




MatrixBandwidth.jl: Fast algorithms for matrix bandwidth minimization and recognition

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DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

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Submitted: 01 January 1970

Published: unpublished

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Summary

The *bandwidth* of an $n \times n$ matrix A is the minimum non-negative integer $k \in \{0, 1, \dots, n-1\}$ such that $A_{i,j} = 0$ whenever $|i - j| > k$. Reordering the rows and columns of a matrix to reduce its bandwidth has many practical applications in engineering and scientific computing: it can improve performance when solving linear systems, approximating partial differential equations, optimizing circuit layout, and more (Mafteiu-Scai, 2014). There are two variants of this problem: *minimization*, which involves finding a permutation matrix P such that the bandwidth of PAP^T is minimized, and *recognition*, which entails determining whether there exists a permutation matrix P such that the bandwidth of PAP^T is less than or equal to some fixed non-negative integer (an optimal permutation that fully minimizes the bandwidth of A is not required). Accordingly, [MatrixBandwidth.jl](#) offers fast algorithms for matrix bandwidth minimization and recognition, written in Julia.

Example

Consider the following 60×60 sparse matrix with initial bandwidth 51:

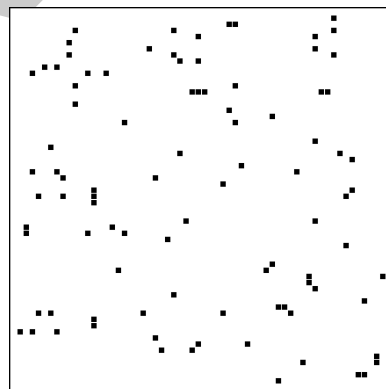


Figure 1: Original 60×60 matrix with bandwidth 51

MatrixBandwidth.jl can both recognize whether the minimum bandwidth of A is less than or equal to some fixed integer (Figure 2) and actually minimize the bandwidth of A (Figure 3):

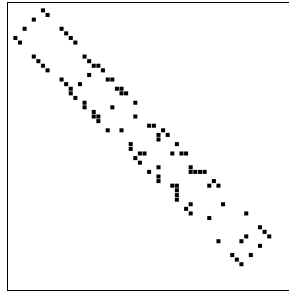


Figure 2: The matrix with bandwidth recognized as ≤ 6 via the Del Corso–Manzini algorithm

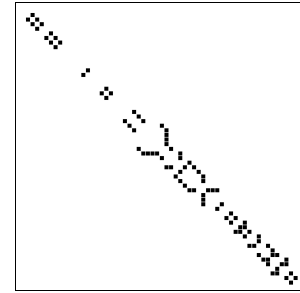


Figure 3: The matrix with bandwidth minimized to 5 via the Gibbs–Poole–Stockmeyer algorithm

(Note that since Gibbs–Poole–Stockmeyer is a heuristic algorithm, 5 may not be the *true* minimum bandwidth of A , but it is likely close.)

Algorithms

As of version 0.2.1, the following matrix bandwidth reduction algorithms are available:

- Minimization
 - Exact
 - * Caprara–Salazar–González (Caprara & Salazar-González, 2005)
 - * Del Corso–Manzini (Del Corso & Manzini, 1999)
 - * Del Corso–Manzini with perimeter search (Del Corso & Manzini, 1999)
 - * Saxe–Gurari–Sudborough (Gurari & Sudborough, 1984; Saxe, 1980)
 - * Brute-force search
 - Heuristic
 - * Gibbs–Poole–Stockmeyer (Gibbs et al., 1976)
 - * Cuthill–McKee (Cuthill & McKee, 1969)
 - * Reverse Cuthill–McKee (Cuthill & McKee, 1969; George, 1971)
- Recognition
 - Caprara–Salazar–González (Caprara & Salazar-González, 2005)
 - Del Corso–Manzini (Del Corso & Manzini, 1999)
 - Del Corso–Manzini with perimeter search (Del Corso & Manzini, 1999)
 - Saxe–Gurari–Sudborough (Gurari & Sudborough, 1984; Saxe, 1980)
 - Brute-force search

Recognition algorithms determine whether any row-and-column permutation of a matrix induces bandwidth less than or equal to some fixed integer. Exact minimization algorithms always guarantee optimal orderings to minimize bandwidth, while heuristic minimization algorithms produce near-optimal solutions more quickly. Metaheuristic minimization algorithms employ iterative search frameworks to find better solutions than heuristic methods (albeit more slowly); no such algorithms are already implemented, but several (e.g., simulated annealing) are currently under development.

Statement of need

Many matrix bandwidth reduction algorithms exist in the literature, but implementations in the open-source ecosystem are scarce, with those that do exist primarily tackling older, less efficient algorithms. The `Boost` libraries in C++ (Lumsdaine et al., 2001), the `NetworkX` library in Python (NetworkX Developers, 2025), and the MATLAB standard library (MATLAB Developers, 2025) all only implement the aforementioned reverse Cuthill–McKee algorithm from 1971. In Julia, the only other relevant package identified by the author is `SymRCM.jl` (Krysl, 2020), which also only implements reverse Cuthill–McKee.

59 Furthermore, not enough attention is given to recognition algorithms or exact minimization
60 algorithms. Although more performant modern alternatives are often neglected, at least reverse
61 Cuthill–McKee is a widely implemented method of approximating a minimal bandwidth ordering
62 (as noted above). However, no such functionality for recognition or exact minimization is
63 widely available, requiring researchers with such needs to fully re-implement these algorithms
64 themselves.

65 These two gaps in the ecosystem not only make it difficult for researchers to benchmark and
66 compare new proposed algorithms but also preclude the application of the most performant
67 modern algorithms in real-life industry settings. MatrixBandwidth.jl aims to bridge this gap by
68 presenting a unified interface for matrix bandwidth reduction algorithms in Julia.

69 Research applications

70 The author either has used or is using MatrixBandwidth.jl to do the following:

- 71 ■ Develop a new polynomial-time algorithm for “bandwidth $\leq k$ ” recognition efficient
72 for both small and large k , and benchmarking it against other approaches (Gurari &
73 Sudborough, 1984; Saxe, 1980)
- 74 ■ Speed up k -coherence checks of quantum states in many cases by confirming that the
75 density matrix’s minimum bandwidth is greater than k (Johnston et al., 2025)
- 76 ■ Compute the spectral graph property of “ S -bandwidth” (Johnston & Plosker, 2025)
77 via the SDiagonalizability.jl package (Varona et al., 2025), which depends critically on
78 MatrixBandwidth.jl for bandwidth recognition
- 79 ■ Investigate the precise performance benefits of reducing the propagation graph’s band-
80 width when training a recurrent neural network, building on Balog et al. (2019)

81 The first three use cases rely on the recognition and exact minimization functionality unique
82 to MatrixBandwidth.jl (indeed, they largely motivated the package’s development). The last
83 (ongoing) research project *could* be facilitated by SymRCM.jl instead, but the author intends to
84 use more performant metaheuristic minimization algorithms currently under development when
85 producing the final computational results, as well as use recognition algorithms to minimize
86 bandwidth to various target levels when quantifying performance improvements.

87 Limitations

88 Currently, MatrixBandwidth.jl’s core functions generically accept any input of the type
89 AbstractMatrix{<:Number}, not behaving any differently when given sparsely stored ma-
90 trices (e.g., from the SparseArrays.jl standard library package). Capabilities for directly
91 handling graph inputs (aiming to reduce the matrix bandwidth of a graph’s adjacency) are
92 also not available. Given that bandwidth reduction is often applied to sparse matrices and
93 graphs, this will be addressed in future releases.

94 Moreover, many of the algorithms only apply to structurally symmetric matrices (i.e., those
95 whose nonzero pattern is symmetric). However, this is a limitation of the algorithms themselves,
96 not the package’s implementation. Future releases with metaheuristic algorithms will include
97 more methods that accept structurally asymmetric inputs.

98 Conflict of interests

99 The author declares no conflict of interest.

Acknowledgements

I owe much to my research supervisors—Nathaniel Johnston, Sarah Plosker, and Craig Brett—for supporting and guiding me in my work. I would also like to thank Liam Keliher, Peter Lelièvre, and Marco Cagnetta for useful discussions. Finally, credit for MatrixBandwidth.jl's telepathic-cat-and-turtle logo goes to Rebekka Jonasson.

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