Data Report

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April 8, 2025

Introduction

In collaboration with the Union of Municipalities of New Brunswick and Dr. Craig Brett of Mount Allison University, we conduct a fixed-effects two-stage least squares (or FE-2SLS) regression analysis of average tax rates on police spending in New Brunswick municipalities, using median household income as an instrumental variable to reduce simultaneity bias. We herein investigate whether police spending is a significant predictor of municipal tax rates and, if so, whether the specific policing provider plays into this correlation. Moreover, we leverage the fact that police expenditure (as per the Provincial Police Service Agreement with the Royal Canadian Mounted Police) is largely an exogenous variable outside of municipal control to use this to approximate tax base elasticity with respect to tax rates. In addition, we consider the relationship between population and this estimated elasticity, observing that smaller municipalities tend to exhibit higher tax base elasticity than larger ones due to a variety of mobility factors.

(Note that this report is intended to be taken together with our GitHub project repository, with repeated references to specific scripts/file paths. However, it is certainly possible to peruse this document independently, as we have made every effort to ensure that all relevant information is encapsulated herein.)

Background of the Problem

Price per unit of public goods—particularly police spending, in the context of this study—varies widely across municipalities in New Brunswick. We herein aim to regress regression municipal tax rates on the costs of several different public goods. We place particular emphasis on the significant variation in per capita cost of municipal bills under the Provincial Police Service Agreement (PPSA)—a contract between the Government of New Brunswick (GNB) and the Royal Canadian Mounted Police (RCMP) to provide smaller municipalities with policing services. As the RCMP provides the province with a single combined bill, the GNB charges different municipalities based on population, safety levels, and other factors, with this formula acting as an exogenous factor in the cost of policing services.

On the other hand, it is common for larger municipalities have their own direct contracts with the RCMP, further obscuring the relationship between municipal spending patterns and taxation. For instance, the Codiac Regional Policing Authority serves the municipalities of Dieppe, Moncton, and Riverview, none of which pay additional fees to the GNB under the PPSA. Others still maintain their own independent police forces like the Bathurst Police Force (although there remains an RCMP presence in Bathurst). (As shall be revealed in our section on Methodology, Amy Anderson of the Union of Municipalities of New Brunswick has provided us with data on municipal policing providers as of 2024 and a way to map this backwards to jurisdictions from previous years.)

[TODO: Explain why this setup is of interest not only insofar as showing how, and why, different level of exogeneity affect tax rates in different ways, but also in terms of ... other stuff?]

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We use a fixed-effects two-stage least squares (FE-2SLS) regression model to investigate the relationships described above. The "fixed-effects" (FE) part of this model allows us to control for time-invariant biases, controlling for unobserved heterogeneity across municipalities. However, this fails to address the problem of simultaneity bias arising from the bidirectional relationship between average tax rate (our response variable) and tax base per capita (one of our explanatory variables).

Hence, the "two-stage least squares" (2SLS) part of our model utilizes the fact that median household income (our instrumental variable) is correlated with tax base per capita but not with average tax rate. Regressing tax base on median income and using our predicted values in the second-stage fixed-effects regression allows us to isolate (to some extent) the effect of tax base on tax rate from the effect of tax rate on tax base. Both aspects of our combined FE-2SLS model are common approaches in econometrics, and we outline both more thoroughly in the **Methodology** section.

Literature Review

[TODO: Elaborate on this now that we have a first draft, especially with regards to what methodology we took away]

We review two relevant articles on municipal taxation and spending (Brett and Pinkse 2003; Gadienne 2017). Brett and Pinkse (2003) investigate the "determinants of municipal tax rates in British Columbia," considering population, distance from major metropolitan centers (namely Vancouver), income, and several other factors as determinants of tax rates. They do not particularly emphasize spending patterns as a potential factor in municipal taxation, but the methodology presented in their paper will provide a useful framework on which to build for our project.

Gadienne (2017) provides a study in how extra tax revenue truly affects the quantity and quality of public services. Again, we are going the "other way" in that we are examining how public spnding costs affect taxation, but several of the case studies here are of interest. The relevant study also investigates municipalities specifically, so the methodology laid out (especially in Section B: Local Public Revenues) proves useful.

[TODO: Add lit. review for stuff like our elasticity estimates, median income as an instrument for tax base/capita, etc. specifically]

[TODO: Moreover, and this is important—add literature review on whether or not police expenditure is typically this exogenous (it should not be), and discuss further how the PPSA affects it. Maybe for the previous section too?]

(TODO: Add Brett and Tardif 2008; Dahlby 2024)

Finally, a review of previous studies of tax rate both as a response variable (Buetter 2003, 116) and as an explanatory one (Ferede 2019, 8) reveals that the inclusion of tax base on the other side of the equation is well-known to cause simultaneity bias. While our other explanatory variables (expenditure, revenue, etc.) are fairly exogenous in that they are determined outside of the model, tax base per capita is an endogenous variable highly bicorrelated with (and thus determined by) tax rate, which creates bias in regression estimates. Findings from Auten and Carroll (1999, 689) indicate that household income is viable as an instrument to reduce this bias, being correlated with tax base (as higher income implies more taxable property) but not tax rate (as Canadian taxation schemes tend not to be overly progressive). This supports the overall structure of our FE-2SLS model described in the **Methodology** section below.

Methodology

We now delineate our data collection process, data organization methods, and econometric models. We use Python (namely the polars and linearmodels/statsmodels ecosystems) to parse and clean data from Statistics Canada and the Government of New Brunswick. Subsequently, we run several fixed-effects and correlated random-effects regression models on the resulting data in combination with median household income as an instrumental variable to account for simultaneity bias.

Data Collection and Sources

We use an unbalanced panel of annual data from 2000–2018 on New Brunswick municipalities, received via personal correspondence with the GNB and Dr. Craig Brett of Mount Allison University; however, this data is also publicly available at ("2000–2018 Annual Reports of Municipal Statistics for New Brunswick" 2000–2018), albeit in a less structured format. (The year 2005 is excluded due to missing/improperly formatted tokens, but we may coordinate further with the GNB to obtain this data in the future.) Each set of annual data contains 95 to 103 municipalities, with a total of 104 unique municipalities across all years.

This is supplemented by 2024 data on municipal policing provider agreements (Anderson 2025). We map this data backwards to municipal jurisdictions and boundaries from previous years and integrate indicators into interaction terms in our panel as described below.

Finally, the instrumental variable in the first stage of our 2SLS regression is median household income, given in census data from Statistics Canada. Data is only available from 2000 ("Table 95F0437XCB2001006" 2001), 2005 ("Table 97-563-XCB2006052" 2006), 2015 ("Table 98-400-X2016099" 2016), and 2020 ("Table 98-10-0061-01" 2021); hence, linear interpolation is applied for the intervening years. The resulting income data (typically correlated with tax base per capita but not with tax rate) is then used to reduce simultaneity bias in our fixed-effects model.

Data Cleaning and Organization

Primary Data

Primary data is cleaned in the data_pipeline/ directory. The original Excel files extracted from .zip archives provided by the GNB and the UMNB are contained in the data_raw/ subdirectory. These contain annual data from 2000–2022 on New Brunswick municipalities, as well as 2024 data on municipal policing providers. Given that some of these files are .xls and .xlw workbooks, we copy and convert them all to .xlsx format in the data_xlsx/ subdirectory. The helper_scripts/_1_raw_to_xlsx_.py script is used for this purpose.

Files in this data_xlsx/subdirectory are cleaned and organized by helper_scripts/_2_xlsx_to_clean_.py. Finding that data from 2005 and 2019–2022 is unusable due to missing/improperly formatted tokens, our output (placed in the data_clean/subdirectory) excludes these time periods. No original data is discarded during this process (save for metadata and notes)—it is all simply reorganized into parseable form.

Addressing inconsistent municipality naming conventions across years/categories and concatenating all annual panels within each category (budget expenditures, budget revenues, comparative demographics, and tax bases), the helper_scripts/_3_clean_to_final_.py script then writes all four resulting worksheets—plus a fifth for provider data—to a single data_final/data_master.xlsx workbook. (The new municipal naming convention is also used to map provider data on newer, reformed 2024 municipalities and districts to past jurisdictions all the way back to 2000.)

All scripts are called and run by the main executable of the associated directory, main.py.

Instrumental Variable Data

Data on the instrumental income data is stored and processed in the data_iv/ directory. There is one folder each for 2001, 2006, 2016, and 2021 (the years in which the census data were released) containing the original files downloaded from the Statistics Canada website. For 2016 and 2021, the downloads are straightforward, nicely formatted .csv files requiring no further processing. For 2001 and 2006, however, full data is only available in .ivt and .xml format; no schemas are available to parse the XML data, so we use the Government of Canada's Beyond 20/20 Browser to extract and download the data in .csv format. (Unfortunately, this process is not easily documentable, as the browser requires manual processing.)

With CSV files for all four years, the main.py executable script is finally used to clean and combine the relevant columns and rows into a single polars DataFrame. This is then saved as an .xlsx file in the results/ subdirectory for immediate usage in the data analysis stage. (The aforementioned data

interpolation—performed using Python's numpy library—is not applied until this stage and is thus not considered part of the data cleaning and organization pipeline.)

It is worth noting that although household income data from Canada censuses is publicly accessible for municipal-level geographic localities in 2000, 2005, 2015, and 2020, the only available source for 2010 is aggregated data from the 2011 National Household Survey. This survey refrained from providing disaggregated data at lower levels of geography, so we are unable to map it to most of the 104 municipalities in our dataset. Hence, linear interpolation is used to estimate the missing data for 2010, just as for all the other missing years. In the future, we may collaborate further with Statistics Canada to obtain the geographically disaggregated data, if it remains in their records.

Data Analysis and Modelling

All data analysis is performed in the data_analysis/ directory. Our included variables are:

- Average Tax Rate, or AvgTaxRate unitless
- Police Spending per Capita, or PolExpCapita 10⁵ CAD / person
- Non-Police Spending per Capita, or OtherExpCapita 10⁵ CAD / person
- Non-Warrant Revenue per Capita, or OtherRevCapita 10⁵ CAD / person
- Tax Base for Rate per Capita, or TaxBaseCapita 10⁵ CAD / person
- Policing Provider boolean, three categories:
 - Provincial Police Service Agreement (excluded control variable)
 - Municipal Police Service Agreement, or Provider_MPSA (included)
 - Municipal Police, or Provider_MPSA (included)
- Median Household Income, or MedHouseInc 10⁵ CAD / person

(These scaling factors are chosen to make our regression coefficients more interpretable, but when visualizing our results in the form of plots, we switch back to % for AvgTaxRate and CAD / person for the remaining expenditure and revenue variables.)

Our response variable is AvgTaxRate, which is calculated as a weighted average of the residential and non-residential tax rates in a municipal jurisdiction. (That is—as per government formulae, non-residential rates are multiplied by a factor of 1.5 before being integrated into the calculated average. Said averages are already available in the raw data ("2000–2018 Annual Reports of Municipal Statistics for New Brunswick" 2000–2018), not calculated by us; we take note of the process simply to clarify the layout of our data.) Our exogenous explanatory variables are PolExpCapita, OtherExpCapita, OtherExpCapita, OtherExpCapita, $PolExpCapita*Provider_MPSA$, and $PolExpCapita*Provider_Muni$. Our sole endogenous explanatory variable is TaxBaseCapita, for which we control simultaneity bias using MedHouseInc as an instrumental variable.

Each of these variables is used throughout our FE-2SLS regression model, carried out by the helper_scripts/_3_fe_2sls_.py script. In addition, we have also included vanilla correlated random-effects (CRE) and fixed-effects (FE) models, run by helper_scripts/_1_cre_.py and helper_scripts/_2_fe_.py, to determine which variables are relevant and to demonstrate the need for an instrumental variable. All helper scripts are called and run by the main executable of the associated directory, main.py.

Our decision to integrate a panel data model with 2SLS, clearly, arose from the factors described above in our **Literature Review**, as the inclusion of TaxBaseCapita in the model creates simultaneity bias if unaddressed. Our ultimate choice of FE over CRE for the base panel OLS was motivated by [TODO: Elaborate]

It is worth noting that we chose not to use non-linear functional forms—with the most obvious candidate for a study in this particular real-world context being log transformation—as summary statistics indicate that both the AvgTaxRate data and explanatory variables are fairly normally distributed and do not exhibit significant skewness. (Although many economic parameters such as income and GDP indeed exhibit right-skewed distributions—hence the popularity of the log transformation—we found that our particular variables of interest do not.)

Finally, we also estimate tax base elasticity by [TODO: Elaborate]

We now turn to describing our vanilla CRE and FE analyses, then proceed to more thoroughly delineate our final FE-2SLS regression model.

Correlated Random-Effects (CRE)

[TODO: Elaborate]

Fixed-Effects (FE)

After deeming the potential benefits of including the policing provider indicators directly (not in interaction terms) insufficient to warrant [TODO: Elaborate]

Fixed-Effects Two-Stage Least Squares (FE-2SLS)

Finally, we decided on [TODO: Elaborate]

Stage 1 We begin by estimating MedHouseInc data for the years missing from the Statistics Canada census data, which we do using simple linear interpolation. (As this project continues to develop, we may investigate more sophisticated approximation approaches, but this shall do for now.) After this is done, we perform an ordinary least squares regression of TaxBaseCapita on MedHouseInc to obtain

$$TaxBaseCapita_{it} = \alpha_0 + \alpha_1 MedHouseInc_{it} + v_{it}.$$

By performing this regression before proceeding to a fixed-effects model, we manage to reduce simultaneity bias, as MedHouseInc is correlated with TaxBaseCapita but not with AvgTaxRate. We use these predicted $TaxBaseCapita_{it} = TaxBaseCapita_{it} - v_{it}$ values in the second-stage regression, where we demean all variables over municipality.

Stage 2 Our primary fixed-effects regression model is now given by

$$\begin{split} AvgT\ddot{a}xRate_{it} &= \beta_1 PolEx\ddot{p}Capita_{it} + \beta_2 OtherE\ddot{x}pCapita + \beta_3 OtherR\ddot{e}vCapita + \\ &\beta_4 TaxBa\ddot{se}Capita_{it} + \beta_5 PolEx\ddot{p}Capita_{it} * Provider_MPSA_{it} + \\ &\beta_6 PolEx\ddot{p}Capita_{it} * Provider_Muni_{it} + \ddot{u}_{it}, \end{split}$$

where we use the notation $\ddot{X}_{it} := X_{it} - \bar{X}_i$ to denote the difference between the value of X for municipality iin year t and the mean value of X for municipality i over all years. (Note that $TaxBaseCapita_{it}$ is not the demeaning of $TaxBaseCapita_{it}$ itself but rather the demeaned prediction from our first-stage regression.)

Our covariance estimator in this model is clustered by municipality, as [TODO: Elaborate]

Tax Base Elasticity Estimates

Given these results, we now approximate tax base elasticity with respect to tax rates by multiplying our obtained coefficient on PolExpCapita—one of the most exogenous expenditure categories, as previously discussed—by TaxBaseCapita. First, we set up the following notation:

- E for government expenditure,
- A for tax base assessed for rate,
- t for tax rate,
- $\beta := \frac{\mathrm{d}t}{\mathrm{d}E}$ for the effect of expenditure on tax rate, and $\eta := \frac{\mathrm{d}}{A} \frac{\mathrm{d}A}{\mathrm{d}t}$ for tax base elasticity w.r.t. tax rate.

Given small deficits/surpluses, expenditure is approximately $E \approx tA$; hence, assuming exogeneity of the expenditure variable so that β is (relatively) free of simultaneity bias, we obtain the following:

$$\begin{split} \frac{\mathrm{d}E}{\mathrm{d}t} &\approx A + t\frac{\mathrm{d}A}{\mathrm{d}t} = A + A\eta = A(1+\eta) \\ & \therefore 1 + \eta \approx \frac{1}{A}\frac{\mathrm{d}E}{\mathrm{d}t} \\ & \therefore \frac{1}{1+\eta} \approx A\frac{\mathrm{d}t}{\mathrm{d}E} = A\beta \\ & \therefore \eta \approx \frac{1}{A\beta} - 1 \end{split}$$

Clearly, the assumption of exogenous expenditure is vital to this calculus; many types of expenditure are endogenously influenced by taxation, so the (relative) exogeneity of police expenditure via the PPSA is a key factor in our approximation. Using $\hat{\beta}$ to represent the coefficient on PolExpCapita in our FE-2SLS regression model and TBC as shorthand for TaxBaseCapita, it therefore follows that for the i^{th} municipality,

$$A_i \beta \approx \overline{TBC}_i \cdot \hat{\beta},$$

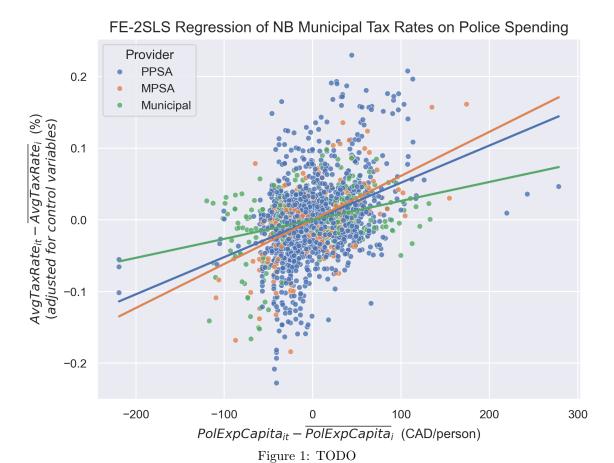
since the per-capita transformations on tax base (averaged over time) and police spending cancel out. Hence, we obtain the tax base elasticity estimate

$$\hat{\eta}_i \coloneqq \frac{1}{\overline{TBC}_i \cdot \hat{\beta}} - 1,$$

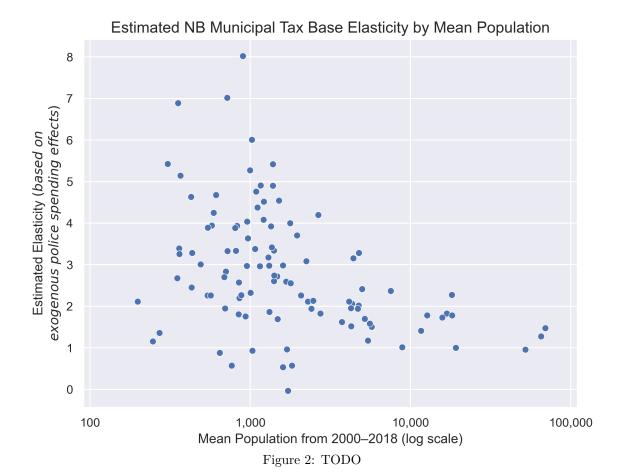
where $\hat{\eta}_i$ the estimated tax base elasticity for municipality i over the period 2000–2018. [TODO: Add time effects here?]

Results

[TODO: Elaborate]



 $\left[\text{TODO: Add explanation of the above figure} \right]$



[TODO: Add explanation of the above figure]

Discussion

[TODO: Elaborate]

Conclusion

[TODO: Elaborate]

Appendix

The raw fixed-effects regression model used in our analysis is given below:

Dep. Variable:	AvgTaxRate	R-squared:	0.7216
Estimator:	PanelOLS	R-squared (Between):	0.1325
No. Observations:	1818	R-squared (Within):	0.7216
Date:	Fri, Apr 04 2025	R-squared (Overall):	0.1356
Time:	18:11:50	Log-likelihood	1.196e + 04
Cov. Estimator:	Clustered		
		F-statistic:	738.02
Entities:	104	P-value	0.0000
Avg Obs:	17.481	Distribution:	F(6,1708)
Min Obs:	6.0000		
Max Obs:	18.000	F-statistic (robust):	66.473
		P-value	0.0000
Time periods:	18	Distribution:	F(6,1708)
Avg Obs:	101.00		
Min Obs:	95.000		
Max Obs:	103.00		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
PolExpCapita	1.3092	0.0990	13.222	0.0000	1.1150	1.5034
Other Exp Capita	0.9665	0.0914	10.575	0.0000	0.7872	1.1458
Other Rev Capita	-0.8964	0.0991	-9.0486	0.0000	-1.0907	-0.7021
TaxBaseCapita	-0.0122	0.0012	-10.440	0.0000	-0.0145	-0.0099
PolExpCapita:Provider_MPSA	-0.5551	0.1951	-2.8459	0.0045	-0.9377	-0.1725
PolExpCapita:Provider_Muni	-0.6083	0.1377	-4.4162	0.0000	-0.8784	-0.3381

F-test for Poolability: 51.098

P-value: 0.0000

Distribution: F(103,1708)

Included effects: Entity

[TODO: Elaborate]

[TODO: Explain first stage regression]

Dep. Variable:	TaxBaseCa	oita R-s	ita R-squared:		0.011
Model:	OLS	\mathbf{Adj}	. R-squa	ared:	0.011
Method:	Least Squa	res \mathbf{F} -s	tatistic:	20.99	
Date:	Fri, 04 Apr	2025 Pro	b (F-sta	4.93e-06	
Time:	18:11:51 Log-Likelih			ood:	-363.29
No. Observations:	1818	AIC	C:		730.6
Df Residuals:	1816	1816 BIC:			741.6
Df Model:	1				
Covariance Type: nonrobust					
c	oef std ei	r t	$\mathbf{P} > \mathbf{t} $	[0.025]	0.975]
Intercept 0.	6668 0.020	33.737	0.000	0.628	0.706
MedHouseInc -11	.2497 2.455	-4.582	0.000	-16.066	-6.434
Omnibus:	920.376	Durbin-V	Vatson:	1.4	34
Prob(Omnibus	s): 0.000	Jarque-B	era (JB)): 8236	5.943
Skew:	2.196	Prob(JB)) :	0.0	00
Kurtosis:	12.458	Cond. N	0.	35	4.

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The final FE-2SLS regression model used in our analysis is given below:

Dep. Variable:	AvgTaxRate	R-squared:	0.4290
Estimator:	PanelOLS	R-squared (Between):	0.9794
No. Observations:	1818	R-squared (Within):	0.4290
Date:	Fri, Apr 04 2025	R-squared (Overall):	0.9783
Time:	18:11:51	Log-likelihood	1.131e+04
Cov. Estimator:	Clustered		
		F-statistic:	213.84
Entities:	104	P-value	0.0000
Avg Obs:	17.481	Distribution:	F(6,1708)
Min Obs:	6.0000		
Max Obs:	18.000	F-statistic (robust):	27.629
		P-value	0.0000
Time periods:	18	Distribution:	F(6,1708)
Avg Obs:	101.00		
Min Obs:	95.000		
Max Obs:	103.00		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
PolExpCapita	0.5188	0.1371	3.7845	0.0002	0.2499	0.7877
OtherExpCapita	0.0301	0.0255	1.1801	0.2381	-0.0199	0.0802
OtherRevCapita	0.1025	0.0644	1.5912	0.1117	-0.0238	0.2288
TaxBaseCapita	0.0225	0.0068	3.2987	0.0010	0.0091	0.0359
PolExpCapita:Provider_MPSA	0.0955	0.1941	0.4923	0.6226	-0.2851	0.4762
${\bf PolExpCapita: Provider_Muni}$	-0.2542	0.1821	-1.3962	0.1628	-0.6113	0.1029

F-test for Poolability: 115.50

P-value: 0.0000

Distribution: F(103,1708)

Included effects: Entity

[TODO: Explain results further; potentially show in the **Results** section instead, and show the other preliminary models here?]

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