218,31 \text{ K.ft}$$

$$M_{CB} = 207,59 + 6,3376 (-5) = 175,9 \text{ K.ft}$$

$$M_{CD} = -207,59 + 6,3376 (-5) = -175,9 \text{ K.ft}$$

$$M_{DC} = -103,77 + 6,3376 (-7,65) = -55,29 \text{ K.ft}$$

HARDY CROSS



F.R

$$R_{AD} = \frac{4EI}{L} = \frac{4EI}{18} = 0,2222 EI$$

$$R_{CB} = \frac{4EI}{L} = \frac{4EI}{12} = 0,3333 EI$$

$$R_{DC} = \frac{4EI}{L} = \frac{4EI}{20} = 0,2 EI$$

F.D

$$D_{DA} = \frac{R_{AD}}{R_{AD} + R_{DC}} = \frac{0,2222}{0,2222 + 0,2} = 0,53$$

$$D_{DC} = \frac{R_{DC}}{R_{AD} + R_{DC}} = \frac{0,2}{0,2222 + 0,2} = 0,47$$

$$D_{CD} = \frac{R_{DC}}{R_{DC} + R_{CB}} = \frac{0,2}{0,2 + 0,3333} = 0,37$$

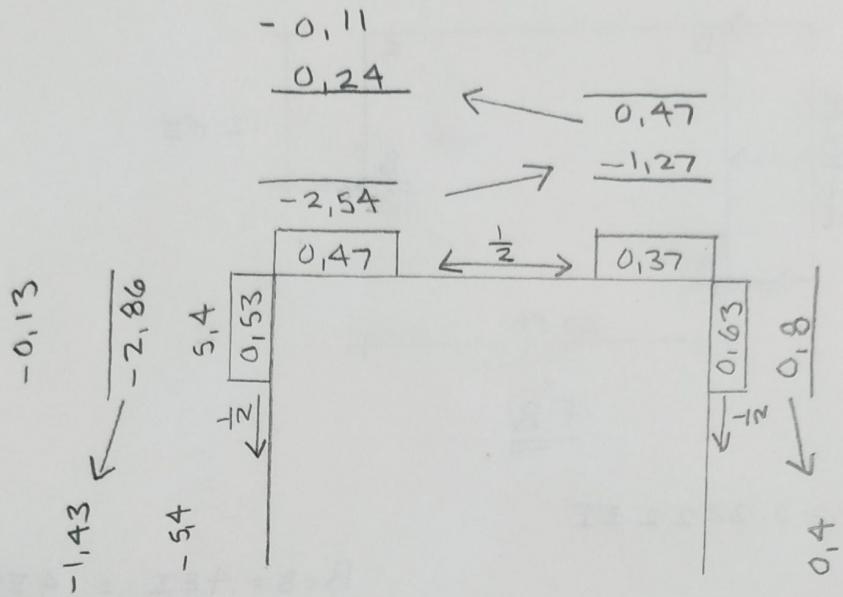
$$D_{CB} = \frac{R_{CB}}{R_{DC} + R_{CB}} = \frac{0,3333}{0,2 + 0,3333} = 0,63$$

FEM

$$AD = -\frac{WL^2}{12} = -\frac{0,2(18)^2}{12} = -5,4$$

$$DA = 5,4$$

MARCO 1



$$M_{AD} = -6.83 \text{ k-ft}$$

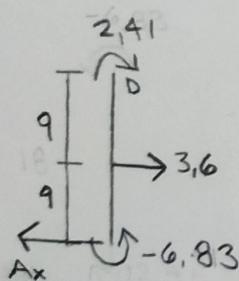
$$M_{DC} = -2.41 \text{ k-ft}$$

$$M_{CB} = 0.8 \text{ k-ft}$$

$$M_{DA} = 2.41 \text{ k-ft}$$

$$M_{CD} = -0.8 \text{ k-ft}$$

$$M_{BC} = 0.4 \text{ k-ft}$$

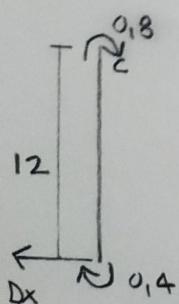


$$\textcircled{R} \sum M_D = 0$$

$$2.41 - 6.83 - 9(3.6) + 18A_x = 0$$

$$A_x = 2.04 \leftarrow$$

$$R_1 = 3.6 - 2.04 + 0.1 = 1.66$$



$$\textcircled{R} \sum M_c = 0$$

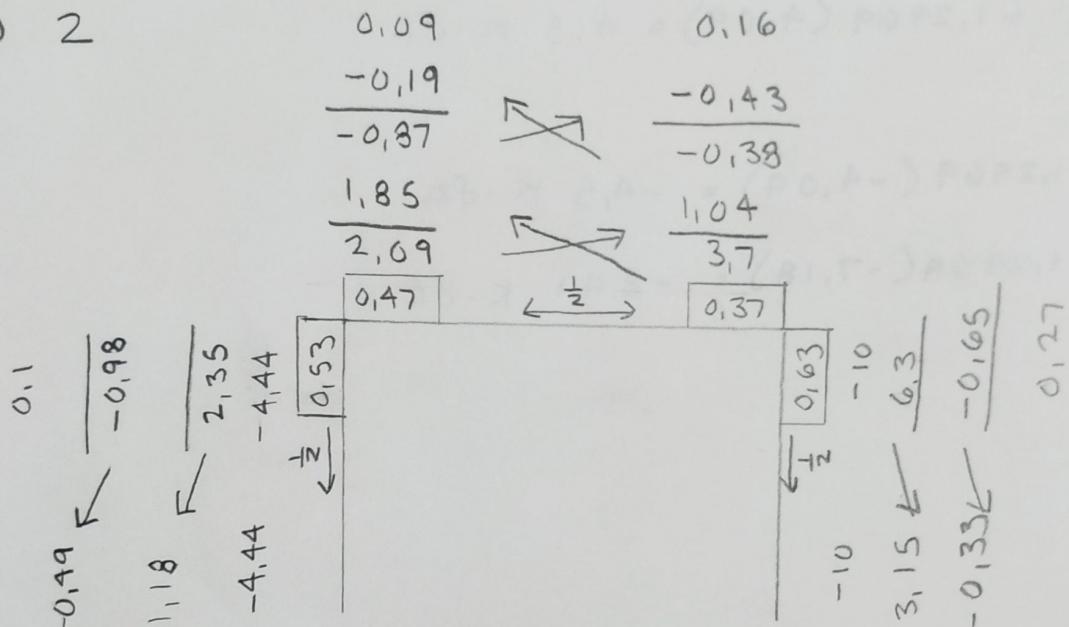
$$0.8 + 0.4 + 12D_x = 0$$

$$D_x = -0.1 \rightarrow$$

$$M_{CB} = M_{BC} = -10 \quad \Delta = \frac{M L^2}{6 EI} = \frac{-10(12)^2}{6 EI} = -\frac{240}{EI}$$

$$M_2 = \frac{6 EI \Delta}{L^2} = \frac{6(-240)}{18^2} = -4,44$$

MARCO 2



$$M_{AD} = -3,75$$

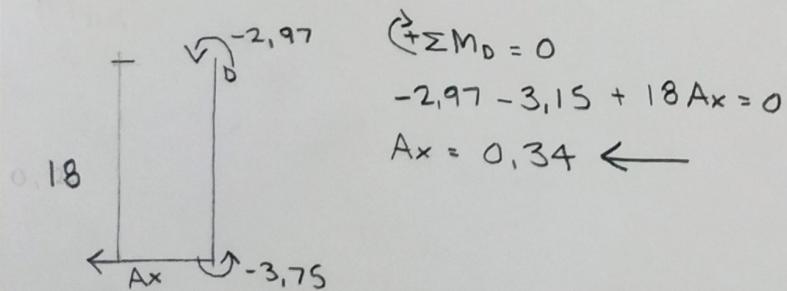
$$M_{DA} = -2,97$$

$$M_{DC} = 2,97$$

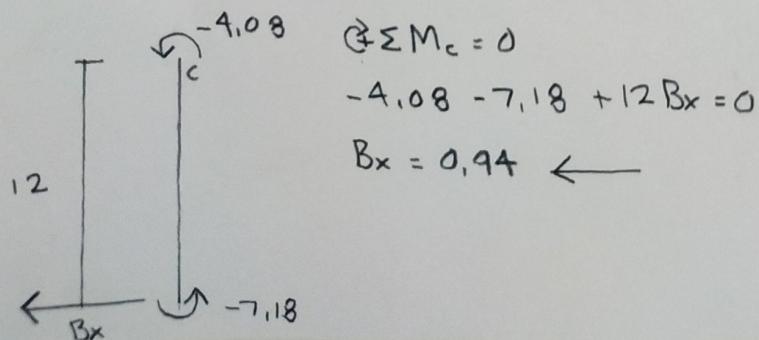
$$M_{CD} = 4,09$$

$$M_{CB} = -4,08$$

$$M_{BC} = -7,18$$



$$R_2 = 0.34 + 0.94 = 1.28$$



MOMENTOS FINALES

$$M_{AD} = -6,83 + 1,2969(-3,75) = -11,69 \text{ K.ft}$$

$$M_{DA} = 2,41 + 1,2969(-2,97) = -1,44 \text{ K.ft}$$

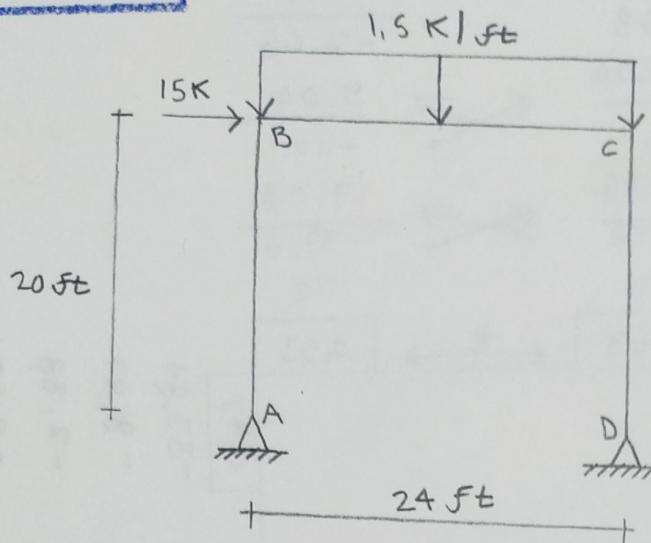
$$M_{DC} = -2,41 + 1,2969(2,97) = 1,44 \text{ K.ft}$$

$$M_{CD} = -0,8 + 1,2969(4,09) = 4,5 \text{ K.ft}$$

$$M_{CB} = 0,8 + 1,2969(-4,09) = -4,5 \text{ K.ft}$$

$$M_{BC} = 0,4 + 1,2969(-7,18) = -8,91 \text{ K.ft}$$

HARDY CROSS



$$R_{AB} = R_{CD} = \frac{3EI}{L} = \frac{3EI}{20} = 0,15 \quad \underline{\text{F.R.}}$$

$$R_{BC} = \frac{4EI}{L} = \frac{4EI}{24} = 0,1667$$

$$D_{BA} = \frac{R_{AB}}{R_{AB} + R_{BC}} = \frac{0,15}{0,15 + 0,1667} = 0,47 \quad \underline{\text{F.D.}}$$

$$D_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} = \frac{0,1667}{0,15 + 0,1667} = 0,53$$

$$D_{CB} = \frac{R_{BC}}{R_{BC} + R_{CD}} = \frac{0,1667}{0,1667 + 0,15} = 0,53$$

$$D_{CD} = \frac{R_{CD}}{R_{CB} + R_{CD}} = \frac{0,15}{0,1667 + 0,15} = 0,47$$

FEM

$$BC = -\frac{WL^2}{12} = -\frac{1,5(24)^2}{12} = -72$$

$$CB = 72$$

MARCO 1

$$\begin{array}{r}
 & \begin{array}{r} 0,18 \\ -0,35 \\ \hline 0,71 \end{array} & \begin{array}{r} -0,18 \\ -0,35 \\ \hline -0,71 \end{array} \\
 & \cancel{\begin{array}{r} -1,34 \\ \hline 2,68 \end{array}} & \begin{array}{r} 1,34 \\ \hline -2,68 \end{array} \\
 & \cancel{\begin{array}{r} -5,06 \\ \hline 10,11 \end{array}} & \cancel{\begin{array}{r} 5,06 \\ \hline -10,11 \end{array}} \\
 & \cancel{\begin{array}{r} -19,08 \\ \hline 38,16 \end{array}} & \cancel{\begin{array}{r} 19,08 \\ \hline -38,16 \end{array}} \\
 & -72 & 72 \\
 & \begin{array}{|c|c|} \hline & 0,53 & \xleftarrow{\frac{1}{2}} & 0,53 \\ \hline 0,47 & | & | & | \\ \hline \end{array} & \begin{array}{|c|c|} \hline & 0,47 & | & -33,84 \\ \hline & | & | & -8,97 \\ \hline \end{array} \\
 & 0,16 & 0,63 & 2,38 & 8,97 & 33,84 & 0,47 & -2,38 & -0,63 & -0,16 \\
 & \downarrow & \downarrow
 \end{array}$$

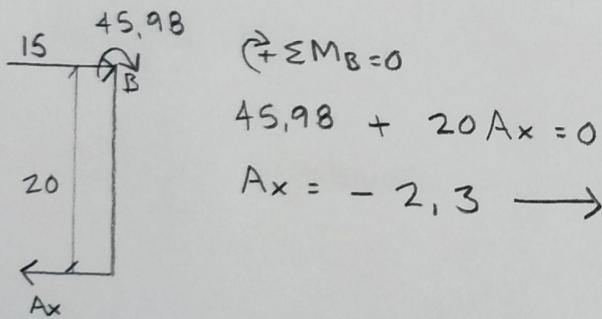
$$M_{AB} = M_{DC} = 0$$

$$M_{BC} = -45,99$$

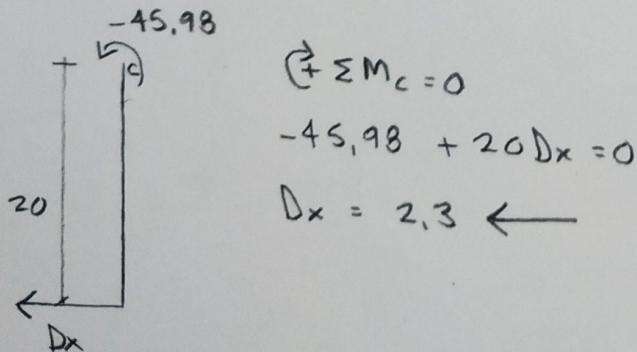
$$M_{CD} = -45,98$$

$$M_{BA} = 45,98$$

$$M_{CB} = 45,99$$



$$R_1 = 2,3 + 15 - 2,3 = 15$$



MARCO 2

$$\begin{array}{r}
 -0,09 \\
 0,18 \\
 \hline
 0,37 \\
 -0,7 \\
 \hline
 -1,40 \\
 2,65 \\
 \hline
 5,3 \\
 \end{array}
 \quad
 \begin{array}{r}
 -0,09 \\
 0,18 \\
 \hline
 0,37 \\
 -0,7 \\
 \hline
 -1,40 \\
 2,65 \\
 \hline
 5,3 \\
 \end{array}$$

-0,09	0,33	1,25	1	4,7	10	0,47	0,53
0,18	0	1	4	-1	0		
0,37							
-0,7							
-1,40							
2,65							
5,3							

0,40	10	4,7	1,25	0,33	-0,09
0	1	4	-1	0	

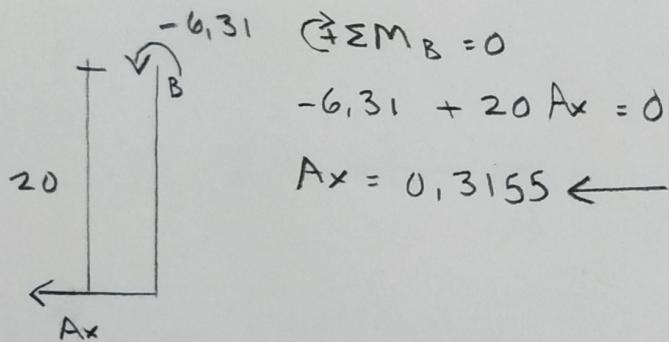
$$M_{AB} = M_{DC} = 0$$

$$M_{BC} = 6,31$$

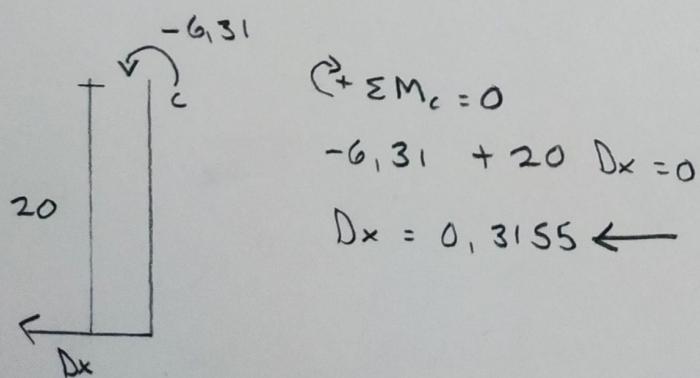
$$M_{CD} = -6,31$$

$$M_{BA} = -6,31$$

$$M_{CB} = 6,31$$



$$R_2 = 0,3155 + 0,3155 = 0,631$$



MOMENTOS FINALES

$$M_{AB} + M_{DC} = 0$$

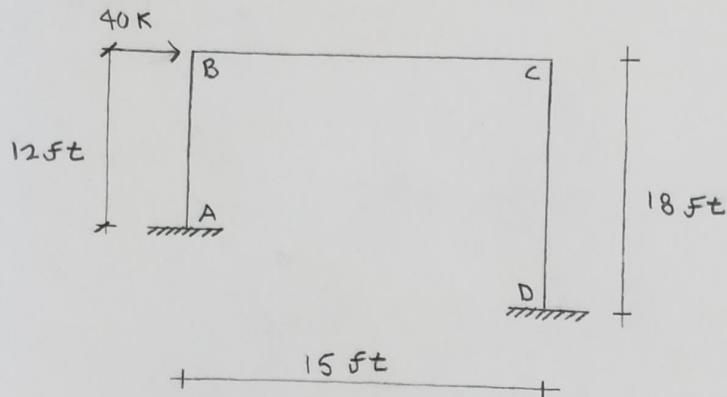
$$M_{BA} = 45,98 + 23,7718 (-6,31) = -104,02 \text{ K.ft}$$

$$M_{BC} = -45,98 + 23,7718 (6,31) = 104,02 \text{ K.ft}$$

$$M_{CB} = 45,99 + 23,7718 (6,31) = 196 \text{ K.ft}$$

$$M_{CD} = -45,98 + 23,7718 (-6,31) = -196 \text{ K.ft}$$

PENDIENTE DEFLEXIÓN



MOMENTOS

$$\Psi_{AB} = \frac{\Delta}{12} ; \quad \Psi_{DC} = \frac{\Delta}{18}$$

$$\Delta = \Delta \Rightarrow 12\Psi_{AB} = 18\Psi_{DC}$$

$$\Psi_{AB} = 1,5 \Psi_{DC}$$

$$M_{AB} = \frac{2EI}{12} (\theta_B - 3(1,5\Psi)) = -212 \text{ K.ft}$$

$$M_{BA} = \frac{2EI}{12} (2\theta_B - 3(1,5\Psi)) = -138 \text{ K.ft}$$

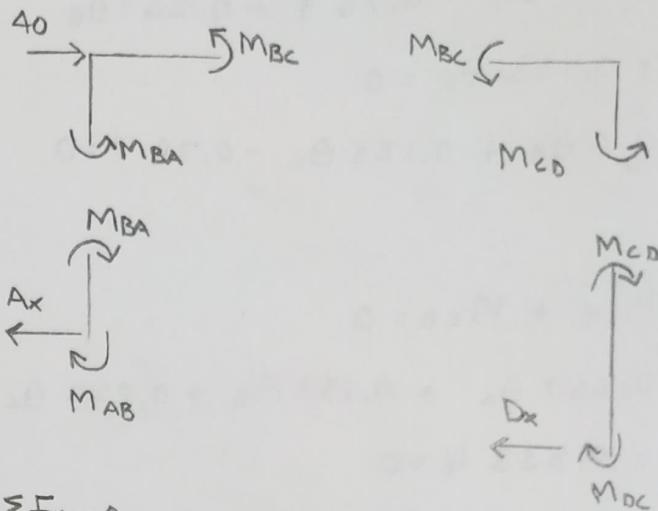
$$M_{BC} = \frac{2EI}{15} (2\theta_B + \theta_C) = 138 \text{ K.ft}$$

$$M_{CB} = \frac{2EI}{15} (2\theta_C + \theta_B) = 97 \text{ K.ft}$$

$$M_{CD} = \frac{2EI}{18} (2\theta_C - 3\Psi) = -97 \text{ K.ft}$$

$$M_{DC} = \frac{2EI}{18} (\theta_C - 3\Psi) = -112 \text{ K.ft}$$

DETERMINE LOS MOMENTOS . EI = CONST



$$\sum F_x = 0$$

$$40 - Ax - Dx = 0$$

$$\sum M_B = 0$$

$$M_{BA} + M_{AB} + 12Ax = 0$$

$$Ax = \frac{-M_{AB} - M_{BA}}{12}$$

$$\sum M_c = 0$$

$$M_{CD} + M_{DC} + 18Dx = 0$$

$$Dx = \frac{-M_{CD} - M_{DC}}{18}$$

$$40 + \frac{M_{AB} + M_{BA}}{12} + \frac{M_{CD} + M_{DC}}{18} = 0$$

$$\frac{40}{EI} + 0,014\theta_B - 0,062\Psi + 0,028\theta_B$$

$$-0,062\Psi + 0,012\theta_C - 0,018\Psi$$

$$+ 0,006\theta_C - 0,018\Psi = 0$$

$$0,042\theta_B + 0,018\theta_C - 0,16\Psi = \frac{-40}{EI}$$

$$M_{BA} + M_{BC} = 0$$

$$0,333 \theta_B - 0,75 \psi + 0,267 \theta_B$$

$$+ 0,133 \theta_C = 0$$

$$0,6 \theta_B + 0,133 \theta_C - 0,75 \psi = 0$$

$$M_{CB} + M_{CD} = 0$$

$$0,267 \theta_C + 0,133 \theta_B + 0,222 \theta_C$$

$$- 0,333 \psi = 0$$

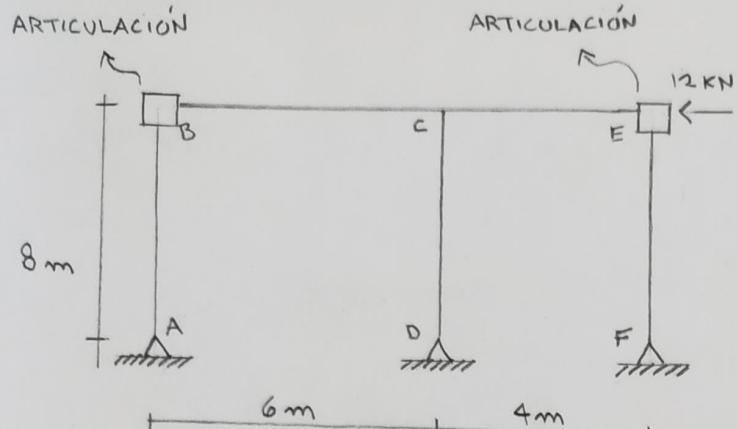
$$0,133 \theta_B + 0,489 \theta_C - 0,333 \psi = 0$$

$$\theta_B = \frac{448,34}{EI}$$

$$\theta_C = \frac{139,1}{EI}$$

$$\psi = \frac{383,34}{EI}$$

PENDIENTE DEFLEXIÓN



DETERMINE LOS MOMENTOS Y DIBUJE LOS DIAGRAMAS DE MOMENTOS PARA LA TRABE BCE, EI = CONSTANTE.

$$\begin{aligned} \sum M_C &= 0 \\ M_{CD} - 8D_x &= 0 \\ D_x &= \frac{M_{CD}}{8} \end{aligned}$$

$$-12 + \frac{M_{CD}}{8} = 0$$

$$-\frac{96}{EI} + 0,375\theta_c - 0,375\psi = 0$$

$$-0,375\theta_c - 0,375\psi = \frac{96}{EI} \quad (2)$$

$$\theta_c = -\frac{76,8}{EI} \quad \psi = -\frac{332,8}{EI}$$

$$M_{CB} + M_{CE} + M_{CD} = 0$$

$$0,5\theta_c + 0,75\theta_c + 0,375\theta_c =$$

$$-0,375\psi = 0$$

$$1,625\theta_c - 0,375\psi = 0 \quad (1)$$

$$\rightarrow \sum F_x = 0$$

$$-12 + D_x = 0$$

Ax y Fx SON IGUAL
A 0, PORQUE SUS
APOYOS NO PERMITEN
MOMENTO

$$M_{DA} + M_{DC} = 0$$

$$1,333\theta_D - 2\Psi + \frac{q}{EI} + 1,333\theta_C$$

$$+ 0,667\theta_C = 0$$

$$0,667\theta_C + 2,666\theta_D - 2\Psi = -\frac{q}{EI} \quad (2)$$

$$M_{CD} + M_{CB} = 0$$

$$1,333\theta_C + 0,667\theta_D + 1,333\theta_C$$

$$- 2\Psi = 0$$

$$2,666\theta_C + 0,667\theta_D - 2\Psi = 0 \quad (3)$$

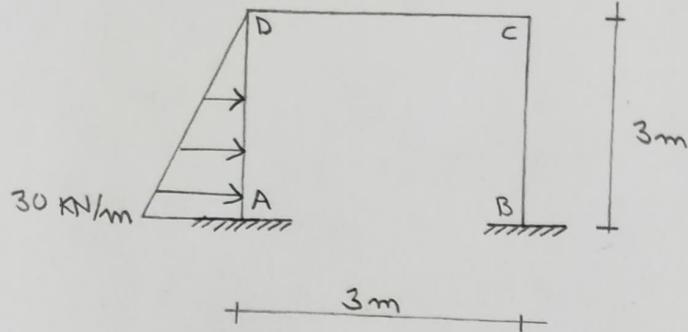
$$\theta_C = \frac{4,66}{EI}$$

$$\theta_D = \frac{0,16}{EI}$$

$$\Psi = \frac{6,27}{EI}$$

PENDIENTE DEFLEXIÓN

DETERMINE LOS MOMENTOS y DIBUJE LOS DIAGRAMAS DE MOMENTOS.



FEM

$$AD = -\frac{WL^2}{20} = -\frac{30(3)^2}{20} = -13,5$$

$$DA = \frac{WL^2}{30} = \frac{30(3)^2}{30} = 9$$

MOMENTOS

$$M_{AD} = \frac{2EI}{3} (\theta_D - 3\psi) - 13,5 = -25,93$$

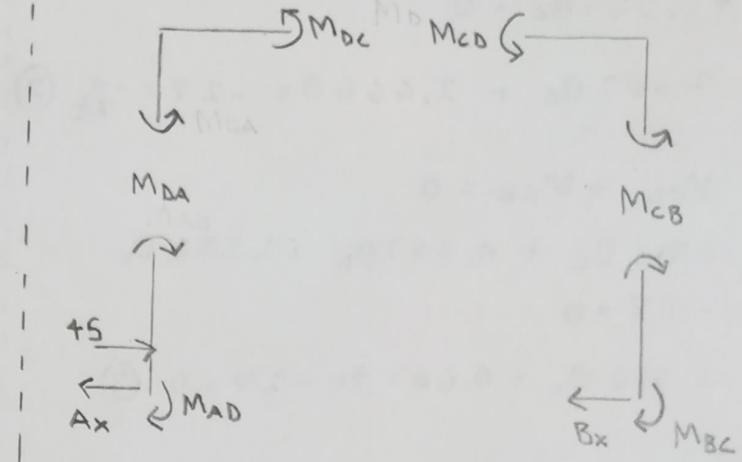
$$M_{DA} = \frac{2EI}{3} (2\theta_D - 3\psi) + 9 = -3,33$$

$$M_{DC} = \frac{2EI}{3} (2\theta_C + \theta_D) = 3,32$$

$$M_{CD} = \frac{2EI}{3} (2\theta_C + \theta_D) = 6,32$$

$$M_{CB} = \frac{2EI}{3} (2\theta_C - 3\psi) = -6,32$$

$$M_{BC} = \frac{2EI}{3} (\theta_C - 3\psi) = -9,43$$



$$\rightarrow \sum F_x = 0$$

$$45 - Ax - Bx = 0$$

$$\text{↶ } \sum M_D = 0$$

$$M_{DA} + M_{AD} - 45(2) + 3Ax = 0$$

$$Ax = 30 - \frac{M_{AD} - M_{DA}}{3}$$

$$\text{↶ } \sum M_C = 0$$

$$M_{CB} + M_{BC} + 3Bx = 0$$

$$Bx = -\frac{M_{BC} - M_{CB}}{3}$$

$$45 - 30 + \frac{M_{AD} + M_{DA}}{3} + \frac{M_{BC} + M_{CB}}{3} = 0$$

$$\frac{45}{EI} + 0,667\theta_D - 2\psi - \frac{13,5}{EI} + 1,333\theta_D$$

$$- 2\psi + \frac{9}{EI} + 1,333\theta_C - 2\psi$$

$$+ 0,667\theta_C - 2\psi = 0$$

$$2\theta_C + 2\theta_D - 8\psi = -\frac{40,5}{EI} \quad ①$$

$$\vec{\oplus} \sum M_0 = 0 \quad -0,701\theta_c - 0,877\theta_D + 0,181\Delta$$

$$M_{AC} + M_{BD} + 42,67 S_{BD} + 53,33 S_{AC} = 800,1 \quad (3)$$

$$-30(26,67) = 0 \quad \theta_c = 67,5$$

$$M_{AC} + M_{BD} + 42,67 S_{BD} + 53,33 S_{AC} = 800,1 \quad \theta_D = 127,5$$

$$\Delta = 5299,6$$

$$\vec{\oplus} \sum M_C = 0$$

$$M_{CA} + M_{AC} + 20 S_{AC} = 0$$

$$S_{AC} = \frac{-M_{CA} - M_{AC}}{20}$$

$$\vec{\oplus} \sum M_D = 0$$

$$M_{BD} + M_{DB} + 16 S_{BD} = 0$$

$$S_{BD} = \frac{-M_{BD} - M_{DB}}{16}$$

$$M_{AC} + M_{BD} + 2,67 M_{BD} - 2,67 M_{DB}$$

$$- 2,67 M_{CA} - 2,67 M_{AC} = 800,1$$

$$- 2,67 M_{AC} - 2,67 M_{CA} - 1,67 M_{BD}$$

$$- 2,67 M_{DB} = 800,1$$

$$- 1,67(0,1\theta_c - 0,0188\Delta) - 2,67(0,2\theta_c$$

$$- 0,0188\Delta) - 1,67(0,125\theta_D - 0,023\Delta)$$

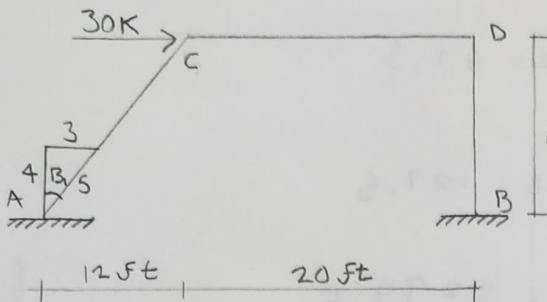
$$- 2,67(0,25\theta_D - 0,0234\Delta) = 800,1$$

$$- 0,167\theta_c + 0,031\Delta - 0,534\theta_c$$

$$+ 0,05\Delta - 0,209\theta_D + 0,038\Delta$$

$$- 0,668\theta_D + 0,062\Delta = 800,1$$

PENDIENTE DEFLEXIÓN



$EI = \text{CONSTANTE}$

$$\Psi_{AC} = \frac{CC'}{L} = \frac{5/4\Delta}{20} = 0,0625$$

$$\Psi_{CD} = -\frac{D_1 D}{L} = \frac{\Delta(3/4)}{20} = -0,0375$$

$$\Psi_{BD} = \frac{DD'}{L} = \frac{\Delta}{16} = 0,0625$$

MOMENTOS

$$M_{AC} = 0,1EI(\theta_C - 0,1875\Delta) =$$

$$M_{CA} = 0,1EI(2\theta_C - 0,1875\Delta) =$$

$$M_{CD} = 0,1EI(2\theta_C + \theta_D + 0,1125\Delta) =$$

$$M_{DC} = 0,1EI(2\theta_D + \theta_C + 0,1125\Delta) =$$

$$M_{DB} = 0,125EI(2\theta_D - 0,1875\Delta) =$$

$$M_{BD} = 0,125EI(\theta_D - 0,1875\Delta) =$$

DETERMINE LOS MOMENTOS EN LOS EXTREMOS DEL ELEMENTO Y LAS REACCIONES DEL MARCO.

ECUACIONES

$$M_{CA} + M_{CD} = 0$$

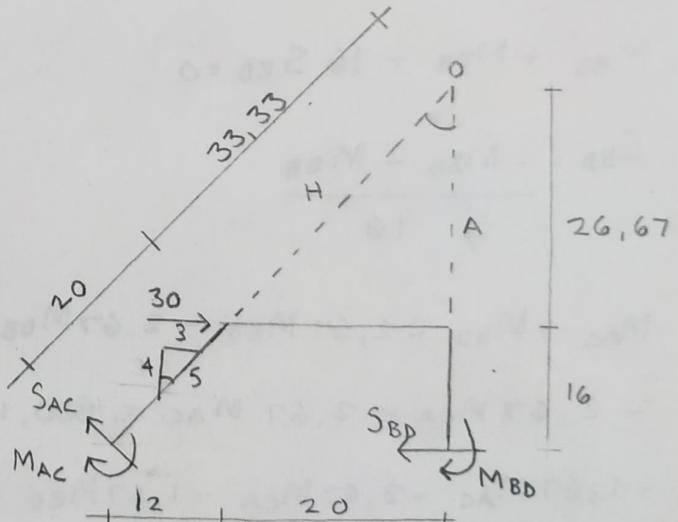
$$0,2\theta_C - 0,0188\Delta + 0,2\theta_C + 0,1\theta_D + 0,0113\Delta = 0$$

$$0,4\theta_C + 0,1\theta_D - 0,0075\Delta = 0 \quad (1)$$

$$M_{DC} + M_{DB} = 0$$

$$0,2\theta_D + 0,1\theta_C + 0,0113\Delta + 0,25\theta_D - 0,0234\Delta = 0$$

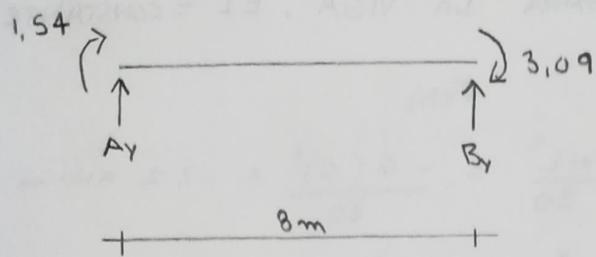
$$0,1\theta_C + 0,45\theta_D - 0,0121\Delta = 0 \quad (2)$$



$$A = \frac{\Delta}{\tan \theta} = \frac{20}{\frac{3}{4}} =$$

$$H = \frac{A}{\cos \theta} = \frac{20}{\frac{4}{5}} = 33,33$$

STRUCTURAL ANALYSIS OF BEAMS AND FRAMES



$$\sum M_A = 0$$

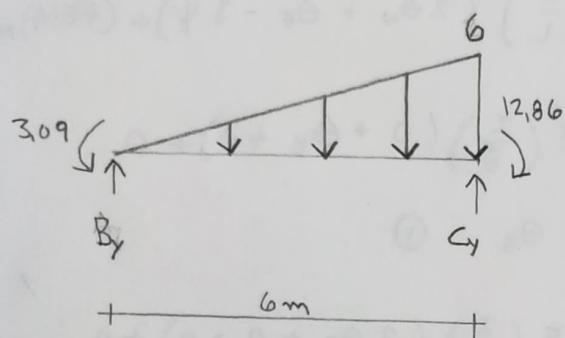
$$1,54 + 3,09 - 8B_y = 0$$

$$B_y = 0,58 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$A_y + 0,58 = 0$$

$$A_y = -0,58 \text{ kN} \downarrow$$



$$\sum M_B = 0$$

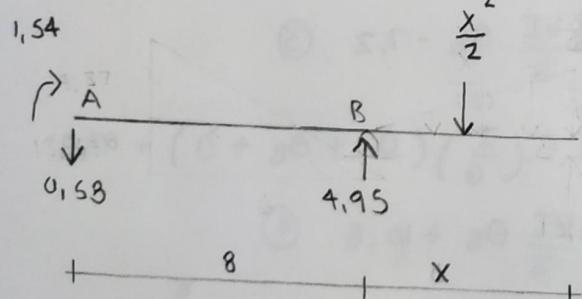
$$-3,09 + 12,86 + \frac{6(6)}{2}(4) - 6C_y = 0$$

$$C_y = 13,63 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$B_y - \frac{6(6)}{2} + 13,63 = 0$$

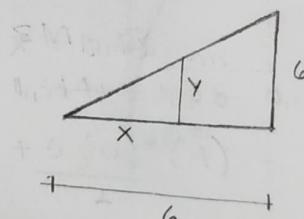
$$B_y = 4,37 \text{ kN} \uparrow$$



$$\sum F_y = 0$$

$$-0,58 + 4,95 - \frac{x^2}{2} = 0$$

$$x = 2,96 \text{ m}$$



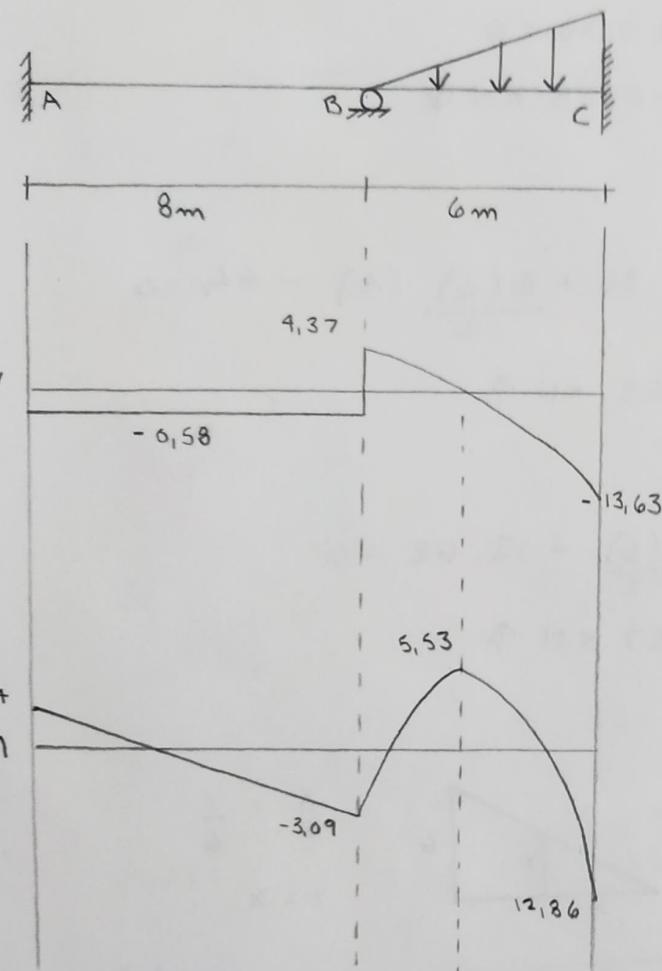
$$\frac{x}{6} = \frac{y}{6}$$

$$y = x$$

$$w = \frac{x^2}{2}$$

M. PENDIENTE DE DEFLEXIÓN

DIBUJE LOS DIAGRAMAS DE FUERZA CORTANTE Y DE MOMENTO PARA LA VIGA. EI = CONSTANTE.



FEM

$$BC = -\frac{wL^2}{30} = -\frac{6(6)^2}{30} = -7,2 \text{ KN}\cdot\text{m}$$

$$CB = \frac{wL^2}{20} = \frac{6(6)^2}{20} = 10,8 \text{ KN}\cdot\text{m}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2E\left(\frac{I}{8}\right)(0 + \theta_B + 0) + 0$$

$$M_{AB} = \frac{EI}{4} \theta_B \quad ①$$

$$M_{BA} = 2E\left(\frac{I}{8}\right)(2\theta_B + 0 + 0) + 0$$

$$M_{BA} = \frac{EI}{2} \theta_B \quad ②$$

$$M_{BC} = 2E\left(\frac{I}{6}\right)(2\theta_B + 0 + 0) + (-7,2)$$

$$M_{BC} = \frac{2EI}{3} \theta_B - 7,2 \quad ③$$

$$M_{CB} = 2E\left(\frac{I}{6}\right)(0 + \theta_B + 0) + 10,8$$

$$M_{CB} = \frac{EI}{3} \theta_B + 10,8 \quad ④$$

$$M_{BA} + M_{BC} = 0$$

$$\frac{EI}{2} \theta_B + \frac{2EI}{3} \theta_B - 7,2 = 0 \Rightarrow \theta_B = \frac{6,17}{EI}$$

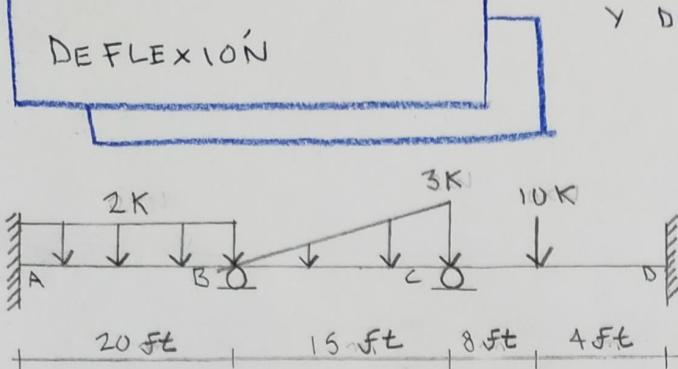
$$M_{AB} = 1,54 \text{ KN}\cdot\text{m}$$

$$M_{BA} = 3,09 \text{ KN}\cdot\text{m}$$

$$M_{BC} = -3,09 \text{ KN}\cdot\text{m}$$

$$M_{CB} = 12,86 \text{ KN}\cdot\text{m}$$

M. PENDIENTE DE DEFLEXIÓN



DIBUJA LOS DIAGRAMAS DE FUERZA CORTANTE Y DE MOMENTO PARA LA VIGA.

FEM

$$AB = -\frac{WL^2}{12} = -\frac{2(20)^2}{12} = -66,67 \text{ K}\cdot\text{ft}$$

$$BA = \frac{WL^2}{12} = \frac{2(20)^2}{12} = 66,67 \text{ K}\cdot\text{ft}$$

$$BC = -\frac{WL^2}{30} = -\frac{3(15)^2}{30} = -22,5 \text{ K}\cdot\text{ft}$$

$$CB = \frac{WL^2}{20} = \frac{3(15)^2}{20} = 33,75 \text{ K}\cdot\text{ft}$$

$$CD = -\frac{Pb^2a}{L^2} = -\frac{10(4)^2(8)}{12^2} = -4,44 \text{ K}\cdot\text{ft}$$

$$DC = \frac{Pa^2b}{L^2} = \frac{10(8)^2(4)}{12^2} = 17,78 \text{ K}\cdot\text{ft}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi + (\text{FEM})_N)$$

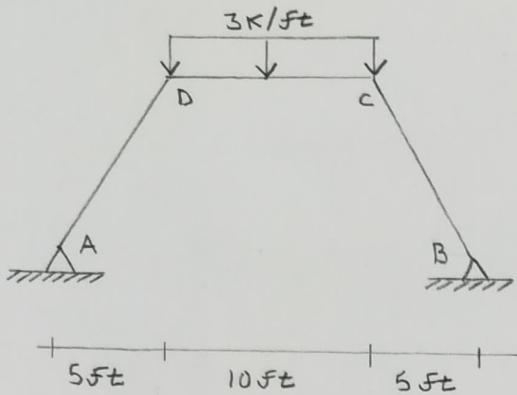
$$M_{AB} = 2E\left(\frac{I}{20}\right)(\theta_B) + (-66,67) = \frac{EI}{10} \theta_B - 66,67$$

$$M_{BA} = 2E\left(\frac{I}{20}\right)(2\theta_B) + 66,67 = \frac{EI}{10} \theta_B + 66,67$$

$$M_{BC} = 2E\left(\frac{I}{15}\right)(2\theta_B + \theta_C) + (-22,5) = \frac{2EI}{15} (2\theta_B + \theta_C) - 22,5$$

PENDIENTE DEFLEXIÓN

DETERMINE LOS MOMENTOS Y DIBUJE EL DIAGRAMA DE MOMENTO.



FEM

$$D_C = -\frac{WL^2}{12} = -\frac{3(10)^2}{12} = -25$$

$$CD = 25$$

MOMENTOS

$$M_{DA} = \frac{3EI}{13}(\theta_D) = 13,4 \text{ K.ft}$$

$$M_{DC} = \frac{2EI}{10}(2\theta_D + \theta_C) - 25 = -13,4 \text{ K.ft}$$

$$M_{CD} = \frac{2EI}{10}(2\theta_C + \theta_D) + 25 = 13,4 \text{ K.ft}$$

$$M_{CB} = \frac{3EI}{13}(\theta_C) = -13,4 \text{ K.ft}$$

ANGULOS

$$\theta_{DA} + \theta_{DC} = 0$$

$$0,23\theta_D + 0,4\theta_D + 0,2\theta_C - \frac{25}{EI} = 0$$

$$0,2\theta_C + 0,63\theta_D = \frac{25}{EI} \quad (1)$$

$$M_{CD} + M_{CB} = 0$$

$$0,4\theta_C + 0,2\theta_D + \frac{25}{EI} + 0,23\theta_C = 0$$

$$0,63\theta_C + 0,2\theta_D = -\frac{25}{EI} \quad (2)$$

$$\theta_C = \theta_D$$

$$\frac{125}{EI} - 3,15\theta_D + \frac{39,68}{EI} + 0,32\theta_D = 0$$

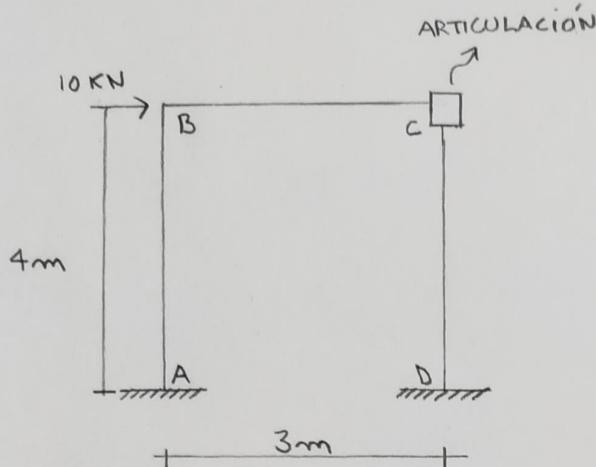
$$\frac{164,68}{EI} - 2,83\theta_D = 0$$

$$\theta_D = \frac{58,2}{EI}$$

$$\theta_C = -\frac{58,2}{EI}$$

PENDIENTE DEFLEXIÓN

DETERMINE LOS MOMENTOS Y DIBUJE EL DIAGRAMA DE MOMENTO.



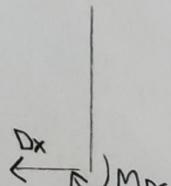
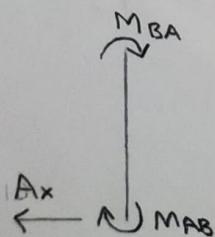
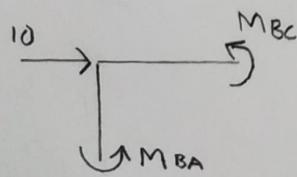
MOMENTOS

$$M_{AB} = \frac{2EI}{4} (\theta_B - 3\psi) = -17,14 \text{ KN}\cdot\text{m}$$

$$M_{BA} = \frac{2EI}{4} (2\theta_B - 3\psi) = -11,43 \text{ KN}\cdot\text{m}$$

$$M_{BC} = \frac{3EI}{3} (\theta_B) = 11,43 \text{ KN}\cdot\text{m}$$

$$M_{DC} = \frac{3EI}{4} (-\psi) = -11,43 \text{ KN}\cdot\text{m}$$



$$\rightarrow \sum F_x = 0$$

$$10 - A_x - D_x = 0$$

$$\textcircled{+} \sum M_B = 0$$

$$M_{BA} + M_{AB} + 4A_x = 0$$

$$A_x = \frac{-M_{AB} - M_{BA}}{4}$$

$$\textcircled{+} \sum M_C = 0$$

$$M_{DC} + 4D_x = 0$$

$$D_x = -\frac{M_{DC}}{4}$$

$$10 + \frac{M_{AB} + M_{BA}}{4} + \frac{M_{DC}}{4} = 0$$

$$\frac{40}{EI} + 0,5\theta_B - 1,5\psi + \theta_B - 1,5\psi = 0$$

$$-0,75\psi = 0$$

$$1,5\theta_B - 3,75\psi = -\frac{40}{EI} \quad \textcircled{1}$$

$$M_{BA} + M_{BC} = 0$$

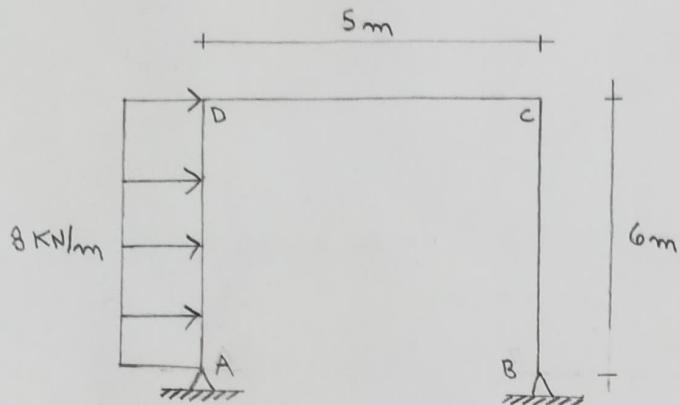
$$\theta_B - 1,5\psi + \theta_B = 0$$

$$2\theta_B - 1,5\psi = 0 \quad \textcircled{2}$$

$$\theta_B = \frac{11,43}{EI} \quad \psi = \frac{15,24}{EI}$$

PENDIENTE DEFLEXIÓN

DETERMINE LOS MOMENTOS EN LAS JUNTAS C Y D Y DIBUJE EL DIAGRAMA DE MOMENTO. SUPONGA QUE A Y B ESTÁN ARTICULADOS.



FEM

$$DA = \frac{WL^2}{8} = \frac{8(6)^2}{8} = 36$$

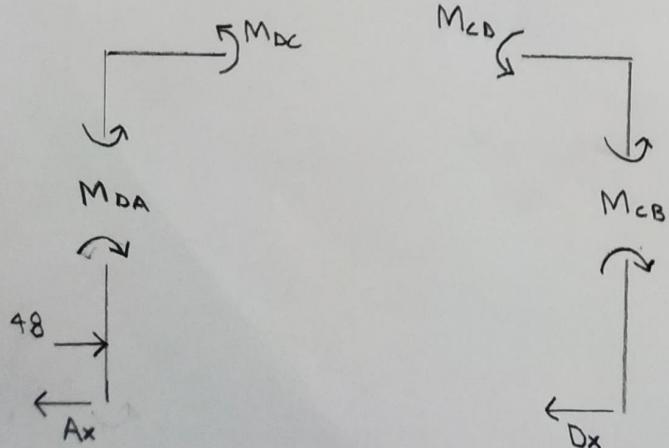
MOMENTOS

$$M_{DA} = \frac{3EI}{6} (\theta_D - \psi) + 36 = -64 \text{ KN}\cdot\text{m}$$

$$M_{DC} = \frac{2EI}{5} (2\theta_D + \theta_C) = 64 \text{ KN}\cdot\text{m}$$

$$M_{CD} = \frac{2EI}{5} (2\theta_C + \theta_D) = 80 \text{ KN}\cdot\text{m}$$

$$M_{CB} = \frac{3EI}{6} (\theta_C - \psi) = -80 \text{ KN}\cdot\text{m}$$



$$\text{(+)} \sum M_D = 0$$

$$M_{DA} - 48(3) + 6Ax = 0$$

$$Ax = 24 - \frac{M_{DA}}{6}$$

$$\text{(+)} \sum M_C = 0$$

$$M_{CB} + 6Dx = 0$$

$$Dx = -\frac{M_{CB}}{6}$$

$$\rightarrow \sum F_x = 0$$

$$48 - Ax - Dx = 0$$

$$48 - 24 + \frac{M_{DA}}{6} + \frac{M_{CB}}{6} = 0$$

$$\frac{144}{EI} + 0,5\theta_D - 0,5\psi + \frac{36}{EI}$$

$$+ 0,5\theta_C - 0,5\psi = 0$$

$$0,5\theta_C + 0,5\theta_D - \psi = -\frac{180}{EI} \quad (1)$$

$$M_{DA} + M_{DC} = 0$$

$$0,5\theta_D - 0,5\psi + \frac{36}{EI} + 0,8\theta_D + 0,4\theta_C = 0$$

$$0,4\theta_C + 1,3\theta_D - 0,5\psi = -\frac{36}{EI} \quad (2)$$

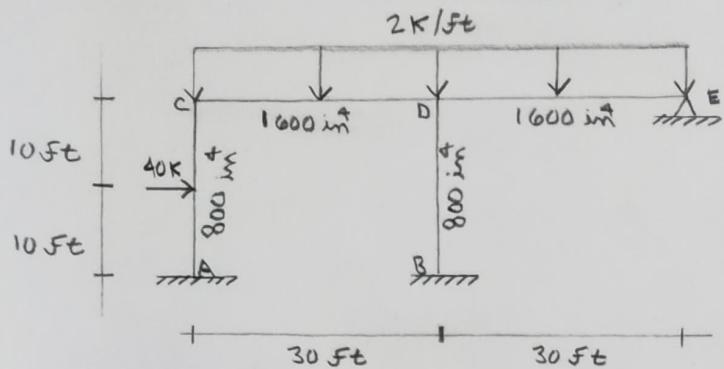
$$M_{CD} + M_{CB} = 0$$

$$0,8\theta_C + 0,4\theta_D + 0,5\theta_C - 0,5\psi = 0$$

$$1,3\theta_C + 0,4\theta_D - 0,5\psi = 0 \quad (3)$$

$$\theta_C = \frac{80}{EI} \quad \theta_D = \frac{40}{EI} \quad \psi = \frac{240}{EI}$$

PENDIENTE DEFLEXIÓN



FEM

$$CD = -\frac{WL^2}{12} = -\frac{2(30)^2}{12} = -150$$

$$DC = 150$$

$$DE = -\frac{WL^2}{8} = -\frac{2(30)^2}{8} = -225$$

$$AC = -\frac{PL}{8} = -\frac{40(20)}{8} = -100$$

$$CA = 100$$

MOMENTOS

$$MAC = \frac{2EI}{20} (\theta_c) - 100 = -92 \text{ K.Ft}$$

$$M_{CA} = \frac{2EI}{20} (2\theta_c) + 100 = 115,9 \text{ K.Ft}$$

$$M_{CD} = \frac{4EI}{30} (2\theta_c + \theta_d) - 150 = -115,9 \text{ K.Ft}$$

$$M_{DC} = \frac{4EI}{30} (2\theta_d + \theta_c) + 150 = 186,4 \text{ K.Ft}$$

$$M_{DB} = \frac{2EI}{20} (2\theta_d) = 19,3 \text{ K.Ft}$$

$$M_{BD} = \frac{2EI}{20} (\theta_d) = 9,7 \text{ K.Ft}$$

DETERMINE LOS MOMENTOS EN LOS EXTREMOS DEL ELEMENTO Y LAS REACCIONES.

$$M_{DE} = \frac{6EI}{30} (\theta_d) - 225 = -205,7 \text{ K.Ft}$$

ANGULOS

$$M_{CA} + M_{CD} = 0$$

$$0,2\theta_c + \frac{100}{EI} + 0,267\theta_c + 0,133\theta_d = 0$$

$$-\frac{150}{EI} = 0$$

$$0,467\theta_c + 0,133\theta_d = \frac{50}{EI}$$

$$M_{DC} + M_{DB} + M_{DE} = 0$$

$$0,267\theta_d + 0,133\theta_c + \frac{150}{EI} + 0,2\theta_d = 0$$

$$+0,2\theta_d - \frac{225}{EI} = 0$$

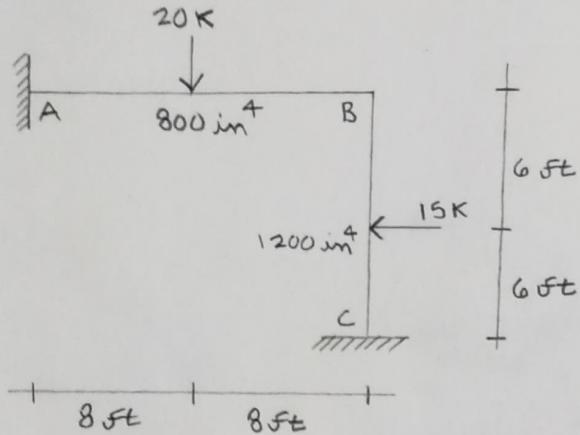
$$0,133\theta_c + 0,667\theta_d = \frac{75}{EI}$$

$$\theta_c = \frac{79,6}{EI}$$

$$\theta_d = \frac{96,6}{EI}$$

PENDIENTE DE DEFLEXIÓN

DETERMINE LOS MOMENTOS EN LOS SOPORTES Y DIBUJE EL DIAGRAMA DE MOMENTO. $E = 29(10)^3 \text{ KSI}$.



ANGULOS

$$M_{BA} + M_{BC} = 0$$

$$0,25 EI \theta_B + 40 + 0,5 EI \theta_B$$

$$-22,5 = 0$$

$$0,75 EI \theta_B = -17,5$$

$$\theta_B = \frac{-23,33}{EI}$$

FEM

$$AB = -\frac{PL}{8} = -\frac{20(16)}{8} = -40$$

$$BA = 40$$

$$BC = -\frac{PL}{8} = -\frac{15(12)}{8} = -22,5$$

$$CB = 22,5$$

MOMENTOS

$$M_{AB} = \frac{2EI}{16} (\theta_B) - 40 = -42,92 \text{ K.Ft}$$

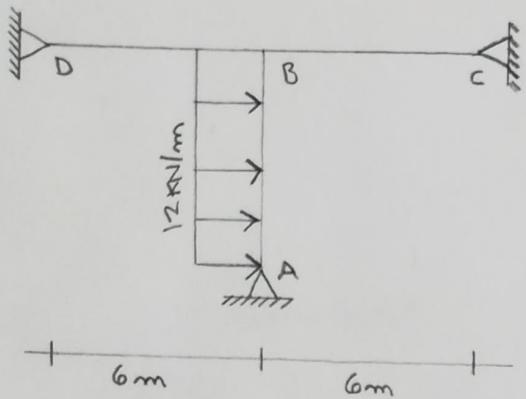
$$M_{BA} = \frac{2EI}{16} (2\theta_B) + 40 = 34,17 \text{ K.Ft}$$

$$M_{BC} = \frac{3EI}{12} (2\theta_B) - 22,5 = -34,17 \text{ K.Ft}$$

$$M_{CB} = \frac{3EI}{12} (\theta_B) + 22,5 = 16,67 \text{ K.Ft}$$

PENDIENTE DEFLEXIÓN

DETERMINE EL MOMENTO QUE EJERCE CADA ELEMENTO SOBRE LA JUNTA EN B Y DIBUJE EL DIAGRAMA DE MOMENTO.



$$BA = -\frac{wL^2}{8} = -\frac{12(8)^2}{8} = 96$$

ANGULOS

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$0,375 EI \theta_B + 96 + 0,5 EI \theta_B + 0,5 EI \theta_B = 0$$

$$1,375 EI \theta_B = -96$$

$$\theta_B = \frac{-69,82}{EI}$$

MOMENTOS

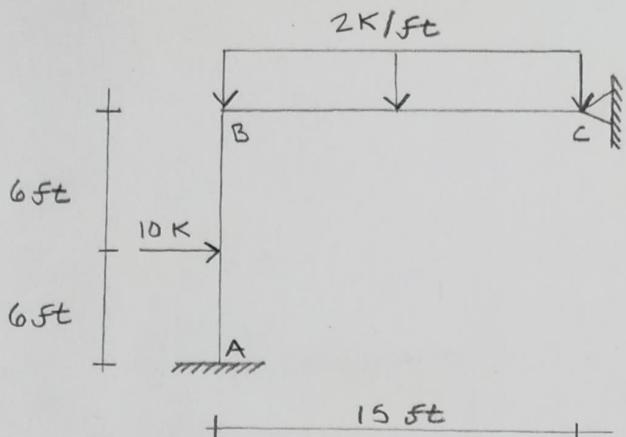
$$M_{BA} = \frac{3EI}{8} (\theta_B) + 96 = 69,82 \text{ KN}\cdot\text{m}$$

$$M_{BC} = \frac{3EI}{6} (\theta_B) = -34,91 \text{ KN}\cdot\text{m}$$

$$M_{BD} = \frac{3EI}{6} (\theta_B) = -34,91 \text{ KN}\cdot\text{m}$$

PENDIENTE DEFLEXIÓN

DETERMINE EL MOMENTO QUE EJERCE CADA ELEMENTO SOBRE LA JUNTA EN B Y DIBUJE EL DIAGRAMA DE MOMENTO.



ANGULOS

$$M_{BA} + M_{BC} = 0$$

$$0,167 EI (2\theta_B) + 15 + 0,2 EI \theta_B = 0$$

$$-56,25 = 0$$

$$0,534 EI \theta_B = 41,25$$

$$\theta_B = \frac{77,25}{EI}$$

FEM

$$AB = -\frac{PL}{8} = -\frac{10(12)}{8} = -15$$

$$BA = 15$$

$$BC = -\frac{WL^2}{8} = -\frac{2(15)^2}{8} = -56,25$$

MOMENTOS

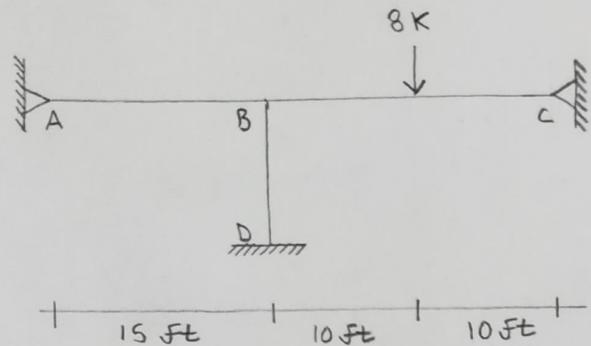
$$M_{AB} = \frac{2EI}{12} (\theta_B) - 15 = -2,125 \text{ K.Ft}$$

$$M_{BA} = \frac{2EI}{12} (2\theta_B) + 15 = 40,75 \text{ K.Ft}$$

$$M_{BC} = \frac{3EI}{15} (\theta_B) - 56,25 = -40,8 \text{ K.Ft}$$

PENDIENTE DEFLEXIÓN

DETERMINE LOS MOMENTOS EN B Y D
Y DIBUJE EL DIAGRAMA DE MOMENTO.



ANGULOS

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$0.2 EI \theta_B + 0.15 EI \theta_B - 30$$

$$0.333 EI \theta_B = 0$$

$$0.683 EI \theta_B = 30$$

$$\theta_B = \frac{43.92}{EI}$$

FEM

$$BC = -\frac{3PL}{16} = -\frac{3(8)(20)}{16} = -30$$

MOMENTOS

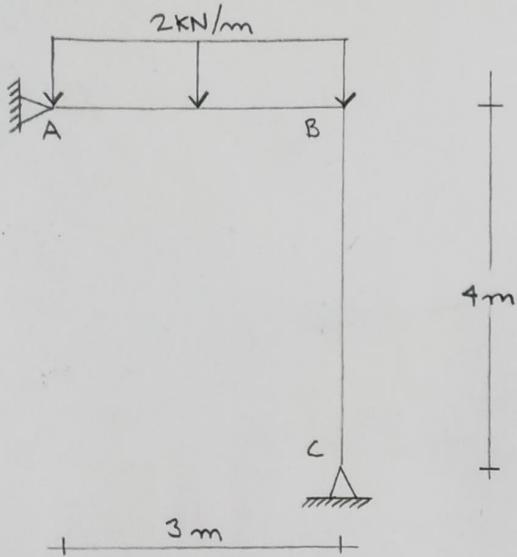
$$M_{BA} = \frac{3EI}{15} (\theta_B) = 8.78 \text{ K.ft}$$

$$M_{BC} = \frac{3EI}{20} (\theta_B) - 30 = -23.41 \text{ K.ft}$$

$$M_{BD} = \frac{2EI}{12} (2\theta_B) = 14.64 \text{ K.ft}$$

$$M_{DB} = \frac{2EI}{12} (\theta_B) = 7.32 \text{ K.ft}$$

PENDIENTE DEFLEXIÓN



DETERMINE EL MOMENTO EN B Y DIBUJE EL DIAGRAMA DE MOMENTO.

ANGULO

$$M_{BA} + M_{BC} = 0$$

$$1,333 EI (\theta_B) + 1,5 + 0,75 EI \theta_B = 0$$

$$2,083 EI \theta_B = -1,5$$

$$\theta_B = -\frac{0,7201}{EI}$$

FEM

$$AB = -\frac{WL^2}{12} = -\frac{2(3)^2}{12} = -1,5$$

$$BA = 1,5$$

MOMENTOS

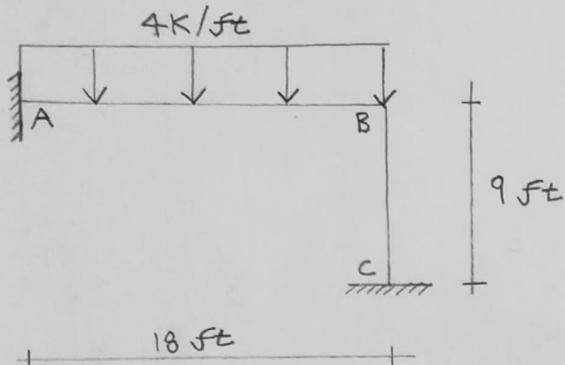
$$M_{AB} = \frac{2EI}{3}(\theta_B) - 1,5 = -1,98$$

$$M_{BA} = \frac{2EI}{3}(2\theta_B) + 1,5 = 0,54$$

$$M_{BC} = \frac{3EI}{4}(\theta_B) = -0,54$$

PENDIENTE DEFLEXIÓN

DETERMINE LOS MOMENTOS Y DIBUJE
EL DIAGRAMA DE MOMENTO



FEM

$$AB = -\frac{WL^2}{12} = -\frac{4(18)^2}{12} = -108$$

$$BA = 108$$

MOMENTOS

$$M_{AB} = \frac{2EI}{18} (\theta_B) - 108 = -126,02$$

$$M_{BA} = \frac{2EI}{18} (2\theta_B) + 108 = 71,96$$

$$M_{BC} = \frac{2EI}{9} (2\theta_B) = -72,07$$

$$M_{CB} = \frac{2EI}{9} (\theta_B) = -36,03$$

ANGULOS

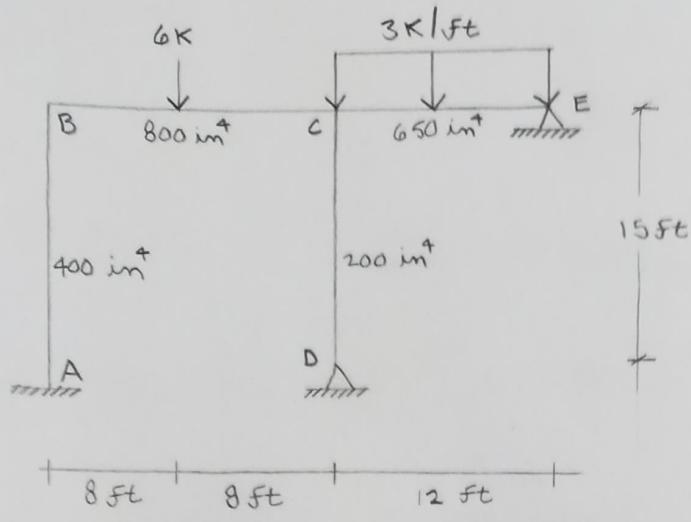
$$M_{BA} + M_{BC} = 0$$

$$\frac{EI}{9} (2\theta_B) + 108 + 0,222 EI (2\theta_B) = 0$$

$$0,222 EI \theta_B + 0,444 EI \theta_B = -108$$

$$\theta_B = \frac{-162,16}{EI}$$

PENDIENTE DE FLEXIÓN



FEM

$$BC = -\frac{PL}{8} = -\frac{6(16)}{8} = -12$$

$$CB = 12$$

$$CE = -\frac{WL^2}{8} = -\frac{3(12)^2}{8} = -54$$

MOMENTOS

$$M_{AB} = \frac{4EI}{15} (\theta_B) = 0,296 \text{ K}\cdot\text{ft}$$

$$M_{BA} = \frac{4EI}{15} (2\theta_B) = 0,592 \text{ K}\cdot\text{ft}$$

$$M_{BC} = \frac{8EI}{16} (2\theta_B + \theta_C) - 12 = -0,592 \text{ K}\cdot\text{ft}$$

$$M_{CB} = \frac{8EI}{16} (2\theta_C + \theta_B) + 12 = 33,1 \text{ K}\cdot\text{ft}$$

$$M_{CE} = \frac{9,75EI}{12} (\theta_C) - 54 = -37,3 \text{ K}\cdot\text{ft}$$

$$M_{CD} = \frac{3EI}{15} (\theta_C) = 4,12 \text{ K}\cdot\text{ft}$$

DETERMINAR LOS MOMENTOS. EL MOMENTO DE INERCIA PARA CADA ELEMENTO SE MUESTRA EN LA FIGURA. $E = 29(10)^3 \text{ ksi}$

ANGULOS

$$M_{BA} + M_{BC} = 0$$

$$\frac{4EI}{15} (2\theta_B) + \frac{8EI}{16} (2\theta_B + \theta_C) - 12 = 0$$

$$0,533 EI \theta_B + EI \theta_B + 0,5 EI \theta_C = 12$$

$$1,533 EI \theta_B + 0,5 EI \theta_C = 12$$

$$M_{CB} + M_{CD} + M_{CE} = 0$$

$$0,5 EI (2\theta_C + \theta_B) + 12 + 0,812 EI (\theta_C) = 0$$

$$-54 + 0,2 EI (\theta_C) = 0$$

$$EI \theta_C + 0,6 EI \theta_B - 42 + 1,012 EI \theta_C = 0$$

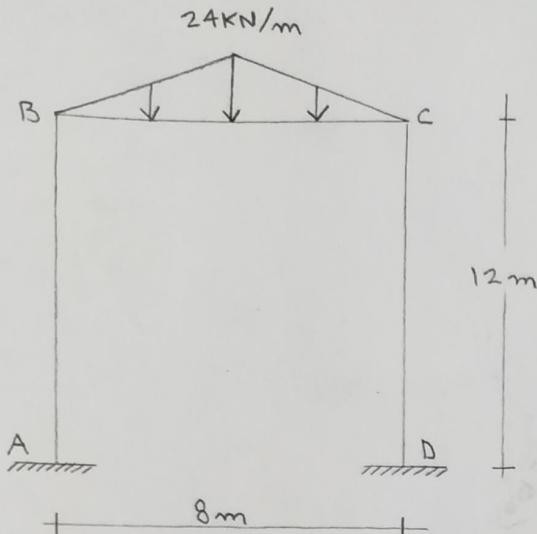
$$0,5 EI \theta_B + 2,012 EI \theta_C = 42$$

$$\theta_B = \frac{1,1092}{EI} \quad \theta_C = \frac{20,5991}{EI}$$

PENDIENTE DEFLEXIÓN

DETERMINAR LOS MOMENTOS

ANGULOS



$$M_{BA} + M_{BC} = 0$$

$$\frac{EI}{6} (2\theta_B) + \frac{EI}{4} (2\theta_B + \theta_C) - 80 = 0$$

$$0,333 EI \theta_B + 0,5 EI \theta_B + 0,25 EI \theta_C = 80$$

$$0,833 EI \theta_B + 0,25 \theta_C = 80$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{EI}{4} (2\theta_C + \theta_B) + 80 + \frac{EI}{6} (2\theta_C) = 0$$

$$0,5 EI \theta_C + 0,25 EI \theta_B + 0,333 EI \theta_C = -80$$

$$0,25 EI \theta_B + 0,833 EI \theta_C = -80$$

$$\theta_B = \frac{137,22}{EI}$$

$$\theta_C = -\frac{137,22}{EI}$$

MOMENTOS

$$M_{AB} = \frac{2EI}{12} (\theta_B) = 22,87 \text{ KN}\cdot\text{m}$$

$$M_{BA} = \frac{2EI}{12} (2\theta_B) = 45,74 \text{ KN}\cdot\text{m}$$

$$M_{BC} = \frac{2EI}{8} (2\theta_B + \theta_C) - 80 = -45,69 \text{ KN}\cdot\text{m}$$

$$M_{CB} = \frac{2EI}{8} (2\theta_C + \theta_B) + 80 = 45,69 \text{ KN}\cdot\text{m}$$

$$M_{CD} = \frac{2EI}{12} (2\theta_C) = -45,74 \text{ KN}\cdot\text{m}$$

$$M_{DC} = \frac{2EI}{12} (\theta_C) = 22,87 \text{ KN}\cdot\text{m}$$