

Whereas classical computers use bits (0 or 1) as their basic units of information, quantum computers instead take advantage of “qubits” (subatomic particles in a superposition between the two binary states). The usage of qubits allows quantum computers to solve certain problems exponentially faster, but their susceptibility to decoherence produces unique roadblocks when ensuring “fidelity” (the degree to which information is preserved after transmission) in quantum information systems. Such systems of coupled qubits, or “quantum spin networks,” are often represented by graphs—nodes represent qubit particles, edges represent quantum couplings, and edge weights represent coupling constants.

There is said to be “perfect state transfer” (or PST) from node u to node v on some quantum network if the wave function of u completely transfers to v over some finite time in the idealized absence of external noise. PST protocols, therefore, are integral in facilitating high-fidelity communication in quantum computing. It has recently been shown that a spectral graph property known as “ S -bandwidth” acts as a heuristic indicator of PST—given some finite set of integers $S \subset \mathbb{Z}$, a graph is said to have an S -bandwidth of k if its discrete Laplacian is diagonalizable by some matrix P with all entries from S such that the matrix bandwidth of $P^T P$ is k . (Recall that a matrix X is said to have a bandwidth of k if $x_{ij} = 0$ whenever $|i - j| \geq k$.)

We herein discuss how to use Heisenberg’s matrix mechanics to ascertain whether PST occurs on a given graph representation of a quantum network, then present the first (non-heuristic) algorithm to compute S -bandwidth. Of particular interest are the cases $S = \{-1, 1\}$ and $S = \{-1, 0, 1\}$, motivated by their relation to Hadamard and weak Hadamard matrices. Finally, having conducted the most extensive survey to date of graph S -bandwidths, we set forth our numerical results on select unweighted graphs up to order 14.